Photoproduction of π^0 mesons off nuclei

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1 Introduction

It was noticed long time ago [1], that production of mesons in the nuclei Coulomb field known as the Primakoff effect, can be isolated for the presence of the Coulomb pole in the production amplitude of the process

$$\gamma(k) + A \to P(p) + A \tag{1}$$

This allows one to measure the radiative decay width of pseudoscalar mesons $P \to \gamma\gamma;\ P = (\pi^0, \eta, \eta\prime)$.
The Primakoff cross section is given by [2]

$$\frac{d\sigma}{dt} = 8\pi\alpha Z^2\Gamma(\frac{\beta}{\mu})^3\frac{k^2 sin^2\theta}{(q^2+\Delta^2)^2} \mid F_{em}(q) \mid^2 \tag{2}$$

Here F_{em} is electromagnetic form factor of nucleus; invariant momentum transfer $t = (k-p)^2 = -q^2 - \Delta^2 = -4kpsin^2(\frac{\theta}{2}) - (\frac{\mu^2}{2E_\gamma})^2$; μ and Z are the pion mass and the nucleus charge number; $\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$ fine structure constant; Γ the radiative decay width of the pseudoscalar meson.
The differential cross section (2) has Coulomb enhancement, which attains its maximum value at $q^2 = \Delta^2$.It rises with energy as $\frac{d\sigma}{dt} \propto 1/\Delta^2 \propto E_\gamma^2$, whereas the position of the peak shifts to lower values of q^2. This property allows one to isolate the Coulomb production cross section from coherent photoproduction (strong part) and measure the decay width Γ (or lifetime $\tau = \frac{1}{\Gamma}$) of pseudoscalar mesons.
The principal point in the production off Coulomb field is that the important transverse dimensions in Coulomb amplitude are very large and grows rapidly with energy

$$\mid \vec{b} \mid \sim \frac{1}{q} \sim \frac{1}{\Delta} = \frac{2E_\gamma}{\mu^2} \tag{3}$$

The electromagnetic form factor of the nucleus [1] with respect for meson absorption is given by [3]:

$$F_{em}(q) = 2\pi \frac{q^2+\Delta^2}{q} \int J_1(qb) \frac{b^2 dbdz}{(b^2+z^2)^{3/2}} e^{i\Delta z}$$
$$\times \exp(-\frac{\sigma\prime A}{2} \int_z^\infty \rho(b,z\prime)dz\prime) \int_0^{\sqrt{b^2+z^2}} x^2 \rho(x)dx \qquad (4)$$

Here $J_1(x)$ is the Bessel function of first order; $\sigma\prime = \sigma(1 - i\frac{Ref(0)}{Imf(0)})$, f(q) and σ are the forward elastic amplitude and total cross section of the process $\pi + N \to \pi + N$. In calculations one can use $\sigma = \sigma_{tot}(\pi N) = 26mb$; $\frac{Ref(0)}{Imf(0)}) = -0.2$. $\rho(r)$ is the charge density of the nucleus.
The above expressions allow to solve in principle the problem of pseudoscalar mesons photoproduction in Coulomb field. Nevertheless the main problem in the lifetime extraction from the experimental data is the correct accounting of the mesons photoproduction off the strong nuclear field. The amplitude of the process (1) is the sum of Coulomb F_C and strong F_S amplitudes and the main task of the theory is the investigation of the strong part F_S ,whose accurate knowledge is extremely crucial for π^0 lifetime extraction.
We adopt the following normalization for amplitudes:

$$\frac{d\sigma}{d\Omega} = \frac{k^2}{\pi}\frac{d\sigma}{dt} = \mid F_C + e^{i\varphi} F_S \mid^2 + \frac{d\sigma_{inc}}{d\Omega} \qquad (5)$$

Here $\frac{d\sigma_{inc}}{d\Omega}$ is the incoherent cross section. φ the relative phase between Coulomb and strong parts.

