

A Precision Measurement of Neutral Pion Lifetime

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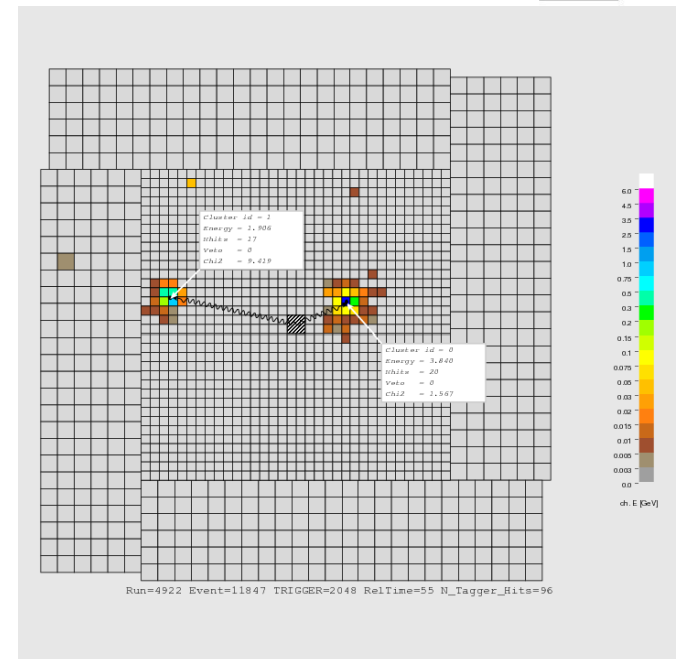
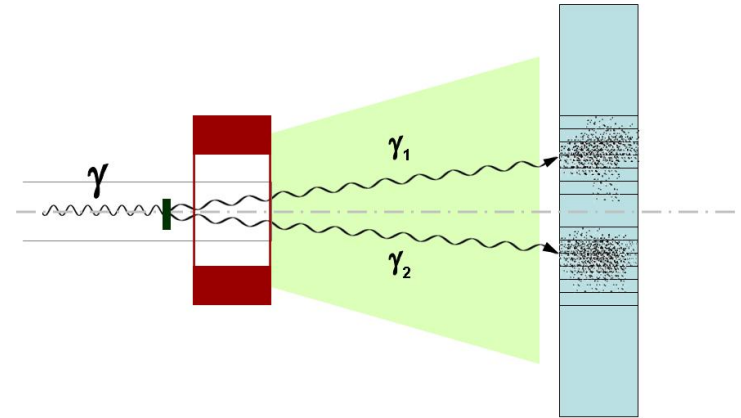
π^0 analysis

Physical quantities measured:

- Incident photon energy and time
- Decay photon energies, coordinates and time

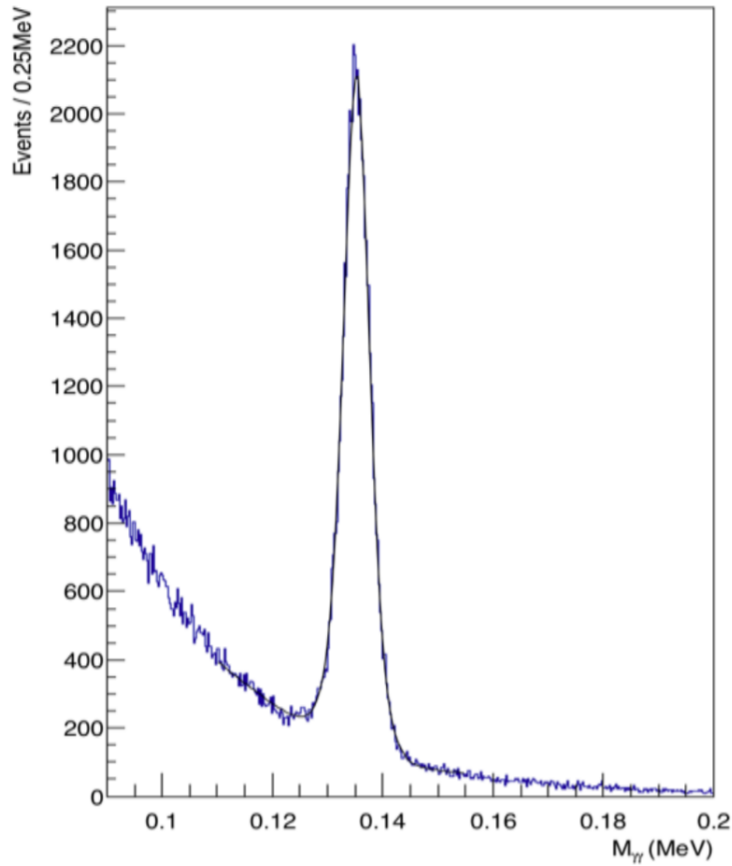
π^0 event selection:

- Requires two HyCal clusters:
 - 1) Every combination are considered as π^0 candidate
 - 2) $E_1, E_2 > 0.5$ GeV
 - 3) $E_1 + E_2 > 3.5$ GeV
- Conservation of energy:
$$E_{\text{beam}} = E_1 + E_2$$
- Invariant mass: $m_{\gamma\gamma} = m_{\pi^0}$
- Coincidence between incident photon and decay photons:
 - 1) “Best tdiff” π^0 candidate selection
 - 2) Cut on the time different between HyCal and Tagger (tdiff)

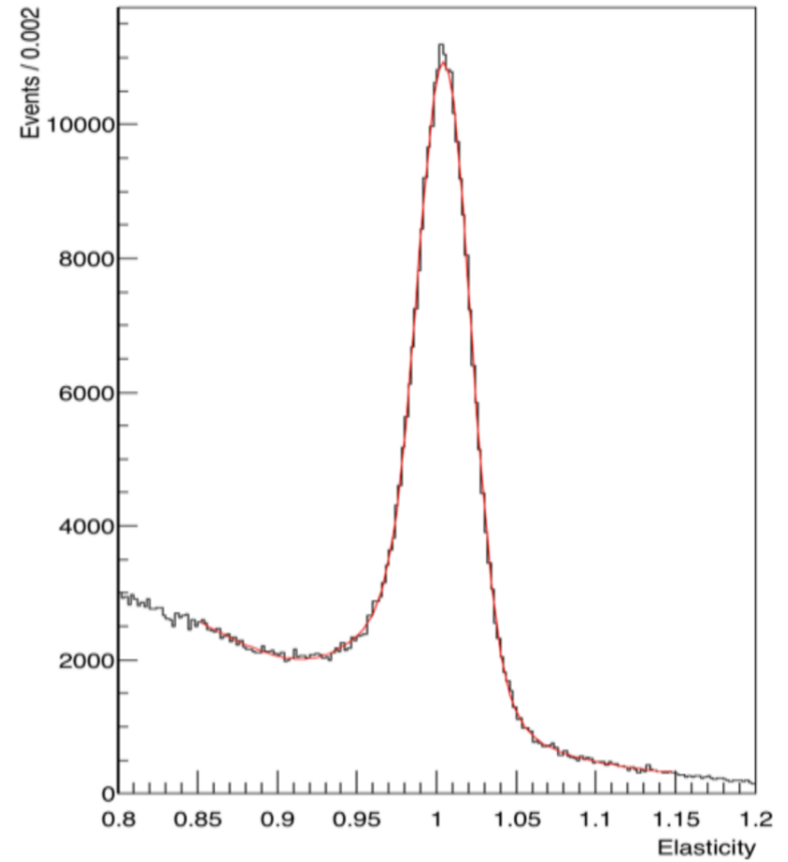


π^0 invariant mass and elasticity

π^0 event signature: two γ invariant mass



π^0 elasticity: $E_{\text{beam}} = E_1 + E_2$



π^0 invariant mass and elasticity

- Invariant mass $m_{\gamma\gamma}$

$$m_{\gamma\gamma HYCAL} = \sqrt{2(E_1 + \Delta E_1)(E_2 + \Delta E_2)(1 - \cos(\theta))}$$

