Incoherent Photoproduction of π^0 Mesons off Nuclei
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1 Introduction

The incoherent photoproduction of pions on nuclei is a process with excitation or break up of the target nucleus:

$$\gamma + A \to \pi^0 + A' \tag{1}$$

where A' includes all possible final states of the target nucleus A except from its ground state. In high energy photoproduction experiments, where the energy resolution is not enough to distinguish the final states of residual nuclei, one can sum up over all final states. The completeness relationship (closure approximation) will provide a compact analytical expressions for the incoherent cross sections [1]. With this assumption, the incoherent π^0 photoproduction cross section can be written in the following form [2]:

$$\frac{d\sigma_{inc}}{d\Omega} = (\frac{Z}{A}\frac{d\sigma_p}{d\Omega} + \frac{N}{A}\frac{d\sigma_n}{d\Omega})N(0,\sigma)(1 - G(t)) \tag{2}$$

where the incoherent cross section is a product of three factors:
1) photoproduction of π^0 on constituent nucleons;
2) a term describing the effect of pion final state interactions in the nuclei;
3) a term restricting the pions production according to Pauli principle [3].

2 Photoproduction off Nucleon

In general the neutral pion photoproduction off nucleon should be described by four invariant amplitudes [5]. Due to angular momentum conservation these amplitudes have to be proportional to the powers of transfered momentum. As a result, in the case of a single exchange in the t-channel the cross section vanishes at small angles. The multiple exchanges (Regge cuts) lead to non-vanishing terms in the differential cross section (term F_3 in (3)). From the other hand the π^0 production off the proton at small angles should include the Coulomb part [4]. Based on these,we adopt the following parameterization of the differential cross section in accordance with [6]:

$$\frac{d\sigma}{dt} = \frac{1}{128\pi m^2}[\mid (t+t_{min})^{0.5}(F_1(0)e^{bt/2} + \frac{g}{t}) \mid^2 +2 \mid F_3(0) \mid^2] \tag{3}$$

where

$$t = -4kpsin^2(\theta/2) - t_{min}; \quad t_{min} = \frac{m_\pi^4}{4k^2} \tag{4}$$

The exponential term in the strong part was added to provide the fall of the cross section at large transfer momenta. The constant in the Coulomb part is determined by the pion radiative decay rate $\Gamma \approx 8eV$:

$$g = \frac{32\pi m\sqrt{\alpha\Gamma}}{m_\pi^{3/2}} \approx 0.014 \tag{5}$$

There are several other approaches to parametrize the elementary differential cross section, described in literature [7, 9, 8]. Parametrization (3) used in present work is based on minimum free parameters and as seen from Fig.1, 2 and 3 fits well the existing experimental data [4] at relatively small momentum transfer.
The differential cross sections of π^0 production on proton had been measured for a wide range of energies (from 4 to 18 GeV) at DESY [4] and SLAC [10]. Energy dependence of the experimental data is shown to exhibit a scaling behavior [10]:

$$\frac{k^2}{\pi}\frac{d\sigma}{dt} = f(t) \tag{6}$$

Since

$$\frac{d\sigma}{d\Omega} = \frac{k^2}{\pi}\frac{d\sigma}{dt} \tag{7}$$

then the dependence of $\frac{d\sigma}{d\Omega}$ on momentum transfer should be independent from the initial energies in this region. For illustration in the Fig.4 we show the results of our fits for $\frac{d\sigma}{d\Omega}$ as a function of momentum transfered for different energies.

