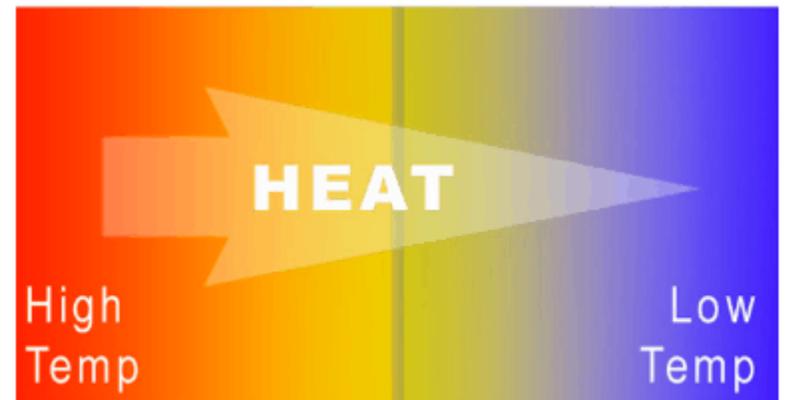
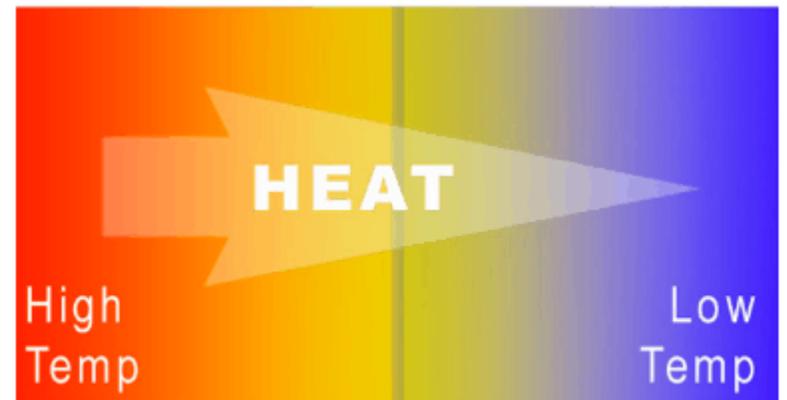


Surprises in Large N_c Thermodynamics



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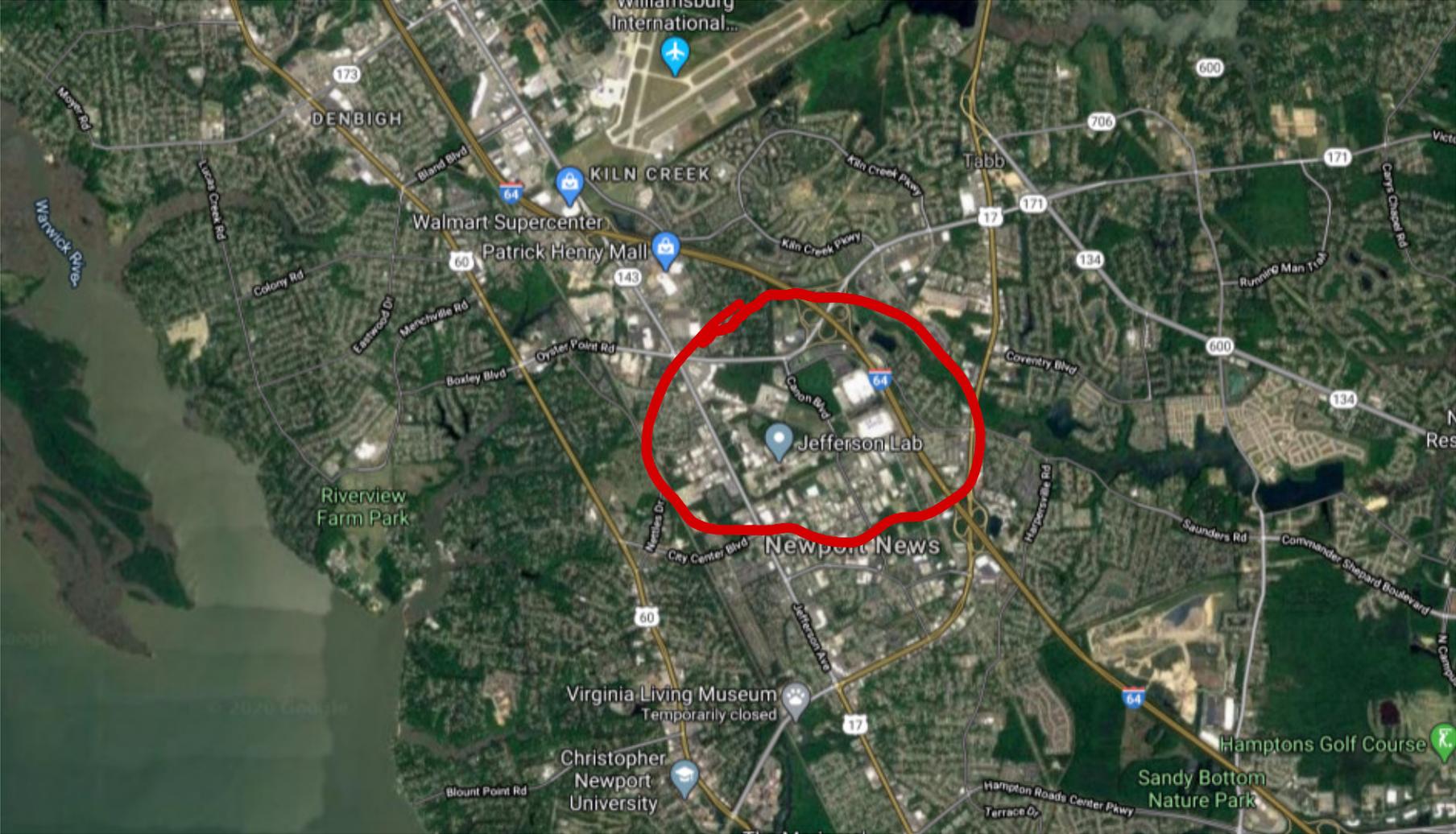


TDC, Scott Lawrence & Yukari Yamauchi (In Preparation)



An Overview

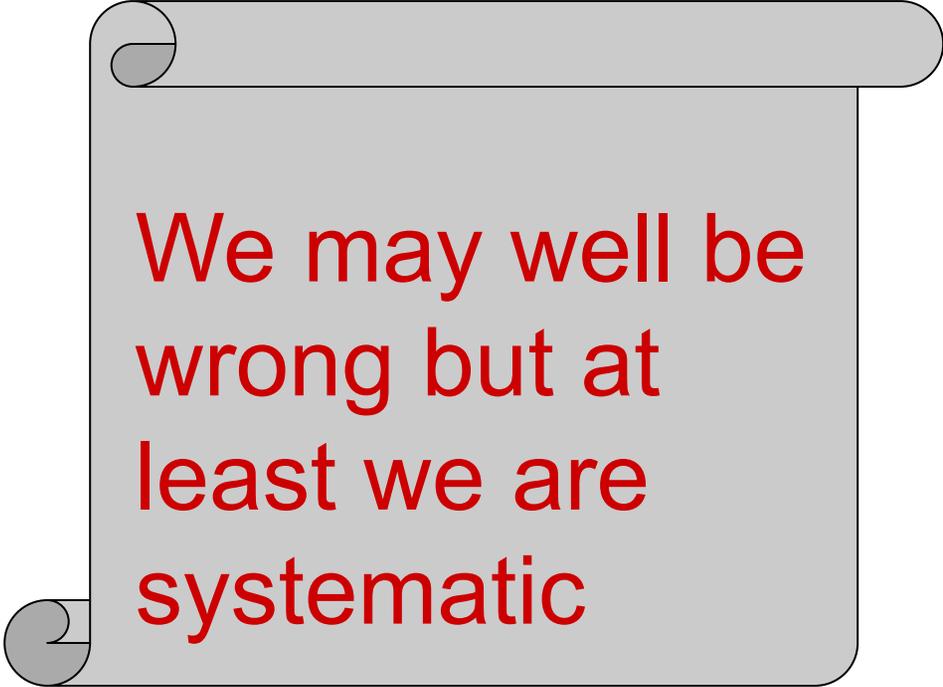
An Overview



An Overview

- **Introduction**
 - Some cautions
 - Standard results
 - Assumptions
- **Some surprises**
 - A metastable supercooled phase with **negative absolute pressure**.
 - A **clean** demonstration of a strongly coupled regime of plasma.
 - Peculiar behavior at the endpoint of the hadronic phase; breakdown of standard thermodynamic limit.

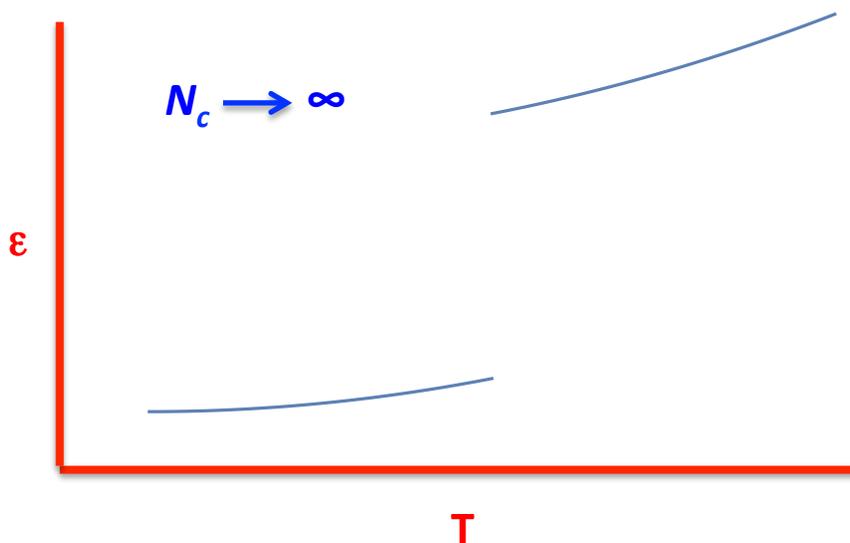
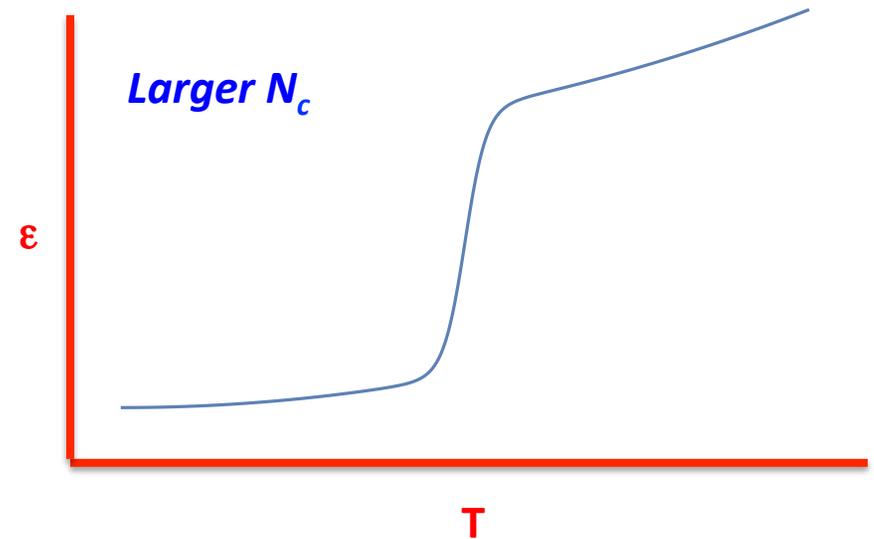
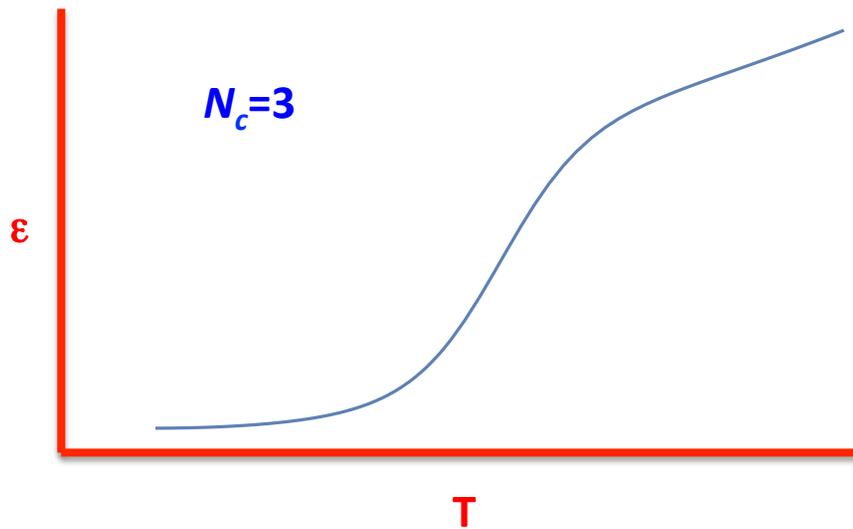
A Motto for $1/N_c$ practitioners



We may well be
wrong but at
least we are
systematic

To the extent that $1/N_c$ corrections are modest, the large N_c world **may** be a useful cartoon version of the physical world.

However **thermodynamic properties around phase transitions** or rapid cross-overs are **likely to be cases where the cartoon is insufficient.**



A crossover for $N_c=3$ can become increasing sharp as N_c increases and as it goes to ∞ , the qualitative behavior can change from being a crossover to a first order transition—a qualitatively different behavior.

This is precisely what we believe happens for QCD.

Despite the qualitative differences there may be useful insights by considering the large N_c limit.

Some standard large N_c results (Witten, 't Hooft 1970s)

- Mesons and glueballs exist as unmixed narrow states with masses of order unity in a $1/N_c$ expansion:

$$m_{\text{meson}} \sim N_c^0, \quad m_{\text{glueball}} \sim N_c^0$$

- Meson-meson, meson-glueball and glueball-glueball interactions vanish as $N_c \rightarrow \infty$. A coupling with n_m mesons and n_g glueballs scales as $N_c^{1-n_g-\frac{1}{2}n_m+d_{0,n_m}}$.

- Widths scales: $\Gamma_{\text{meson}} \sim N_c^{-1}, \quad \Gamma_{\text{glueball}} \sim N_c^{-2}$
- Meson-meson & meson-glueball cross-section scales as $\sim N_c^{-1}$
- glueball-glueball cross-section scales as $\sim N_c^{-2}$

- There are an infinite number of glueballs and mesons with any given fixed quantum numbers as $N_c \rightarrow \infty$

- Baryons have masses that scale as $m_{\text{baryon}} \sim N_c^1$

Some standard large N_c thermodynamic results:

- Previous results imply that in a hadronic phase the system becomes a weakly coupled hadronic gas composed of mesons and glueballs with the energy density scaling as N_c^0 .
- RG analysis indicates that the QCD becomes weakly coupled at a momentum transfer that scale as $\sim N_c^0$.
 - The system enters a quark-gluon plasma regime at temperature that scales as $\sim N_c^0$.
 - The energy density in the quark-gluon plasma regime scales as $\sim N_c^2$.
- The discrepancy between the N_c^0 behavior in the hadronic regime and the N_c^2 behavior in the plasma regime implies that there must be a phase transition (first or second order)—at least as $N_c \rightarrow \infty$.

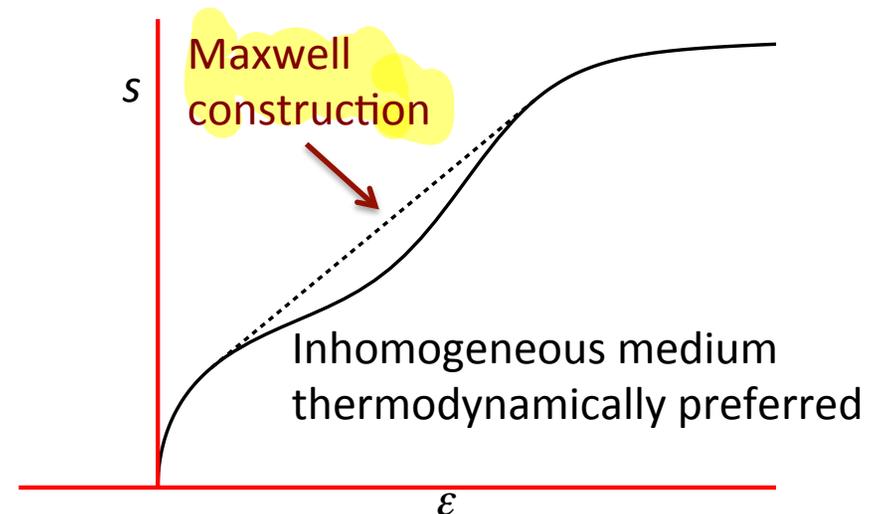
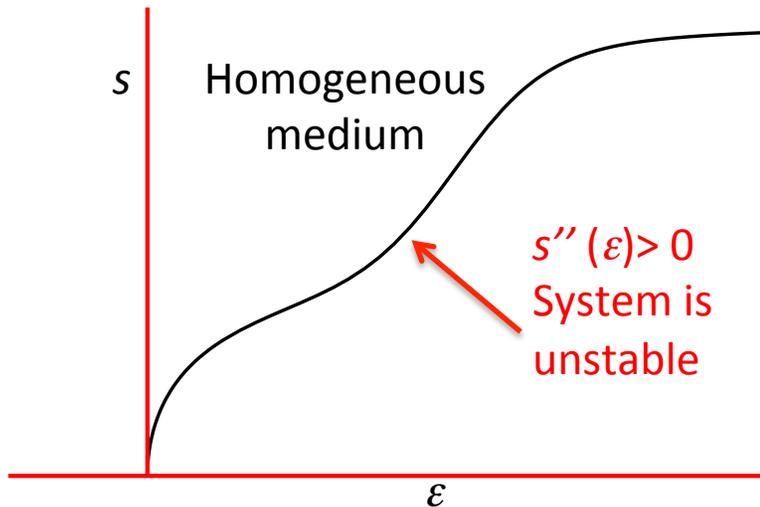
Some standard large N_c thermodynamic results:

– There is a strong reason to believe that this phase transition should be first order.

