The κ and $K^{\star}(892)$ resonances from lattice QCD

Gumaro Rendon

June 29, 2020

STRANGE MESONS

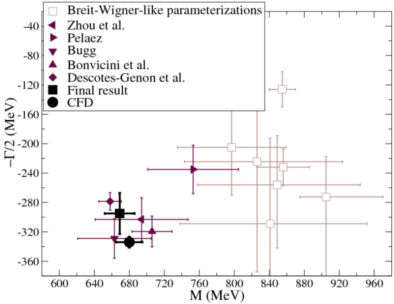
($S = \pm1$, $C = B = 0$)

$K^+ = u\bar{s}$, $K^0 = d\bar{s}$, $\overline{K}^0 = \bar{d}s$, $K^- = \bar{u}s$, similarly for K^*'s

$K_0^*(700)$ $\quad$ $I(J^P) = 1/2(0^+)$

also known as κ; was $K_0^*(800)$

Needs confirmation See the mini-review on sc



[a]

a. **Rodas:2017snm**.

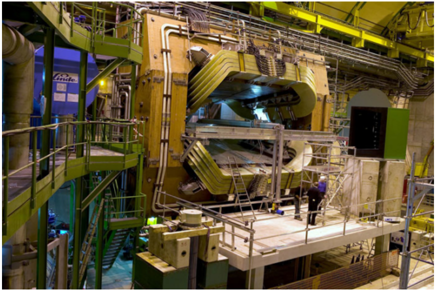
Motivation : $b \to s\ell^+\ell^-$ weak decays

physicsworld

PARTICLES AND INTERACTIONS RESEARCH UPDATE

Has LHCb spotted physics beyond the Standard Model?

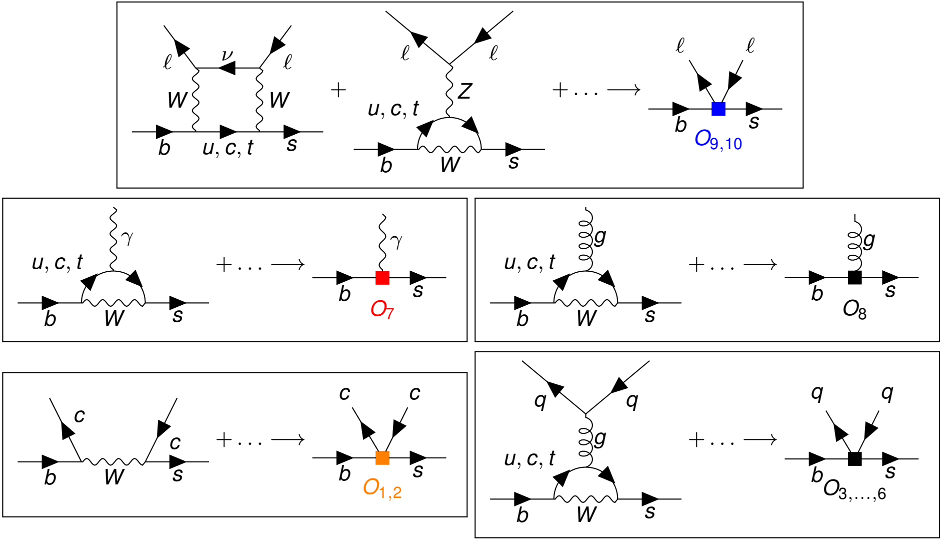
02 Aug 2013 Hamish Johnston



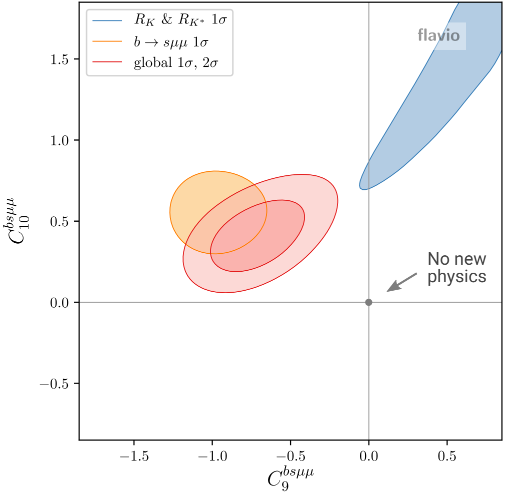
Is physics beyond the Standard Model lurking in LHCb?