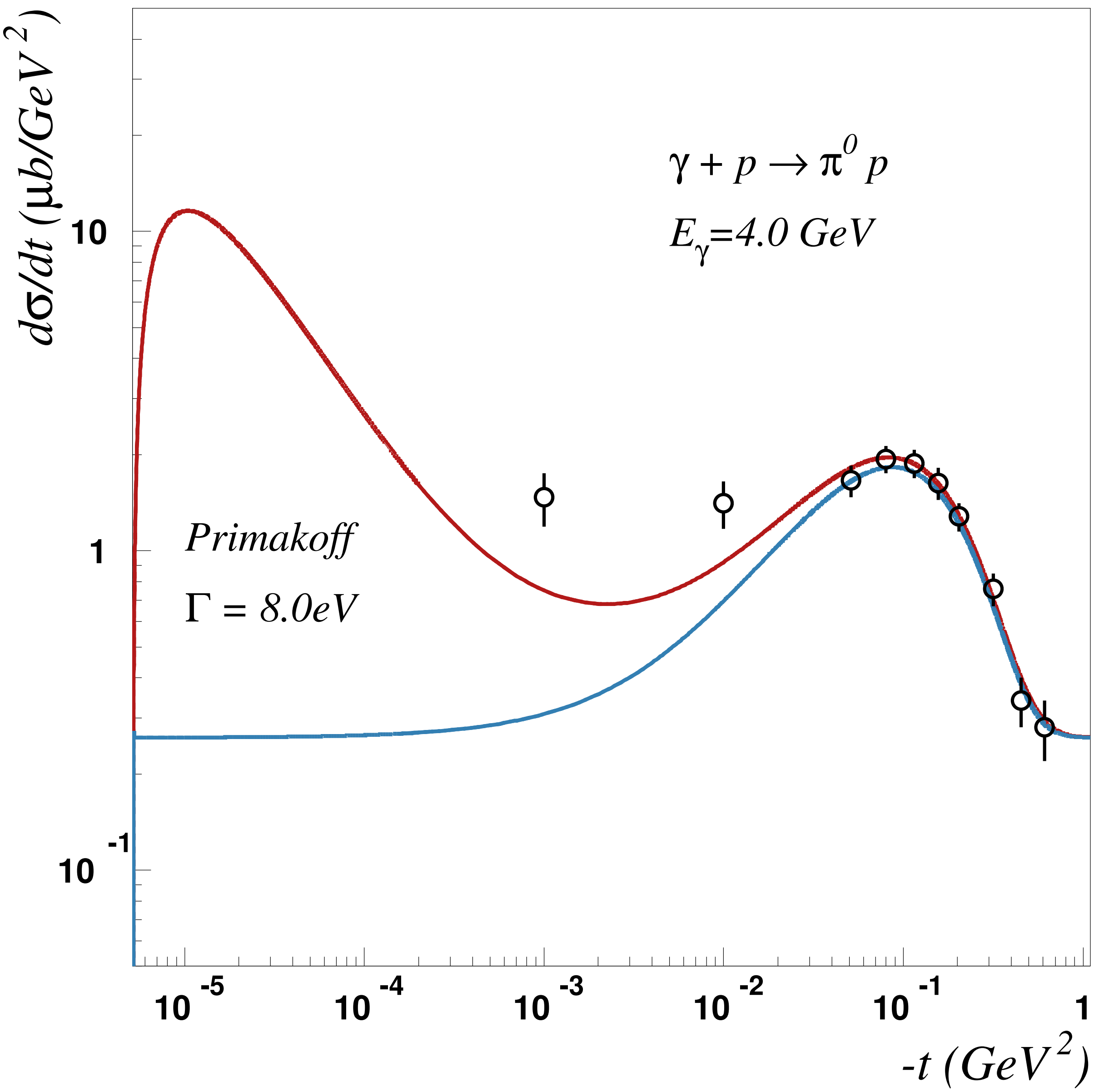

Figure 1: Differential cross section of π^0 photoproduction on proton at 4.0 GeV. Experimental points are from [7]. Blue curve is representing the strong part of elementary amplitude, the red points - including Coulomb part also.

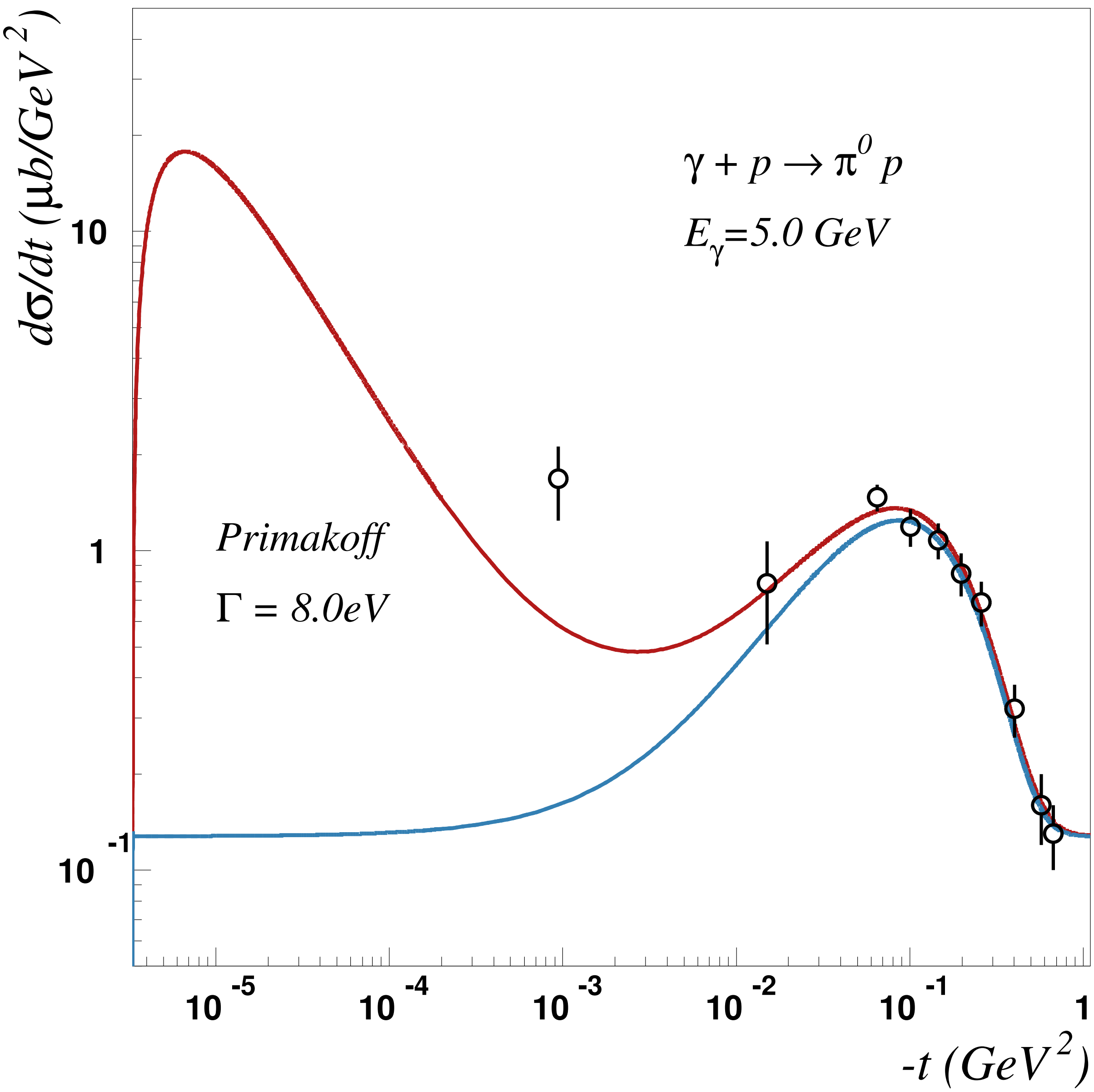

Figure 2: Differential cross section of π^0 photoproduction on proton at 5.0 GeV. Experimental points are from [7]. Blue curve is representing the strong part of elementary amplitude, the red points - including Coulomb part also.