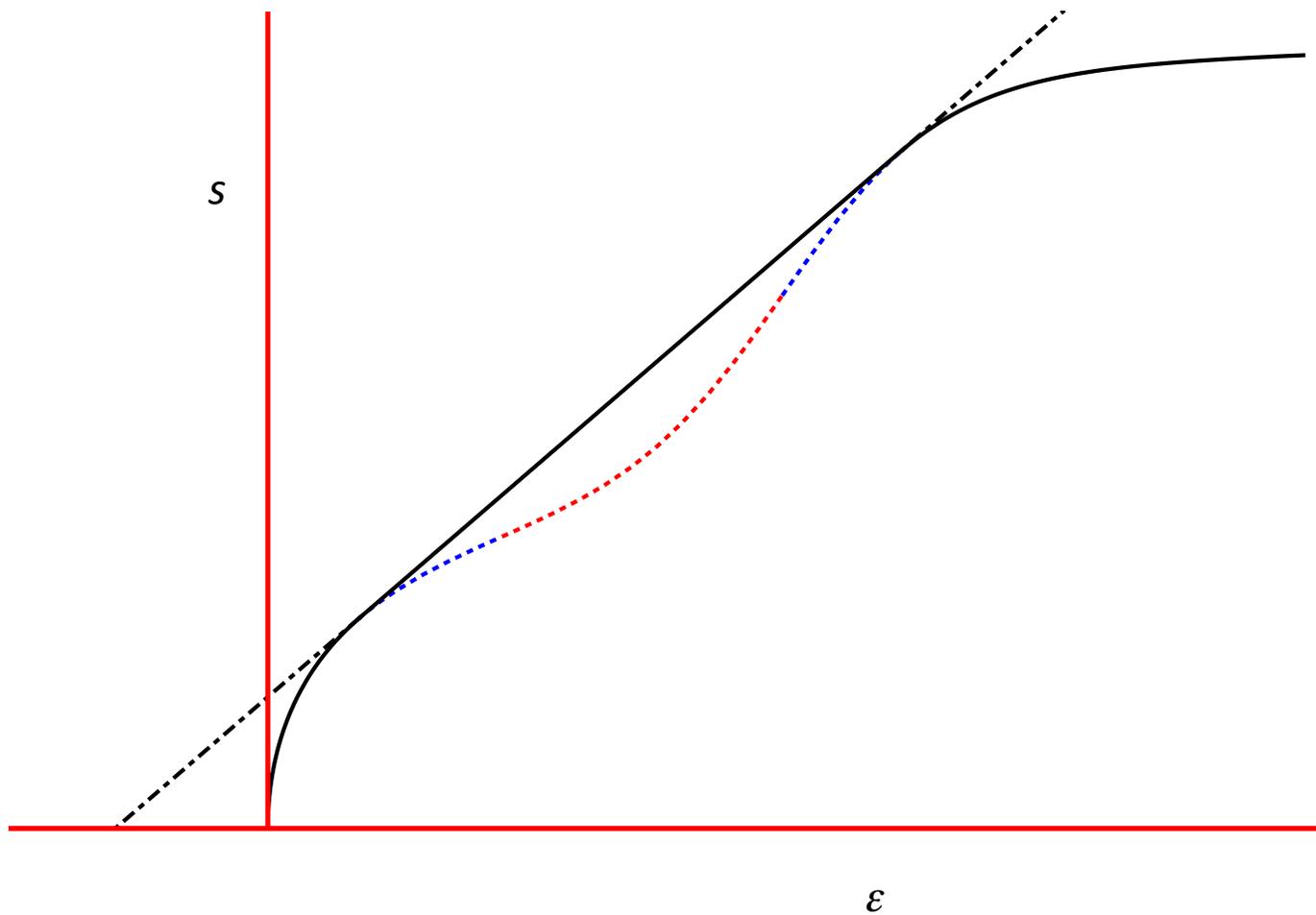
- In the large N_c limit quark loops are suppressed. Thus one expects the thermodynamics of QCD to become equivalent to Yang-Mills as N_c gets large.
- Yang Mills is known to have a first order transition at $N_c=3$.
- Lattice simulations by the Oxford Group (*Teper and collaborators*) in the early 2000s indicate that the first order transition persists at larger N_c with latent heat growing as N_c^2 as one would expect if the first-order transition persisted up to infinity.

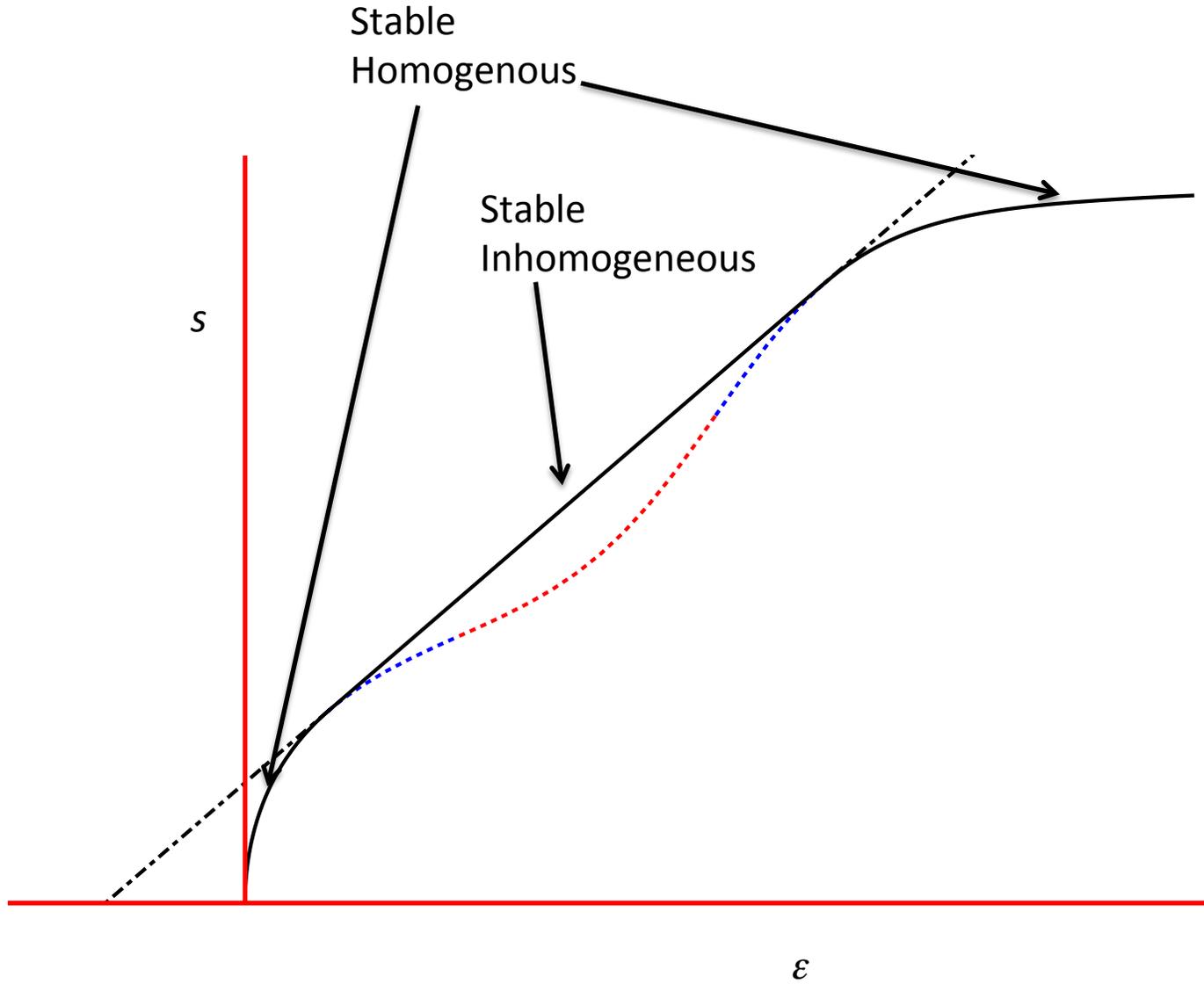
Throughout this talk, it will be assumed that a first order transition exists between a hadron and plasma phase

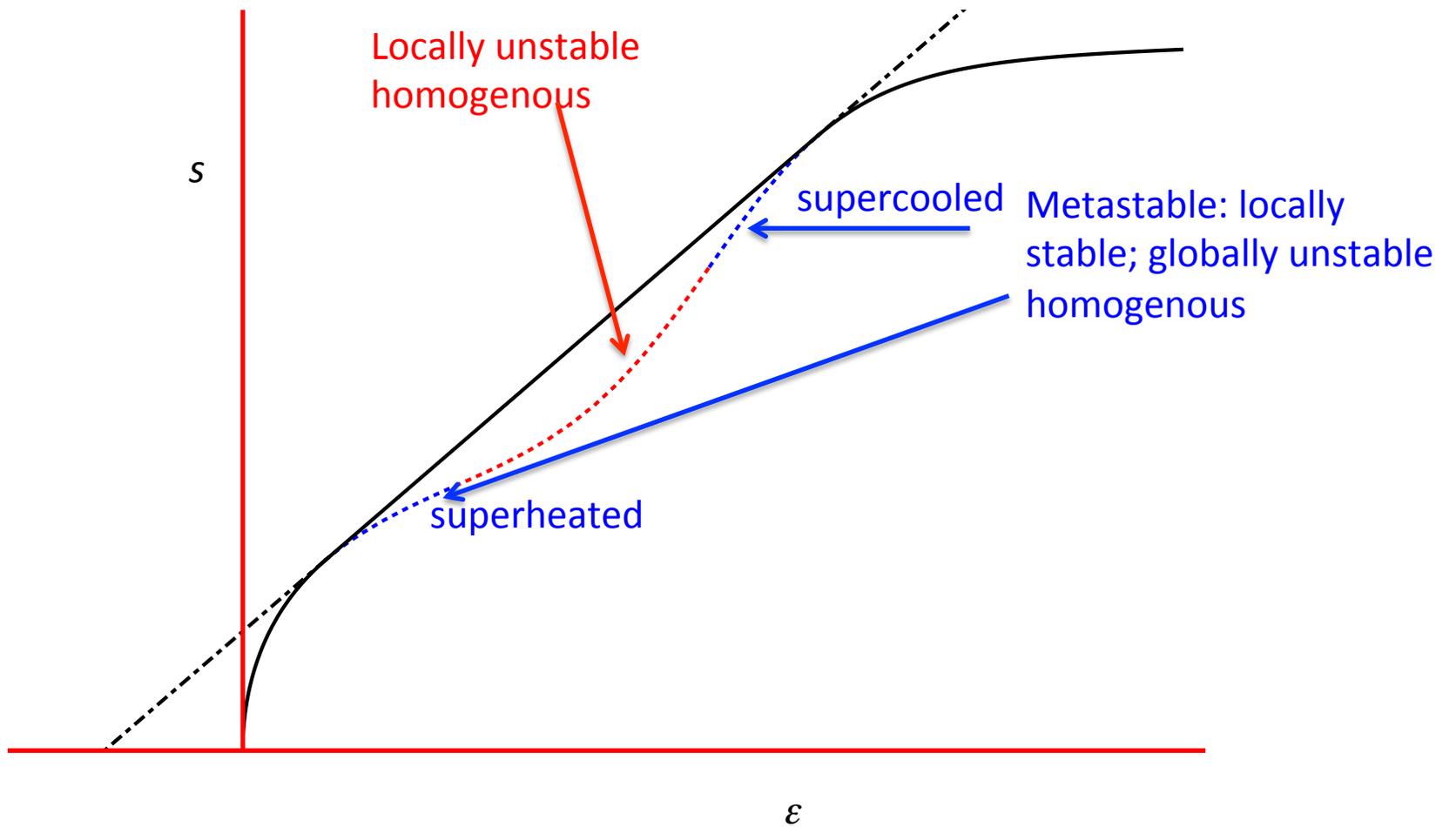
- This talk will mostly use the **microcanonical** ensemble as this provides the most insight for these problems.
 - Key quantity $S(E)$ where $S(E)$ is the log of the number of accessible states at E .
 - $S'(E)=1/T$
 - In thermodynamic limit of large volumes relevant quantities are entropy density, s , and energy density ε : $s(\varepsilon) = \lim_{V \rightarrow \infty} S(\varepsilon V)/V$
 - Thermodynamic stability implies $s''(\varepsilon) \leq 0$.

