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Form Factors for the Photoproduction of Pseudoscalar Mesons

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Abstract

The form factors defined by Morpurgo are rewritten for evaluation on a computer. Graphs of the form factors are given for a convenient range of parameters, and the program used to do the calculations is described.

We have recently determined the decay rate of the π^o and the η^o mesons into photon pairs by the Primakoff effect.[1] The experiment consisted of measuring the near forward photoproduction cross section for these mesons on complex nuclear targets. The experimental cross sections were then fitted with theoretical expressions calculated from the sum of an amplitude for production in the Coulomb field and an amplitude for production in the nuclear field. The partial decay rate into photon pairs is proportional to the square of the Coulomb amplitude. A discussion of the proper procedure for carrying through this analysis have been given by Morpurgo.[2] The amplitude for a complex nuclear target is obtained by summing the amplitude from the individual nucleons within the nucleus. The resulting expression may be written as a product of the nucleon amplitude times an appropriate form factor. In this note the form factors given by Morpurgo are recast for numerical evaluation on a computer. Some of their properties are discussed. Since these form factors are functions of many parameters, it is not practical to tabulate their numerical values. One will always rely on a computer for accurate evaluation. However, graphs of the form factors are given for typical values of the parameters. These should be useful for estimating their effects.

Definition of the Form Factors

The differential cross section for spin 0 meson photoproduction from a nucleus of Z protons and A-Z neutrons may be written as

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} \sum_{\substack{\text{photon} \\ \text{polar.}}} \left| T_C + T_N \right|^2 \qquad (1)$$

where T_C is the amplitude for production in the Coulomb field and T_N, the amplitude for production in the nuclear field. To calculate T_N one begins with the nucleon amplitude, $\mathcal{T}_N^{\tau_3}$

$$\mathcal{T}_N^{\tau_3} = (2\pi\rho_f)^{1/2} \int \frac{d^3r}{V} \Big\{ \big((\vec{\mathcal{E}} \cdot \vec{\sigma} + L) + (\vec{\mathcal{M}} \cdot \vec{\sigma} + N)\tau_3 \big) \phi_{\vec{q}}^{(-)*}(\vec{r}) \Big\} e^{i\vec{k}\cdot\vec{r}} \qquad (2)$$

Here and throughout the paper $\vec{k}$ is the photon momentum and $\vec{q}$ is the meson momentum. The meson energy, $\omega = \sqrt{q^2+m^2}$, is taken to be the same as that of the photon (Heavy nucleus approximation). The quantity $\rho_f = (kq/(2\pi)^3)$ is the density of final states factor. τ_3 refers to the nucleon isospin state (+1 for the proton, -1 for the neutron). The factor V representing the volume of the nucleon hadronic matter is introduced for dimensional convenience. The operators $\vec{\mathcal{E}}$ and $\vec{\mathcal{M}}$ induce spin flip transitions, while L and N induce spin-non-flip transitions.

Since we will be interested in those transitions which leave the nucleus unchanged, we drop the terms with $\vec{\mathcal{E}}$ and $\vec{\mathcal{M}}$. For pseudoscalar mesons and for a small angular range near the forward direction, the operators L and N can be shown to be of the form

$$L\varphi_{\vec{q}}^{(-)}(\vec{r}) = i\,L_o\,(\hat{e}\times\hat{k})\cdot\vec{\nabla}\varphi_{\vec{q}}^{(-)}(\vec{r})$$

$$N\varphi_{\vec{q}}^{(-)}(\vec{r}) = i\,N_o(\hat{e}\times\hat{k})\cdot\vec{\nabla}\varphi_{\vec{q}}^{(-)}(\vec{r}) \tag{3}$$

where L_o and N_o are angle independent complex constants. The range of integration in (2) is supposed to cover the region of the nucleon matter. Using (3) and dropping the spin flip terms leaves

$$\mathcal{T}_N^{\tau_3} = i(2\pi\rho_f)^{1/2}(L_o+N_o\tau_3)(\hat{e}\times\hat{k})\cdot\int\frac{d^3r}{V}\,e^{i\vec{k}\cdot\vec{r}}\,\vec{\nabla}\varphi_{\vec{q}}^{(-)*}(\vec{r}) \tag{4}$$

If the Born approximation is used for $\varphi_{\vec{q}}^{(-)}(\vec{r})$, we can evaluate the gradient and (4) is reduced to an elementary integral over the volume of the nucleon matter.

$$\mathcal{T}_N^{\tau_3} = (2\pi\rho_f)^{1/2}(L_o+N_o\tau_3)(\hat{e}\times\hat{k})\cdot\vec{q}\int\frac{d^3r}{V}\,e^{i(\vec{k}-\vec{q})\cdot\vec{r}} \tag{5}$$

We naively picture the nucleon as an interacting sphere of radius, a, and obtain

$$\mathcal{T}_N^{\tau_3} = (2\pi\rho_f)^{1/2}(L_o+N_o\tau_3)(\hat{e}\times\hat{k})\cdot\hat{q}\left\{\frac{3}{(a\Delta)^3}(\sin a\Delta - a\Delta\cos a\Delta)\right\} \tag{6}$$

where we have set

$$\vec{\Delta} = \vec{k} - \vec{q}, \qquad \Delta \equiv |\vec{\Delta}| \tag{7}$$

The factor in curly brackets goes to unity for $a\Delta \to 0$, and hence we can define the small angle, spin-non-flip, point nucleon photoproduction amplitude as

$$\mathcal{A}_N^{p,n} = (2\pi\rho_f)^{1/2}(L_o \pm N_o)(\hat{e}\times\hat{k})\cdot\vec{q} \tag{8}$$

For unpolarized photons the "spin-non-flip cross section" is

$$\left.\frac{d\sigma}{d\Omega}\right)_N^{p,n} = \pi\rho_f|L_o \pm N_o|^2\, q^2 \sin^2\theta_q \tag{9}$$

The upper sign in (8) and (9) refers to protons and the lower to neutrons. It should be kept in mind that L_o and N_o are independent of angle but there is no restriction on their energy dependence. Hence the only important content in Eq. (9) is the $\sin(\theta_q)$ factor (the origin of this factor is angular momentum conservation).