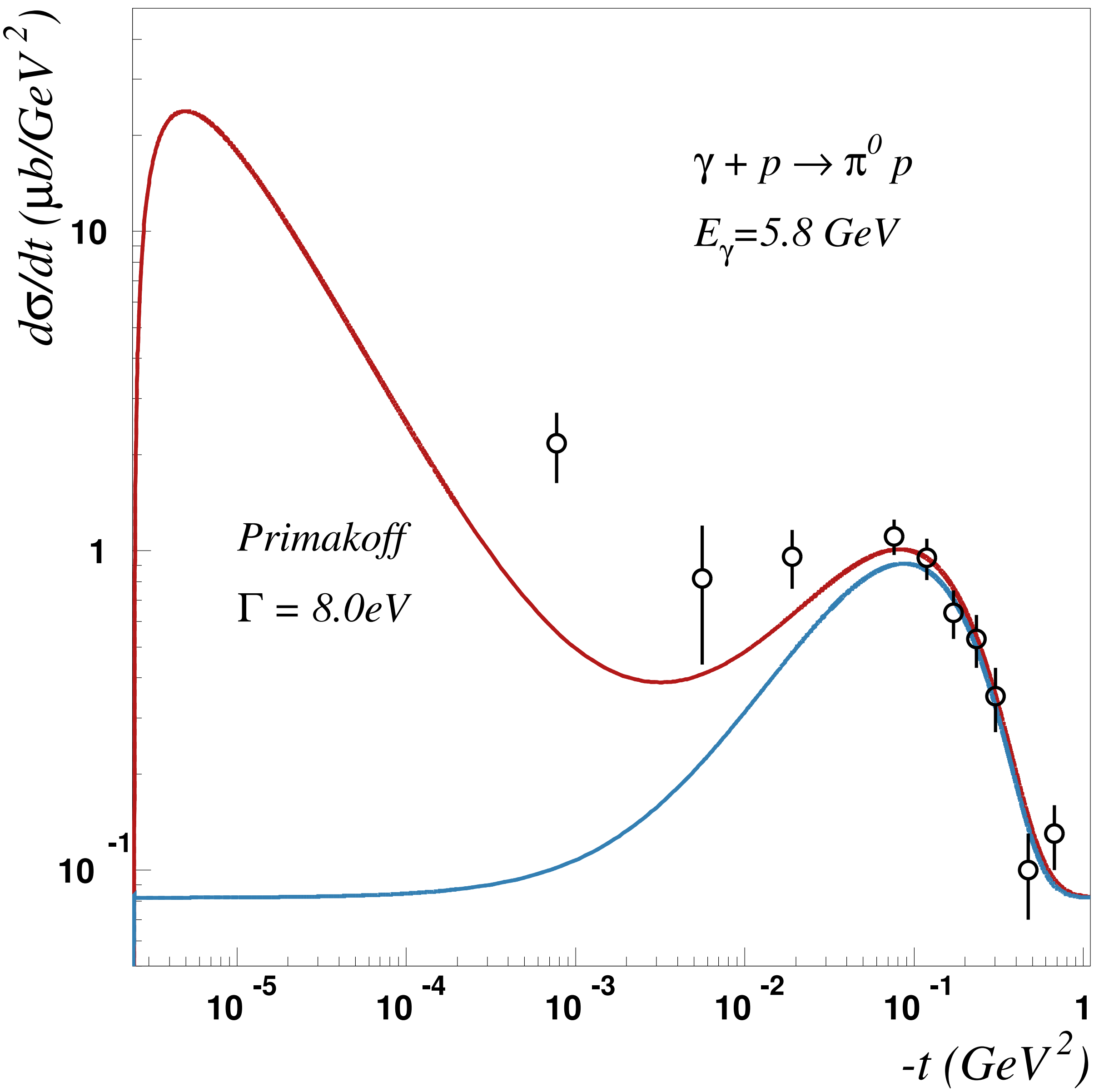

Figure 3: Differential cross section of π^0 photoproduction on proton at 5.8 GeV. Experimental points are from [7]. Blue curve is representing the strong part of elementary amplitude, the red points - including Coulomb part also.