$$\approx \sqrt{2E_1 E_2 (1 - \cos(\theta))} \sqrt{1 + \frac{\Delta E_1}{E_1} + \frac{\Delta E_2}{E_2}}$$

$$\approx m_{\pi^0} \left[1 + \frac{1}{2} \left(\frac{\Delta E_1}{E_1} + \frac{\Delta E_2}{E_2} \right) \right]$$

- Elasticity:

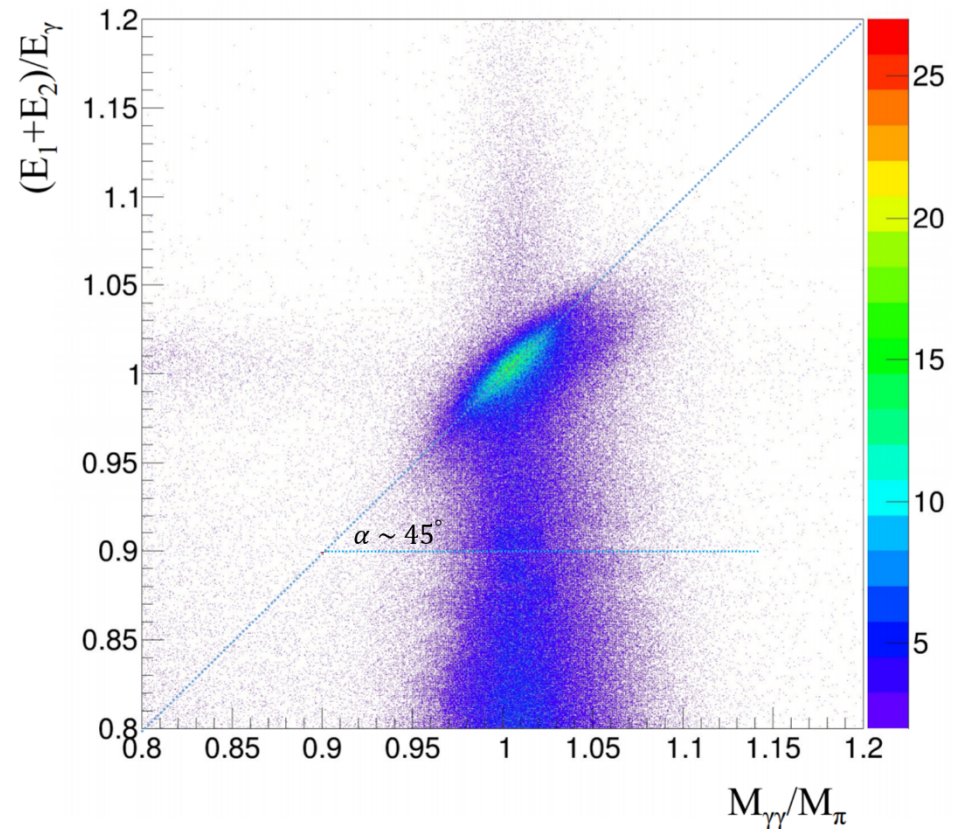
$$Elasticity_{HYCAL} = \frac{E_1 + \Delta E_1 + E_2 + \Delta E_2}{E_1 + E_2}$$

$$= 1 + \frac{\Delta E_1 + \Delta E_2}{E_1 + E_2}$$

- In case $E_1 = E_2$

$$\frac{\Delta E_1}{E_1} = \frac{\Delta E_2}{E_2} = \frac{\delta E_1 + \delta E_2}{E_1 + E_2}$$

$$\Rightarrow \alpha = 45^\circ$$

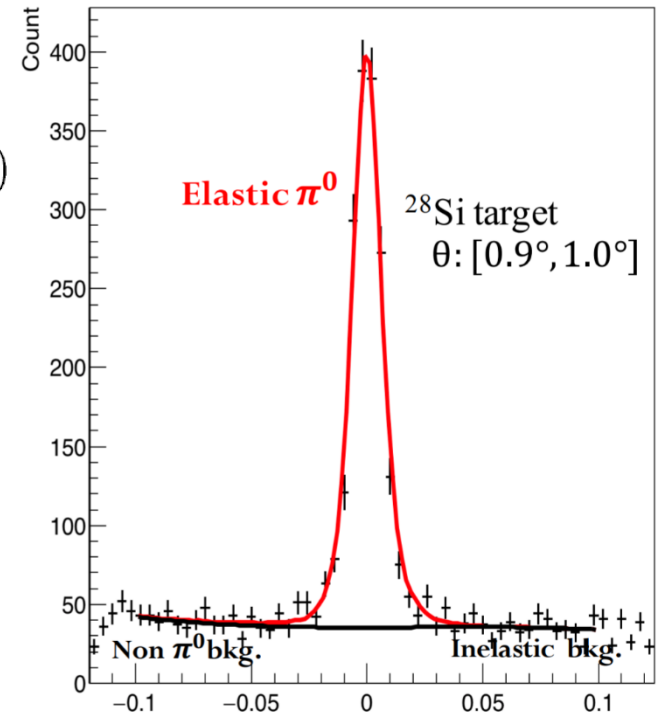


π^0 hybrid mass

- π^0 hybrid mass:

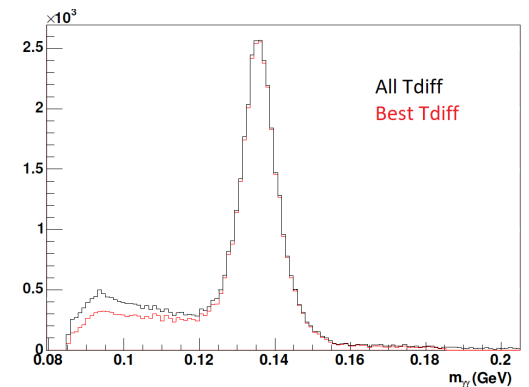
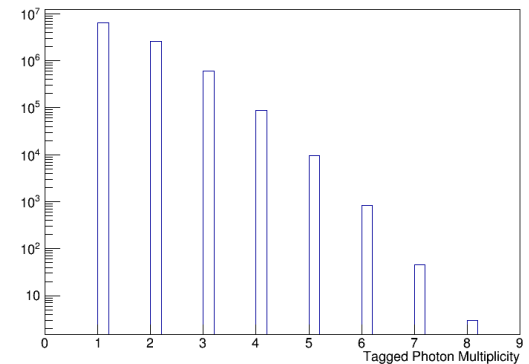
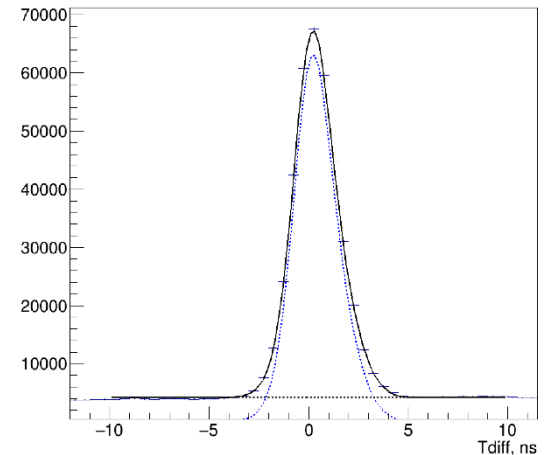
$$\text{hybridmass} = \frac{m_{\gamma\gamma}}{m_{\pi^0}} \cos(\alpha) - \text{Elasticity} \sin(\alpha)$$

