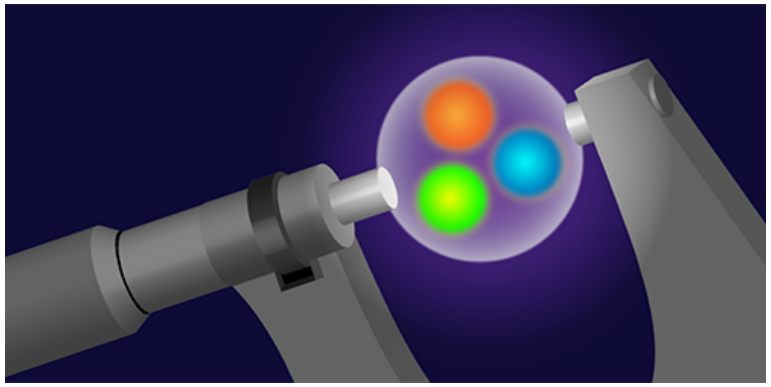
Defining the proton radius: A unified treatment

Gerald A. Miller, U. of Washington
PRC 99,035202 (2019)

What is the proton radius?



Physics

Synopsis: Putting the Proton Radius in Its Proper Place

March 7, 2019
An analysis of the proton radius puzzle helps to define what the proton radius really means.
APS/Alan Stonebraker

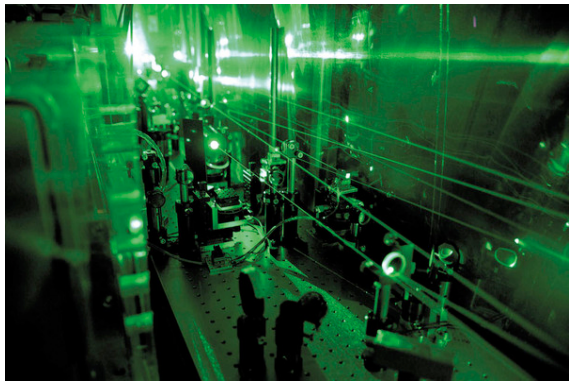
The size of the proton

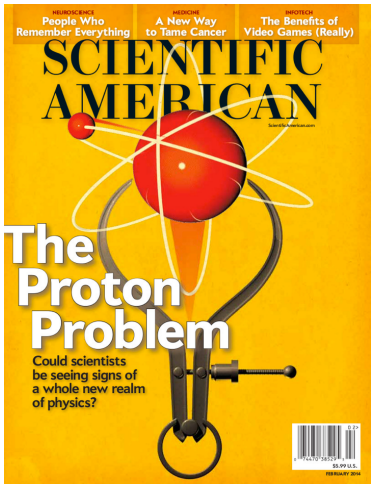
Nature Physics

Stefanie Reichert
*Nature Physics***volume 15**, page306 (2019) | Download Citation *Phys. Rev. C* **99**, 035202 (2019)

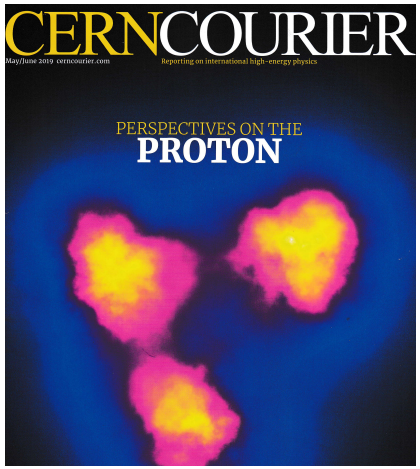
Why care?- The proton radius puzzle

Pohl et al Nature 466, 213 (8 July 2010)





Feb. 2014



May/June 2019

4 % Difference

$$r_p^2 \equiv -6 \frac{dG_E(Q^2)}{dQ^2}\bigg|_{Q^2=0}$$

muon H r_p =0.84184 (67) fm Small

electron H r_p =0.8768 (69)fm Large

~~electron-p scattering r_p =0.875 (10)fm~~

Pohl, Gilman, Miller, Pachucki (ARNPS63, 2013)

C Carlson PPNP 82,59(2015)

PRad at JLab- lower Q^2

$$3 \times 10^{-4} \leq Q^2 \leq 5 \times 10^{-2}\,\mathrm{GeV}^{-2}$$

$$7.7 \times 10^{-3} \leq Q^2 \leq 0.13\,\mathrm{fm}^{-2}$$

error bar

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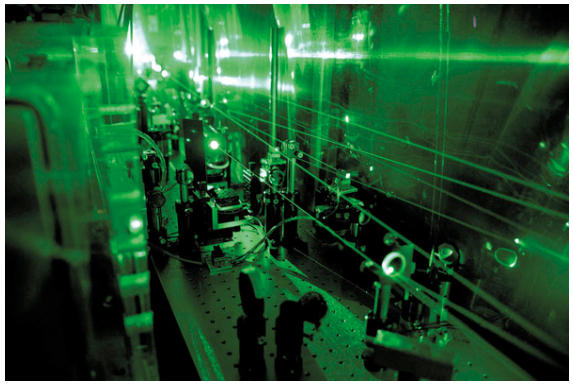

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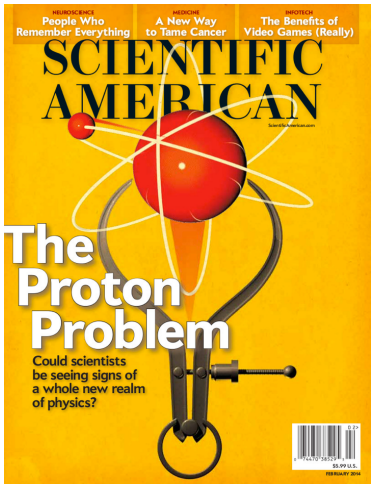

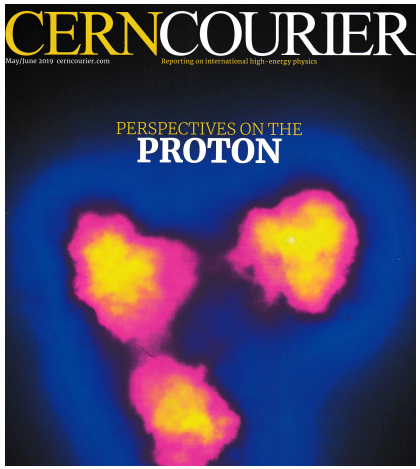

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4 % in radius: why care?

- Can't be calculated to that accuracy- current lattice QCD give too small radius by about 20-25%

Is the muon-proton interaction the same as the electron-proton interaction?
violation of universality
connections with muon g-2?
connections with LHCb ?

PHYSICAL REVIEW C **99**, 035202 (2019)

Editors' Suggestion Featured in Physics

Defining the proton radius: A unified treatment

Gerald A. Miller
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Why I wrote the paper

- Many practitioners do not understand that

$$r_p^2 \equiv -6 \frac{dG_E(Q^2)}{dQ^2}\bigg|_{Q^2=0}$$

Definition- G_E
Depends on magnetic moment

- there is no three-dimensional charge density of the proton
- proton radius is part of a unified, relativistic formalism of proton properties - GPDs, TMDs, etc
- proton radius depends on the probe- axial is not charge two-gluon is not charge

Aims:

- unite the hydrogen spectroscopy literature with lepton-proton scattering
- show how $r_p^2 = -6G'_E(0)$ emerges from the separate bodies of work
- show three-dimensional charge density cannot be defined
- r_p^2 is **not** a second moment of a density distribution.
- show honest two-dimensional charge density can be defined, measured
- place two-dimensional charge density in modern context of EIC physics

Hydrogen Atom

$$H = \frac{\vec{p}^2}{2m_r} - \frac{\alpha}{r} + V_{\text{Breit}}, \text{ to order } v^2/c^2 \qquad v/c \sim \alpha$$

First-Order Perturbation Theory Point proton

$$E_{nj} = -\frac{m_r\alpha^2}{2n^2} - \frac{m_r\alpha^4}{2n^3}\left(\frac{1}{j+\frac{1}{2}} - \frac{3}{4n} + \frac{m_r}{4n(m+M)}\right) + \frac{\alpha^4 m_r^3}{2n^3M^2}\left(\frac{1}{j+\frac{1}{2}} - \frac{1}{l+\frac{1}{2}}\right)(1-\delta_{l0}),$$

Darwin term comes with point proton

$\bar{u}(\vec{p}', s')\Gamma^0 u(\vec{p}, s) = (1 - \frac{\vec{q}^2}{8M^2})G_E(\vec{q}^2)$ Frame independent to given order

Expand $\bar{u}(\vec{p}', s')\Gamma^0 u(\vec{p}, s) \to 1 - \frac{\vec{q}^2}{8M^2} + \vec{q}^2 G'_E(0)$

$$V_C(\vec{q}^2) = -\frac{4\pi\alpha}{\vec{q}^2}(1 + \vec{q}^2 G'_E(0) = -4\pi\alpha(\frac{1}{\vec{q}^2} + G'_E(0))$$

$\vec{q}^2 = Q^2$ plus higher order

$$V_C(r) = -\frac{\alpha}{r} - 4\pi\alpha G'_E(0)\delta(\vec{r})$$

$\Delta E = \langle\psi_{nl}|-4\pi\alpha G'_E(0)\delta(\vec{r})|\psi_{nl}\rangle$ Energy shift due to non-zero proton size

History $G'_E(0) = -\frac{r_p^2}{6}$, $G_(Q^2) = 1 - \frac{r_p^2}{6}Q^2$

$$\boxed{\Delta E = 4\pi\alpha\frac{r_p^2}{6}|\psi_{0l}(0)|^2}$$

No 3 dim charge density needed

Electron-Proton Scattering

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_M \times [G_E^2 + \frac{\tau}{\epsilon} G_M^2] \frac{1}{1+\tau}$$

G_E and G_M separated at low Q^2 by plotting against kinematic factor $\frac{\tau}{\epsilon}$

History $G_E(Q^2) \approx 1 - \frac{Q^2 r_p^2}{6}$ A definition

Sachs *et. al.* (1960,1962) introduced Breit frame, $q^0 = 0$ $Q^2 = \vec{q}^2$ Lorentz scalar

Sachs 1962 argued in this frame density is Fourier transform

$\rho_{\rm NR}(r) = \int \frac{d^3Q}{(2\pi)^3} e^{-i\mathbf{Q}\cdot\mathbf{r}} G_E(Q^2)$.

NR means non-relativistic: impossible for nearly massless quarks

$\rho_{\rm NR}$ (a function of the magnitude of a three-vector) is not a density

$$\rho_{\rm NR}(r) = \int \frac{d^3Q}{(2\pi)^3} e^{-i\mathbf{Q}\cdot\mathbf{r}} G_E(Q^2).$$ not a density-physics

Charge radius of neutron magnetic moment $=\mu_n$:

$$G_{En}(Q^2) = F_{1n}(Q^2) - \frac{Q^2}{4M^2}\mu_n F_2(Q^2) \to -Q^2 R_1^2/6 - \frac{Q^2}{4M^2}\mu_n \approx -\frac{Q^2}{4M^2}\mu_n$$

Charge radius of neutron determined by its magnetic moment. Gasp!

$r_n^2 = -\frac{6}{4M^2}\mu_n = -0.126\,\mathrm{fm}^2$. Experimental value $= -0.116\,\mathrm{fm}^2$

Charge radius of pion

$$F_\pi(Q^2) \approx \frac{1}{1+\frac{Q^2}{m_\rho^2}}$$

Fourier transform $\rho_{\rm NR}(r) \propto \frac{e^{-m_\rho r}}{r}$

Singular at $r = 0$. Gasp!

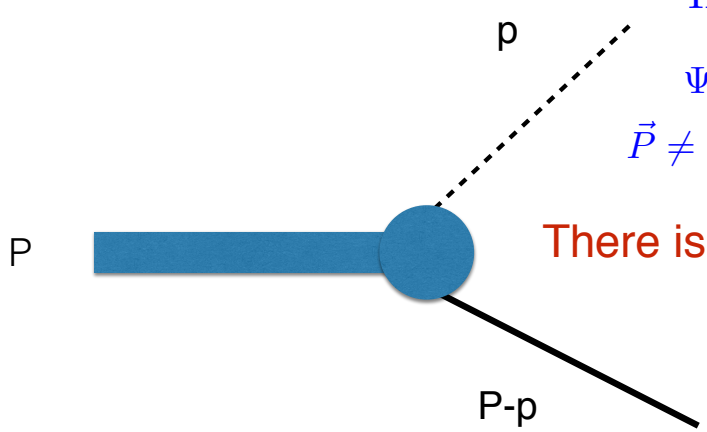
$$\rho_{\rm NR}(r) = \int \frac{d^3Q}{(2\pi)^3} e^{-i\mathbf{Q}\cdot\mathbf{r}} G_E(Q^2).$$ not a density formal reasons

- Correct non-relativistic because center-of-mass and internal coordinates are independent- $e^{iP\cdot R}g(r)$ factorization
- Relativistic: Internal wave functions depend on total momentum. No factorization.
- Momentum q is absorbed by proton of momentum p. Initial and final wave functions have different momenta, so no square of wave function appears
- NO $\Psi^*\Psi$, NO density

Initial state 4-momentum =P, final state =P+q

$$\Psi_{\rm B.S.}(P,p) = \frac{1}{p^2-m_q^2+i\epsilon}\frac{1}{(P-p)^2-M_S^2+I\epsilon}$$

$\vec{P} \neq 0$ is called a 'Boost'. Different boosts different wave functions



There is no density in the square of wave function sense

Where Sachs (1962) went wrong

Appendix claims three-dimensional density exists

Tries to get around boost by using wave packet

$|\Psi\rangle = \int d^3P g(\vec{P})|P, s\rangle$

$|g(\vec{P})|^2 = \delta(\vec{P})$

Gets rid of boost- both initial and final wave function are at 0 momentum

But defining momentum precisely means position is spread over all of space!

Technically- Sachs ignored derivative of $\delta(\vec{P})$, dropping an infinite term

Summary so far

- Both hydrogen spectroscopy and electron scattering measure slope of G_E at its origin
- Both define this slope as being proportional to r^2_p
- There is no 3-dimensional density
- (Later discuss honest 2 dimensional density)

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Donnelly, Hassell, Milner ask (1806.10475)
Are the Muonic Hydrogen and Electron Scattering Experiments Measuring the Same Observable?

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Are the Muonic Hydrogen and Electron Scattering Experiments Measuring the Same Observable?