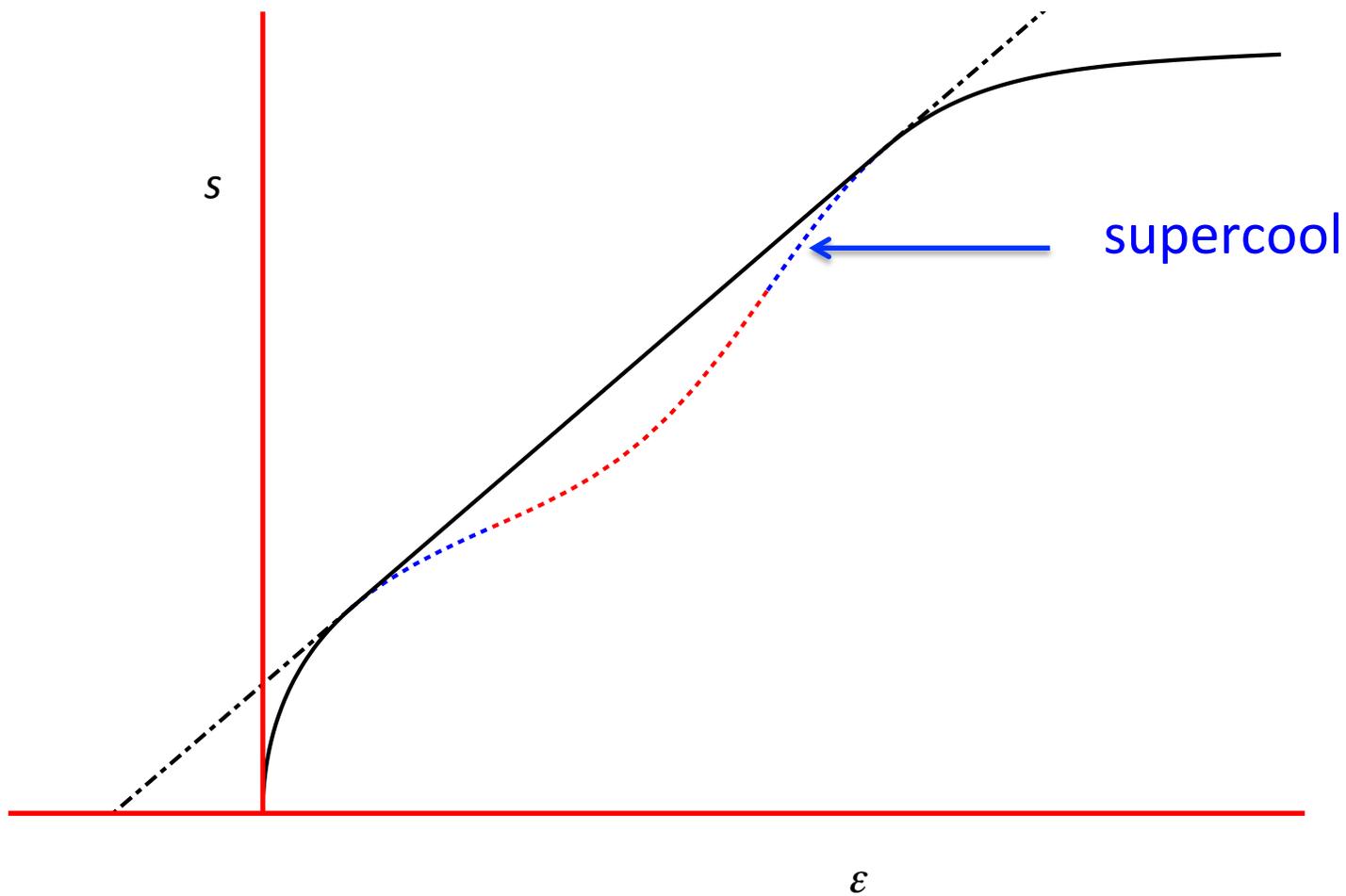


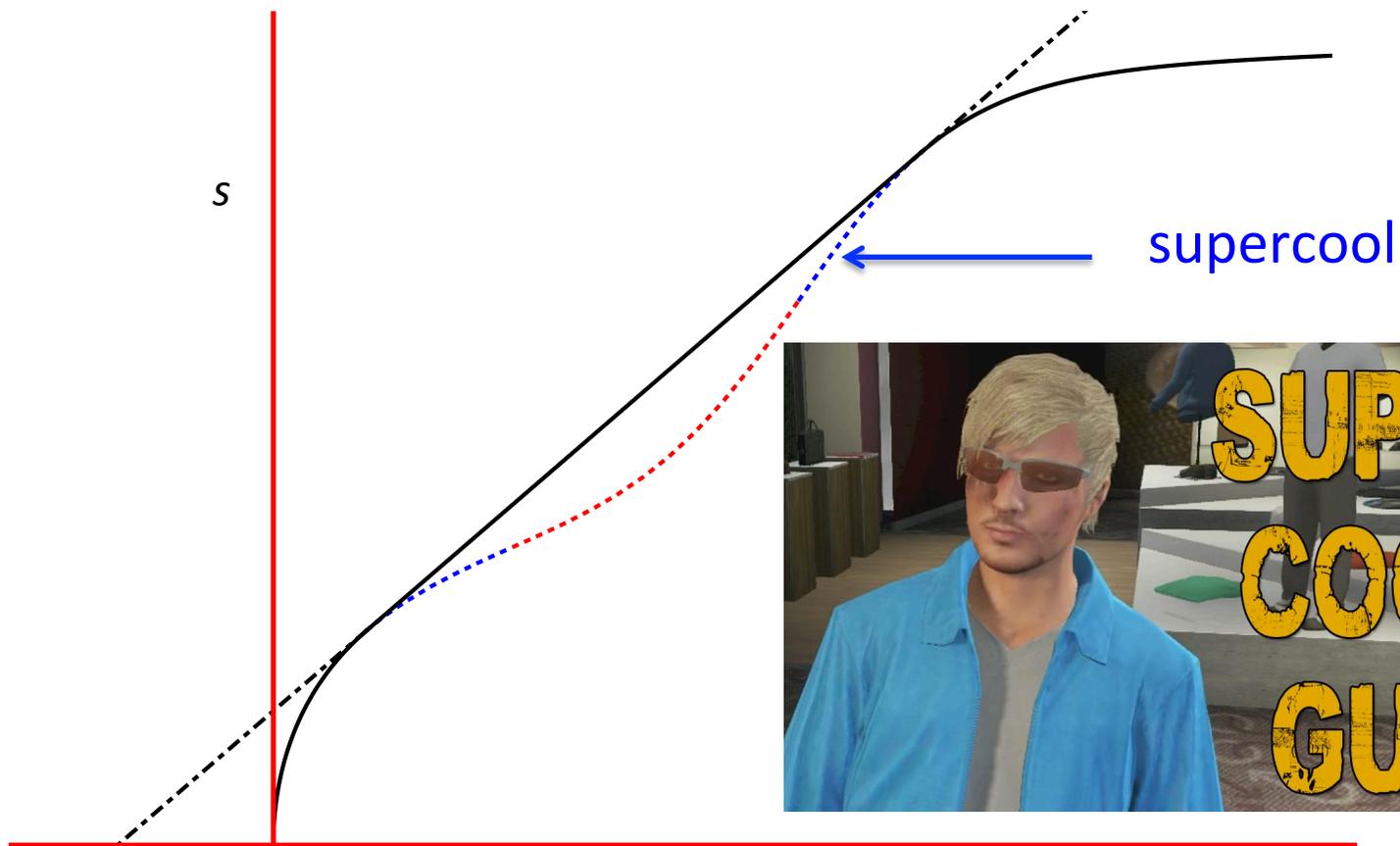
Generic first order transition



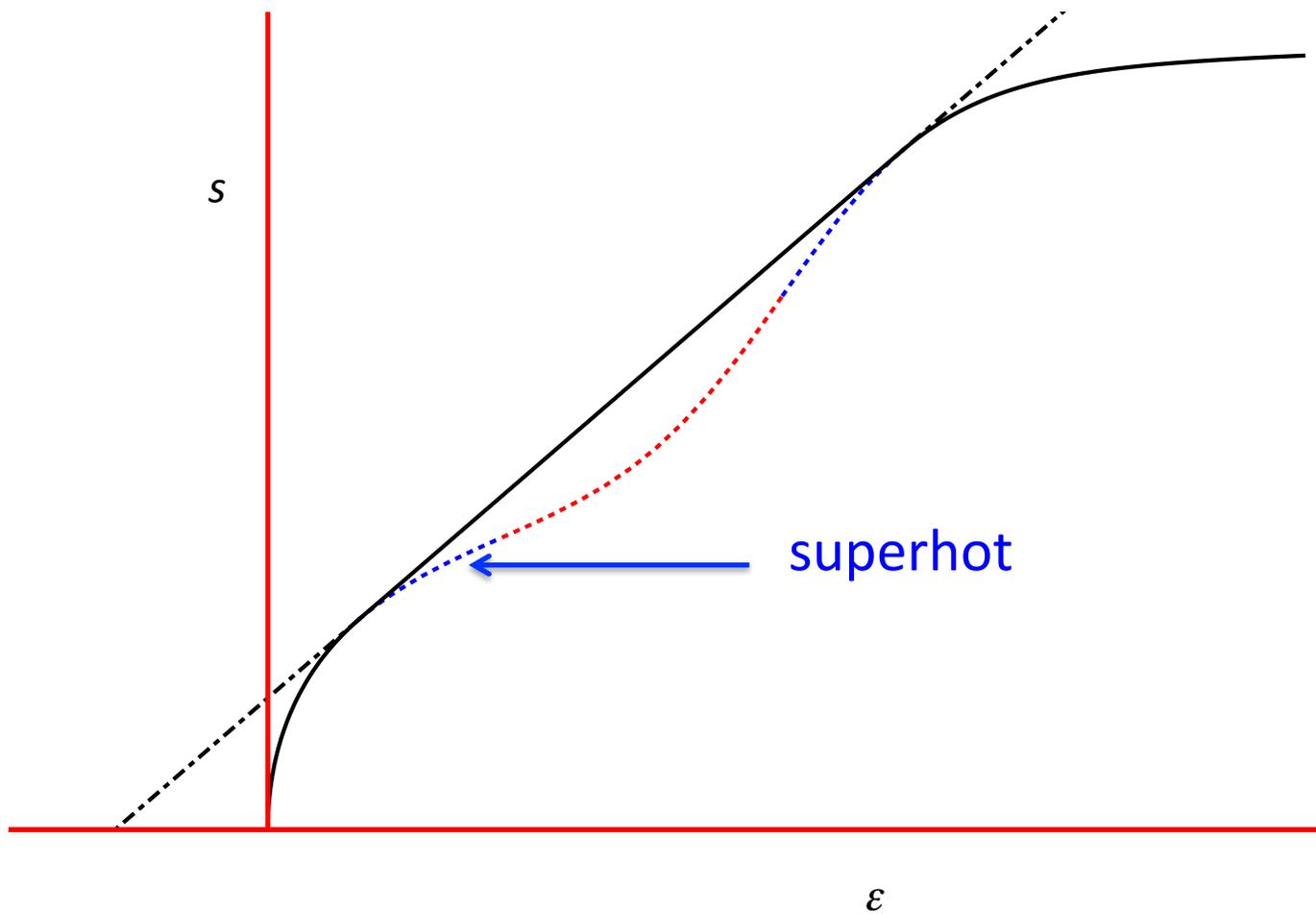


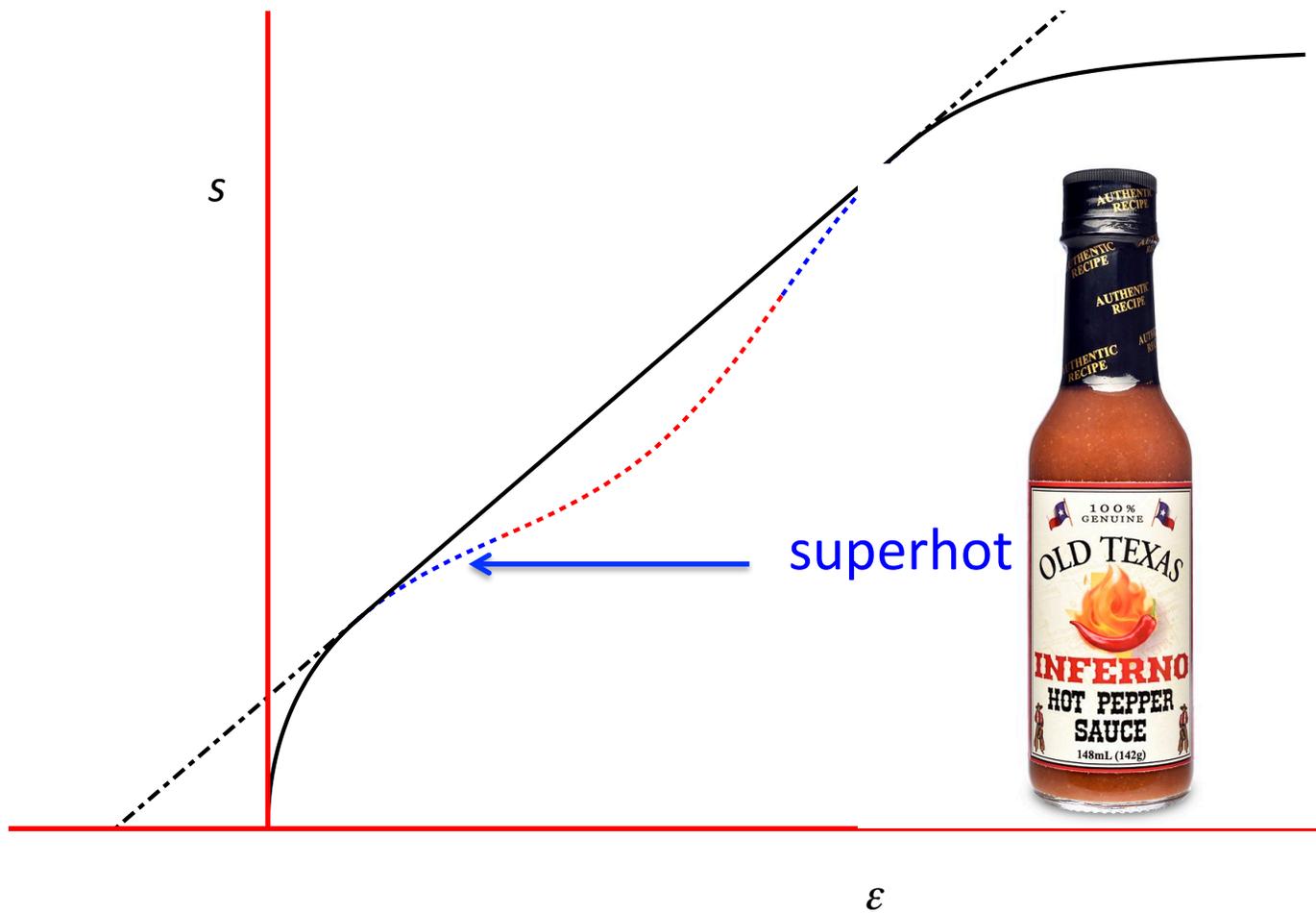


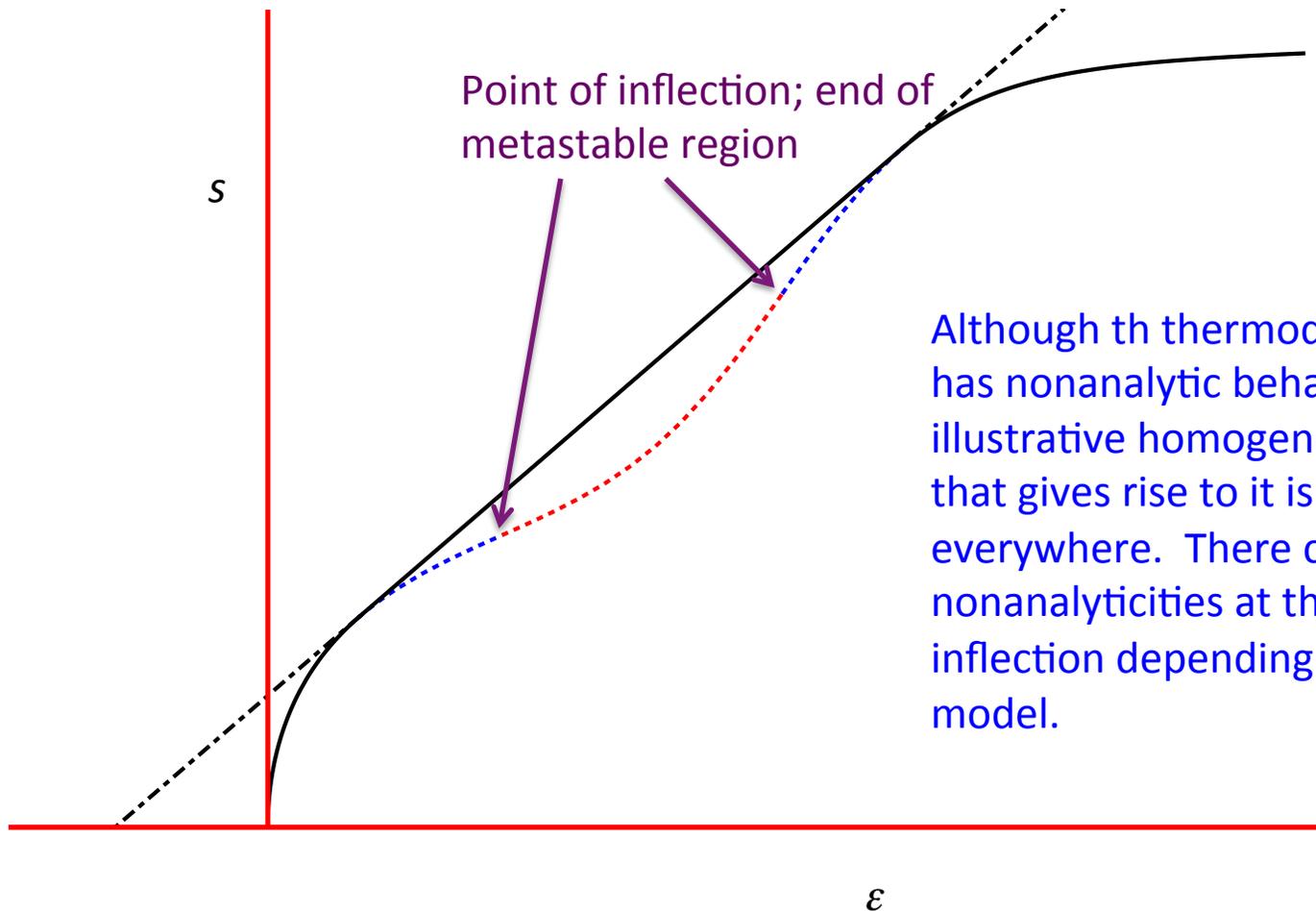




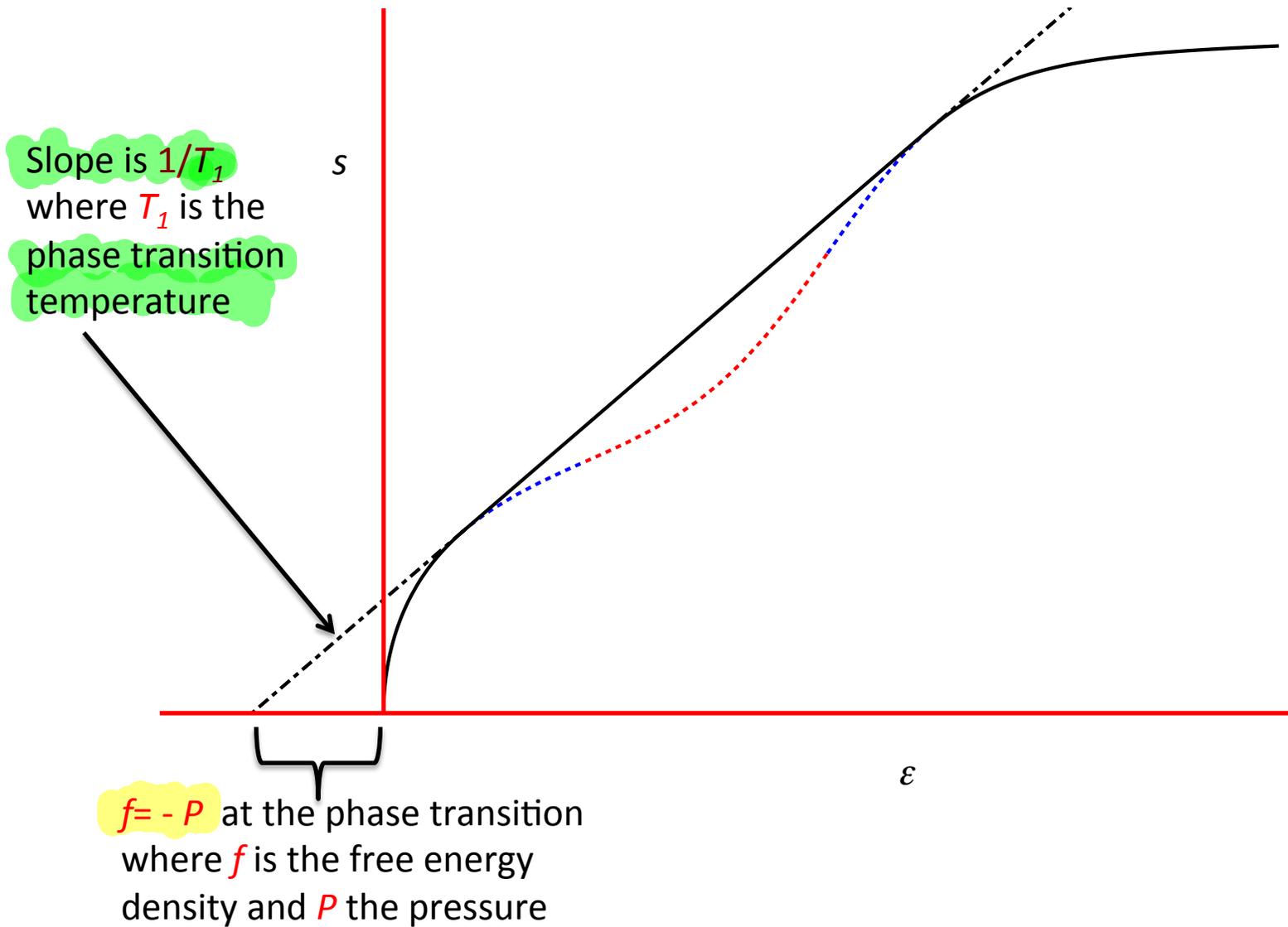
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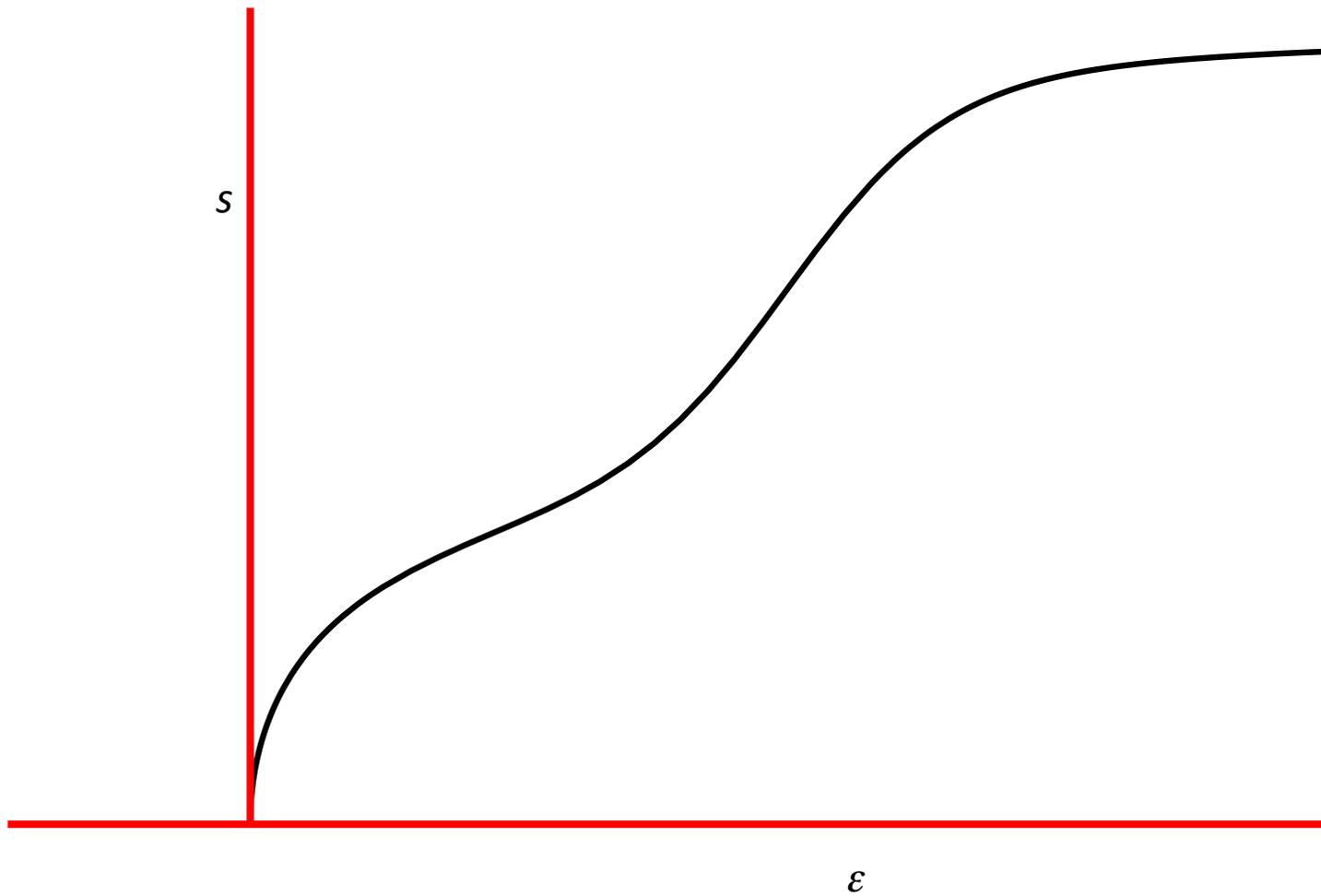