- $\alpha = 45^\circ$
- By projecting π^0 events onto hybrid mass, both non π^0 and inelastic backgrounds are pushed away from the signal



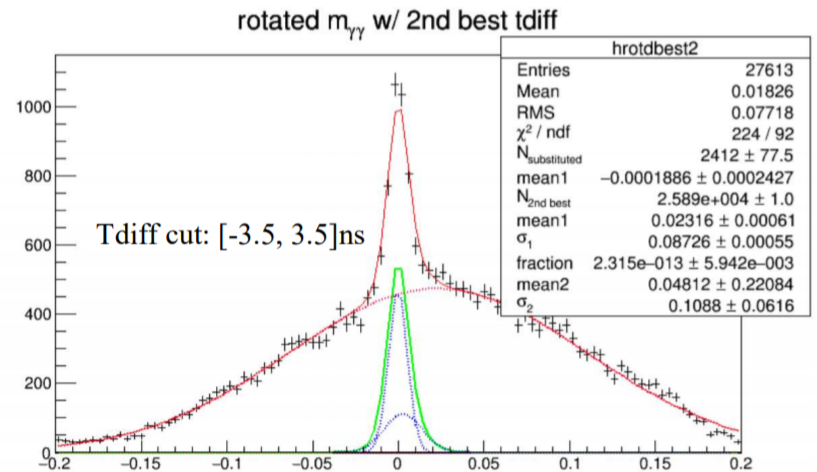
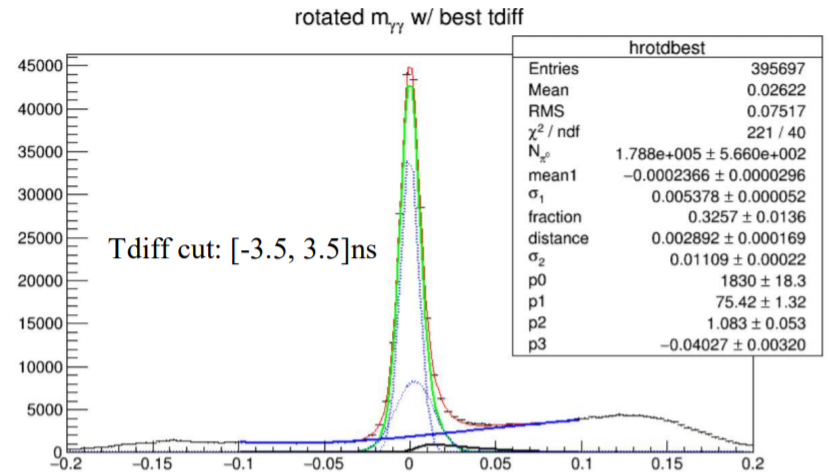
Timing Coincidence

- HyCal total sum trigger:
 - Total $E_{\text{deposit}} > 2 \text{ GeV}$
- Tagger Master OR:
 - Master OR of all T-channels
- Coincidence between these two triggers is required (tdiff cut)
- Tagged photon multiplicity
- “Best tdiff” or “all tdiff”:
 - “Best tdiff” method only accounts tagged photon with the smallest $|tdiff|$
 - Effect on signal is minimal but “best tdiff” removes more background



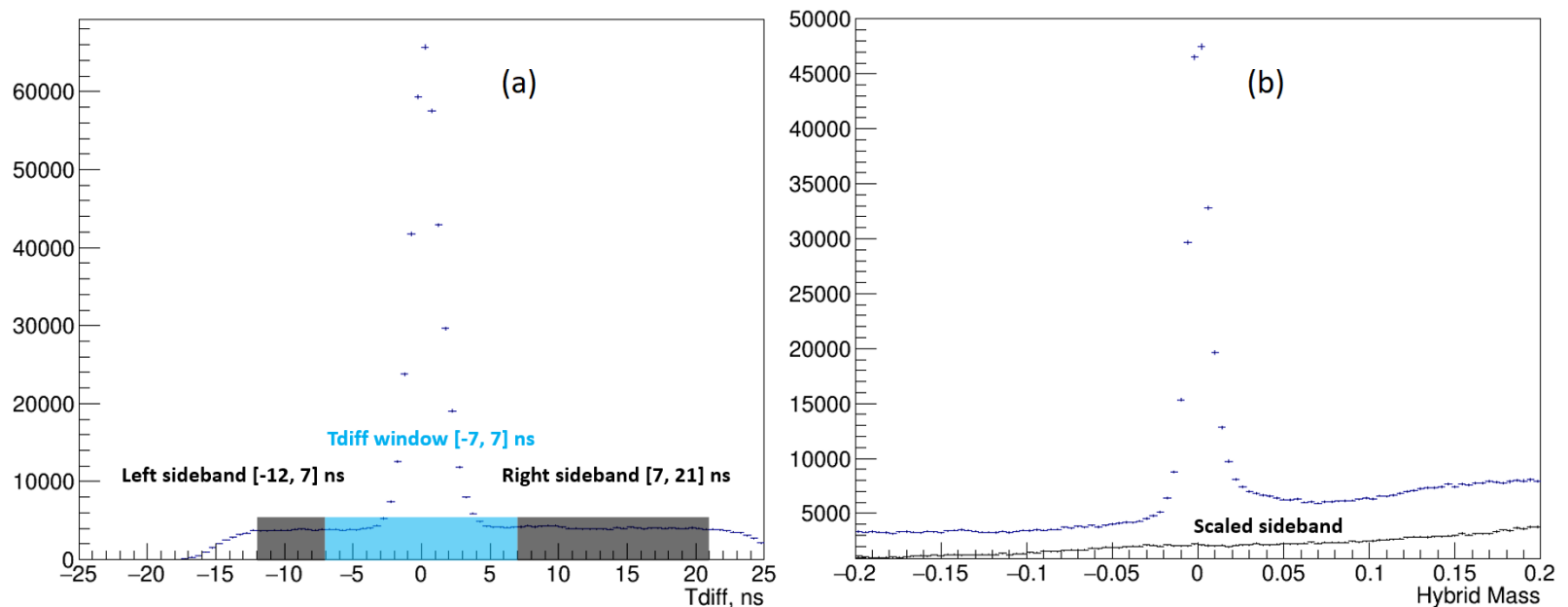
Best Tdiff and 2nd Best Tdiff

- Tagged photons selected by mistake using “best tdiff”:
 - actual incident photon has bigger tdiff
- 2nd “best tdiff”:
 - 1.16% of total π^0 for silicon
 - 1.45% of total π^0 for carbon
 - 3rd “best tdiff” only accounts for $\sim 100 \pi^0$ over the full acceptance and is ignored