YES

Proton radius puzzle Electronic Hydrogen -Pohl

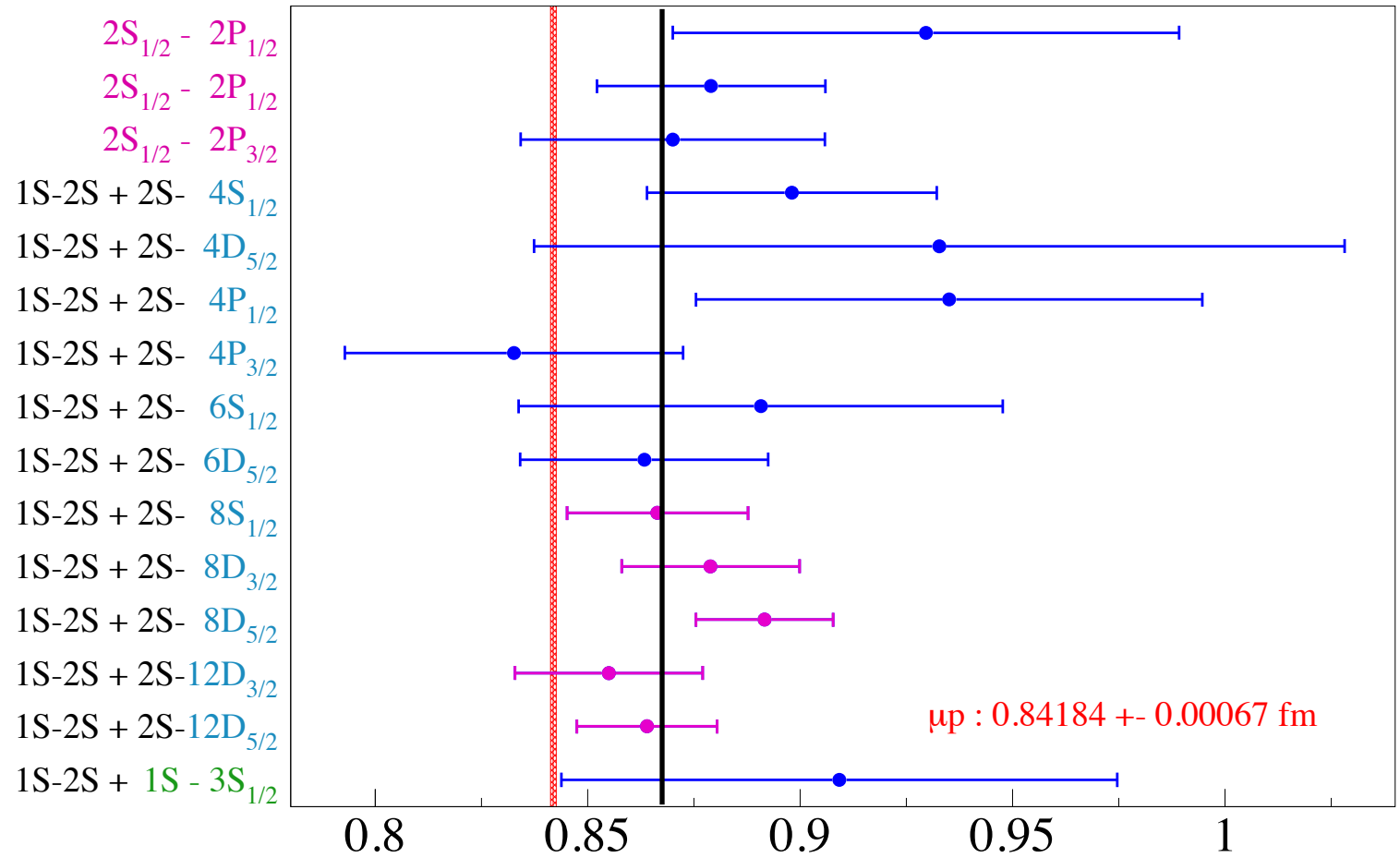
- Need two levels to get Rydberg and Lamb shift-have ~ 20 available

$$E(nS) \approx \frac{R_\infty}{n^2} + \frac{L_{1S}}{n^3}$$

1s-2s gives R_∞

12 figs change 5 st dev no prp

CODATA updated R May20

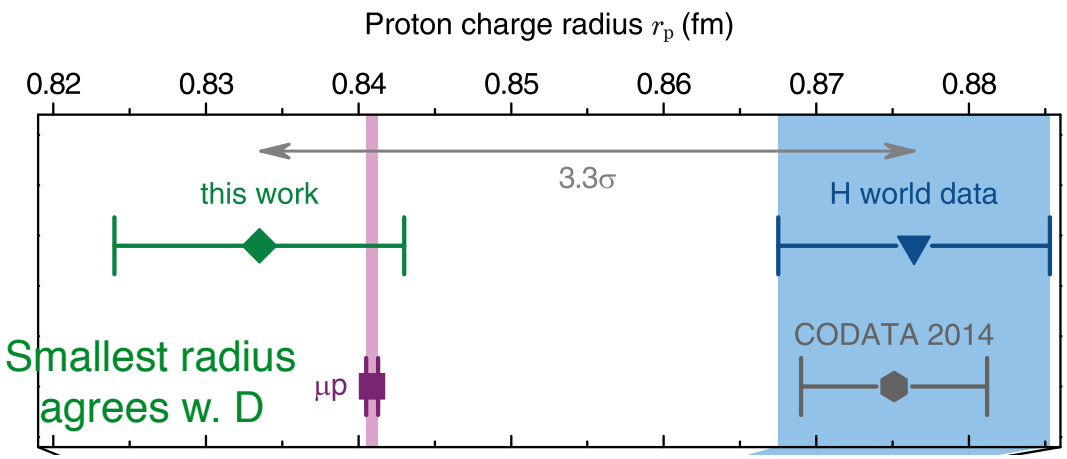


2 levels needed except for 2S-2P results ARE correlated

New Electron H 2017

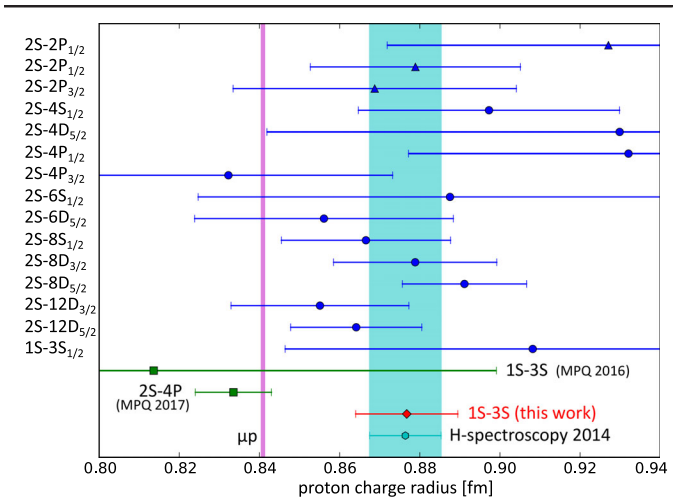
Beyer et al Science 358,79 electron H -small radius

Munich
2S-4P transition



Smallest radius
agrees w. D

Is there a puzzle?



Fleurbaey et al

Phys. Rev. Lett. 120, 183001 (2018)

0.877 (13) fm
large radius

Maybe

MAMI arXiv:1905.11182 [nucl-ex]
(0.870±0.014stat.±0.024sys.±0.003mod.)fm.

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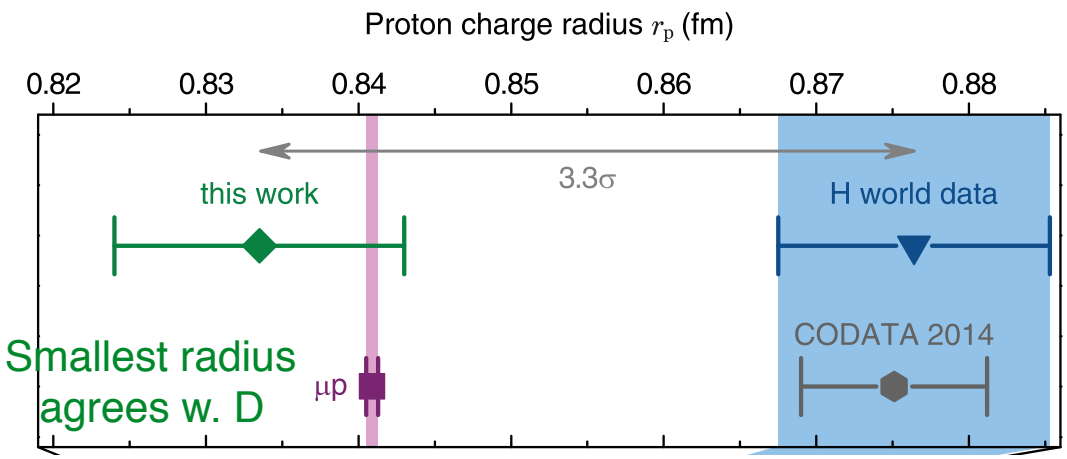

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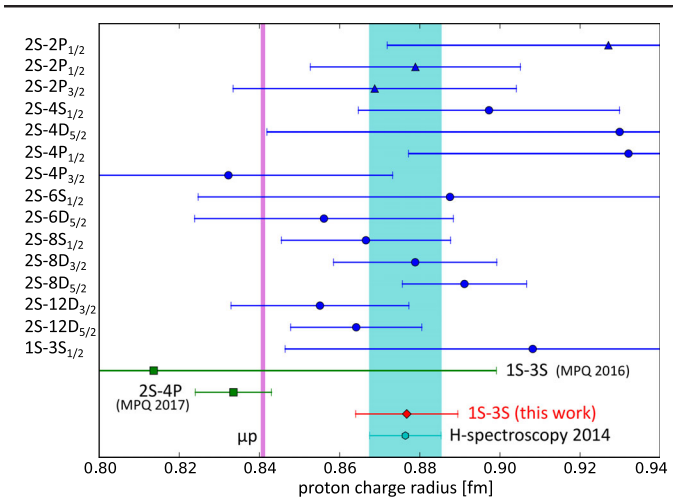

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Data So far

- potential difference between muon and electron hydrogen spectroscopy re the proton radius, perhaps Rydberg is wrong by 5 st dev
- 2017 eH Munich experiment gives small radius , PRAD rumor small radius
- 2018 eH Paris experiment gives large radius Paris
- 2019 MAMI large radius
- deuteron is smaller too, almost accounted for by proton being smaller, but some effect on neutron exists
- no effect in 3He ion (large error bars) → strong constraints

Coming soon?

- PRad experiment at JLab very low Q^2
- gets small radius-preliminary still working the error bar
- Hessels at York, $2s_{1/2}$-$2P_{1/2}$ no Rydberg
- Get's small radius- preliminary submit to Science July '18 no answer by April '19
- Likely explanation eH folks too optimistic about error bars
- But what are the new error bars ????

Possible resolutions

- Electron H spectroscopy not so accurate
- Strong interaction effect in two photon exchange diagram
- Muon interacts differently than electron!- new scalar boson

Shift from history to our efforts to explain

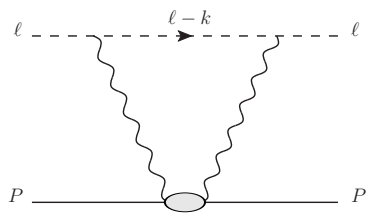
Suppose radii extracted in earlier H experiments is correct, some new muon effect is responsible

What energy difference corresponds to 4% in radius?

Measured =206.2949(32)= 206.0573(45)-5.2262 r_p^2+0.0347 r_p^3 meV
computed

Explain puzzle with radius of earlier H atom:
increase 206.0573 meV by 0.31 meV-attraction in 2S state

Miller PLB 2012

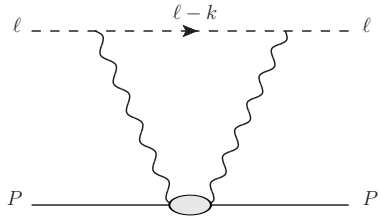


energy shift proportional to lepton mass^4

Graph is Real virtual Compton scattering
Im part is measured - use dispersion relations
but unknown subtraction function is needed

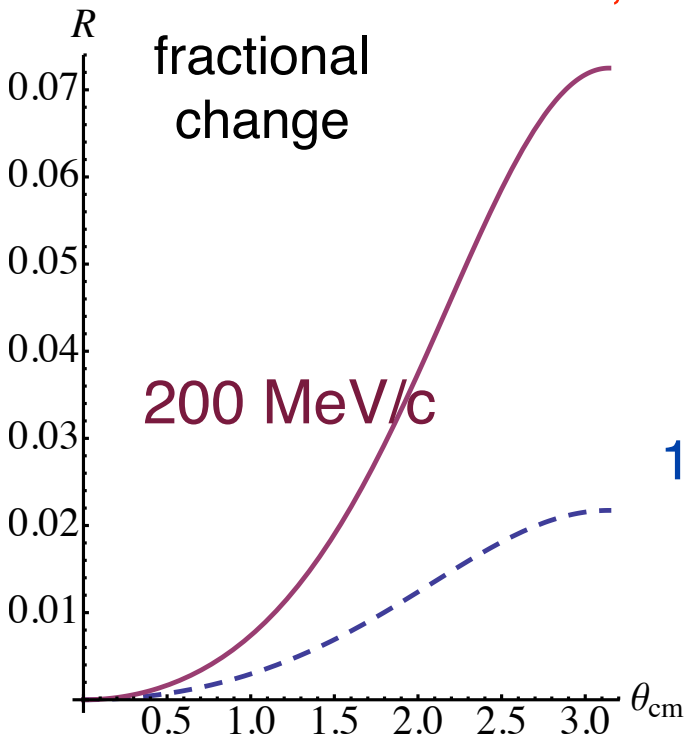
Gives 0.31 meV

Unknown function can give smaller values too



energy shift proportional to lepton mass^4

Explain puzzle with radius as in H atom increase 206.0573 meV by 0.31 meV-attractive effect on 2S state, reproduce Lamb shift in proton, deuterium



10-20% effect in scattering

Radians

fails in 4He

$$\delta E_L^{\mu A} \propto Z^3 (Z \delta E_L^p + N \delta E_L^n)$$

4 standard deviations off ^{4}He

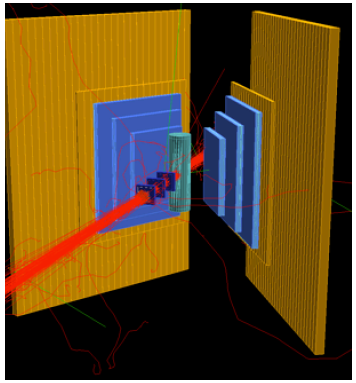
or 1 st dev

Still the largest uncertainty

e^+/e^- and μ^+/μ^- scattering on proton

So what? MUSE expt

http://www.physics.rutgers.edu/~rgilman/elasticmup/



PSI proposal R-12-01.1

- constrains two photon effect (biggest uncertainty) by measuring l^+-p l^- -p scattering separately
- if large radius correct, remove two photon all leptons see large radius
- if small radius correct and remove two photon all leptons see small radius
- will not see a new light particle, but all leptons see large radius -

BSM? muon anomalous moment

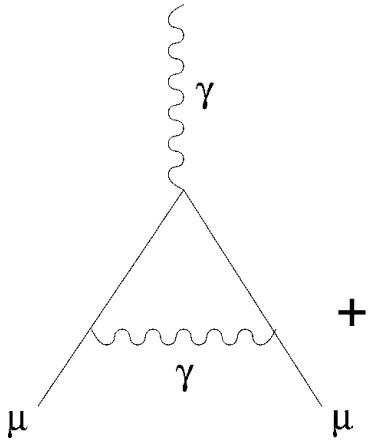
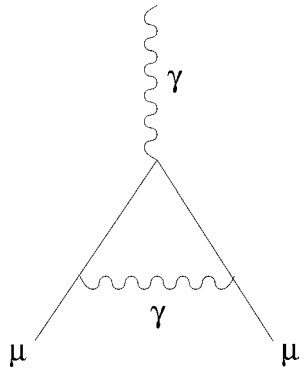


Figure 1 The first-order QED correction to g-2 of the muon.

