Towards Quantum Simulation of Non-Abelian Lattice Gauge Theories

JLab Theory Seminar

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Motivation for Quantum Computing/Simulation

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- **Quantum computation is expected to efficiently handle the exponential growth of information in entangled quantum systems that overwhelms classical computers.**
- **Despite of tremendous success of lattice QCD calculations, there are some forbidden regions to explore even with the largest supercomputers.**
- **For lattice gauge theories, quantum computers offer hope for *ab initio* studies of non-zero density, topological properties, and real-time phenomena, which are exponentially hard to solve classically due to sign problems.**

State of the art

Quantum computers are still at infancy just like classical computers 40 years ago.

Present day's effort:

- Contructing proposals for digital and analog quantum simulation.
- NISQ era computation: quantum noise.
- Experimental implementation in both digital and analog for simple and/or toy models.

Quantum computation for gauge theories

Suitable Framework: Hamiltonian Lattice Gauge Theory

Schwinger model: QED in $1+1$ dimensions

- Super simple to analyze yet contains rich physics
- Real time simulation of Schwinger model shows dynamics of pair production, string breaking etc.
- Many analog proposals has been made in past five years. European review: arXiv:1911.00003, Davoudi et al. arXiv:1908.03210 and more.
- Digital computation with Schwinger model: N. Klco et al: arXiv:1803.03326 and more.
- First experimental realization: Martinez et al, Nature'16
- Many ongoing projects across the globe.

Long Term Goal: Quantum Simulating QCD

QCD: Non-abelian $(SU(3))$ gauge theory in $3+1$ dimensions.

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- ONLY A FEW ANALOG PROPOSALS

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- **WHY?**

Drawbacks of Conventional formalism

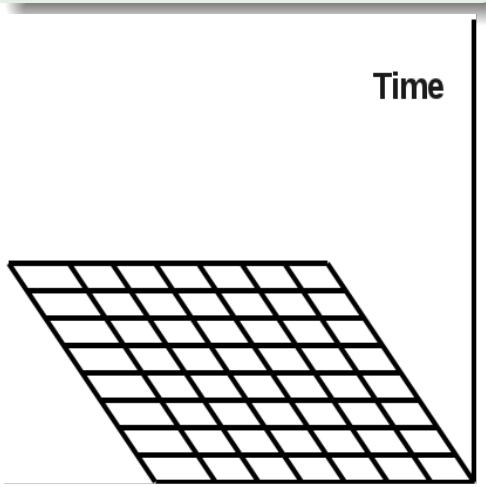
- Non-Abelian LGT : local Hamiltonian+ gauge theory Hilbert space+ Gauss law for gauge invariance.
- States in the Hilbert space are predominantly unphysical, and a noisy quantum computer would get lost among them.
- Gauss's law is nontrivial on a quantum computer: color components are not simultaneously diagonalizable.
- Different representations to be mapped onto a register of qubits are on different footings under (and mixed by) the action of the Hamiltonian.
- Crafting the action of the Hamiltonian in terms of quantum computer operations $\rightarrow$ straightforward in principle but rather unnatural to do.

Alternate formulation

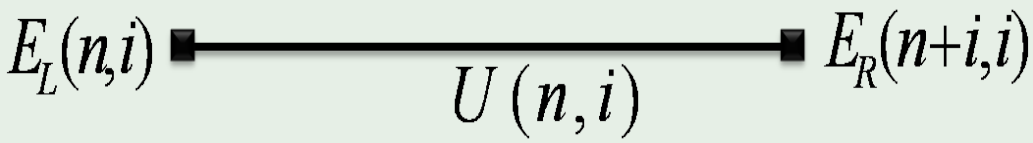
- Prepotential formulation of LGT is developed over last decade starting with Mathur'05, '07.
- prepotential formulation of LGT uses gauge invariant towers of states, characterized by integer quantum numbers.
- The Hamiltonian acts as a sum of ladder operators on those towers of states, which seems far more natural for quantum computation.