For a nucleon located at position $\vec{r}'$, the phase shifted amplitude is given by

$$\mathcal{T}_N^{\tau_3}(\vec{r}') = i(2\pi\rho_f)^{1/2}\int\frac{d^3r}{V}\left\{(L_o+N_o\tau_3)(\hat{e}\times\hat{k})\cdot \vec{\nabla}\varphi_{\vec{q}}^{(-)}(\vec{r}+\vec{r}')\right\} e^{i\vec{k}\cdot(\vec{r}+\vec{r}')} \tag{10}$$

where the range of the $\vec{r}$ integration is the same as in (2) (i.e. for our spherical model, the integration is over a sphere of radius a about the origin). If the meson wave function is approximated as a plane wave, we obtain

$$\mathcal{T}_N^{\tau_3}(\vec{r}\,') = e^{i\vec{\Delta}\cdot\vec{r}\,'}\mathcal{T}_N^{\tau_3}(0) \tag{11}$$

One may view the coherent nuclear amplitude, T_N, as a superposition of the individual nucleon amplitudes.

$$T_N = \sum_{i=1}^{A} \mathcal{T}_N^{\tau_{3i}}(\vec{r}_i) \tag{12}$$

The sum in (12) can be written as an integral. Assuming the density distribution of neutrons and protons within the nucleus to be identical, we denote this distribution by $\rho(\vec{r})$ with the normalization understood to be

$$\int d^3r\, \rho(\vec{r}) = 1 \tag{13}$$

For a plane wave final meson, Eq. (12) may be written as

$$T_N = \frac{AL_o + (2Z-A)N_o}{L_o + N_o}\mathcal{T}_N^{p}\int d^3r\, \rho(\vec{r})e^{i\vec{\Delta}\cdot\vec{r}} \tag{14}$$

where $\mathcal{T}_N^{p}$ is given by (6) with $\tau_3 = +1$. Recognizing that (8) contains as much generality as (6) we can write this as

$$T_N = (2\pi\rho_f)^{1/2}(AL_o + (2Z-A)N_o)(\hat{e} \times \hat{k})\cdot\vec{q}\int d^3r\,\rho(r)\,e^{i\vec{\Delta}\cdot\vec{r}} \qquad (15)$$

It should be emphasized that (14) was obtained from Born approximation in which the final meson wave function is assumed to be unchanged by the nuclear potential. As a consequence, we have lost the effects arising from absorption and rescattering of the meson leaving the nucleus. To include these, it is convenient to begin from (4) and treat the nucleus as a continuum rather than a collection of neutrons and protons. Accordingly we have

$$T_N = i(2\pi\rho_f)^{1/2}(AL_o + (2Z-A)N_o)\int d^3r\,\rho(\vec{r})$$
$$\left\{(\hat{e} \times \hat{k})\cdot\vec{\nabla}\phi^{(-)*}_{\vec{q}}(\vec{r})\right\}e^{i\vec{k}\cdot\vec{r}} \qquad (16)$$

with the region of integration being the entire nucleus. Denoting the total meson-nucleon cross section by σ, and the ratio of the real to imaginary part of the forward scattering amplitude by λ, the eikonal model approximation for $\phi^{(-)}_{\vec{q}}(\vec{r})$ is

$$\phi^{(-)}_{\vec{q}}(\vec{r}) = e^{i\vec{q}\cdot\vec{r} - \frac{\sigma' A}{2}\int_0^\infty \rho(\vec{r} + \hat{q}\xi)d\xi} \qquad (17)$$

with

$$\sigma' = \sigma(1 + i\lambda) \qquad (18)$$

Morpurgo defines the nuclear production form factor as

$$F_N(k,\theta_q) = \frac{i}{(\hat{e}\times\hat{k})\cdot\vec{q}}\int d^3r\,\rho(\vec{r})\left\{(\hat{e}\times\hat{k})\cdot\vec{\nabla}\phi^{(-)*}_{\vec{q}}(\vec{r})\right\}e^{i\vec{k}\cdot\vec{r}} \qquad (19)$$

and thereby writes T_N as given in Eq. (16) in the form analogous to the point nucleon amplitude.

$$T_N = (2\pi\rho_f)^{1/2}(AL_o + (2Z-A)N_o)(\hat{e}\times\hat{k})\cdot\vec{q}\;F_N(k,\theta) \qquad (20)$$

The Coulomb amplitude, T_C, is given by

$$T_C = \left(\frac{16}{4\pi}\Gamma_{2\gamma}\right)^{1/2}(\beta/m)^{3/2}\left(\frac{k^2}{q}\right)\int d^3r\left\{\phi^{(-)*}_{\vec{q}}(\vec{r})\right.$$
$$\left.(\hat{e}\times\hat{k})\cdot\left(\frac{1}{i}\vec{\nabla}V(\vec{r})\right)e^{i\vec{k}\cdot\vec{r}}\right\} \qquad (21)$$

In Eq. (21), $\Gamma_{2\gamma}$ is the partial width into photon pairs, β is the meson velocity, and $V(\vec{r})$ is the electrostatic potential. The amplitude is proportional to the electric field strength. To have an expression for $\mathcal{T}_C$ analogous to that for $\mathcal{T}_N$ we can evaluate (21) for the potential of a point charge.

$$V(\vec{r}) = \frac{e}{4\pi|\vec{r}|} \qquad (22)$$

There is no absorption of the final meson for a point charge, and since the meson is neutral, there is no distortion of the outgoing wave. Consequently, we use a plane wave for the meson and are able to evaluate the integral in (21) explicitly. The resulting expression for $\mathcal{T}_C$ is

$$\mathcal{T}_C = (16\alpha\Gamma_{2\gamma})^{1/2}(\beta/m)^{3/2}((\hat{e}\times\hat{k})\cdot\hat{q})\frac{k^2}{\Delta^2} \tag{23}$$

The amplitude for a finite charge density corresponding to a nucleus of charge Z may be obtained either by superimposing phase shifted point charge amplitudes with density $Z\rho(\vec{r})$, or by integrating (21) directly with the potential given by

$$V(\vec{r}) = \frac{Ze}{4\pi}\int d^3r' \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|} \tag{24}$$

The result is

$$T_C = (16\alpha Z^2\Gamma_{2\gamma})^{1/2}(\beta/m)^{3/2}((\hat{e}\times\hat{k})\cdot\hat{q})$$

$$\frac{k^2}{\Delta^2}\int d^3r\ \rho(\vec{r})e^{i\vec{\Delta}\cdot\vec{r}} \tag{25}$$

The inclusion of absorption for the Coulomb amplitude is more delicate because only a fraction of the production takes place within (or in front of) the nucleus. The amplitude is calculated from (21) using the eikonal approximation for $\varphi_{\vec{q}}^{(-)}(\vec{r})$ (i.e. Eq. (17)). The Coulomb form factor as defined by Morpurgo is

$$F_C(k,\theta_q) = \frac{\Delta^2}{(\hat{e}\times\hat{k})\cdot\vec{q}}\int d^3r\left\{\varphi_{\vec{q}}^{(-)*}(\vec{r})(\hat{e}\times\hat{k})\cdot(\frac{1}{Zei}\vec{\nabla}V(\vec{r}))\right.$$

$$\left. e^{i\vec{k}\cdot\vec{r}}\right\} \tag{26}$$

so that

$$T_C = \left(\frac{16}{137} Z^2 \Gamma_{2\gamma}\right)^{1/2} (\beta/m)^{3/2} ((\hat{e} \times \hat{k}) \cdot \hat{q}) \frac{k^2}{\Delta^2} F_C(k, \theta_q) \qquad (27)$$