An analysis of data from the LHCb experiment at the CERN particle-physics lab suggests that the B-meson could decay in a way not predicted by the Standard Model of particle physics, according to theoretical physicists in Spain and France. The researchers believe that the deviation from the Standard Model has been measured with a confidence of 4.5σ – which is approaching the gold standard of 5σ required for a discovery in particle physics.

Motivation : $b \to s\ell^+\ell^-$ weak decays



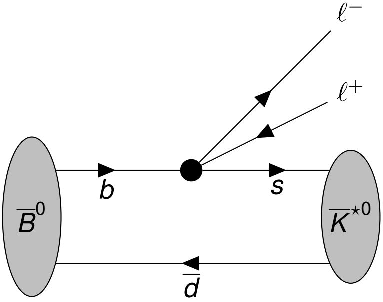
Motivation : $b \to s\ell^+\ell^-$ weak decays



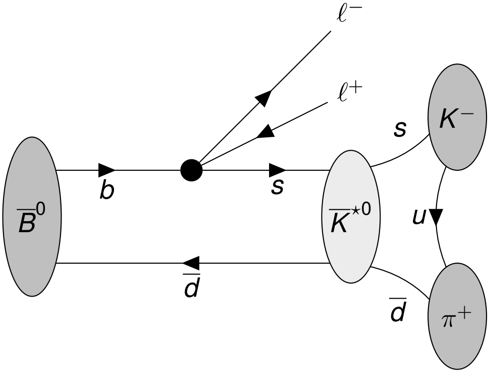
[1]

1. Jason AEBISCHER et al. "*B*-decay discrepancies after Moriond 2019". In : **Eur. Phys. J. C** 80.3 (2020), p. 252. DOI : 10.1140/epjc/s10052-020-7817-x. arXiv : 1903.10434 [hep-ph].

$B \to K^\star \ell^+ \ell^-$



$B \to K^\star(\to K\pi)\ell^+\ell^-$



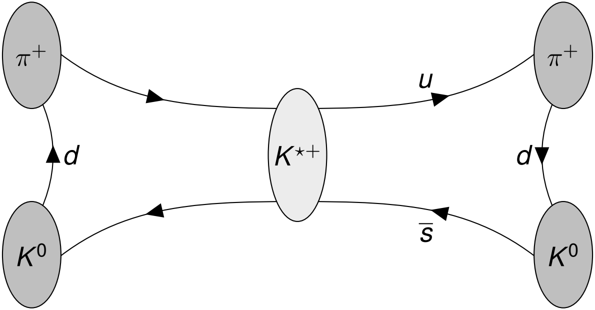
Finite volume corrections

$$\frac{|\langle B, \vec{p}_B | J_\mu(0) | s, \vec{P},\ \Lambda,\ r\rangle_{IV}|^2}{|\langle B, \vec{p}_B | J_\mu(0,\vec{q}) | n, \vec{P},\ \Lambda,\ r\rangle_{FV}|^2}$$
$$= \frac{1}{2E_n^{\vec{P},\Lambda}} \frac{16\pi\sqrt{s_n^{\vec{P},\Lambda}}}{k_n^{\vec{P},\Lambda}} \left(\frac{\partial \delta}{\partial E} + \frac{\partial \phi^{\vec{P},\Lambda}}{\partial E} \right)\Big|_{E=E_n^{\vec{P},\Lambda}}.$$

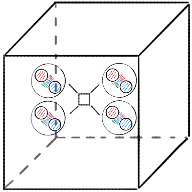
a

a. Raúl A. Briceño, Maxwell T. Hansen et André Walker-Loud. "Multichannel 1 → 2 transition amplitudes in a finite volume". In : **Phys. Rev. D** 91.3 (2015), p. 034501. DOI : 10.1103/PhysRevD.91.034501. arXiv : 1406.5965 [hep-lat].