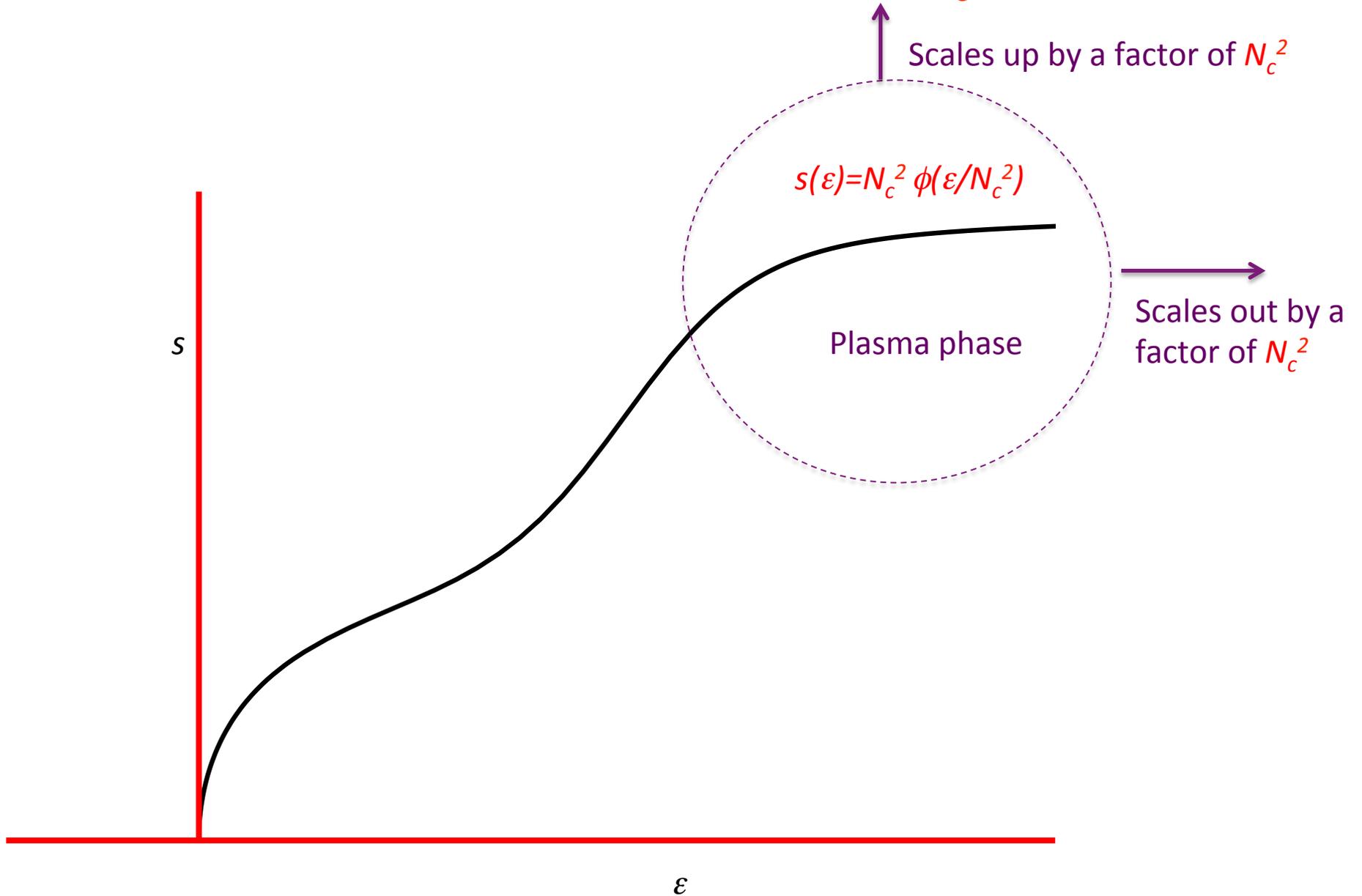
Although th thermodynamics has nonanalytic behavior, his illustrative homogenous curve that gives rise to it is analytic everywhere. There can be nonanalyticities at the points inflection depending on the model.



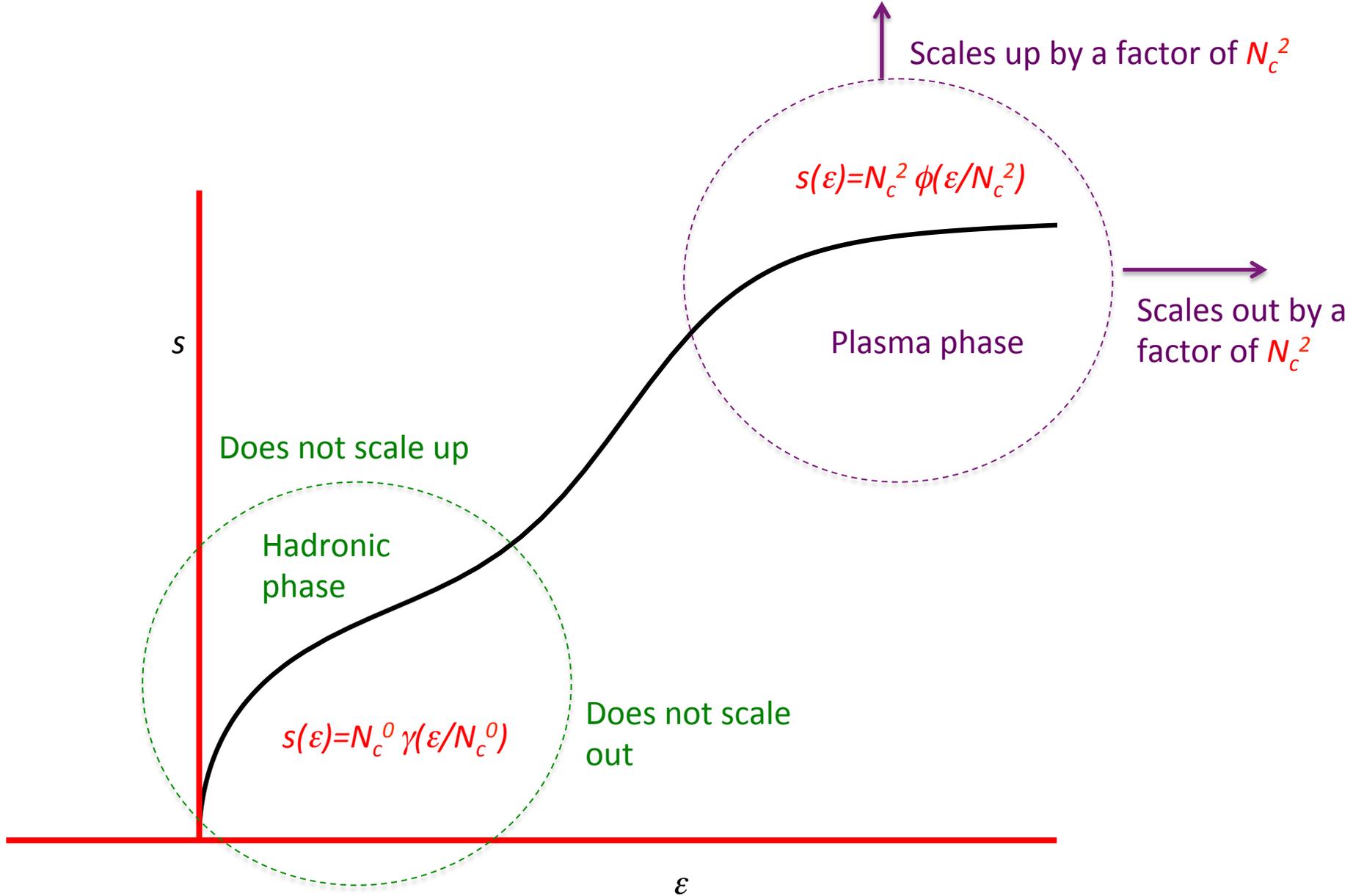
What happens at large N_c ?