Simplication of the Nuclear Form Factor

We are required to calculate the nuclear form factor (Eq. (19)) using (17) for the meson wave function. Although it is possible to handle the 4-fold integration on a computer (the 3 dimensional volume integral and the 1 dimensional integral in the exponent of $\phi_{\vec{q}}^{(-)*}(\vec{r})$), there is a considerable saving if one can take advantage of the inherent azimuthal symmetry of the problem. This may be done with some approximation. We first do a partial integration to move the gradient off of $\phi_{\vec{q}}^{(-)*}(\vec{r})$.

$$F_N(k,\theta) = \frac{(\hat{e} \times \hat{k}) \cdot}{i(\hat{e} \times \hat{k}) \cdot \vec{q}} \int d^3r \; \phi_{\vec{q}}^{(-)*}(\vec{r}) \vec{\nabla} (\rho(\vec{r}) e^{i(\vec{k} \cdot \vec{r})}) \qquad (28)$$

The gradient of the photon wave function is orthogonal to $(\hat{e} \times \hat{k})$, and we are left with

$$F_N(k,\theta) = \frac{(\hat{e} \times \hat{k}) \cdot}{i(\hat{e} \times \hat{k}) \cdot \vec{q}} \int d^3r \left\{ e^{+i\vec{\Delta} \cdot \vec{r} - \frac{\sigma'^* A}{2} \int_0^\infty d\xi \rho(\vec{r} + \hat{q}\xi)} \vec{\nabla} \rho(\vec{r}) \right\} \qquad (29)$$

We approximate the absorption term in the exponent by performing the integration along the direction $\hat{k}$ rather than $\hat{q}$.

For small production angles the difference is not important in the cases we've examined. To save some writing, we define

$$\Sigma(\vec{r}) = \frac{\sigma'^{*}A}{2} \int_0^\infty d\xi \; \rho(\vec{r} + \hat{k}\xi) \tag{30}$$

The two benefits of this approximation are

1) The absorption integral in the exponent becomes independent of $\vec{q}$ and hence need only be evaluated once for each value of $\vec{r}$.

2) The azimuthal dependence of the entire exponent is simplified so that the integration over azimuth reduces to a Bessel function.

To evaluate the integral over azimuth, we choose an explicit coordinate system, $\hat{k} = \hat{z}$, $\hat{e} = \hat{x}$, and use the assumed spherical symmetry

$$\vec{\nabla}\rho(\vec{r}) = \left(\frac{x}{r}\,\hat{x} + \frac{y}{r}\,\hat{y} + \frac{z}{r}\,\hat{z}\right)\frac{\partial\rho}{\partial r} \tag{31}$$

The $(\hat{e} \times \hat{k})$ factor picks out the y component leaving

$$F_N(k,\theta) = \frac{1}{(\hat{e} \times \hat{k})\cdot\vec{q}} \int d^3r \; e^{i\vec{\Delta}\cdot\vec{r} - \Sigma(\vec{r})} \; \frac{y}{r}\,\frac{\partial\rho}{\partial r} \tag{32}$$

Using cylindrical coordinates b, z, Φ for the position vector we have

$$F_N(k,\theta_q) = \frac{1}{(\hat{e}\times\hat{k})\cdot\vec{q}}\int_{-\infty}^{\infty} dz \int_0^{\infty} b\, db \int_0^{2\pi} d\phi \left\{ \frac{b\sin\phi}{r}\left(\frac{\partial}{\partial r}\rho(b,z)\right)\right.$$

$$\left. \exp(i(\Delta_z z - q_x b\cos\phi - q_y b\sin\phi) - \Sigma(b,z))\right\} \tag{33}$$

$$F_N(k,\theta_q) = \frac{2\pi}{q\sin\theta_q}\int_0^{\infty} b^2\, db\, J_1(qb\sin\theta_q)\int_{-\infty}^{\infty}\frac{dz}{\sqrt{b^2+z^2}}$$

$$\left\{ e^{i(k-q\cos\theta_q)z}\left(-\frac{\partial\rho(b,z)}{\partial r}\right)e^{-\Sigma(b,z)}\right\} \tag{34}$$

where $J_1(u)$ is the Bessel function of order 1. To proceed further with the remaining integrals requires a specific function for $\rho(\vec{r})$.

Simplification of the Coulomb Form Factor

Since absorption takes place only in nuclear matter, whereas the Coulomb production occurs over a larger range of space, it is advantageous to rewrite Eq. (26) as the sum of two terms, one with absorption and one without[3],

$$F_C(k,\theta_q) = F_1(k,\theta_q) + F_2(k,\theta_q) \tag{35}$$

$$F_1(k,\theta_q) = \frac{i\Delta^2}{(\hat{e}\times\hat{k})\cdot\vec{q}}\int d^3r\; e^{i\vec{\Delta}\cdot\vec{r}}(\hat{e}\times\hat{k})\cdot\vec{\varepsilon}'(\vec{r}) \tag{36}$$

$$F_2(k,\theta_q) = \frac{i\Delta^2}{(\hat{e}\times\hat{k})\cdot\vec{q}}\int d^3r\; e^{i\vec{\Delta}\cdot\vec{r}}(\hat{e}\times\hat{k})\cdot\vec{\varepsilon}'(\vec{r})(e^{-\Sigma(\vec{r})}-1) \tag{37}$$

Here $\vec{\varepsilon}'(\vec{r})$ is the electric field divided by the total charge, Ze. Eq. (36) for $F_1(k,\theta_q)$ can be simplified by partial integration in the same way that (25) was derived. The result is

$$F_1(k,\theta_q) = \int d^3r\ \rho(\vec{r})e^{i\vec{\Delta}\cdot\vec{r}} \tag{38}$$

The integration over polar and azimuth angles may be performed most easily by choosing $\vec{\Delta}$ along the z axis, and one obtains

$$F_1(k,\theta_q) = \frac{4\pi}{\Delta}\int_0^\infty dr\ r\rho(r)\sin(\Delta r) \tag{39}$$