Accidental Subtraction

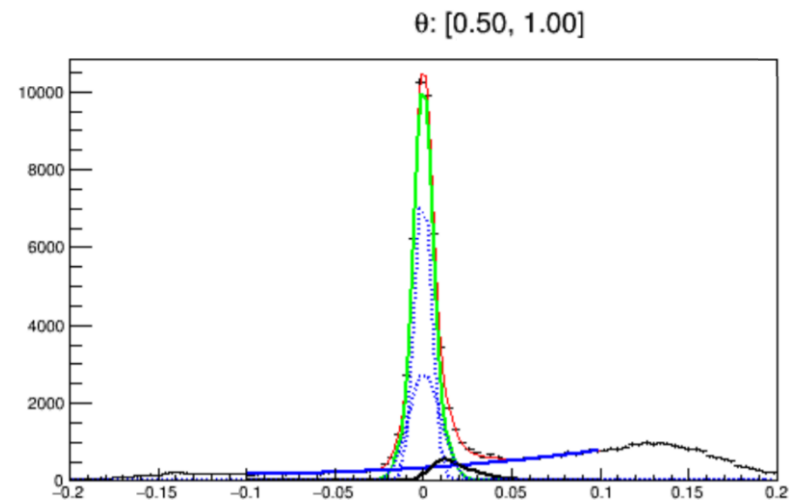
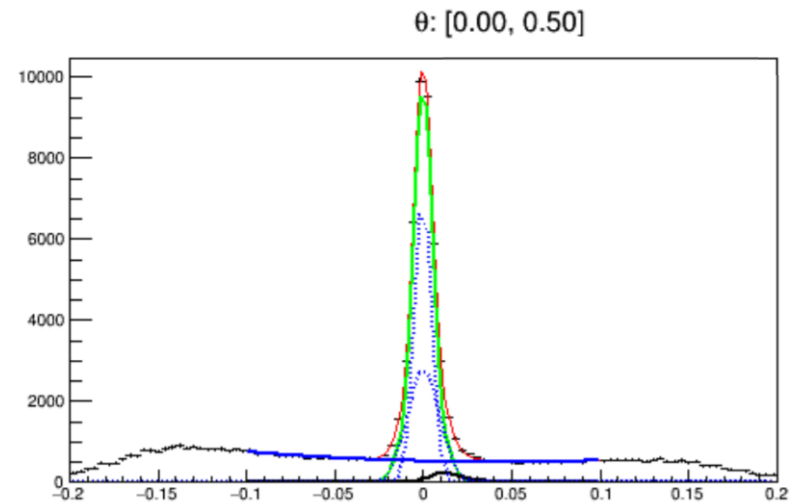
- T_{diff} cut $[-7, 7]$ ns to remove accidental backgrounds
- Assuming background structure stays the signal under the signal and in the sidebands
- Use $[-12, 7]$ and $[7, 21]$ ns sidebands to further remove accidentals



ω background

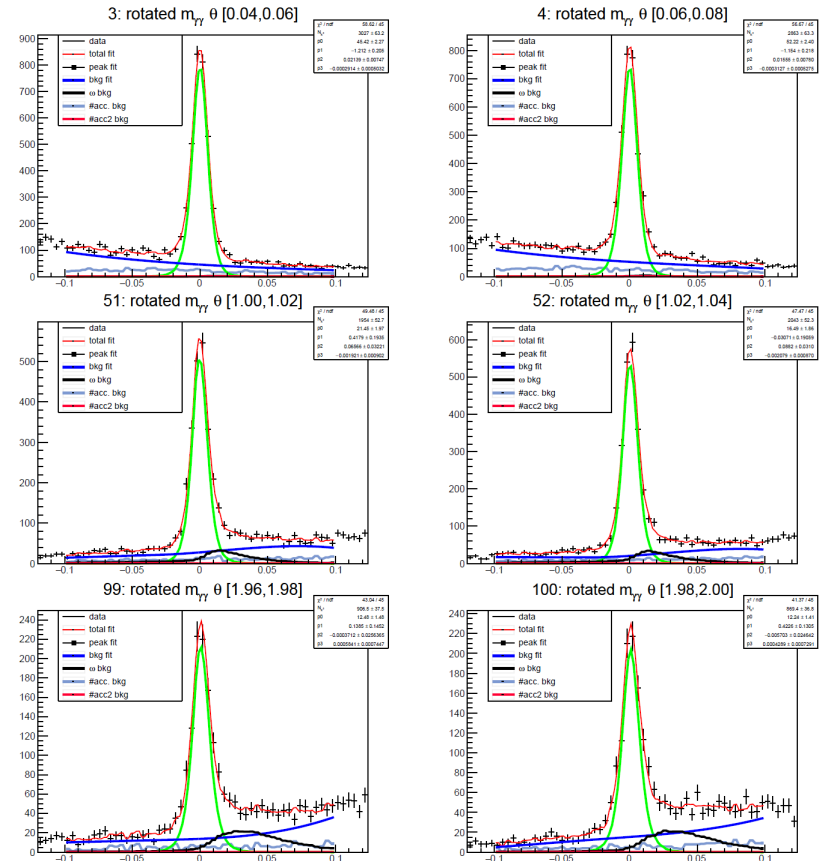
$\omega \rightarrow \pi^0 \gamma$:

- π^0 may carry most of the energy
- Gives rise to significant background off hybrid mass peak
- Simulated by Monte Carlo w/ very large statistics
- Analyzed same as experimental data and obtain hybrid mass



Hybrid mass fitting

- Hybrid mass is binned by 0.02° ($0 \sim 2.5^\circ$)
- Hybrid mass is fitted to extract π^0 yield
- Signal shape from Monte Carlo
- Background component:
 - Accidental sidebands
 - ω backgrounds (M.C.)
 - Polynomial



Fits to π^0 Yields

$$\frac{dN_{\pi^0}}{d\theta_{rec}}(\theta_{rec}, \mathfrak{P}) = N_{\gamma} \times t \times \epsilon \times \sum_{E\text{-channel } i, \theta} \frac{d\bar{\sigma}}{d\theta}(\theta, \mathfrak{P}, i) \times \omega_{flux}(i) \times M(i, \theta, \theta_{rec})$$