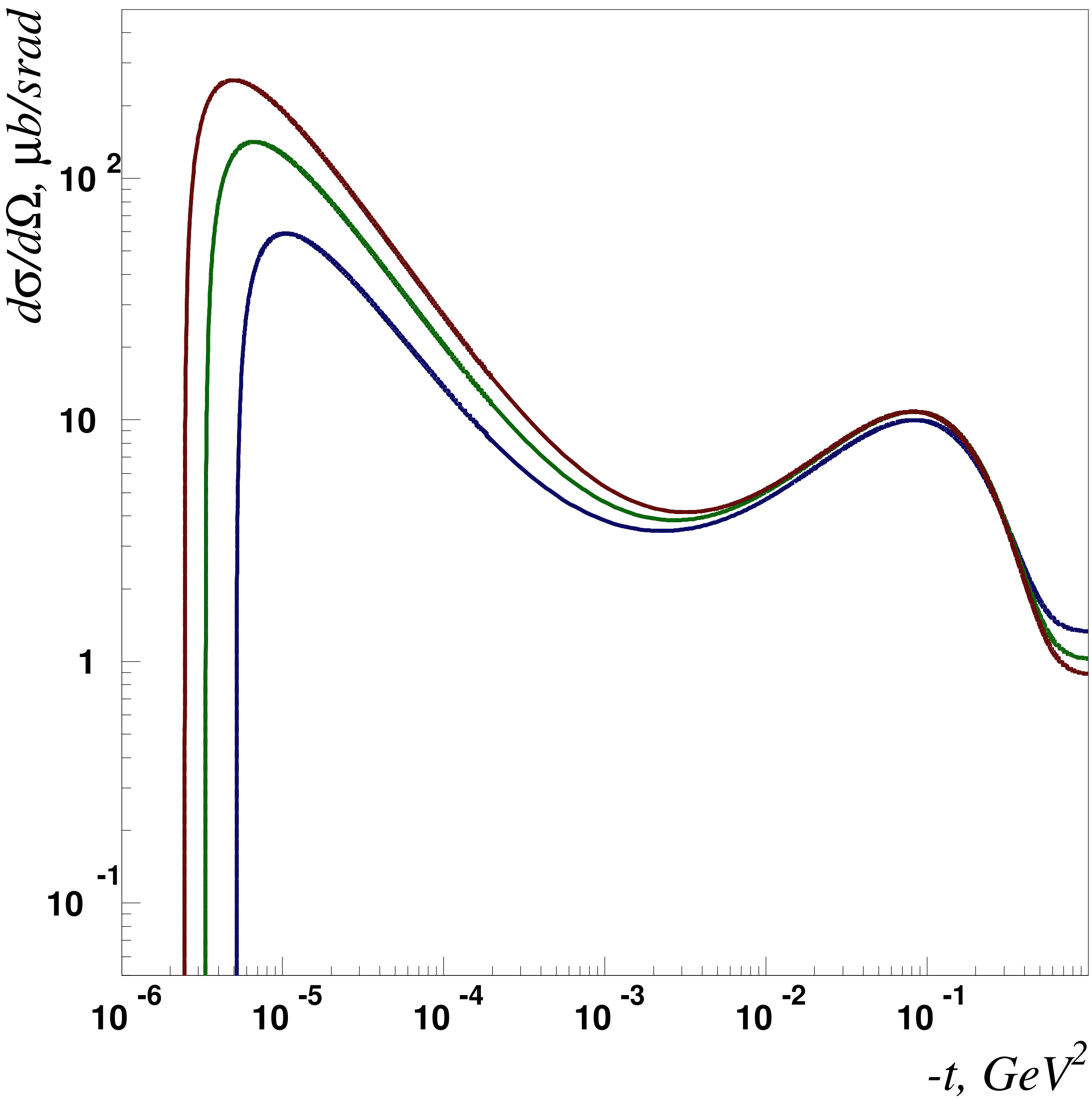

Figure 4: Distribution of fitted differential cross sections at three energies vs. momentum transefered. As it seen, the strong part of the cross section (in the limits of experimental errors) does not depend from energy. See the text for details.

3 The Pauli Suppression Factor

The Pauli blocking effect for the pion incoherent photoproduction reaction in nuclei has been discussed in the work of Engelbrecht [3] for both light and heavy nuclei. Using the Fermi gas model it is shown that for heavy nuclei the suppression factor can be expressed by:

$$\begin{aligned} 1 - G(t) &= \frac{3}{4}(\frac{q}{k_F}) - \frac{1}{16}(\frac{q}{k_F})^3, \quad p < 2P_F \\ 1 - G(t) &= 1, \qquad\qquad p > 2p_F \end{aligned} \tag{8}$$

where the Fermi momenta $k_F = 0.26GeV$
For light nuclei using the harmonic oscillator wave functions the same work is suggesting the following expression:

$$G(t) = (1 + (\frac{qR}{\sqrt{15}})^4)\exp(-\frac{2q^2R^2}{15}) \tag{9}$$

where q is the three dimensional momentum transfer and parameter $R = r_0 A^{\frac{1}{3}} \approx 14GeV^{-1}$ for Carbon nucleus.

4 Final State Interactions

In general,the factor $N(\sigma_1, \sigma_2)$ which is the effective nucleons number, is responsible for both interaction of the initial particle and for the interaction of produced pion with nucleons in the nucleus:

$$\begin{aligned} N(\sigma_1, \sigma_2) &= 2\pi \int \frac{\exp(-\sigma_1 T(b)) - \exp(-\sigma_2 T(b))}{\sigma_2 - \sigma_1} bdb \\ T(b) &= \int \rho(b, z)dz \end{aligned} \tag{10}$$

For our case neglecting the effect of photon shadowing in the nucleus, the expression (10) becomes:

$$\begin{aligned} N(0, \sigma) &= 2\pi \int \frac{1 - \exp(-\sigma T(b))}{\sigma} bdb \\ T(b) &= \int \rho(b, z)dz \end{aligned} \tag{11}$$

Here $\rho(r)$ is the nuclear density, σ is the pion nucleon interaction total cross section, $\sigma = 26mb$. With the 3 parameter Fermi distribution the

N(0,26)=7.36 for carbon nucleus, N(0,26)=62 for lead.
The incoherent cross sections are shown in Fig. 5 (^{12}C) and Fig. 6 (^{208}Pb) calculated using the expression (2).
Finally, in Fig.7 and Fig.8 the results of fit of experimental data from Primex for carbon and lead nuclei are presented.