2 Coherent π^0 photoproduction on nuclei

The main impact on the lifetime extraction comes from our precision knowledge of strong amplitude F_S relevant to the coherent process

$$\gamma A \to \pi^0 A \qquad (6)$$

Later on we will try to explain in detail the approximations,which have been done in obtaining the formulas and the boundaries of applicability of obtained expressions.
In the Glauber multiple scattering theory, which we adopt as a basis for our

[1] The nucleus electromagnetic form factor in [4] is cited only for the case, when the primarily and produced particles has the same attenuation in nuclei.Moreover there is a misprint in [4]: lose of factor 2π in expression (3.4)

calculations, the amplitude for meson coherent photoproduction off nuclei with atomic number A (the nucleus is left in its ground state) is given by:[2]

$$\begin{aligned} F_S(q,q_L) &= A\frac{ik}{2\pi}\int e^{i\vec{q}\vec{b}+iq_L z}\Gamma_p(\vec{b}-\vec{s})\rho(\vec{s},z) \\ &\times [1-\int \Gamma_s(\vec{b}-\vec{s}\prime)\rho(\vec{s}\prime,z\prime)d^2s\prime dz\prime]^{A-1}d^2bd^2sdz \end{aligned} \tag{7}$$

The two dimensional vectors $\vec{b},\vec{s}$ are the impact parameter and the nucleon coordinate in the plane transverse to the primary photon momenta; z is the longitudinal coordinate of nucleon in the nucleus and integration goes up to infinity,as a nuclear distributions have an infinite tail. $\rho(\vec{s},z)$ is the nuclear density distribution at the point $\vec{r}(\vec{s},z)$ normalized as

$$\int \rho(\vec{s},z)d^2sdz = 1$$

The profile functions $\Gamma_p(\vec{b}-\vec{s})$ by definition are the two dimensional Fourier transformation of elementary amplitudes for the pion photoproduction off the nucleon $f_p = f(\gamma + N \to \pi^0 + N)$ and elastic pion-nucleon scattering $f_s = f(\pi + N \to \pi + N)$

$$\begin{aligned} \Gamma_p(\vec{b}-\vec{s}) &= \frac{1}{2\pi ik}\int e^{i\vec{q}(\vec{b}-\vec{s})}f_p(q)d^2q \\ \Gamma_s(\vec{b}-\vec{s}) &= \frac{1}{2\pi ik}\int e^{i\vec{q}(\vec{b}-\vec{s})}f_s(q)d^2q \end{aligned} \tag{8}$$

It is widely accepted approximation to take out the nuclear density distributions from the integrals in expression (7) at point $\vec{s} = \vec{b}$, as a nucleons distribution in nuclei $\rho(r)$ is slowly varying function in respect to profile functions.[3] As a result for elastic scattering one gets:

$$\int \Gamma_s(\vec{b}-\vec{s})\rho(s,z)d^2sdz = \frac{\sigma\prime}{2}\int \rho(\vec{b},z)dz \tag{9}$$

[2]The two well grounded assumptions have been done in obtaining of this expression:
1. The overall phase-shift function $\lambda(\vec{b},\vec{s}_1,\vec{s}_2,...\vec{s}_A)$ is the sum of the phase-shift functions λ_j for collisions with individual nucleons.
2. The ground state of the nucleus can be described by means of the independent particle model, i.e.

$$\mid \Psi(\vec{r}_1,\vec{r}_2,...\vec{r}_A) \mid^2 = \prod_{j=1}^{A}\rho_j(r_j)$$

[3]The slopes of elementary amplitudes f_s, f_p are much less than the square of nuclear radius for middle and heavy nuclei

where $\sigma\prime = \sigma(1 - i\frac{Ref(0)}{Imf(0)}) = \frac{4\pi}{ik} f_s(0)$. On the other hand the same procedure for production amplitudes, which are zero at zero angle (as in the considered reaction $\gamma + N \to \pi^0 + N$) has to be done with a great care. The receipt for this specific case is well known [4]. Isolating near zero angle part of the production amplitude $f_p(q) = (\vec{h}\vec{q})f(q)$ ($f(0)\neq 0$ $\vec{h} = [\vec{k} \times \vec{\epsilon}]$ where ϵ the photon polarization vector)[4] one obtains

$$\int \Gamma_p(\vec{b} - \vec{s})\rho(s, z)d^2 s dz = \frac{2\pi}{k} f(0)\vec{h} \int \frac{\partial\rho(\vec{b}, z)}{\partial\vec{b}} dz \tag{10}$$