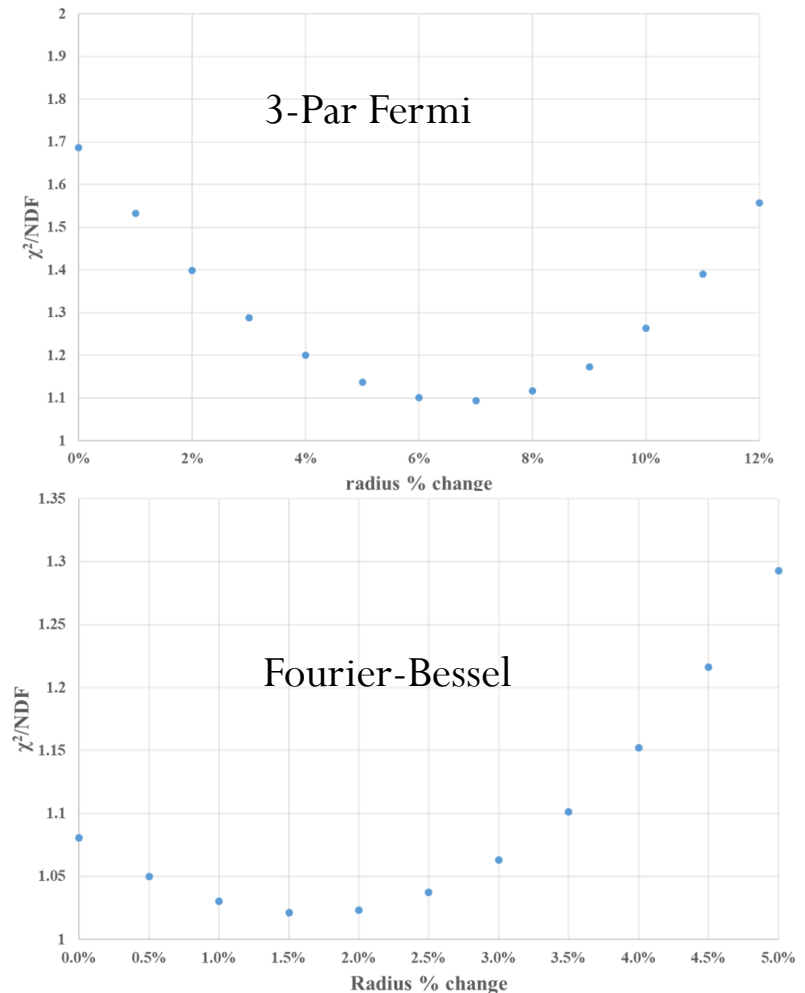
- θ_{rec} : reconstructed π^0 production angle
- θ : actual π^0 production angle
- N_{γ} : total number of tagged photons
- t : target thickness
- $\epsilon = \epsilon_1 \epsilon_2 \dots$: other constant factors
- $\frac{d\bar{\sigma}}{d\theta}(\theta, \mathfrak{P}, i)$: average π^0 production differential cross section for i^{th} E channel; \mathfrak{P} are the fitting parameters ($\Gamma(\pi \rightarrow \gamma\gamma)$, C_1 , C_2 and ϕ)
- $\omega_{flux}(i)$: fraction of tagged photon flux for i^{th} E channel
- $M(i, \theta, \theta_{rec})$: acceptance and angular resolution matrix

Constant Factors

Parameter or correction	Target [Sample]	Value	Error
Flux, [$\times 10^{12}$]	Si (all runs)	5.5420	~0.8%
	Si (reduced set)	5.2821	
	C ¹²	2.41782	
Number of Atoms per square unit, [barn ⁻¹]	Si	0.049735	0.35%
	C ¹²	0.177009	0.02%
BR($\pi^0 \rightarrow \gamma\gamma$)	All	0.98823	0.00034
γ -beam absorption in target	Si	0.9605	0.0008
	C ¹²	0.97008	0.0006
$\pi^0 \rightarrow \gamma\gamma$ decay products absorption in target	C ¹² vs Si ratio	1.018	N/A
Signal fraction out of $ Tdif < 7.0$ ns cut	Si (all runs)	0.0014	0.0003
	Si (reduced set)	0.0020	0.0003
	C ¹²	0.0068	0.0017
Best-in-time correction within $ Tdif < 3.5$ ns window	Si (reduced set)	0.0116	0.0014
	C ¹²	0.0145	0.0017
Events with HyCal ADC error	applied to flux	0.002...0.014	<0.001
HyCal energy response function	All	0.9956	0.0044

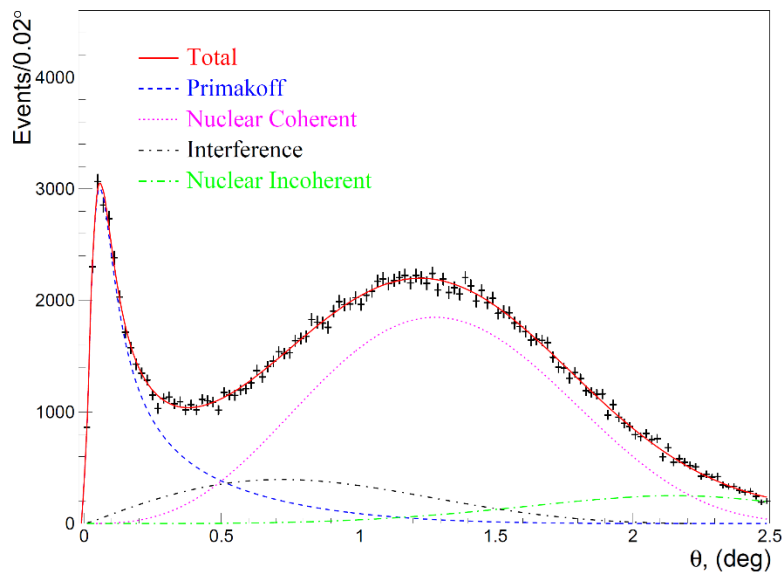
Nuclear density models

- Different nuclear density models:
 - 3-parameter Fermi
 - Fourier-Bessel
- Form factors calculated with both models
- Silicon radius increased in fit:
 - 6% in 3-par Fermi
 - 2% in Fourier-Bessel
- Best fits achieved using Fourier-Bessel density model for both silicon and carbon targets