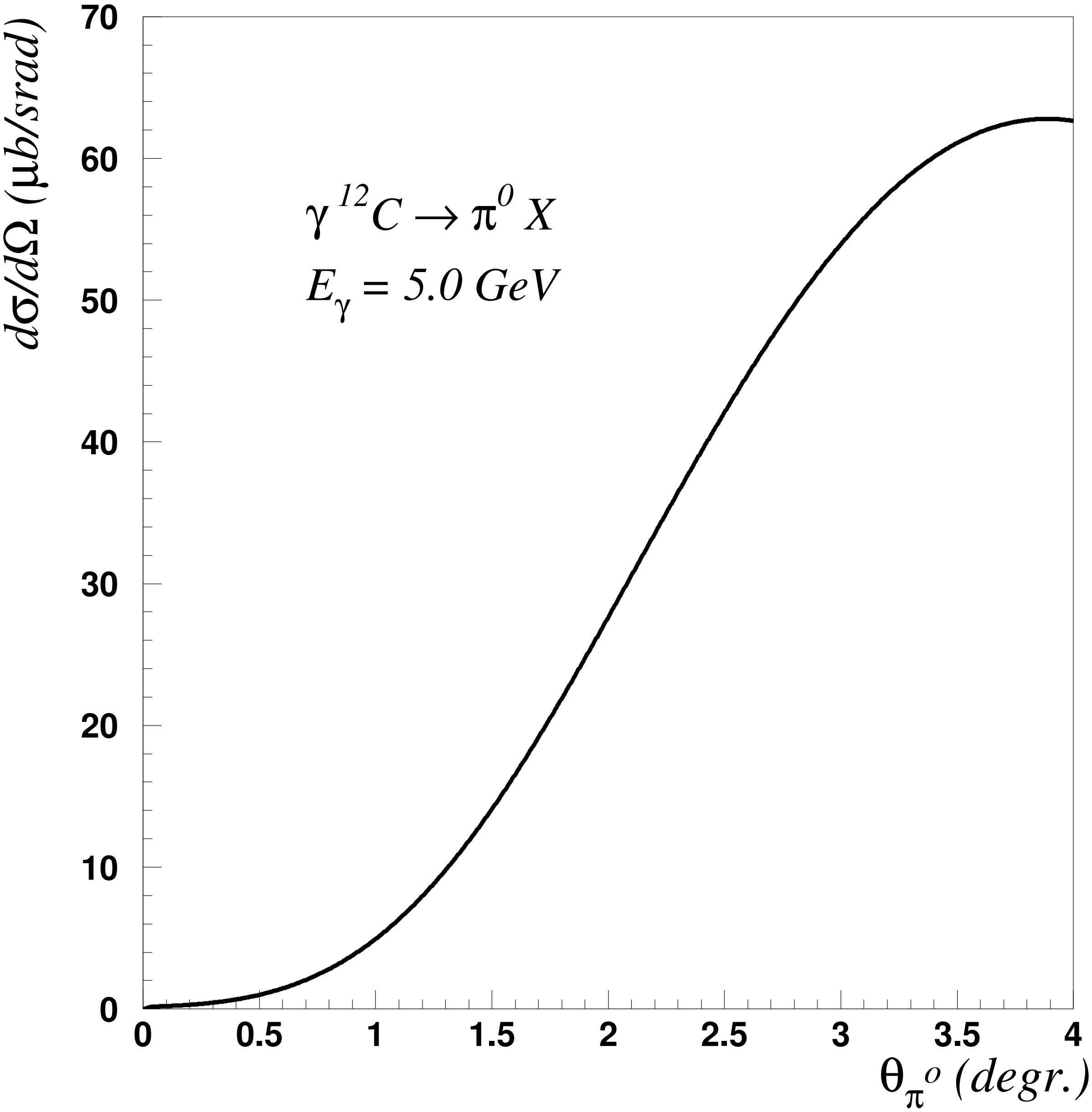

Figure 5: Incoherent cross section for π^0 photoproduction off ^{12}C calculated by formula 2, page 1.

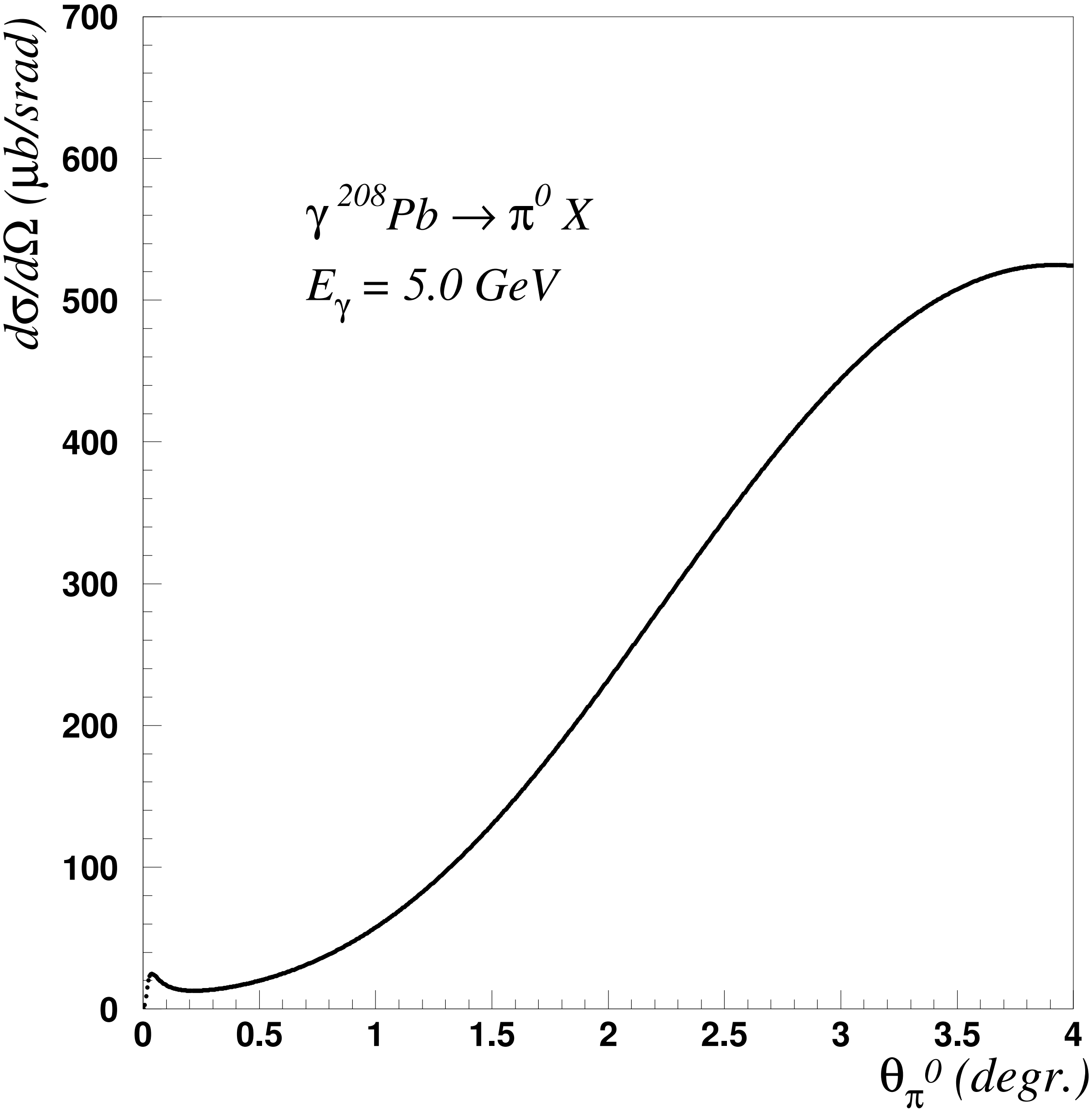

Figure 6: Incoherent cross section for π^0 photoproduction off ^{208}Pb calculated by formula 2, page 1.