+



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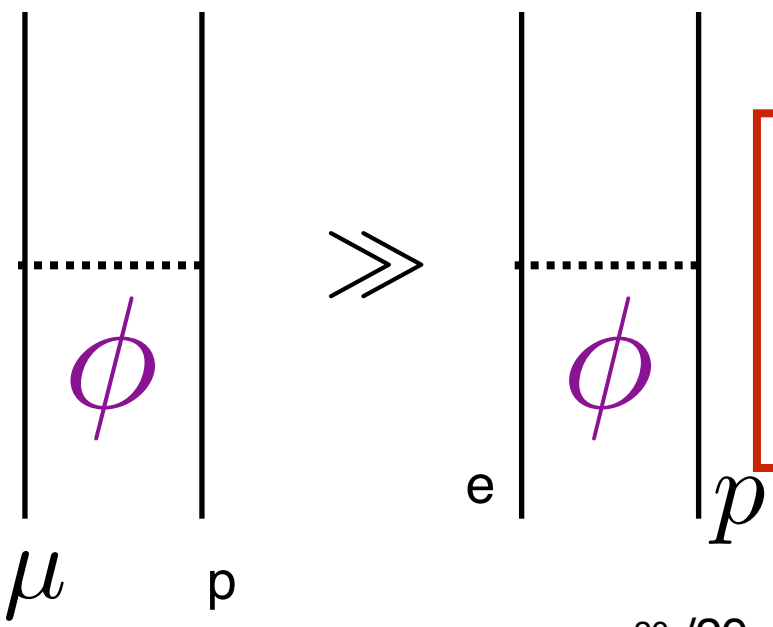
H

3.6 st. dev anomaly now
fix add heavy photon
interacts preferentially with muon

Muon data is g-2 - BNL exp't, Hertzog- FermiLab now...

Maybe dark matter, energy particles show up in muon physics!



Postulate new **scalar** boson!
accounts for magnetic moment and r_p

New scalar bosons

assumes puzzle exists

ϕ

- give μ-p Lamb shift
- almost no hyperfine in μ proton
- small effect for D, almost no effect ^{4}He
- consistent with g-2 of μ and electron
- avoid many other constraints
- be found

PRL **117,** 101801 (2016) PHYSICAL REVIEW LETTERS week ending 2 SEPTEMBER 2016

Electrophobic Scalar Boson and Muonic Puzzles

Yu-Sheng Liu,* David McKeen,† and Gerald A. Miller‡

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Electrophobic Scalar Boson and Muonic Puzzles

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Constraining ϕ

Beam dump experiments do not see

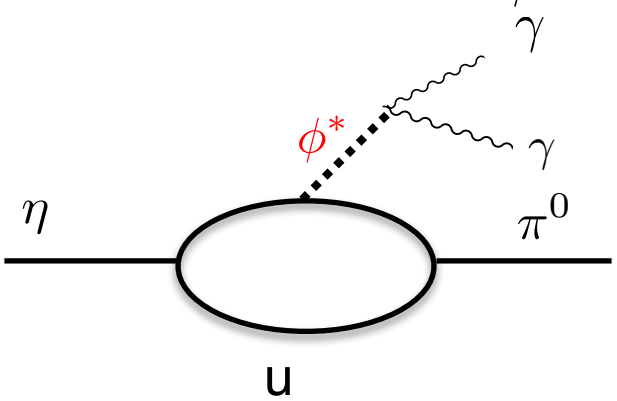
$\phi \to e^+e^-$, $\phi \to \gamma\gamma$

Eta decay and muonic puzzles 1805.01028

Yu-Sheng Liu, Ian Cloet, GAM

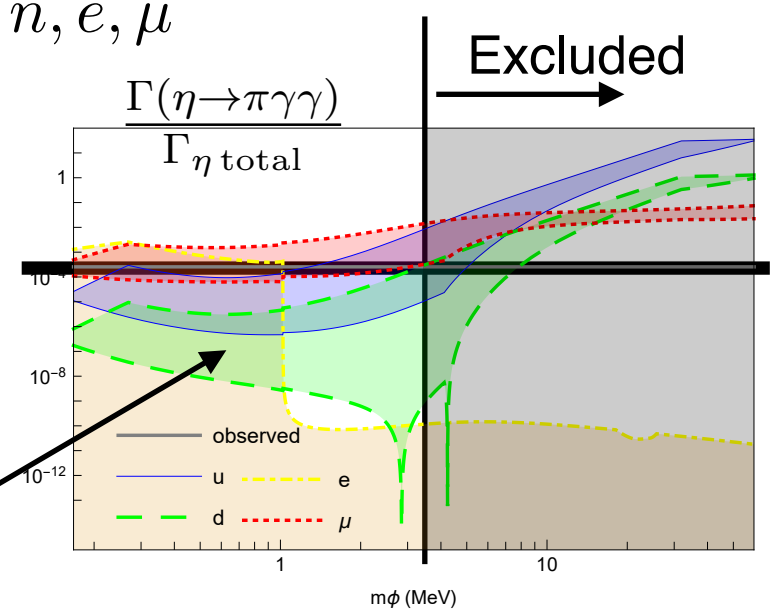
Previously ϕ couples to p, n, e, μ

Now ϕ couples to $u, d, e\mu$



Observed rate

Allowed shaded



Liping Gan suggestion

Low mass excluded by nuclear physics

ϕ decay to $\gamma\gamma$ via triangle diagram

Allowed mass range from 200 KeV to 3 MeV

Density in EIC Context-I Non-relativistic Wigner distributions (one-body)

$$W(\mathbf{R},\mathbf{P}) = \int \frac{d^3r}{(2\pi)^3} \langle \mathbf{R}+\mathbf{r}/2|\Psi\rangle\langle\Psi|\mathbf{R}-\mathbf{r}/2\rangle e^{-i\mathbf{P}\cdot\mathbf{r}}$$

3+3=6 variables

$$\int W(\mathbf{R},\mathbf{P})d^3P = |\langle\mathbf{R}|\Psi\rangle|^2, \text{ coordinate} - \text{space density}$$

$$\int W(\mathbf{R},\mathbf{P})d^3R = |\langle\mathbf{P}|\Psi\rangle|^2, \text{ momentum} - \text{space density unintegrated parton distributio}$$

$$\int W(\mathbf{R},\mathbf{P})d^3R\, d^2P_T = |\langle P_z|\Psi\rangle|^2, \text{ parton distribution}$$

$$\int W(\mathbf{R},\mathbf{P})dR_z d^2\, P_T = |\langle \mathbf{R}_T, P_z|\Psi\rangle|^2, \text{ impact parameter parton distribution}$$

EIC white paper p29

5 variables not 6

Where is charge density hiding?

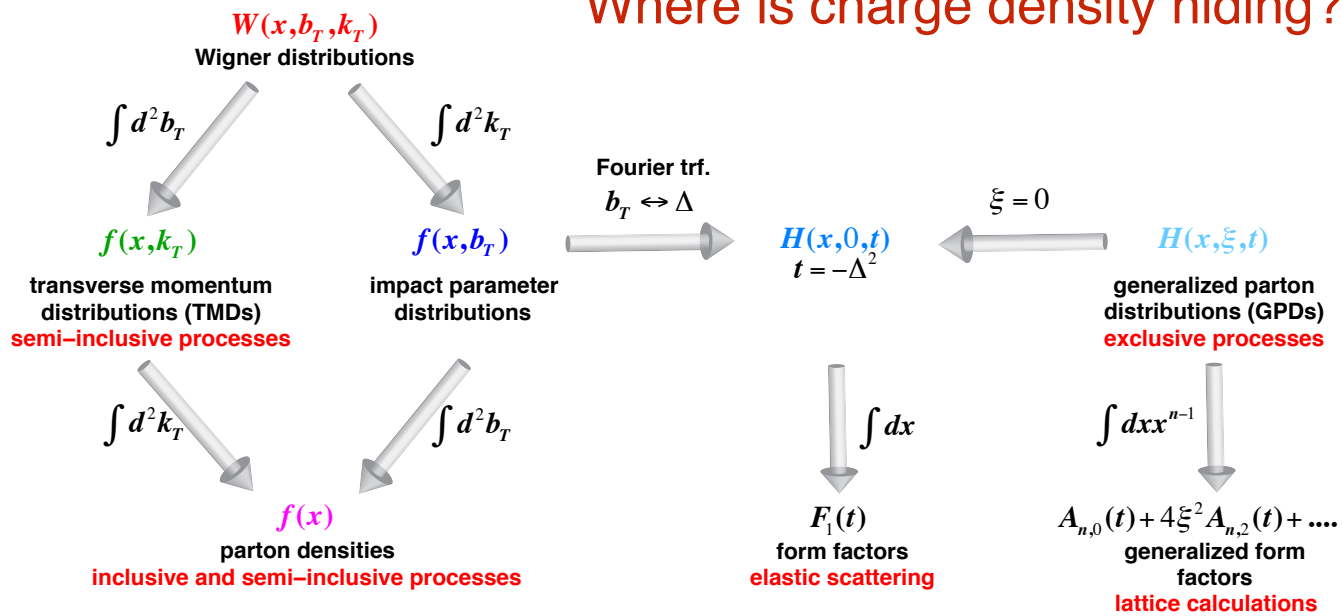


Figure 2.2: Connections between different quantities describing the distribution of partons inside the proton. The functions given here are for unpolarized partons in an unpolarized proton; analogous relations hold for polarized quantities.

EIC white paper p29

5 variables not 6

Where is charge density hiding?

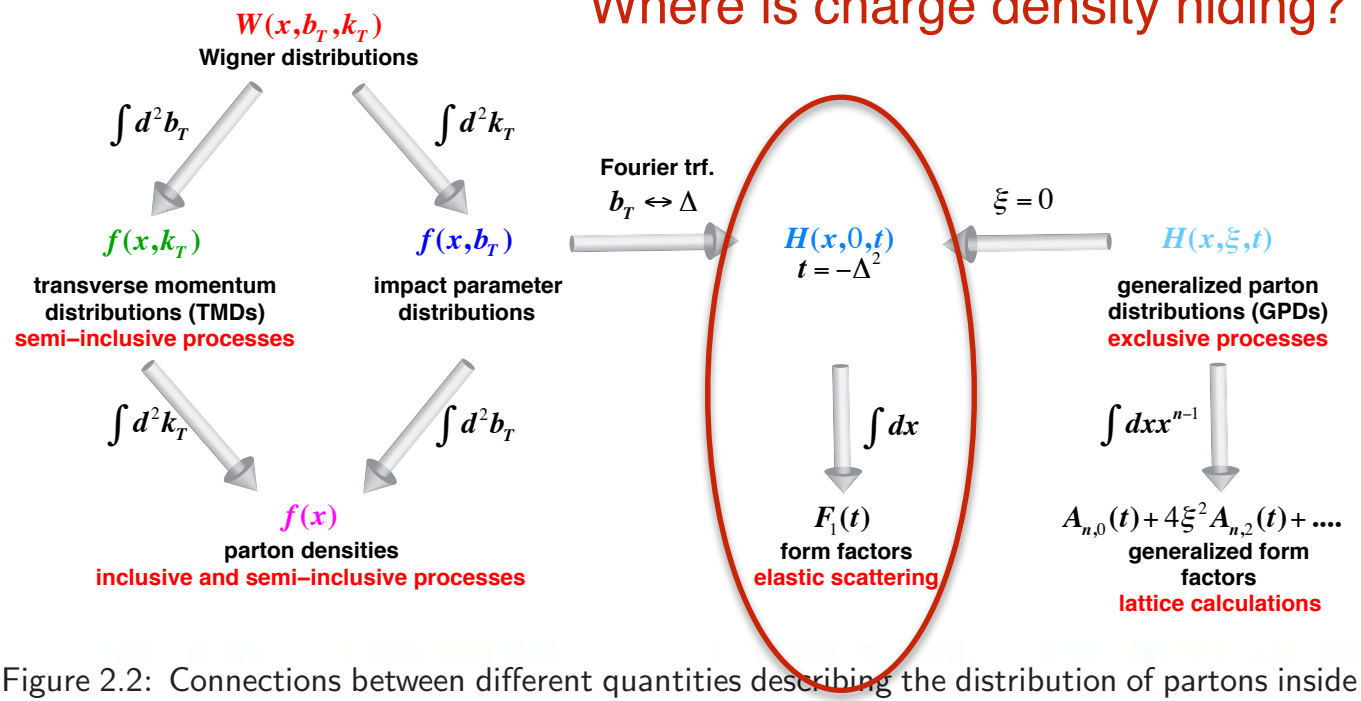


Figure 2.2: Connections between different quantities describing the distribution of partons inside the proton. The functions given here are for unpolarized partons in an unpolarized proton; analogous relations hold for polarized quantities.

Light front, Infinite momentum frame

$$\text{“}Time\text{”},\ x^+ = x^0 + x^3, \quad \text{“}Evolve\text{”},\ p^- = p^0 - p^3$$

$$\text{“}Space\text{”},\ x^- = x^0 - x^3, \quad \text{“}Momentum\text{”},\ p^+(\text{Bjorken})$$

$$Transverse\ position,\ momentum \quad \mathbf{b}, \mathbf{p}$$

These variables are used in GPDs, TMDs, standard variables

transverse boosts in kinematic subgroup

$$\mathbf{k} \to \mathbf{k} - k^+ \mathbf{v}$$

$$\text{space} - \text{like}\ q^\mu, q^+ = 0,$$

$$\text{momentum transfer in transverse direction}$$

then density is 2 Dimensional Fourier Transform

TRUE CHARGE DENSITY

IMF: electromagnetic charge density $J^0 \to J^+$: True Density

$$J^+(x^-, \mathbf{b}) = \sum_q e_q \overline{q}(x^-, \mathbf{b})\gamma^+ q(x^-, \mathbf{b}) = \sum_q e_q q_+^\dagger(x^-, \mathbf{b}) q_+(x^-, \mathbf{b}) \tag{1}$$

Density

Relate to GPD

$$H_q(x,t) = \int \frac{dx^-}{4\pi} \langle p^+, \mathbf{p}', \lambda | \bar{q}(-\frac{x^-}{2}, \mathbf{0})\gamma^+ q(\frac{x^-}{2}, \mathbf{0}) | p^+, \mathbf{p}, \lambda \rangle e^{ixp^+x^-}, \qquad A^+ = 0 \tag{2}$$

$$-t = -(p' - p)^2 = (\mathbf{p}' - \mathbf{p})^2 = \mathbf{q}^2 \ .$$

$$F_1(\mathbf{q}^2) = \sum_q e_q \int dx H_q(x,t) = \frac{\langle p'^+, \mathbf{p}', \lambda | J^+(0) | p^+, \mathbf{p}, \lambda \rangle}{2p^+} \tag{3}$$

$$\left| p^+, \mathbf{R} = \mathbf{0}, \lambda \right\rangle \equiv \mathcal{N} \int \frac{d^2\mathbf{p}}{(2\pi)^2 \sqrt{2p^+}} \left| p^+, \mathbf{p}, \lambda \right\rangle, \tag{4}$$

Transverse distance $\mathbf{b}$ relative to $\mathbf{R} = 0$ defined
Use (4) in (3), 2Dim Fourier transform, translational invariance + integrate:

Transverse Density

$$\boxed{\int d^2q e^{-i\mathbf{q}\cdot\mathbf{b}} F_1(\mathbf{q}^2) = \int dx^- \left\langle p^+, \mathbf{R} = \mathbf{0}, \lambda \right| q_+^\dagger(x^-, \mathbf{b}) q_+(x^-, \mathbf{b}) \left| p^+, \mathbf{R} = \mathbf{0}, \lambda \right\rangle \equiv \rho(\mathbf{b})} \tag{5}$$

TRUE *VS.* NON-RELATIVISTIC DENSITY

$\rho(b)$ is a true density: matrix element of density operator between identical initial and final states.

Depends only on the two-dimensional transverse variable **b**- longitudinal boosts depend on interactions

$$\rho_{\mathbf{NR}}(r) \equiv \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} G_E(\vec{q}^{\,2}), \quad (r = \sqrt{b^2+z^2})$$

phony

$$\rho_{\mathbf{NR},\mathbf{T}}(b) = \int_{-\infty}^{\infty} dz\, \rho_{\mathrm{NR}}(\sqrt{b^2+z^2} = \frac{1}{2\pi}\int_0^{\infty} dQQJ_0(bQ)G_E(Q^2)$$

$$\rho(b) = \frac{1}{2\pi}\int_0^{\infty} dQQJ_0(bQ)F_1(Q^2)$$

Get true transverse density from non-rel $G_E \to F_1$.