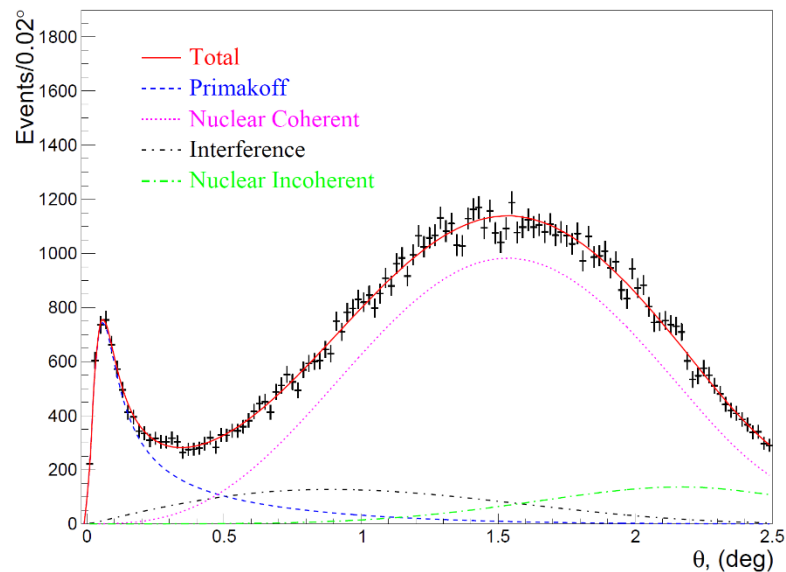


Fits to π^0 Yields

Silicon Target



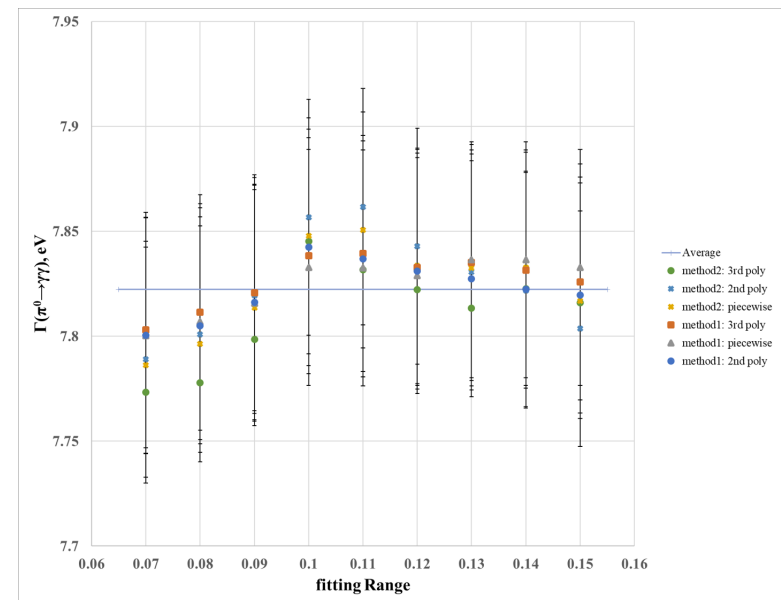
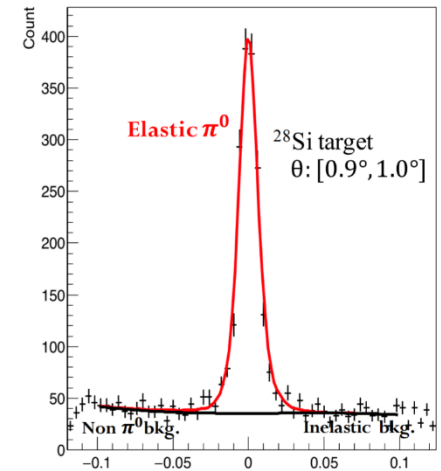
Carbon Target



Target	χ^2/NDF	$\Gamma_{\gamma\gamma}$ (eV)
Silicon	1.02	7.83 ± 0.06
^{12}C	1.11	7.78 ± 0.12

Systematic Uncertainties Due to Yield Extraction

- Signal background separated by fitting hybrid mass
- Three different background function:
 - 3rd order polynomial
 - 2nd order polynomial
 - Piecewise polynomial (2nd order + 3rd order)
- Two methods to count N_{π^0} :
 - Count N_{π^0} under fitted signal directly
 - Total N_{π^0} - fitted bkg.
- 0.77% uncertainty for silicon
- 1.0% uncertainty for carbon



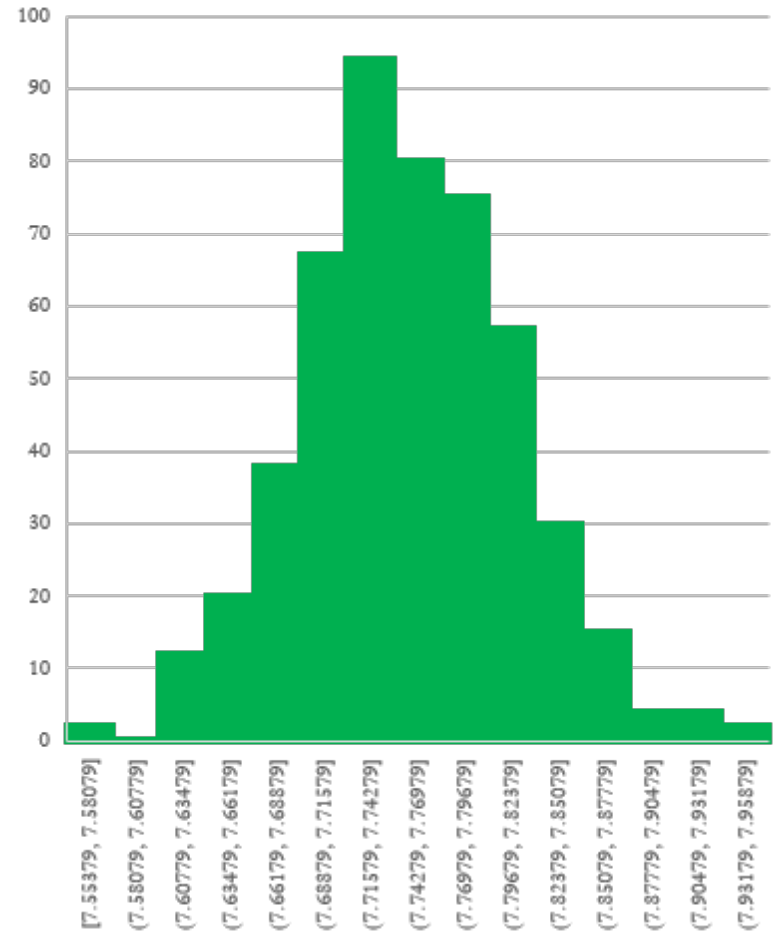
Systematic Uncertainties

- Realistic π^0 decay in M.C.
- Add bkg from data
- Extract M.C. yields and fitted
- 500 dataset, each w/ same statistics as experiment data
- Conclusion:

Preset $\Gamma(\pi^0 \rightarrow \gamma\gamma)$: 7.70 eV

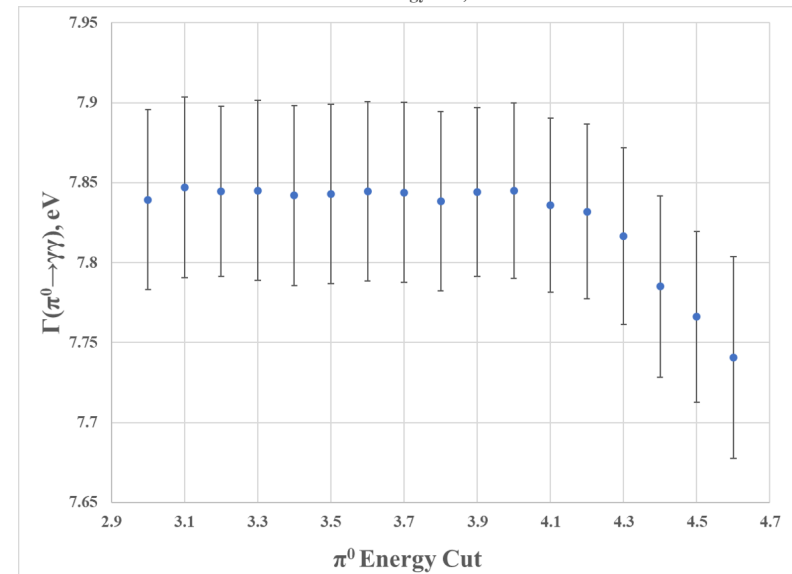
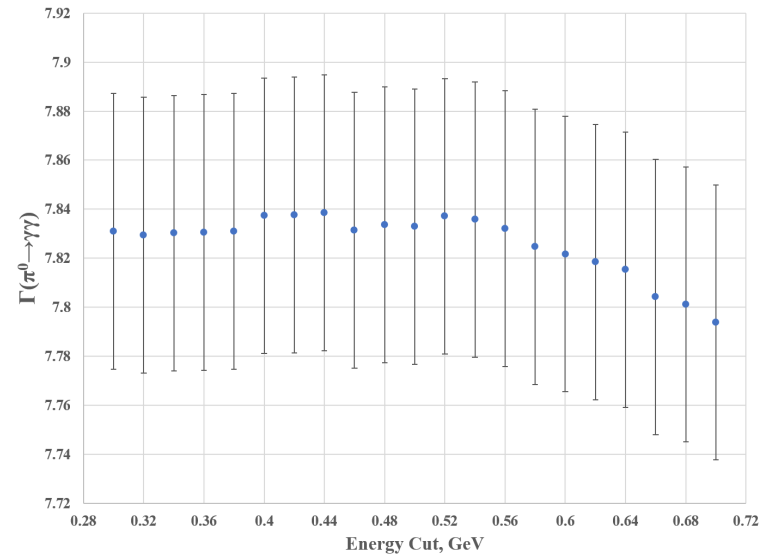
Average fitted $\Gamma(\pi^0 \rightarrow \gamma\gamma)$: 7.75 eV