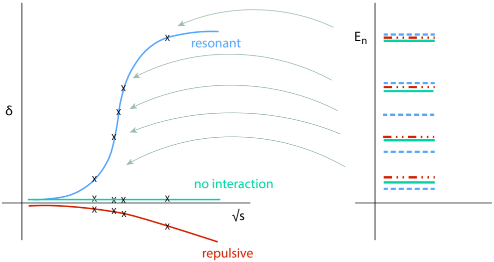
$K\pi \to K\pi$



Elastic Scattering on the Lattice



Energy shift + boundary conditions $\rightarrow$ Infinite volume phase shift



Lüscher's formalism :

Quantization Condition Lüscher NPB1991 :

$$
det \begin{bmatrix} \overset{\text{infinite volume}}{\boxed{\cot \delta_J}} - \overset{\text{finite volume}}{\boxed{\mathcal{M}(k)}} \\ \\ \end{bmatrix} = 0
$$

Determine $\delta(s)$ at discrete values of s_n.

S and P wave mixing. Simultaneous fits.

$\frac{L}{2\pi}\vec{P}$	Little Group ($LG^{\vec{P}}$)	irrep ($\Lambda^{\vec{P},r}$)	spin content	dimension
(0,0,0)	O_h	A_{1g}	J=0,4,...	1
(0,0,0)	O_h	T_{1u}	J=1,3,...	3
(0,0,1)	C_{4v}	$A_1(A_2)$	J=0,1,...	1
(0,0,1)	C_{4v}	E	J=1,2,...	2
(0,1,1)	C_{2v}	$A_1(B_3)$	J=0,1,...	1
(0,1,1)	C_{2v}	B_1	J=1,2,...	1
(0,1,1)	C_{2v}	B_2	J=1,2,...	1
(1,1,1)	C_{3v}	$A_1(A_2)$	J=0,1,...	1
(1,1,1)	C_{3v}	E	J=1,2,...	2

Quantization Conditions

No Mixing

$$\vec{p}_{K\pi} = \frac{2\pi}{L}(0,0,0), \quad LG = O_h, \quad \Lambda = A_{1g}:$$

$$\boxed{\cot\delta_0} = w_{00},$$

$$\vec{p}_{K\pi} = \frac{2\pi}{L}(0,0,0), \quad LG = O_h, \quad \Lambda = T_{1u}:$$

$$\boxed{\cot\delta_1} = w_{00},$$

Mixing

$$\vec{p}_{K\pi} = \frac{2\pi}{L}(0,0,1), \quad LG = C_{4v}, \quad \Lambda = A_1:$$

$$\left(\boxed{\cot\delta_0} - w_{00}\right)\left(\boxed{\cot\delta_1} - w_{00} - 2w_{20}\right) - 3w_{10}^2 = 0,$$

$$\chi^2 = \sum_{\vec{P},\Lambda,n} \sum_{\vec{P}',\Lambda',n'} [C^{-1}]_{\vec{P},\Lambda,n;\vec{P}',\Lambda',n'}$$
$$\times \quad \left(\sqrt{s_n^{\Lambda,\vec{P}}}^{[\mathrm{data}]} - \sqrt{s_n^{\Lambda,\vec{P}}}^{[\mathrm{model}]}\right)$$
$$\times \quad \left(\sqrt{s_{n'}^{\Lambda',\vec{P}'}}^{[\mathrm{data}]} - \sqrt{s_{n'}^{\Lambda',\vec{P}'}}^{[\mathrm{model}]}\right).$$