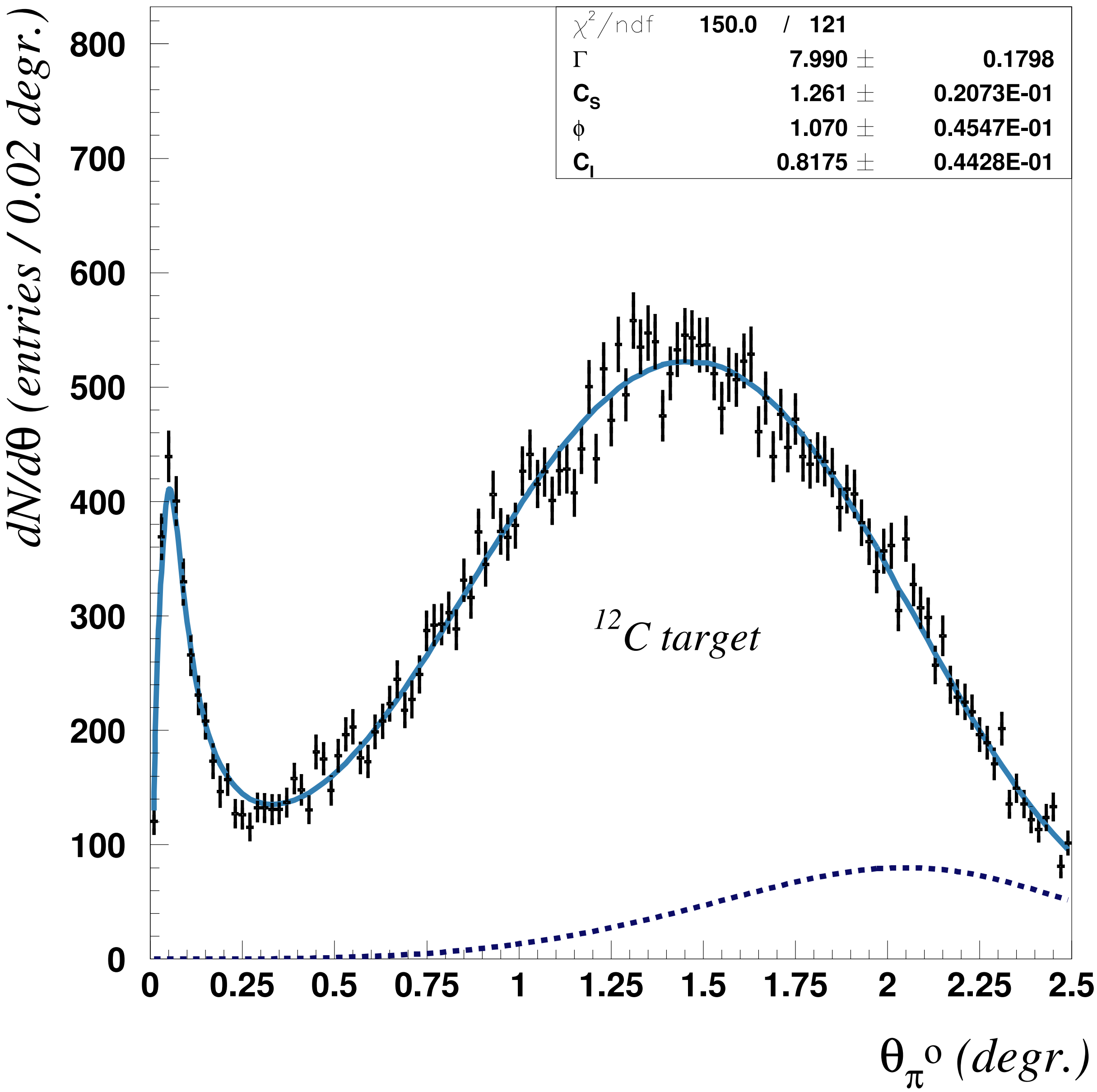

Figure 7: Fit of the PrimEx experimental data for the Carbon target (the solid line). The contribution of incoherent photoproduction is shown separately with dashed line.