0.65% uncertainty



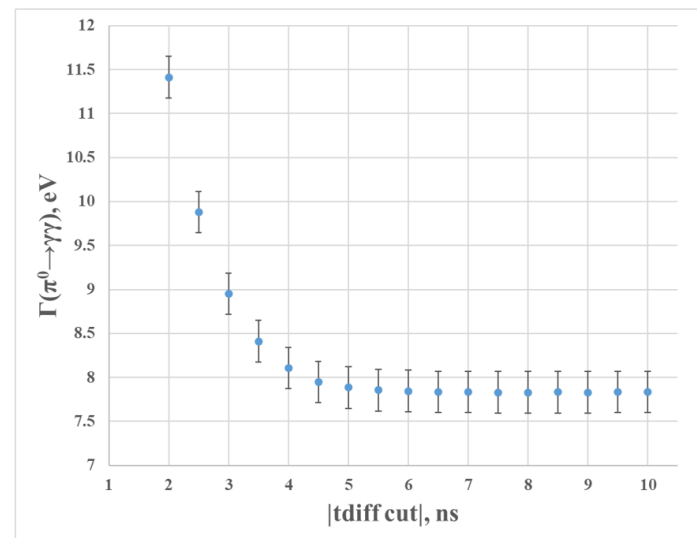
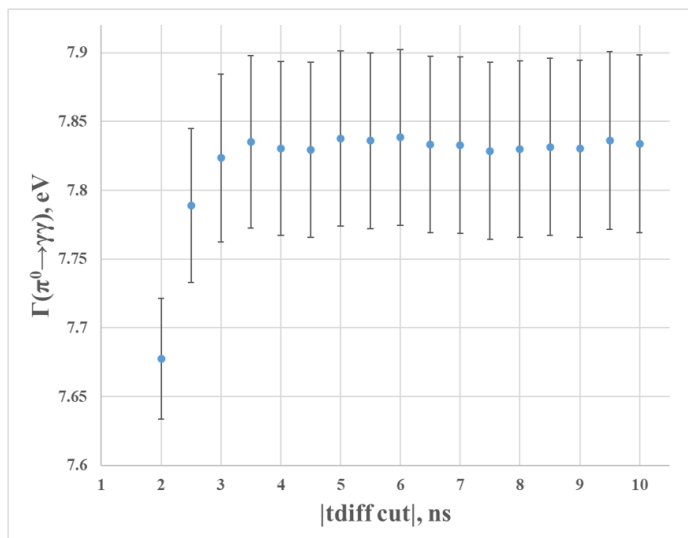
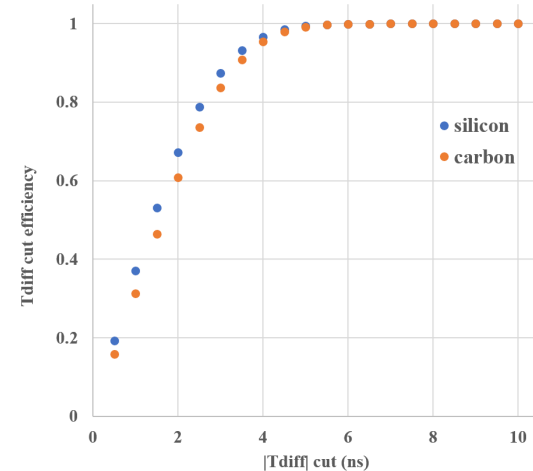
Systematic Uncertainties

- Single γ energy cut:
 - 0.5 GeV cut
 - Decay width plateaus until 0.56 GeV cut
 - 0.05% uncertainty
- π^0 energy cut (two γ):
 - 3.5 GeV cut
 - Decay width plateaus until 4.1 GeV cut
 - 0.05% uncertainty



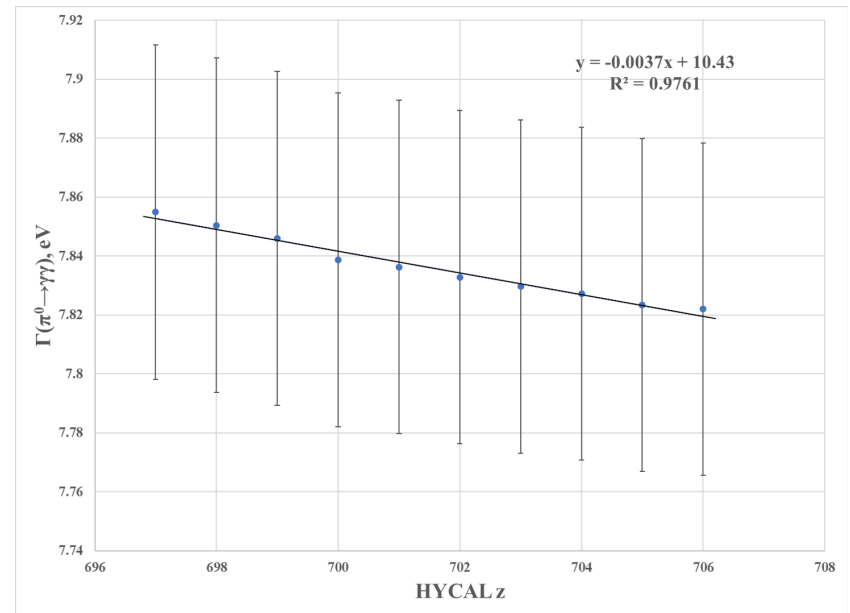
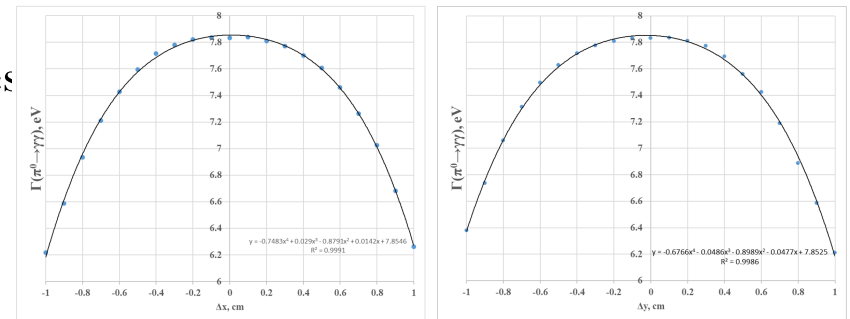
Systematic Uncertainties