True is narrower

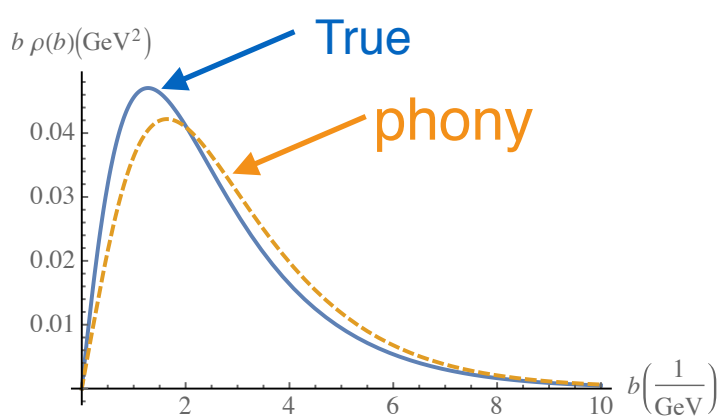


FIG. 1. True (solid) vs non-relativistic (dashed) density

Relativistic Moment Expansion- RME

$$F_1(Q^2) = \int d^2b\rho(b)e^{i\mathbf{Q}\cdot\mathbf{b}}$$

Power series in $i\mathbf{Q}\cdot\mathbf{b}$ and Taylor expand $F_1(Q^2)$ in powers of Q^2. Equate

$$F_1(Q^2) = \sum_{n=0}^{\infty}(-1)^n \frac{(2n-1)!!}{2^n\, n!(2n)!}\langle b^{2n}\rangle Q^{2n} F_1^{(n)}(0)),$$

$F_1^{(n)}(0)$ is n'th derivative of F_1 with respect to Q^2 at $Q^2 = 0$:

$$F_1(Q^2) \approx 1 - \frac{Q^2}{4}\langle b^2\rangle F_1^{(1)}(0) + \frac{Q^4}{64}\langle b^4\rangle F_1^{(2)}(0) + \cdots .$$

$\langle b^{2n}\rangle$ Lorentz invariant.

$$\langle b^{2n}\rangle = \int d^2b b^{2n}\rho(b)$$

Hydrogen spectroscopy gets the moments $\langle r^{2,4}\rangle_{\mathbf{NR}}$.

$$\langle b^2\rangle = \frac{2}{3}\langle r^2\rangle_{\rm NR} - \frac{\kappa}{4M^2}, \quad \langle b^4\rangle = \frac{8}{15}\langle r^4\rangle_{\mathbf{NR}} + \frac{8}{3M^2}\langle r^2\rangle_{\mathbf{NR}} - \frac{4\mu}{3M^2}\langle r_M^2\rangle_{\rm NR} - \frac{4\kappa}{M^4},$$

$\mu = 1 + \kappa$, $\langle r_M^2\rangle_{\mathbf{NR}} \equiv -6G'_M(0)$.

Summary

- Hydrogen spectroscopy and electron scattering both measure the slope of G_E.
- If the proton radius puzzle exists: new scalar boson of mass from 300 KeV to 3 MeV exist? narrow target JLab

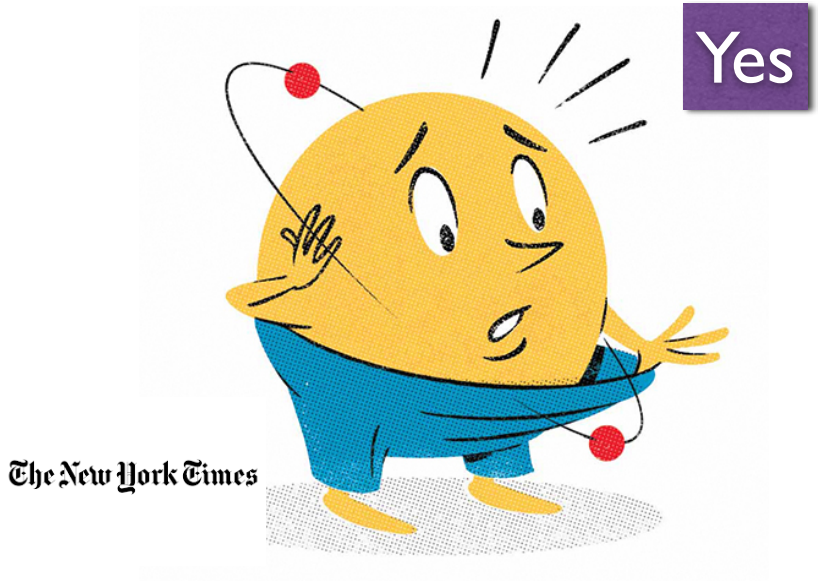
η decay

- Direct detection is needed.
- Need PRad & Hessels data to publish to better limits on 2photon and scalar mesons
- True honest density is two-dim Fourier transform of F_1. $\rho(b)$

$\rho(b)$ is closely connected to EIC formalism

Does 4% matter?

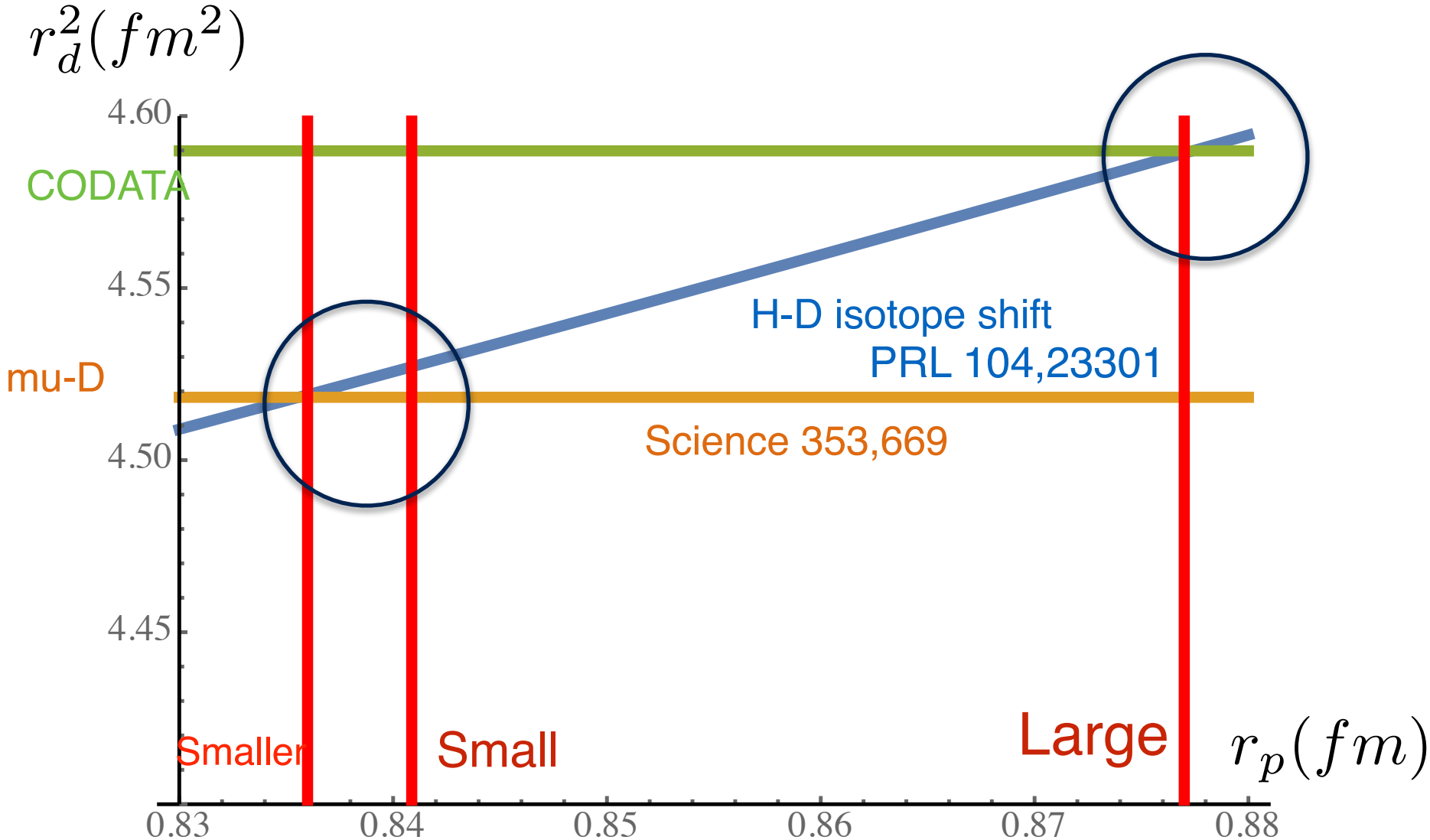
Does 4% matter?



Yes

Spares follow

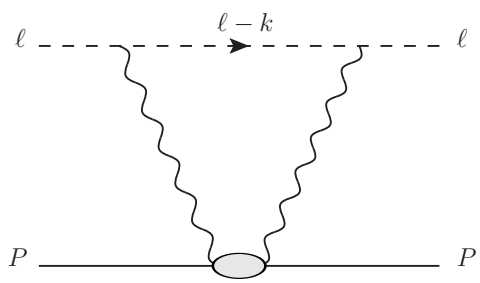
Deuteron is smaller too



Two photon exchange

Measured =206.2949(32)= 206.0573(45)-5.2262 r_p^2+0.0347 r_p^3 meV
computed

Miller PLB 2012



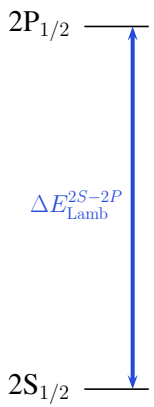
energy shift proportional
to lepton mass^4

Re part of virtual Compton scattering
Im part is measured
use dispersion relations
but unknown subtraction function is needed

Can account for 0.31 meV, no conflict with e-H

no conflict with np mass difference

Muonic hydrogen experiment and r_p



The Lamb shift is the splitting of the degenerate $2S_{1/2}$ and $2P_{1/2}$ eigenstates

Proton radius in Lamb shift

$$\Delta E = \langle \Psi_S | V_C - V_C^{\rm pt} | \Psi_S \rangle = \frac{2}{3}\pi\alpha |\Psi_S(0)|^2 \underbrace{(-6G'_E(0))}_{= r_p^2}$$

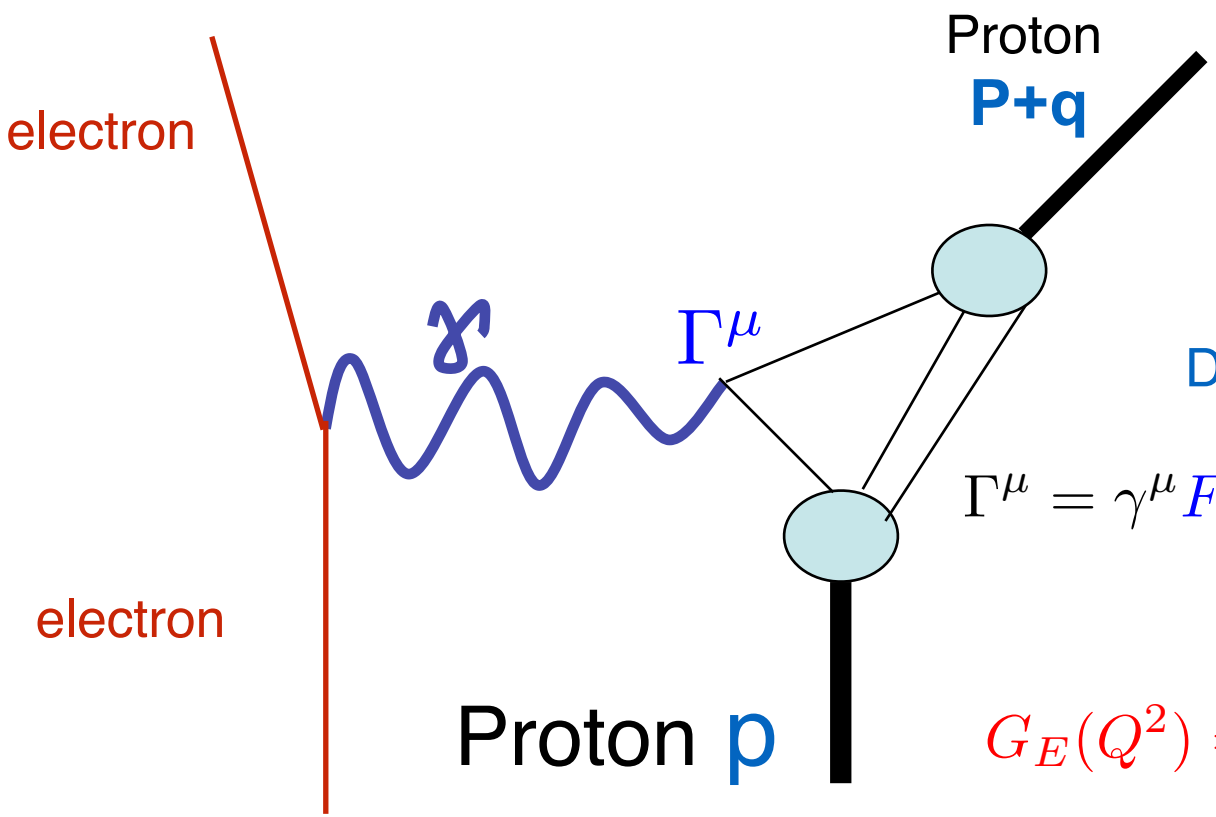
Muon/electron mass ratio 205! 8 million times larger for muon

What **is** the proton radius? QFT system – nearly massless quarks with gluons?

Measured in hydrogen atom spectroscopy and electron-proton elastic scattering.

Separate, clear, literature -both fields: $-6G'_E(0) \equiv r_p^2$ Sachs electric form factor

Probability amplitude that photon q^μ $(Q^2 = -q^2)$ absorbtion leaves proton in ground state.



