

# Determination of QGP Parameters from a Global Bayesian Analysis

**Steffen A. Bass**

# Properties of QCD: Transport Coefficients

**shear** and **bulk** viscosity are defined as the coefficients in the expansion of the stress tensor in terms of the **velocity fields**:

$$T_{ik} = \varepsilon u_i u_k + P (\delta_{ik} + u_i u_k) - \eta \left( \nabla_i u_k + \nabla_k u_i - \frac{2}{3} \delta_{ik} \nabla \cdot u \right) + \zeta \delta_{ik} \nabla \cdot u$$

## $\eta/s$ from Lattice QCD:



The confines of the Euklidian Formulation:

- extracting  $\eta/s$  formally requires taking the zero momentum limit in an infinite spatial volume, which is numerically not possible...

• preliminary estimates:

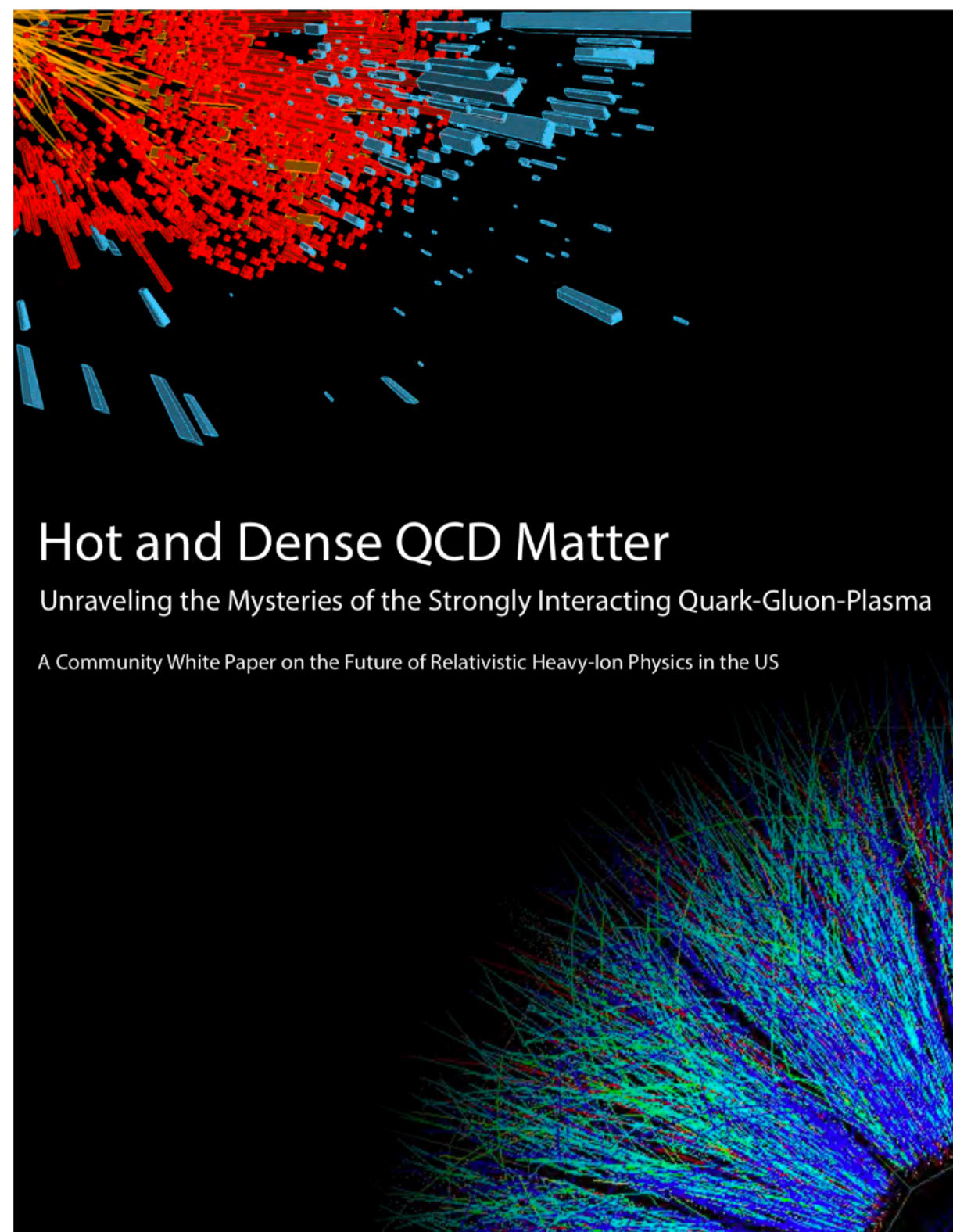
T	1.58 T <sub>c</sub>	2.32 T <sub>c</sub>
$\eta/s$	0.2-0.25	0.25-0.5

A. Nakamura & S. Sakai: Phys. Rev. Lett. **94** (2005) 072305  
Harvey B. Meyer: Phys. Rev. **D79** (2009) 011502  
Harvey B. Meyer: [arXiv:0809.5202](#) [hep-lat]

**The determination of the QCD transport coefficients is one of the key goals of the global relativistic heavy-ion effort!**



# An Effort by the Heavy-Ion Community

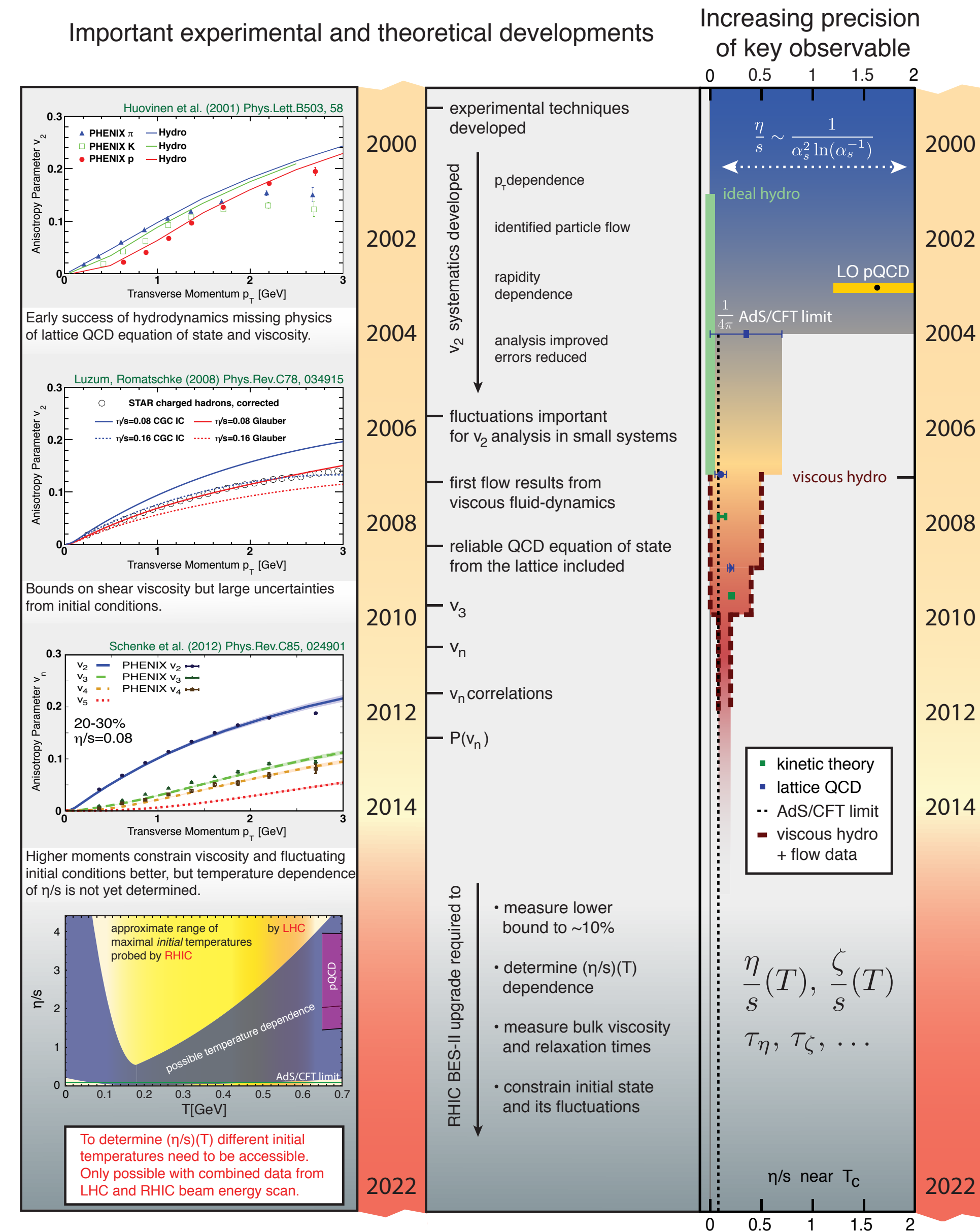


2012 response of the US relativistic heavy-ion community to the request for comments by the NSAC subcommittee, that was tasked to recommend *optimizations* to the US Nuclear Science Program over the following five years.

2012 RHIC community White Paper identified key developments and laid out milestones for the determination of QGP properties:

**Goal:** by 2022 determine the temperature dependence of  $\eta/s$  and  $\zeta/s$  as well as relaxation times and other QGP transport coefficients of interest (e.g.  $q$ -hat and  $e$ -hat)

**We are well on our way deliver on these goals!**





# Standing on the Shoulders of Giants

PRL **97**, 152303 (2006)

PHYSICAL REVIEW LETTERS

week ending  
13 OCTOBER 2006

## Strongly Interacting Low-Viscosity Matter Created in Relativistic Nuclear Collisions

Laszlo P. Csernai,<sup>1,2</sup> Joseph I. Kapusta,<sup>3</sup> and Larry D. McLerran<sup>4</sup>

<sup>1</sup>Section for Theoretical Physics, Department of Physics, University of Bergen, Allegaten 55, 5007 Bergen, Norway

<sup>2</sup>MTA-KFKI, Research Institute of Particle and Nuclear Physics, 1525 Budapest 114, P. O. Box 49, Hungary

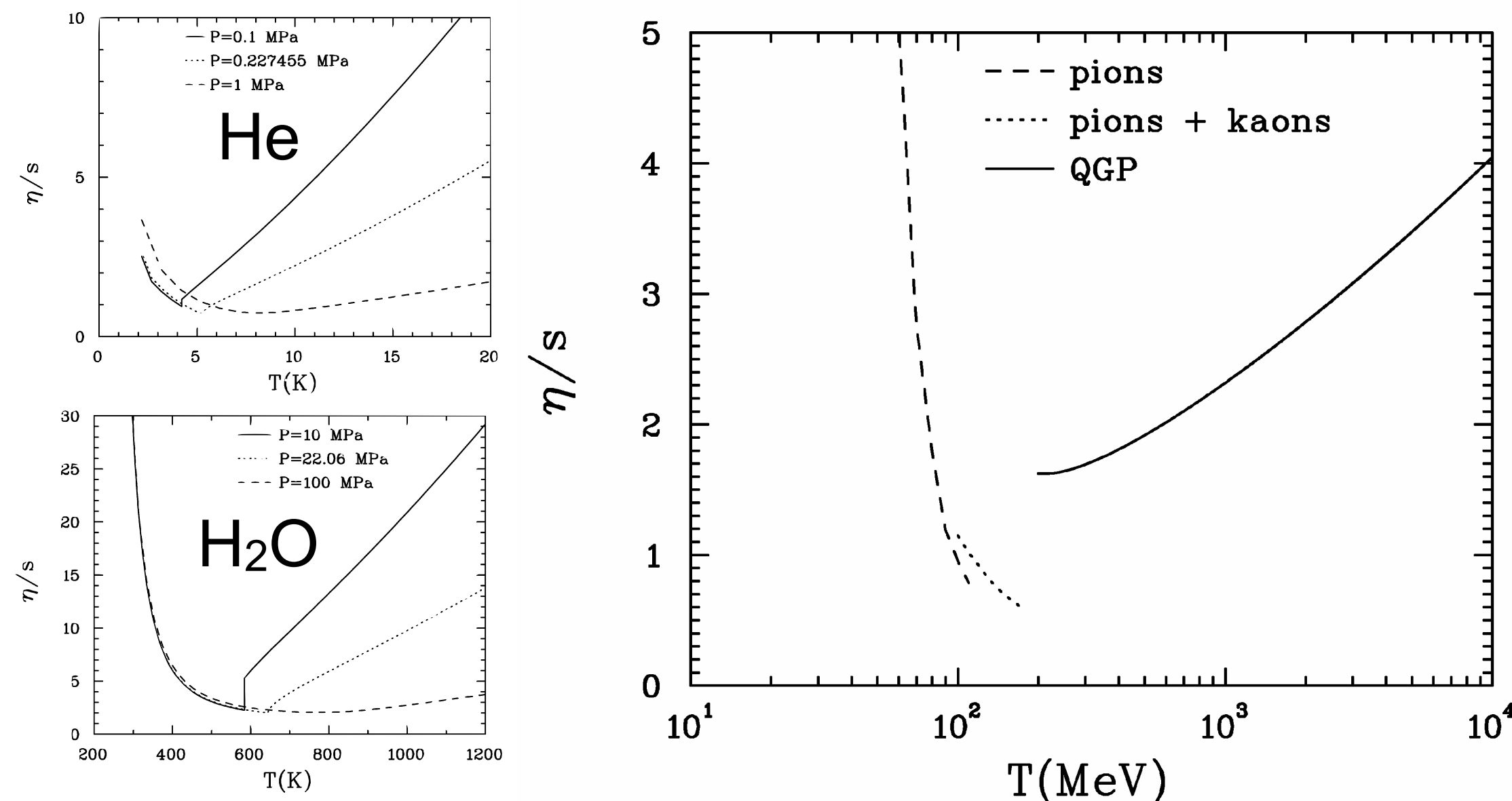
<sup>3</sup>School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455, USA

<sup>4</sup>Nuclear Theory Group and Riken Brookhaven Center, Brookhaven National Laboratory, Bldg. 510A, Upton, New York 11973, USA  
(Received 12 April 2006; published 12 October 2006)

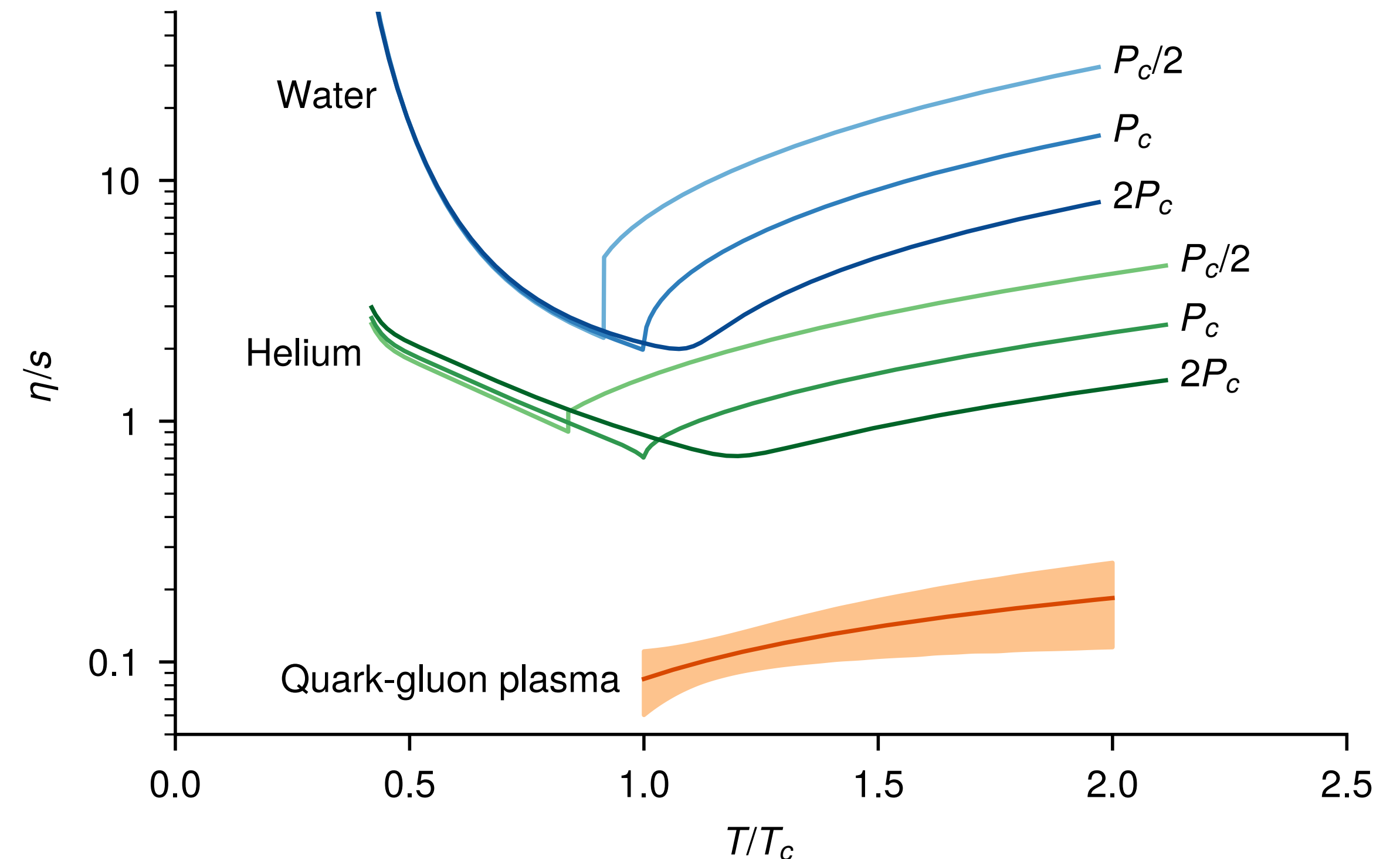
Substantial collective flow is observed in collisions between large nuclei at BNL RHIC (Relativistic Heavy Ion Collider) as evidenced by single-particle transverse momentum distributions and by azimuthal correlations among the produced particles. The data are well reproduced by perfect fluid dynamics. A calculation of the dimensionless ratio of shear viscosity  $\eta$  to entropy density  $s$  by Kovtun, Son, and Starinets within anti-de Sitter space/conformal field theory yields  $\eta/s = \hbar/4\pi k_B$ , which has been conjectured to be a lower bound for any physical system. Motivated by these results, we show that the transition from hadrons to quarks and gluons has behavior similar to helium, nitrogen, and water at and near their phase transitions in the ratio  $\eta/s$ . We suggest that experimental measurements can pinpoint the location of this transition or rapid crossover in QCD.

DOI: [10.1103/PhysRevLett.97.152303](https://doi.org/10.1103/PhysRevLett.97.152303)

PACS numbers: 12.38.Mh, 24.10.Nz, 25.75.Nq, 51.20.+d



Jonah E. Bernhard, J. Scott Moreland & Steffen A. Bass,  
Nature Physics **15** (2019) 1113-1117



- more than a decade of hard work by multiple research groups
- cooperation between theory & experiment
- significant investment by the funding agencies

# **Telescopes for the Early Universe: Heavy-Ion Collider Facilities**



# Heating & Compressing QCD Matter

The only way to heat & compress QCD matter under controlled laboratory conditions is by colliding two heavy atomic nuclei!



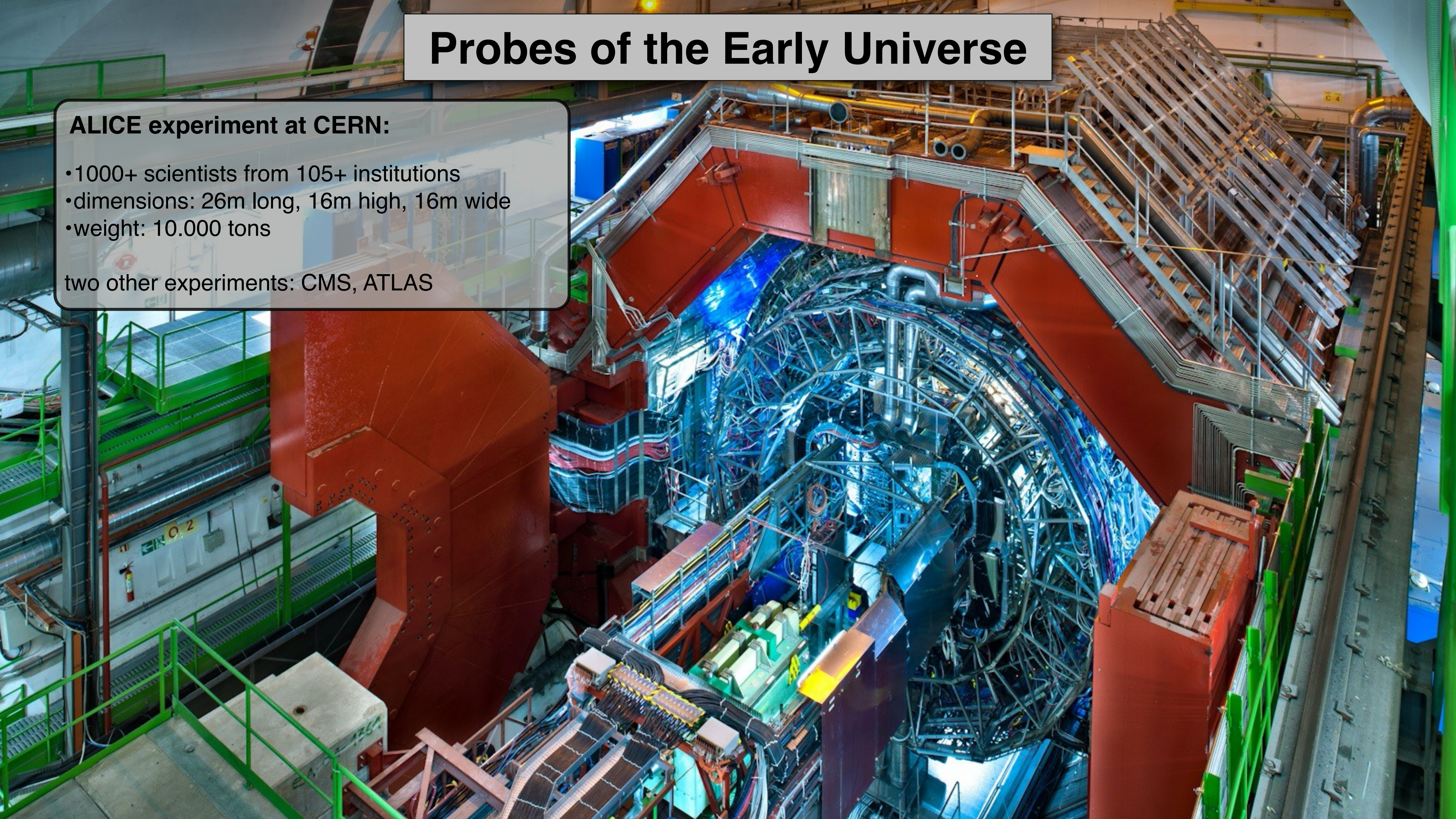


# Probes of the Early Universe

## ALICE experiment at CERN:

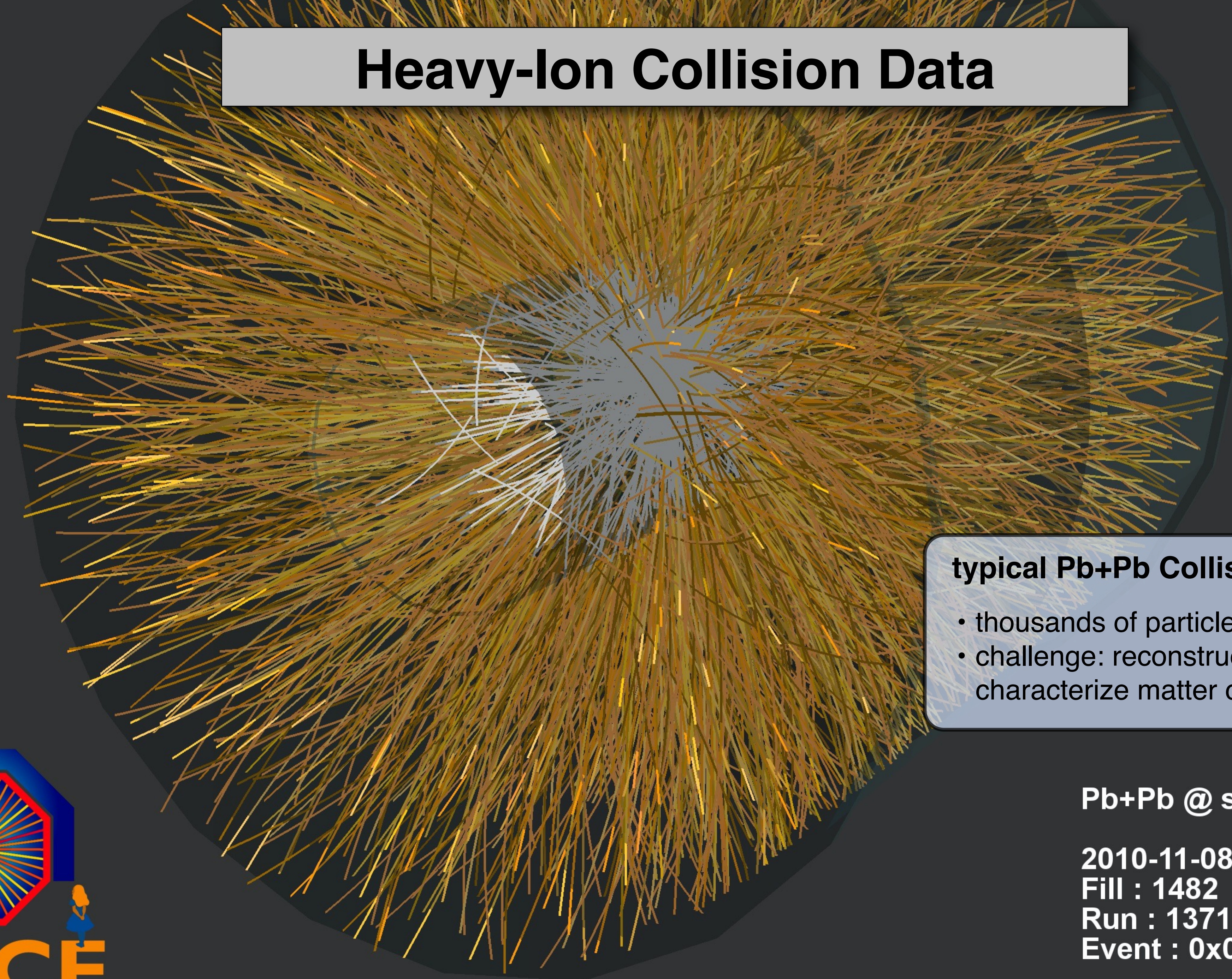
- 1000+ scientists from 105+ institutions
- dimensions: 26m long, 16m high, 16m wide
- weight: 10.000 tons

two other experiments: CMS, ATLAS





# Heavy-Ion Collision Data



## typical Pb+Pb Collision at the LHC:

- thousands of particle tracks
- challenge: reconstruction of final state to characterize matter created in collision



**Pb+Pb @  $\sqrt{s} = 2.76$  ATeV**

**2010-11-08 11:29:52**

**Fill : 1482**

**Run : 137124**

**Event : 0x0000000042B1B693**



# **Transport Theory: Connecting Data to Knowledge**

# Transport Theory

## microscopic transport models based on the Boltzmann Equation:

- transport of a system of microscopic particles
- all interactions are based on **binary scattering**

$$\left[ \frac{\partial}{\partial t} + \frac{\vec{p}}{E} \times \frac{\partial}{\partial \vec{r}} \right] f_1(\vec{p}, \vec{r}, t) = \sum_{\text{processes}} C(\vec{p}, \vec{r}, t)$$

## diffusive transport models based on the Langevin Equation:

- transport of a system of microscopic particles in a thermal medium
- interactions contain a **drag term** related to the properties of the medium and a **noise term** representing random collisions

$$\vec{p}(t + \Delta t) = \vec{p}(t) - \frac{\kappa}{2T} \vec{v} \cdot \Delta t + \vec{\xi}(t) \Delta t$$

## (viscous) relativistic fluid dynamics:

- transport of macroscopic degrees of freedom
- based on conservation laws:

$$\begin{aligned} \partial_\mu T^{\mu\nu} &= 0 \\ T_{ik} &= \varepsilon u_i u_k + P (\delta_{ik} + u_i u_k) \\ &\quad - \eta \left( \nabla_i u_k + \nabla_k u_i - \frac{2}{3} \delta_{ik} \nabla \cdot u \right) \\ &\quad + \zeta \delta_{ik} \nabla \cdot u \end{aligned}$$

(plus an additional 9 eqns. for dissipative flows)

## hybrid transport models:

- combine microscopic & macroscopic degrees of freedom
- current state of the art for RHIC modeling

Each transport model relies on roughly a dozen physics parameters to describe the time-evolution of the collision and its final state. These physics parameters act as a representation of the information we wish to extract from RHIC & LHC.

# Computational Modeling

Time: 0.10

3+1D Hydro + Boltzmann Hybrid

rapidity

5.9

5

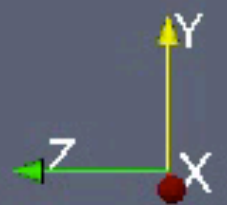
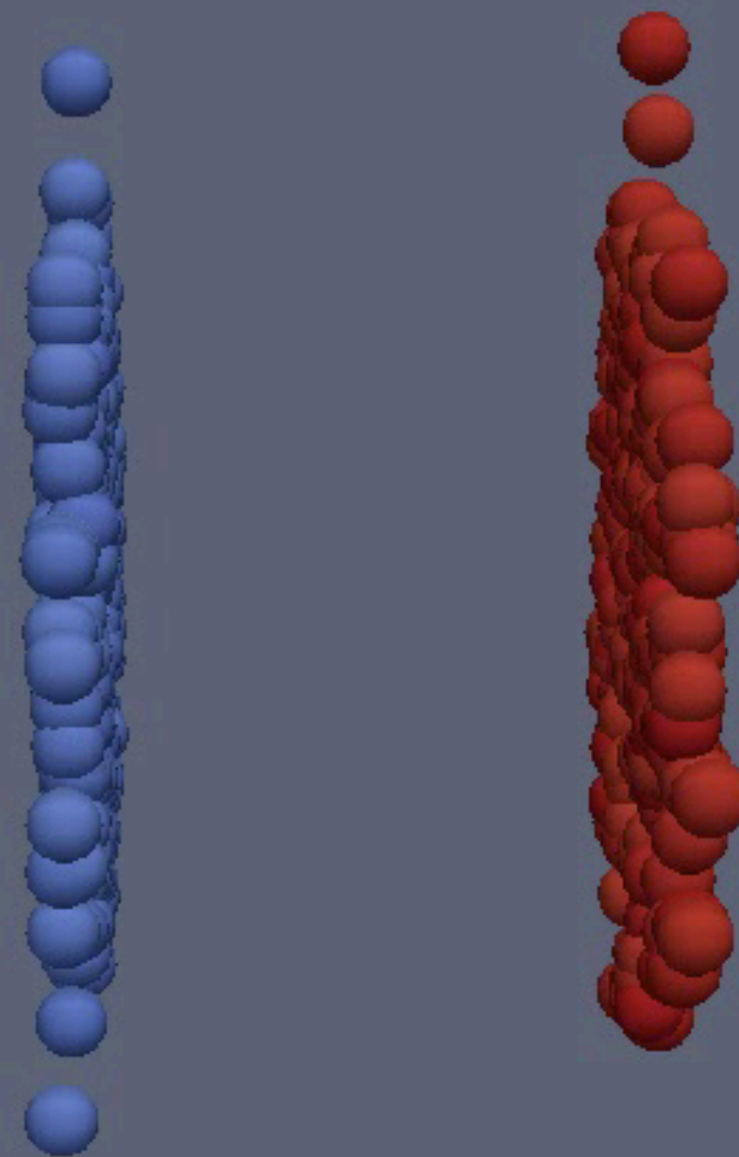
2.5

0

-2.5

-5

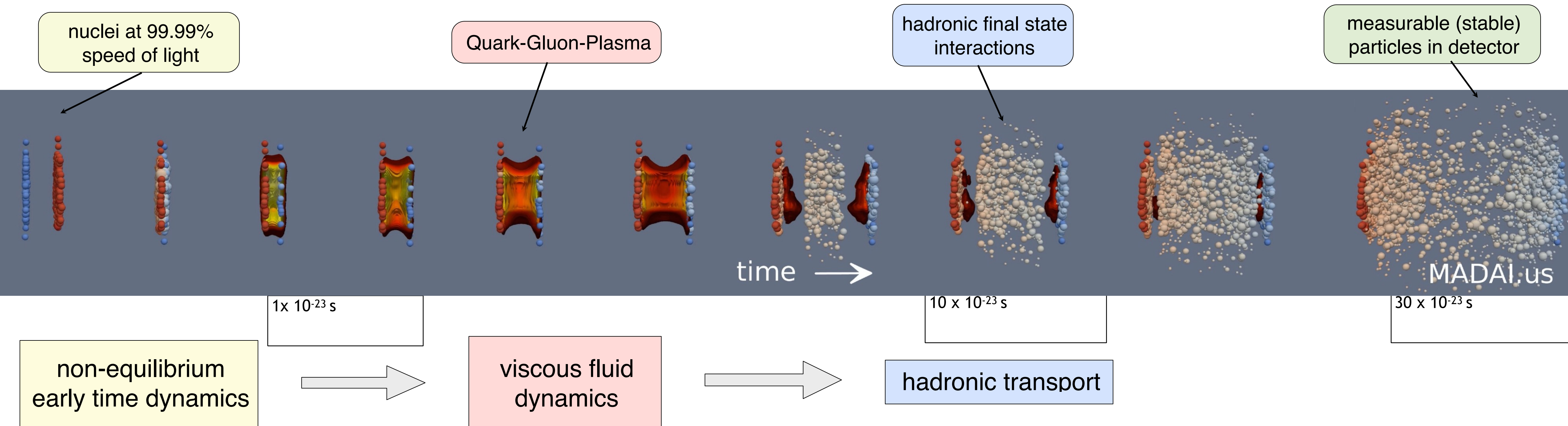
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*MADAI.us*