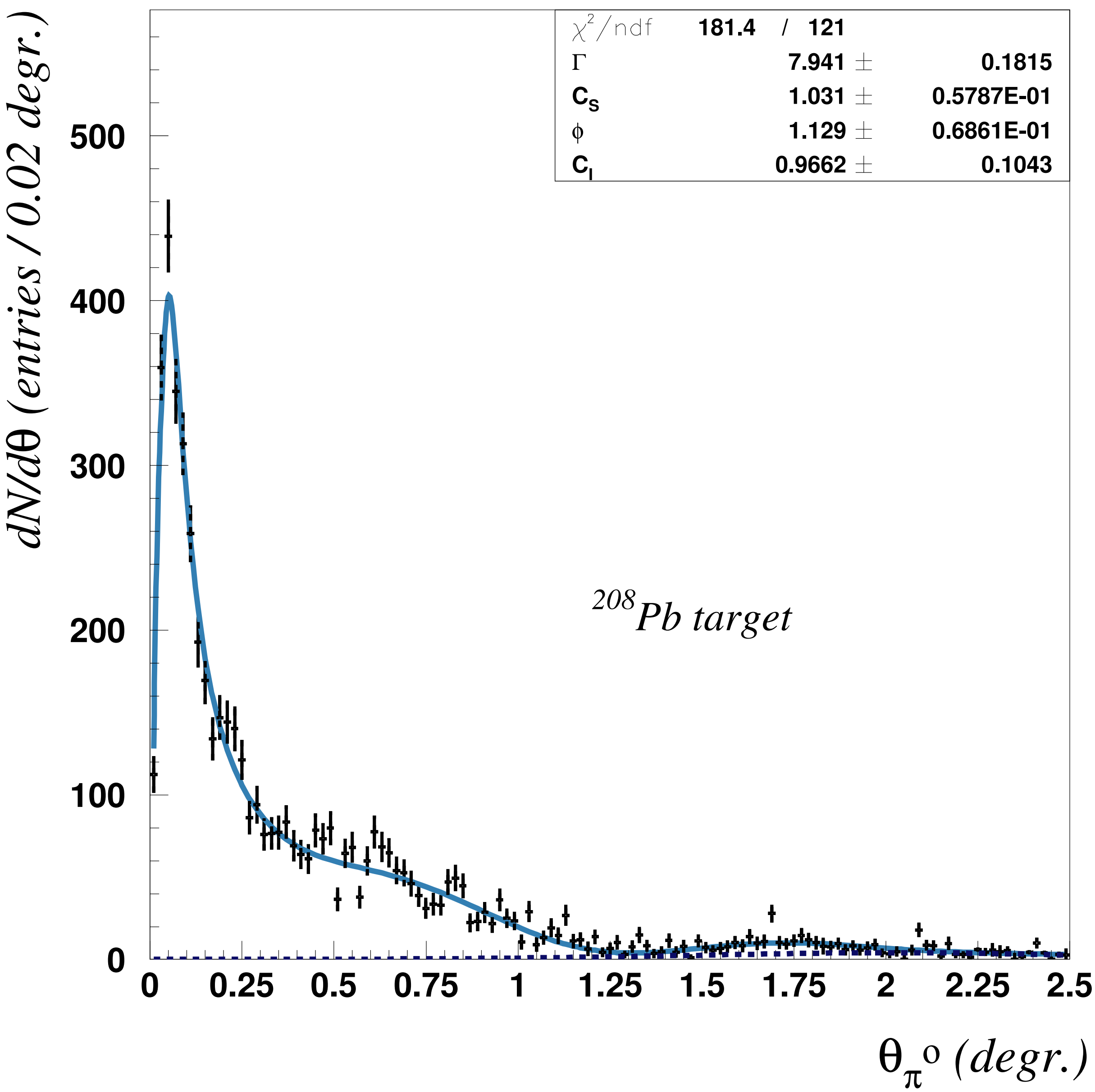

Figure 8: Fit of the PrimEx experimental data for the Lead target (the solid line). The contribution of incoherent photoproduction is shown separately with dashed line.

5 Incoherent Rescattering of Pion in the Nucleus

We generalized the incoherent cross section (expression (2)), in order to take into account the pion multiple scatterings changing its direction (incoherent re-scatterings [11]). This effect would be significant for considered process as such restrings change the pions yield at small angles.
The expression for the generalized incoherent cross section is given by:

$$\frac{d\sigma_{inc}}{d\Omega} = \frac{k^2}{4\pi^2}\int d^2\beta d^2 B e^{i\vec{\beta}\vec{q}}\omega_p(\beta)\frac{1-e^{-(\sigma-\omega_s(\beta))T(B)}}{\sigma-\omega_s(\beta)} \tag{12}$$

$$\begin{aligned} \omega_p(\beta) &= \int e^{i\vec{\beta}\vec{\Delta}}\frac{d\sigma_p}{d\Omega}\frac{d^2\Delta}{k^2} \\ \omega_s(\beta) &= \int e^{i\vec{\beta}\vec{\Delta}}\frac{d\sigma_s}{d\Omega}\frac{d^2\Delta}{k^2} = \sigma_{el}e^{-\frac{\beta^2}{4a}} \end{aligned} \tag{13}$$

where $\frac{d\sigma_p}{d\Omega}$ is the cross section for reaction $\gamma + N \rightarrow \pi^0 + N$ whereas $\frac{d\sigma_s}{d\Omega}$ is the cross section for elastic process $\pi^0 + N \rightarrow \pi^0 + N$.
Integrating the expression (12) by momentum transfer we get the total cross section for incoherent photoproduction on nuclei:

$$\sigma_{incoh} = \int \frac{d\sigma_{inc}}{dt}dt = \sigma_p N(0, \sigma_{inel}) \tag{14}$$

The absorption of total incoherent cross section σ_{inc} is determined by inelastic pion nucleon cross section σ_{inel}. It is result of the fact that the elastic scattering cannot change the total yield of pions in the incoherent production.
The incoherent cross sections calculated by expression (12) for Carbon and Lead are plotted in Fig. 6,7.
As it seen from the plots the effect of incoherent re-scatterings at small angles is not relatively large.The Pauli suppression factor is not included in (12) in contrary to expression (2). The work is in progress to implement the Pauli blocking effect in the derivation of $\frac{d\sigma_{inc}}{d\Omega}$ in (12).
One more effect which can potentially change the incoherent cross section is the photon shadowing in the nucleus. This effect is well known and widely explored for the diffractive processes [12], but purely investigated for the non-difractive processes like the π^0 photoproduction. We are carefully working on this subject also.