Dirac

Pauli

$$\Gamma^\mu = \gamma^\mu F_1(Q^2) + i\frac{\sigma^{\mu\nu}}{2M}\kappa F_2(Q^2)$$

Sachs

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2}\kappa F_2$$

Model independent transverse charge density

Charge Density
b is impact parameter

$$\rho_\infty(x^-, \mathbf{b}) = \langle p^+, \mathbf{R} = \mathbf{0}, \lambda | \sum_q e_q q_+^\dagger(x^-, b) q_+(x^-, b) | p^+, \mathbf{R} = \mathbf{0}, \lambda \rangle$$

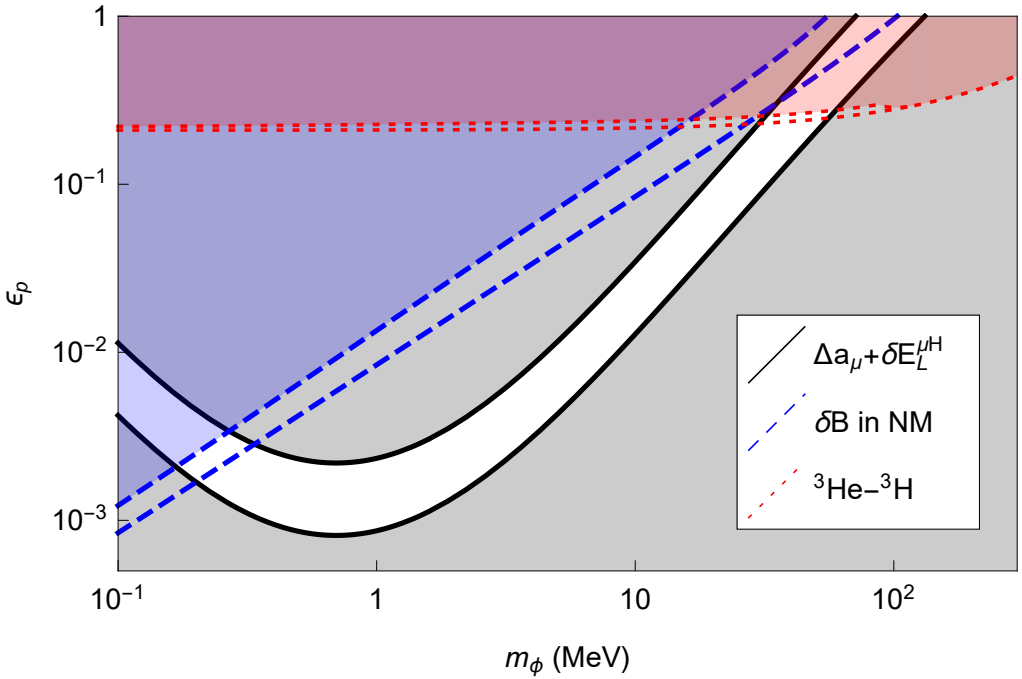
$$\boxed{\rho(b) \equiv \int dx^- \rho_\infty(x^-, \mathbf{b}) = \int \frac{Q dQ}{2\pi} F_1(Q^2) J_0(Qb)}$$

Soper '77

Density is $u - \bar{u},\ d - \bar{d}$

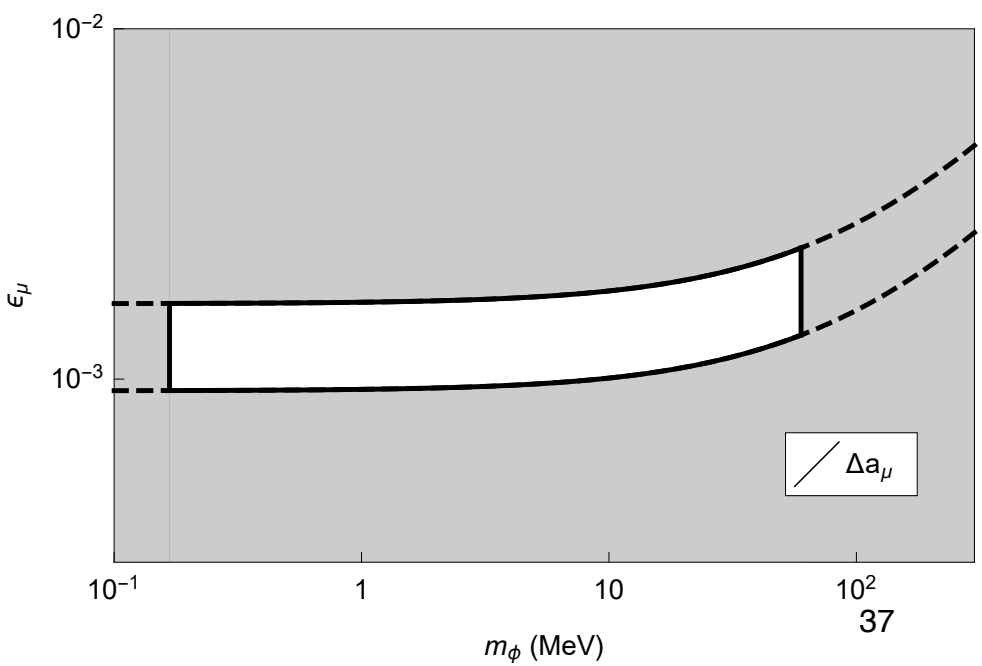
$$F_1(Q^2) = \int d^2 b \rho(b) e^{i\mathbf{q}\cdot\mathbf{b}}$$

Two dimensional FT

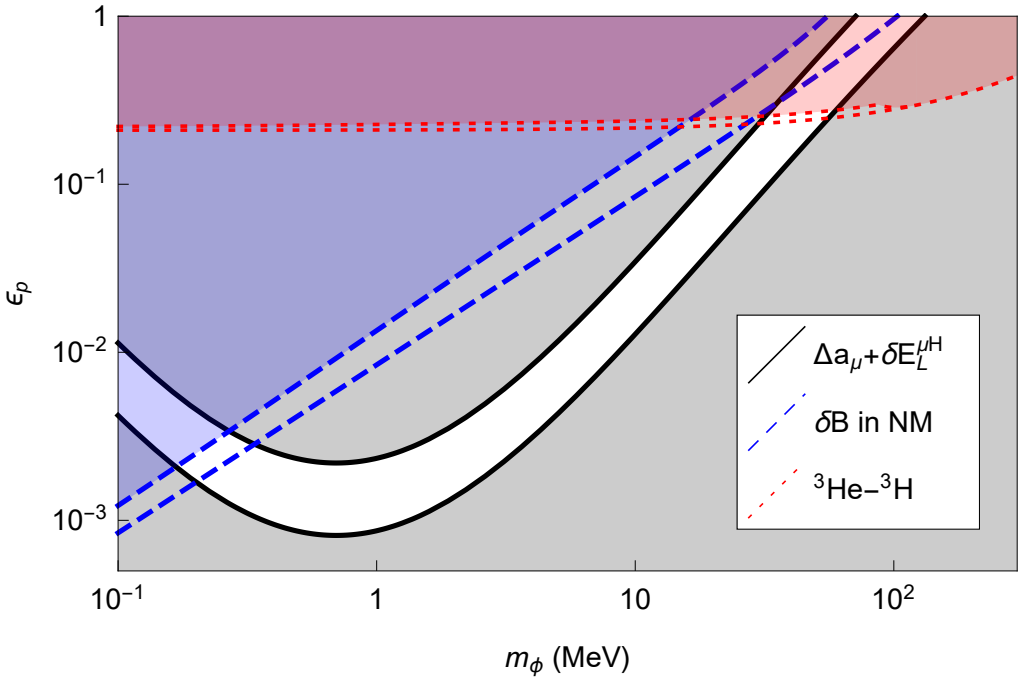


Unshaded allowed by muon g-2 and muon-p Lamb shift

Two anomalies have same scale



Unshaded allowed by muon g-2



Unshaded allowed by muon g-2 and muon-p Lamb shift

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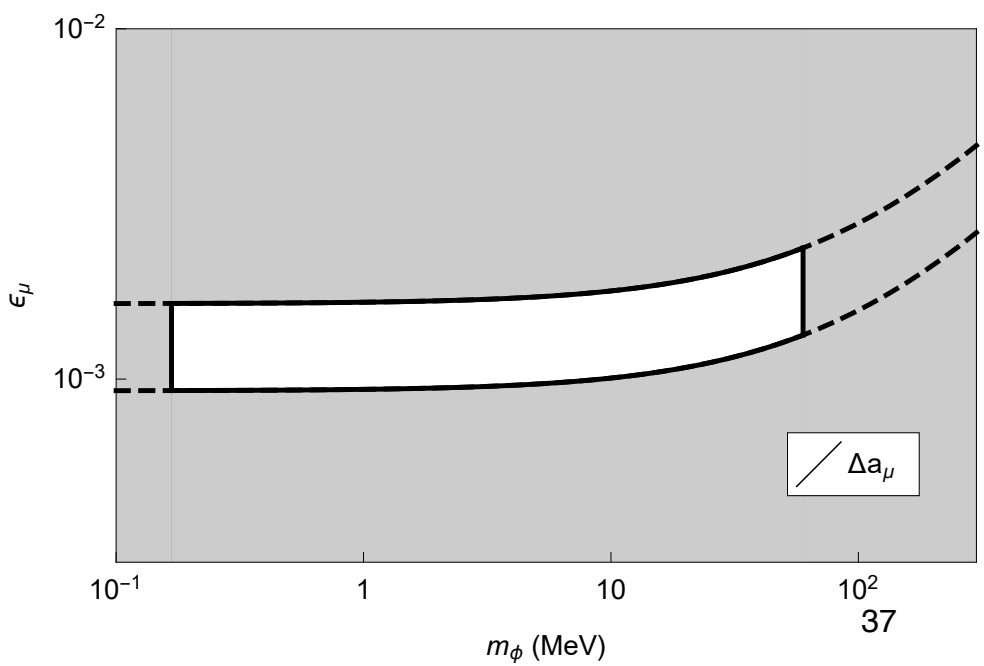


Unshaded allowed by muon g-2

Using new eH experiment $\epsilon_\mu \epsilon_p$ is reduced by a factor of 3: barely visible in loglog plot

Nuclear Physics constraints

$$\epsilon_n/\epsilon_p$$

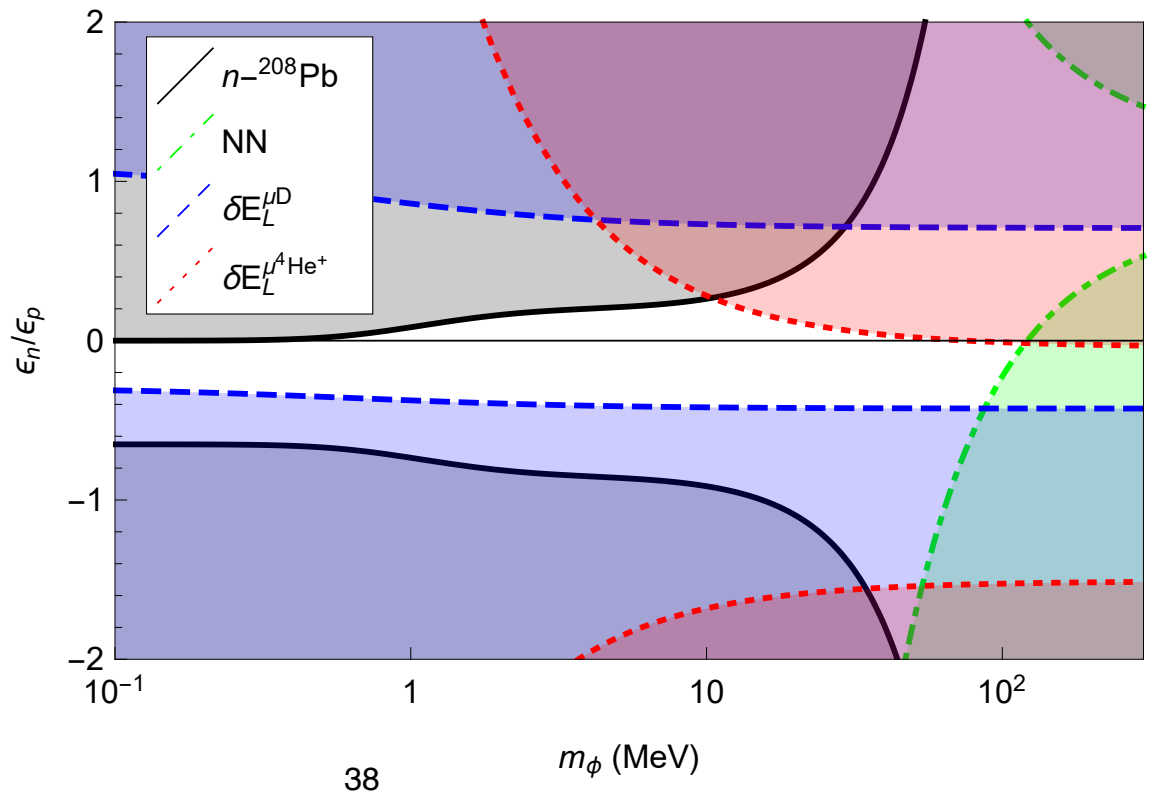
If $\epsilon_n = \epsilon_p$ scalar is ruled out by n-^{208}Pb scattering

If ϵ_n has opposite sign as ϵ_p parameter space widens

Other constraints:

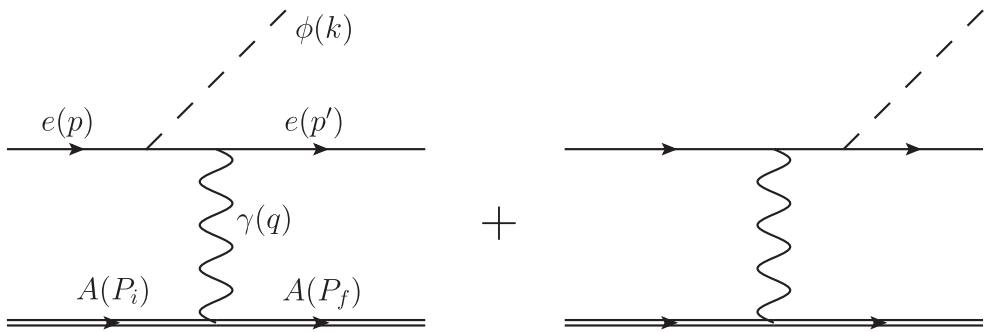
NN scattering, nuclear matter & ^{3}He $-^3$ H binding

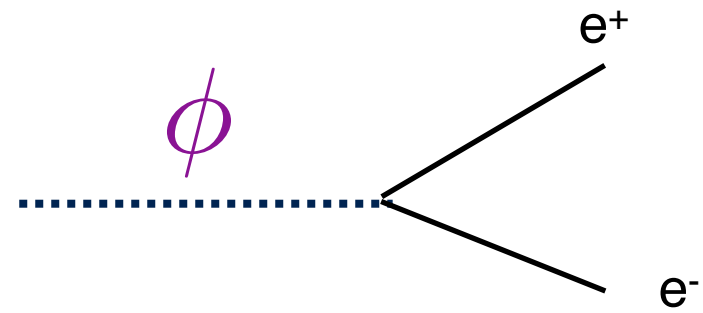
muonic D, ^{3}He

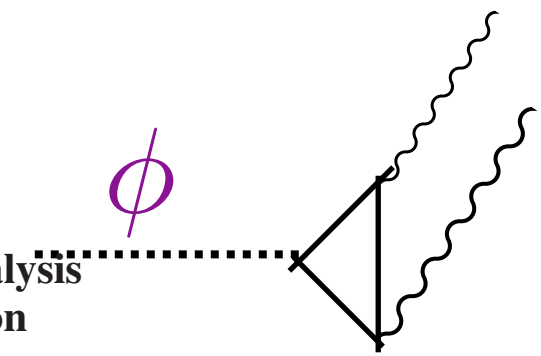


Beam dump experiments

Beam dumps absorb beam of charged particles
e+^{27}Al to dissipate energy
hope e+^{27}Al makes penetrating new particles







PHYSICAL REVIEW D **95,** 036010 (2017)

Validity of the Weizsäcker-Williams approximation and the analysis of beam dump experiments: Production of a new scalar boson