# Probing the QGP in Relativistic Heavy-Ion Collisions



## Principal Challenges of Probing the QGP with Heavy-Ion Collisions:

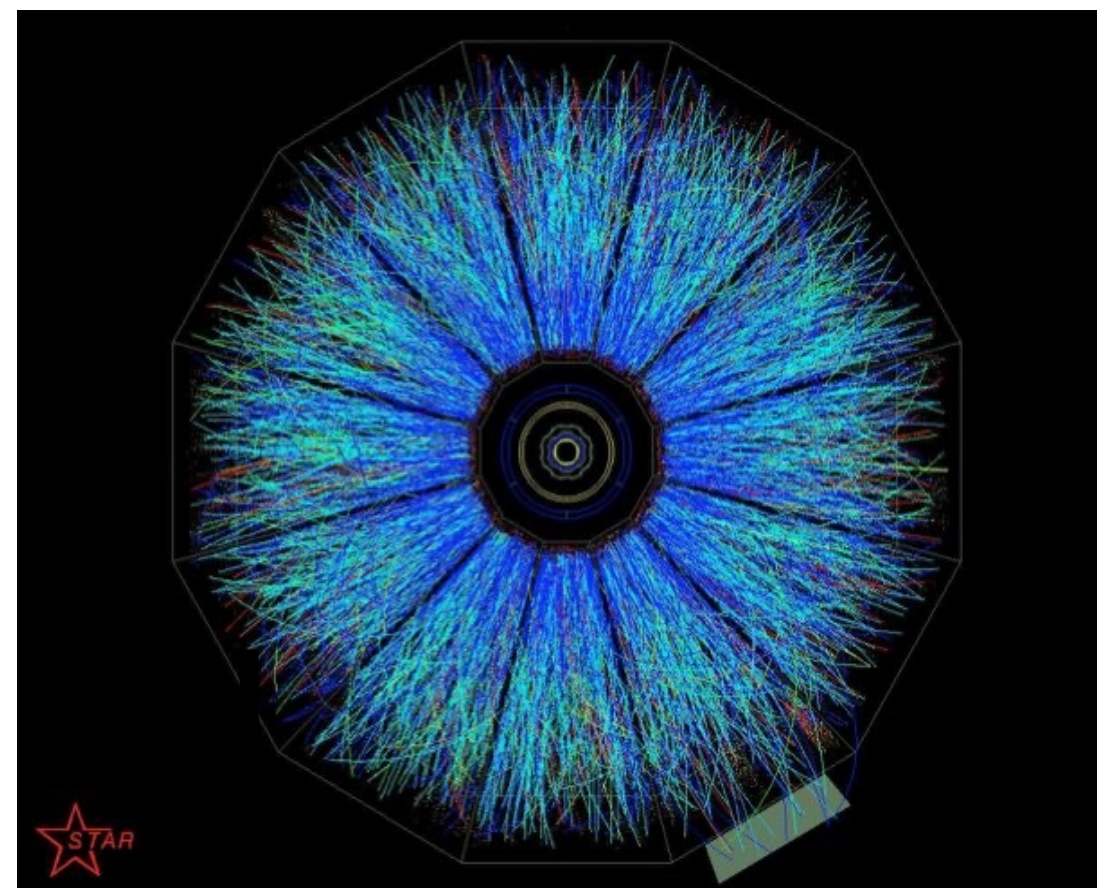
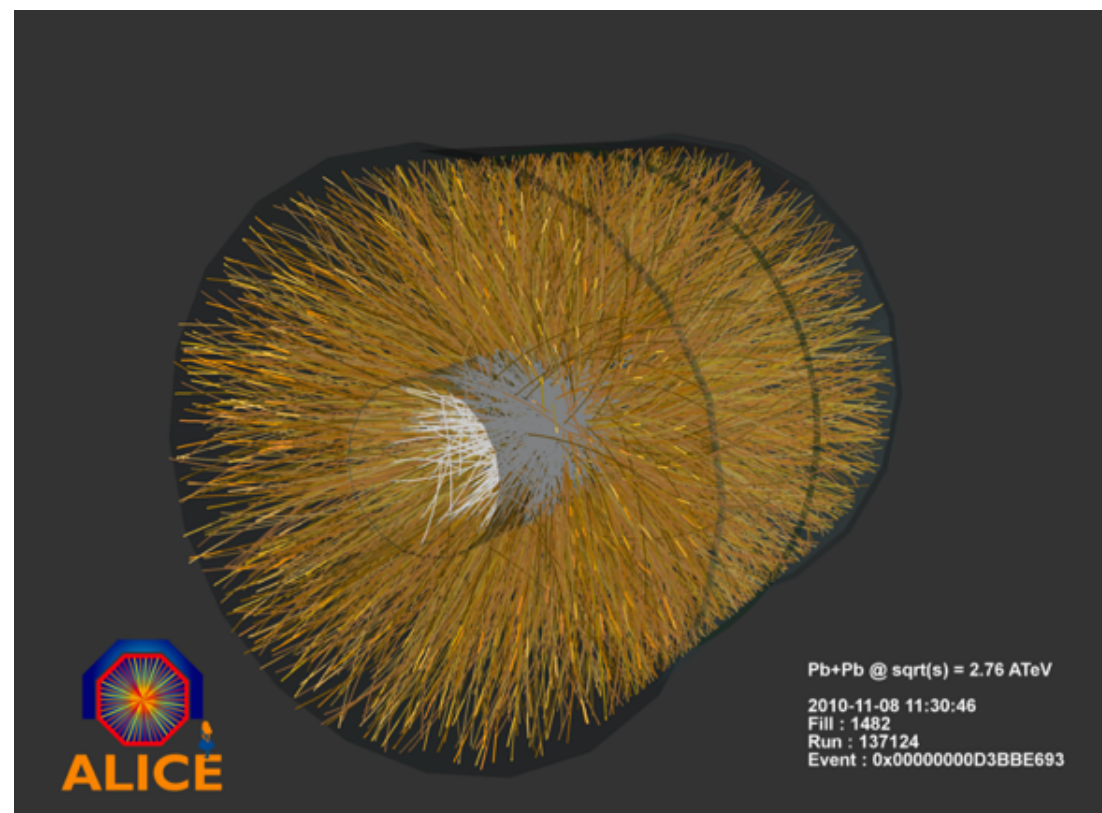
- time-scale of the collision process:  $10^{-24}$  seconds! [too short to resolve]
  - characteristic length scale:  $10^{-15}$  meters! [too small to resolve]
  - confinement: quarks & gluons form bound states, experiments don't observe them directly
- **computational models are needed to connect the experiments to QGP properties!**

# **Knowledge Extraction from Relativistic Heavy-Ion Collisions**



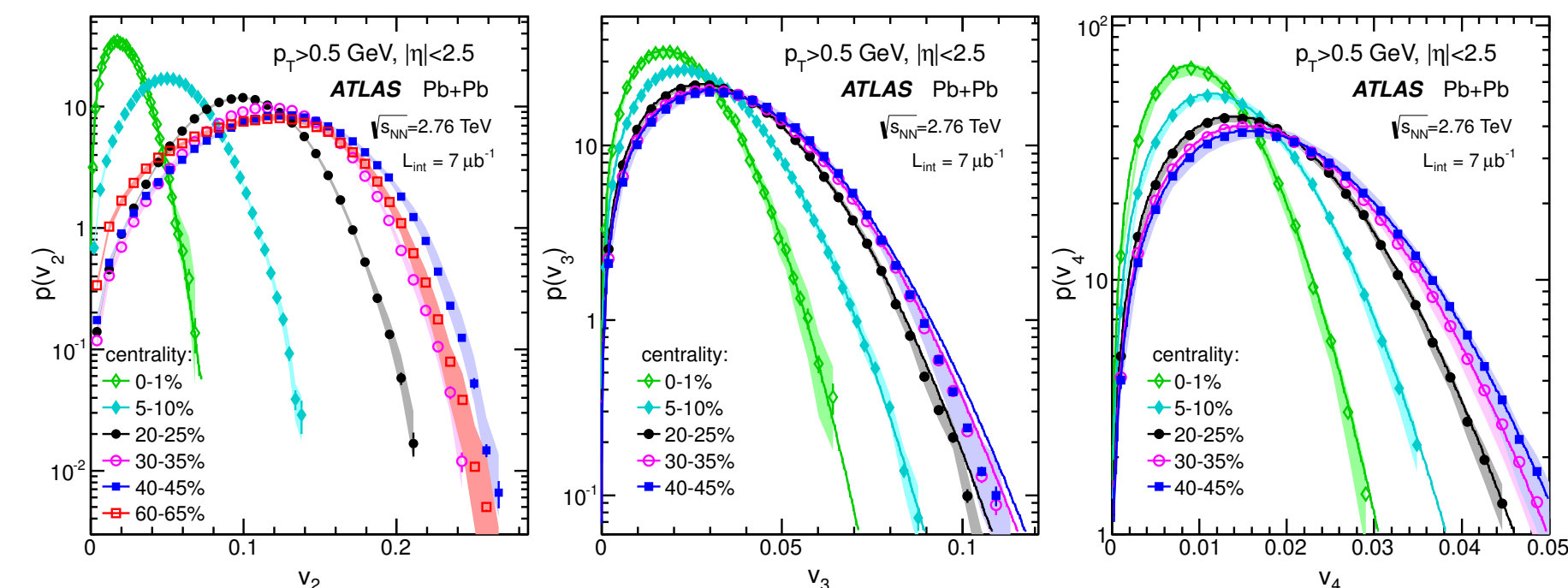
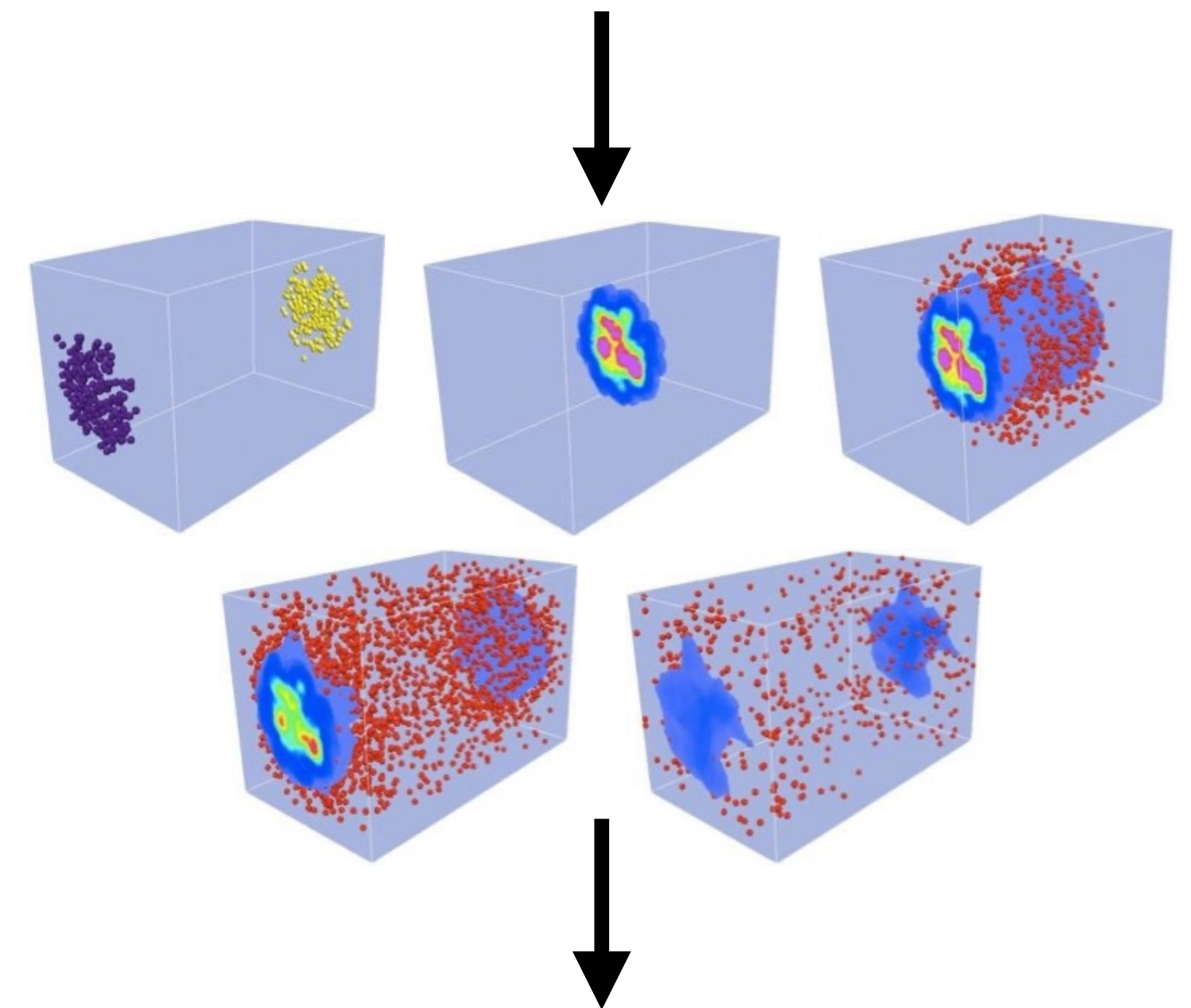
# Probing QCD in Heavy-Ion Collisions

**Data:**

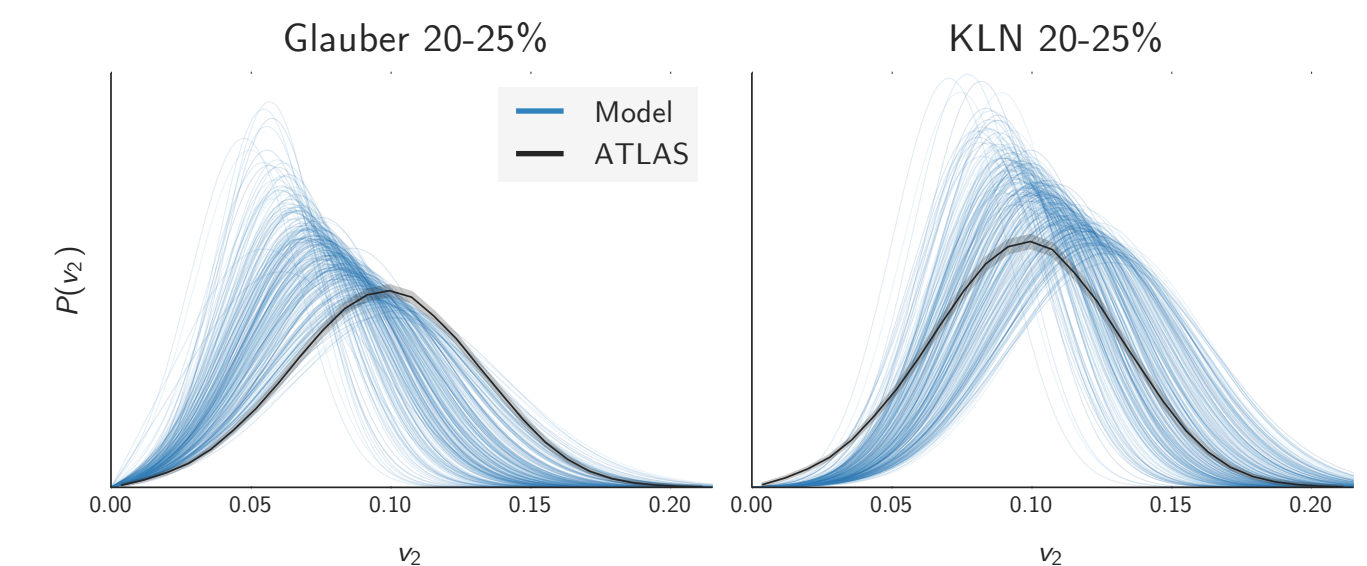


**Model:**

initial conditions,  $\tau_0$ ,  $\eta/s$ ,  $\zeta/s$ , ....

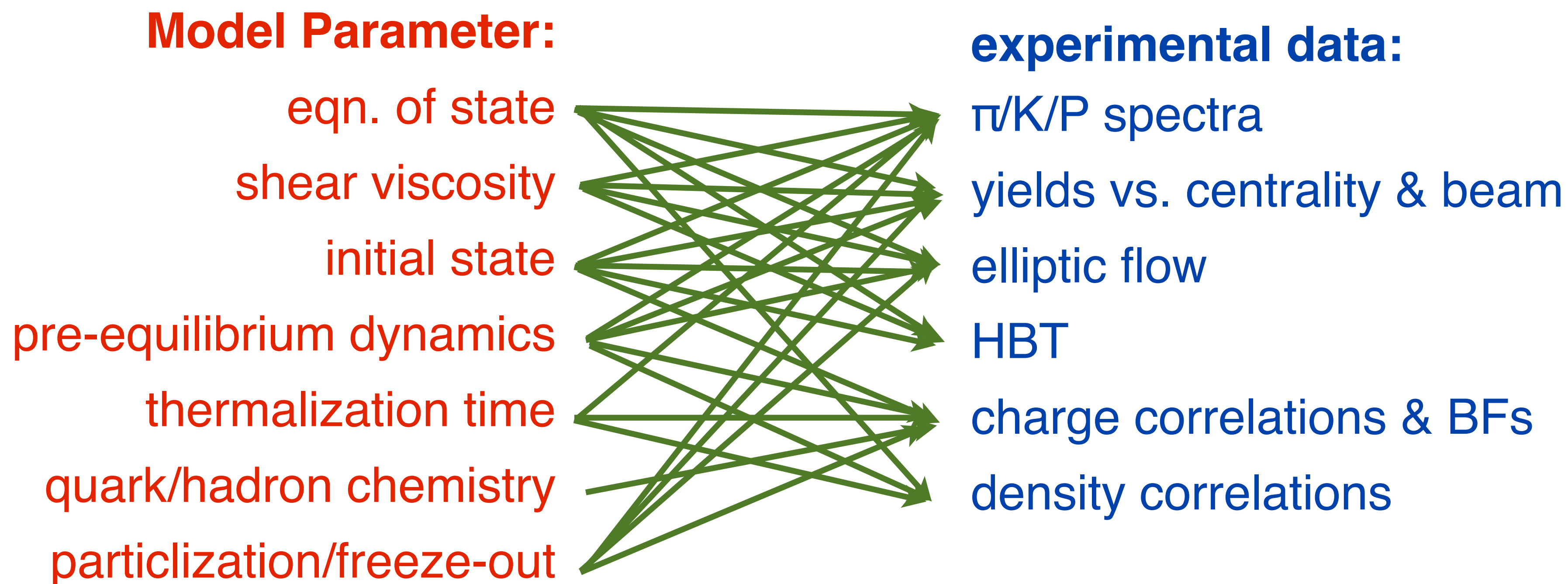


**extracted QGP properties:  $\eta/s$ , ...**





# Determining the QGP Properties via a Model to Data Comparison



- large number of interconnected parameters w/ non-factorizable data dependencies
  - data have correlated uncertainties
  - develop novel optimization techniques: Bayesian Statistics and MCMC methods
  - transport models require too much CPU: need new techniques based on emulators
  - general problem, not restricted to RHIC Physics
- **collaboration with Statistical Sciences**

# Bayesian Analysis

Each computational model relies on a set of physics parameters to describe the dynamics and properties of the system. These physics parameters act as a representation of the information we wish to extract from RHIC & LHC.

## Model Parameters - System Properties

- initial state
- temperature-dependent viscosities
- hydro to micro switching temperature

## Physics Model:

- Trento
- iEBE-VISHNU

Bayesian analysis

## Experimental Data

- ALICE flow & spectra

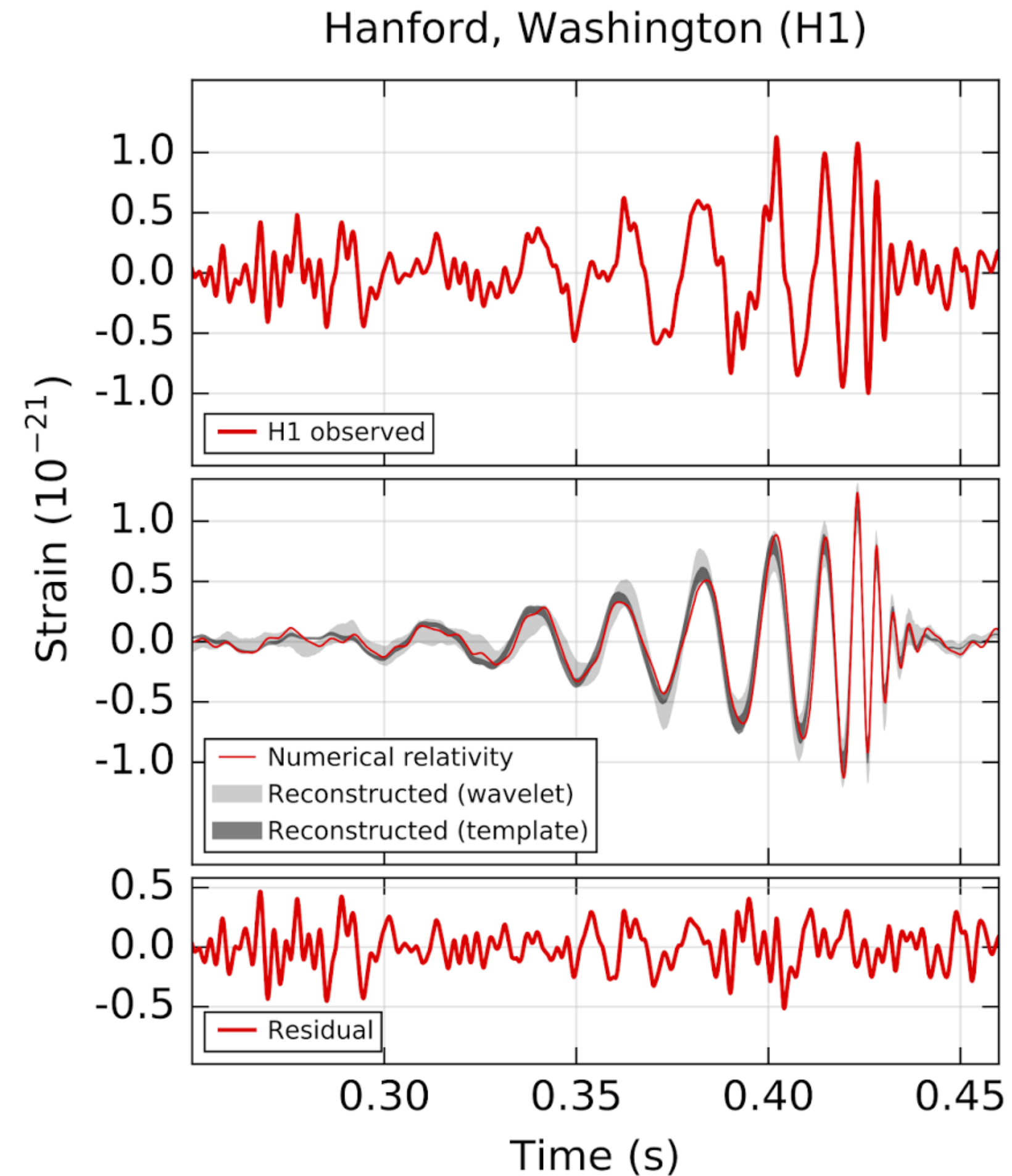
calculate observables & compare to data

estimate or calculate parameters

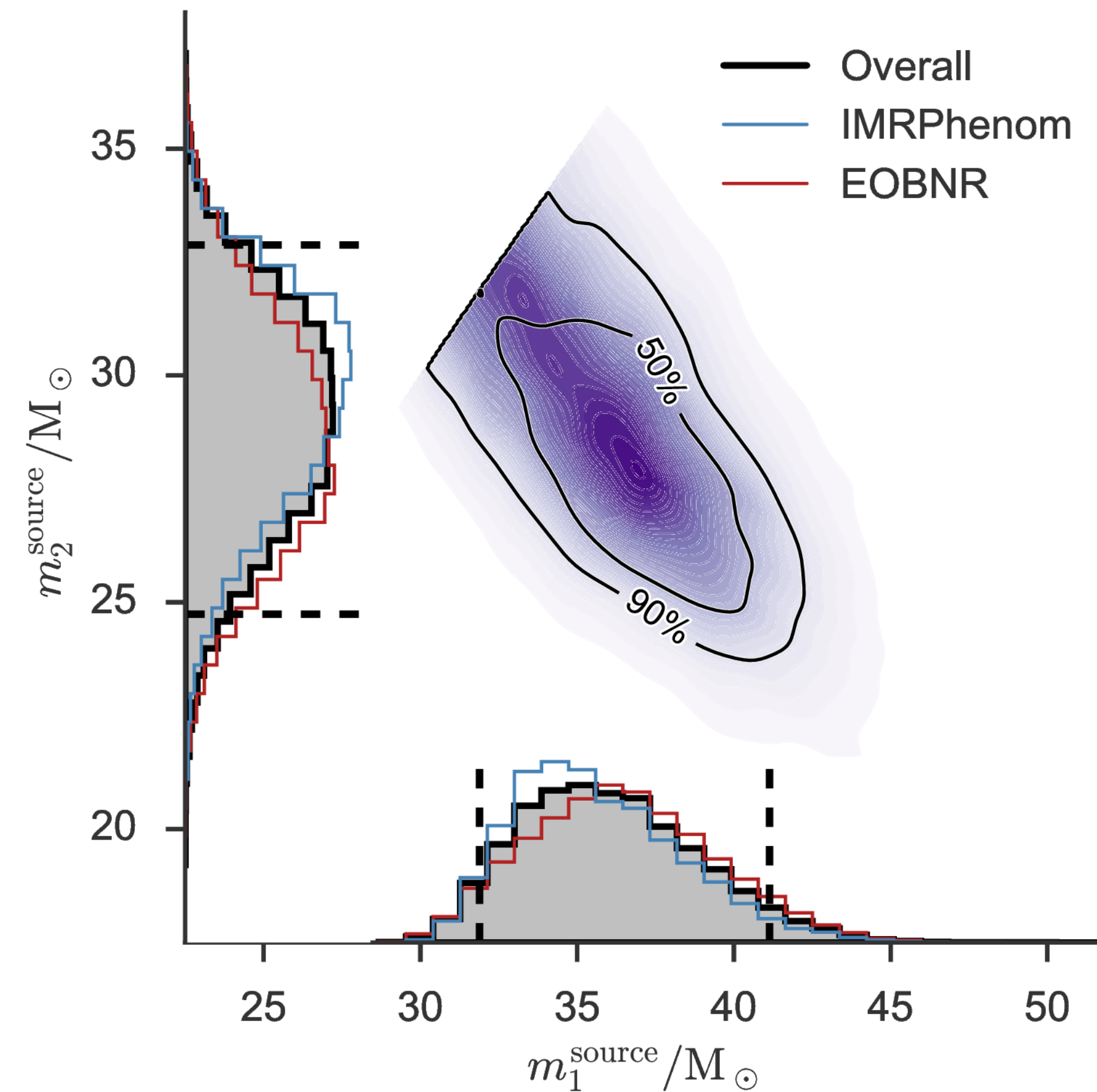
- Bayesian analysis allows us to simultaneously calibrate all model parameters via a model-to-data comparison
- determine parameter values such that the model best describes experimental observables
- extract the probability distributions of all parameters

# Example: Gravitational Waves

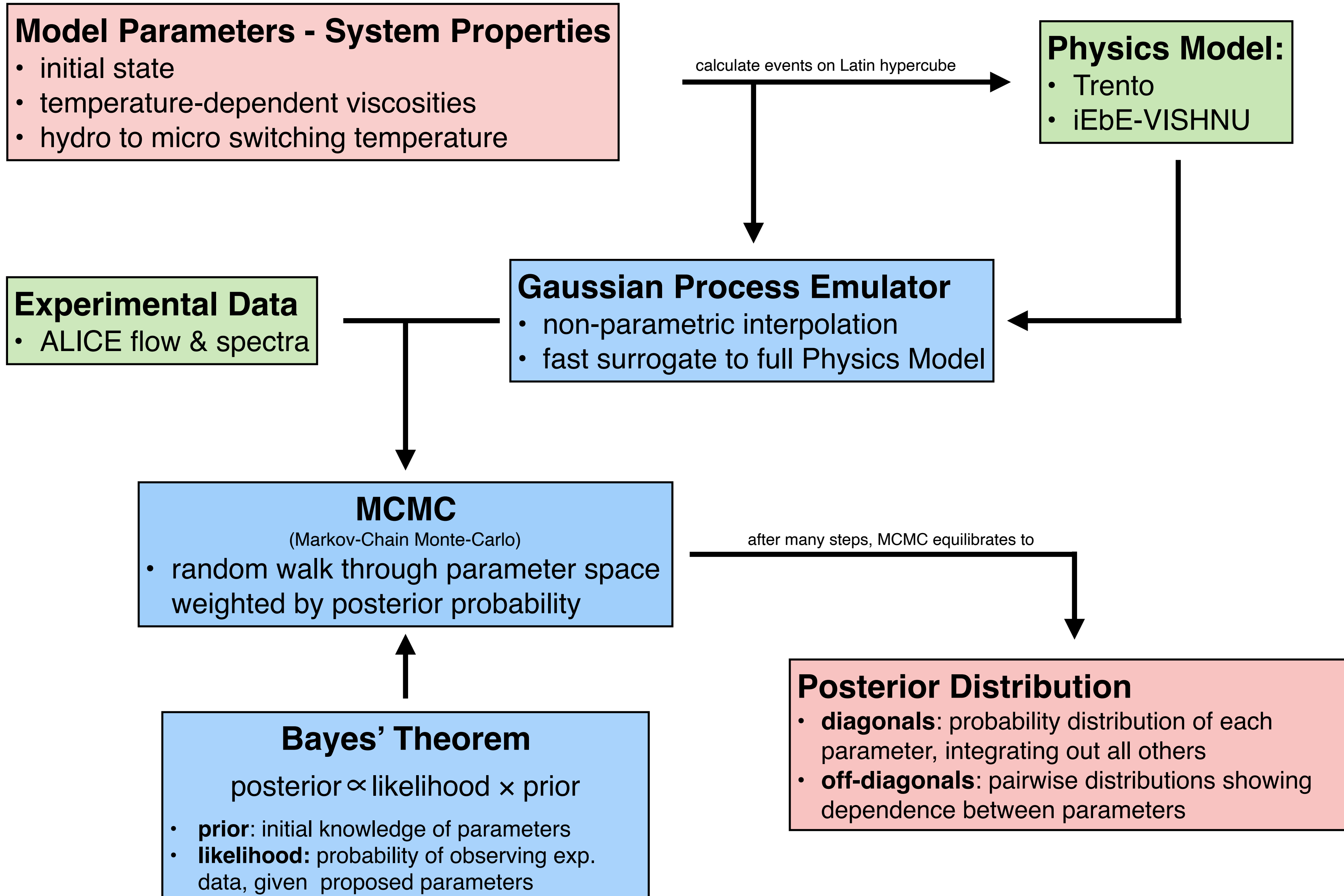
LIGO gravitational wave signal:



Bayesian analysis of GR model of merging black holes of masses  $m_1$  and  $m_2$  that is capable of reproducing LIGO data:



# Setup of a Bayesian Statistical Analysis





# **Components of the Bayesian Analysis**

# Methodology

## Model Parameters - System Properties

- initial state
- temperature-dependent viscosities
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calculate events on Latin hypercube

## Physics Model:

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- iEbE-VISHNU

## Experimental Data

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## Gaussian Process Emulator

- non-parametric interpolation
- fast surrogate to full Physics Model

## MCMC

(Markov-Chain Monte-Carlo)

- random walk through parameter space weighted by posterior probability

after many steps, MCMC equilibrates to

## Posterior Distribution

- **diagonals:** probability distribution of each parameter, integrating out all others
- **off-diagonals:** pairwise distributions showing dependence between parameters

## Bayes' Theorem

$$\text{posterior} \propto \text{likelihood} \times \text{prior}$$

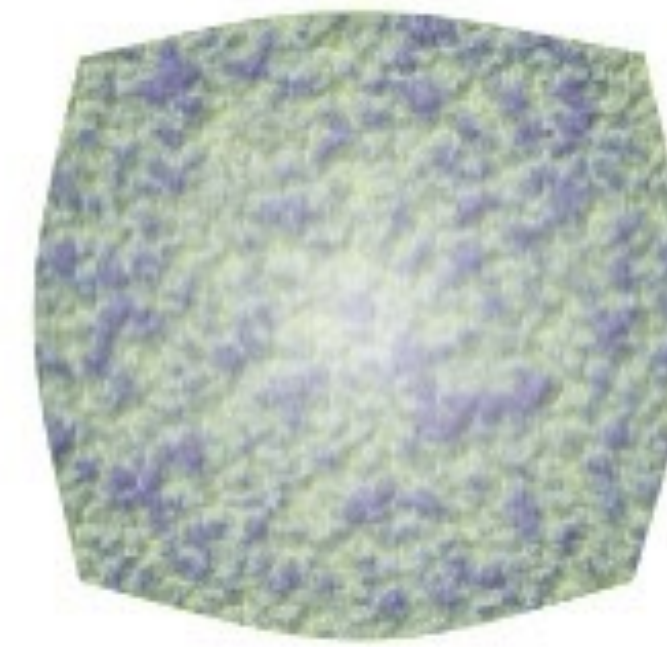
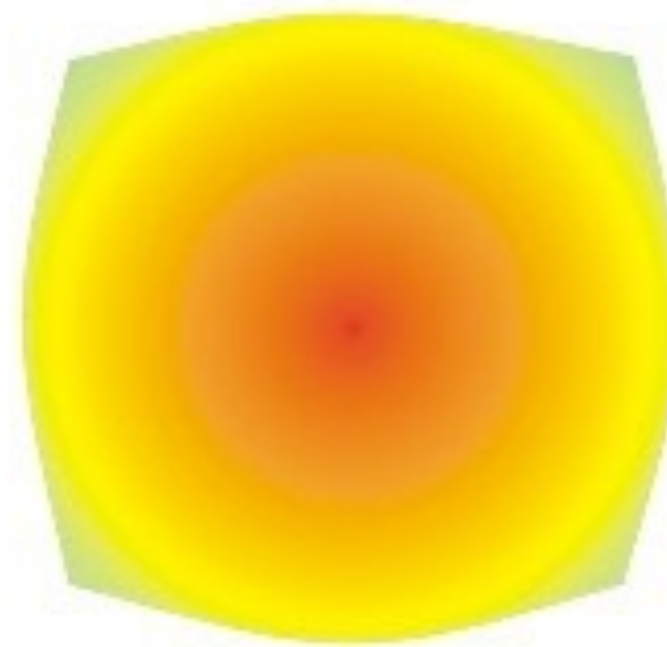
- **prior:** initial knowledge of parameters
- **likelihood:** probability of observing exp. data, given proposed parameters

