$$a_\mu^{\mathrm{HLbL};\pi^0(1)} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau\, w_1(Q_1, Q_2, \tau) \mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q_1^2, -(Q_1+Q_2)^2) \mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q_2^2, 0), \tag{11}$$

$$a_\mu^{\mathrm{HLbL};\pi^0(2)} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau\, w_2(Q_1, Q_2, \tau) \mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q_1^2, -Q_2^2) \mathcal{F}_{\pi^0\gamma^*\gamma^*}(-(Q_1+Q_2)^2, 0). \tag{12}$$

$$a_\mu^{\mathrm{HLbL};\pi^0} = \left(\frac{\alpha}{\pi}\right)^3 \left[a_\mu^{\mathrm{HLbL};\pi^0(1)} + a_\mu^{\mathrm{HLbL};\pi^0(2)}\right],$$

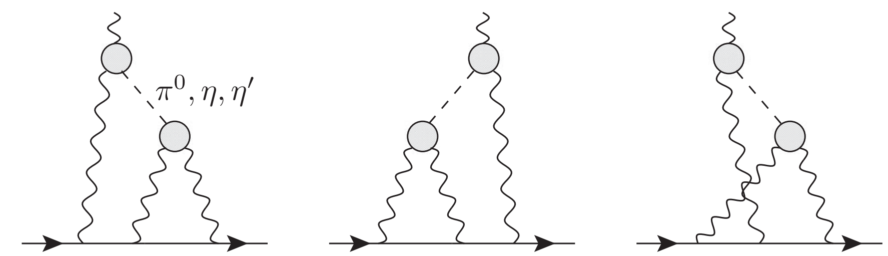

FIG. 1. The pseudoscalar-pole contribution to hadronic light-by-light scattering. The shaded blobs represent the transition form factor $\mathcal{F}_{\mathrm{P}\gamma^*\gamma^*}(q_1^2, q_2^2)$, where $\mathrm{P} = \pi^0, \eta, \eta'$.

Another model for the TFF, which was intensively used in the literature [80,81,84] is the so called LMD+V (lowest meson dominance + vector) parameterization [85]. It incorporates certain short-distance constraints from the operator product expansion and has the following form

$$F_{\pi^0}^{\mathrm{LMD+V}}(-Q_1^2, -Q_2^2) = \frac{f_\pi}{3} \frac{Q_1^2 Q_2^2 (Q_1^2 + Q_2^2) - h_2 Q_1^2 Q_2^2 + h_5 (Q_1^2 + Q_2^2) - h_7}{(Q_1^2 + M_{V_1}^2)(Q_1^2 + M_{V_2}^2)(Q_2^2 + M_{V_1}^2)(Q_2^2 + M_{V_2}^2)} . \tag{60}$$

The values of the parameters used for the π^0 in Refs. [80,81] are given by:

$$\begin{aligned}
M_{V_1} &= M_\rho = 0.7755 \text{ GeV}, \\
M_{V_2} &= M_{\rho'} = 1.465 \text{ GeV}, \\
h_2 &= -10.63 \text{ GeV}^2, \\
h_5 &= (6.93 \pm 0.26) \text{ GeV}^4, \\
h_7 &= -\frac{3 M_{V_1}^4 M_{V_2}^4}{4\pi^2 f_\pi^2} = -14.83 \text{ GeV}^6,
\end{aligned}$$

$$h_7 = -\frac{3M_1^4 M_2^4}{F_\pi}\sqrt{\frac{4\Gamma_{\pi^0\to\gamma\gamma}}{\pi\alpha^2 m_\pi^3}}$$

TFF expansion to $O(Q^6)$:

$$F_{TFF}(-Q_1^2, -Q_2^2) = \frac{1}{4\pi^2 F_\pi}\left[1 - a\left(Q_1^2 + Q_2^2\right) + b\left(Q_1^4 + Q_2^4\right) + cQ_1^2Q_2^2 + d\left(Q_1^6 + Q_2^6\right) + e\left(Q_1^4Q_2^2 + Q_1^2Q_2^4\right) + \dots\right]$$

$$F_{TFF}(-Q_1^2, -Q_2^2) = \sqrt{\frac{4\Gamma_{\pi^0\to\gamma\gamma}}{\pi\alpha^2 m_\pi^3}}\left[1 - a\left(Q_1^2 + Q_2^2\right) + b\left(Q_1^4 + Q_2^4\right) + cQ_1^2Q_2^2 + d\left(Q_1^6 + Q_2^6\right) + e\left(Q_1^4Q_2^2 + Q_1^2Q_2^4\right) + \dots\right]$$

$$a = \frac{1}{M_1^2} + \frac{1}{M_2^2} + \frac{h_5}{h_7} = 1.66147\ GeV^{-2}$$

$$b = \frac{1}{M_1^4} + \frac{1}{M_2^4} + \frac{1}{M_1^2M_2^2} + \frac{h_5}{h_7}\left(\frac{1}{M_1^2} + \frac{1}{M_2^2}\right) = 2.76210\ GeV^{-4}$$

$$c = \left(\frac{1}{M_1^2} + \frac{1}{M_2^2}\right)^2 + \frac{h_2}{h_7} + 2\frac{h_5}{h_7}\left(\frac{1}{M_1^2} + \frac{1}{M_2^2}\right) = 3.25890\ GeV^{-6}$$

$$d = -4.59252\ GeV^{-6}$$

$$e = -5.58285\ GeV^{-6}$$

Compare HLbL integrals for $Q_1^2 < 0.1\ GeV^2$, $Q_2^2 < 0.1\ GeV^2$ for (i) the full LMD+V form factor and (ii) the $O(Q^6)$ low Q^2 expansion (labeled TFF here)

```
Integral 1 (LMD+V):     0.0163223       Sigma: 1.95857e-06
Integral 2 (LMD+V):     0.000982305     Sigma: 1.93489e-07
Integral 1 (TFF)  :     0.0166974       Sigma: 1.81352e-06
Integral 2 (TFF)  :     0.000993877     Sigma: 2.16397e-07
Final LMD+V       :     2.16875e-10
Final TFF         :     2.2172e-10

% Error = 2.23397
```

Use simulated data to determine sensitivity to the low Q^2 parameters

- Fix “d” and “e” from $O(Q^6)$ expansion of LMD+V TFF
- Fit “a”, “b” and “c” to our data

Simulation:

i. Use values for a, b, c, d and e to produce pseudo-data

ii. Fit pseudo-data to obtain errors in a, b and c

iii. Propogate errors in a, b and c back into the HLbL calculation: find projected error in $a_\mu^{HLbL;\pi^0}$