Hamiltonian LGT: Variables

Discrete Space and Continuous time



On a link of the spatial lattice



$$E_R^a(n+i,i) = E_L^b(n,i) R^{ab}(U(n,i))$$

$$\hat{H} = \underbrace{g^2 \sum_{(x,i)} \hat{E}^2(x,i)}_{H_E} + \underbrace{\frac{1}{g^2} \sum_{\square} \mathrm{Tr}(2 - \hat{U}_{\square} - \hat{U}_{\square}^{\dagger})}_{H_B} + \underbrace{\sum_{(x,i)} \hat{\psi}^{\dagger}(x) \hat{U}(x,i) \hat{\psi}(x+e_i) + m \sum_{x} (-)^x \hat{\psi}^{\dagger}(x) \hat{\psi}(x)}_{H_M} . \quad (1)$$

Canonical conjugate variables satisfy:

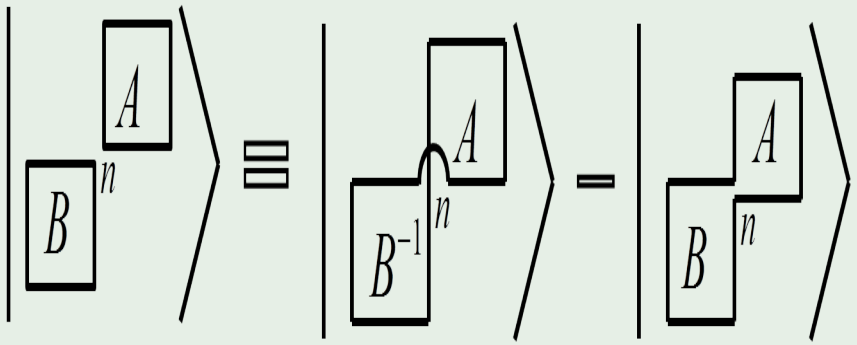
$$[\hat{E}^{\mathrm{a}}_L , \hat{U}] = -\tfrac{1}{2} \sigma^{\mathrm{a}} \hat{U} \ , \ [\hat{E}^{\mathrm{a}}_R , \hat{U}] = \tfrac{1}{2} \hat{U} \sigma^{\mathrm{a}} . \quad (2)$$

The Gauss law: $\hat{G}^{\mathrm{a}} |\mathrm{phys}\rangle_x = 0$ where,

$$\hat{G}^{\mathrm{a}}(x) = \sum_{i=1}^{d} (\hat{E}^{\mathrm{a}}_{L,i}(x) + \hat{E}^{\mathrm{a}}_{R,i}(x)) - \tfrac{1}{2} \hat{\psi}^{\dagger}_{\alpha}(x) \sigma^{\mathrm{a}}{}_{\alpha\beta} \hat{\psi}_{\beta}(x), \quad (3)$$

Wilson loops and Mandelstam Constraints: SU(2)

Involving two loops, each carrying one unit of flux



ONLY NON-INTERSECTING LOOPS ARE PHYSICAL

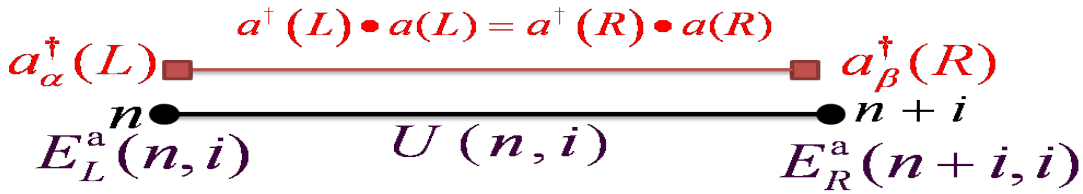
Wilson loops and Mandelstam Constraints: SU(2)

- **Increasing number of Loops $\Rightarrow$ Increasing number of Mandelstam Identities!**
- **In prepotential formulation these fundamental Mandelstam identities becomes local and can be analyzed as well as solved to get Orthonormal Loop states.**