To evaluate F_2 we note that the integrand vanishes except for points, $\vec{r}$, within or directly in front of the nucleus. Therefore we can limit the range of integration to a semi-infinite cylinder. The Φ integration may be carried through in the same way that was done to derive (34). The result is

$$F_2(k,\theta_q) = \frac{2\pi\Delta^2}{q\ \sin\theta_q}\int_{-\infty}^{\infty} b^2 db\ J_1(q\,b\sin\theta_q)\int_{-\infty}^{\infty}\frac{dz}{\sqrt{b^2+z^2}}$$

$$\left\{e^{i(k-q\cos\theta_q)z}\ [e^{-\Sigma(b,z)} - 1]\ |\vec{\varepsilon}'(b,z)|\right\} \tag{40}$$

Calculation of $\varepsilon'(\vec{r})$ may be performed using Gauss' law.

$$|\vec{\varepsilon}'(r)| = \frac{1}{r^2}\int_0^r r'^2\, dr'\, \rho(r') \tag{41}$$

General Features of the Form Factors

The form factors are transforms of the nuclear density function and its derivatives. As such, they provide information about the nuclear density. In this note, we assume the nuclear density function to be known and we shall try to describe some features of the form factors. To have something specific, we take the density, $\rho(\vec{r})$, as

$$\rho(r) = \frac{\rho_o}{1 + \exp(\frac{r - a}{c})} \tag{42}$$

where a is the nuclear radius at half the central density, and c is the skin thickness parameter. ρ_o is a normalization constant. Experimental values for a and c are available.[4] Within 10%, one may take $a = 1.1\ A^{1/3}$ fermi and $c = 0.55$ fermi for A greater than 10.

To discuss the form factors, we consider the independent variables to include the parameters a,c, defining the density function, the kinematic variables k, θ_q, m, the atomic weight A, and the parameters σ, λ, which describe the meson-nucleon interaction. (The atomic weight enters in two ways. First, the radius a, varies with A, but since we take the radius as an independent variable here we need not consider the atomic weight. Second, the absorption is proportional to A.)

We have been writing the form factors as functions of k

and θ_q, the energy and production angle. These, together with the meson mass are usually the most convenient parameters for experimenters. Some arithmetic simplification is obtained by reexpressing m and θ_q in terms of the longitudinal and transverse components of the momentum transfer.

$$\Delta^2 = |\vec{k} - \vec{q}|^2 = \Delta_L^2 + \Delta_T^{2} \tag{43}$$

$$\Delta_L = k - q\cos\theta_q \simeq \frac{m^2}{2k} + \frac{1}{2}k\theta_q^{\,2} \tag{44}$$

$$\Delta_T = q\sin\theta_q \simeq k\theta_q \tag{45}$$

(In the expressions on the right of (44) and (45), we neglect terms in $(m/k)^2\theta_q$.) In the absence of absorption and rescattering corrections, both the nuclear and the Coulomb form factor reduce to the Fourier transform of the matter density.

$$F_C(k,\theta_q) = F_N(k,\theta_q) = F_1(k,\theta_q) \tag{46}$$

which is a real quantity. For a thin skin, ($c \ll a$), the integral is a function of $a\Delta$ alone. In Figure 1 the absorptionless form factors are plotted vs. $a\Delta$ for beryllium, copper, and uranium. We are mainly interested in the range of values for which $|F| > 0.1$, and hence we may restrict our attention to $a\Delta < 4$. The radius, a, varies from 2.2 f = 11 GeV^{-1} in beryllium to 6.8 f = 34 GeV^{-1} in uranium, giving upper limits

to the interesting momentum transfer range of 0.35 and 0.12 GeV, respectively. Absorption corrections are more important than those for rescattering. The variation of the form factors with the meson-nucleon cross section is illustrated in Figure 2, where Re F_C and Re F_N of uranium are shown for σ = 20 and 40 mb. Near the forward direction, absorption depresses Re F_C less than Re F_N. This is a reflection of the Coulomb production taking place to a large extent external to the nucleus. The relative transparency of beryllium is also illustrated.

For high energy, $k \gg m$, the longitudinal momentum transfer is much smaller than the transverse except for the most forward direction. As a consequence, we may view Δ_T as the only important kinematic variable at high energy. The real parts of F_C and F_N are plotted vs. Δ_T in Figure 3. The curves were calculated for incident photon energy of 1.0 GeV and π^o mesons. Calculations for higher energy show the form factors change by less than 0.02 as $\Delta_L \rightarrow 0$.

The imaginary part of the form factors is more difficult to describe in terms of one or two parameters. They arise from the absorption term combined with longitudinal momentum transfer. In Figures 4, the Im F_C and Im F_N are shown vs. $a\Delta_T$ for k = 1.0 GeV incident photons. At higher energies, the longitudinal momentum transfer at a given transverse momentum decreases (as

k^{-1}), so the imaginary part of the form factors is reduced by a corresponding amount. Figure 5 illustrates the variation of the imaginary part of the form factors with absorption. The most salient feature of the imaginary part is its minuteness. Finally, for semi-quantitative estimates, curves of $|F_C|^2$ and $|F_N|^2$ vs. Δ_T are provided in Figure 6.

Appendix

Computer Program for Evaluating F_C and F_N

A program has been written to calculate F_C and F_N using a Woods-Saxon density function (see Eq. (42)). The program is driven by an input file, INFORM.DRV, in which the following parameters are specified.

Z,A atomic number and atomic weight of the nucleus

a,c mean radius and skin thickness for the W-S function (fermi)

m meson mass (GeV)

k incident photon energy (GeV)

ist set to 1,2 to step in equal angle, transverse momentum intervals

smn minimum value of angle (radians) or transverse momentum (GeV)

stp step size in angle (radians) or transverse momentum (GeV)

nst number of points to calculate

sig total meson-nucleon cross section (millibarns)

lam ratio of the real to imaginary part of the forward elastic scattering amplitude

phm multiplier for calculating the coherent nuclear cross section

rt2 partial width (eV) for the two photon decay of the meson

iot output mode, 1 = line printer, 2 = binary file, 3 = both

eps accuracy parameter

The coding has not been written for high speed and hence is not suitable as an inline routine. There are two output modes and the user may request either or both. The line printer output consists of a table of the input parameters followed by the real and imaginary parts of each form factor and the invariant cross sections $d\sigma/d\Delta^2)_C$, $d\sigma/d\Delta^2)_N$. The Coulomb cross section is calculated using the value of rt2 for $\Gamma_{2\gamma}$.

$$\left.\frac{d\sigma}{d\Delta^2}\right)_C = \left(\frac{\pi}{qk}\right) \frac{8Z^2}{137} \Gamma_{2\gamma} \left(\frac{\beta}{m}\right)^3 \frac{k^4 \sin^2\theta_q}{\Delta^4} |F_C|^2$$

The calculation of the coherent nuclear production cross section is done assuming only $I = 0$ exchange (i.e. $N_o = 0$), and includes a user supplied phenomenological multiplier, phm, to set the scale.

$$\left.\frac{d\sigma}{d\Delta^2}\right)_N = (\text{phm}) \cdot \frac{16\pi A^2 q^2 \sin^2\theta_q}{M_p^4 k^2} |F_N|^2$$

Here M_p is the proton mass in GeV. Fits to data[1] suggest phm = 6.8 μb-GeV^2 for π^o and 0.9 μb-GeV^2 for η^o production. The cross sections are given in μb/GeV^2.