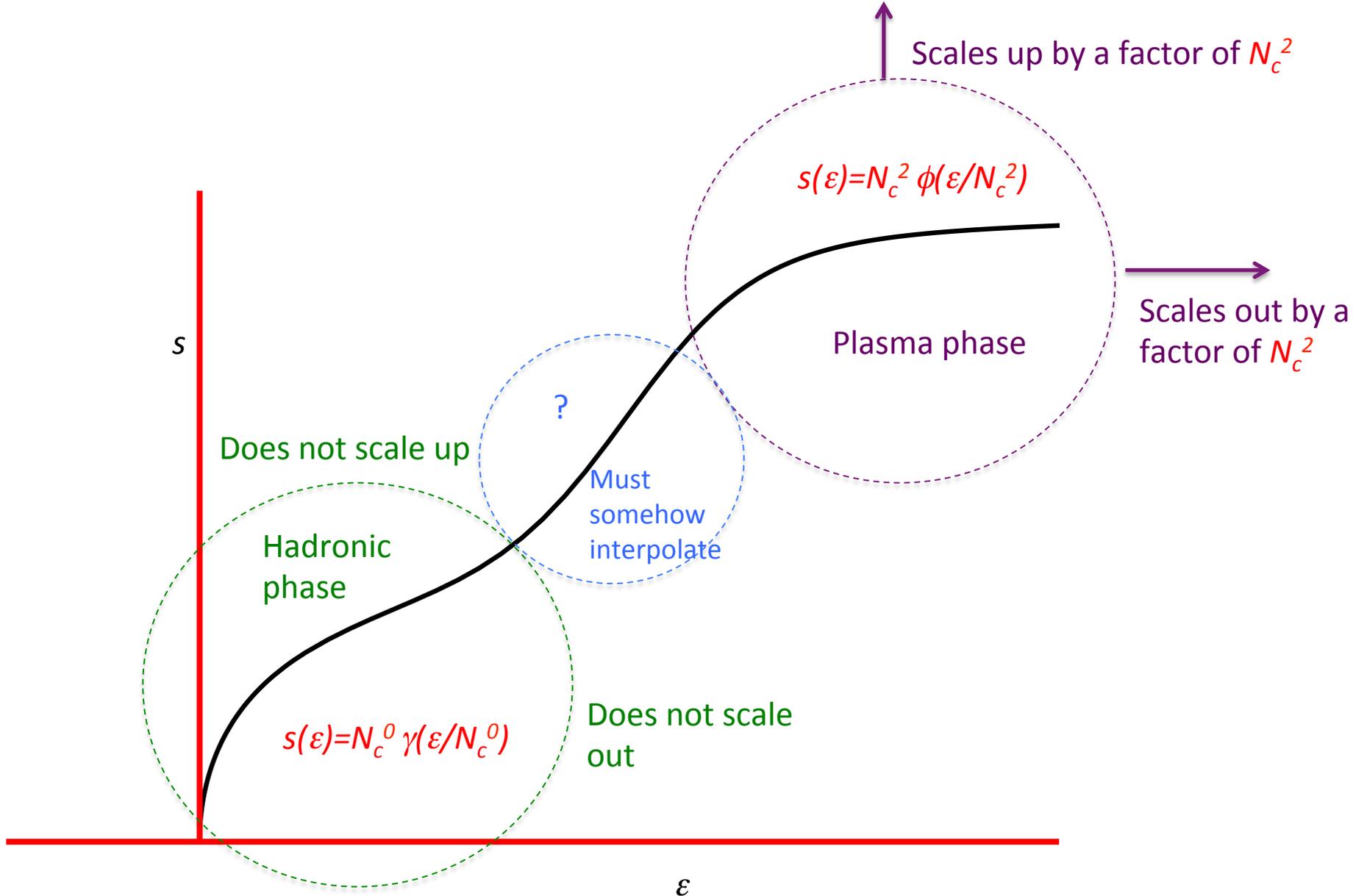
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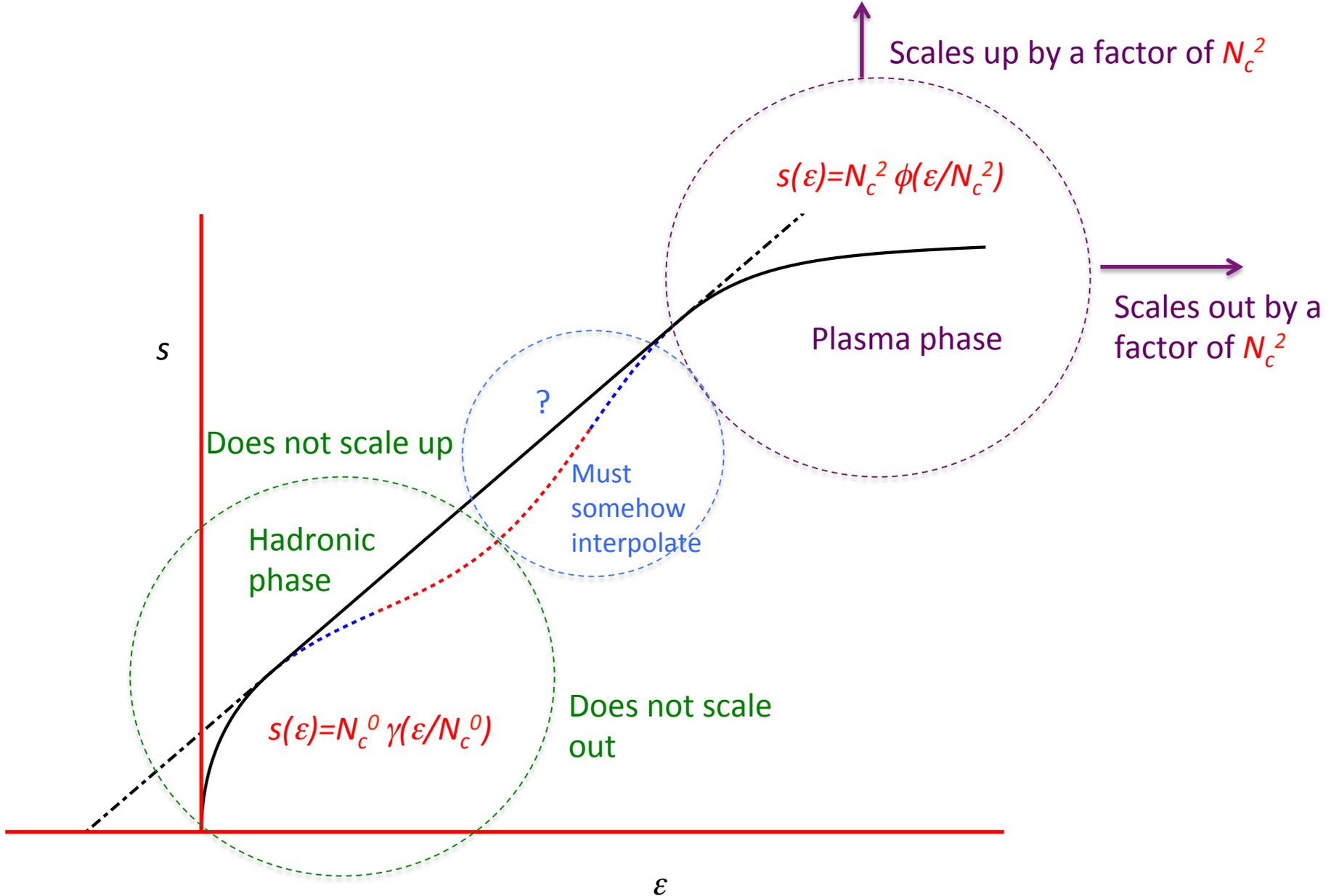
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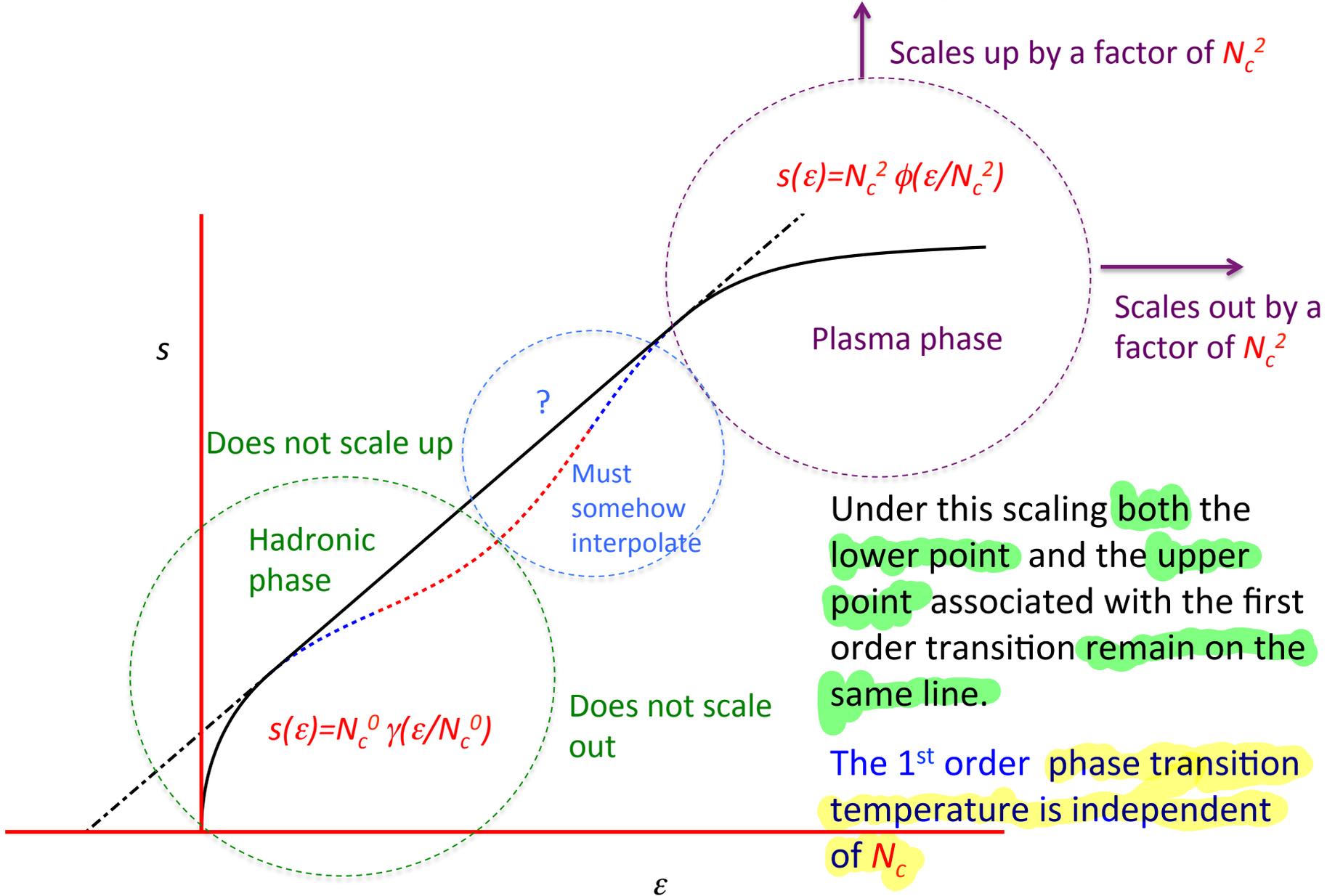
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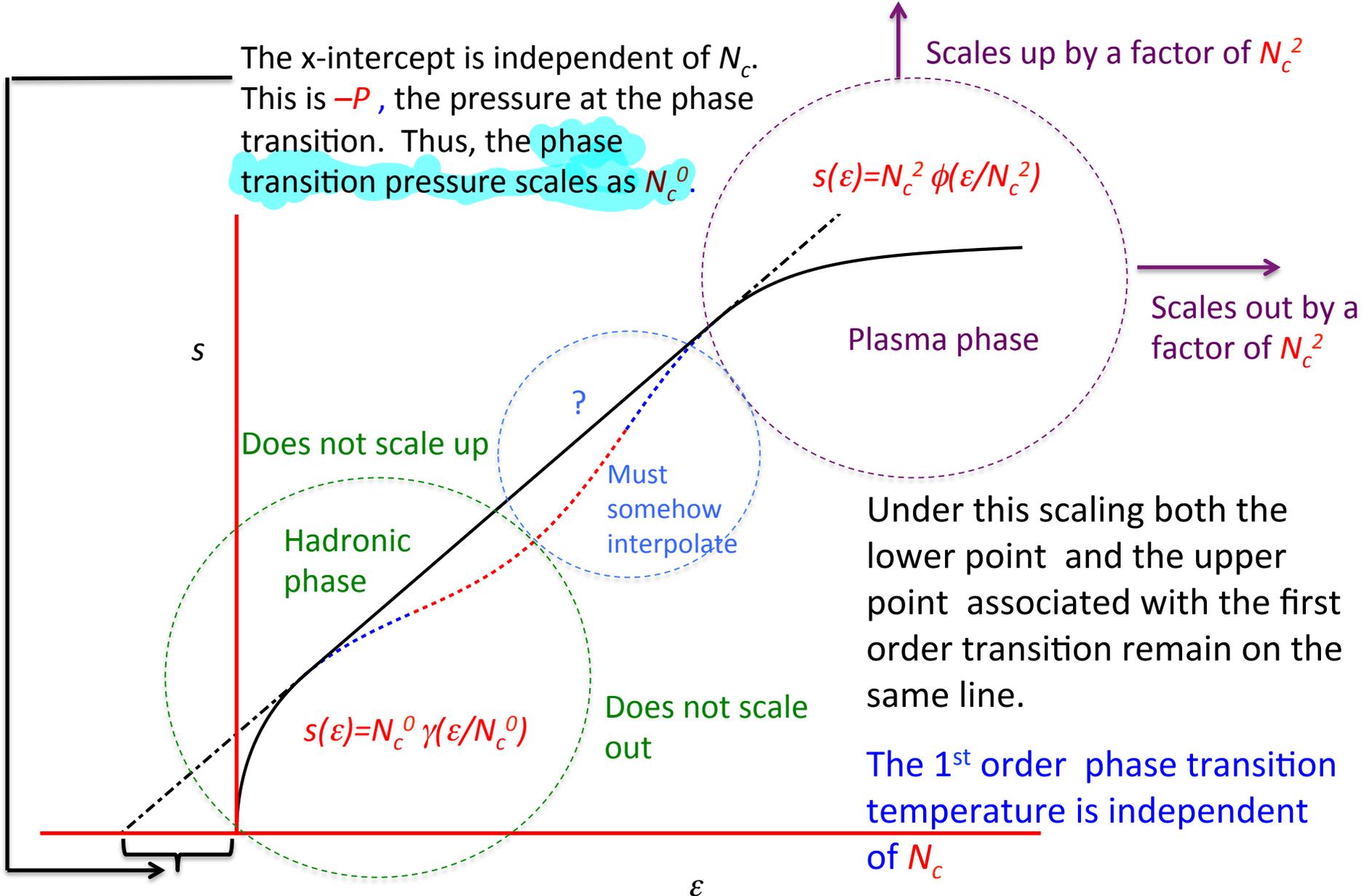


What happens at large N_c ?



What happens at large N_c ?

The x-intercept is independent of N_c . This is $-P$, the pressure at the phase transition. Thus, the phase transition pressure scales as N_c^0 .



Implications for the plasma phase

- The phase transition temperature and pressure are each order N_c^0 (i.e. independent of N_c).
- But $P = -f = Ts - \varepsilon$ and in plasma phase s and ε are each $\mathcal{O}(N_c^2)$ and T is $\mathcal{O}(N_c^0)$. Thus generically P is expected to be $\mathcal{O}(N_c^2)$.
- However, near the phase transition but still in the plasma phase, Ts and ε cancel almost exactly, up to relative order N_c^{-2} .
- This cancelation is rather remarkable and leads to some quite surprising results.

This cancellation gives insight into an almost philosophical issue about about the nature of matter created in heavy ion collisions



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 - Evidence: Analysis based on hydrodynamics suggests that η/s is small (of order $(4 \pi)^{-1}$). This implies that whatever the medium is its components must be strongly coupled.
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Others use different criteria for what constitutes a perfect fluid!

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 - Evidence: Analysis based on hydrodynamics suggests that η/s is small (of order $(4\pi)^{-1}$). This implies that whatever the medium is its components must be strongly coupled.
 - Based on this people have described the medium formed in these collisions as a (nearly) perfect fluid.
 - While there is strong evidence that this medium is strongly coupled, the evidence that it is a “plasma” is more problematic.
 - There is no phase transition in QCD between the plasma and hadronic phases. The medium is called a plasma largely because it is far too dense to be a weakly couple hadronic gas.
 - But it is equally not a weakly couple quark-gluon plasma. So why “strongly coupled plasma” & not “strongly coupled hadronic gas”

- A possible cynical answer: RHIC was sold as a machine to discover the QGP and whatever it discovered would be labeled as a QGP!

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A question approaching philosophy: Is it even possible to find a medium in any system which is both clearly in the plasma regime and also clearly strongly coupled?

- Yes! The high temperature phase of Large N_c QCD just above the phase transition

- Unlike QCD at $N_c=3$, there is a phase transition which cleanly delineates the hadronic from plasma phases. The high temperature phase is clearly a plasma.
- While there is no practical way to test η/s for this system to demonstrate that the constituents were strongly coupled, if the plasma is composed of massless constituents (eg. gluons) there is another useful measure

$$\Omega = \frac{\varepsilon}{3P}$$

– Note that for an noninteracting system of massless constituents $P=\varepsilon/3$ so $\Omega=\varepsilon/(3P)=1$.

- Very weakly acting systems of massless constituents will thus have close to Ω unity.

- If however, $\Omega = \frac{\varepsilon}{3P} \gg 1$ the system is clearly strongly coupled.

- Just above the first order phase transition in the plasma phase.

$$\Omega = \frac{\varepsilon}{3P} \sim N_c^2$$

At large N_c QCD unambiguously is both strongly coupled and in a plasma phase!

This is modulo the very reasonable assumption that a first order transition persists.

- It may be somewhat surprising that large N_c analysis gives a clean answer to this question.
- However, it makes a much more striking prediction about the supercooled phase:
Negative absolute pressure.

Absolute pressure is the pressure relative to the vacuum

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- **Gauge pressure is not the pressure of a gauge theory the pressure but rather the pressure read by a pressure gauge—which measure pressure relative to the ambient atmospheric pressure**

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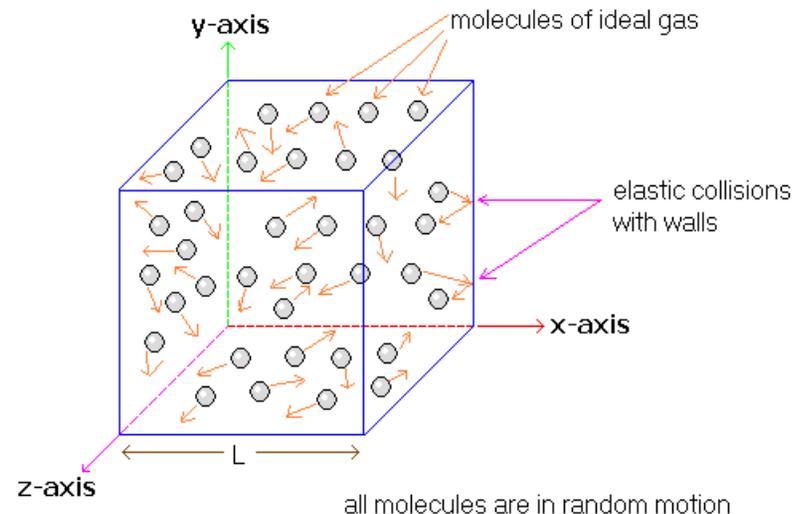
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- However, it makes a much more striking prediction about the supercooled phase:
Negative absolute pressure.
 - Negative absolute pressure violates our naïve kinetic theory intuition based on particles bouncing around in a gas.
 - Intuition based on experience with stable phases in which we never come across negative absolute pressure makes negative pressure seem weird.
 - **No go theorem:** systems with no chemical potentials or fixed densities of conserved quantities and with positive temperatures everywhere that are in a stable phase **cannot** have negative absolute pressure.