Prepotentials

- **Harmonic oscillators belonging to the fundamental representation of the gauge group defined at each lattice site.**
- **Prepotentials transform as matter fields $\rightarrow$ construct local gauge invariant variables and states from them!**
- **Local Mandelstam constraints $\Rightarrow$ Exact solution is non-trivial but possible.**
- **Prepotential formulation of SU(2), SU(3) and arbitrary SU(N) exists** *(ref:IR, PhD Thesis)* **, but we will confine ourselves to SU(2) only in this talk.**

SU(2) Prepotentials



Left electric fields: $$E^a_L(n,i) \equiv a^\dagger(n,i;L)\frac{\sigma^a}{2}a(n,i;L),$$

Right electric fields: $$E^a_R(n+i,i) \equiv a^\dagger(n+i,i;R)\frac{\sigma^a}{2}a(n+i,i;R).$$

Under SU(2) gauge transformation

$$a^\dagger_\alpha(L) \to a^\dagger_\beta(L)\,\left(\Lambda^\dagger_L\right)^\beta{}_\alpha, \qquad a^\dagger_\alpha(R) \to a^\dagger_\beta(R)\,\left(\Lambda^\dagger_R\right)^\beta{}_\alpha$$

$$a^\alpha(L) \to \left(\Lambda_L\right)^\alpha{}_\beta\, a^\beta(L), \qquad a^\alpha(R) \to \left(\Lambda_R\right)^\alpha{}_\beta\, a^\beta(R).$$

Link Operator

- **From $SU(2) \otimes U(1)$ gauge transformations of the prepotentials,**

$$U^{\alpha}{}_{\beta} = \tilde{a}^{\dagger\alpha}(L)\,\eta\,a^{\dagger}_{\beta}(R) + a^{\alpha}(L)\,\theta\,\tilde{a}_{\beta}(R)$$

- **Calculating the coefficients from $U^{\dagger}U = UU^{\dagger} = 1$,**

$$U = \underbrace{\frac{1}{\sqrt{\hat{n}_L + 1}} \begin{pmatrix} a_2^{\dagger}(L) & a_1(L) \\ -a_1^{\dagger}(L) & a_2(L) \end{pmatrix}}_{U_L} \underbrace{\begin{pmatrix} a_1^{\dagger}(R) & a_2^{\dagger}(R) \\ a_2(R) & -a_1(R) \end{pmatrix} \frac{1}{\sqrt{\hat{n}_R + 1}}}_{U_R}$$

Abelian Weaving, Non-abelian Intertwining and Loop States

Link operator: $U^{\alpha}{}_{\beta} = \frac{1}{\sqrt{\hat{n}+1}} \left(\tilde{a}^{\dagger\alpha}(L)\, a^{\dagger}_{\beta}(R) + a^{\alpha}(L)\, \tilde{a}_{\beta}(R) \right) \frac{1}{\sqrt{\hat{n}+1}}$

Four basic gauge invariant operators constructed by $U^{\alpha}{}_{\beta}(n,i)U^{\beta}{}_{\gamma}(n+i,j)$ at site $(n+i)$:

$$a^{\dagger}_{\beta}(i)\frac{1}{\sqrt{\hat{n}_i+1}}\frac{1}{\sqrt{\hat{n}_j+1}}\tilde{a}^{\dagger\beta}(j) = \frac{1}{\sqrt{\hat{n}_i}}\frac{1}{\sqrt{\hat{n}_j+1}}a^{\dagger}(i)\cdot\tilde{a}^{\dagger\beta}(j) \equiv \frac{1}{\sqrt{\hat{n}_i(\hat{n}_j+1)}}k^{ij}_{+} \equiv \hat{\mathcal{O}}^{i_+ j_+}$$

$$a^{\dagger}_{\beta}(i)\frac{1}{\sqrt{\hat{n}_i+1}}\frac{1}{\sqrt{\hat{n}_j+1}}a^{\beta}(j) = \frac{1}{\sqrt{\hat{n}_i}}\frac{1}{\sqrt{\hat{n}_j+1}}a^{\dagger}(i)\cdot a(j) \equiv \frac{1}{\sqrt{\hat{n}_i(\hat{n}_j+1)}}\kappa^{ij} \equiv \hat{\mathcal{O}}^{i_+ j_-}$$

$$\tilde{a}_{\beta}(i)\frac{1}{\sqrt{\hat{n}_i+1}}\frac{1}{\sqrt{\hat{n}_j+1}}\tilde{a}^{\dagger\beta}(j) = \frac{1}{\sqrt{\hat{n}_i+2}}\frac{1}{\sqrt{\hat{n}_j+1}}a(i)\cdot a^{\dagger}(j) \equiv \frac{1}{\sqrt{(\hat{n}_i+2)(\hat{n}_j+1)}}\kappa^{ji} \equiv \hat{\mathcal{O}}^{j_+ i_-}$$