A binary file may be written which contains the input parameters and the form factors. This may be used together with an interpolation routine for inline calculations. The routines are written in Fortran for execution on the laboratory PDP10 computer. However, since input and output are isolated, it should be adaptable to other computers. It is also worth noting that the nuclear density and its radial derivative are calculated in separate subroutines, so that the user may reload the program with his own routines to calculate form factors for other density functions.

References

1. A. Browman et al., Phys. Rev. Lett. 32, 1067 (1974).
   A. Browman et al., Phys. Rev. Lett. 33, 1400 (1974).
2. G. Morpurgo, Nuovo Cimento 31, 569 (1964). See also G. Faldt, Nucl. Phys. 43, 591 (1972).
3. The separation of the Coulomb form factor into an external absorptionless contribution and an internal contribution, which is suggested in Morpurgo's paper, produces a significant improvement when one uses numerical methods to evaluate the integrals. I'm indebted to D. Yennie for pointing this out.
4. H. Collard and R. Hofstadter, "Nuclear Radii Determined by Electron Scattering", HEPL Report TN-67-12, September 1967, Stanford University.

Figure Captions

Form factors for the photoproduction of π^0 mesons ($m = 0.135$ GeV) from beryllium, copper, and uranium are presented in the following graphs. The radius, a, used for these nuclei was 2.2, 4.28, and 6.8 fermi, and the skin thickness was taken as 0.55 fermi for each. (The atomic weight used was 3.01, 63.54, and 238.02, respectively.) In call cases, λ, the ratio of real to imaginary part of the forward elastic scattering amplitude was set to 0.

1. The form factors with no absorption ($\sigma = 0$).
2. The real part of the form factors for uranium and beryllium illustrating the importance of absorption.
3. The real part of F_C and F_N vs. Δ_T for a total meson nucleon cross section of 30 mb.
4. The imaginary part of F_C and F_N for an incident photon energy of 1.0 GeV and $\sigma = 30$ mb.
5. The imaginary part of F_C and F_N for uranium and beryllium illustrating the dependence on the meson nucleon cross section. (Photon energy = 1 GeV).
6. $|F_C|^2$ and $|F_N|^2$ vs. Δ_T^2 for all three elements, calculated for k = 1 GeV and $\sigma = 30$ mb. For small values of Δ_T^2, these curves may be approximated by

	$\lvert F_C \rvert^2$	$\lvert F_N \rvert^2$
Beryllium	$1.e^{-72\Delta_T^2}$	$0.77e^{-60\Delta_T^2}$
Copper	$1.e^{-210\Delta_T^2}$	$0.56e^{-150\Delta_T^2}$
Uranium	$1.e^{-520\Delta_T^2}$	$0.42e^{-320\Delta_T^2}$