Yu-Sheng Liu,* David McKeen,† and Gerald A. Miller‡

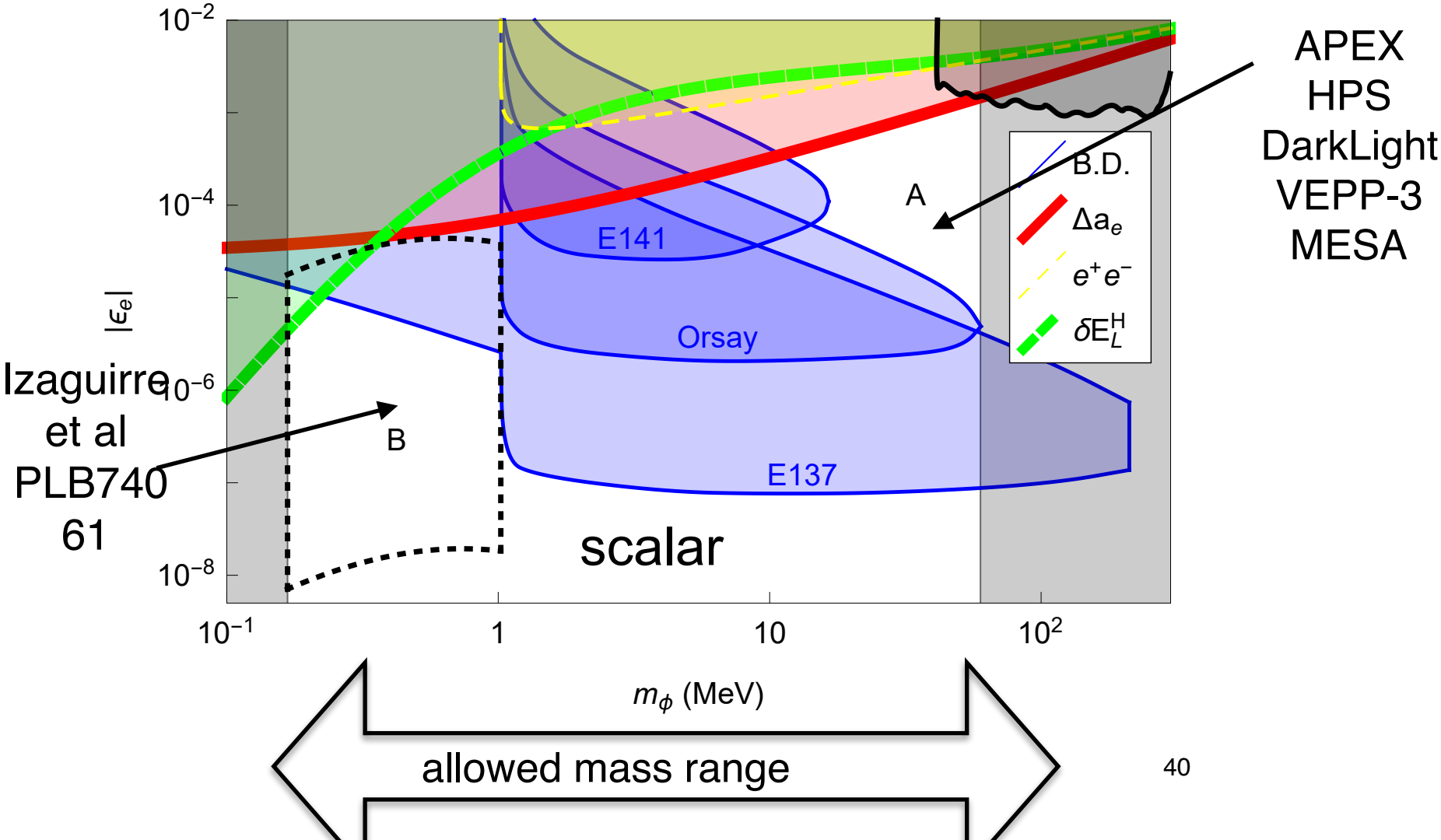
Better analysis if discovery

PHYSICAL REVIEW D **96,** 016004 (2017)

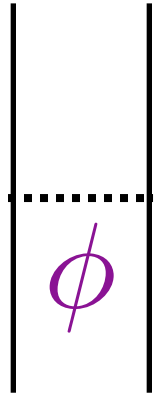
Validity of the Weizsäcker-Williams approximation and the analysis of beam dump experiments: Production of an axion, a dark photon, or a new axial-vector boson

Yu-Sheng Liu* and Gerald A. Miller†

electron Exclusion plot



Our approach



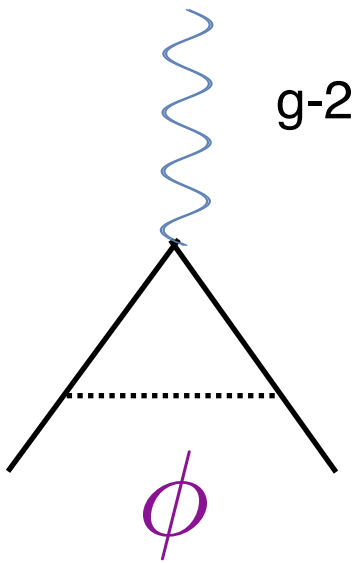
any signs

$$V(r) = -\epsilon_{f_1}\epsilon_{f_2}\alpha\frac{e^{-m_\phi r}}{r}. \qquad |\epsilon_e| \ll \epsilon_\mu$$

Bohr radius

$$\delta E_L^{\ell \mathrm{N}} = -\frac{\alpha}{2a_{\ell\mathrm{N}}}\epsilon_\ell[Z\epsilon_p + (A-Z)\epsilon_n]\frac{(a_{\ell\mathrm{N}}m_\phi)^2}{(1+(a_{\ell\mathrm{N}}m_\phi))^4}$$

g-2



$\Delta a_\mu = 287(80)\times 10^{-11}$, $\Delta a_e = 1.5\times 10^{-12}$ From ^{87}Rb

For light mass- evades 4He data

More constrains Coupling to electron

ϵ_e

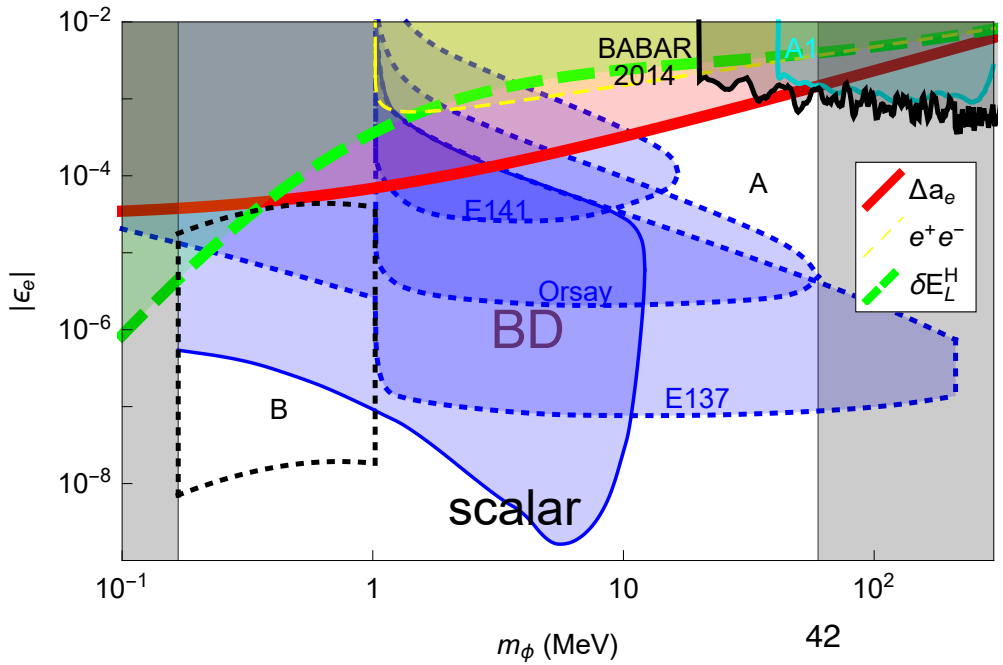
Hydrogen atom Lamb shift

$\delta E_L^H < 45$ kHz

e^+e^- resonant (?) scattering

not seen

electron anomalous magnetic moment



BD is beam dump discussed next

Electrophobic

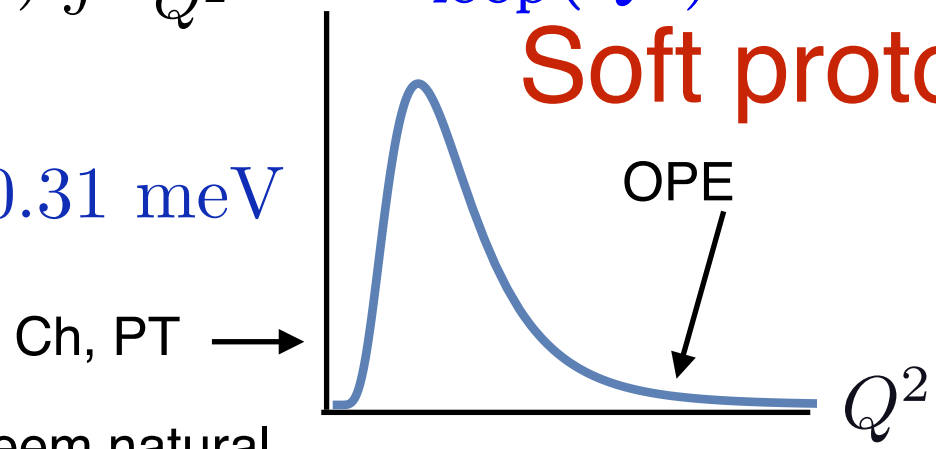
Almost unknown $\bar{T}_1(0, Q^2)$ Miller PLB 2012

$$\Delta E^{\rm subt} \propto \alpha^2 m \Psi_S^2(0) \int \frac{dQ^2}{Q^2} \cdots F_{\rm loop}(Q^2)$$

Soft proton

number of

Infinite $F_{\rm loop}(Q^2)$ give 0.31 meV satisfy all constraints



Recast in EFT- parameters seem natural

So far size of this term cannot be determined from theory-experiment is needed

lattice QCD may disprove above sentence

Nuclear Physics constraints

$$\epsilon_n / \epsilon_p$$

- Low energy scattering of neutrons on ^{208}Pb using ϕ-nucleon coupling g_N.
  $\frac{g_N^2}{e^2} \to \frac{A-Z}{A}\epsilon_n^2 + \frac{Z}{A}\epsilon_p\epsilon_n$ cancellation evades previous limits

- NN charge-independence breaking scattering length

  $\Delta a = (a_{pp} + a_{nn})/2 - a_{np}$, measured: 5.64(60) fm, theory: 5.6(5)

  Scalar boson exchange: $\Delta a_\phi \propto \int_0^\infty \Delta V \bar{u} u_{np} dr \leq 1.6$ fm (2 S.D.)

- Change in binding energy/A infinite nuclear matter: less than 1 MeV

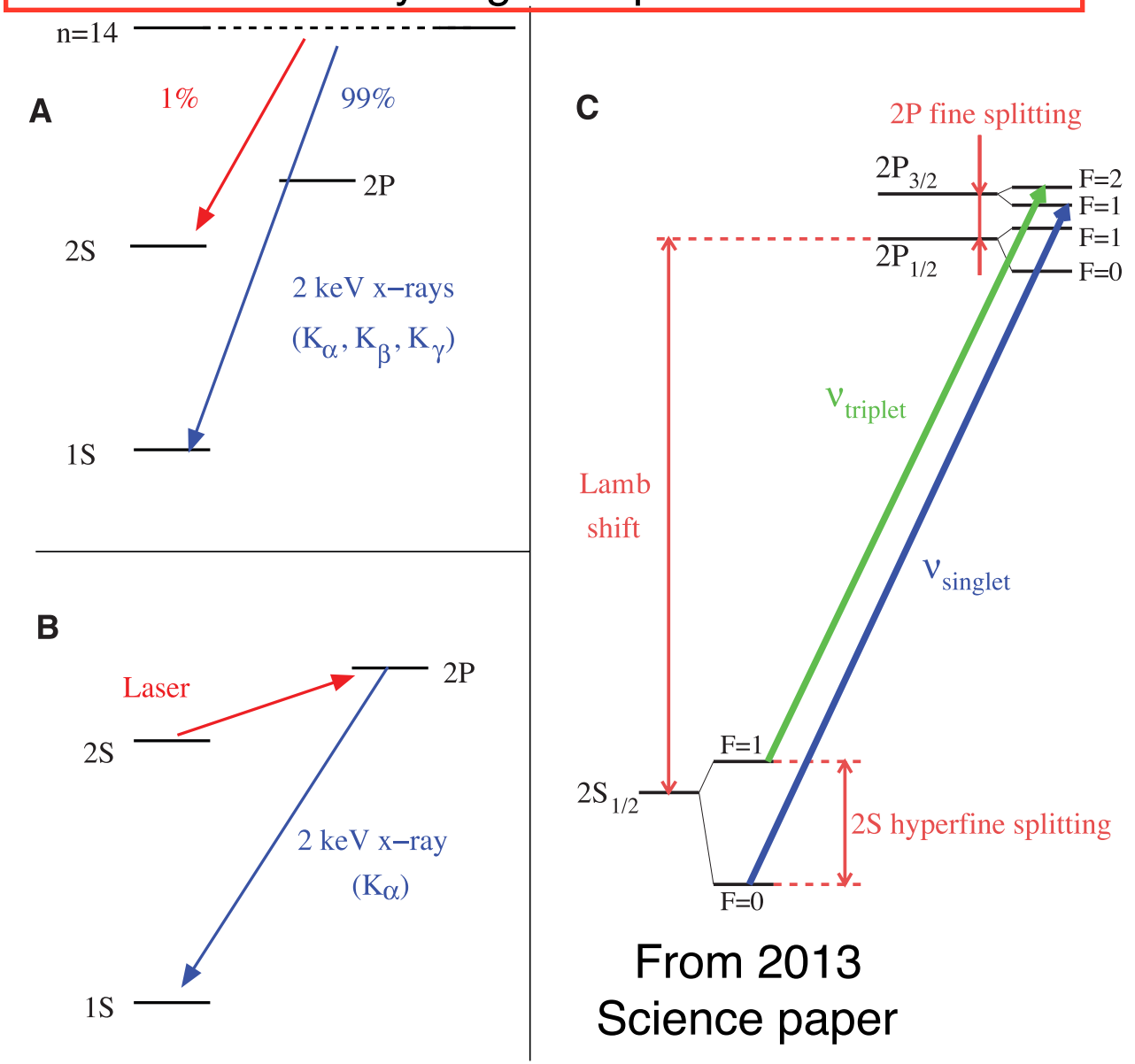
- binding energy B(^3He) $- B(^3H)$= 763.76 keV due to Coulomb (693 keV) + strong force charge symmetry breaking (68 keV) ϕ exchange $<$ 30 keV

MUSE and scalar

$$V_\phi(r) = -1.7 \times 10^{-6}\, \alpha \frac{e^{-m_\phi\, r}}{r}$$

- No scattering experiment can detect a coupling this weak
- If this scalar exists (and other e H large radius experiments correct) MUSE will find electrons/positrons see the same large radius and
- muons and anti-muons will see the same large radius
- If no puzzle: all leptons see small radius
- In any case MUSE limits two photon (biggest uncertainty)

Muonic Hydrogen Experiment



From 2013 Science paper

Randolf Pohl

Aldo Antognini

Fig. 1. (**A**) Formation of μp in highly excited states and subsequent cascade with emission of "prompt" $K_{\alpha,\beta,\gamma}$. (**B**) Laser excitation of the 2S-2P transition with subsequent decay to the ground state with K_α emission. (**C**) 2S and 2P energy levels. The measured transitions ν_s and ν_t are indicated together with the Lamb shift, 2S-HFS, and 2P-fine and hyperfine splitting.