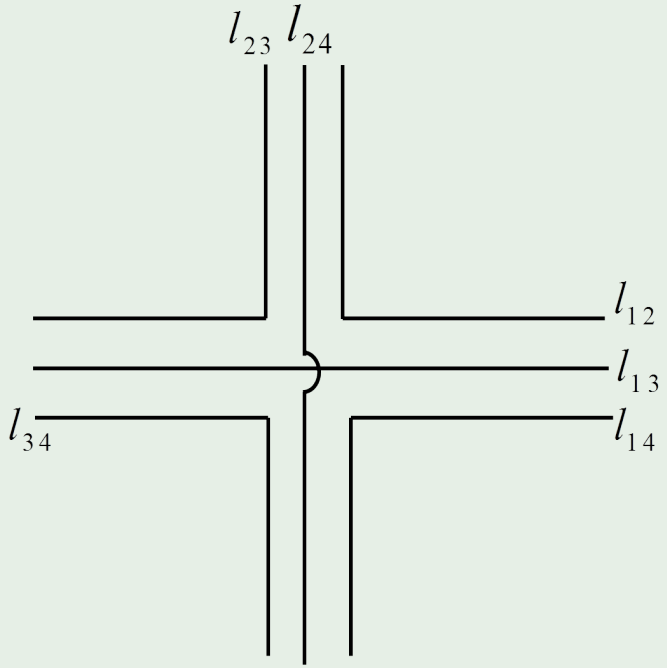
$$\tilde{a}_{\beta}(i)\frac{1}{\sqrt{\hat{n}_i+1}}\frac{1}{\sqrt{\hat{n}_j+1}}a^{\beta}(j) = \frac{1}{\sqrt{\hat{n}_i+2}}\frac{1}{\sqrt{\hat{n}_j+1}}\tilde{a}(i)\cdot a(j) \equiv \frac{1}{\sqrt{(\hat{n}_i+2)(\hat{n}_j+1)}}k^{ji}_{-} \equiv \hat{\mathcal{O}}^{i_- j_-}$$

for i, j different directions at each site

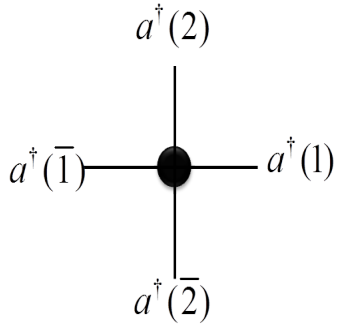
Loop States and Linking Numbers

$$|\{l_{ij}\}\rangle = \prod_{i \neq j} \frac{(k_+)^{l_{ij}}}{l_{ij}!} |0\rangle$$

Linking numbers in 2d

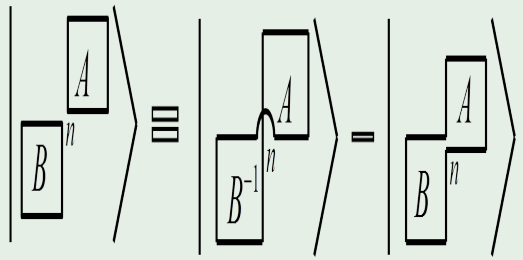


Mandelstam Constraints



$$\left(a^\dagger(1)\cdot\tilde{a}^\dagger(2)\right)\left(a^\dagger(\bar{1})\cdot\tilde{a}^\dagger(\bar{2})\right)\equiv\left(a^\dagger(1)\cdot\tilde{a}^\dagger(\bar{1})\right)\left(a^\dagger(2)\cdot\tilde{a}^\dagger(\bar{2})\right)-\left(a^\dagger(1)\cdot\tilde{a}^\dagger(\bar{2})\right)\left(a^\dagger(2)\cdot\tilde{a}^\dagger(\bar{1})\right)$$