This follows from the condition $s(\epsilon)$ is everywhere concave down, and the facts that

$$s(0) = 0, \quad s'(\epsilon) = T^{-1} > 0, \quad \text{and} \quad P = Ts - \epsilon.$$

– Concavity means that $s'(\epsilon_1) > s'(\epsilon_2)$ if $\epsilon_1 < \epsilon_2$

Thus

$$s(\epsilon_a) = \int_0^{\epsilon_a} d\epsilon s'(\epsilon) > s'(\epsilon_a)\epsilon_a$$
$$P_a = \frac{s(\epsilon_a)}{s'(\epsilon_a)} - \epsilon_a > \frac{s'(\epsilon_a)\epsilon_a}{s'(\epsilon_a)} - \epsilon_a > 0$$

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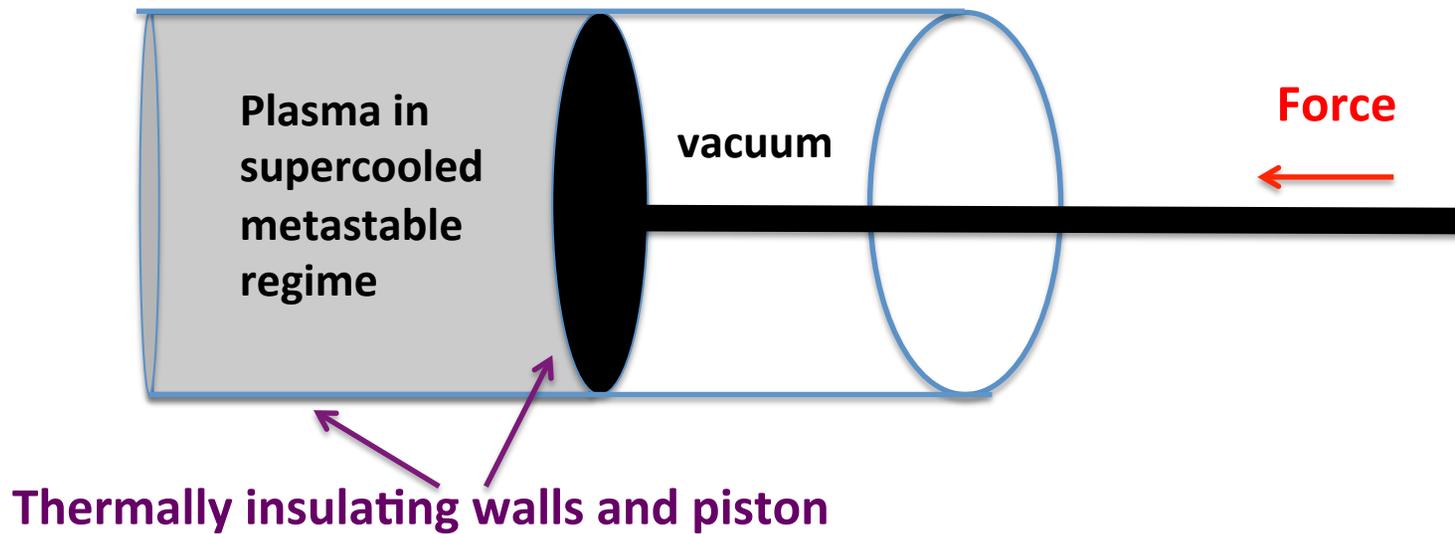
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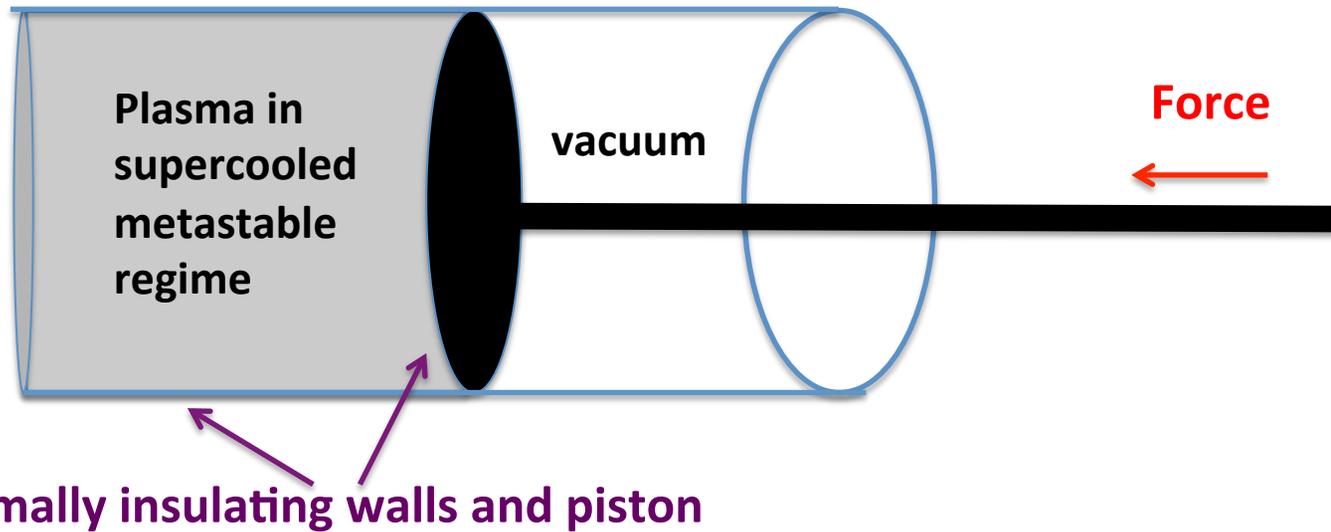
$$P_a = \frac{s(\epsilon_a)}{\epsilon_a} - \epsilon_a > \frac{s'(\epsilon_a)\epsilon_a}{\epsilon_a} - \epsilon_a > 0$$

However this argument does not apply to metastable phases. Thus, it is possible that they have negative absolute pressure and at large N_c , they do provided a generic 1st order transitions survives at large N_c

Negative absolute pressure is remarkable

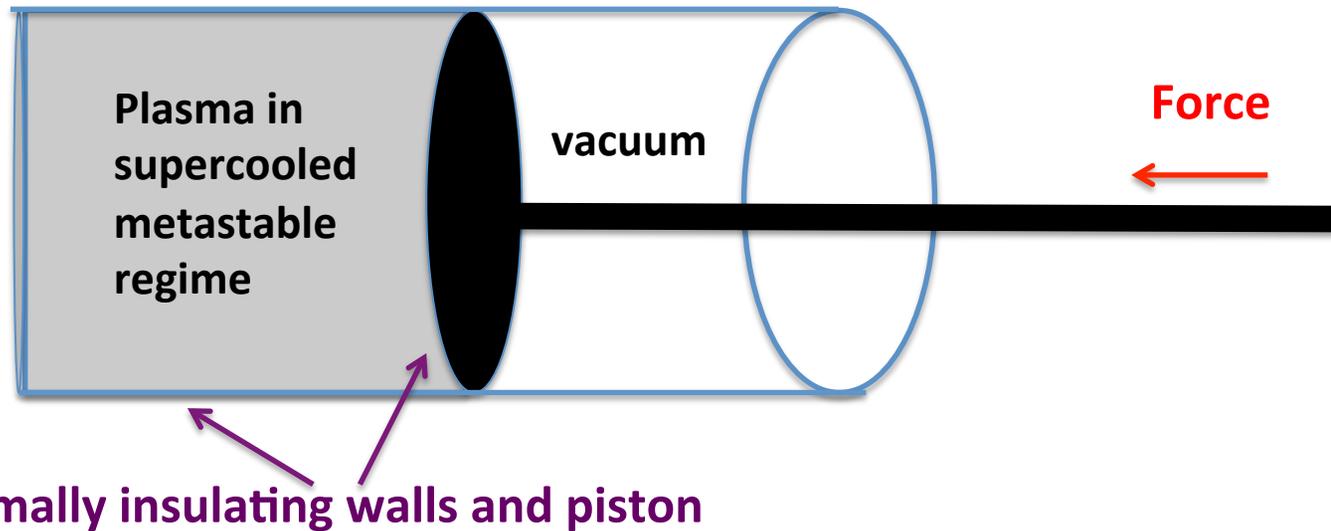


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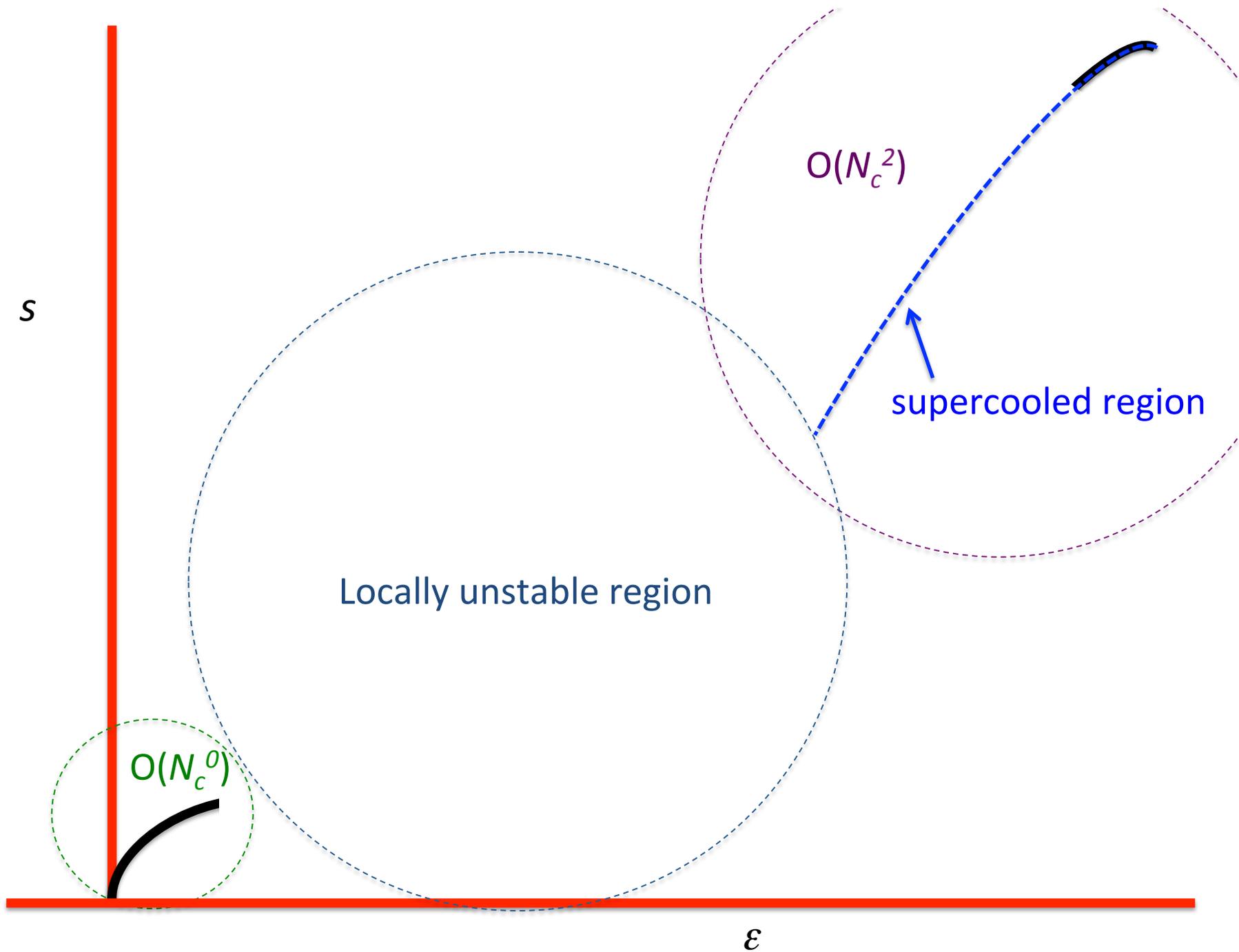
The medium is not just weird—it sucks!

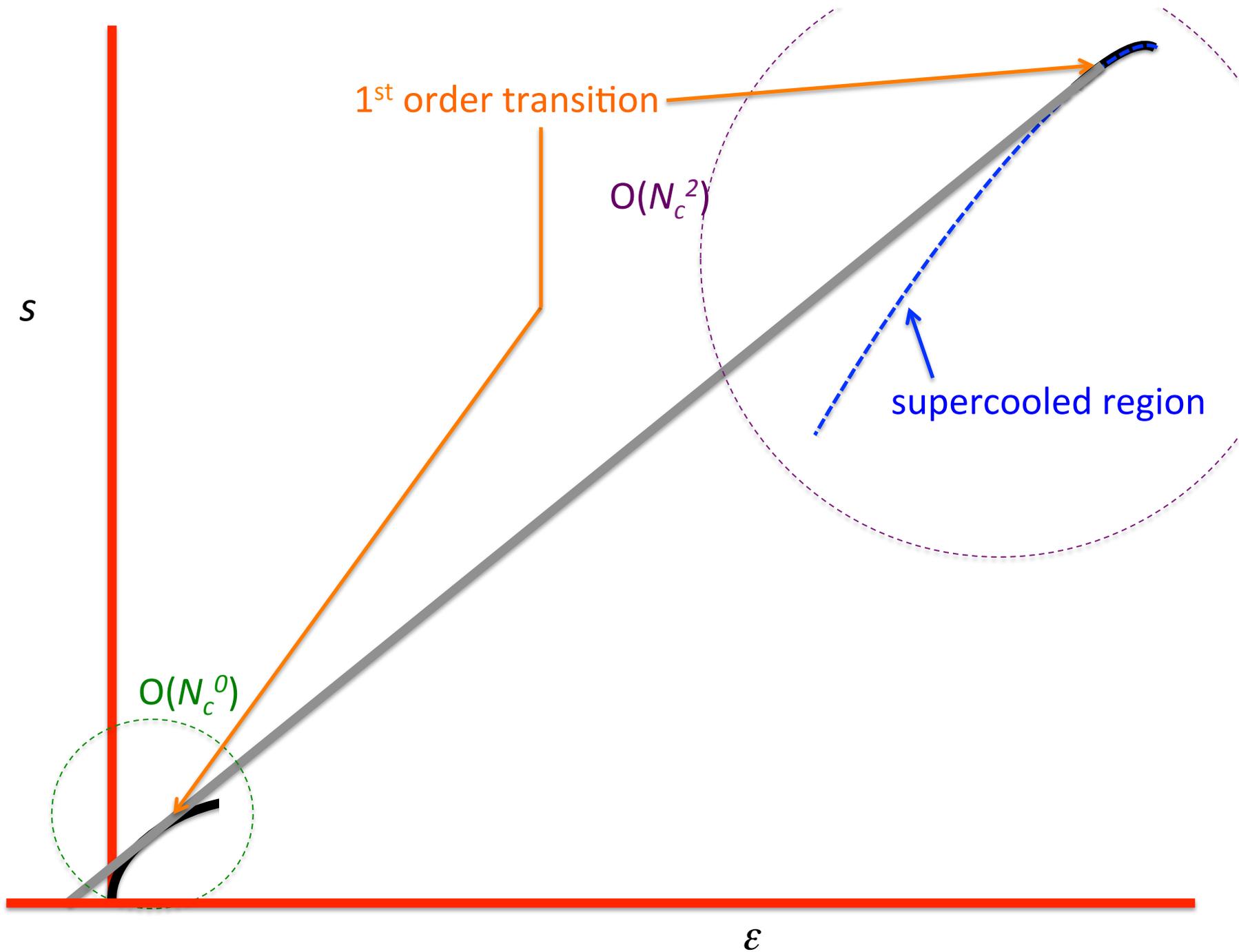
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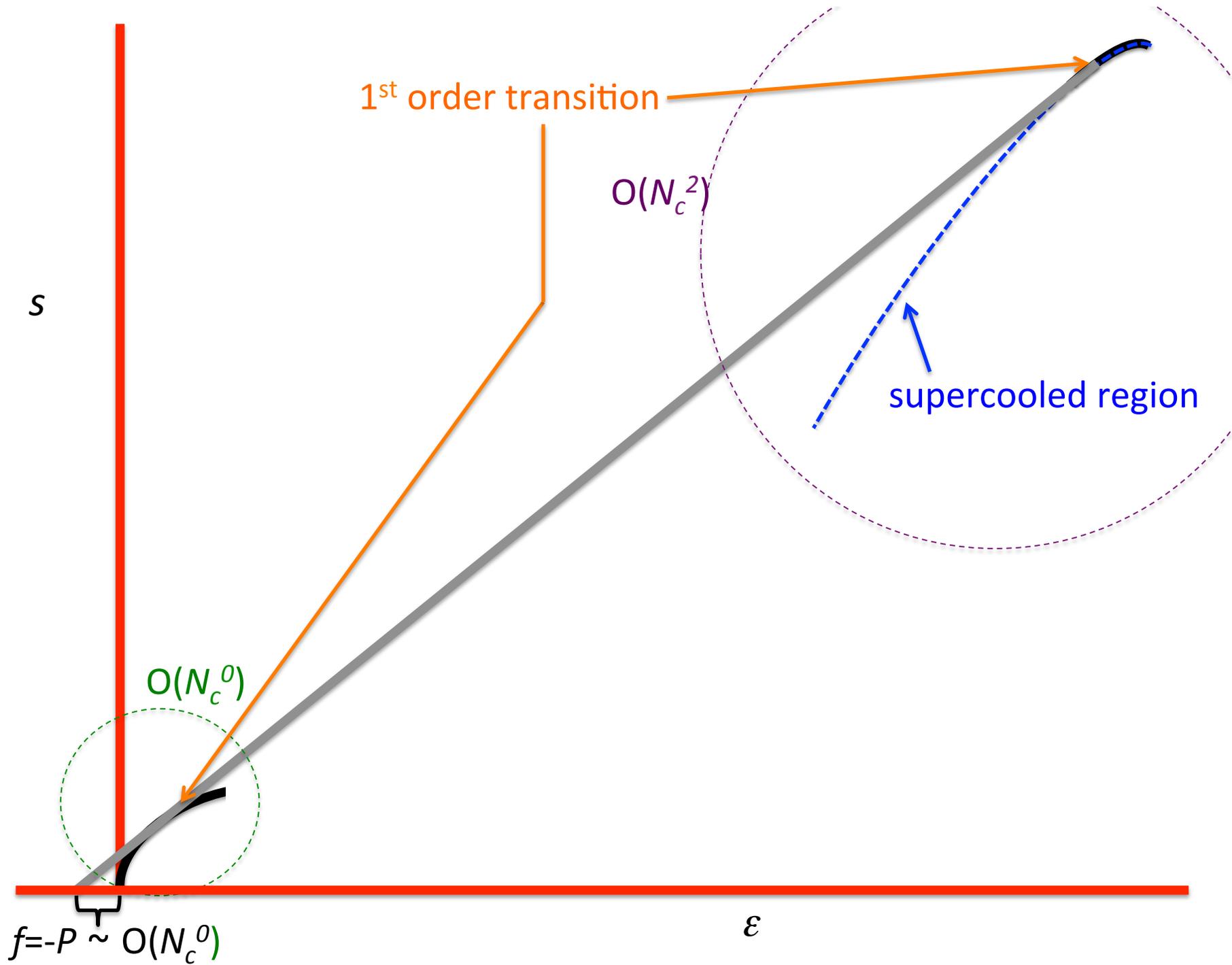