PHYSICAL REVIEW D **95,** 036010 (2017)

Validity of the Weizsäcker-Williams approximation and the analysis of beam dump experiments: Production of a new scalar boson

Yu-Sheng Liu,* David McKeen,† and Gerald A. Miller‡

- previous cross sections obtained w. WW approximation
- cross sections not accurate
- exclusion plots changed substantially
- if discovery, WW gives wrong parameters
- not necessary to assume mass of new particle is much much greater than mass of electron

Lepton-universality violating one boson exchange

- Tucker-Smith & Yavin PRD83, 101702 new particle scalar or vector coupling
- Brax & Burrage scalar particles PRD 83, 035020 &'14
- Batell, McKeen & Pospelov PRL 107, 011803 new gauge boson kinetically mixing with $F^{\mu\nu}$ plus scalar for muon mag. mom. 1401.6154 W decays enhanced
- Carlson Rislow PRD 86, 035013 fine tune scalar pseudoscalar or polar and axial vector couplings
- Barger et al PRL106,153001 - new particles ruled out but assumes universal coupling
- Kaon decays provide constraints

Looking for new scalars is not new
Low mass Higgs searches

$$p(1.88\,\mathrm{MeV}) +^{19} F \rightarrow \alpha +^{16} O^*(6.05)$$
$$^{16}O^*(6.05) \rightarrow^{16} O(GS) + \phi$$

Kohler et al PRL 33, 1628 (1974)

Freedman et al. PRL 52, 240 (1984)

$$p +^3 H \rightarrow^4 He(20.1) \rightarrow 4He(GS) + \phi$$

No Scalars found, but assumed coupling constants were much larger than what we will use

Nuclear dependence of short-ranged mu-p effects

- Energy shift is proportional to square of muon wave function at the origin

GAM
1501.01036

- Suppose you have effect that gives energy shifts Ep (on proton) E_n (on neutron)

$$E_A = \left(\frac{1+\frac{m_\mu}{m_p}}{1+\frac{m_\mu}{Am_p}}\right)^3 Z^3(ZE_p + NE_n)\left(1 - \mathcal{O}(\frac{R_A^2}{a_\mu^2})\right) \approx \left(\frac{1+\frac{m_\mu}{m_p}}{1+\frac{m_\mu}{Am_p}}\right)^3 Z^3(ZE_p + NE_n),$$

Nuclear shift

Square of wave fun

Counting

Size of nucleus

My model: ~0.3 meV (1+0.3)(8)(2) =-6.3 meV about 6 st. dev off

RIP any short range idea

Almost unknown $\overline{T}_1(0,Q^2)$

Miller PLB 2012

$$\Delta E^{\rm subt} = \frac{\alpha^2}{m}\Psi_S^2(0)\int_0^\infty dQ^2\,\frac{h(Q^2)}{Q^2}\overline{T}_1(0,Q^2)$$

Soft proton

$$\lim_{Q^2\to\infty} h(Q^2) \sim \frac{2m^2}{Q^2},\ \text{chiral PT}:\ \overline{T}_1(0,Q^2) = \frac{\beta_M}{\alpha}Q^2 + \cdots$$

$\rightarrow$ Logarithmic divergence

Typo bleow

$$\overline{T}_1(0,Q^2) \rightarrow \frac{\beta_M}{\alpha}Q^2 F_{\rm loop}(Q^2)\ \text{Cuts off integral}$$

$$\text{Birse \& McGovern assume dipole}:\ \Delta E^{\rm subt} = 0.004\,\text{meV very small}$$

$$\text{Miller}\quad F_{\rm loop}(Q^2) = \left(\frac{Q^2}{M_0^2}\right)^n \frac{1}{(1+aQ^2)^N}, n \geq 2, N \geq$$

n+3

Infinite parameter set gets needed 0.31meV, NO constraint on neutron

Choose parameters so shift in proton mass <0.5 MeV
(current uncertainty)

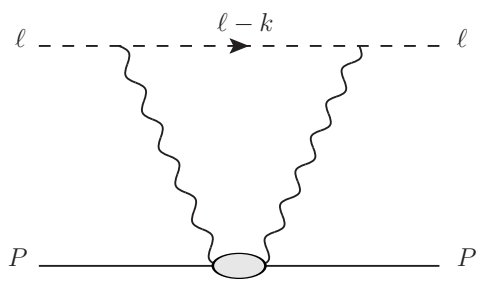
Recast in EFT- parameters seem natural

Two photon exchange

Measured =206.2949(32)= 206.0573(45)-5.2262 r_p^2+0.0347 r_p^3 meV computed

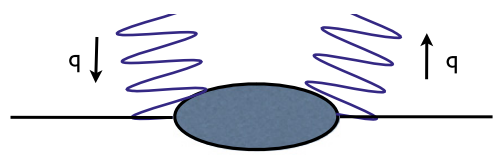
Explain puzzle with radius as in H atom increase 206.0573 meV by 0.31 meV- attractive effect on 2S state needed

Our idea



energy shift proportional to lepton mass^4

$T^{\mu\nu} =$



$$= -(g^{\mu\nu} - \cdots)T_1 + (P^\mu - \cdots)(P^\mu - \cdots)T_2$$

Dispersion relation: $Im[T_1] \propto W_1$ measured

Large virtual photon energy ν, $W_1 \sim \nu$ integral over energy diverges

Subtraction function needed: $\bar{T}_1(0, Q^2)$ zero energy

Hill & Paz- big uncertainty in dispersion approach

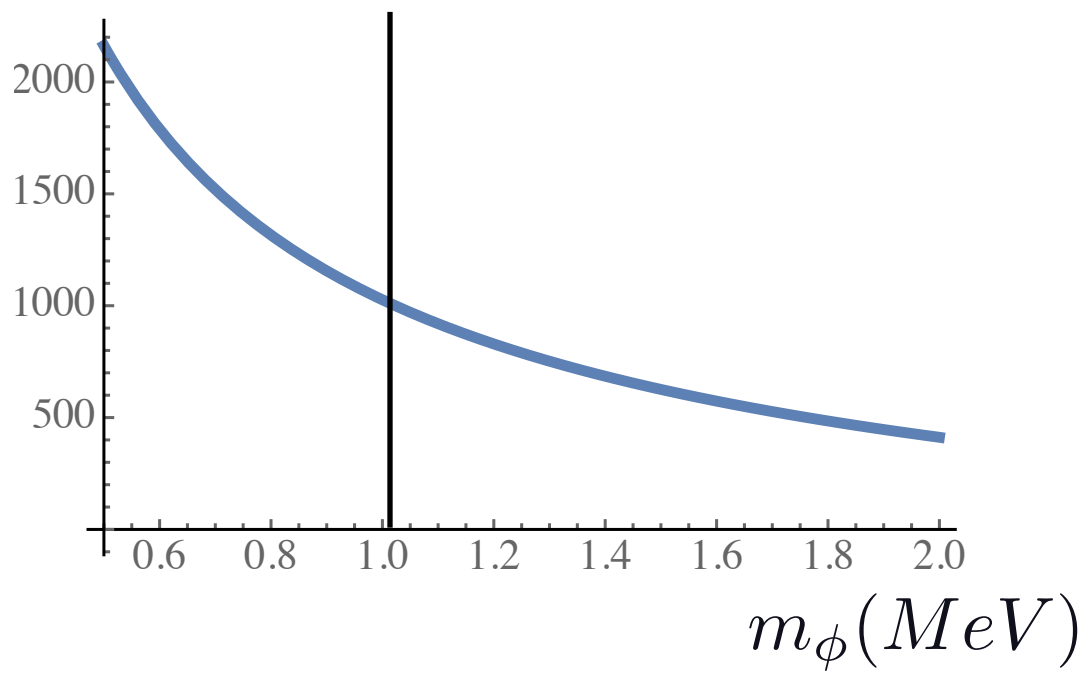
$$^{16}O(6.05, 0^+) \rightarrow {}^{16}O(\mathrm{GS}, 0^+) + \phi, \quad \text{No single photon decay}$$

From electron g-2

$$\frac{\tau(A^* \rightarrow A + e^+e^-)}{\tau(A^* \rightarrow A + \phi} = 3.3 \times 10^3 \frac{g^2_{\phi e \bar{e}}}{e^2} \left(1 - \left(\frac{m_\phi}{6\mathrm{MeV}}\right)^2\right)^{5/2}$$

$$\tau(A^* \rightarrow A + e^+e^-): \quad \text{lifetime is } 10^{-10}\mathrm{s}$$

Decay length (m)
nuclear emission of
scalar boson



Arbitrary functions

$$\overline{T}_1(0,Q^2) = \frac{\beta_M}{\alpha} Q^2 F_{\rm loop}(Q^2)\,.$$

$$F_{\rm loop}(Q^2) = \left(\frac{Q^2}{M_0^2}\right)^n \frac{1}{(1+aQ^2)^N},\ n \geq 2,\ N \geq n+3,$$

$$\overline{T}_1(0,Q^2) \sim \frac{1}{Q^4} \text{ or faster, } \beta_M \to \beta$$

$$\Delta E^{\rm subt} \approx 3\alpha^2 m \Psi_S^2(0) \frac{\beta}{\alpha} \gamma^n B(N,n), \gamma \equiv \frac{1}{M_0^2 a}$$

3 parameters: n, N, a $(M_0 = M_\beta)$

Choose parameters such that shift in proton mass < electromagnetic uncertainty of 0.5 MeV

Almost unknown $\overline{T}_1(0,Q^2)$

Miller PLB 2012

$$\Delta E^{\rm subt} = \frac{\alpha^2}{m}\Psi_S^2(0)\int_0^\infty dQ^2\,\frac{h(Q^2)}{Q^2}\overline{T}_1(0,Q^2)$$

Soft proton

$$\lim_{Q^2\to\infty} h(Q^2) \sim \frac{2m^2}{Q^2},\ \text{chiral PT}:\ \overline{T}_1(0,Q^2) = \frac{\beta_M}{\alpha}Q^2 + \cdots$$

$\rightarrow$ Logarithmic divergence

Typo bleow

$$\overline{T}_1(0,Q^2) \rightarrow \frac{\beta_M}{\alpha}Q^2 F_{\rm loop}(Q^2)\ \text{Cuts off integral}$$

$$\text{Birse \& McGovern assume dipole}:\ \Delta E^{\rm subt} = 0.004\,\text{meV very small}$$

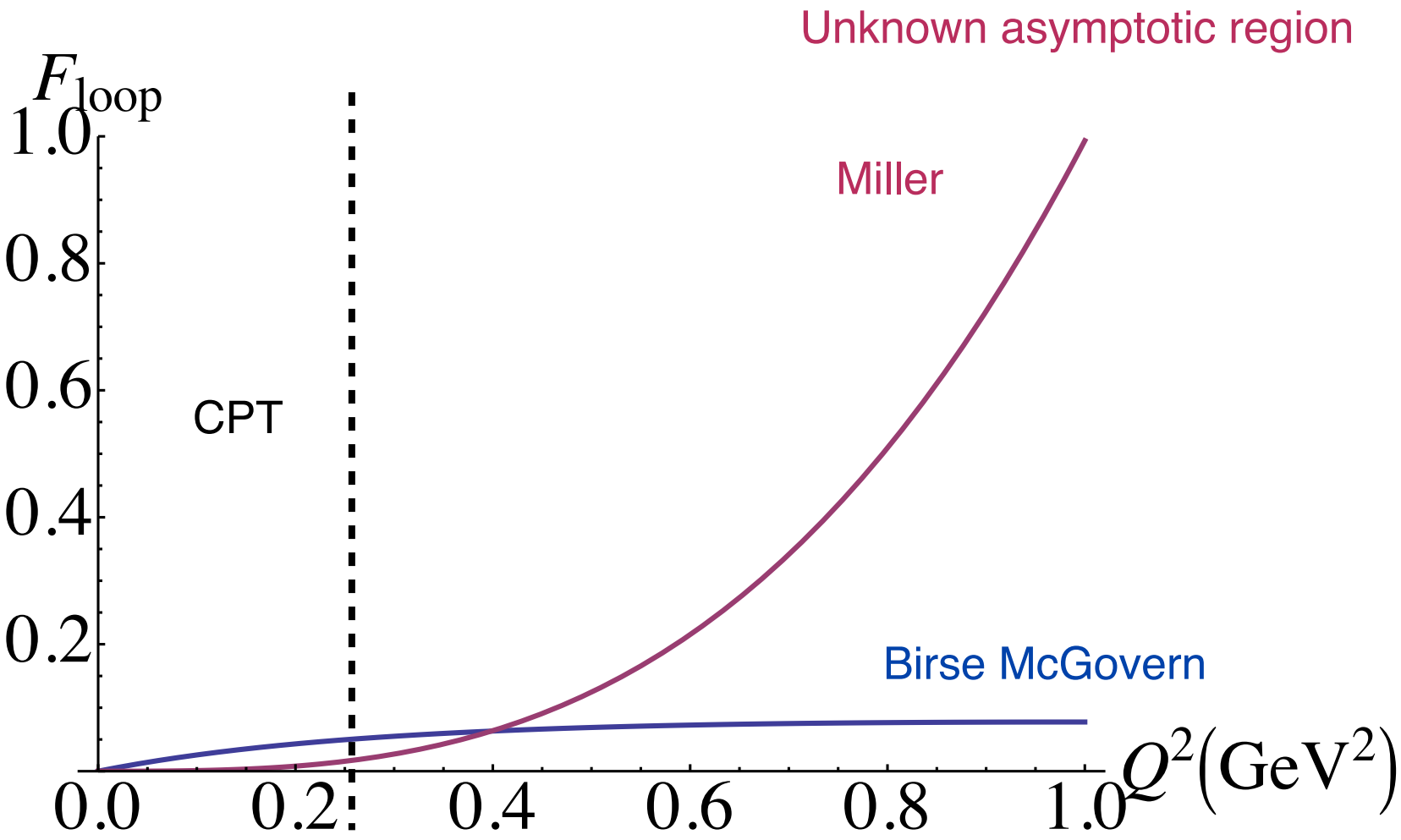
$$\text{Miller}\quad F_{\rm loop}(Q^2) = \left(\frac{Q^2}{M_0^2}\right)^n \frac{1}{(1+aQ^2)^N}, n \geq 2, N \geq N+3$$

Infinite parameter set gets needed 0.31meV, NO constraint on neutron

Choose parameters so shift in proton mass <0.5 MeV
(current uncertainty)

Recast in EFT- parameters seem natural

Form factors

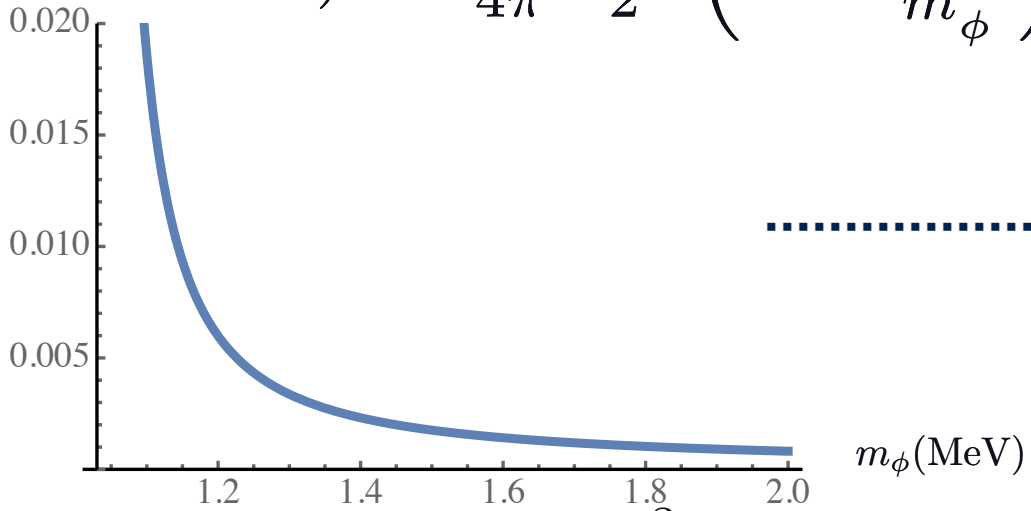


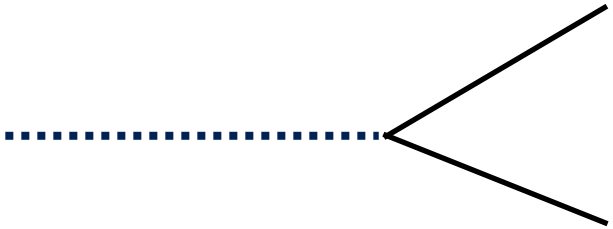
If recast into effective field theory strength seems natural