Equivalent to the fundamental Mandelstam identity



Linking Numbers and Constraints

Loop State characterized by 6 linking numbers

$$|l_{12}, l_{1\bar{1}}, l_{1\bar{2}}, l_{2\bar{1}}, l_{2\bar{2}}, l_{\bar{1}\bar{2}}\rangle \equiv |\{l\}\rangle = \frac{\left(k_+^{12}\right)^{l_{12}}}{l_{12}!} \frac{\left(k_+^{1\bar{1}}\right)^{l_{1\bar{1}}}}{l_{1\bar{1}}!} \frac{\left(k_+^{1\bar{2}}\right)^{l_{1\bar{2}}}}{l_{1\bar{2}}!} \frac{\left(k_+^{2\bar{1}}\right)^{l_{2\bar{1}}}}{l_{2\bar{1}}!} \frac{\left(k_+^{2\bar{2}}\right)^{l_{2\bar{2}}}}{l_{2\bar{2}}!} \frac{\left(k_+^{\bar{1}\bar{2}}\right)^{l_{\bar{1}\bar{2}}}}{l_{\bar{1}\bar{2}}!} |0\rangle \tag{4}$$

with $n_1 = l_{12} + l_{1\bar{1}} + l_{1\bar{2}}$, $n_2 = l_{2\bar{1}} + l_{2\bar{2}} + l_{12}$, $n_{\bar{1}} = l_{\bar{1}\bar{2}} + l_{1\bar{1}} + l_{2\bar{1}}$, $n_{\bar{2}} = l_{1\bar{2}} + l_{2\bar{2}} + l_{\bar{1}\bar{2}}$

One Mandelstam constraint

$$k_+^{12} k_+^{\bar{1}\bar{2}} - k_+^{1\bar{2}} k_+^{2\bar{1}} + k_+^{1\bar{1}} k_+^{2\bar{2}} = 0$$