# Physics Model: Trento + iEbE-VISHNU

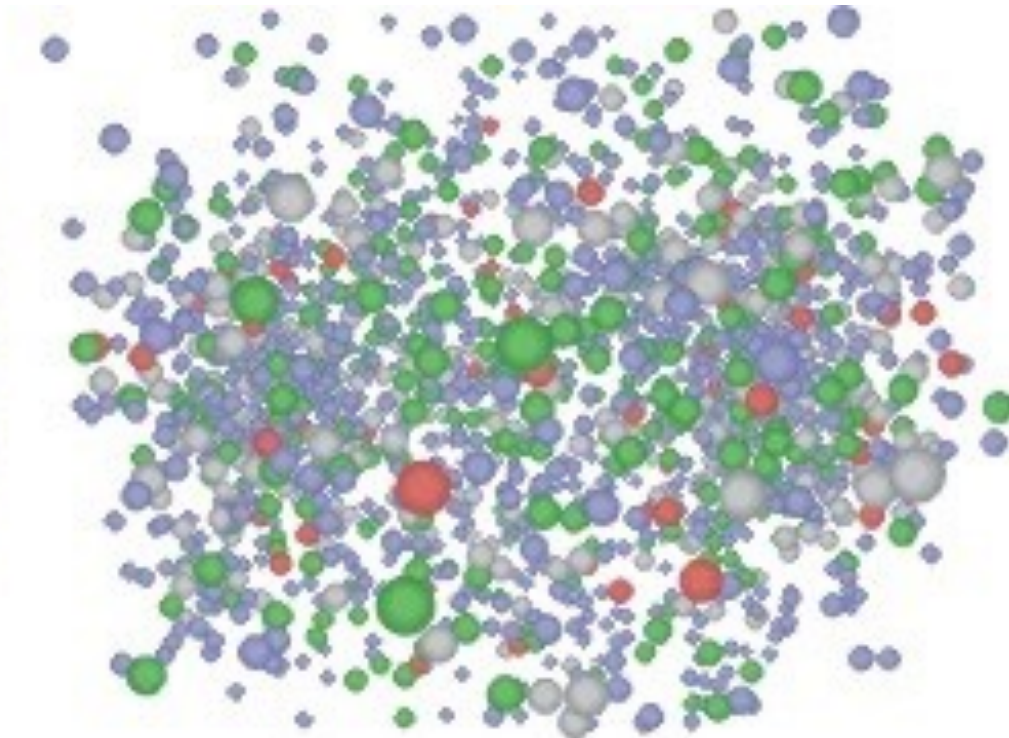
QGP and  
hydrodynamic expansion

hadronic phase  
and freeze-out

parameterized  
initial QGP state

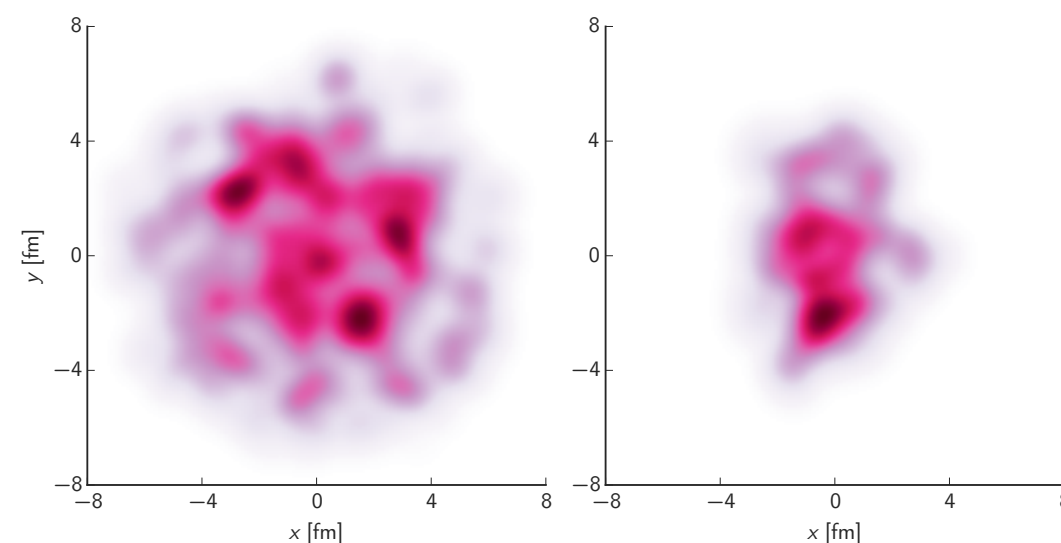


hadronization



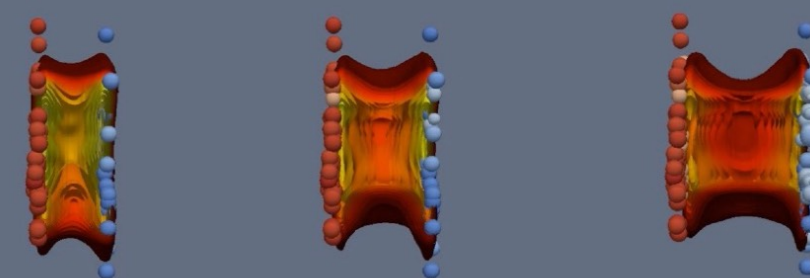
## Trento:

- parameterized initial condition model based on phenomenological concepts for entropy deposition to a QGP



## iEbE-VISHnew:

- EbE 2+1D viscous RFD
- describes QGP dynamics & hadronization
- EoS from Lattice QCD
- temperature-dependent shear and bulk viscosity as input



## UrQMD:

- non-equilibrium evolution of an interacting hadron gas
- hadron gas shear & bulk viscosities are implicitly contained in calculation



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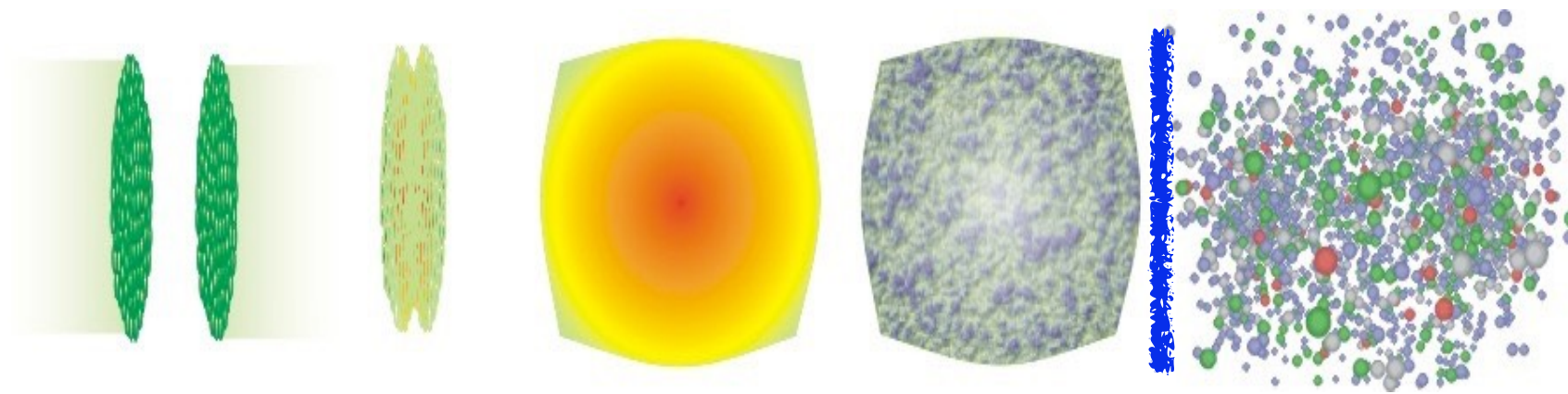
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- **prior:** initial knowledge of parameters
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# Calibration Parameters

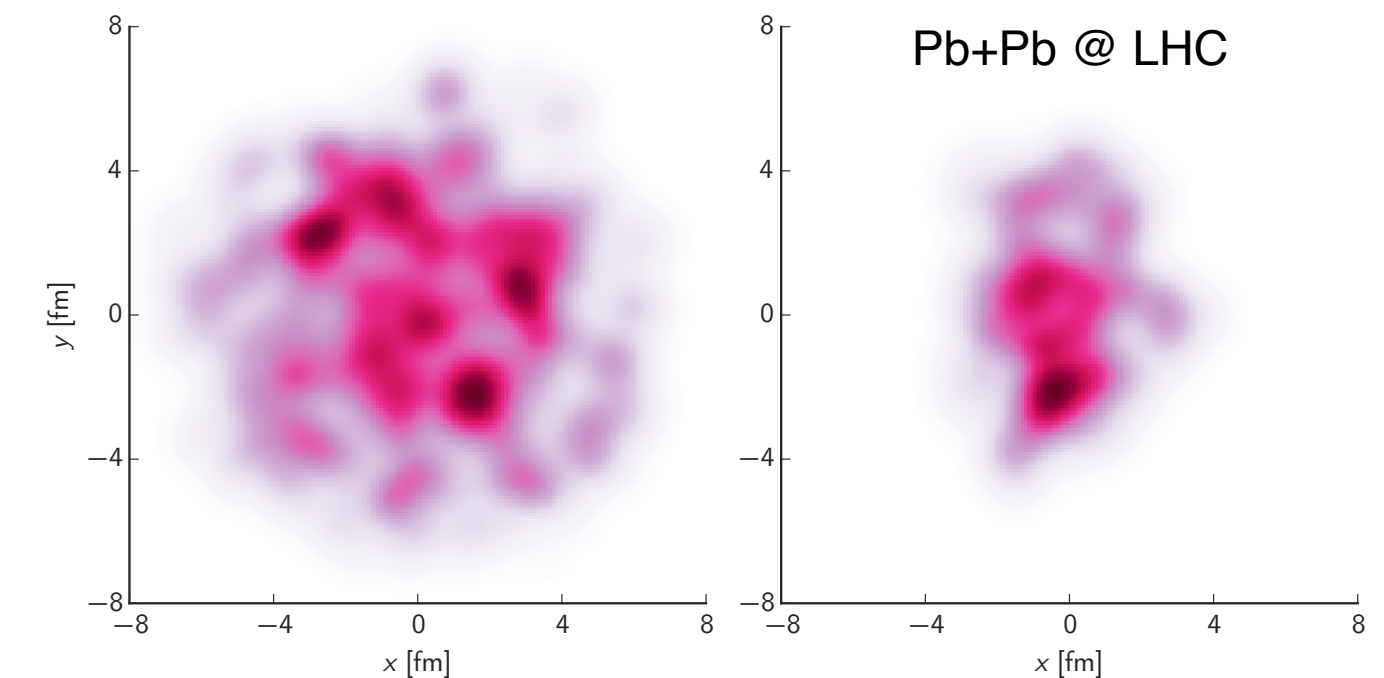
- the calibration parameters are the model parameters that codify the physical properties of the system that we wish to characterize with the analysis

- hydro to micro switching temperature  $T_{sw}$



## Trento initial condition:

- $p$ : attenuation parameter - entropy deposition
- $k$ : governs fluctuation in nuclear thickness
- $w$ : Gaussian nucleon width

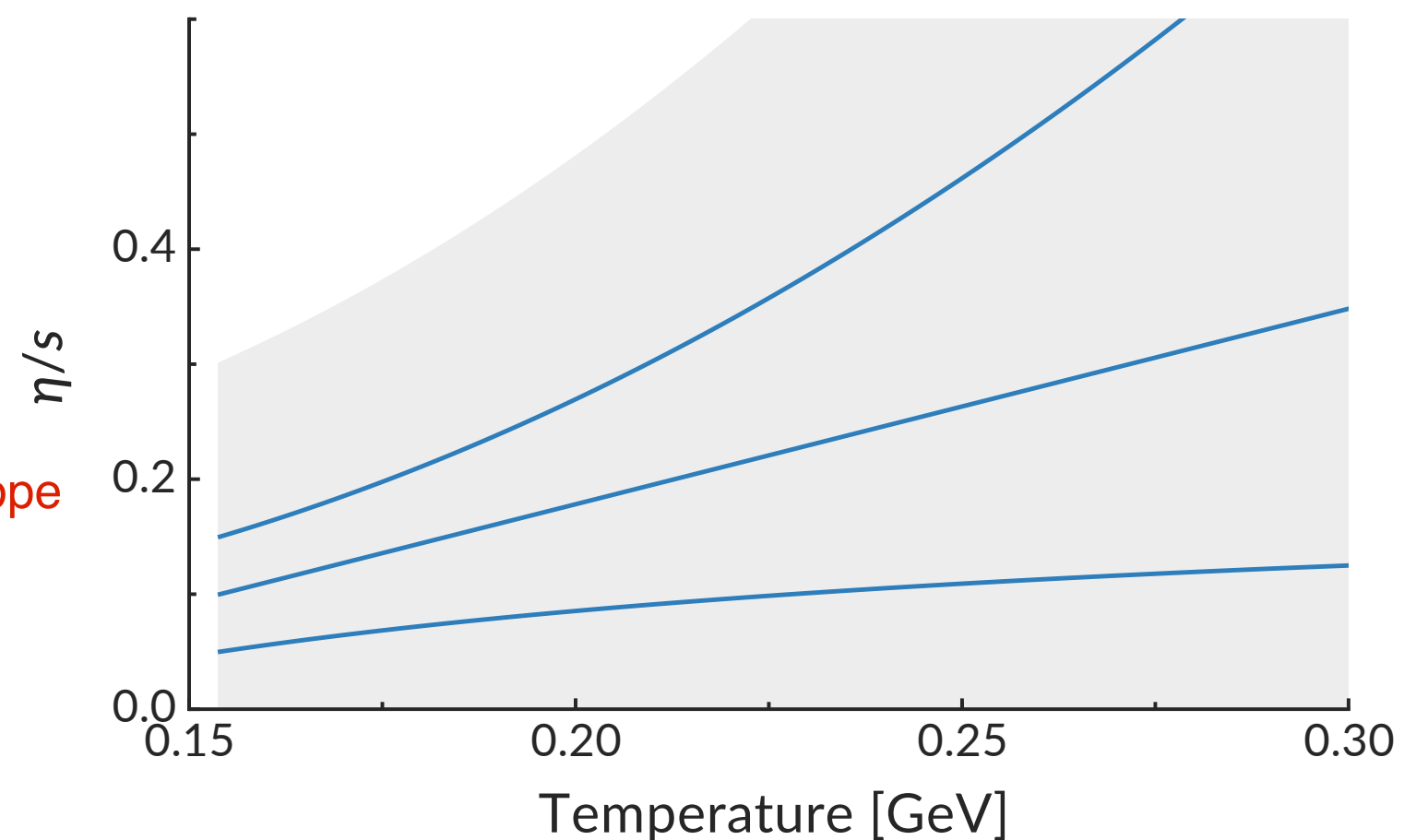


## temperature dependent shear viscosity:

$$\eta/s(T) = (\eta/s)_{\min} + (\eta/s)_{\text{slope}} \times (T - T_C) \times (T/T_C)^\beta$$

### parameters:

- intercept:  $(\eta/s)_{\min}$  at  $T_C$
- slope:  $(\eta/s)_{\text{slope}}$
- curvature:  $\beta$

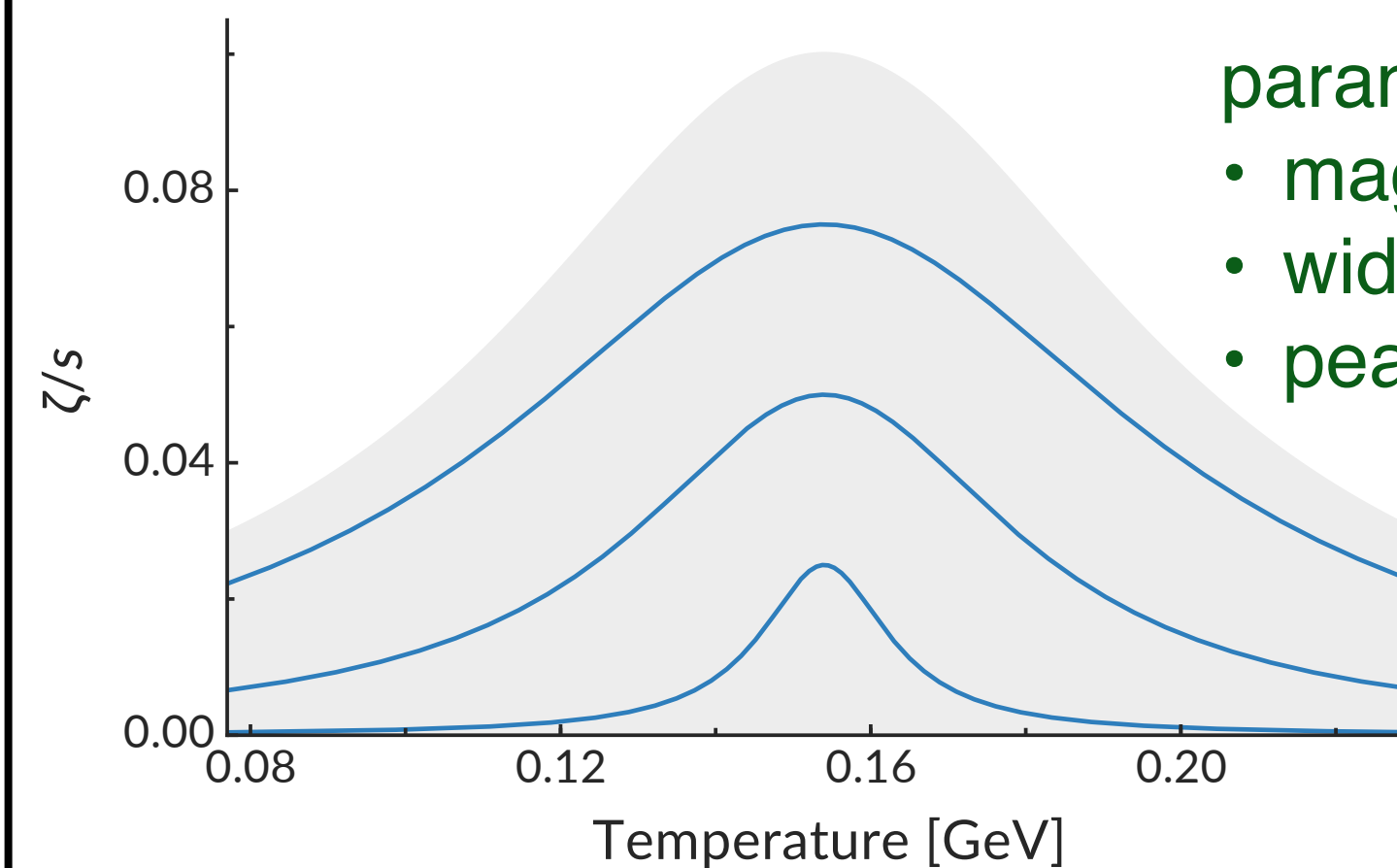


## temperature dependent bulk viscosity:

$$\zeta/s(T) = (\zeta/s)_{\max} / [1 + (T - (\zeta/s)_{\text{peak}})^2 / \Gamma^2]$$

### parameters:

- magnitude  $(\zeta/s)_{\max}$
- width:  $\Gamma$
- peak position:  $(\zeta/s)_{\text{peak}}$



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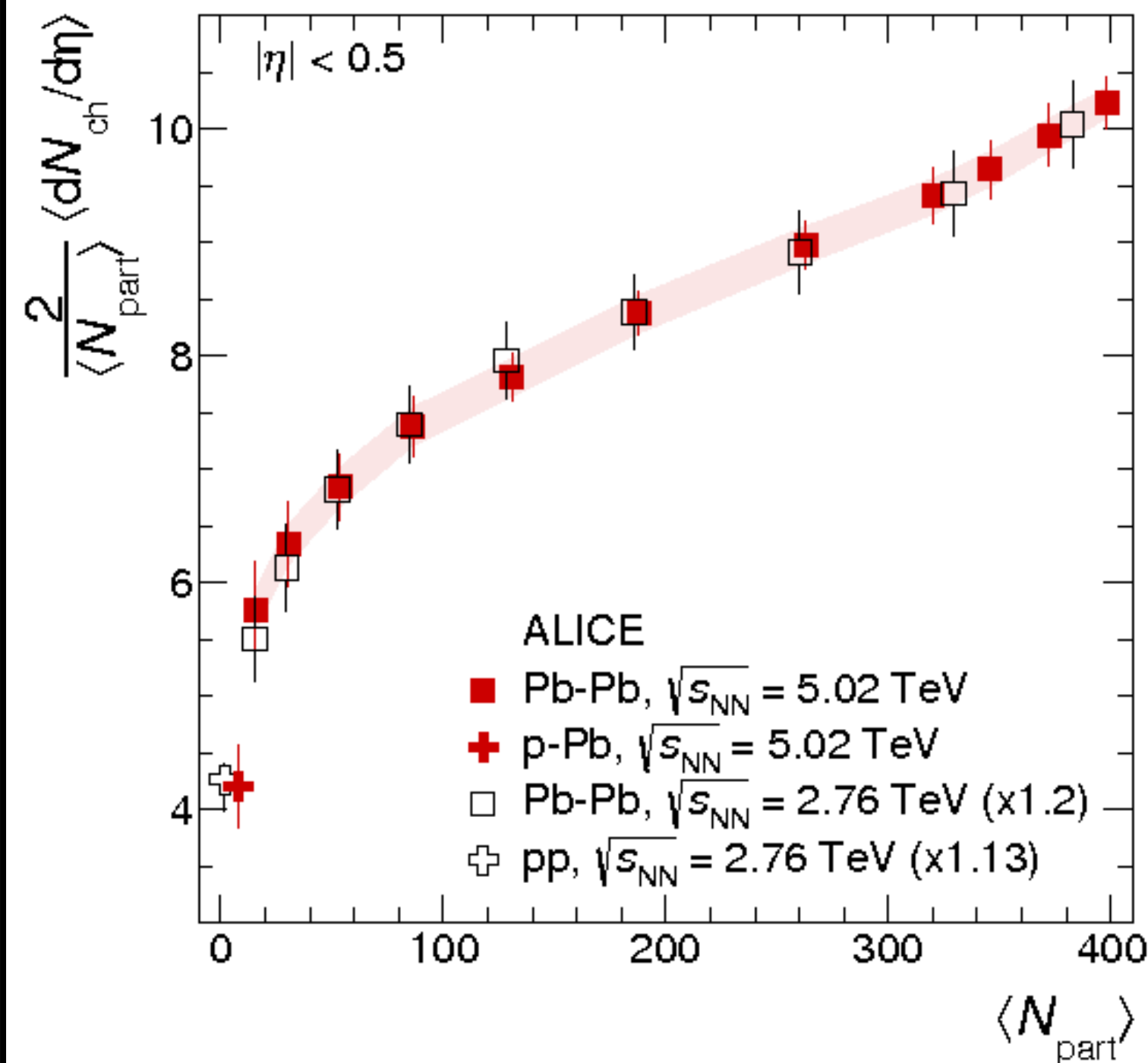
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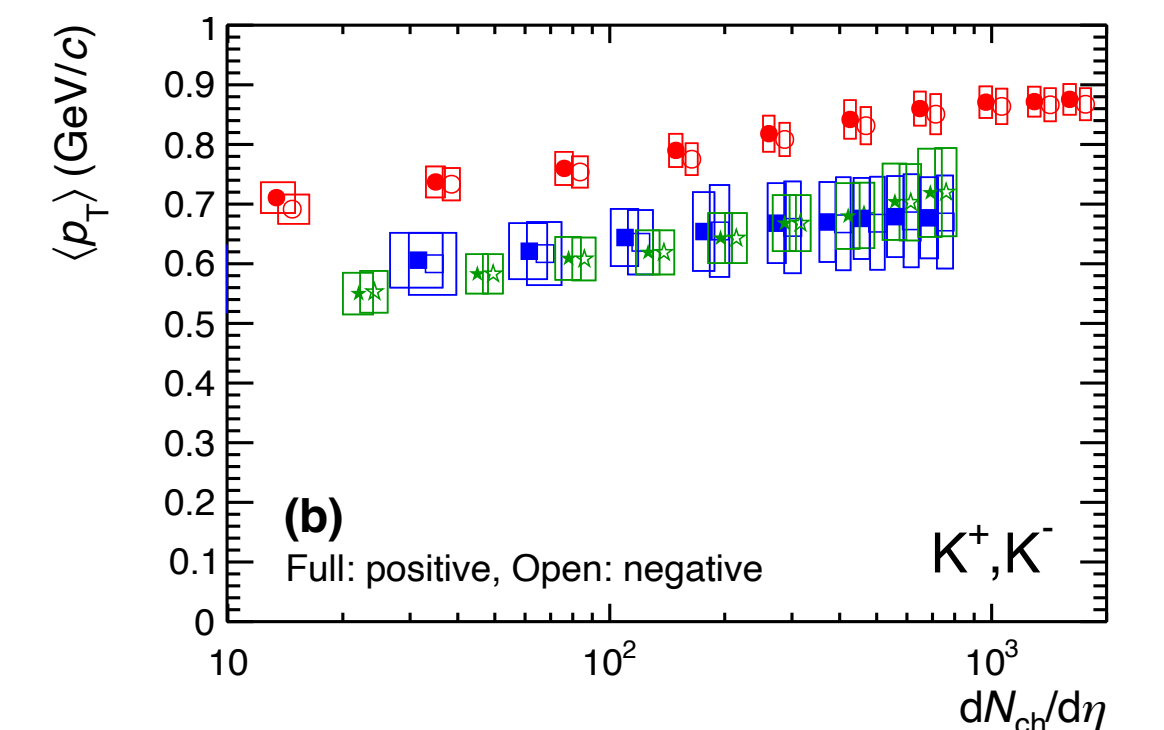
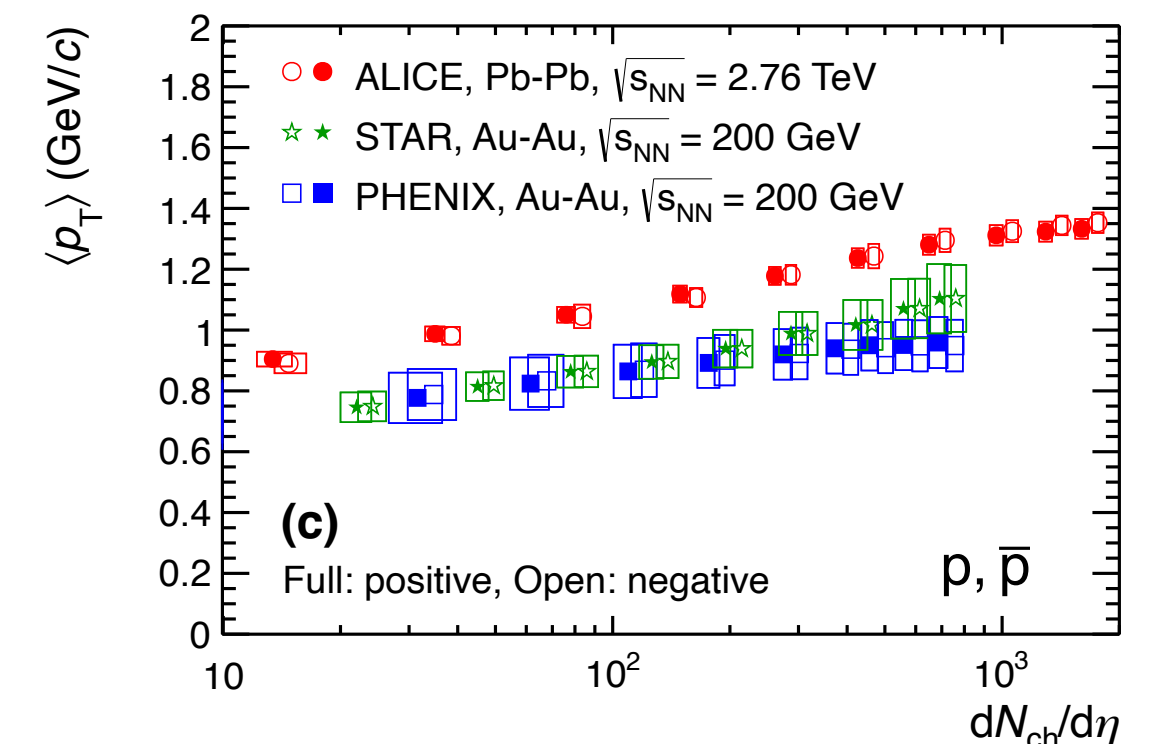
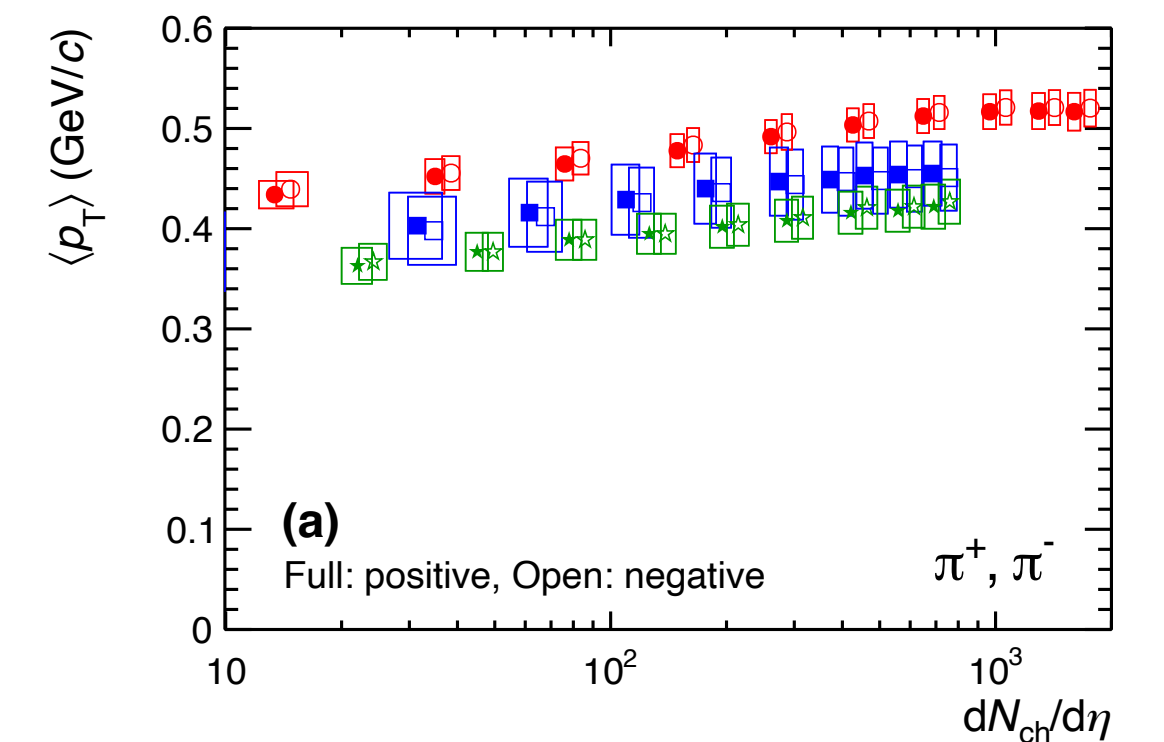
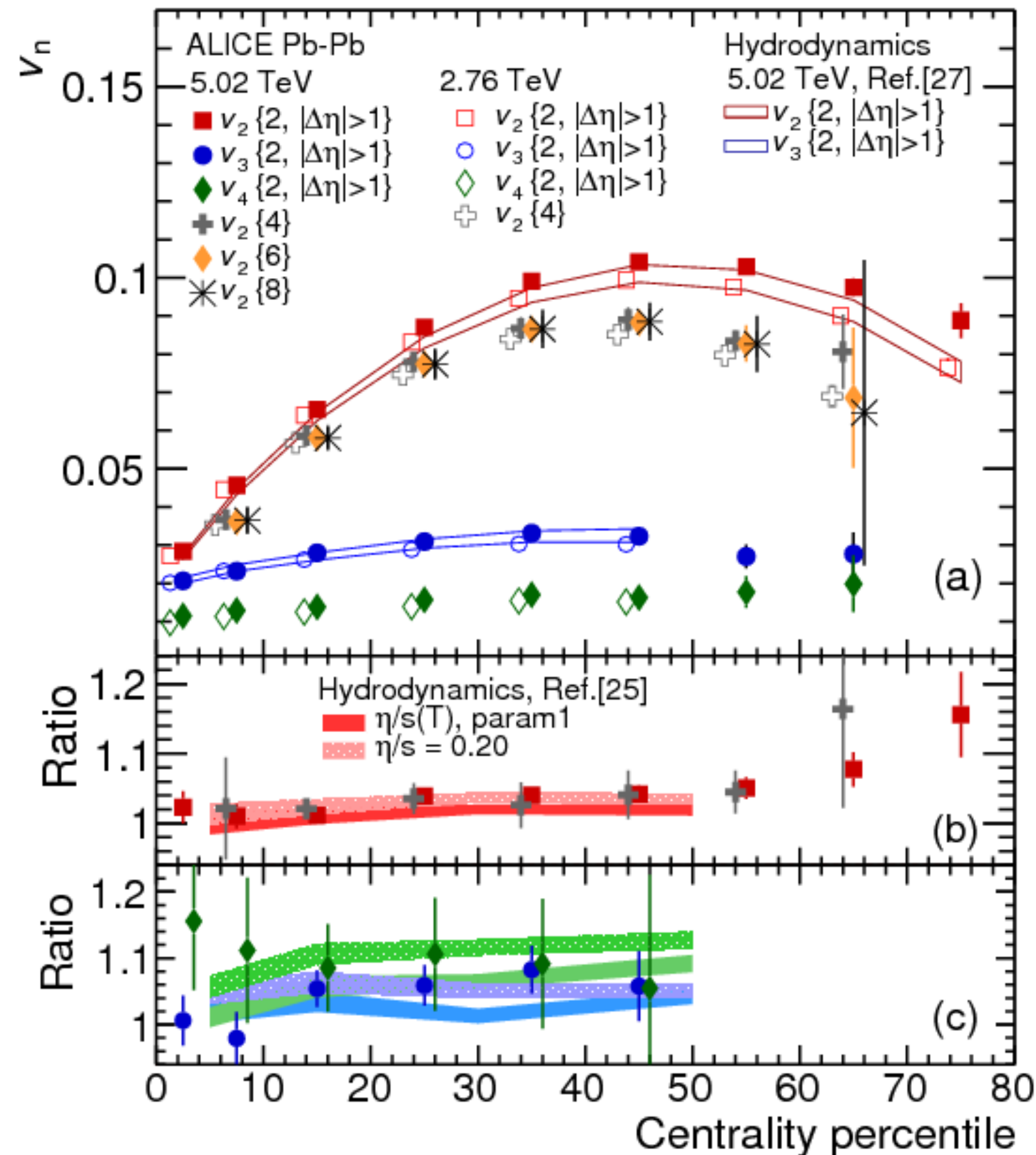
# Training Data

## Data:

- ALICE  $v_2$ ,  $v_3$  &  $v_4$  flow cumulants
- identified & charged particle yields
- identified particle mean  $p_T$
- 2 beam energies:  
2.76 & 5.02 TeV