Lattice ensembles

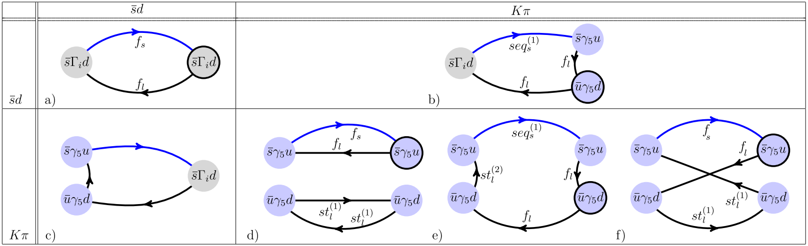
	C13	D6
$N_s^3 \times N_t$	$32^3 \times 96$	$48^3 \times 96$
β	6.1	6.3
$am_{u,d}$	-0.285	-0.2416
am_s	-0.245	-0.205
c_{SW}	1.2493	1.2054
a [fm]	0.11403(77)	0.08766(79)
L [fm]	3.649(25)	4.208(38)
am_π	0.18332(29)	0.07816(35)
am_K	0.30475(17)	0.22803(15)
m_π [MeV]	317.2(2.2)	175.9(1.8)
m_K [MeV]	527.4(3.6)	513.3(4.6)
N_{config}	896	328

Irrep projection

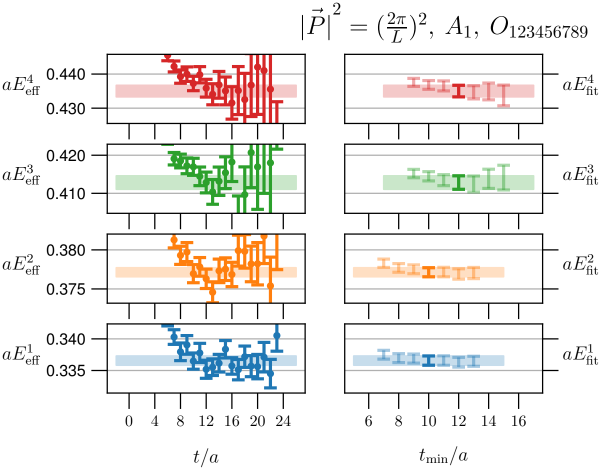
$$O_{K\pi}^{\Lambda,\vec{P}} = \frac{\dim(\Lambda)}{\mathrm{order}(LG(\vec{P}))} \sum_{\vec{p}} \sum_{R\in LG(\vec{P})} \chi^{\Lambda}(R) \times O_{K\pi}(R\,\vec{p}, \vec{P} - R\,\vec{p}),$$

$$O_{K^*,i}^{\Lambda,\vec{P}} = \frac{\dim(\Lambda)}{\mathrm{order}(LG(\vec{P}))} \sum_{R\in LG(\vec{P})} \chi^{\Lambda}(R) \times \sum_{j} R_{ij}\, K_j^{*+}(\vec{P}\,).$$

Wick contractions



$K\pi \to K\pi$: Spectrum



$a\Delta E_{\mathrm{bwd}} = 15(9) \times 10^{-4}$
$a\Delta E_{\mathrm{fwd}} = 4(9) \times 10^{-4}$

$a\Delta E_{\mathrm{bwd}} = 1(1) \times 10^{-3}$
$a\Delta E_{\mathrm{fwd}} = 0(1) \times 10^{-3}$

$a\Delta E_{\mathrm{bwd}} = 4(1) \times 10^{-4}$
$a\Delta E_{\mathrm{fwd}} = -0(2) \times 10^{-4}$

$a\Delta E_{\mathrm{bwd}} = 3(2) \times 10^{-4}$
$a\Delta E_{\mathrm{fwd}} = -0(2) \times 10^{-4}$

K-matrix formalism

$$S^{(\ell)}(s) = 1 + 2\, T^{(\ell)}(s),$$

$$\{\, T^{(\ell)}{}^{-1}\}_{ij} = \{\, K^{(\ell)}{}^{-1}\}_{ij} - i\,\theta(s - s_{\text{thr}}^{(i)})\delta_{ij},$$

$$K^{(\ell)} = \rho^{1/2}\hat{K}^{(\ell)}\rho^{1/2}.$$

Parametrizations : K-matrix pole with barrier factors

$$\hat{K}^{(\ell)} = \sum_{\alpha} \frac{g^0_{\ell,\alpha} g^0_{\ell,\alpha} B^{\ell}_{\alpha}(k, k_\alpha) B^{\ell}_{\alpha}(k, k_\alpha)}{(m^2_{\ell,\alpha} - s)}.$$

$$F_0(k) = 1,$$

$$F_1(k) = \sqrt{\frac{2(k\,R_{1,\alpha})^2}{1 + (k\,R_{1,\alpha})^2}},$$

Parametrizations : Effective Range Expansion

$$\hat{K}^{(\ell=0)^{-1}} = \frac{\rho}{k}\left(\frac{1}{a} + \frac{1}{2} r_0 k^2\right)$$