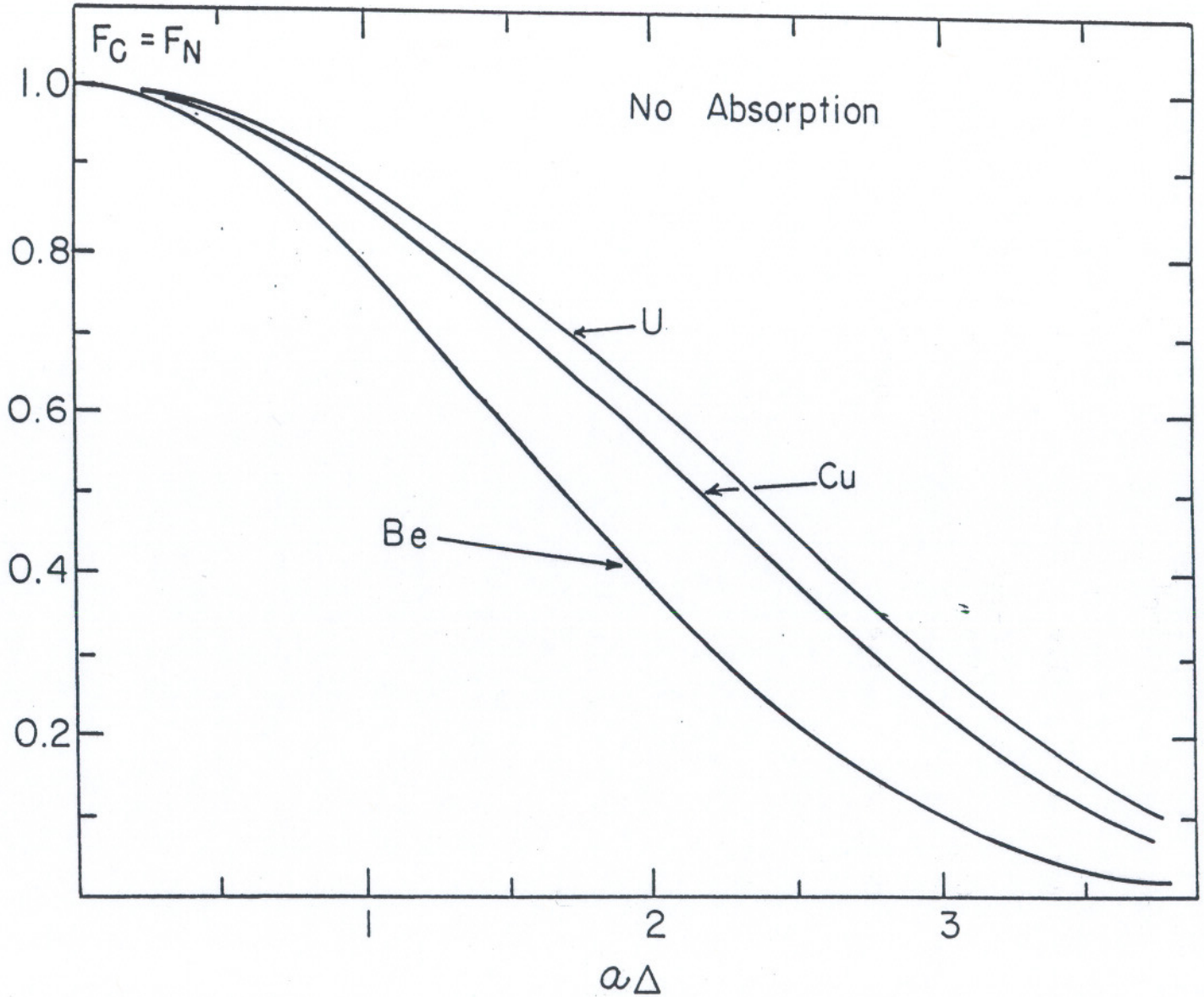

figure 1

0250275

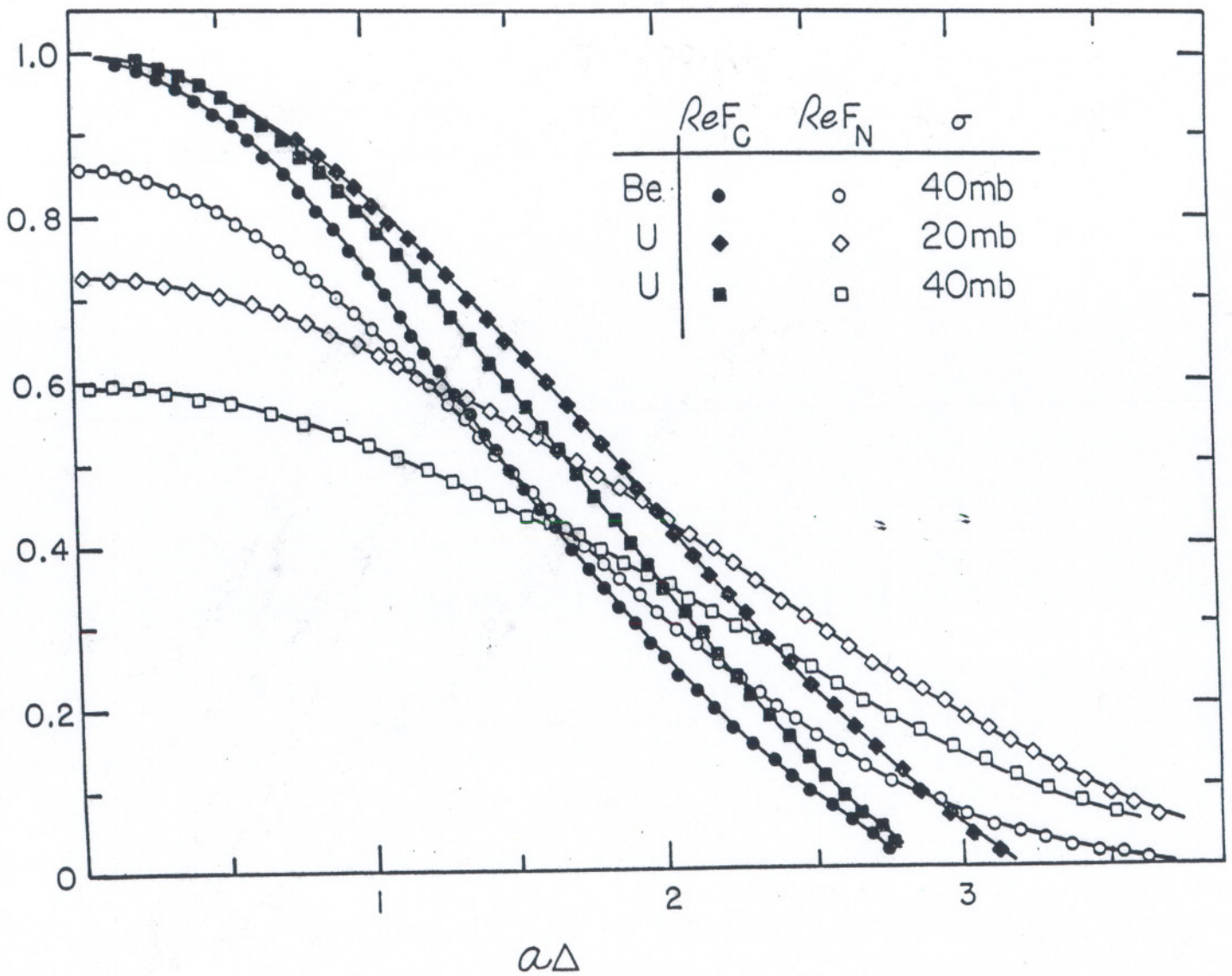

figure 2

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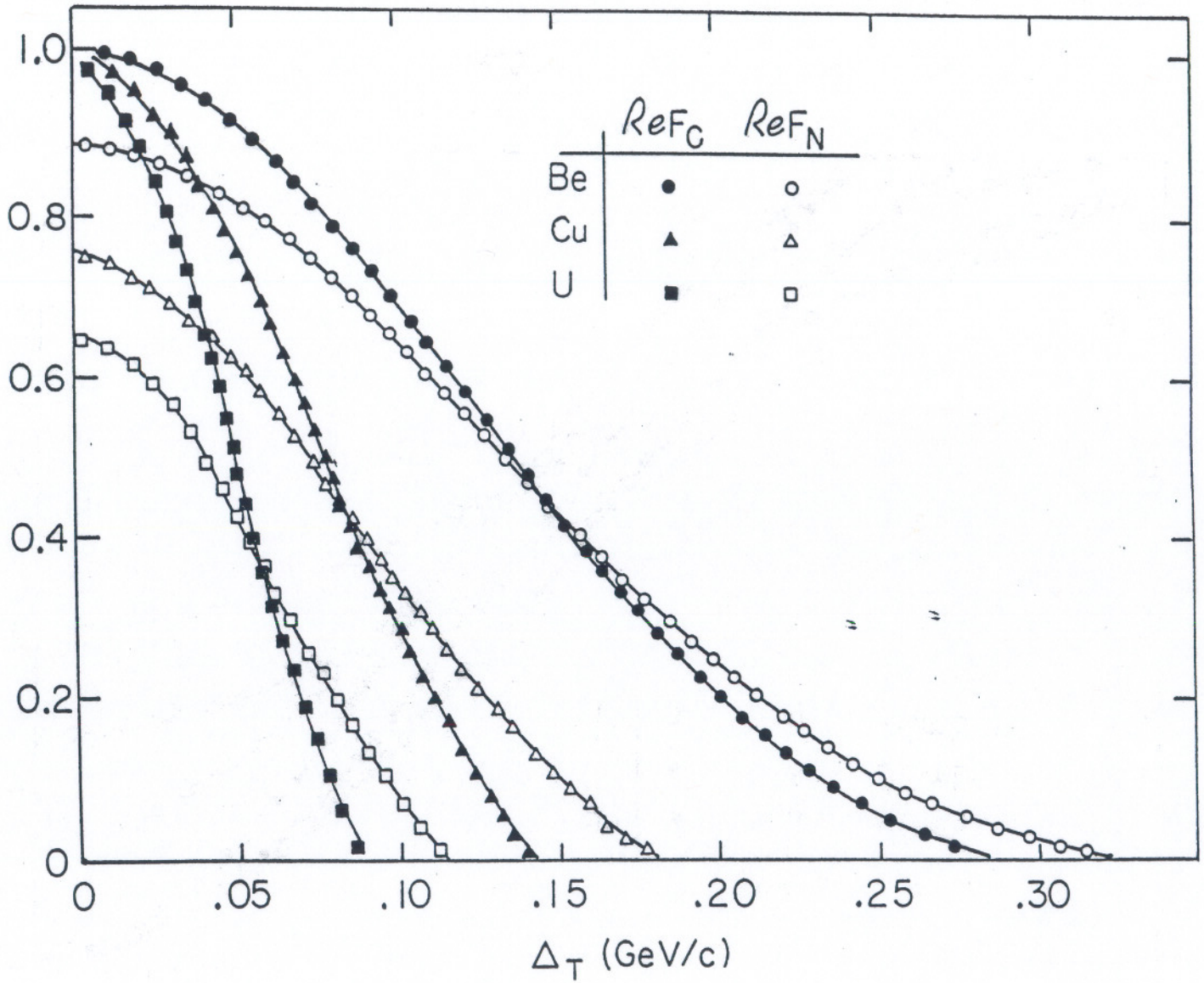