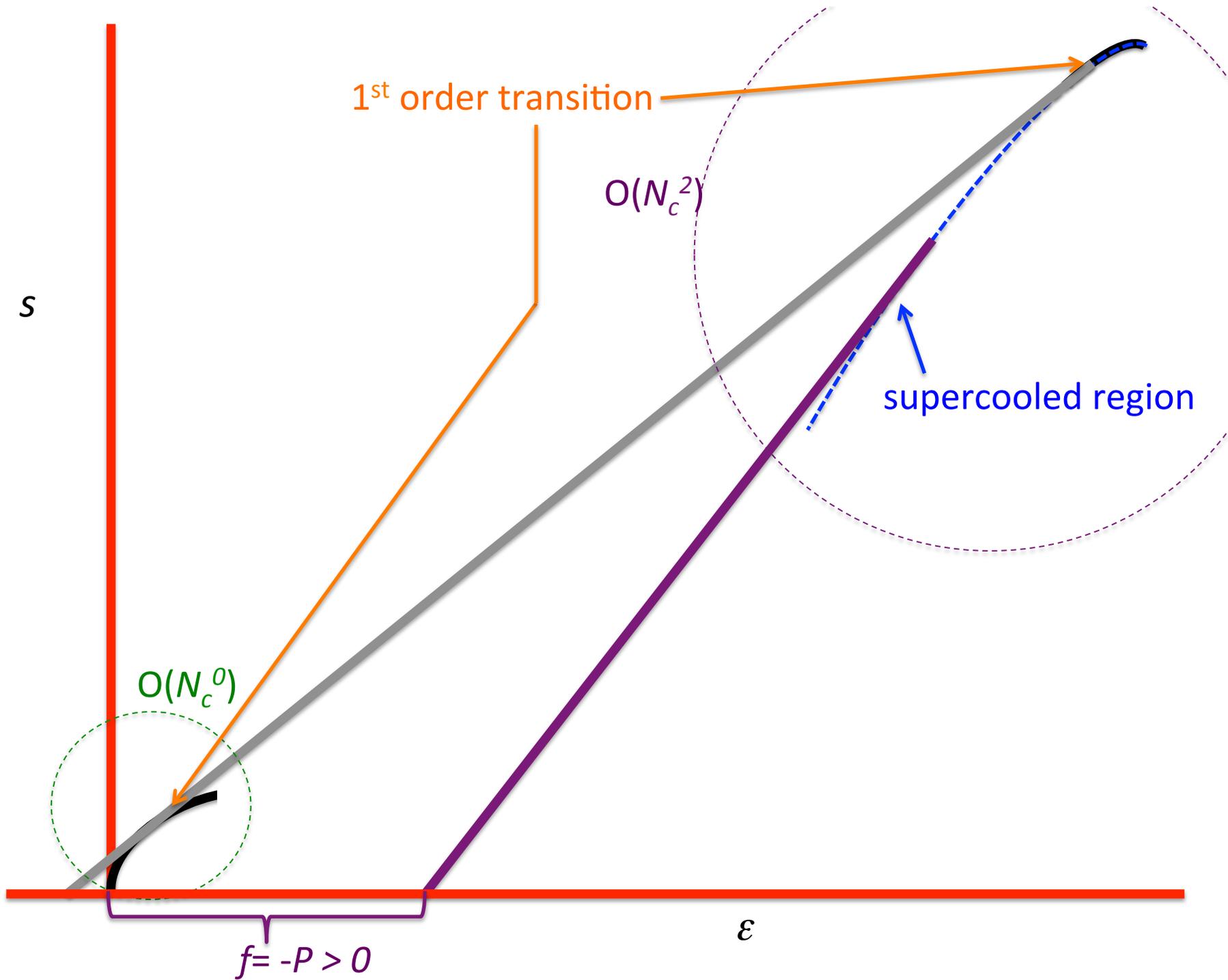
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The break down of intuition based on kinetic theory, indicates that whatever this medium is, the pressure is not describable in terms of particles or quasiparticles that strike the wall transferring momentum and imparting an outward pressure. This requires a strongly coupled theory where the quasiparticle motion is not dominant.









An algebraic way to see this:

Assume that there is a supercooled phase that exists over a range of temperature of order N_c^0 and let T_{sc} be in the supercool phase with $T_c - T_{sc} \sim N_c^0$

$$\begin{aligned} P(T_{sc}) &= P(T_c) - \int_{T_{sc}}^{T_c} dT \frac{dP}{dT} \\ &= P(T_c) - \int_{T_{sc}}^{T_c} dT s(T) && \text{since } \frac{dP}{dT} = -\frac{df}{dT} = s \\ &< \underbrace{P(T_c)}_{\sim N_c^0} - \underbrace{(T_c - T_{sc})}_{\sim N_c^0} \underbrace{s(T_{sc})}_{\sim N_c^2} && \text{since } s(T) \text{ increases monotonically} \end{aligned}$$

$T_c - T_{sc} > 0$ and $s(T_{sc}) > 0$, thus $P(T_{sc}) < 0$ at large N_c

- There is a caveat.
 - There must be a supercooled regime with negative absolute pressure at large N_c , provided that a metastable supercooled regime exists.
 - Logically a first order transition could exist in which the phase transition point happens to coincide with a point of inflection at large N_c ; if this happens there is no metastable regime.
 - There is no reason to expect this to happen based on large N_c analysis; and it would be even more interesting than negative absolute pressure.

**So we can conclude something cool happens!
Either the metastable supercooled phase does not exist or it has negative absolute pressure.**

- The focus so far has been on the plasma phase. Are there any surprises in the hadronic regime?

Yes!

- The key to understanding them is the fact that large N_c QCD should have a Hagedorn spectrum.
 - This is a very old expectation (Thorn 1981) . It is based, in part on the belief that large N_c QCD becomes a string theory, and string theories have Hagedorn spectra
 - There is a generic argument without recourse to stringy models (a sort of “physicist’s proof”) that large N_c QCD is based on the growth of the number of independent operators and natural assumptions about the onset of a viable perturbation for correlators (TDC 2009)

- a Hagedorn spectrum for the density of hadrons as a function of mass asymptotically $N(m) \sim m^{-d} \exp(m/T_H)$, where $N(m)$ is the number of mesons and glueballs with mass less than m , T_H , the Hagedorn temperature and is a mass parameter and $-d$ fixes power law prefactor.
- In the large N_c limit, T_H , corresponds to an upper bound on the temperature of hadronic matter

The Hagedorn Spectrum



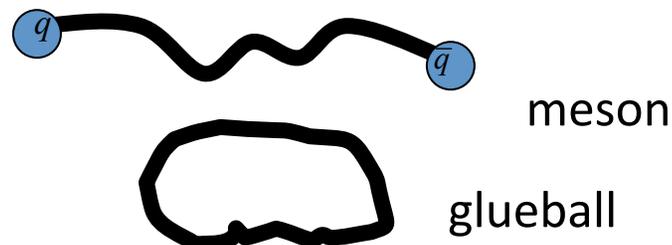
The Hagedorn Spectrum



Strictly, it only makes sense for large N_c , since only at large N_c do hadrons become narrow and their masses well defined

- The value of the prefactor power $-d$ plays a nontrivial role in the large N_c thermodynamics (TDC 2006, Thorn 1981)
 - If $d > 7/2$, the system can reach T_H with a finite energy density and entropy density; for $d \leq 7/2$ they diverge.

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 - If $d > 7/2$, the system can reach T_H with a finite energy density and entropy density; for $d \leq 7/2$ they diverge.
 - There is a good reason to believe that $d = 4$.
 - $d = 4$ is the result for a bosonic string.
 - Highly excited mesons and glueballs are expected to look like excitations of flux tubes which become increasingly stringy as the flux tubes get long—as they do for highly excited states.



Historical Note

Modern string theory grew out of the failed attempt in pre-QCD days to treat strong interactions as a string theory.

It was ultimately abandoned

- Phenomenological issues (a pesky massless spin-2 meson etc.)
- Theoretical consistency (negative norm states, tachyons)
- Emergence of QCD as a viable field theory for strong interactions

String theory reemerged, phoenix-like from the ashes of this failure, as a putative theory of everything

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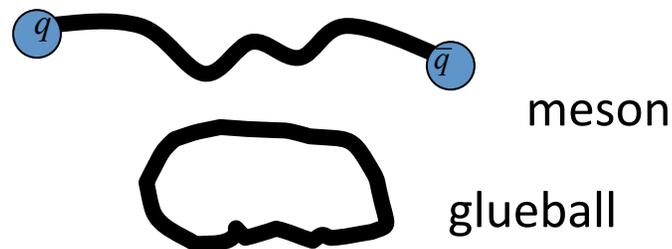


Historical Note

The $d=4$ suggested by stringy dynamics differs from Hagedorn's original proposal from 1965 which had $d=5/2$. This had the T_H unreachable since it required an infinite energy. T_H was viewed as a maximum possible temperature roughly analogous to absolute zero.

$d=5/2$ was predicted based on Hagedorn's "statistical bootstrap model". To modern eyes (or at least mine) it does not seem at all compelling but it led to a very important insight

- The value of the prefactor power $-d$ plays a nontrivial role in the large N_c thermodynamics (TDC 2006, Thorn 1981)
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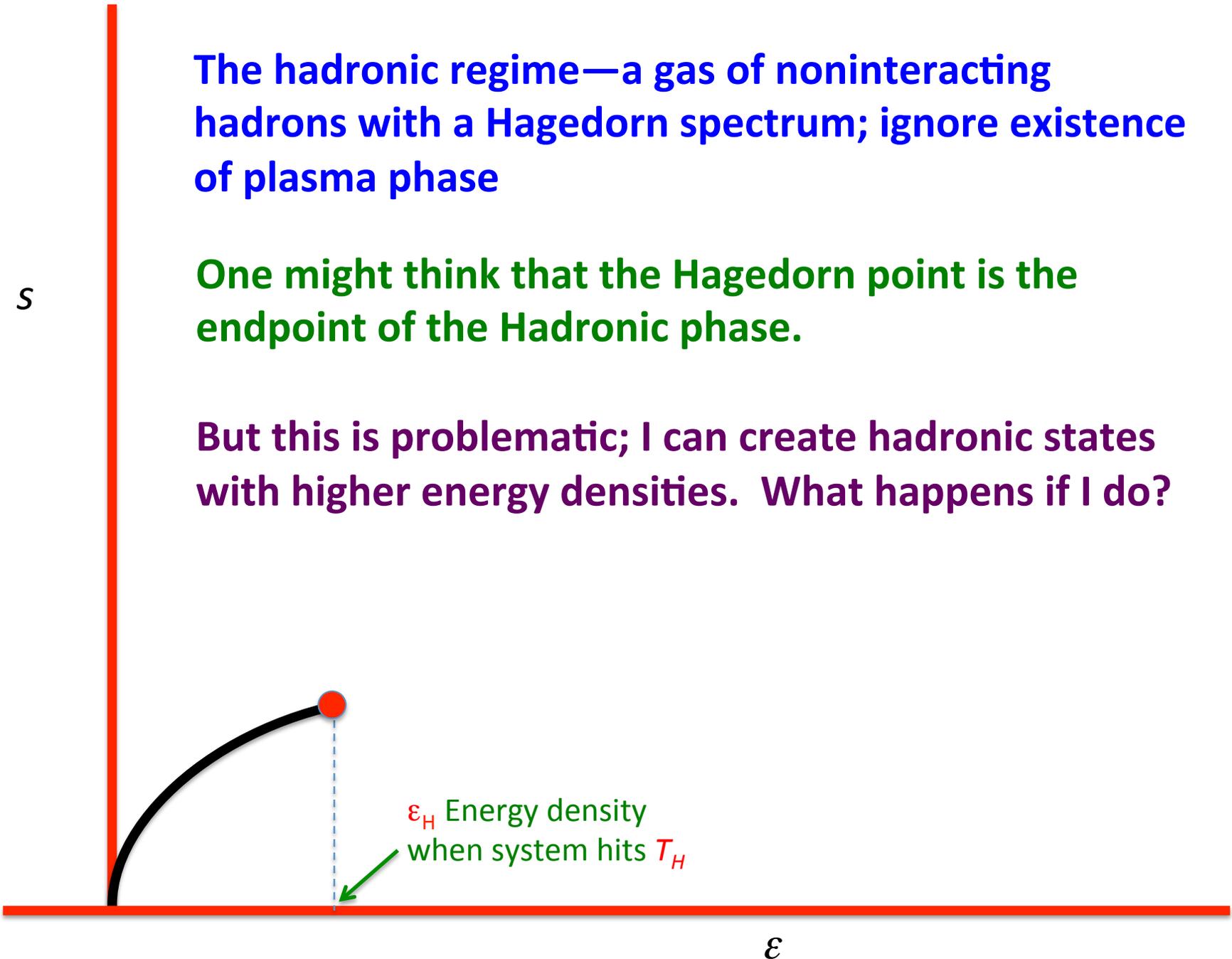


- $d > 7/2$ is assumed in what follows.