Adler zero and χPT

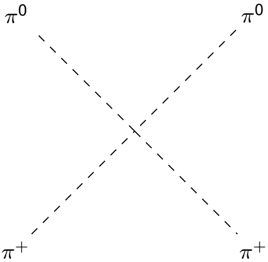
$$\mathcal{L}_2 = \frac{1}{2}\langle \partial_\mu \Phi\, \partial^\mu \Phi \rangle + \frac{1}{12F^2}\langle (\Phi \overset{\leftrightarrow}{\partial}_\mu \Phi)(\Phi \overset{\leftrightarrow}{\partial^\mu} \Phi) \rangle + \mathcal{O}(\Phi^6/F^4),$$

$$\Phi(x) \equiv \frac{\vec{\lambda}}{\sqrt{2}}\,\vec{\phi} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix}.$$

Adler zero and χPT

$$T(\pi^+\pi^0 \to \pi^+\pi^0) = \frac{t}{F^2}$$

$$t \equiv (p'_+ - p_+)^2$$



Parametrizations : K-matrix pole with Adler zero

$$\hat{K}^{(\ell=0)} = \frac{[G_0(s)]^2}{m_0^2 - s},$$
$$G_0(s) = G_0^0\sqrt{(s - s_A)}.$$

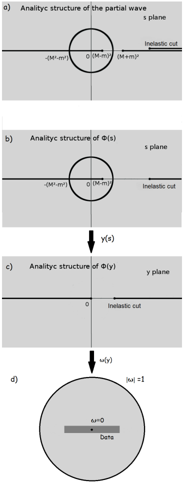
Parametrizations : Conformal map with Adler zero

$$K^{(\ell=0)\,-1} = \frac{\sqrt{s}}{2k} F(s) \sum_n B_n\, \omega^n(s)$$

$$F(s) = \frac{1}{s - s_A},$$

$$\omega(y) = \frac{\sqrt{y} - \alpha\sqrt{y_0 - y}}{\sqrt{y} + \alpha\sqrt{y_0 - y}}$$

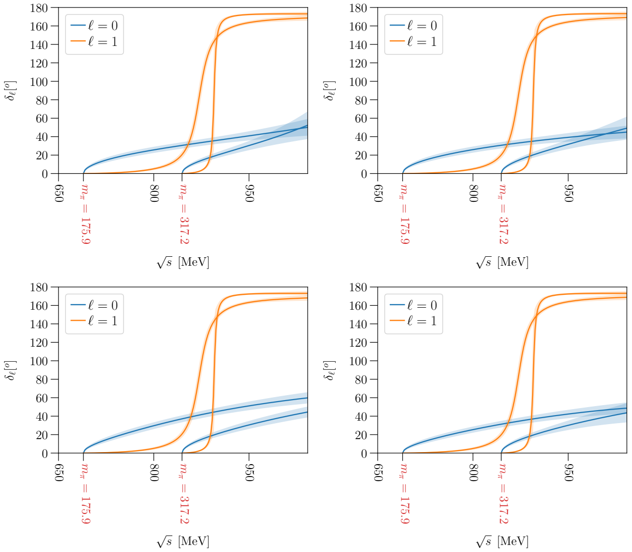
$$y(s) = \left(\frac{s - \Delta_{K\pi}}{s + \Delta_{K\pi}}\right)^2$$