**the entire success of the analysis depends on the quality of the exp. data!**



# Methodology

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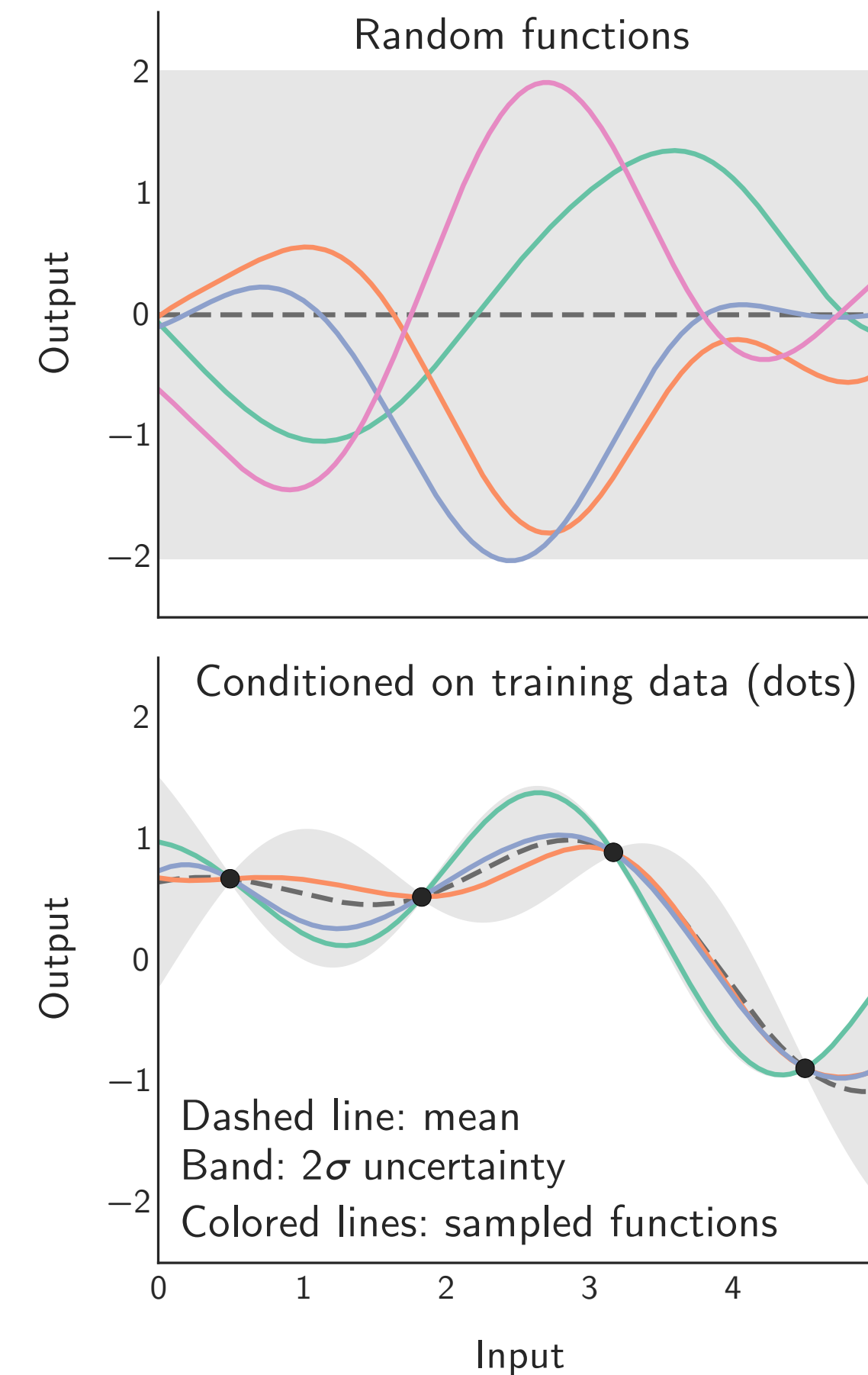
# Exploring the Model Parameter-Space

## Gaussian process:

- stochastic function:  
maps inputs to normally distributed outputs
- specified by mean and covariance functions

## GP as a model emulator:

- non-parametric interpolation of physics model
- predicts probability distributions for model output at any given input value
  - narrow near training points, wide in gaps
- needs to be conditioned on training data (Latin hypercube points)
- fast *surrogate* to actual model





# Computer Experiment Design

## Latin hypercube:

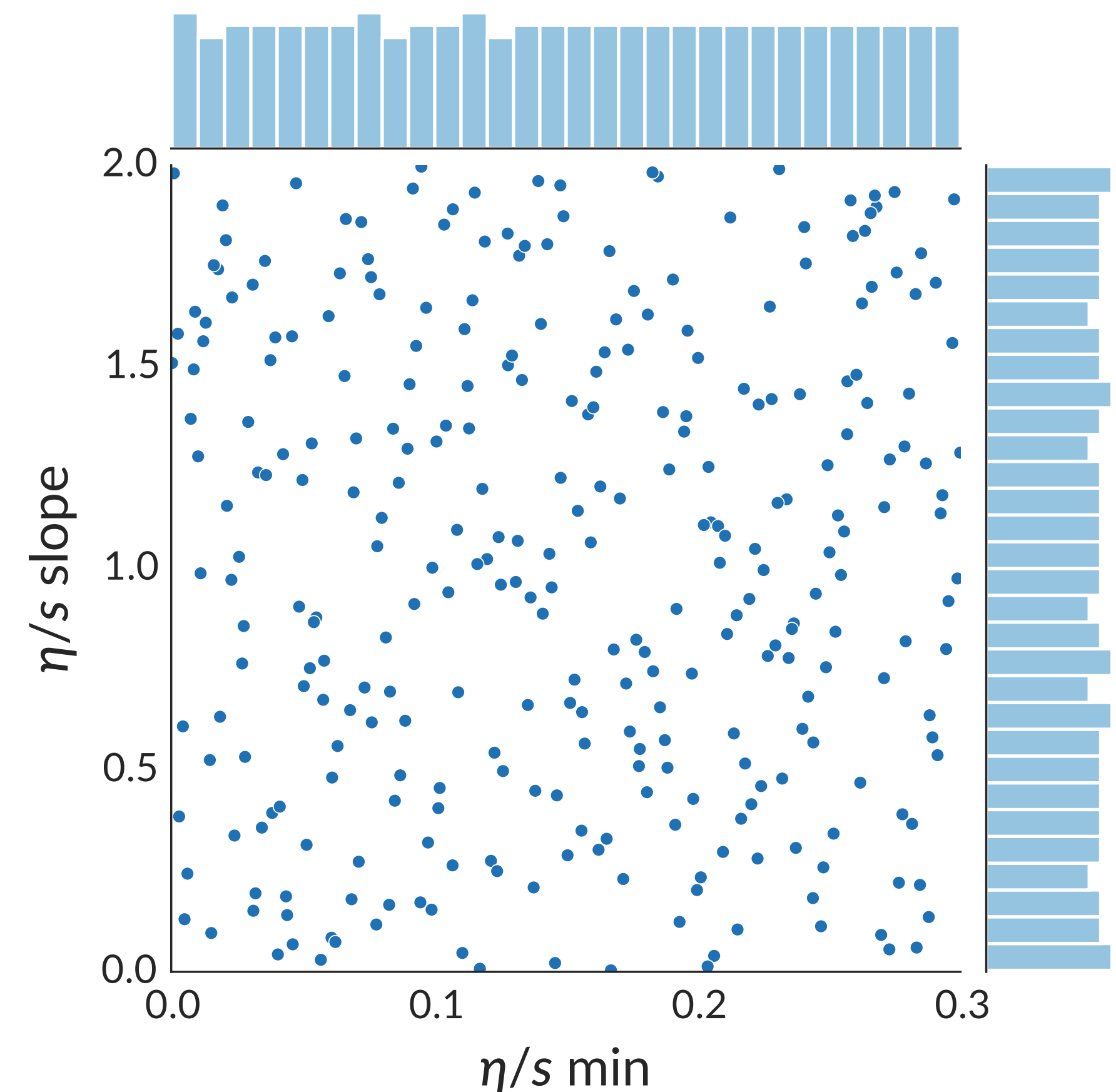
- algorithm for generating semi-randomized, space-filling points (here: maximin Latin hypercube)
- avoids large gaps and tight clusters
- all parameters varied simultaneously
- needs only  $m \geq 10n$  points, with  $n$ : number of model parameters

## this design:

- $n=15$  model parameters
- 9 centrality bins, 2 energies
- Latin hypercube with  $m=500$  points
- $\mathcal{O}(10^4)$  events per point, for a total of approx. 35,000,000 events
- use Gaussian Process Emulators to interpolate between points

## Example:

- Latin-hypercube projection for  $\eta/s$  parameters





# Computer Experiment Execution

Edison @ NERSC:

- Cray XC30: 5586 nodes w/ 24 cores each
- 2 hyperthreads per core
- 2.57 Petaflops/s

Duke QCD workflow:

- 1000 nodes per job: running on 48K cores simultaneously
- entire model design with 30M events can be computed in 1 day

## NOW COMPUTING

A small sample of massively parallel scientific computing jobs running right now at NERSC.

PROJECT	MACHINE	NODES	NERSC HOURS USED
NERSC Staff Accounts PI: Sudip S. Dosanjh, Lawrence Berkeley National Lab - NERSC	Cori KNL	1,008	115,874.8
NERSC Staff Accounts PI: Sudip S. Dosanjh, Lawrence Berkeley National Lab - NERSC	Cori KNL	1,008	77,866.5
Extraction of QCD transport coefficients from ultra-relativistic heavy-ion collisions through a Bayesian model to data analysis PI: Steffen A. Bass, Duke University	Edison	1,000	443,890.9
Extraction of QCD transport coefficients from ultra-relativistic heavy-ion collisions through a Bayesian model to data analysis PI: Steffen A. Bass, Duke University	Edison	1,000	399,224.3
Extraction of QCD transport coefficients from ultra-relativistic heavy-ion collisions through a Bayesian model to data analysis PI: Steffen A. Bass, Duke University	Edison	750	229,928.2
NERSC Staff Accounts PI: Sudip S. Dosanjh, Lawrence Berkeley National Lab - NERSC	Cori KNL	512	282,594.2



# Calibration

Vector of input parameters:  $\mathbf{x}=[p,k,w,(\eta/s)_{\min},(\eta/s)_{\text{slope}},(\zeta/s)_{\text{norm}},T_{\text{sw}},\dots]$

- assume true parameters  $\mathbf{x}_\star$  exist  $\Rightarrow$  find probability distribution for  $\mathbf{x}_\star$

- X: training data design points
- Y: model output on X

**Bayes' Theorem:**  $P(\mathbf{x}_\star | X, Y, \mathbf{y}_{\text{exp}}) \propto P(X, Y, \mathbf{y}_{\text{exp}} | \mathbf{x}_\star) P(\mathbf{x}_\star)$

- $P(\mathbf{x}_\star | X, Y, \mathbf{y}_{\text{exp}})$  = posterior  
 $\Rightarrow$  probability of  $\mathbf{x}_\star$  given observations  $(X, Y, \mathbf{y}_{\text{exp}})$

- $P(X, Y, \mathbf{y}_{\text{exp}} | \mathbf{x}_\star)$  = likelihood  
 $\Rightarrow$  probability of observing  $(X, Y, \mathbf{y}_{\text{exp}})$  given proposed  $\mathbf{x}_\star$

- $P(\mathbf{x}_\star)$  = prior  
 $\Rightarrow$  initial knowledge of  $\mathbf{x}_\star$

## Markov-Chain Monte-Carlo:

- random walk through parameter space weighted by posterior
- large number of samples  
 $\Rightarrow$  chain equilibrates to posterior distribution
- flat prior within design range, zero outside
- posterior  $\sim$  likelihood within design range, zero outside

## Likelihood and Uncertainty Quantification:

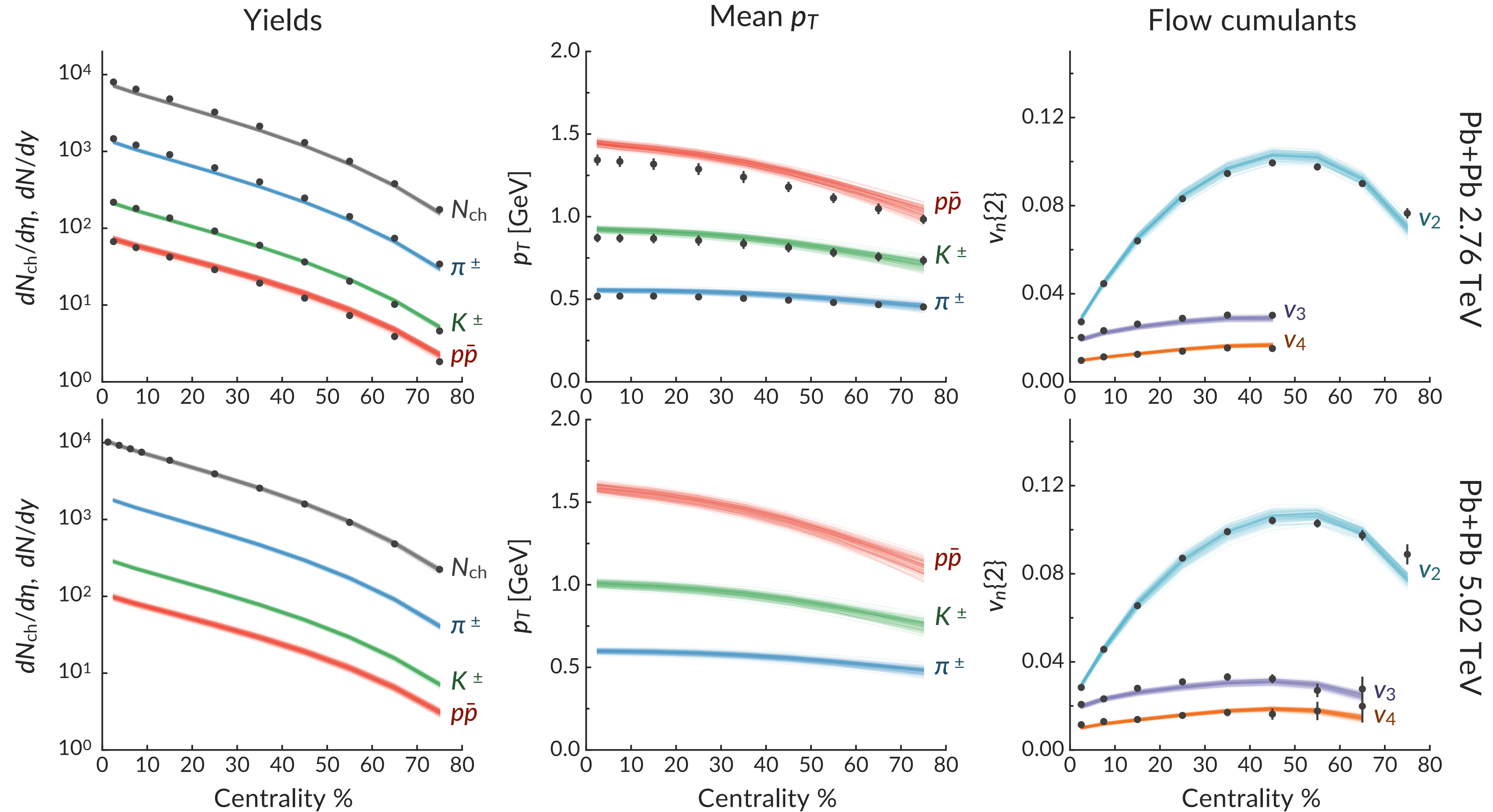
$$\text{Likelihood} \propto \exp[-1/2 (\mathbf{y} - \mathbf{y}_{\text{exp}})^\top \mathbf{\Sigma}^{-1} (\mathbf{y} - \mathbf{y}_{\text{exp}})]$$

- covariance matrix  $\mathbf{\Sigma} = \mathbf{\Sigma}_{\text{experiment}} + \mathbf{\Sigma}_{\text{model}}$
- $\mathbf{\Sigma}_{\text{experiment}} = \text{stat}(\text{diagonal}) + \text{sys}(\text{non-diagonal})$
- $\mathbf{\Sigma}_{\text{model}}$  conservatively estimated as 5%



# Prior vs. Posterior

**Posterior:** emulator predictions for highest likelihood parameter values





# Analysis Results



# Methodology

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calculate events on Latin hypercube

## Physics Model:

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## Experimental Data

- ALICE flow & spectra

## Gaussian Process Emulator

- non-parametric interpolation
- fast surrogate to full Physics Model

## MCMC

(Markov-Chain Monte-Carlo)

- random walk through parameter space weighted by posterior probability

after many steps, MCMC equilibrates to

## Posterior Distribution

- **diagonals:** probability distribution of each parameter, integrating out all others
- **off-diagonals:** pairwise distributions showing dependence between parameters

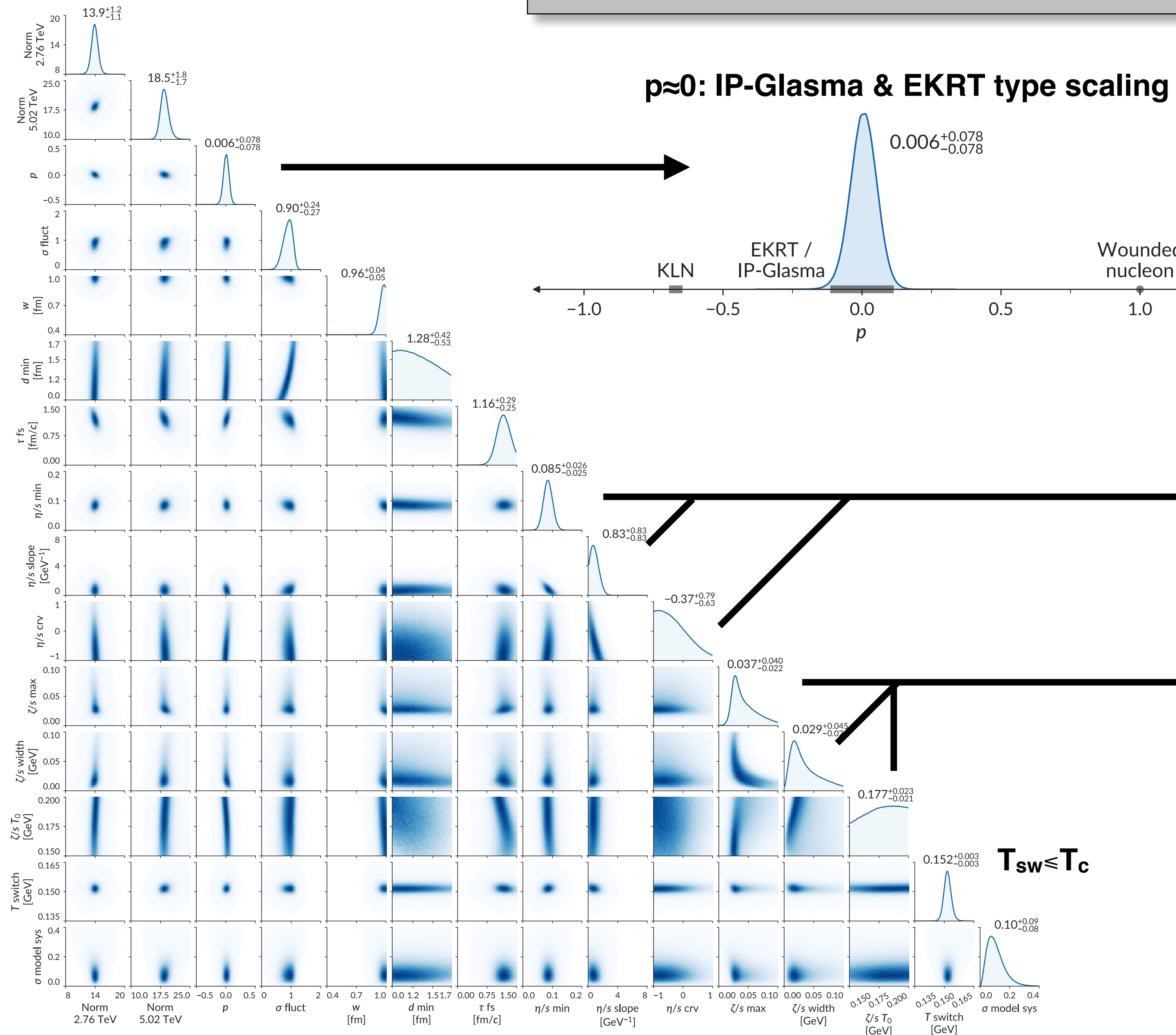
## Bayes' Theorem

$$\text{posterior} \propto \text{likelihood} \times \text{prior}$$

- **prior:** initial knowledge of parameters
- **likelihood:** probability of observing exp. data, given proposed parameters

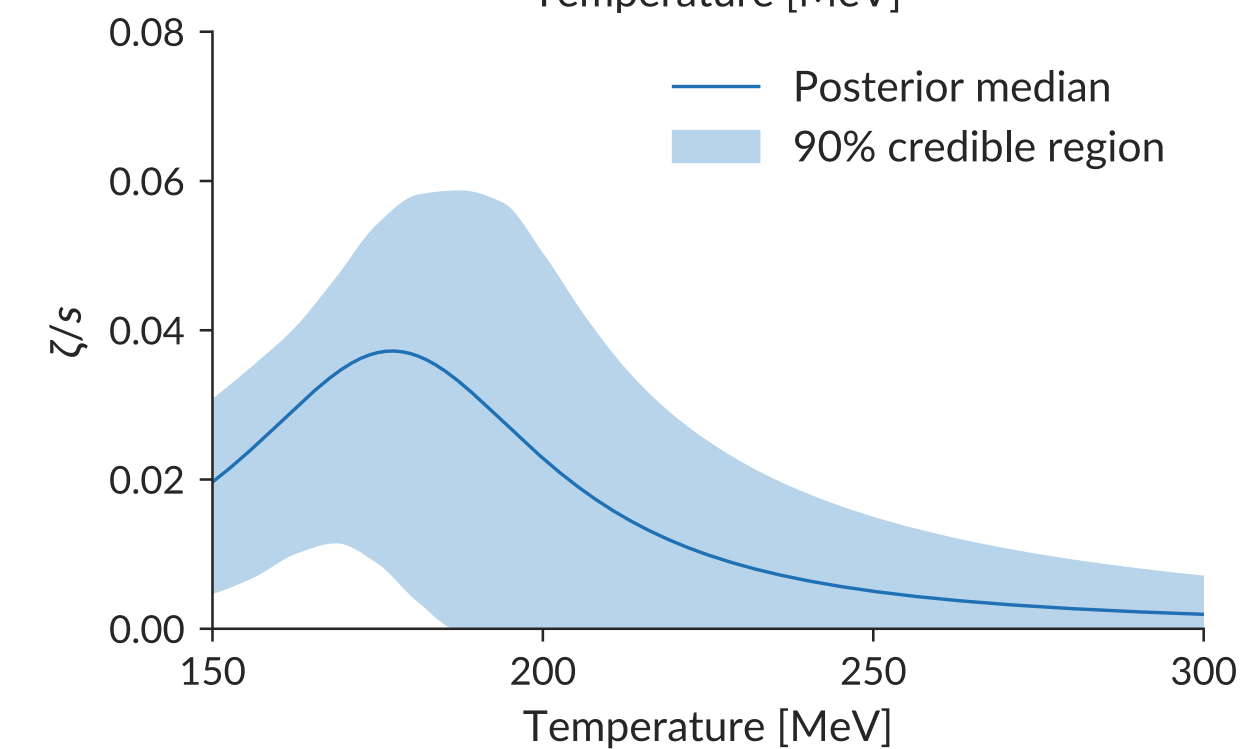
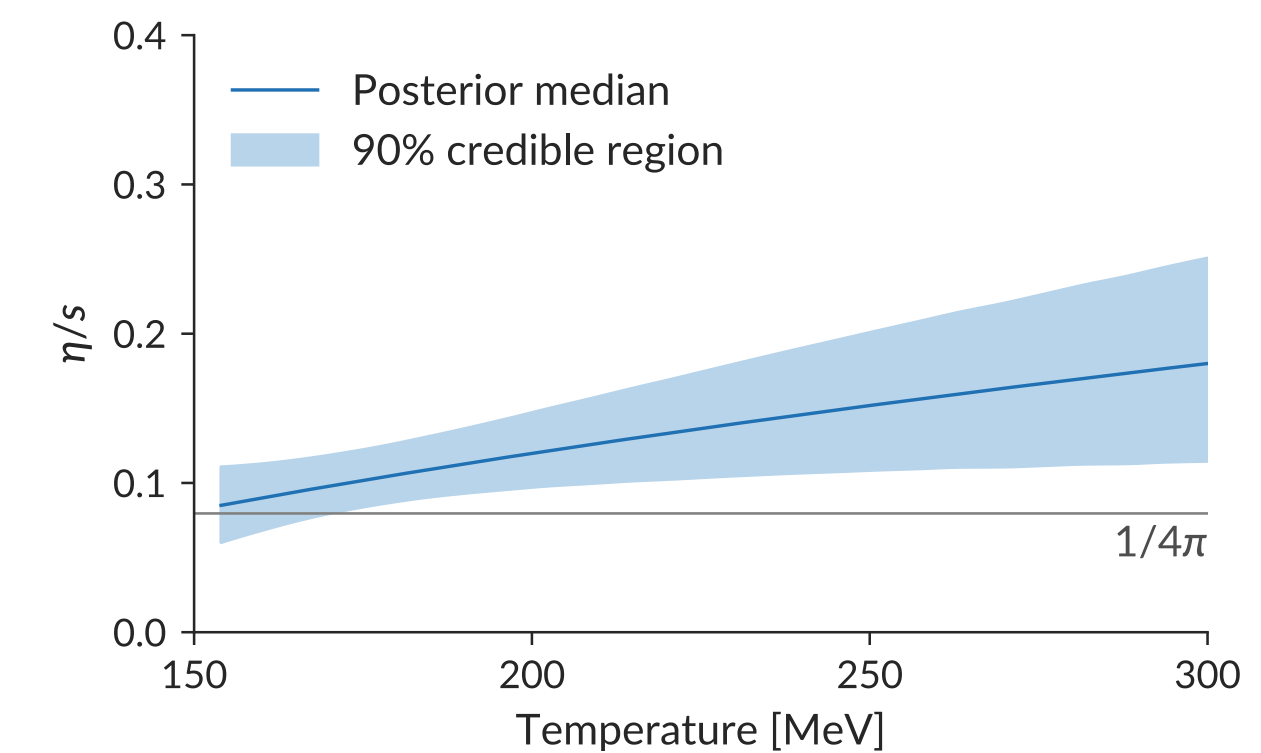


# Calibrated Posterior Distribution



- diagonals:** probability distribution of each parameter, integrating out all others
- off-diagonals:** pairwise distributions showing dependence between parameters

## temperature-dependent viscosities:

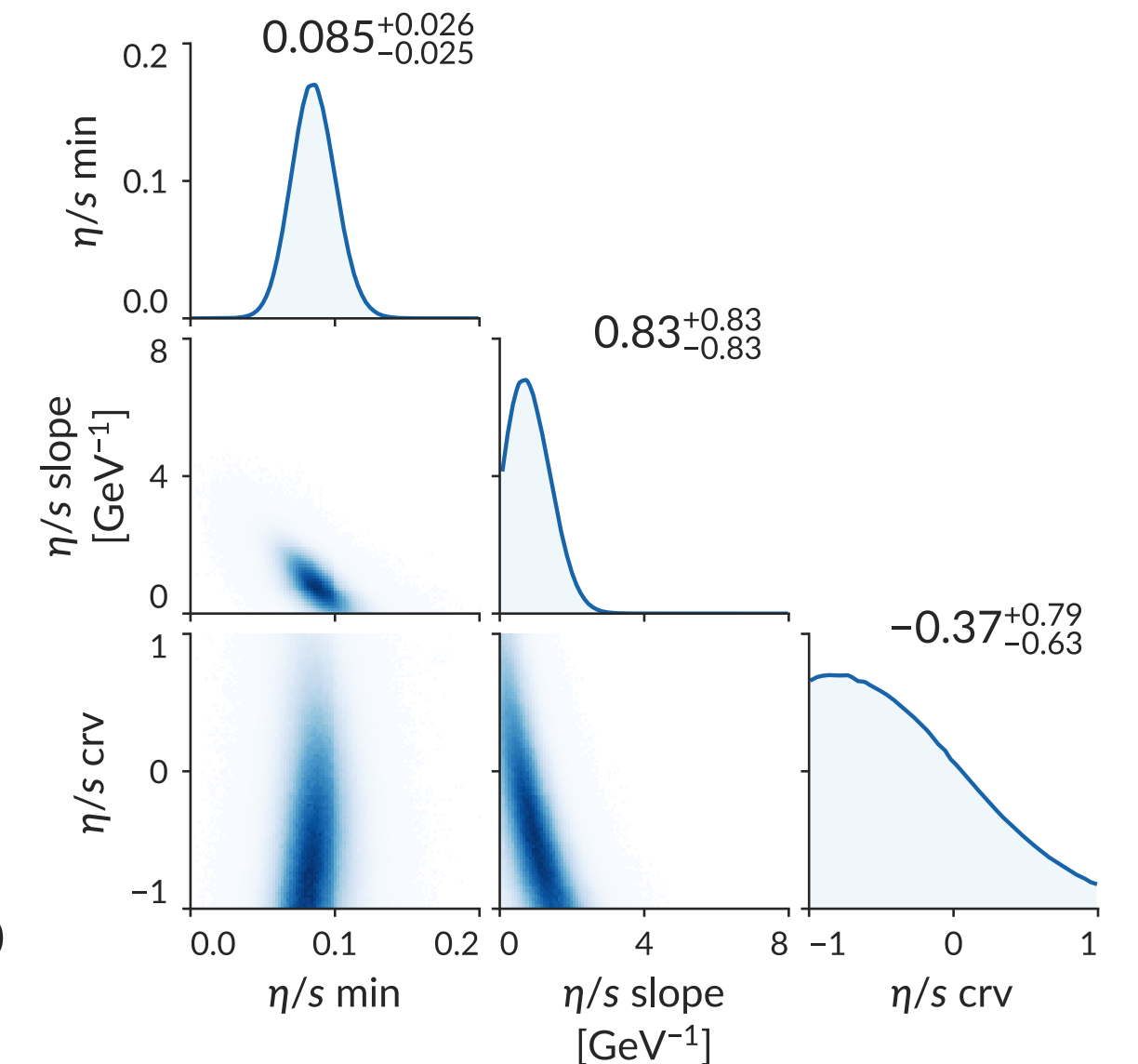
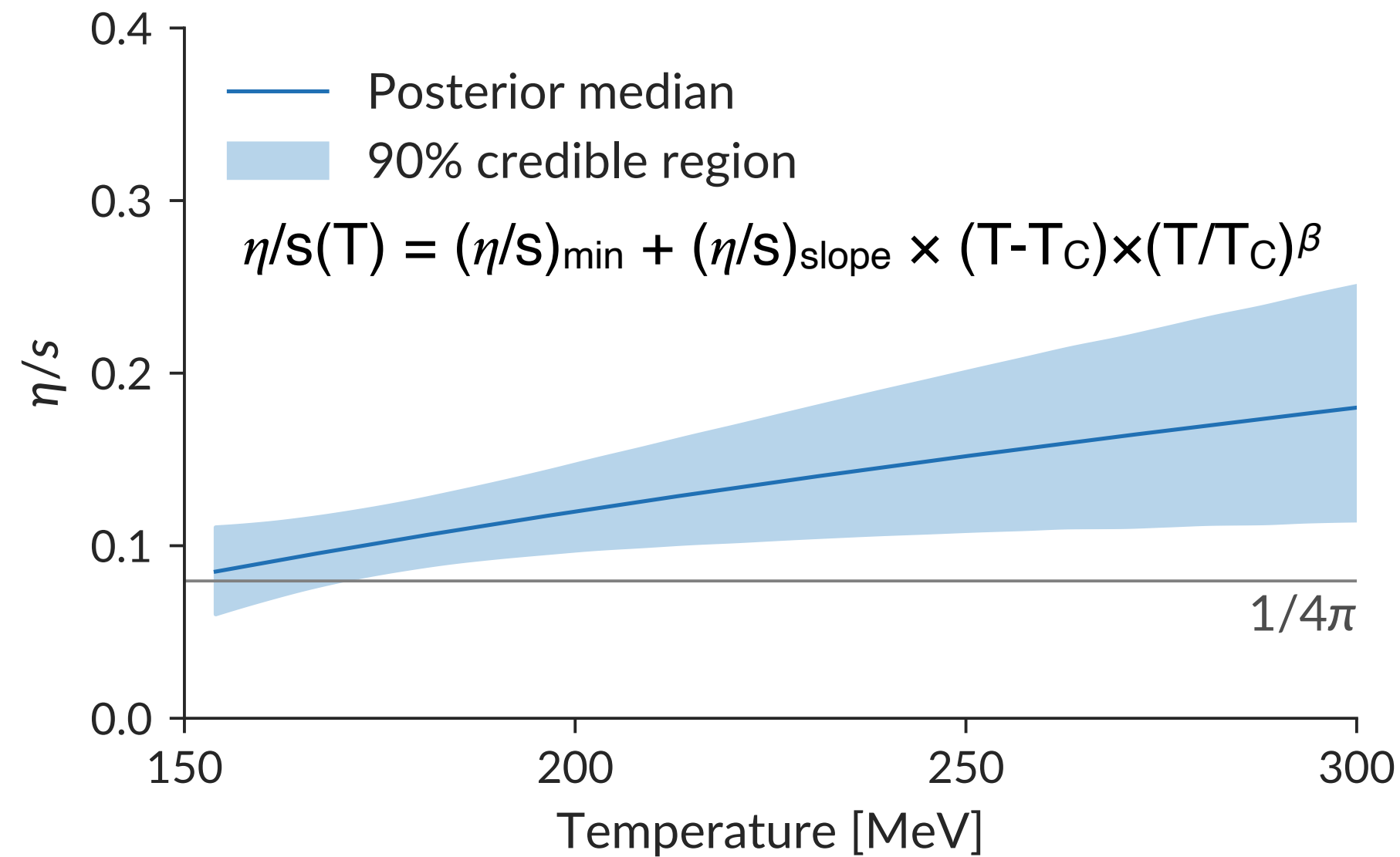




# Temperature Dependence of Shear & Bulk Viscosities

## temperature dependent shear viscosity:

- analysis favors small value and shallow rise
- results do not fully constrain temperature dependence:
  - inverse correlation between  $(\eta/s)_{\text{slope}}$  slope and intercept  $(\eta/s)_{\text{min}}$
  - insufficient data to obtain sharply peaked likelihood distributions for  $(\eta/s)_{\text{slope}}$  and curvature  $\beta$  independently
- current analysis most sensitive to  $T < 0.23$  GeV
  - RHIC data may disambiguate further