Two $U(1)$ Gauss Law constraints

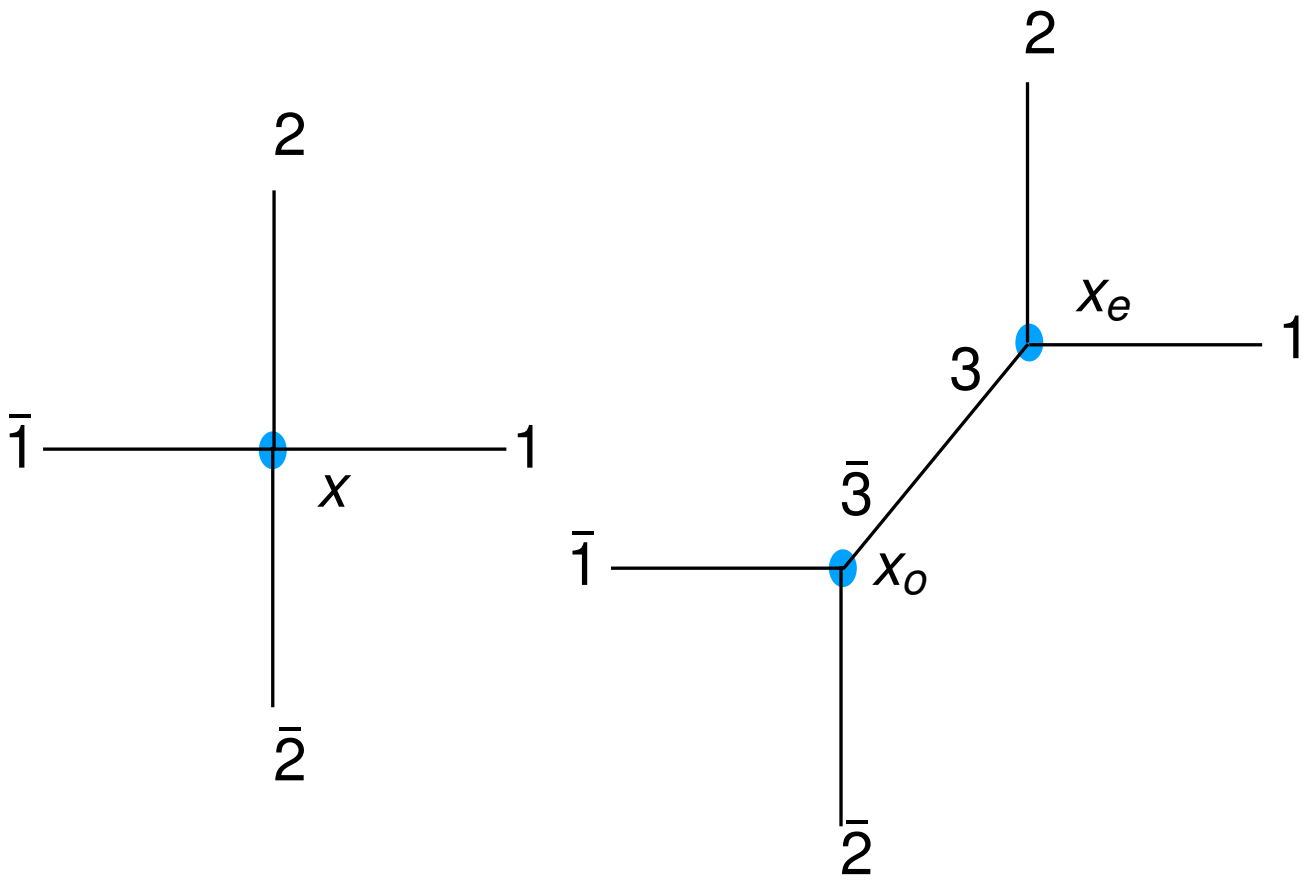
$$n_1(x) = n_{\bar{1}}(x + e_1) \;\; \& \;\; n_2(x) = n_{\bar{2}}(x + e_2)$$

Motivation

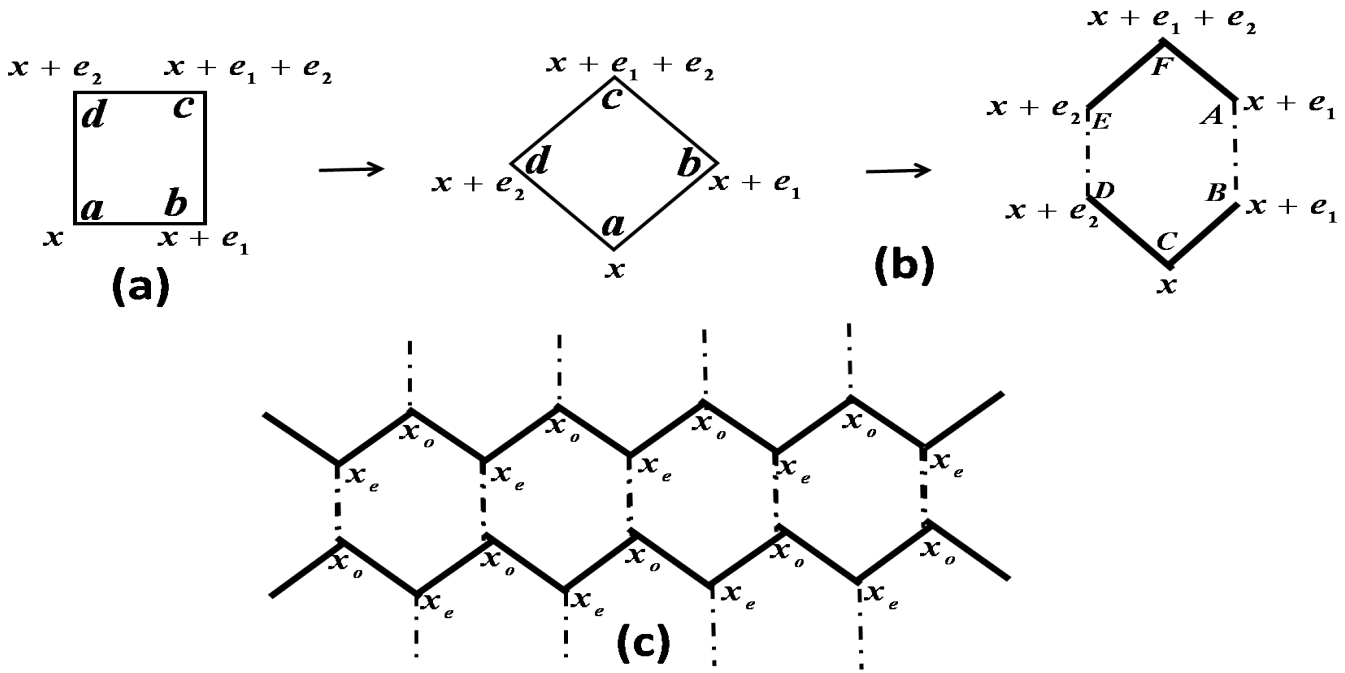
- In two spatial dimension, at each site there is exactly three physical degrees of freedom.
- In terms of linking variables or fusion variables, identifying these three quantum numbers to characterize a loop state is not straightforward.
- Non-linear Constraints: difficult to analyze
- An observation: dynamics on a square plaquette is identical to the dynamics on hexagonal plaquette with only linear constraints (IR '18)