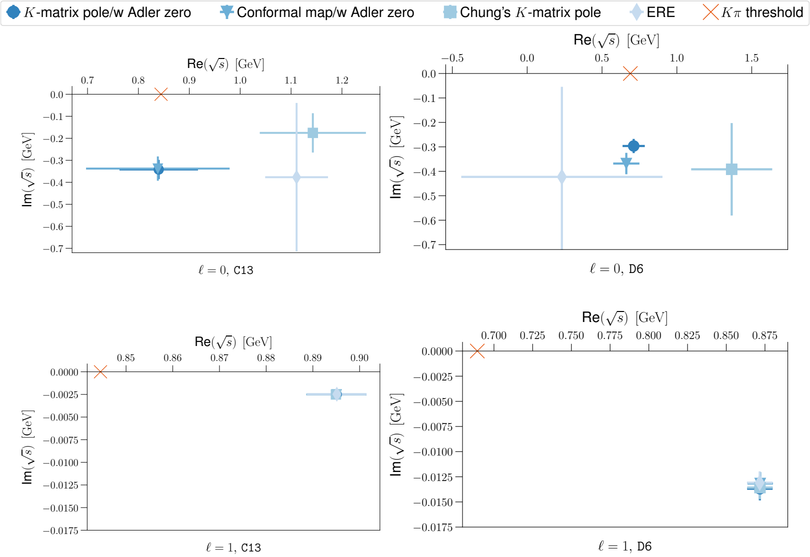
2

2. J.R. Pelaez et A. Rodas. "Pion-kaon scattering amplitude constrained with forward dispersion relations up to 1.6 GeV". In : **Phys. Rev. D** 93.7 (2016), p. 074025. DOI : 10.1103/PhysRevD.93.074025. arXiv : 1602.08404 [hep-ph].

Phase shift results



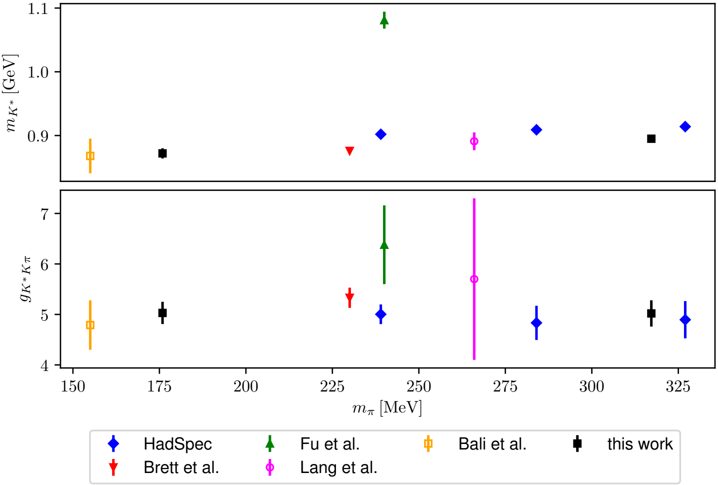
T-matrix poles



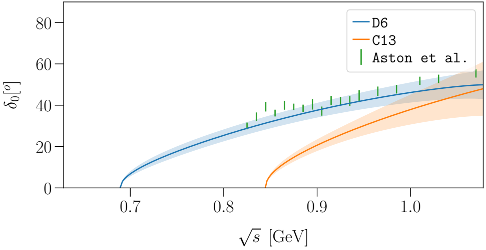
$$\Gamma_{K^*\to K\pi} = \frac{g^2_{K^*K\pi}}{6\pi}\frac{k_*^3}{\mathrm{Re}(\sqrt{s_R})^2},$$

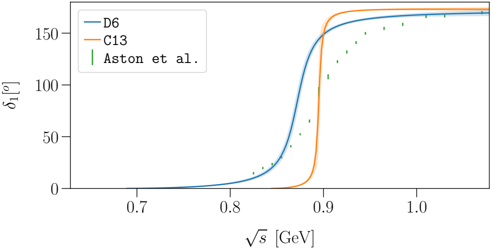
$$g^{\mathtt{C13}}_{K^*K\pi} = 5.02(26),$$
$$g^{\mathtt{D6}}_{K^*K\pi} = 5.03(22).$$

Coupling comparison with other Lattice results



Comparison with LASS experiment





Conclusions

- Adler zero helps describe κ pole
- χPT informs well phenomenology of κ
- Next : $B \to K\pi\ell^+\ell^-$

Thank you !