As a result our basic expression (7) can be written in the factorized form:

$$\begin{aligned} F_S(q) &= A(\vec{h}\vec{q})f(0)F_A \\ F_A &= -\frac{2\pi}{q}\int J_1(qb)\frac{\partial\rho(b, z)}{\partial b} bdbdze^{i\Delta z} \exp(-\frac{\sigma\prime A}{2}\int_z^\infty \rho(b, z\prime)dz\prime) \end{aligned} \tag{11}$$

where F_A is the nuclear form factor[5] The coherent amplitude for the non-diffractive processes, for which the production cross section on nucleon is zero at zero angles, differ substantially from the coherent amplitudes in diffractive production (see e.g. [6]), due to the derivative of nuclear density in (11). The formfactor (11) can be expressed as a sum of two terms:

$$\begin{aligned} F_A &= \int e^{i\vec{q}\vec{b}}\rho(b, z)bdbdze^{i\Delta z} \exp(-\frac{\sigma\prime A}{2}\int_z^\infty \rho(b, z\prime)dz\prime) \\ - & \frac{A\pi\sigma'}{q}\int J_1(qb)\rho(b, z_1)\frac{\partial\rho(b, z_2)}{\partial b} bdbdz_1 dz_2 e^{i\Delta z_1} \\ \times & \exp(-\frac{\sigma\prime A}{2}\int_{z_1}^\infty \rho(b, z')dz') \end{aligned} \tag{12}$$

The first term is the usual nuclear formfactor and the second one is so called Faldt correction [4].The second term is positive ($\frac{\partial\rho(b,z)}{\partial b} < 0$) and can be interpreted as the result of final state interaction of pions produced at nonzero angles coming to forward direction after multiple scatterings.

3 Intermediate channels

It is well known [5] that high energy photons are shadowed in nuclei. According to the theory developed in [8] the photons shadowing is a result

[4] $(\vec{h}\vec{q})^2 = \frac{k^2 sin^2\theta}{2}$ i.e. our normalization coincides with the relevant one in [7]

[5] We replace the binomial in (7) to exponent using the relation $(1 - \frac{x}{A})^A \approx exp(-x)$, which is valid for A¿10

of a two step processes:primary photon produces a vector meson on one of the nucleons and turns to the photon again after multiple scatterings in nucleus.In the π^0 production the same effect can take place: high energy photon can produce the vector meson on one of the nucleons and then produce the pion. The main contribution to such two step pion production comes from the ρ mesons,as the cross section for ρ photoproduction from the nucleon is almost in order higher than for the ω one.Moreover in coherent processes one has to sum the production amplitudes from different nucleons, as a result the intermediate omega contribution has additional suppression,because the reaction $\omega + N \to \pi^0 + N$ provided mainly by isospin one exchange trajectory (ρ-exchange) for which the production amplitudes on protons and neutrons have a different signs thus canceling each other.
Using the same technique as described above, we get the contribution from intermediate ρ channel:

$$F_I(q) = -\frac{\pi\sigma A}{q}\int J_1(qb)\rho(b,z_1)\frac{\partial\rho(b,z_2)}{\partial b}\theta(z_2-z_1)bdbdz_1dz_2$$
$$\times e^{i\Delta_\rho(z_1-z_2)+i\Delta z_2}\exp(-\frac{\sigma\prime A}{2}\int_{z_1}^{\infty}\rho(b,z\prime)dz) \qquad (13)$$

Here $\Delta_\rho = \frac{m^2\rho}{2E}$ is the longitudinal momentum transfer in the ρ meson photoproduction off nucleon. In deriving of this expression, we used the relations between elementary amplitudes in accordance with the vector dominance model for the reaction $\rho + N \to \pi^0 + N$,which works well for the real photoproduction.
The expression (13) and Fäldt correction (second term in (12)) are very similar and only difference is in their dependence on longitudinal momenta transfer.
It is interesting to note that at very high energy ($\Delta = \Delta_\rho = 0$) two terms cancel each other, thus the strong form factor in this limit is determined by the first term in (12) only. In this limit the integration over z in (13) can be done analytically

$$F_A = \frac{2}{\sigma\prime A}\int d^2be^{i\vec{q}\vec{b}}(1-\exp(-\frac{\sigma\prime A}{2}\int\rho(b,z\prime)dz)) \qquad (14)$$

This expression looks similar to the coherent production of π^0 by primarily ρ, a fact well known for the diffractive processes [8].