- Tdiff cut: [-7, 7] ns
- Calculated Tdiff cut efficiency
- Decay width plateaus if $|Tdiff| > 5ns$
- 0.025% uncertainty



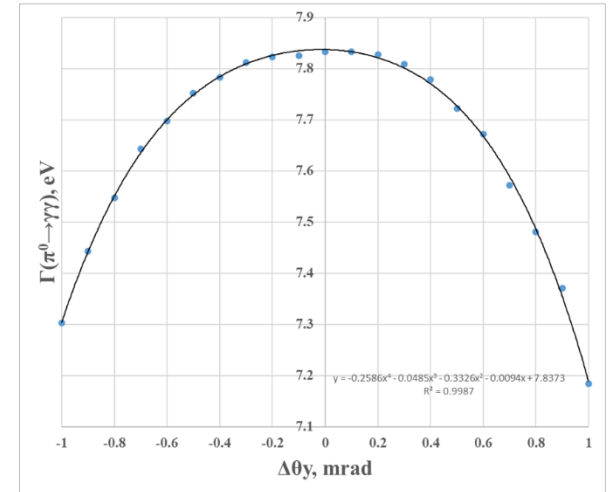
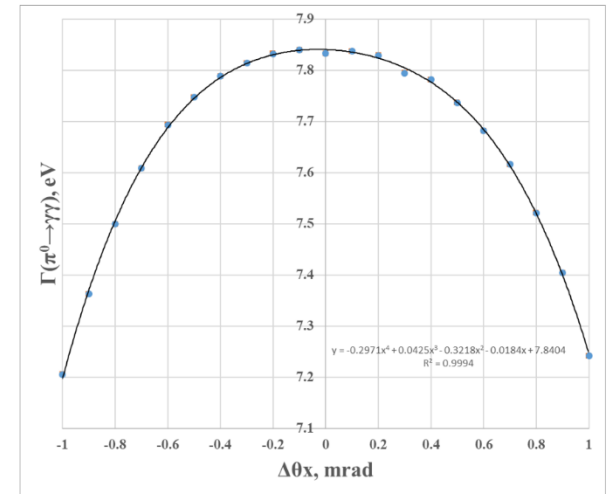
Systematic Uncertainties

- HyCal position from survey is a reference to determine the coordinates of HyCal clusters
 - π^0 production angle
 - HyCal acceptance
- HyCal coordinates misalignment
 - x-y coordinates shifted by up to 1 cm to check effect on the decay width
 - HyCal z is shifted from 697 to 706 cm (702 cm from survey)
- 0.31% uncertainty due to HyCal x-y
- 0.047% uncertainty due to HyCal z



Systematic Uncertainties

- Beam energy:
 - Worst uncertainty 0.13%
 - Translate to 0.32% change in decay width
- Beam width:
 - Affect angular resolution
 - 0.2% uncertainty
- Beam direction:
 - Projection of beam angle to θ_x, θ_y
 - Apply $\Delta\theta_x, \Delta\theta_y$ (± 1 mrad)
 - 0.05% uncertainty due to θ_x
 - 0.09% uncertainty due to θ_y

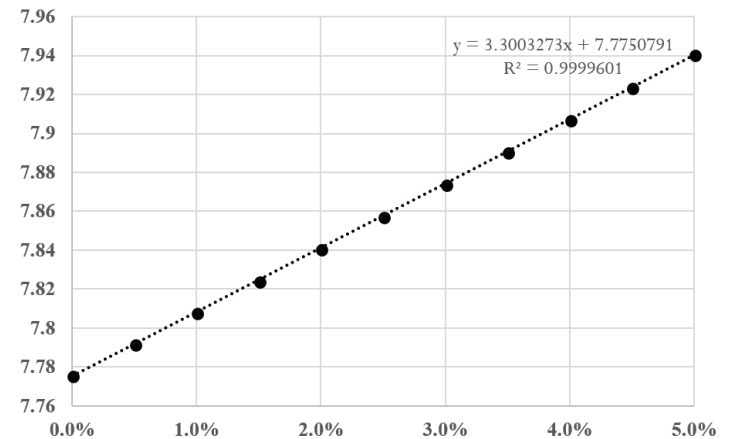
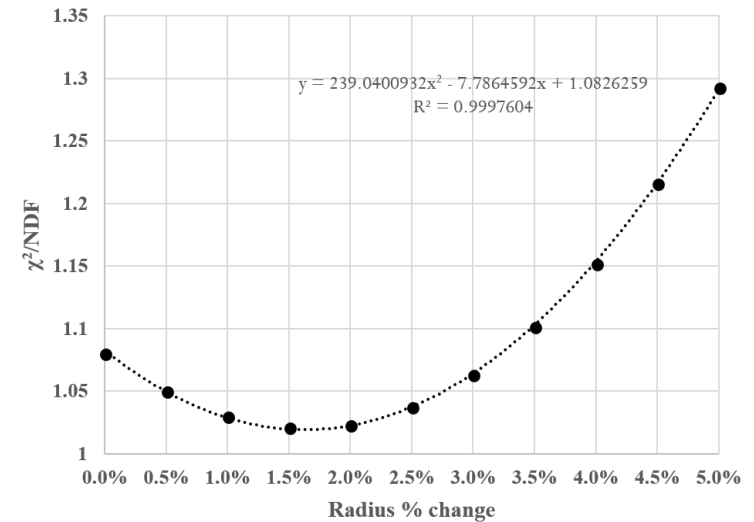


Systematic Uncertainties

- Vary radius in Fermi-Bessel model
- Syst. Uncertainties based on radius change:

$$\Delta\chi^2=1 \Rightarrow \Delta R \text{ 1.0\% } \sim \text{2.2\%}$$

\Rightarrow 0.25% uncertainty



Experimental Uncertainties

Item		Silicon (%)	Carbon (%)	Common	Ref.
Stat. Uncertainties		0.77	1.54		
Branching Ratio				0.03	[7]
Photon Beam Flux				0.83	[25, 26]
Target Measurement		0.35	0.02		[24]
Target Absorption		0.2	0.2		
Trigger Efficiency				0.1	
HYCAL Acceptance				0.31	
HYCAL Energy Response Function				0.45	[41]
Beam Parameters				0.35	
Yield Extraction	Single γ Energy Cut			0.04	
	π^0 Energy Cut			0.06	
	Tdiff Cut			0.03	
	Best Tdiff Selection	0.1	0.2		
	Signal Background Separation	0.76	1.0		
	π^0 yield binning	0.05	0.1		
	realistic M.C.	0.65	0.65		
	Total (Yield Extraction)	1.01	1.23		
ω background		0.14	0.16		
Model Errors (theory)				0.39	
Total (syst.)		1.58	1.69		

Total Uncertainties: Si 1.76%, ^{12}C 2.29%

Result

- π^0 decay width from two targets:

Target	$\Gamma_{\gamma\gamma}$ (eV)
Silicon	$7.83 \pm 0.06(stat.) \pm 0.12(syst.)$
^{12}C	$7.78 \pm 0.12(stat.) \pm 0.13(syst.)$

- Average π^0 decay width:

$$\Gamma(\pi^0 \rightarrow \gamma\gamma) = 7.82 \pm 0.05 \pm 0.12 \text{ eV} \quad (\pm 1.8\% \text{ total})$$

π^0 Yields w/ Extended Angles