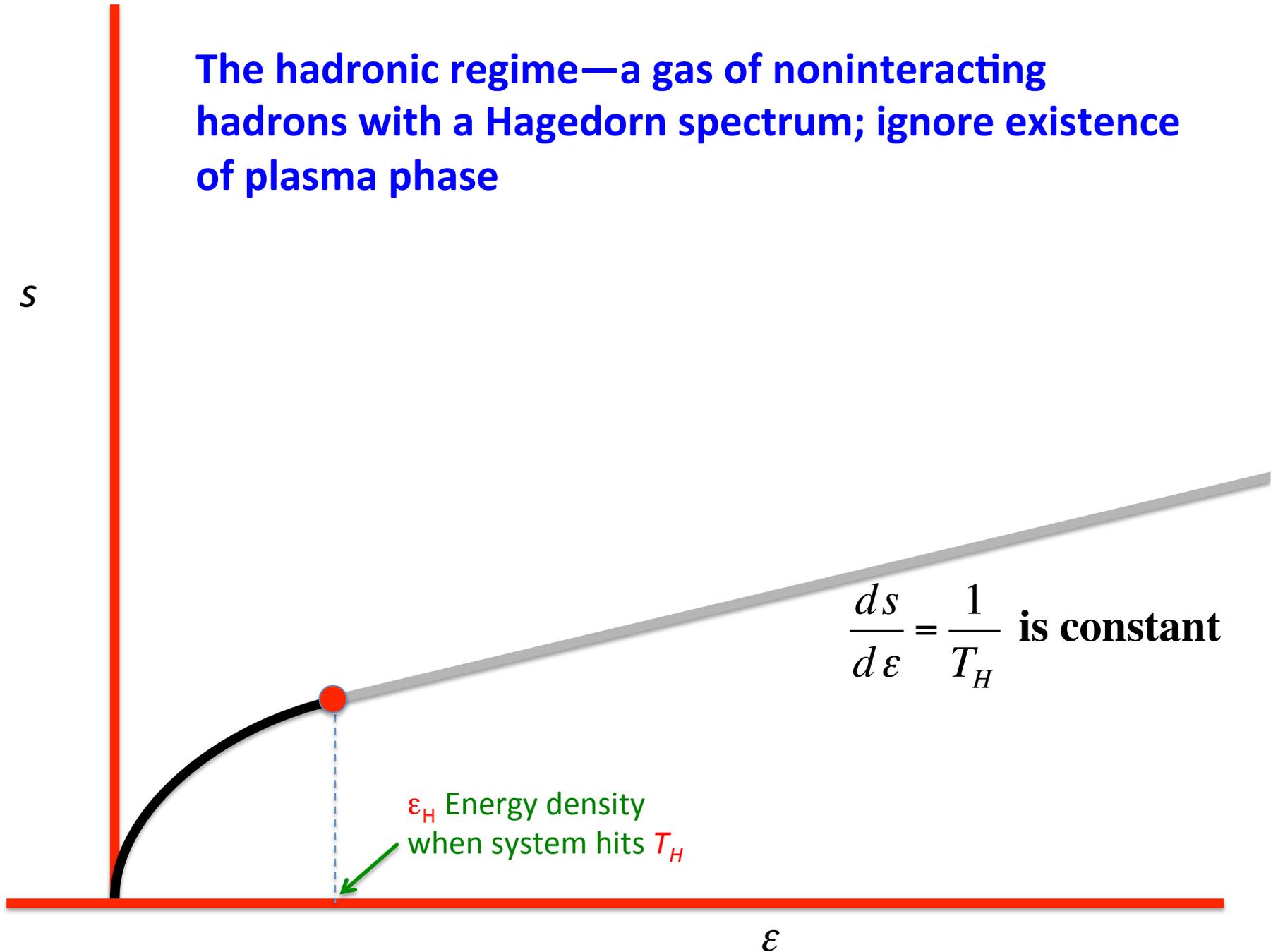
The hadronic regime—a gas of noninteracting hadrons with a Hagedorn spectrum; ignore existence of plasma phase

One might think that the Hagedorn point is the endpoint of the Hadronic phase.

But this is problematic; I can create hadronic states with higher energy densities. What happens if I do?



The hadronic regime—a gas of noninteracting hadrons with a Hagedorn spectrum; ignore existence of plasma phase



Beyond the Hagedorn point

- For $\varepsilon > \varepsilon_H$, $O(N_c^0)$, $s(\varepsilon)$ continues as a straight line; thus $T = 1/s'(\varepsilon)$ remains constant at T_H as energy is added.

Beyond the Hagedorn point

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Why?

Look at thermodynamics subject to an artificial constraint that only masses greater than some large value m_{max} are included. The physical result is when the constraint is removed and m_{max} goes to ∞ .

$$s(\varepsilon) = \lim_{m_{max} \rightarrow \infty} \sum_{k=1}^{k_{max}} s(\varepsilon, m_k)$$

with $m_{k_{max}} = m_{max}$

This is just an ideal gas with a finite number of components until limit is taken. Thus, the canonical and microcanonical descriptions are equivalent. Canonically, these systems have a well-defined T .

Beyond the Hagedorn point

As $m_{max} \rightarrow \infty$

- T is bounded from above by T_H since otherwise ε diverges.
- For $\varepsilon > \varepsilon_H$, is also bounded from below by T_H . This follows from the fact that at any finite m_{max} , the system is a relativistic ideal gas with a finite (but very large) number of species. Such systems have $dT/d\varepsilon = -T^2 s''(\varepsilon) \geq 0$ everywhere.

Ergo for $\varepsilon > \varepsilon_H$ $T = T_H$ and $ds/d\varepsilon = 1/T_H$ is constant. As seen in the figure.

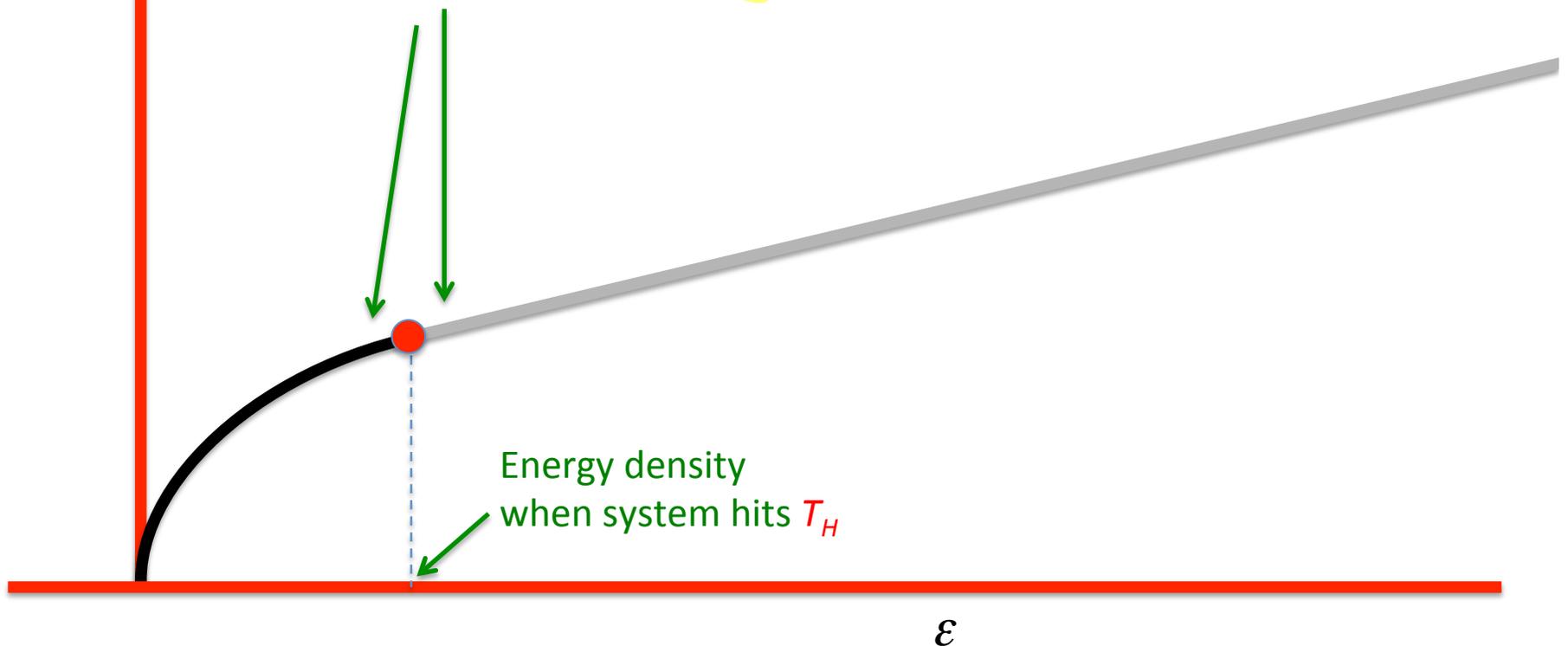
We have verified this behavior via a sophisticated analytic calculation and via numerical experiments by fixing energy and directly summing over hadrons up to a cutoff which we increase until it is very large.

The hadronic regime—a gas of noninteracting hadrons with a Hagedorn spectrum; ignore existence of plasma phase

s

This is very peculiar.

- There is a discontinuity in the $s'''(\epsilon)$ as one sees in 2nd order transition

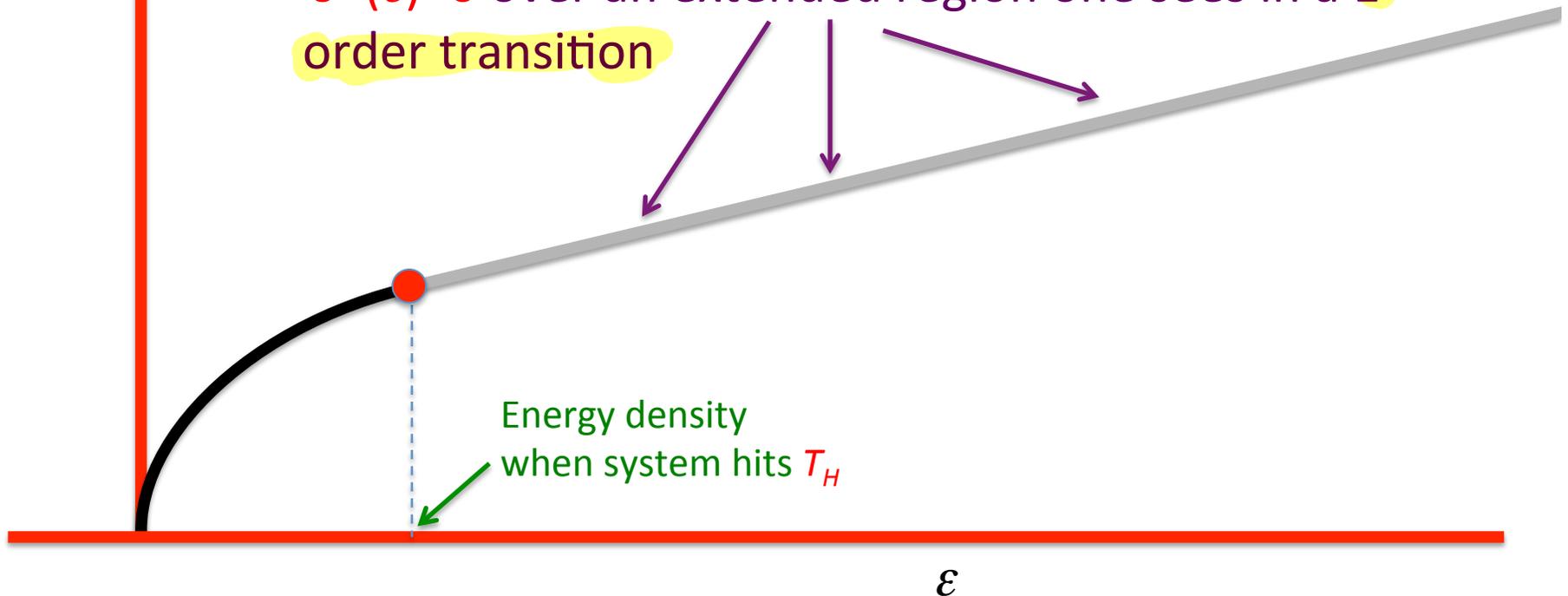


The hadronic regime—a gas of noninteracting hadrons with a Hagedorn spectrum; ignore existence of plasma phase

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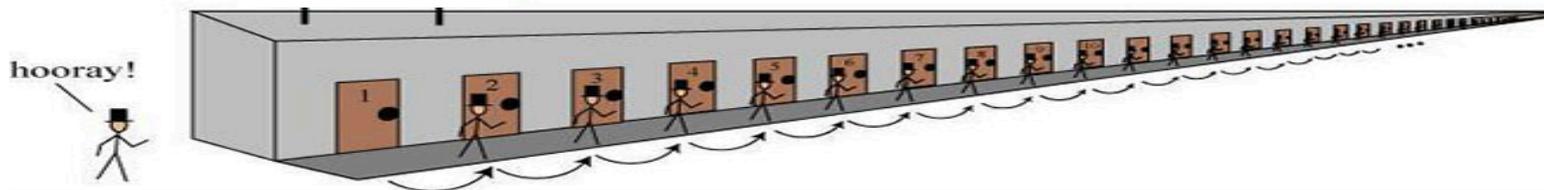
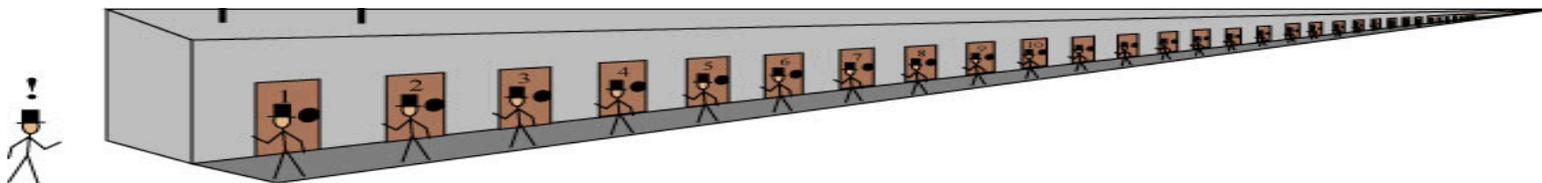
- There is a discontinuity in the $s'''(\epsilon)$ as one sees in 2nd order transition
- $s''(\epsilon)=0$ over an extended region one sees in a 1st order transition



- What is going on?
 - There appears to be a breakdown of the thermodynamic limit in which the microcanonical and canonical descriptions become identical at large volumes.
 - While work is still in progress, currently all indications suggest that in a large but finite volume, all of the excess energy beyond $V\varepsilon_H$ is typically contained in a single extremely massive particle.
 - When the volume doubles with the same energy density, instead of doubling the number of heavy particles one still has one but it is twice as massive.

- Dynamically if the system were infinite in size and at an energy density of ϵ_H and energy in the form of low mass hadrons is added to the system.
 - The lower mass states would thermalize at T_H and the excess energy would be pushed upward to higher masses.
 - This would happen continually and the energy would be pushed to ever higher mass hadrons.
 - The process would not stop if the system is of infinite size.

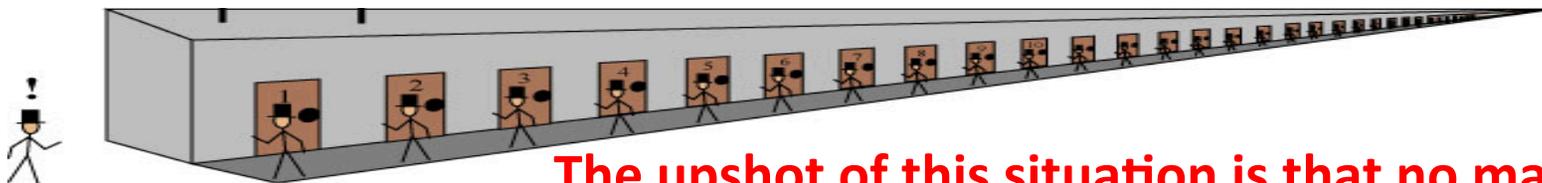
Situation is a bit reminiscent of Hilbert's Grand Hotel



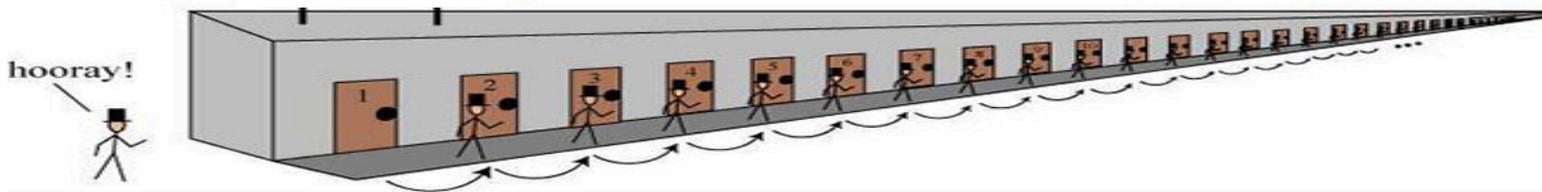
Situation is a bit reminiscent of Hilbert's Grand Hotel



Although the hadronic regime is thermodynamically full at T_H , there is always room for more hadronic ε .



The upshot of this situation is that no matter how big the system, the thermodynamic limit is never reached.



Conclusions/Surprises



There is a clean way to show that a regime exists which is both clearly strongly coupled and clearly a QGP plasma

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The metastable supercooled plasma phase of large N_c QCD has negative absolute pressure.

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The metastable supercooled plasma phase of large N_c QCD has negative absolute pressure.



The endpoint of the metastable hadronic phase of large N_c QCD is odd and indicates a breakdown of the usual thermodynamic limit