ϕ Decay modes

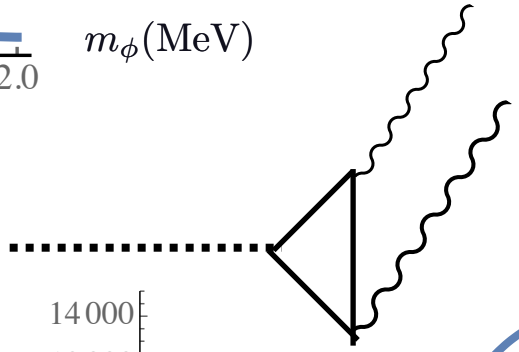
$$\Gamma(\phi \to e^+e^-) = \frac{g_{\phi e}^2}{4\pi}\frac{m_\phi}{2}\left(1 - \frac{4m_e^2}{m_\phi^2}\right)^{3/2}, m_\phi > 2m_e$$

$c\tau$(m)





$$\Gamma(\phi \to \gamma\gamma) = g_{\phi e}^2 \frac{\alpha^2}{144\pi^3}\frac{m_\phi^3}{m_e^2}$$



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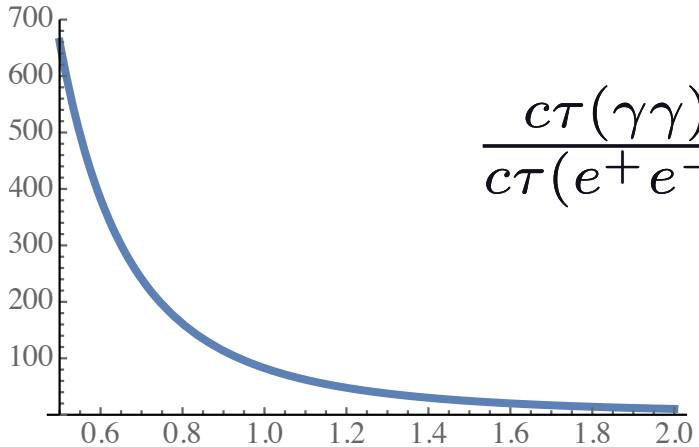
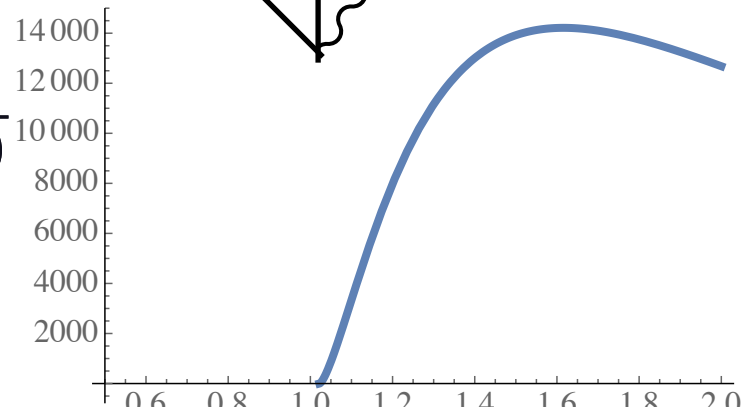


$$\frac{c\tau(\gamma\gamma)}{c\tau(e^+e^-)}$$



m_ϕ(MeV)

Pohl et al. Table of calculations

Lamb shift: vacuum polarization many, many terms

#	Contribution		Our selection		Pachucki[1–3]		Borie[5]	
		Ref.	Value	Unc.	Value	Unc.	Value	Unc.
1	NR One loop electron VP	1,2			205.0074			
2	Relativistic correction (corrected)	1–3,5			0.0169			
3	Relativistic one loop VP	5	205.0282				205.0282	
4	NR two-loop electron VP	5,14	1.5081		1.5079		1.5081	
5	Polarization insertion in two Coulomb lines	1,2,5	0.1509		0.1509		0.1510	
6	NR three-loop electron VP	11	0.00529					
7	Polarisation insertion in two and three Coulomb lines (corrected)	11,12	0.00223					
8	Three-loop VP (total, uncorrected)				0.0076		0.00761	
9	Wichmann-Kroll	5,15,16	−0.00103				−0.00103	
10	Light by light electron loop contribution (Virtual Delbrück scattering)	6	0.00135	0.00135			0.00135	0.00015
11	Radiative photon and electron polarization in the Coulomb line $\alpha^2(Z\alpha)^4$	1,2	−0.00500	0.0010	−0.006	0.001	−0.005	
12	Electron loop in the radiative photon of order $\alpha^2(Z\alpha)^4$	17–19	−0.00150					
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14	Hadronic polarization $\alpha(Z\alpha)^4 m_r$	21–23	0.01077	0.00038	0.0113	0.0003	0.011	0.002
15	Hadronic polarization $\alpha(Z\alpha)^5 m_r$	22,23	0.000047					
16	Hadronic polarization in the radiative photon $\alpha^2(Z\alpha)^4 m_r$	22,23	−0.000015					
17	Recoil contribution	24	0.05750		0.0575		0.0575	
18	Recoil finite size	5	0.01300	0.001			0.013	0.001
19	Recoil correction to VP	5	−0.00410				−0.0041	
20	Radiative corrections of order $\alpha^n(Z\alpha)^k m_r$	2,7	−0.66770		−0.6677		−0.66788	
21	Muon Lamb shift 4th order	5	−0.00169				−0.00169	
22	Recoil corrections of order $\alpha(Z\alpha)^5 \frac{m}{M} m_r$	2,5–7	−0.04497		−0.045		−0.04497	
23	Recoil of order α^6	2	0.00030		0.0003			
24	Radiative recoil corrections of order $\alpha(Z\alpha)^n \frac{m}{M} m_r$	1,2,7	−0.00960		−0.0099		−0.0096	
25	Nuclear structure correction of order $(Z\alpha)^5$ (Proton polarizability contribution)	2,5,22,25	0.015	0.004	0.012	0.002	0.015	0.004
26	Polarization operator induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	23	0.00019					
27	Radiative photon induced correction to nuclear polarizability $\alpha(Z\alpha)^5 m_r$	23	−0.00001					
	Sum		206.0573	0.0045	206.0432	0.0023	206.05856	0.0046

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Total $< r_p^2 >$ contribution		−5.22619	$< r_p^2 >$	−5.2256	−5.2244
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Resolution 1-QED calcs not OK

α

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Mostly irrelevant- theory replaced by experiment

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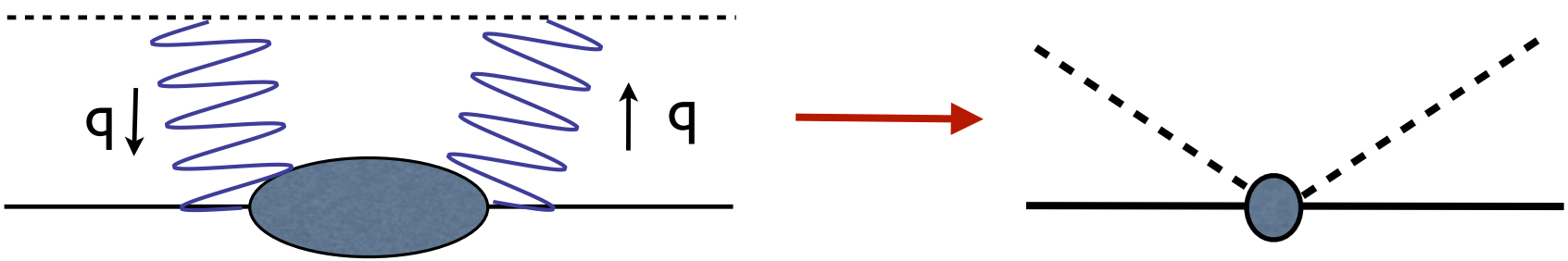
Resolution 1- QED calcs not OK

QED calcs expand in α

EFT of μp interaction

Caswell Lepage '86

- Compute Feynman diagram, remove log divergence using dimensional regularization
- include counter term in Lagrangian



$$\mathcal{M}_2^{DR} = \frac{3}{2} i\, \alpha^2 m \frac{\beta_M}{\alpha} \left[\frac{2}{\epsilon} + \log\frac{\mu^2}{m^2} + \frac{5}{6} - \gamma_E + \log 4\pi\right] \overline{u}_f u_i \overline{U}_f U_i,$$

$$= i\, \alpha^2 m \frac{\beta_M}{\alpha} (\lambda + 5/4)\, \overline{u}_f u_i \overline{U}_f U_i$$

Choose λ to get 0.31 meV shift

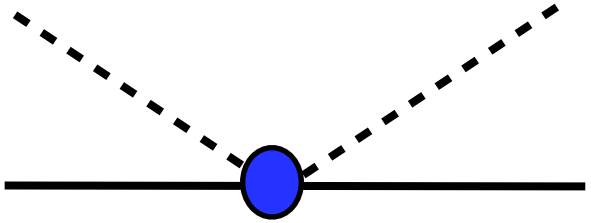
$$\Delta E^{\rm subt}(DR) = \alpha^2 m \frac{\beta_M}{\alpha} \Psi_S^2(0)(\lambda + 5/4)$$

$$\Delta E^{\rm subt}(DR) = 0.31 \text{ meV} \rightarrow \boxed{\lambda = 769}$$

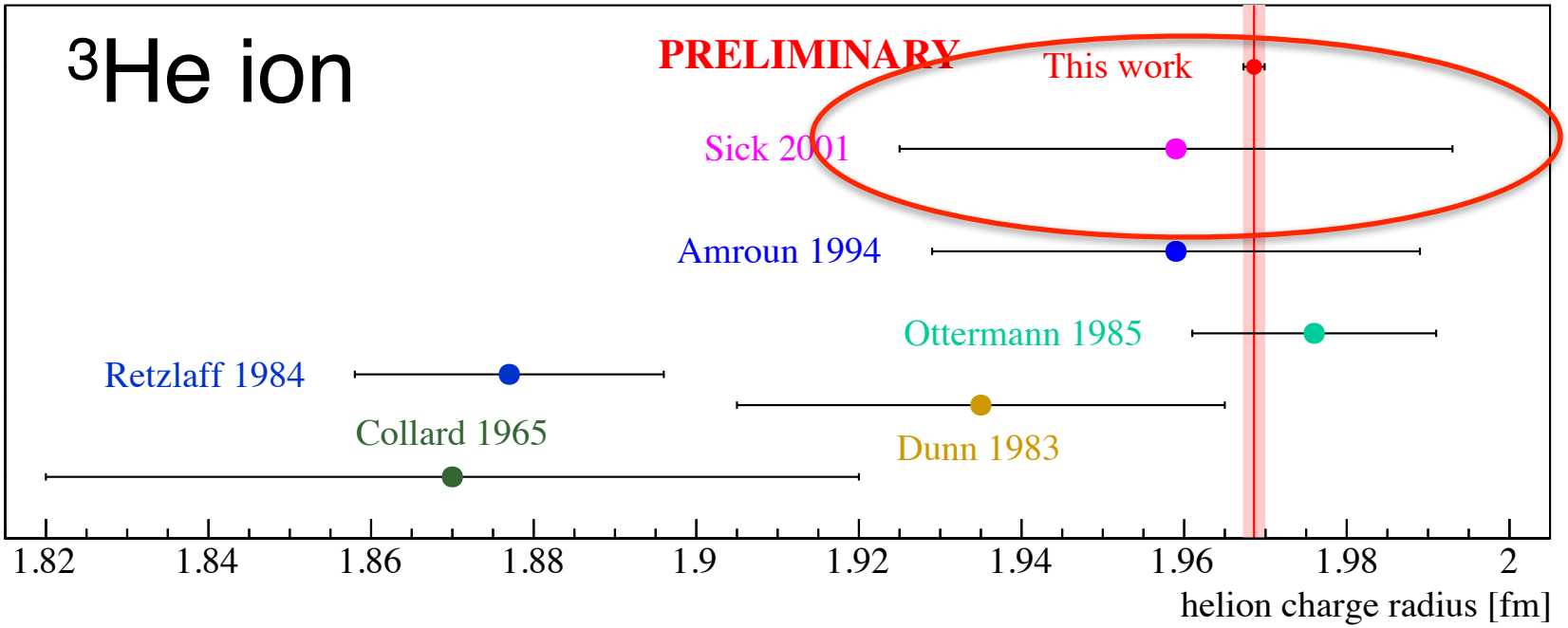
β_M(magnetic polarizability) $= 3.1 \times 10^{-4}\text{fm}^3$ very small

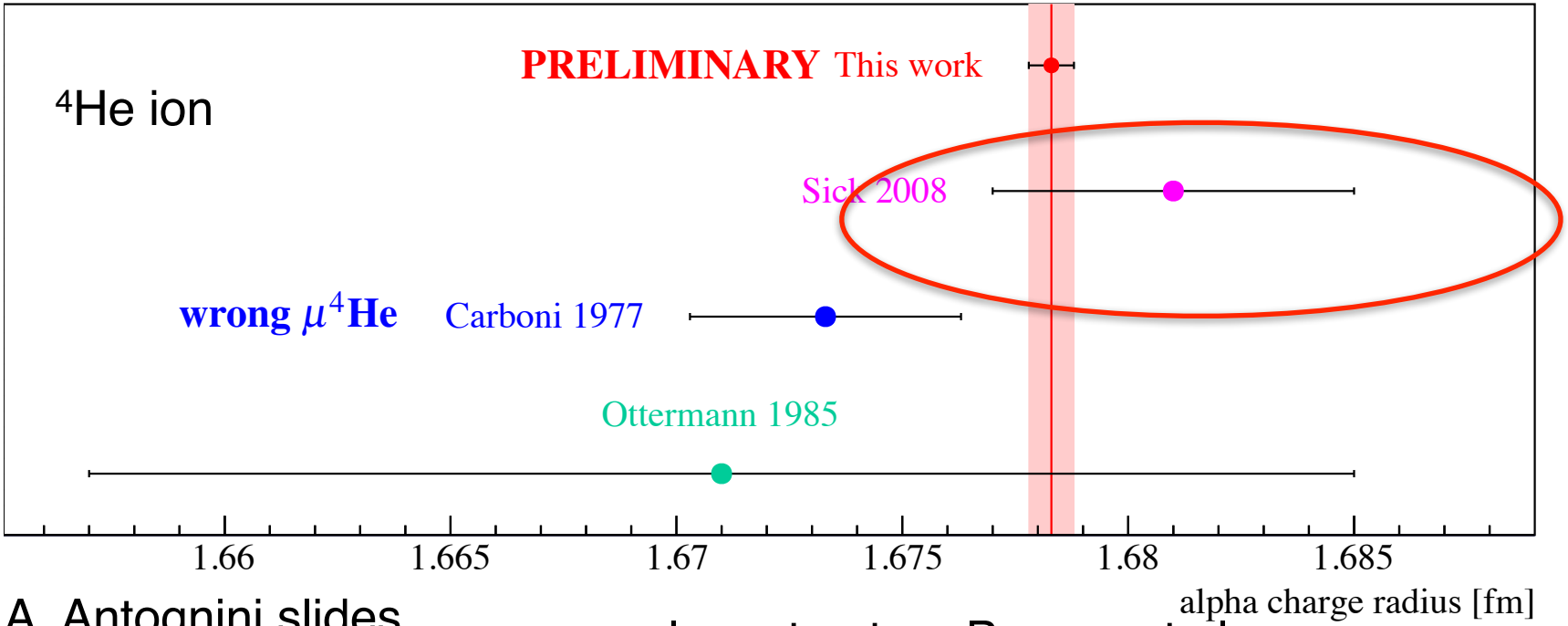
Natural units $\beta_M/\alpha \sim 4\pi/(4\pi f_\pi)^3$ Butler & Savage $'92$

$$\mathcal{M}_2^{DR} = i\, 3.95\, \alpha^2 m \frac{4\pi}{\Lambda_\chi^3} \overline{u}_f u_i \overline{U}_f U_i.$$



3.95 =natural





A. Antognini slides

nuclear structure Barnea et al