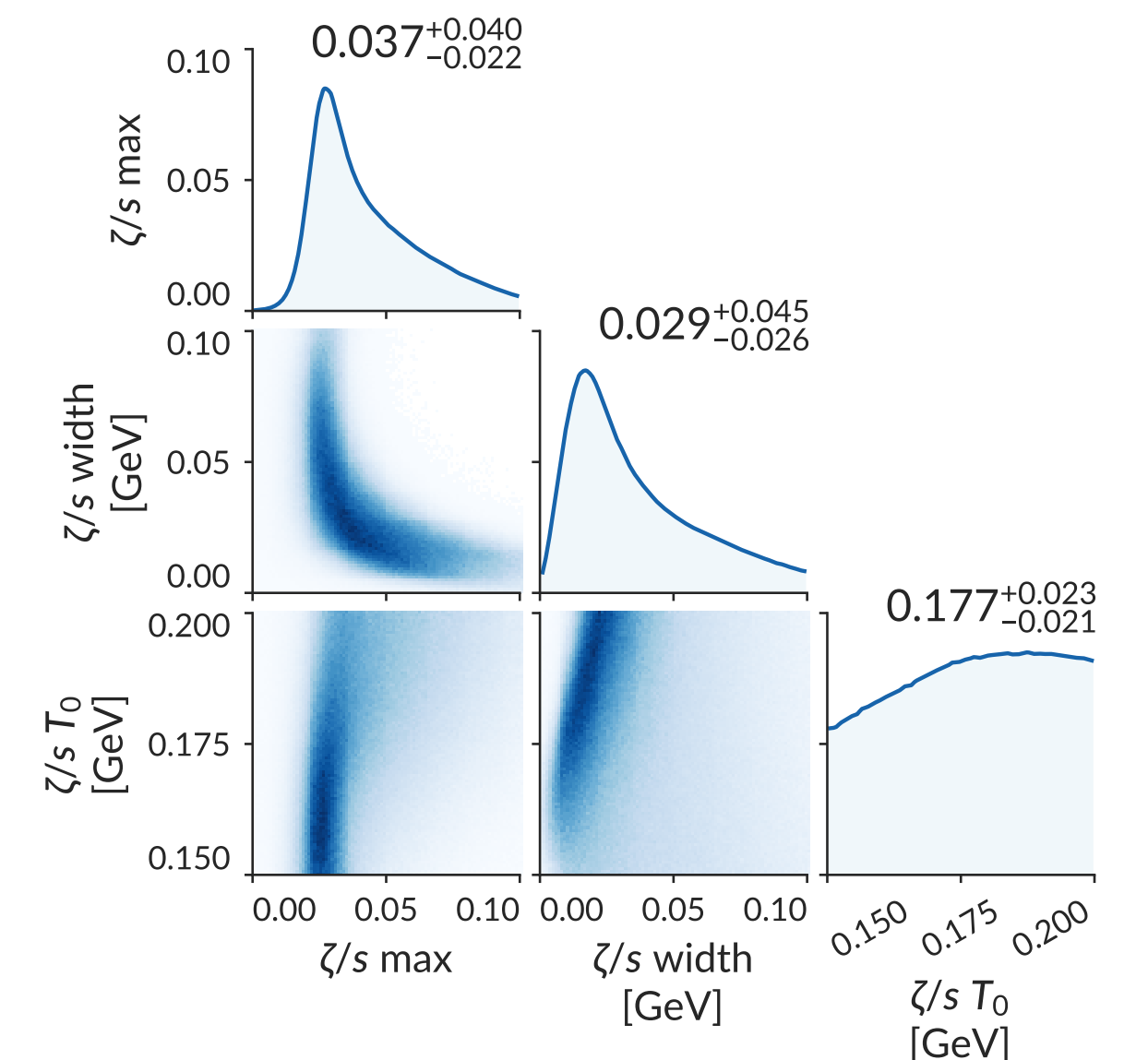
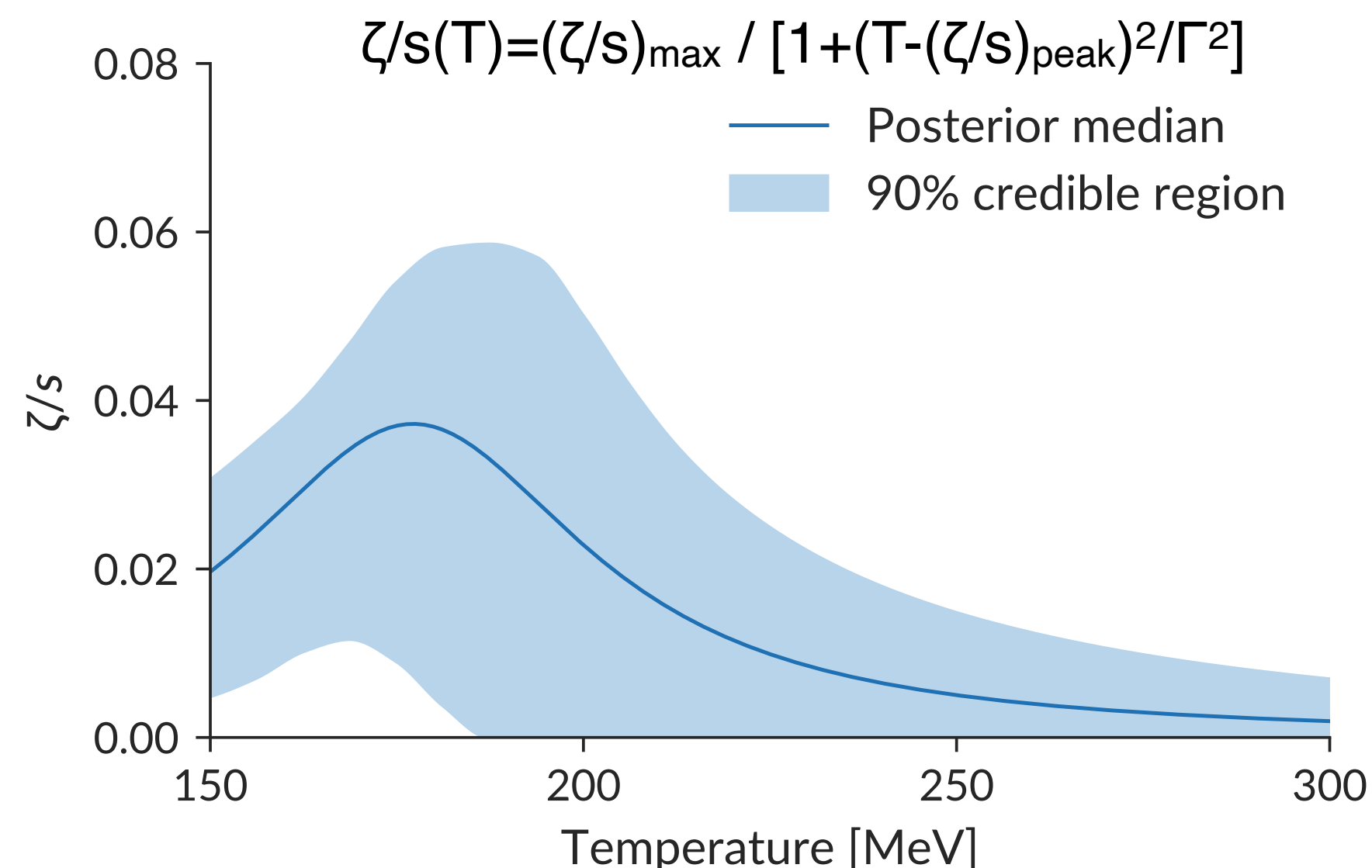


## temperature dependent bulk viscosity:

- setup of analysis allows for vanishing value of bulk viscosity
- significant non-zero value near  $T_c$  favored, confirming the presence / need for bulk viscosity

### caveat of current analysis:

- bulk-viscous corrections are implemented using relaxation-time approximation & regulated to prevent negative particle densities





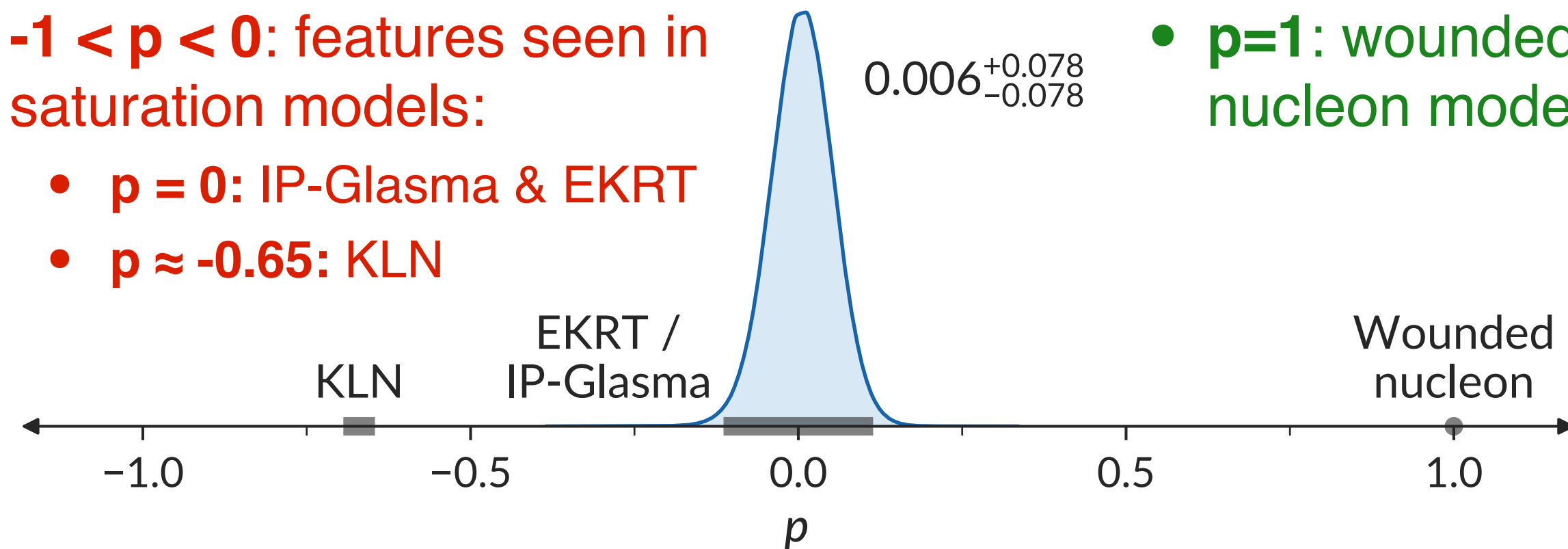
# Constraining the Initial State

$p$  quantifies the attenuation of entropy production in the off-diagonal regions of  $dS/dy \propto T_R(p; T_A, T_B)$ :

- $-1 < p < 0$ : features seen in saturation models:

- $p = 0$ : IP-Glasma & EKRT
- $p \approx -0.65$ : KLN

- $p=1$ : wounded nucleon model



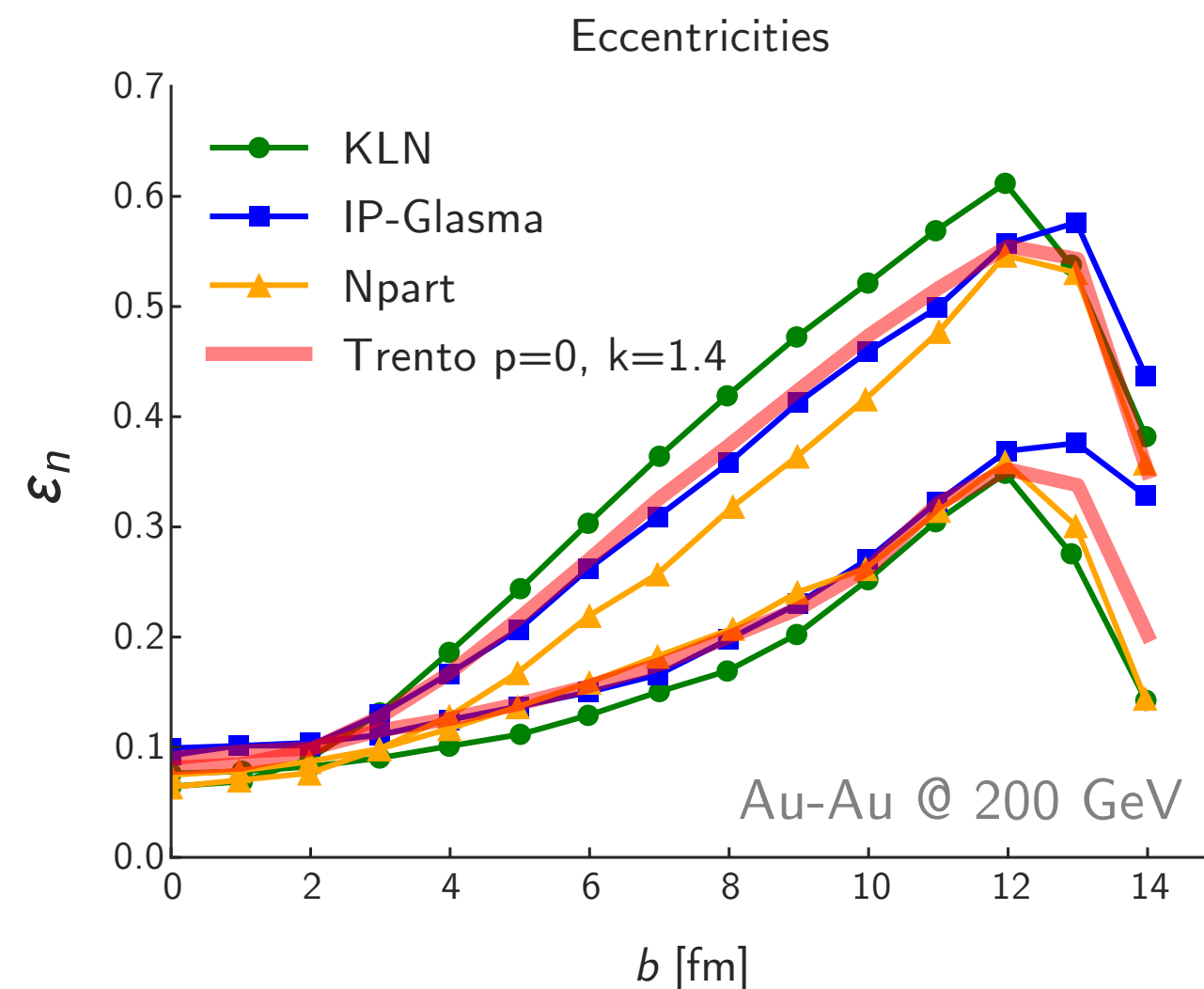
► **analysis strongly favors eccentricity scaling and entropy deposition seen in the EKRT & IP-Glasma models**

► **wounded nucleon and KLN models disfavored**

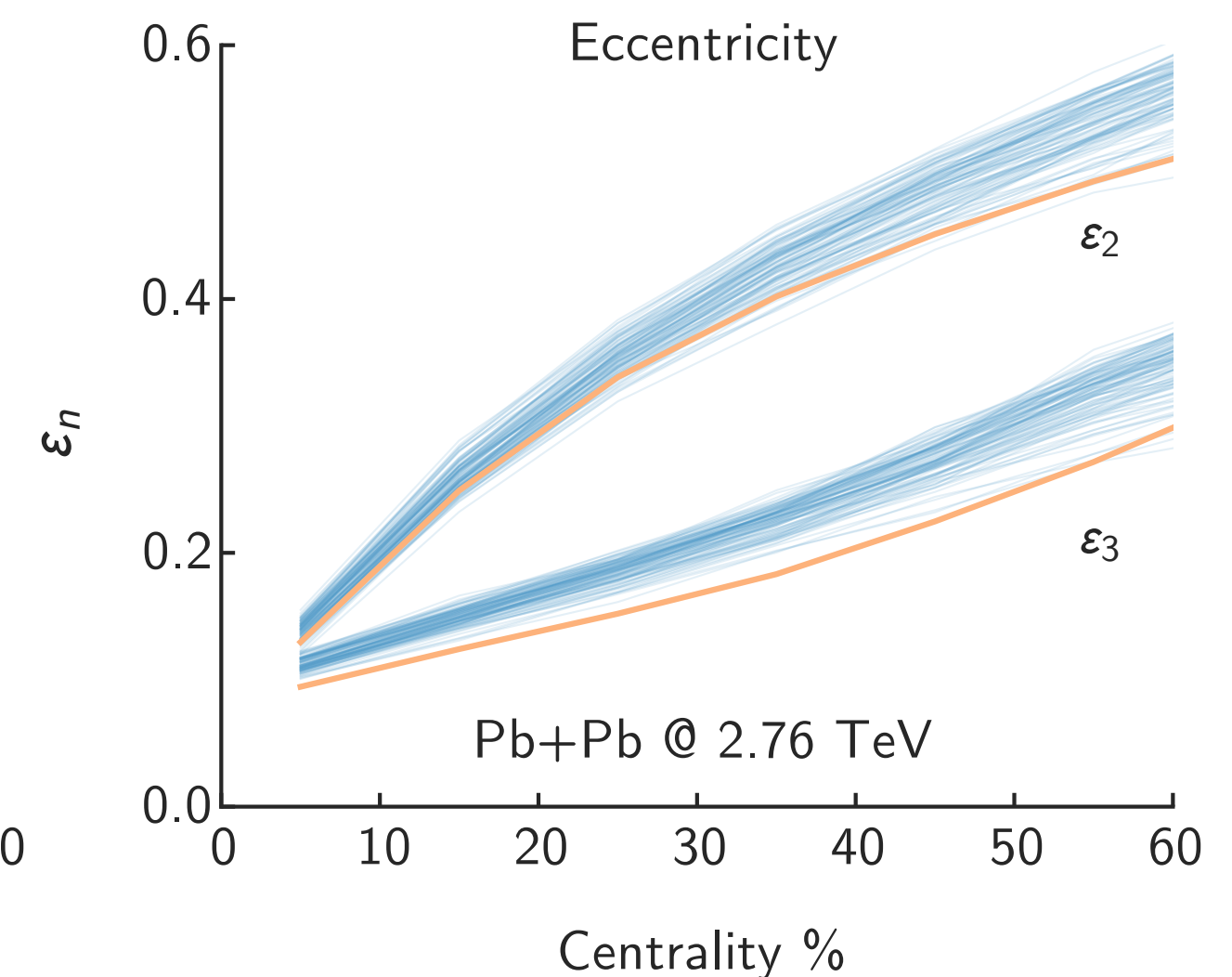
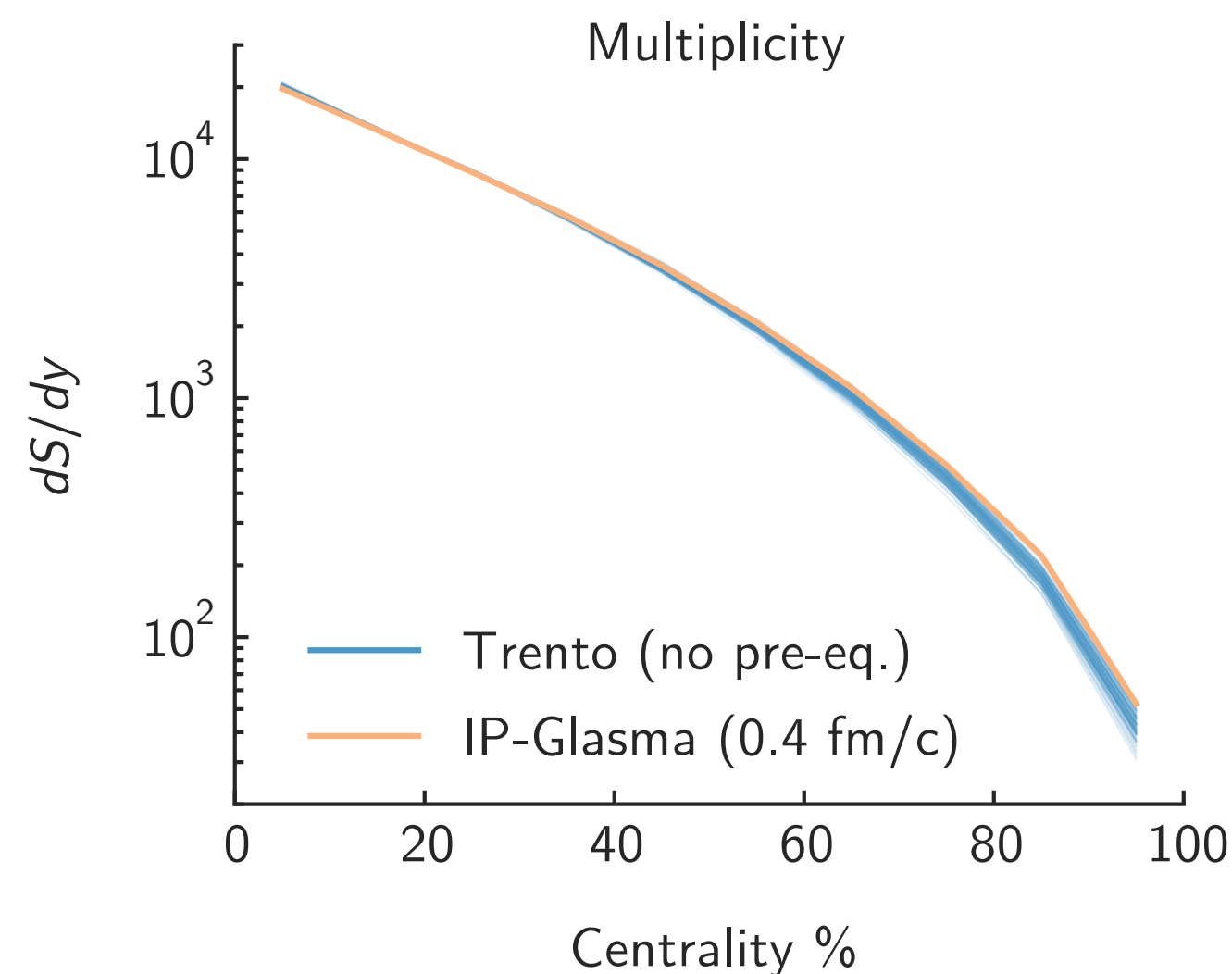
► **no conclusion yet on 2 component WN+BC model**

- **still need to corroborate scale of fluctuations being probed**

## Trento vs. IP-Glasma:



note: no pre-equilibrium flow in the current Trento analysis, may account for larger  $\varepsilon_n$



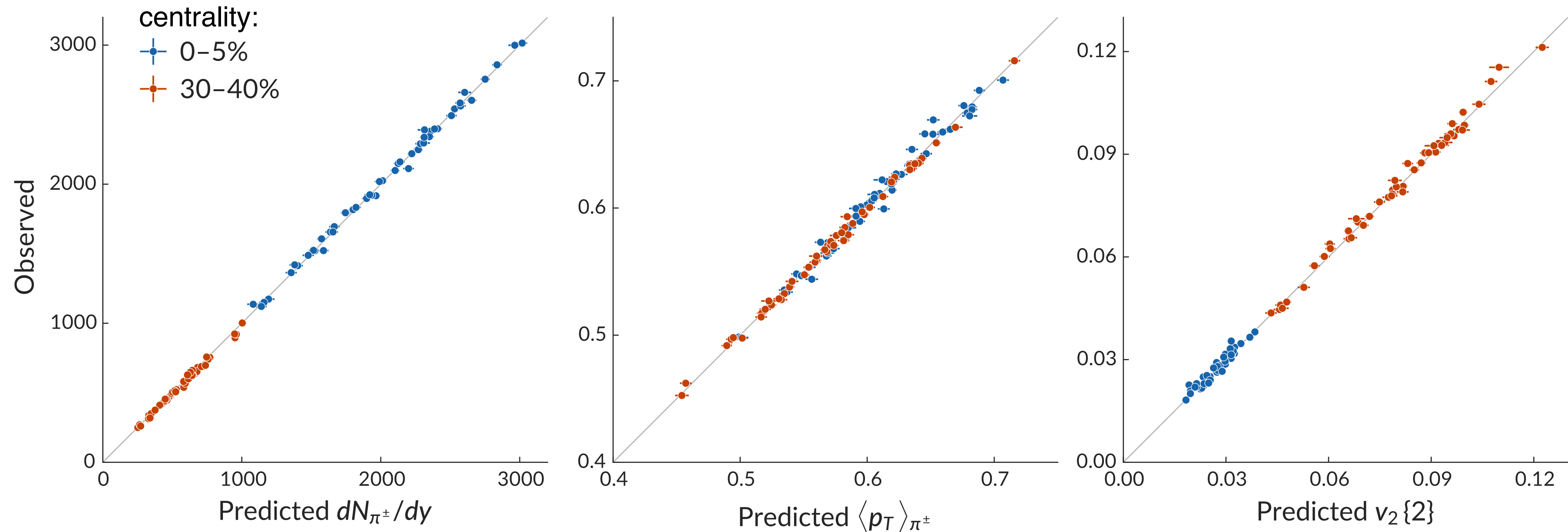


**Precision Science  
or  
“Smoke & Mirrors”?**



# Validation

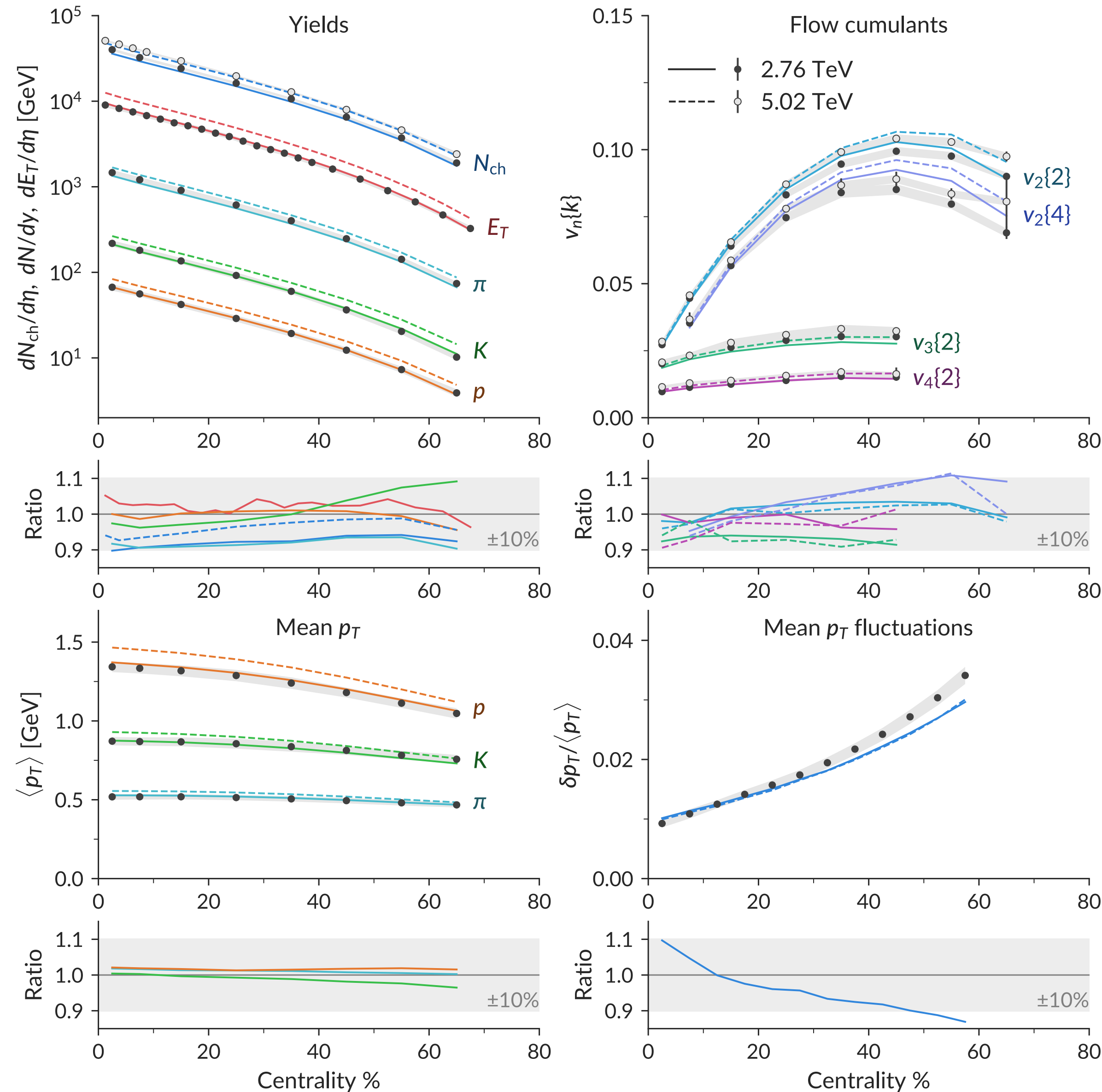
- generate a separate Latin hypercube validation design with 50 points
- evaluate the full physics model at each validation point
- compare physics model output to that of the previously conditioned GP emulators:



- note that since GPEs are stochastic functions, only ~68% of predictions need to fall within 1 standard deviation



# Verification: Explicit Model Calculation



- explicit physics model calculations (no emulator) with parameter values set to the maximum of the posterior probability distributions yield excellent agreement with data!
- description of data to within  $\pm 10\%$  accuracy

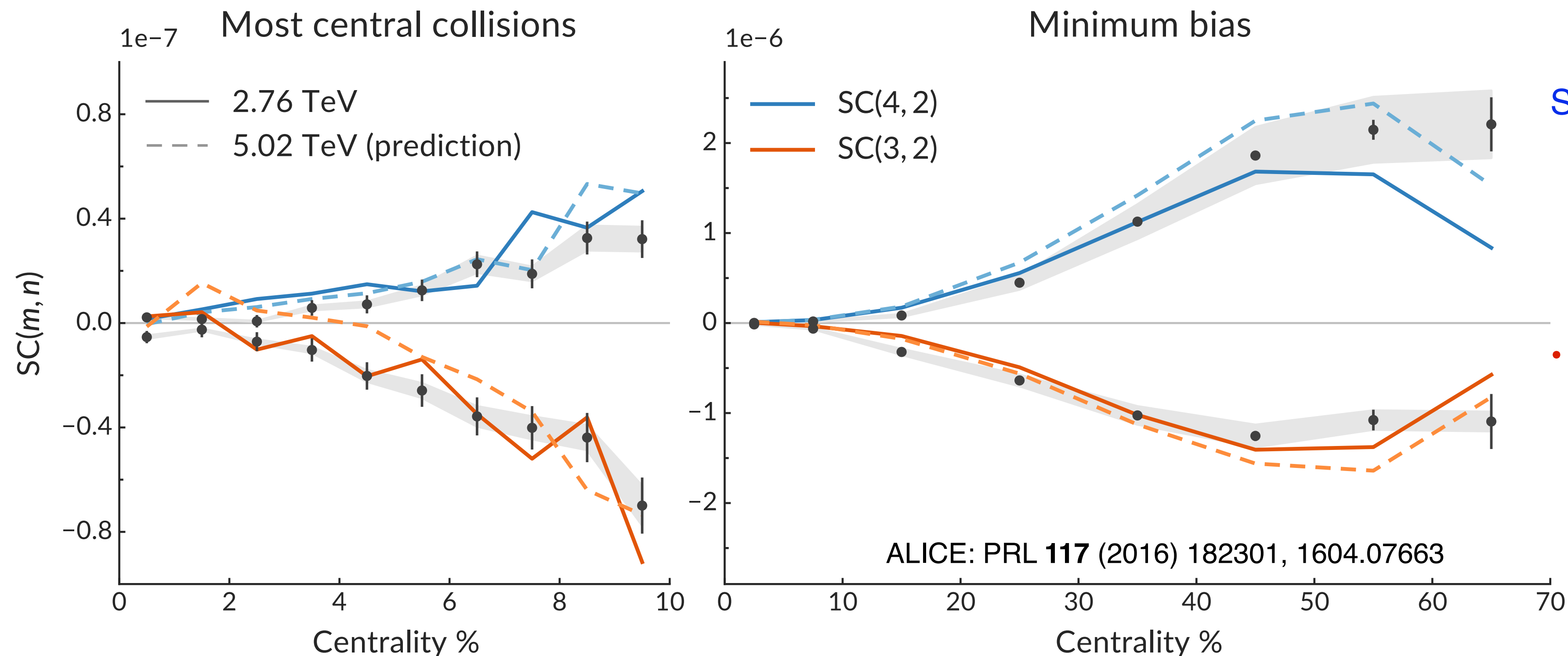


# Non-Calibrated Observables

The robustness and quality of the Physics Model can be tested by making predictions on observables not used during calibration using highest likelihood parameter values.