figure 3

0250275

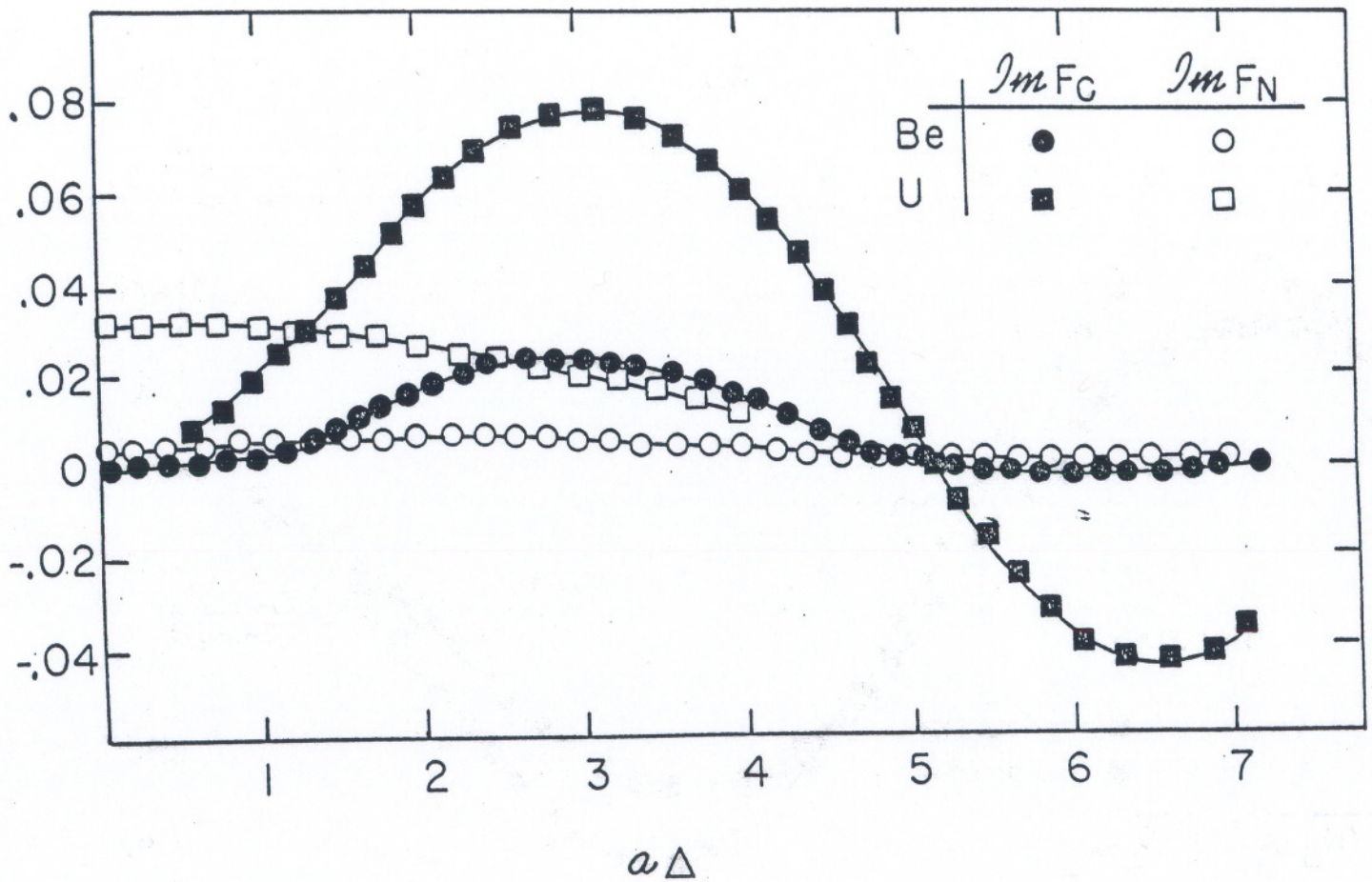

figure 4

0250275