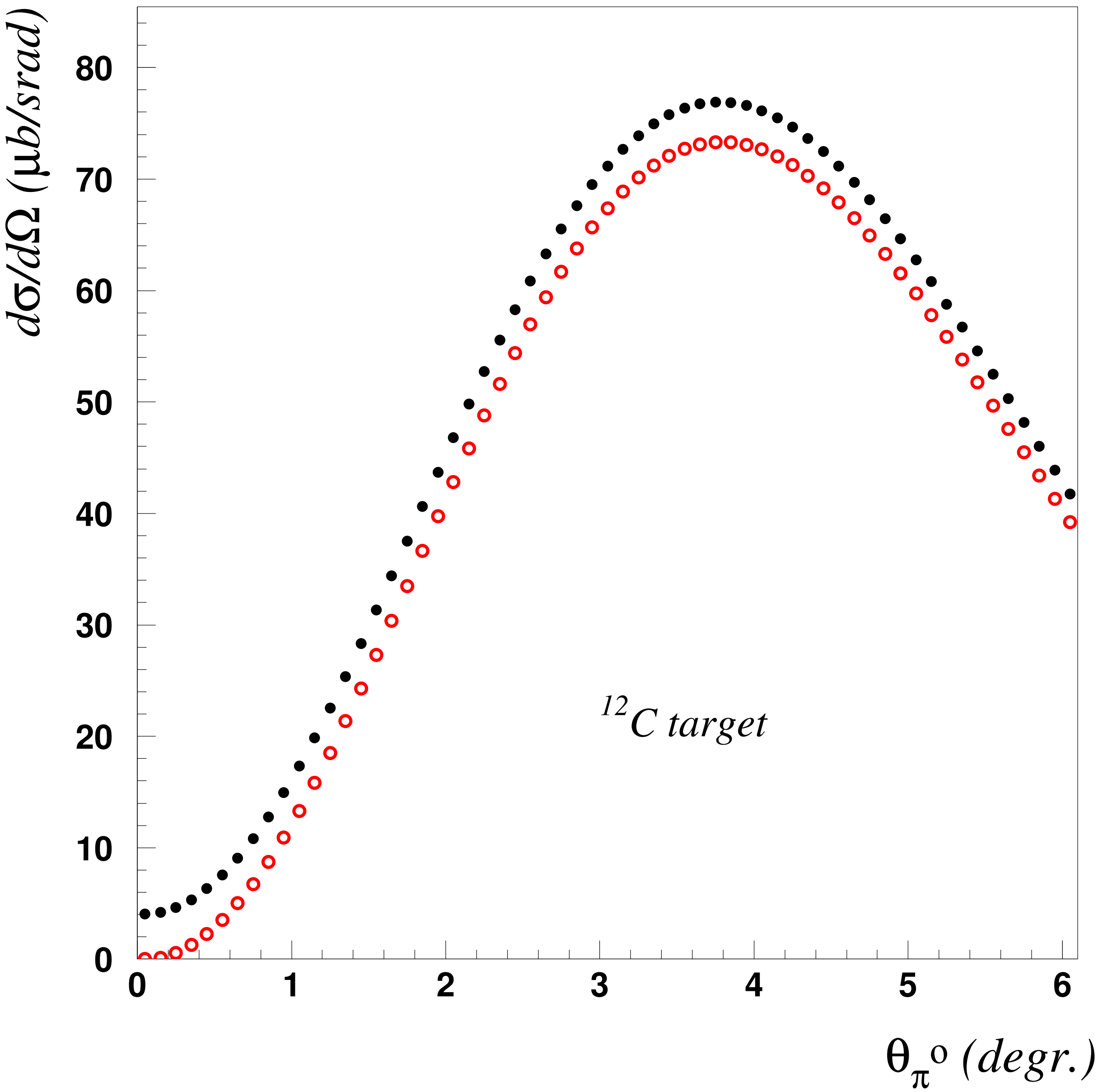

Figure 9: Contribution of incoherent rescattering effect on Carbon (black solid dots), calculated according to formula 12, page 13. Red, open dots - calculation of formula 2, page 1.

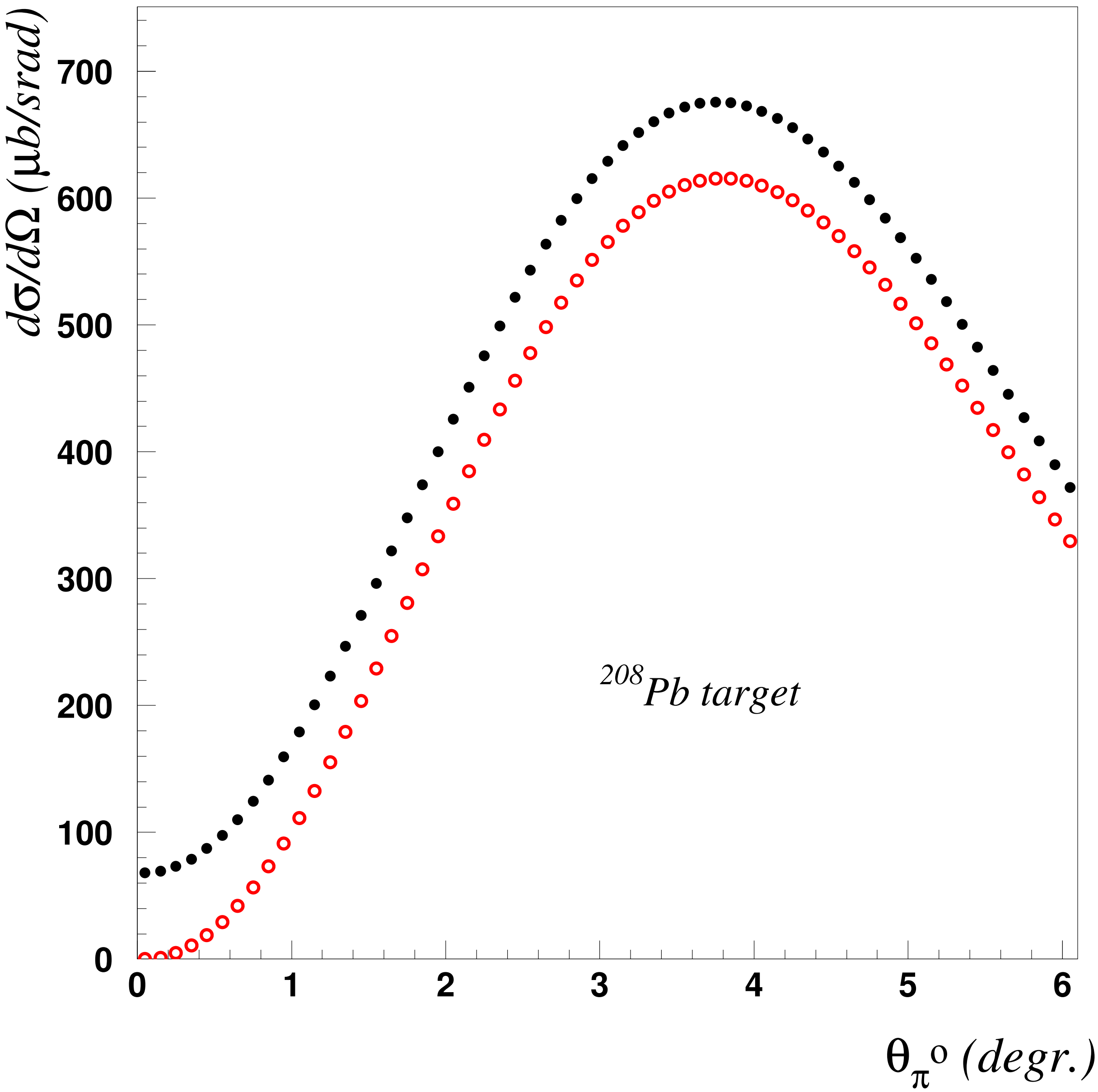

Figure 10: Contribution of incoherent rescattering effect on Lead (black solid dots), calculated according to formula 12, page 13. Red, open dots - calculation of formula 2, page 1.

References

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