**Example: correlations between event-by-event fluctuations of flow harmonics**

$$SC(m,n) = \langle v_m^2 v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle$$



$SC(m,n)$  are sensitive to:

- initial conditions
- evolution model
- QGP transport coefficients

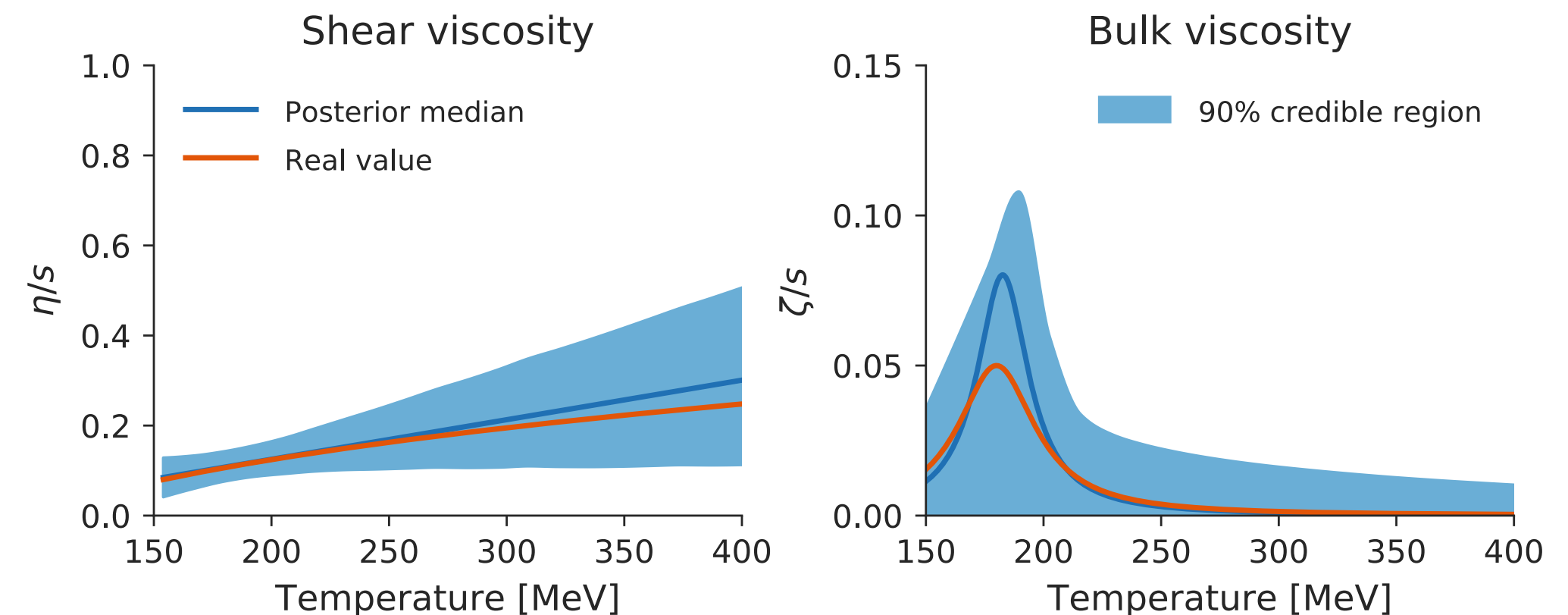
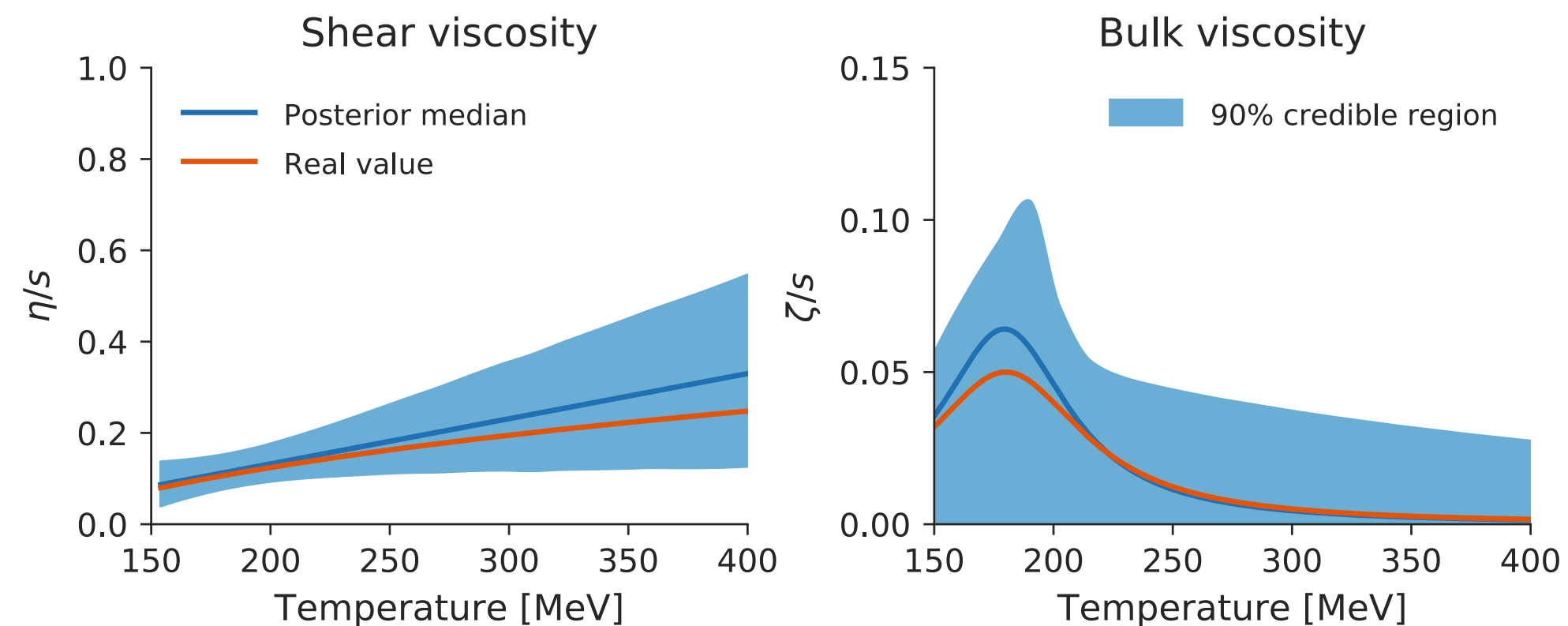
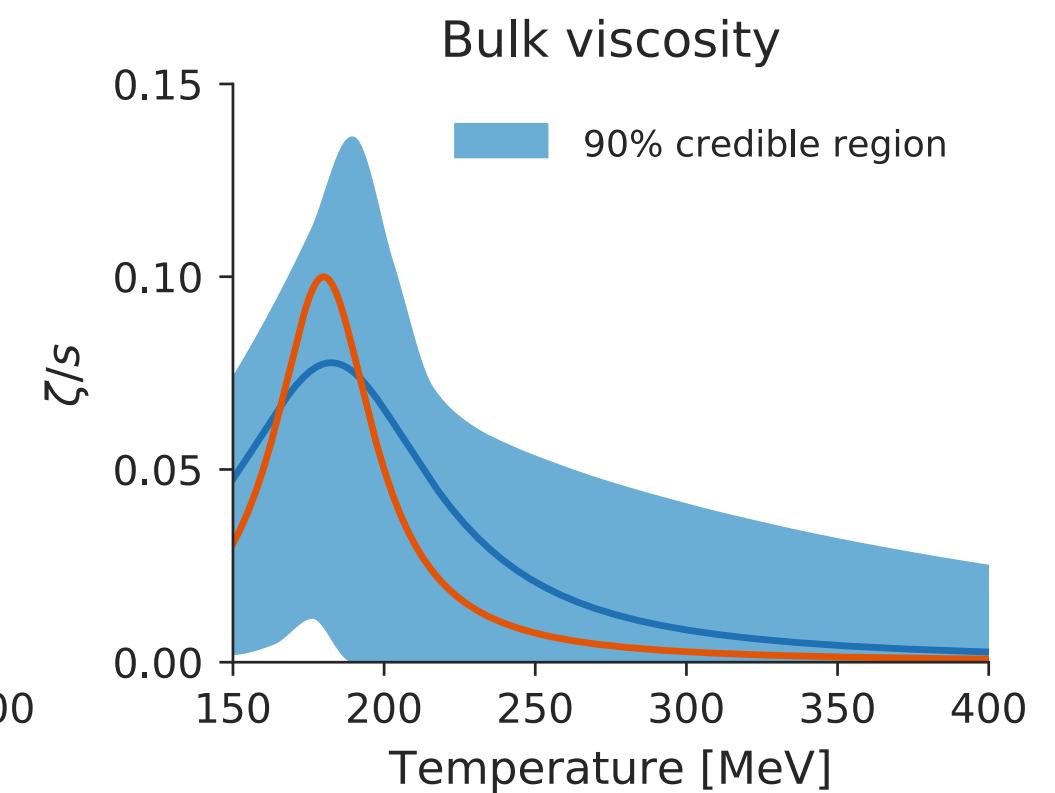
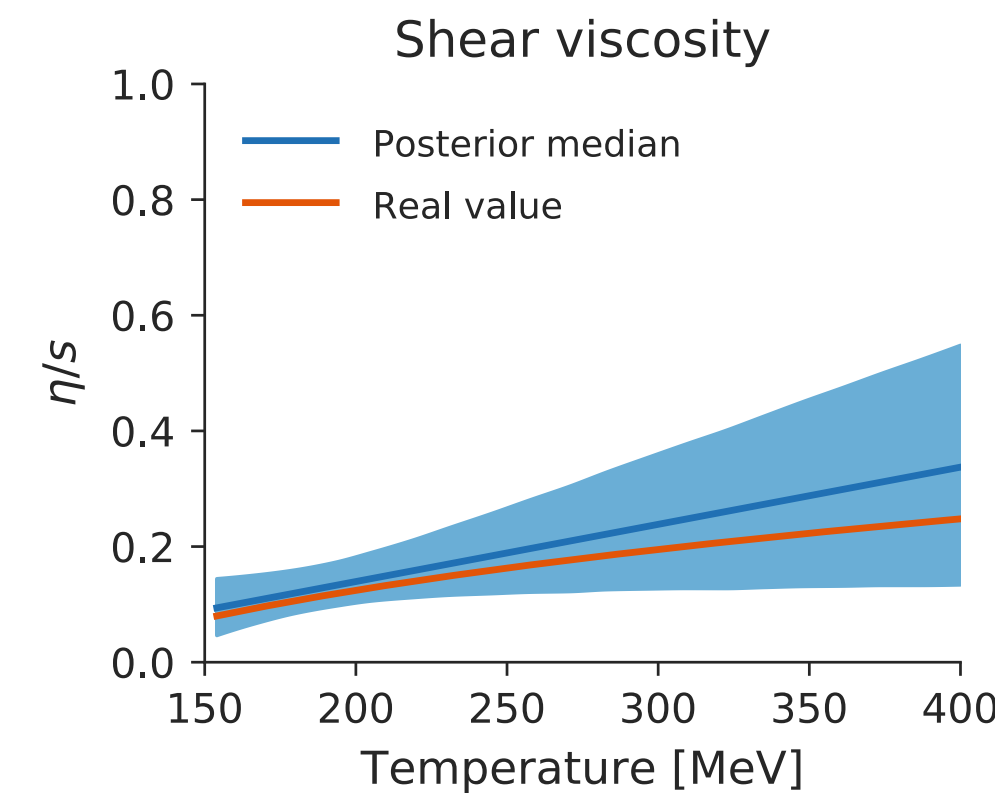
• excellent agreement of model prediction to data!



# Closure Test

Need to verify that analysis can recover “true” values for the parameters: run physics model with chosen set of parameters, generate “fake data” from model output and then conduct analysis on that fake data to test if the input parameters can be recovered!

- both, smooth functions as well as peaked functions, can be reproduced well within the 90% CR
- note: due to reduction of information when going from model output to observables & model/GP uncertainties one should not expect a one-to-one reconstruction
- bulk analysis is mostly sensitive to area under bulk peak, not peak position, height & width independently

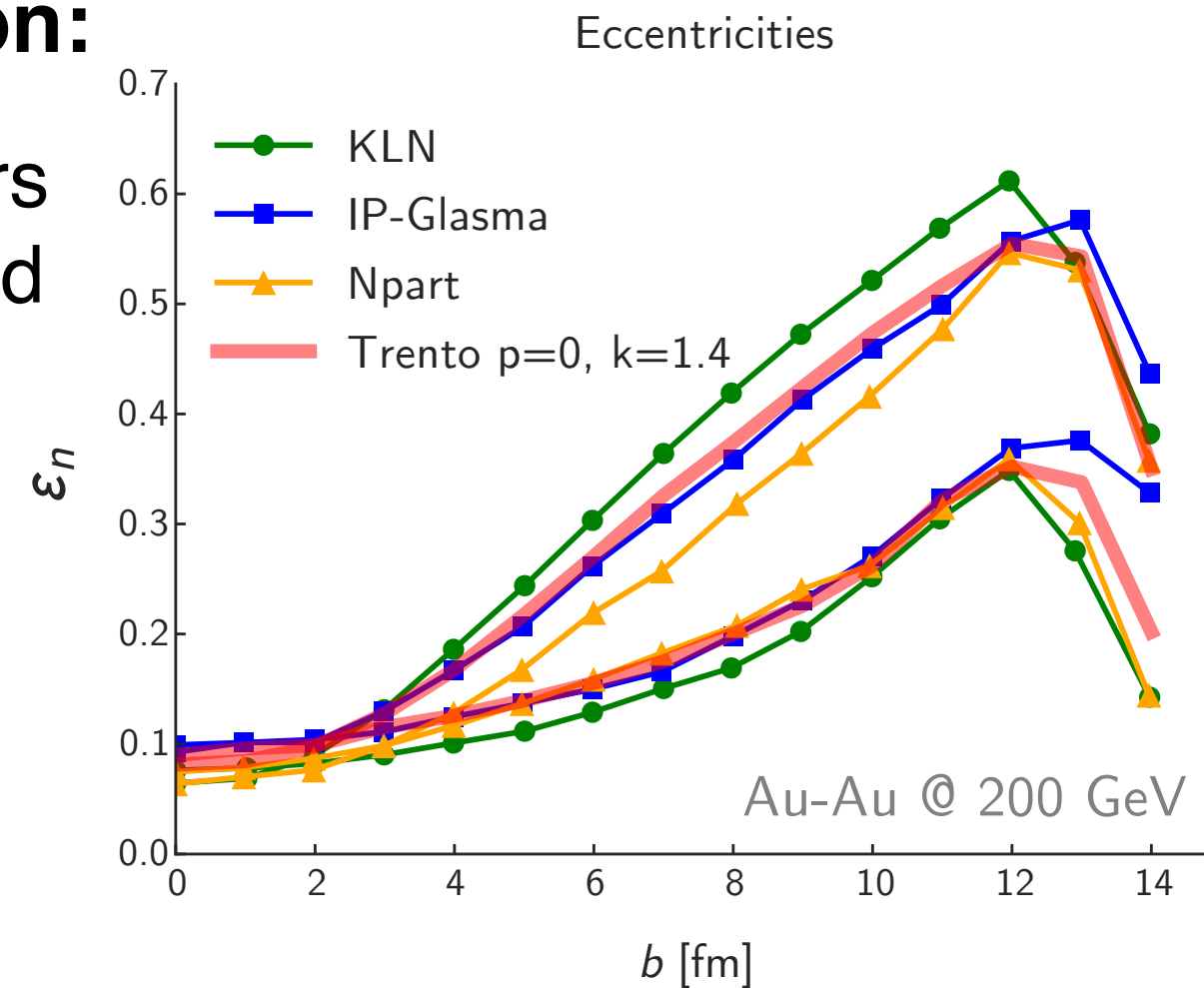




# Summary I: Key Physics Results

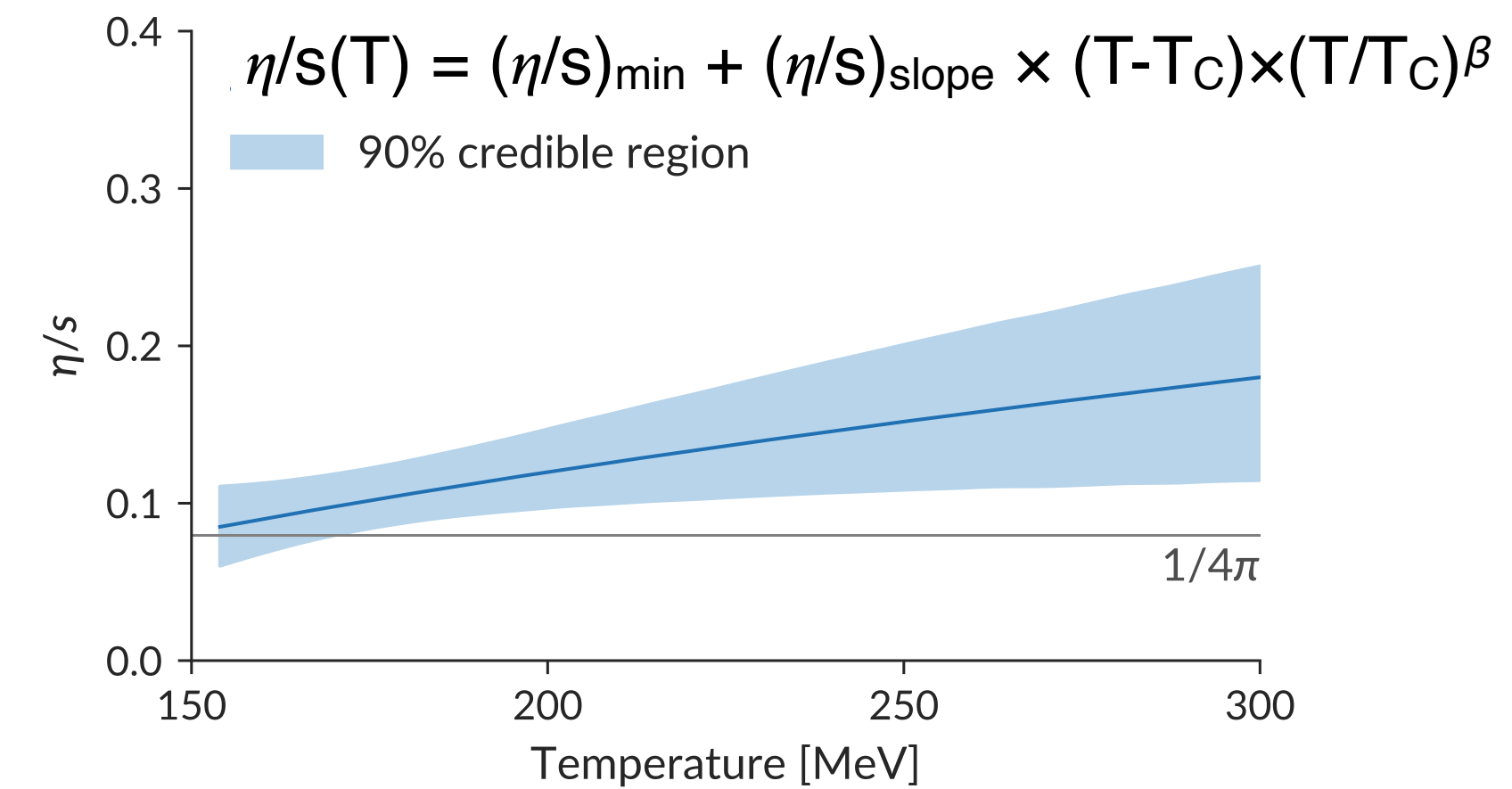
## Trento initial condition:

- analysis strongly favors eccentricity scaling and entropy deposition of EKRT & IP-Glasma model
- Glauber and KLN models strongly disfavored



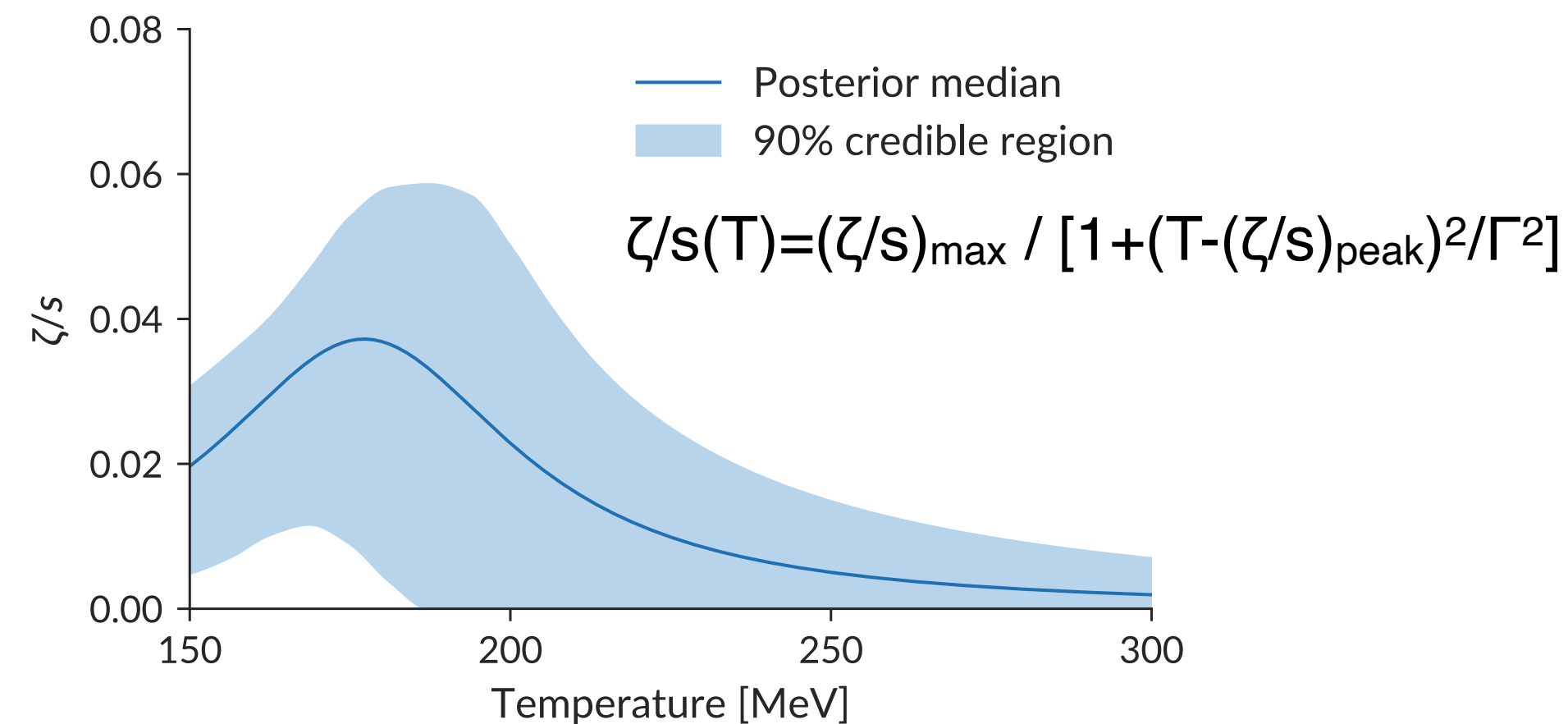
## temperature dependent shear viscosity:

- analysis favors small value and shallow rise
- slope vs. curvature needs disambiguation



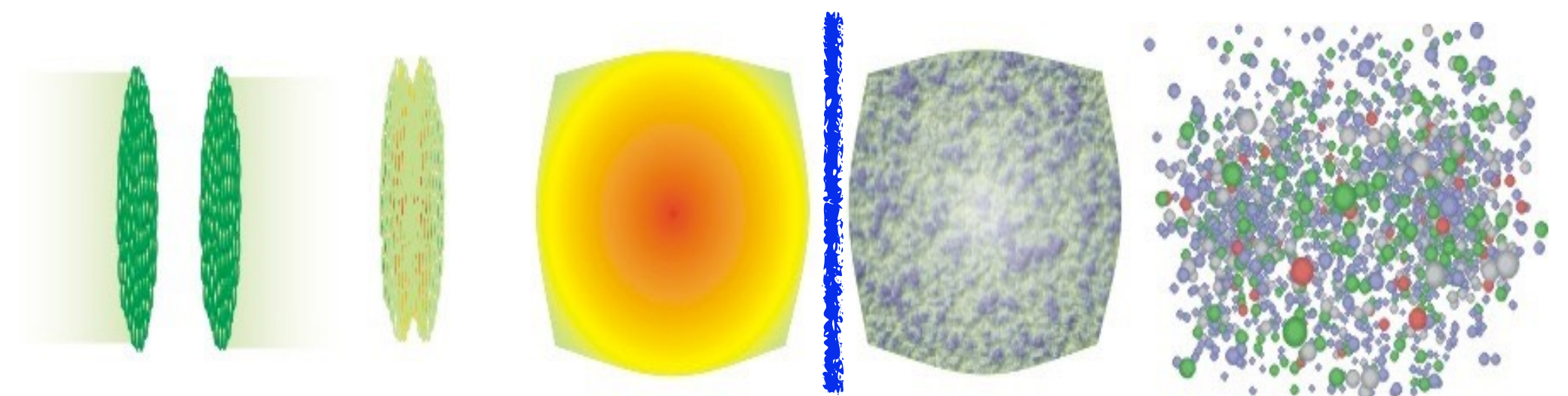
## temperature dependent bulk viscosity:

- non-zero value near  $T_C$  favored
- ambiguities exist for peak height vs. width



## hydro to micro switching temperature $T_{\text{sw}}$

- strong likelihood for a value of  $T_{\text{sw}}$  just around  $T_C$
- indicative of the non-equilibrium nature and dynamical breakup of the hadronic system





# Summary II: Methodology

## Model Parameters - System Properties

- initial state
- temperature-dependent viscosities
- hydro to micro switching temperature

calculate events on Latin hypercube

## Physics Model:

- Trento
- iEbE-VISHNU

## Experimental Data

- ALICE flow & spectra

## Gaussian Process Emulator

- non-parametric interpolation
- fast surrogate to full Physics Model

## MCMC

(Markov-Chain Monte-Carlo)

- random walk through parameter space weighted by posterior probability

after many steps, MCMC equilibrates to

## Posterior Distribution

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## Bayes' Theorem

$$\text{posterior} \propto \text{likelihood} \times \text{prior}$$

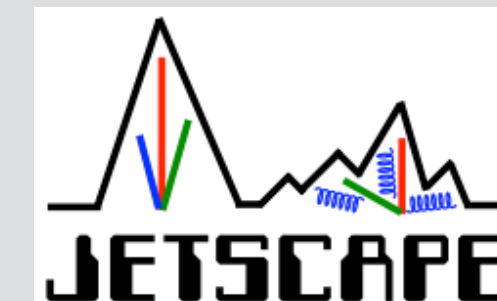
- **prior:** initial knowledge of parameters
- **likelihood:** probability of observing exp. data, given proposed parameters



# Outlook & Future Directions

current analysis focus was on the properties of bulk QCD matter and utilized only LHC data on soft hadrons. The analysis needs to be extended to:

- **include data from lower beam energies**
  - ▶ necessary for determination of the temperature and  $\mu_B$  dependence of transport coefficients
- **include asymmetric collision systems (p+A, d+A, 3He+A, A+B)**
  - ▶ generate improved understanding of the initial state
- **include hard probes (jets and heavy quark observables)**
  - ▶ consistent determination of jet and heavy flavor transport coefficients
- **include other physics models**
  - ▶ analysis is model agnostic, allows for quantitative comparison among different models and verification/falsification of models/conceptual approaches



this work has been made possible through support by



National Energy Research  
Scientific Computing Center



# Past & Present Collaborators & Sponsors

## Duke QCD Group:

- Jonah Bernhard (now Lowe's Corporate)
- J. Scott Moreland
- Weiyao Ke
- Yingru Xu
- Jean-Francois Paquet

## Duke Dept. of Statistical Sciences:

- Robert E. Wolpert
- Jake Coleman

## Ohio State Nuclear Theory:

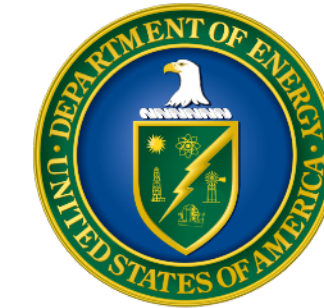
- Ulrich W. Heinz
- Jia Liu (now SAP)
- Chun Shen (now BNL)

## U. of Wyoming Dept. of Statistics:

- Snehalata Huzurbazar
- Peter W. Marcy (now LANL)

This work was made possible  
through support by:

US Dept. of Energy



National Science  
Foundation



Open Science Grid



NERSC



SAMSI



Pioneering work by the MADAI Collaboration, led by Scott E. Pratt, MSU (2009-2014)



# Resources

## **Trento:**

- J. Scott Moreland, Jonah E. Bernhard & Steffen A. Bass: Phys. Rev. C 92, 011901(R)
- <https://github.com/Duke-QCD/trento>

## **iEbE-VISHNU:**

- Chun Shen, Zhi Qiu, Huichao Song, Jonah Bernhard, Steffen A. Bass & Ulrich Heinz: Computer Physics Communications in print, arXiv:1409.8164
- <http://u.osu.edu/vishnu/>

## **UrQMD:**

- Steffen A. Bass et al. Prog. Part. Nucl. Phys. 41 (1998) 225-370 , arXiv:nucl-th/9803035
- Marcus Bleicher et al. J.Phys. G25 (1999) 1859-1896 , arXiv:hep-ph/9909407
- <http://urqmd.org>

## **MADAI Collaboration:**

- Visualization and Bayesian Analysis packages
- <https://madai-public.cs.unc.edu>

## **Duke Bayesian Analysis Package:**

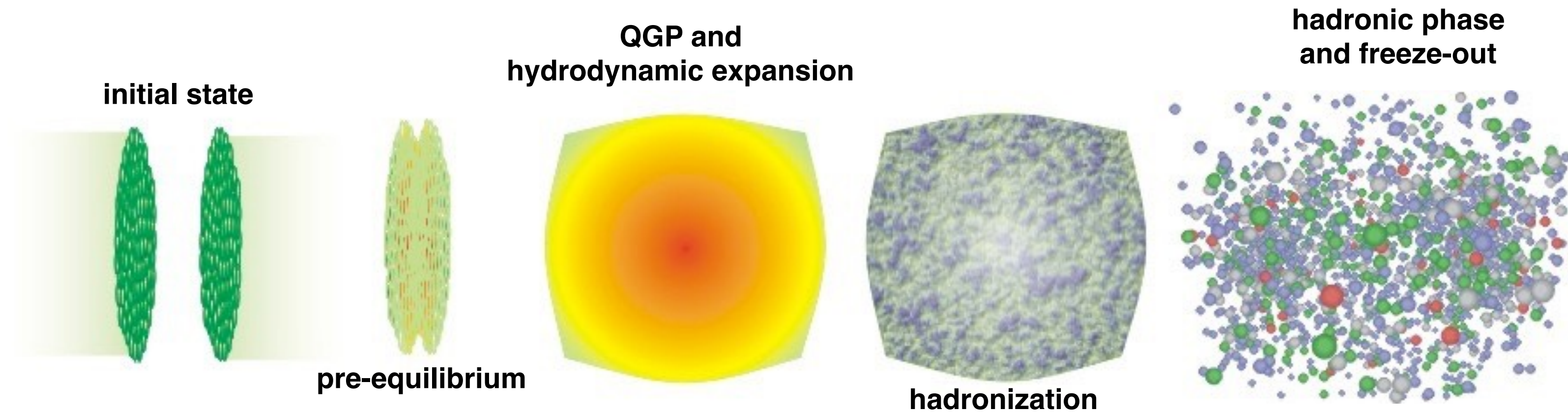
- <https://github.com/jbernhard/mtd>



**The End**



# Time Evolution of a Heavy-Ion Collision



## • Initial State:

- fluctuates event-by-event
- classical color-field dynamics

## • QGP and hydrodynamic expansion:

- proceeds via 3D viscous RFD
- EoS from Lattice QCD

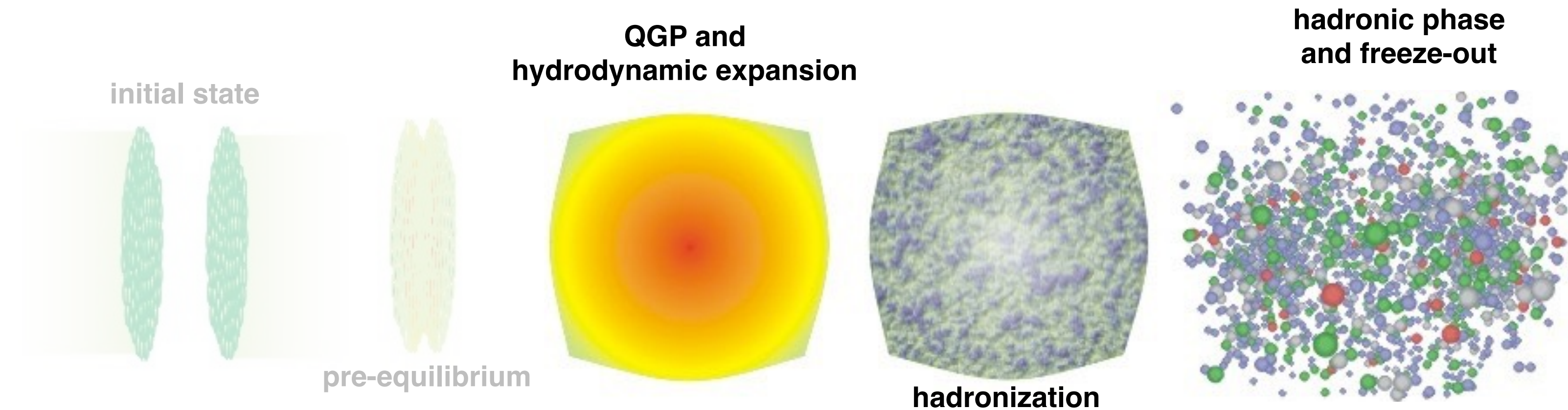
## • Pre-equilibrium:

- rapid change-over from glue-field dominated initial state to thermalized QGP
- time scale: 0.15 to 2 fm/c in duration
- build-up of transverse velocity fields?

## • hadronic phase & freeze-out

- interacting hadron gas
- separation of chemical and kinetic freeze-out

# Constraining the IS of Heavy-Ion Collisions



- treatment of QGP evolution and hadronic freeze-out is well established and largely understood
- major success: first extraction of QGP properties such as  $\eta/s$
- major challenges:
  - quantify uncertainties in extracted QGP properties
  - temperature dependence of transport coefficients

- **QGP and hydrodynamic expansion:**

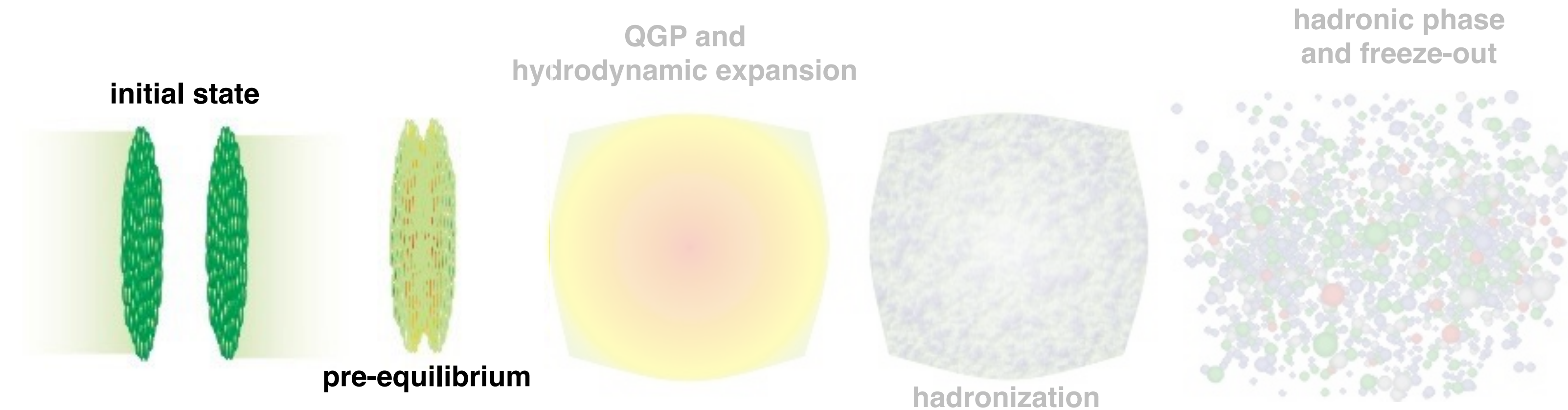
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# Constraining the IS of Heavy-Ion Collisions



- **Initial State:**

- fluctuates event-by-event
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- **Pre-equilibrium:**

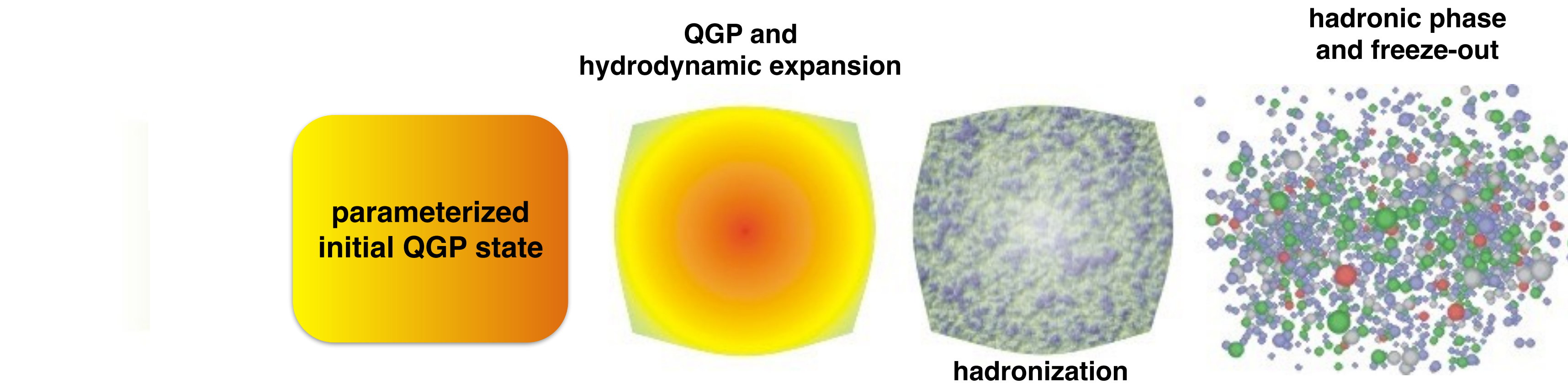
- rapid change-over from glue-field dominated initial state to thermalized QGP
- time scale: 0.15 to 2 fm/c in duration
- build-up of transverse velocity fields?