4 Nuclear density parameterization

For the charge and nuclear densities we have used the Fermi parameterization:

$$\begin{aligned}
\rho(r) &= \rho_0 \frac{1 + w\frac{r^2}{R^2}}{1 + \exp(\frac{r-R}{c})}; \qquad r^2 = b^2 + z^2 \\
\frac{\partial \rho(b,z)}{\partial b} &= \frac{b\rho_0}{crR^2(1 + \exp(\frac{r-R}{c}))} \\
&\times \quad [2cwr + (2cwr - R^2 - wr^2)\exp(\frac{r-R}{c})] \qquad (15)
\end{aligned}$$

with the following parameters [9]:
Carbon(A=12):w=-0.149;R=2.355f;c=0.5224f;$\rho_0 = 0.01519 f^{-3}$
Lead (A=208):w=0;R=6.624f;c=0.549f;$\rho_0 = 0.0007696 f^{-3}$
We are in the process to implement new density distribution functions from the recent electron scattering experiments [14].

5 Incoherent production

The process of incoherent photopoduction

$$\gamma + A \to \pi^0 + A' \qquad (16)$$

where A' includes any excitations of target nuclei and its break up in the first approximation can be calculated according to relations:

$$\begin{aligned}
\frac{d\sigma_{inc}}{d\Omega} &= \frac{d\sigma_N}{d\Omega} N(0,\sigma)(1 - G(t)) \\
N(0,\sigma) &= 2\pi \int \frac{1 - \exp(-\sigma T(b))}{\sigma} b db \\
T(b) &= \int \rho(b,z) dz \qquad (17)
\end{aligned}$$

The Pauli blocking factor G(t) can be taken from [10]. For heavy nucleus it is :

$$1 - G(t) = \frac{3}{4}(\frac{q}{k_F}) - \frac{1}{16}(\frac{q}{k_F})^3 \qquad (18)$$

For light nuclei

$$1 - G(t) = (1 + (\frac{qR}{\sqrt{15}})^4)\exp(-\frac{2q^2R^2}{15}) \qquad (19)$$

6 Photoproduction of π^0 on nucleon

In this part we want to make a short discussion of the π^0 photoproduction off the nucleon i.e.

$$\gamma + N \to \pi^0 + N \tag{20}$$

The amplitude of this process goes to zero at small angles and is proportional to the photon energy [7] $f = Nksin\theta$. Such an energy dependence is a straight result of approximate energy independence of the invariant product of $s^2\frac{d\sigma}{dt}$ where $\frac{d\sigma}{dt}$ is the cross section of (20) and invariant energy $s = (k + P_N)^2 = M_N^2 + 2M_N k$ (lab.system).Our normalization (5) coincides with the one from [7] i.e.

$$k^2\frac{d\sigma}{dt} = \pi\frac{d\sigma}{d\Omega} = \pi N^2 k^2 sin^2\theta \approx (t_{min} - t) \tag{21}$$

On the other hand the experimental data on reaction (20) at small angles [12], has a tendency to have a stronger than linear dependence of amplitude in accordance with Regge theory.
The phase φ between the Coulomb and strong amplitudes (see (5)) would be considered as a free parameter [7].Nevertheless to estimate its value one can use the Regge theory, which is completely suitable for our kinematic (high energies,small transfered momenta).
According to Regge theory the real and imaginary parts of amplitude are determined by signature factor which is determined by Regge trajectory α_ω

$$\frac{1 - e^{-i\pi\alpha_\omega}}{2\sin(\pi\alpha_\omega)} = \frac{e^{\frac{i\pi\alpha_\omega}{2}} - e^{\frac{-i\pi\alpha_\omega}{2}}}{2\sin(\pi\alpha_\omega)}e^{\frac{-i\pi\alpha_\omega}{2}} = \frac{\sin(\frac{\pi\alpha_\omega}{2})}{\sin(\pi\alpha_\omega)}e^{\frac{i\pi(1-\alpha_\omega)}{2}} \tag{22}$$

Thus the relative phase factor between Coulomb and strong amplitudes becomes $e^{\frac{i\pi(1-\alpha_\omega)}{2}}$.For intercept of ω exchange one can take [13] $\alpha_\omega(0)$=0.44 and obtain $\varphi_\omega = \frac{\pi(1-\alpha_\omega)}{2}$=0.88 in agreement with the value obtained from the fit of experimental data [7].

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