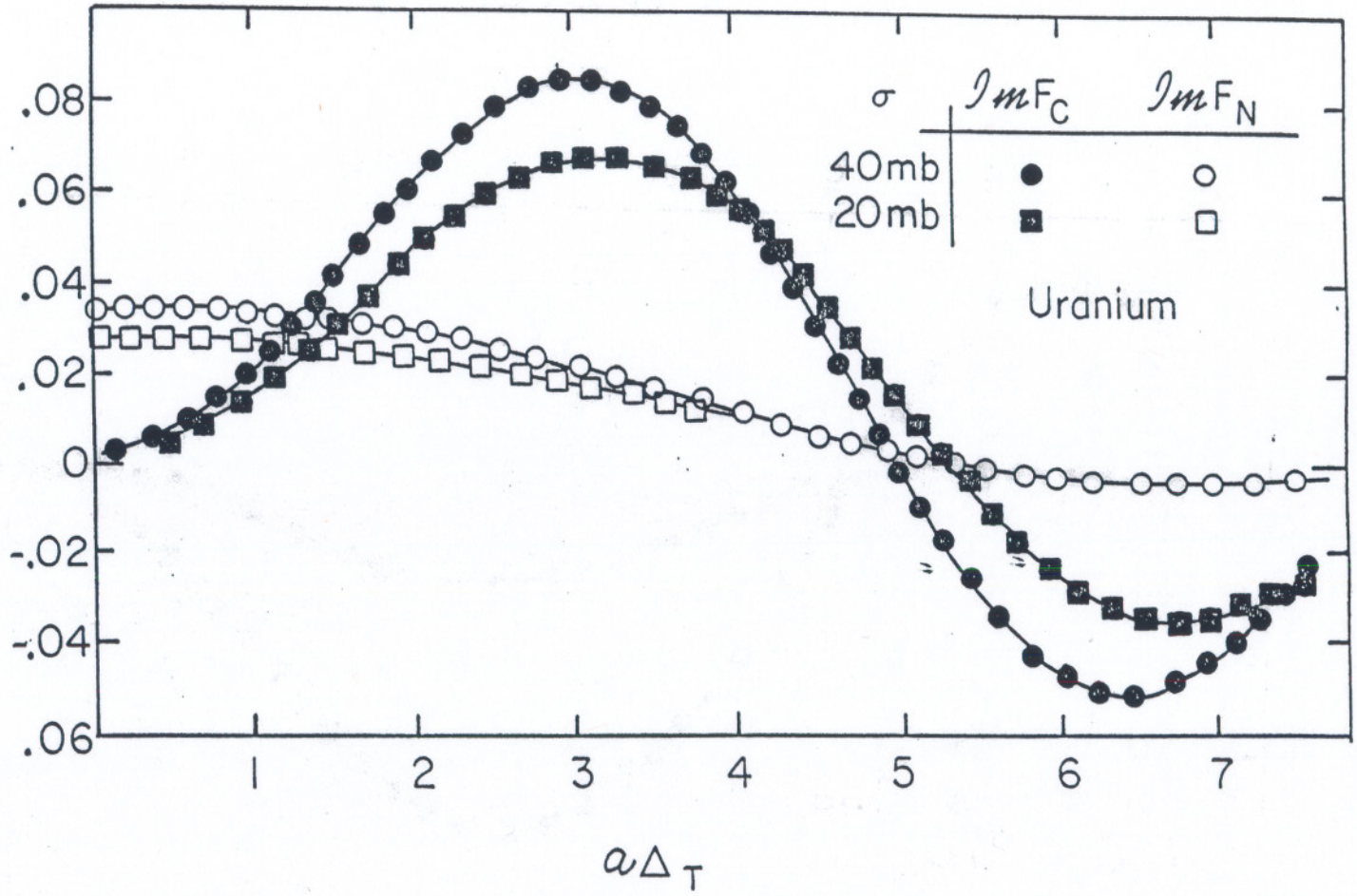

figure 5

0250275

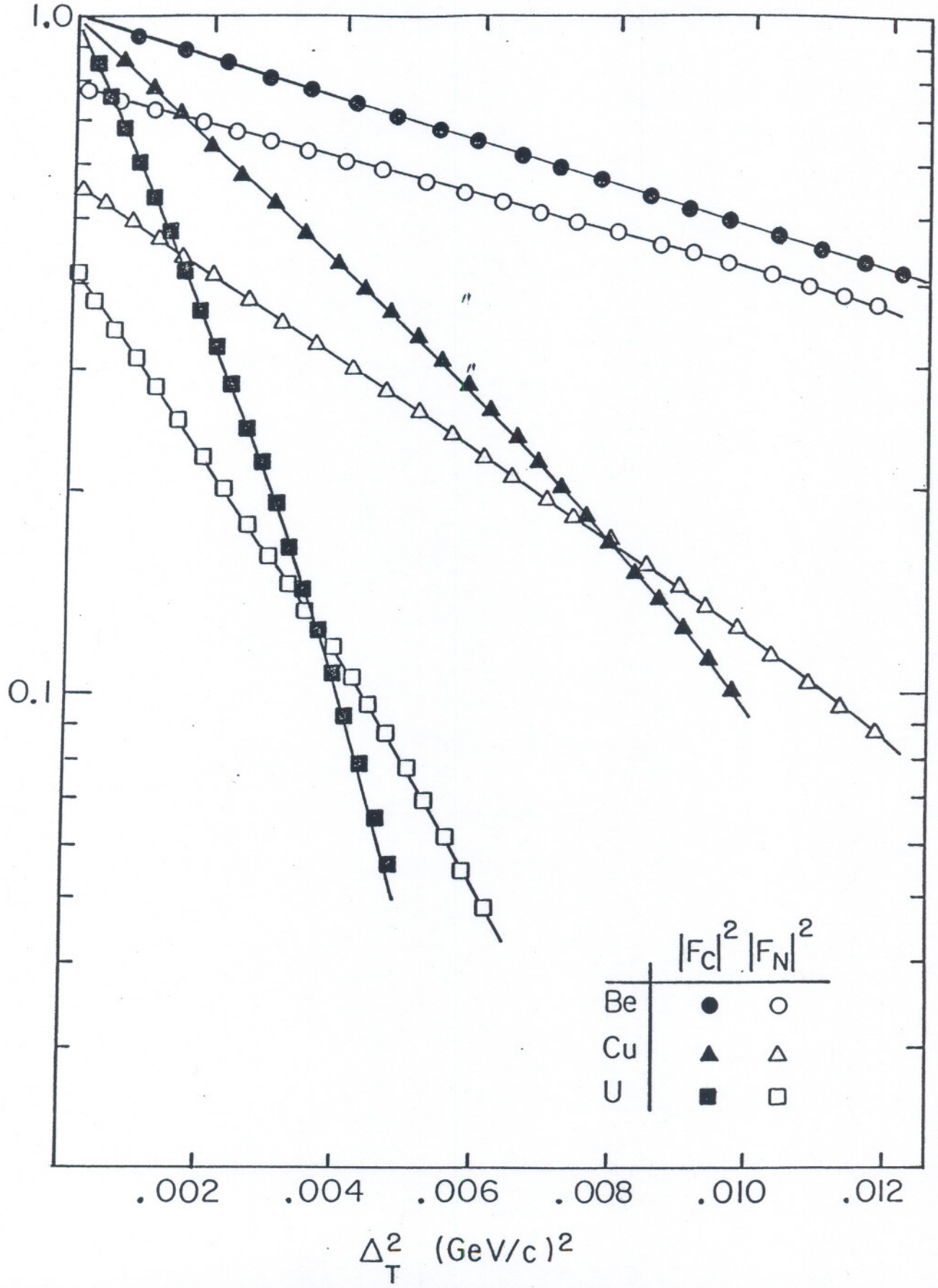

figure 6