- physics of initial state and pre-equilibrium dynamics are still conceptually challenging with many open questions

- what processes drive the system towards equilibration?
- on what timescale?
- ...

- **a major source of uncertainty for the extraction of QGP properties**

# Constraining the IS of Heavy-Ion Collisions



## parameterized initial QGP state:

- based on simple phenomenological ideas for entropy deposition
- constrained by global model to data fit
- provides guidance to ab-initio IS models on features needed to describe the data

## • QGP and hydrodynamic expansion:

- proceeds via 3D viscous RFD
- EoS from Lattice QCD

## • hadronic phase & freeze-out

- interacting hadron gas
- separation of chemical and kinetic freeze-out



# Initial Condition Model: Trento

- effective, parametric, description of entropy production prior to thermalization
- based on **reduced thickness\***  $T_R$  as ansatz for  $dS/dy$ :

$$dS/dy|_{\tau=\tau_0} \propto T_R(p; T_A, T_B) \equiv \left( \frac{T_A^p + T_B^p}{2} \right)^{1/p}$$

- determine participant nucleons in A, B by sampling for each nucleon pair:

$$P_{\text{coll}} = 1 - \exp \left[ -\sigma_{gg} \int dx dy \int dz \rho_A \int dz \rho_B \right]$$

**Nuclear Thickness\*:**

$$T_A = \sum_i \gamma_i \int dz \rho_{\text{nucleon}}(x - x_i, y - y_i, z - z_i)$$

- sum is over participant nucleons with positions sampled from an uncorrelated Woods-Saxon distribution or correlated nuclear configurations when available
- introduce fluctuations via  $\gamma_i$ , sampled from a gamma distribution with unit mean:
- nucleon density  $\rho_{\text{nucleon}}$  modeled as Gaussian in transverse plane

$$P_k(\gamma) = \frac{k^k}{\Gamma(k)} \gamma^{k-1} e^{-k\gamma}$$

$$\int dz \rho_{\text{proton}} = \frac{1}{2\pi w^2} \exp \left( -\frac{x^2 + y^2}{2w^2} \right)$$

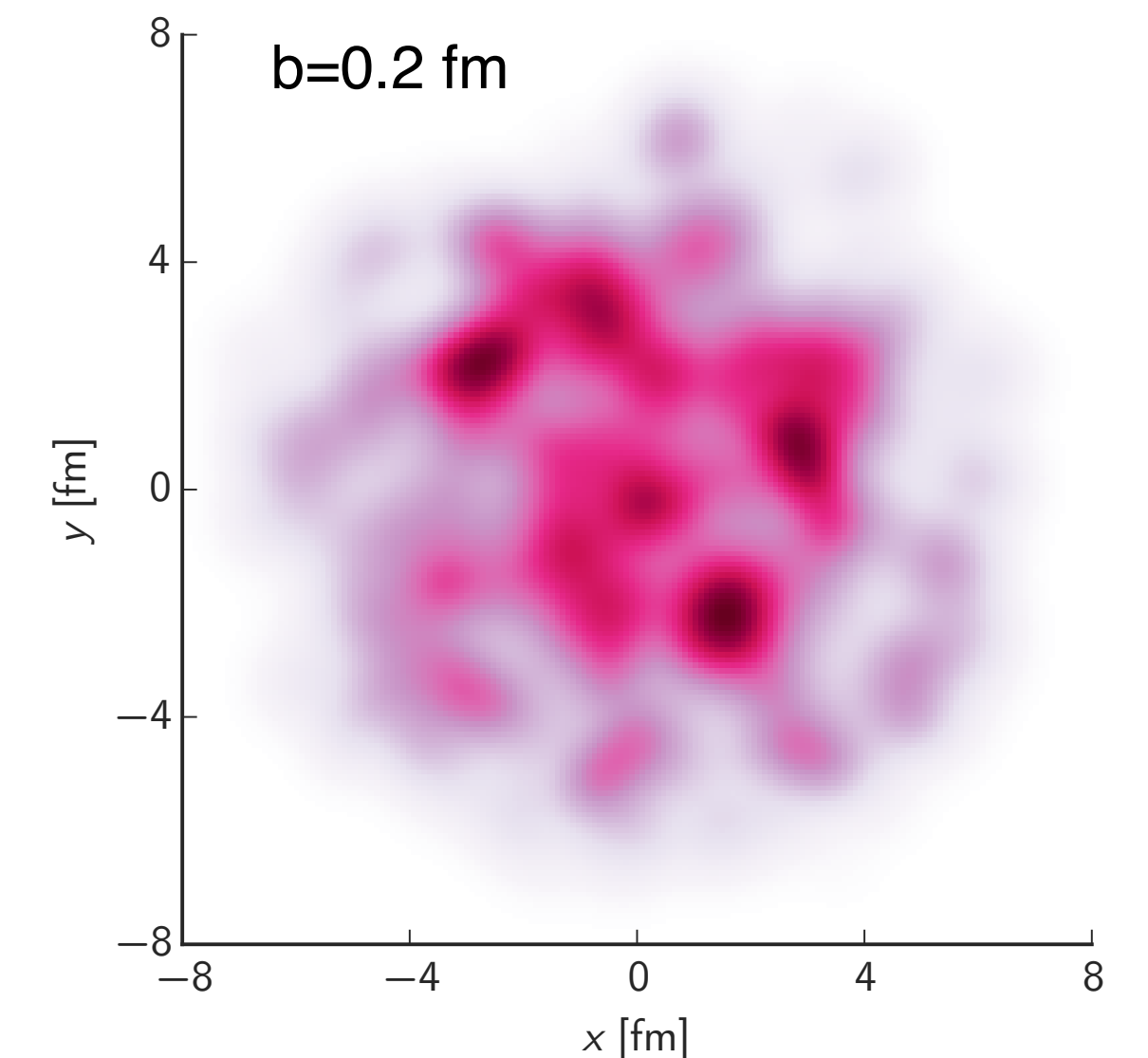
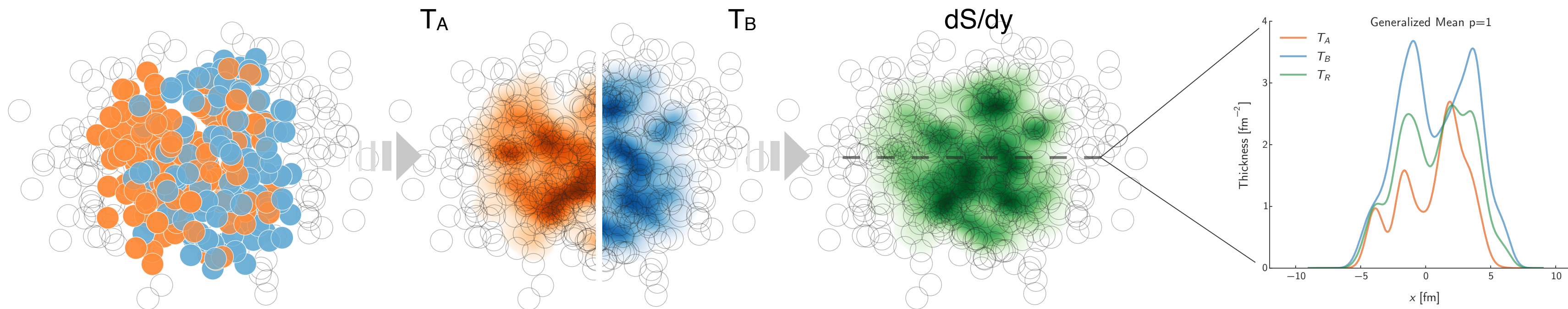
**model parameters:**

- attenuation parameter:  $p$
- fluctuation parameter:  $k$
- width of nucleon:  $w$
- overall normalization:  $C_{\text{norm}}$

**model output:**

- event by event spatial entropy density distribution at mid-rapidity at thermalization time  $\tau_0$

(1) determine participants:      (2) construct thickness functions:      (3) calculate entropy deposition:



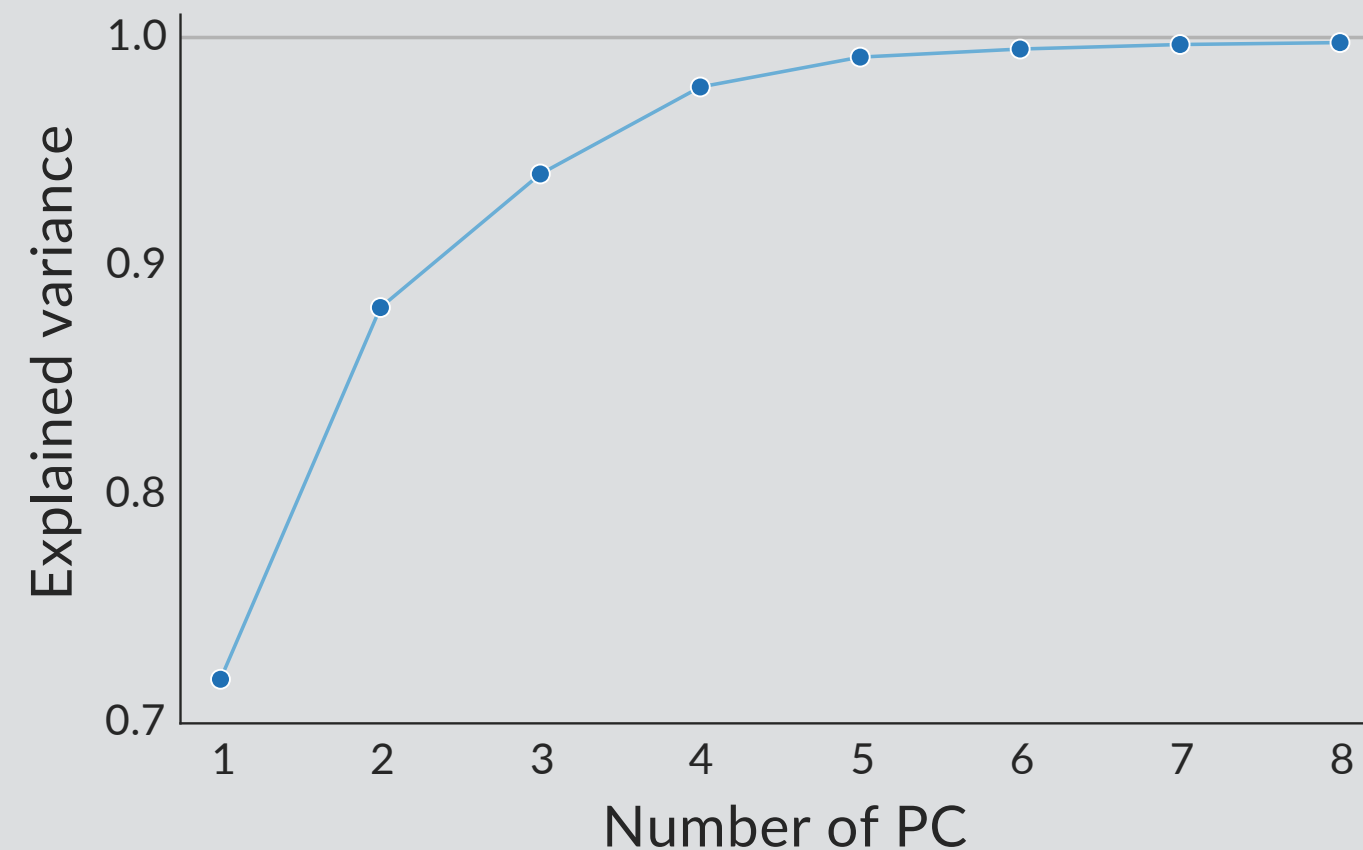
# Multivariate Output

## Scaling of analysis with # of observables:

- independent emulators for each output?
- neglects correlations among outputs
- what if # of outputs scales to 100?
- ▶ training of individual GPE's may become unfeasible and unnecessary in case of strong correlations

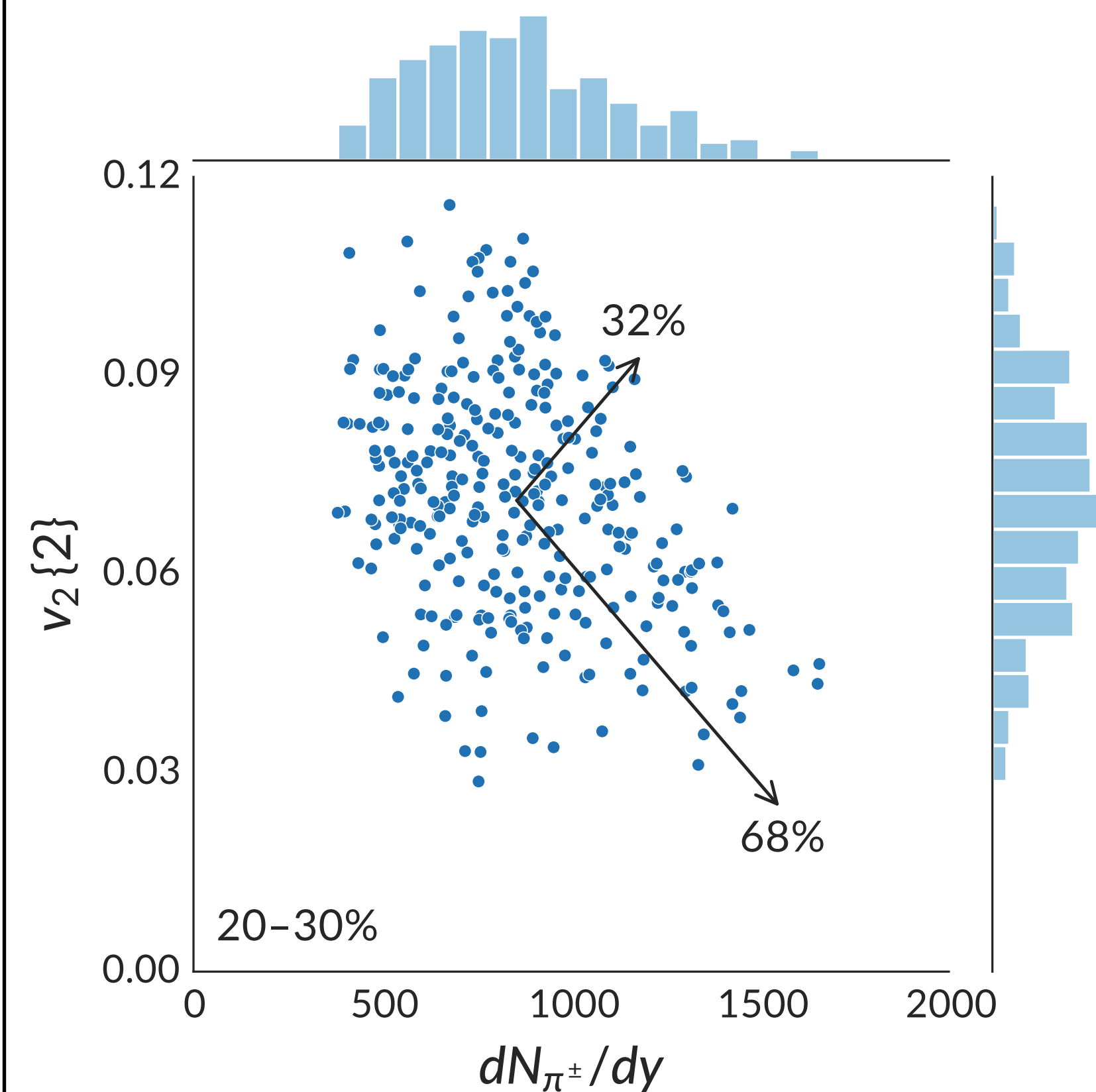
## this analysis:

- model outputs are yields,  $\langle p_T \rangle$ ,  $v_2$ ,  $v_3$  and  $v_4$
- 68 original output dimensions
- 8 principal components used



## Principal Components:

- linear combinations of model output
  - orthogonal and uncorrelated
- ⇒ emulate each PC





## **Next steps:**

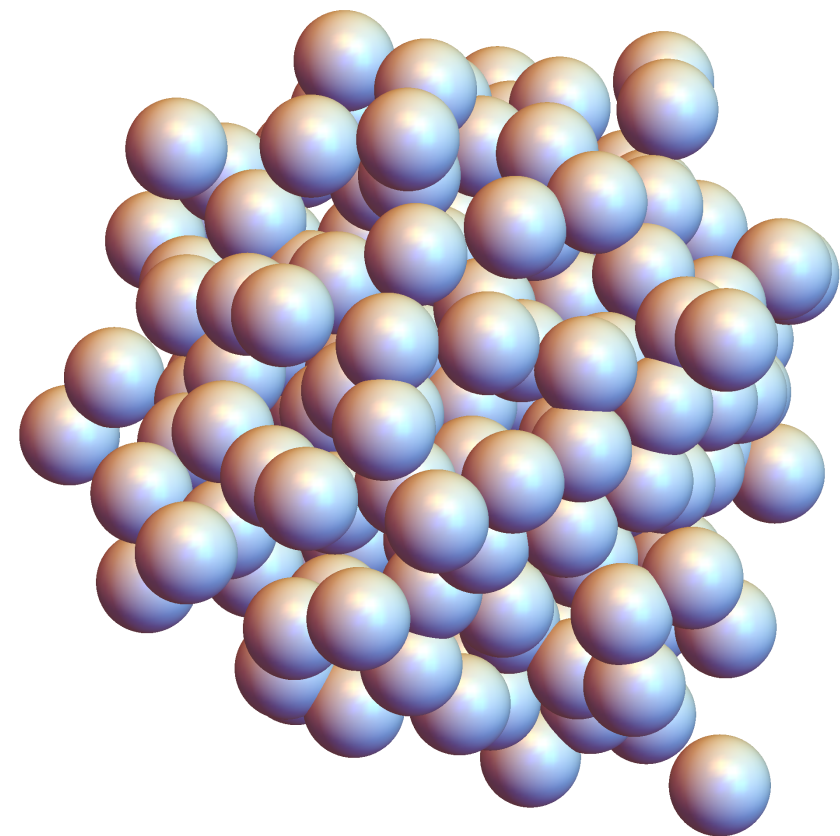
- **sub-nucleon degrees of freedom**
- **forward/backward rapidity**

# Nucleon Substructure

## Original Trento model:

- sample nucleon positions from spherical or deformed Woods-Saxon distributions
- solid angles resampled to preserve minimum distance  $d_{\min}$
- Gaussian nucleons of width  $w$
- works very well for large nuclei

$^{208}\text{Pb}$  nucleus

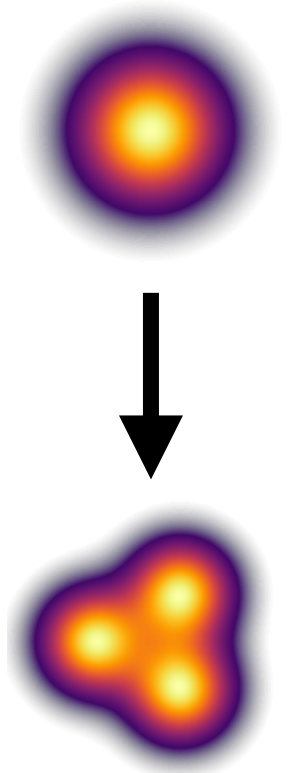


## Caveat:

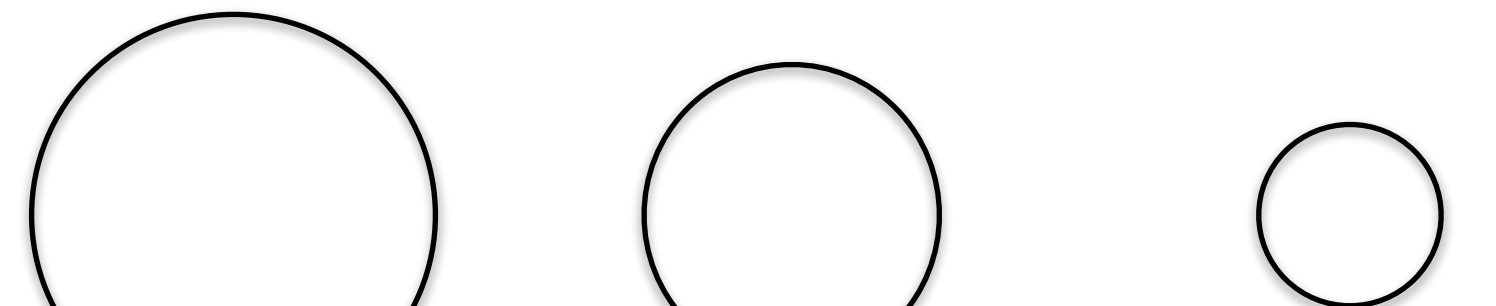
- spherical protons do not allow for proper eccentricities in p+A or small/asymmetric collision systems

## Trento with nucleon substructure:

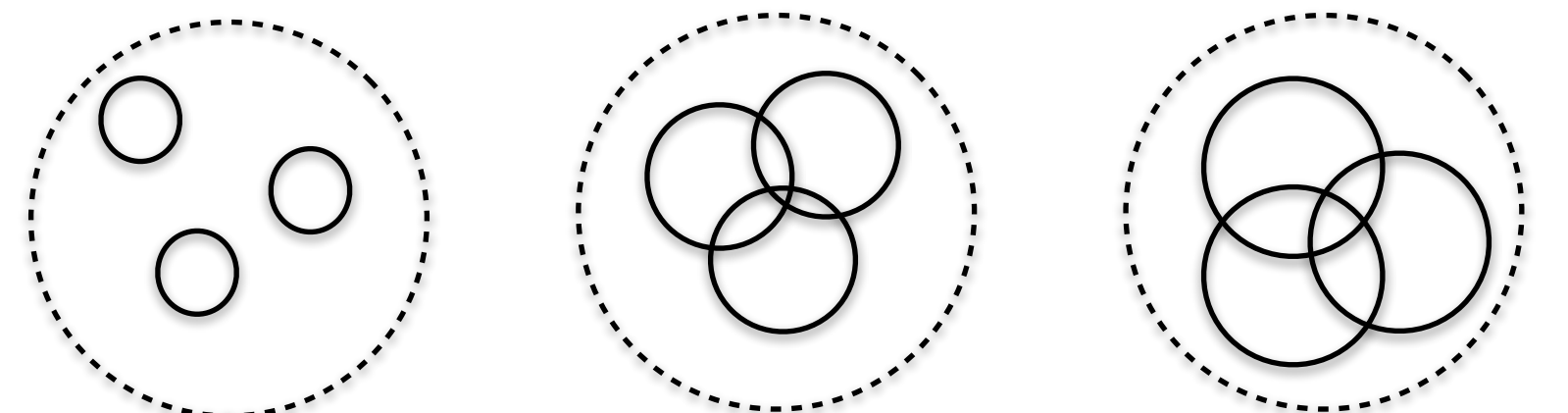
- trade Gaussian nucleons for lumpy nucleons
- additional parameters:
  - sampling radius of constituent positions
  - constituent Gaussian width
  - number of constituents in each nucleon



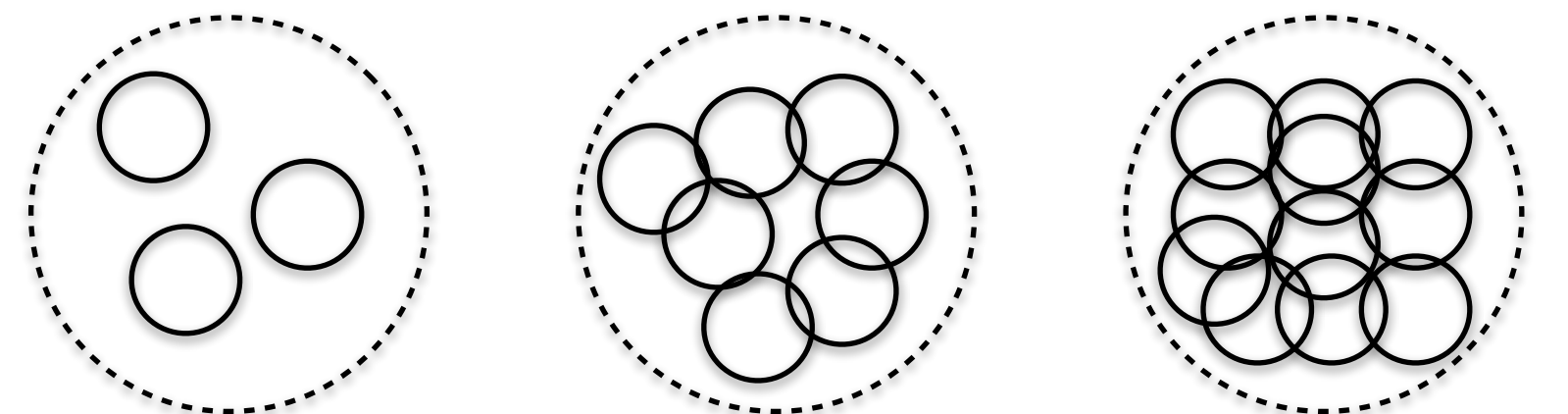
sampling radius:



constituent width:

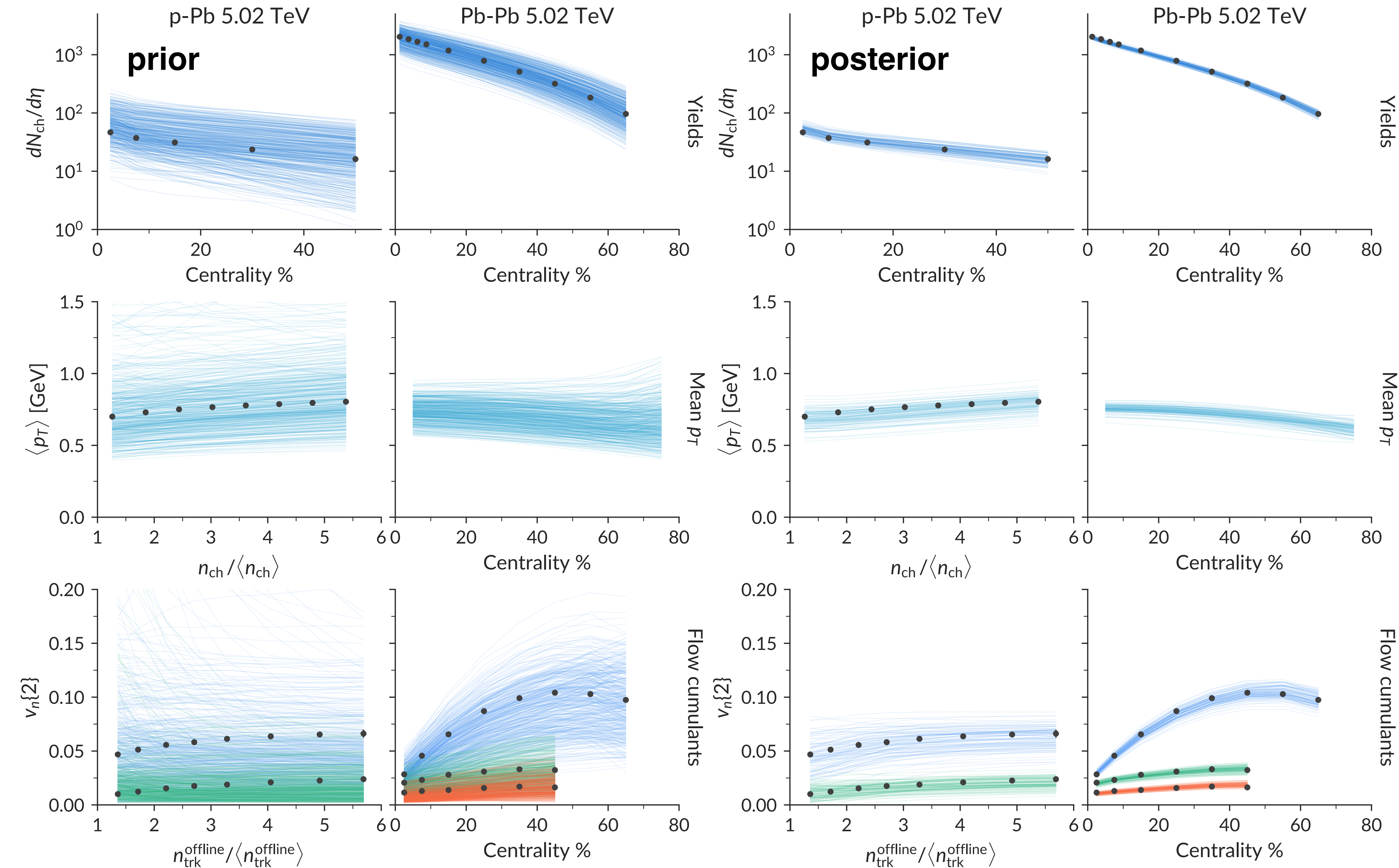


# of constituents:





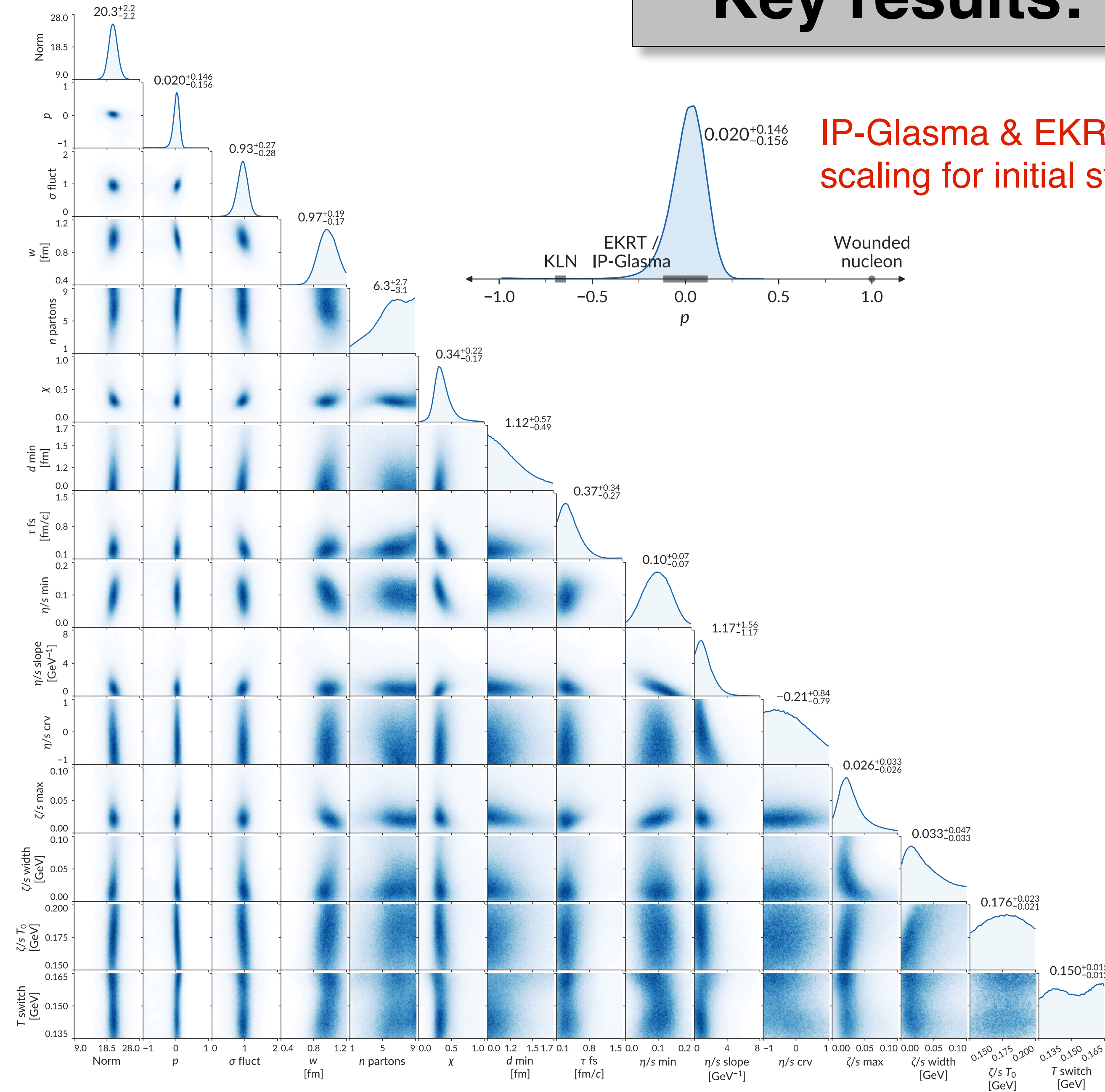
# Simultaneous Calibration on AA and pA



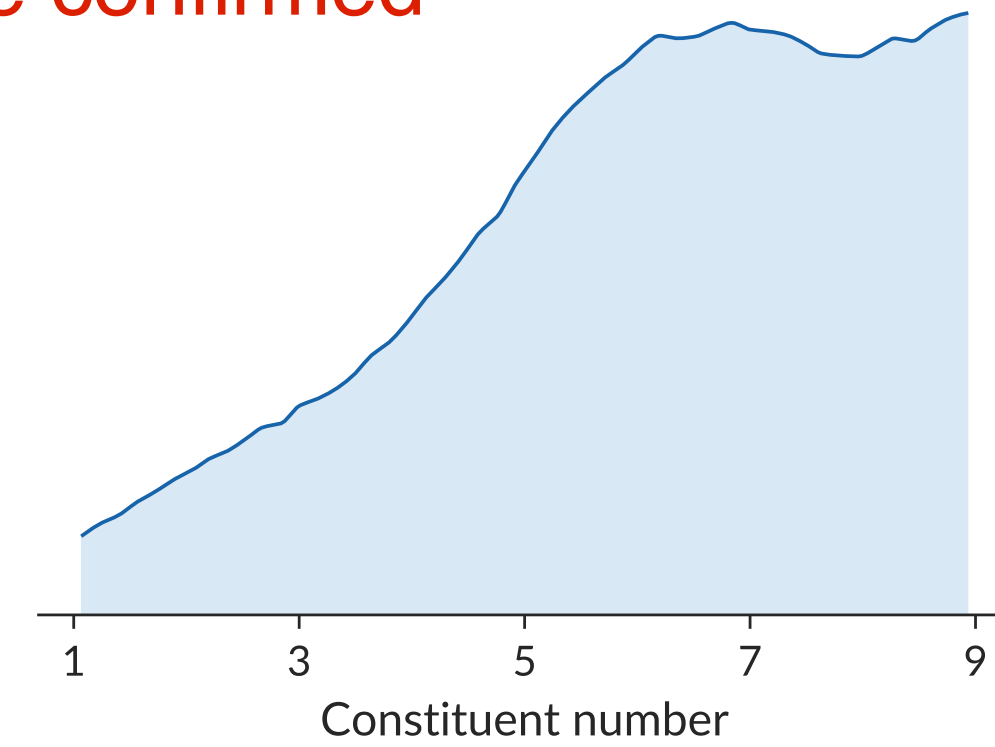
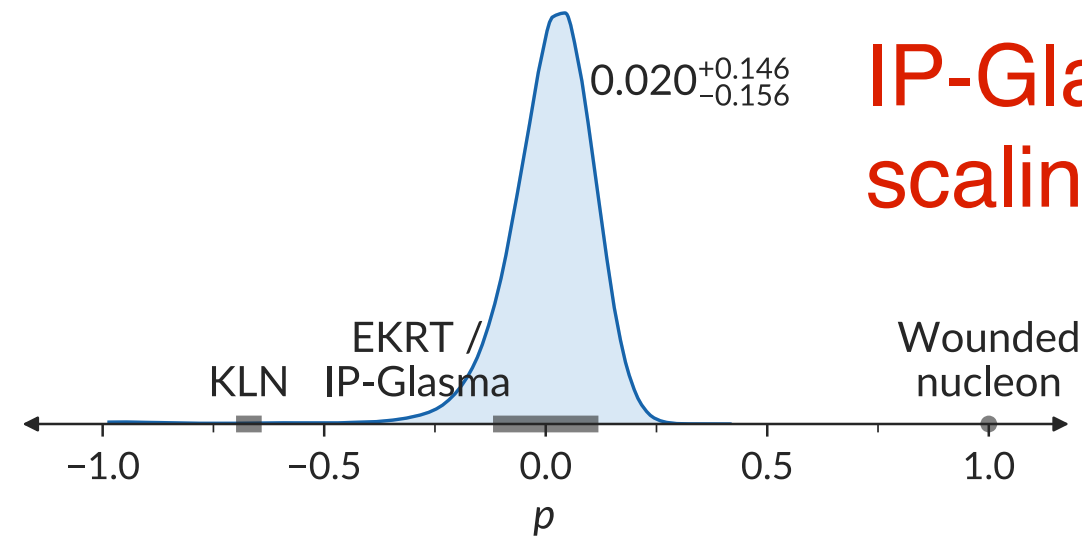
- ALICE & CMS data for AA & pA at 5.02 TeV
- calibration on 15 parameters, for initial state, shear and bulk viscosities
- restriction on 1 energy to keep computational effort reasonable
- generally larger uncertainties in posterior, due to less data than in the AA calibrations for 2 energies...



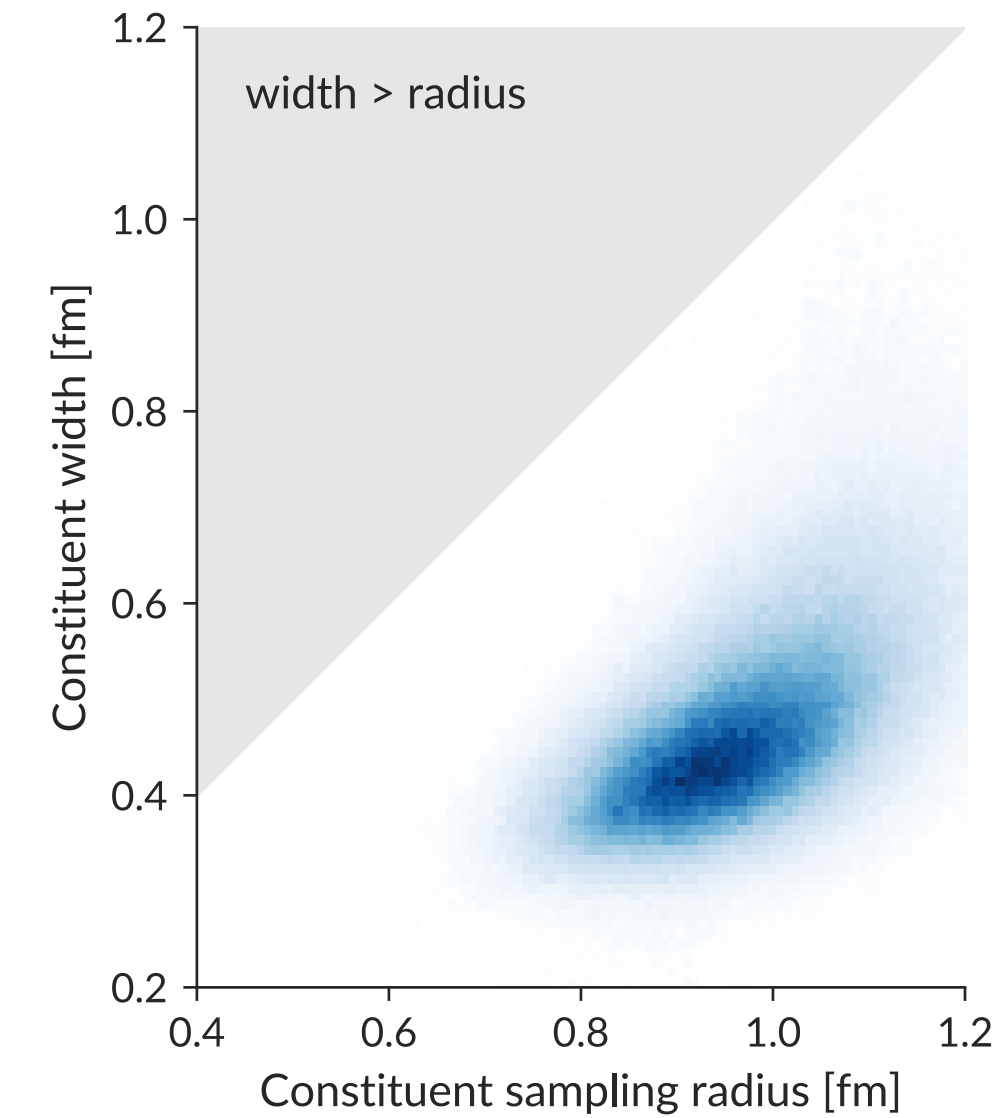
# Key results: initial state



IP-Glasma & EKRT eccentricity scaling for initial state confirmed



no strong preference for a particular constituent # as long as  $n > 3$

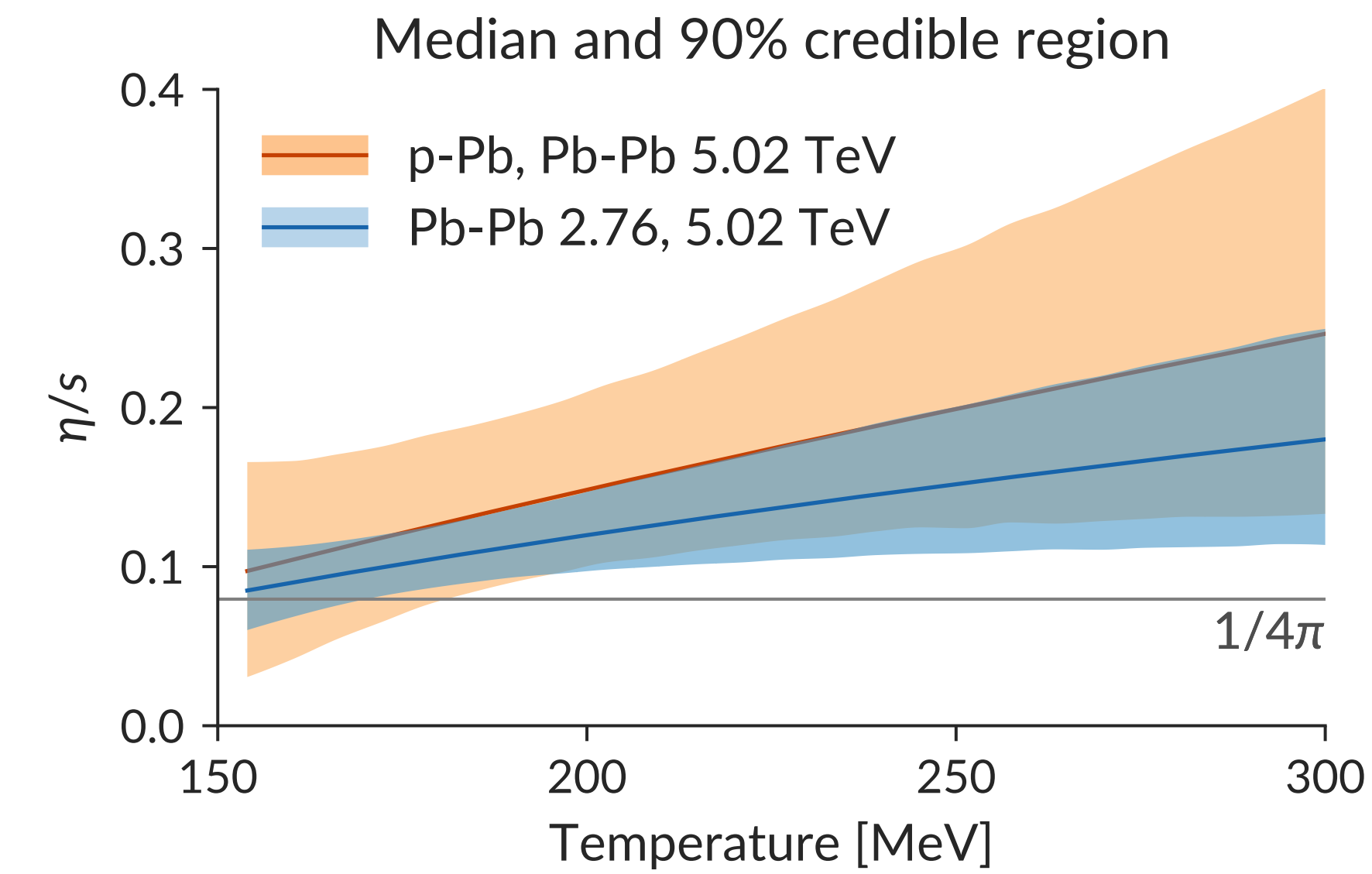
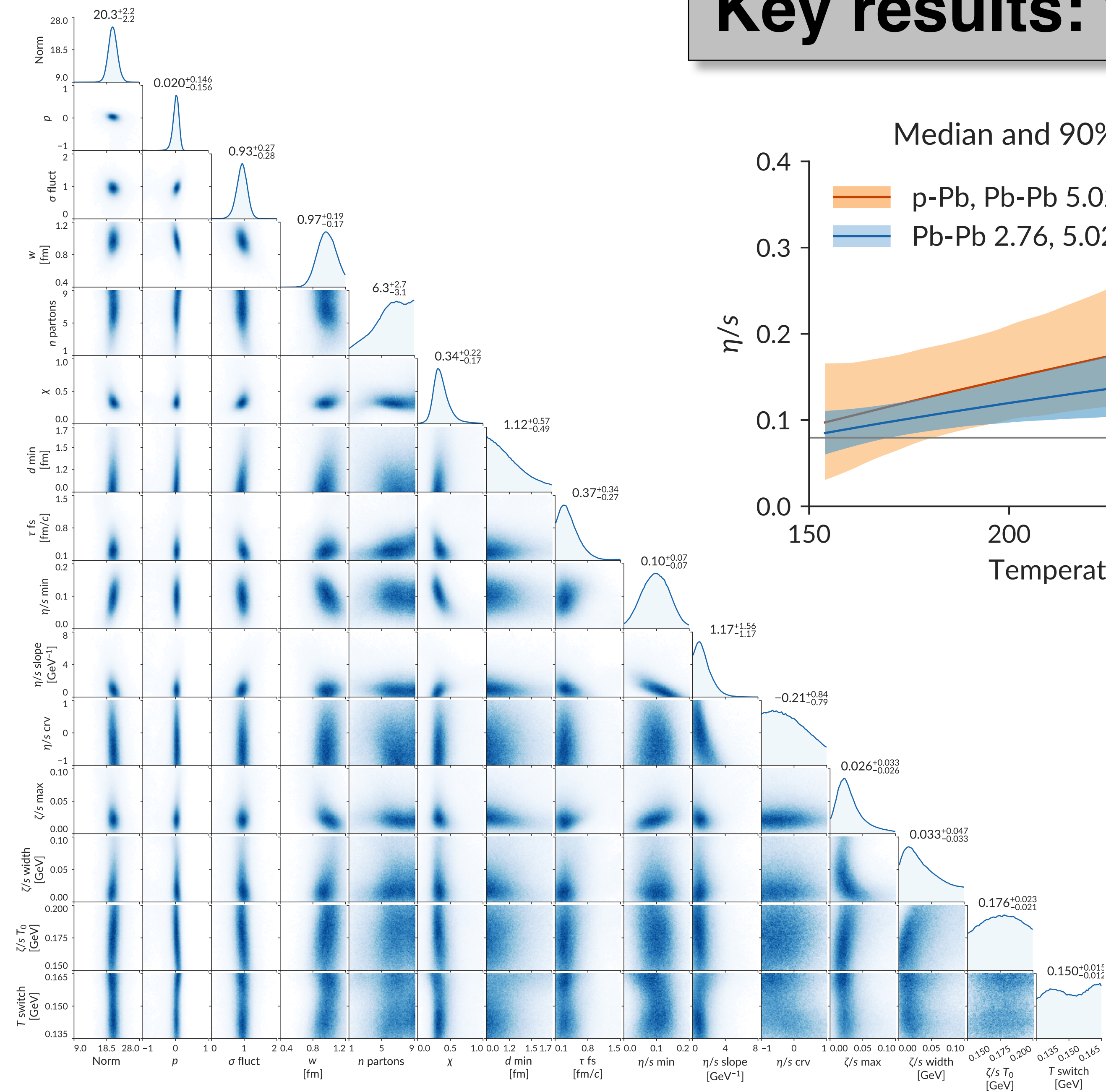


constituent width & sampling radius are well constrained to

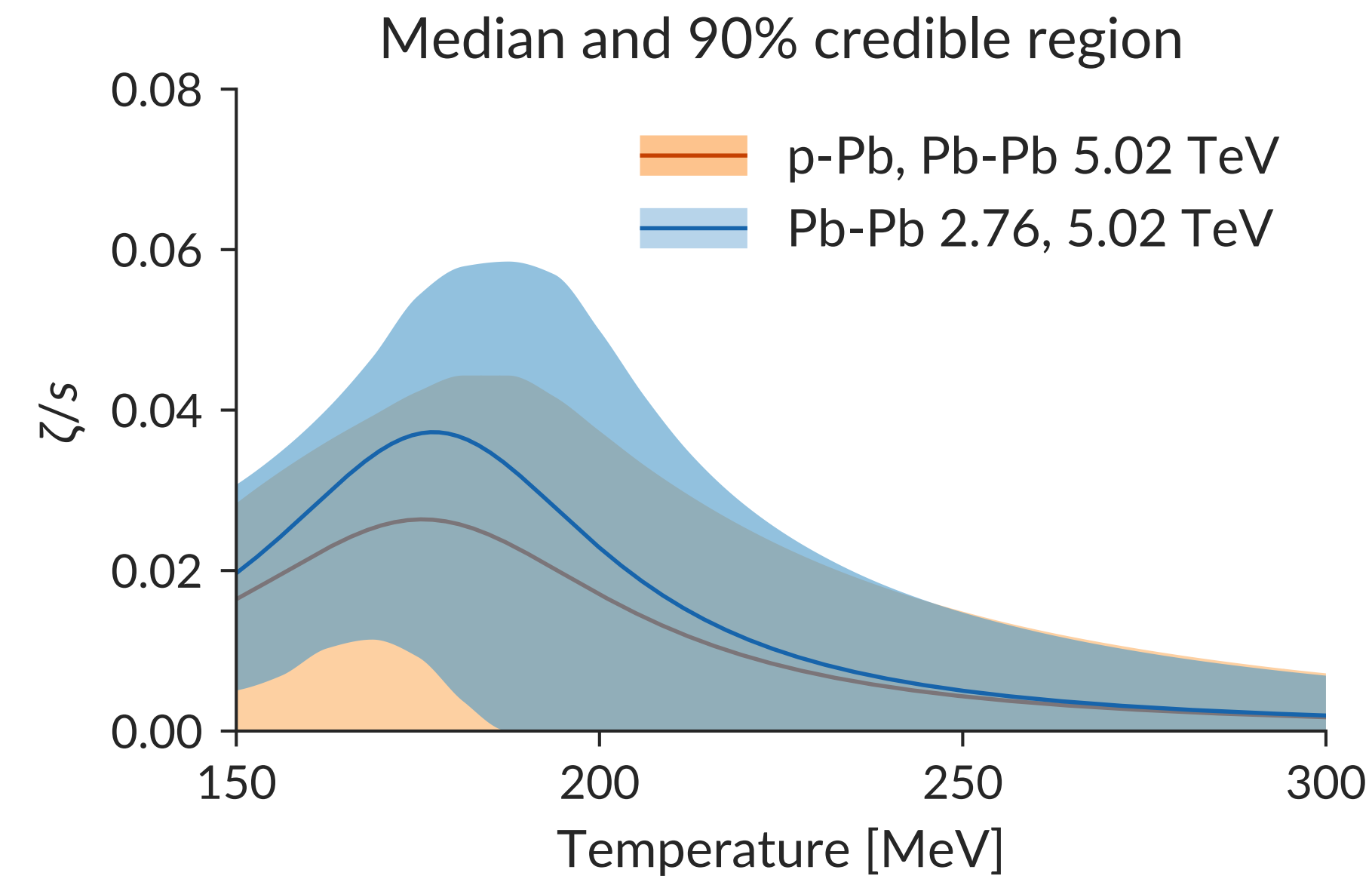
- $r = 0.99 \pm 0.16$
- $w = 0.47 \pm 0.18$



# Key results: viscosities



- shear and bulk viscosities are fully compatible with previous calibration on Pb+Pb @ 2.76 TeV & 5.02 TeV
- uncertainty bands are larger in AA + pA analysis due to focus on single beam energy
- for bulk properties, multiple beam energies are more important than inclusion of small systems



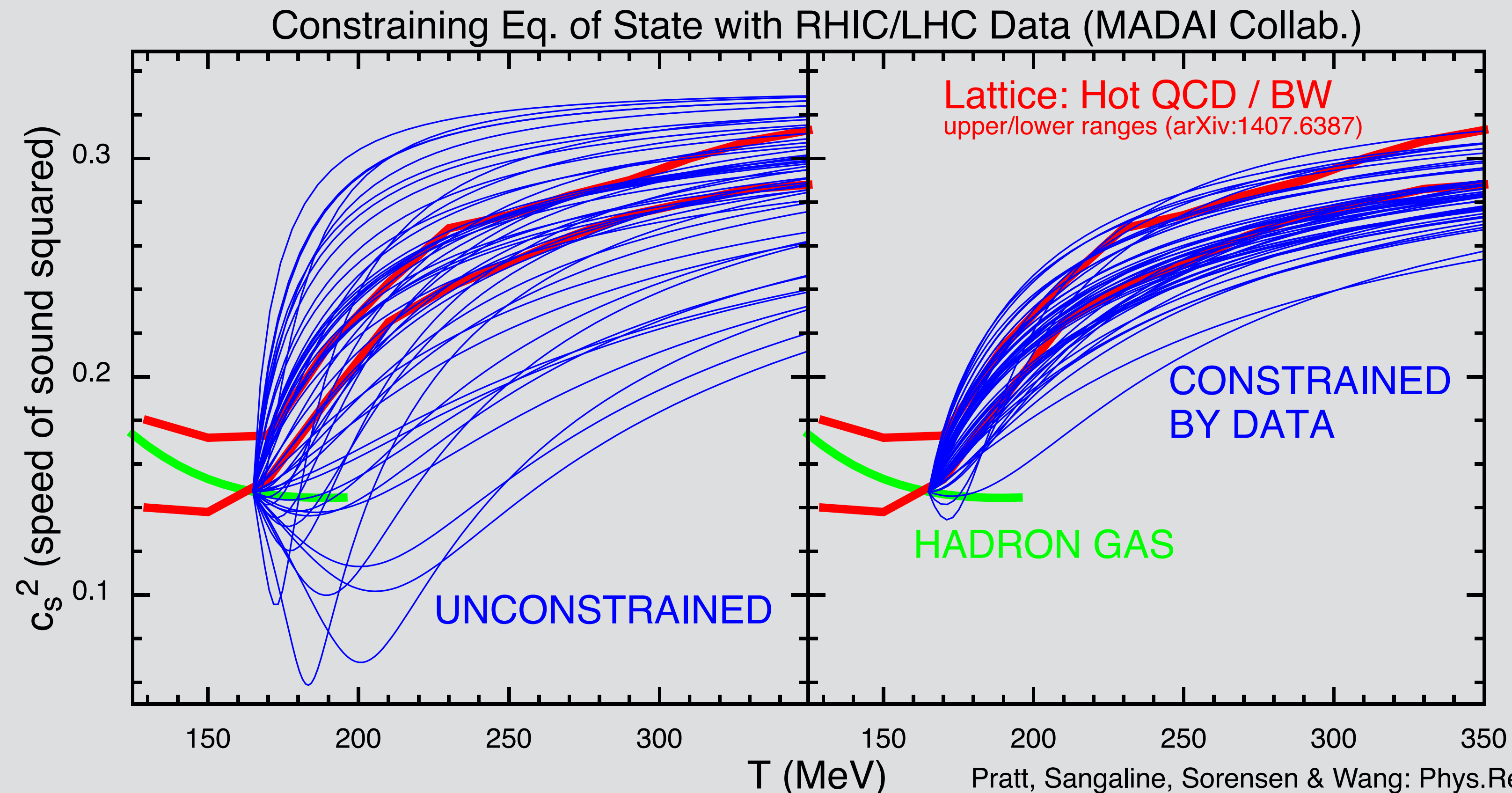


# Other Examples: Equation of State

## Example: determine the EoS of QGP matter from experimental measurements

what equation of state would the physics model choose to best describe the experimental data?

- create set of QCD Equations of State (aka the *prior*)
- run physics model with each EoS
- use comparison with RHIC/LHC data to determine which Equations of State are consistent with data (i.e. the *posterior*)
- ▶ **posterior is very similar to Lattice EoS!!**

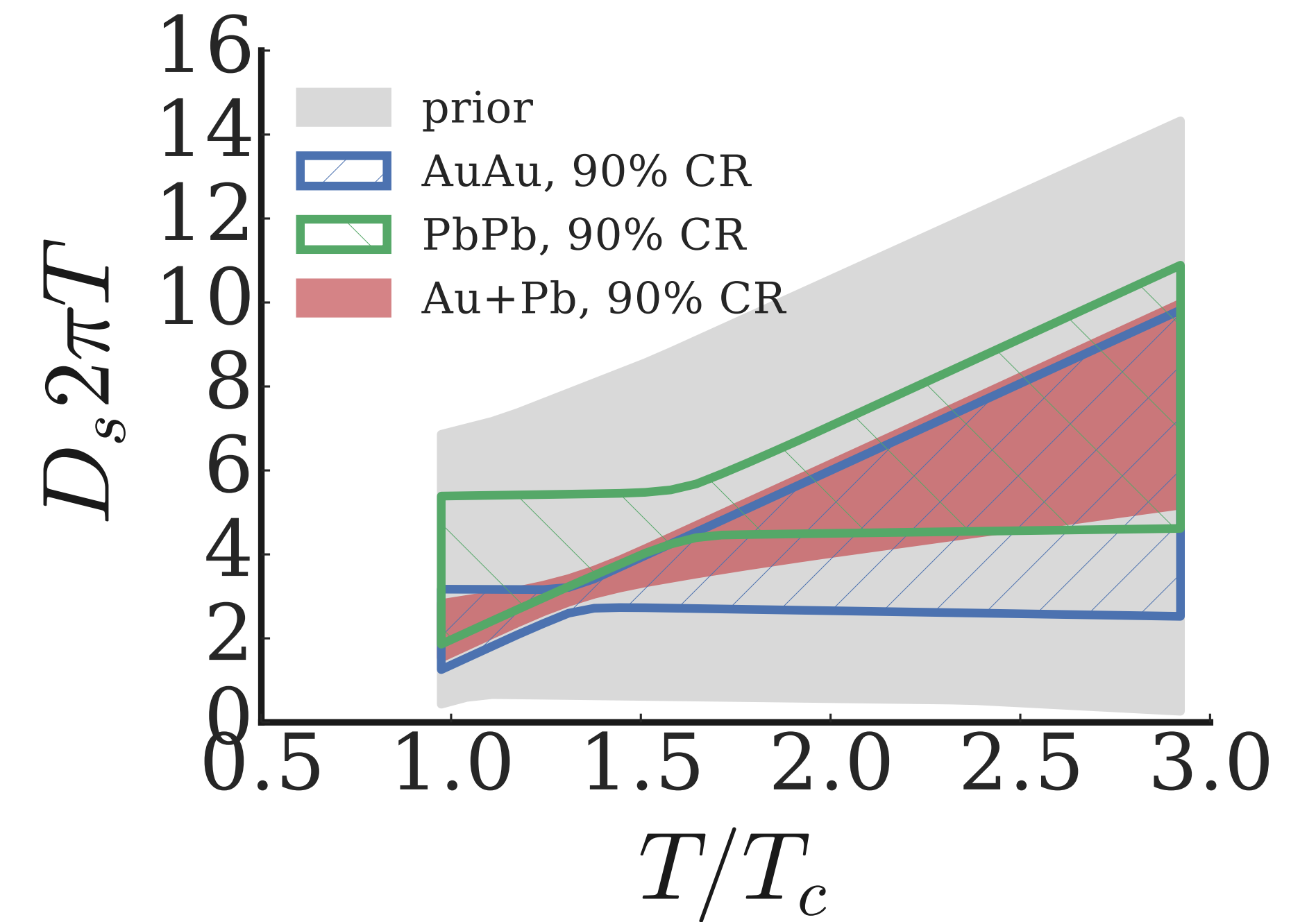
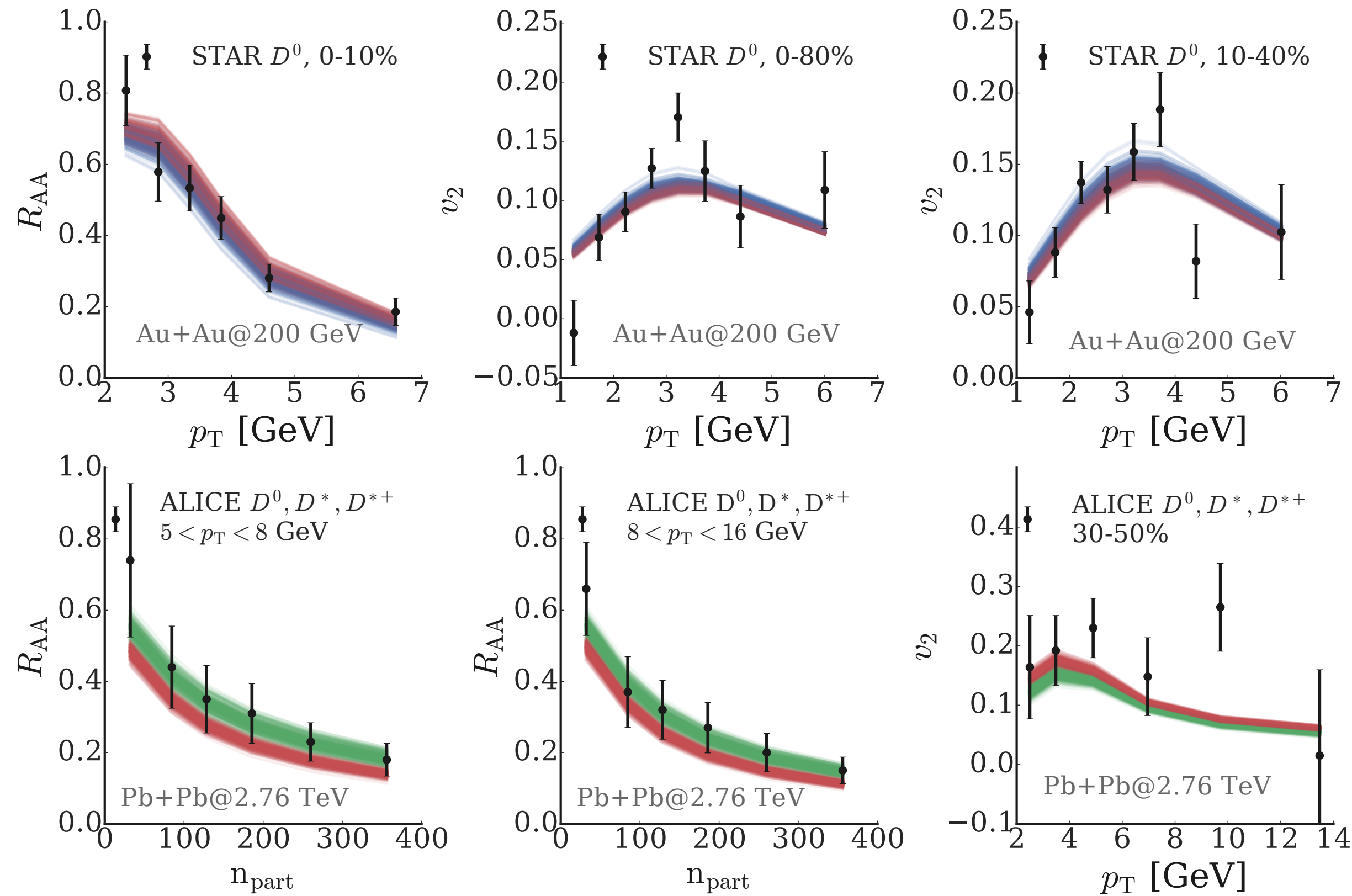




# Other Examples: Heavy Quarks

## Extraction of the Heavy Quark Diffusion Coefficient

- calibration on heavy quark  $v_2$  and  $R_{AA}$

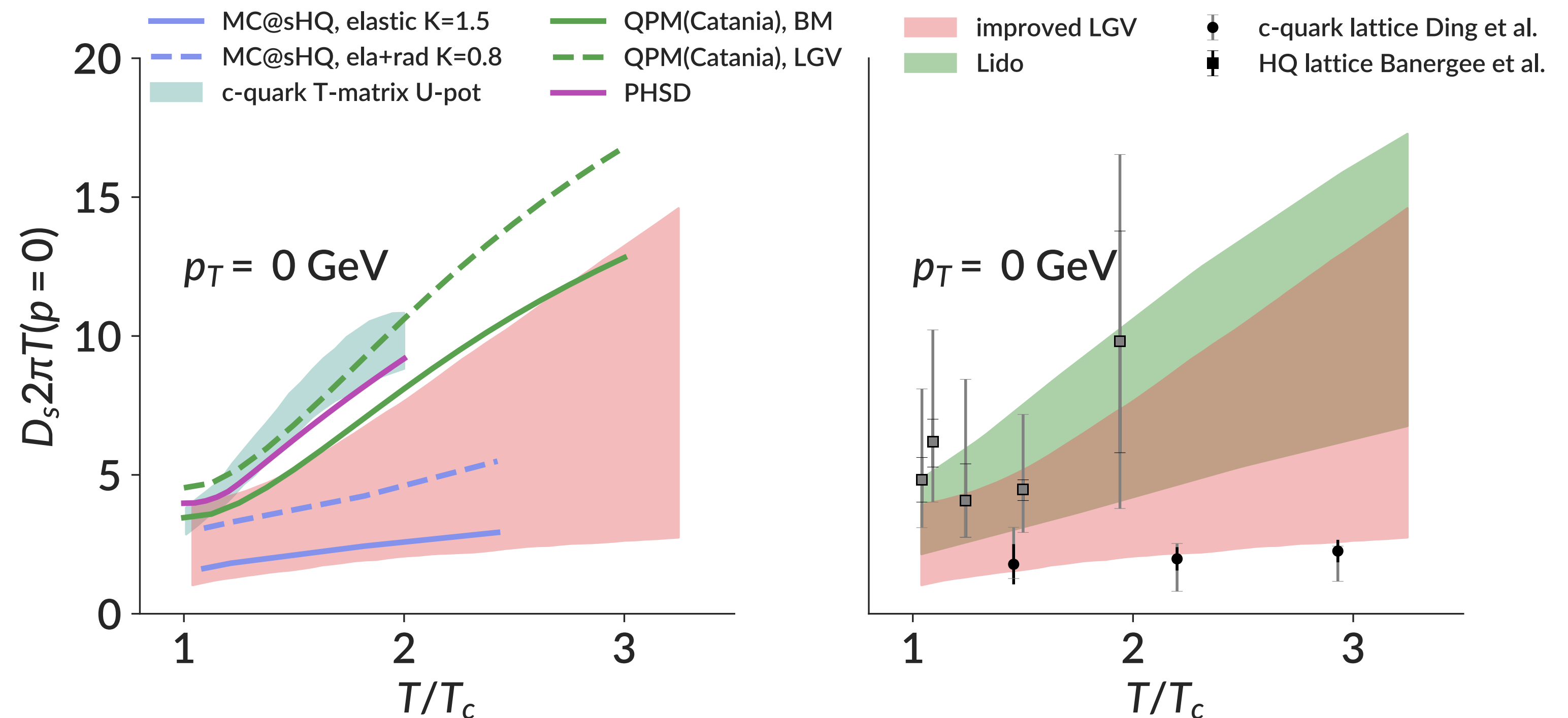


- combining RHIC and LHC data yields significant improvement for the extraction of  $D_s(T)$

# Other Applications: Heavy-Quark Transport Coefficient

## first data-driven extraction of temperature & momentum dependence of $D_s$

- $D_s$  significantly smaller than pQCD baseline at temperatures that can be probed at RHIC & LHC ( $T < 4T_c$ )
- extracted  $D_s$  compatible with Lattice QCD within (large) uncertainties
- Lido prefers slightly larger  $D_s$  values than Langevin



### caveats:

- need better data to reduce experimental uncertainties (& uncertainty-band)
- need additional observables to better constrain  $D_s$

### outlook:

- add more observables to analysis
- run analysis on different physics- and medium models to test robustness of  $D_s$  extraction