Example: point splitting in 2D

Site 'x' on a square lattice is virtually split into two sites 'x_e & x_o' connected by a third virtual direction $3 - \bar{3}$



The virtual Hexagonal Lattice



Abelian Gauss law

$$n_i(x) = n_{\bar{i}}\,(x + e_i)$$

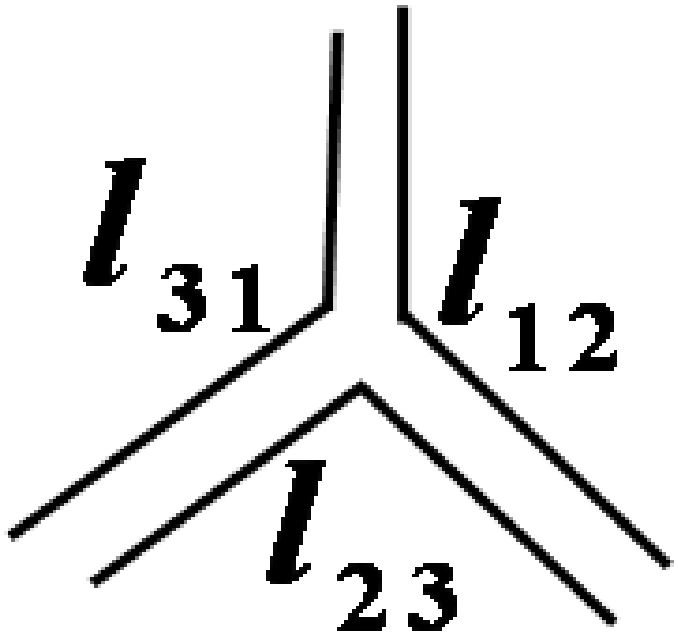
Prepotential Formulation

Local Loop Operators

$$\hat{\mathcal{L}}^{++}_{ij} \equiv \hat{a}^{\dagger}_{\alpha}(i)\hat{\tilde{a}}^{\dagger}_{\alpha}(j) \quad , \quad \hat{\mathcal{L}}^{+-}_{ij} \equiv \hat{a}^{\dagger}_{\alpha}(i)\hat{a}_{\alpha}(j) \tag{5}$$

Above, $\hat{\tilde{a}}^{\dagger\alpha} \equiv \epsilon^{\alpha\beta}\hat{a}^{\dagger}_{\beta}$, $\hat{\tilde{a}}_{\alpha} \equiv \epsilon_{\alpha\beta}\hat{a}^{\beta}$, and i, j are direction indices with $i \neq j$.

Local loop state:



$$|l_{12}, l_{23}, l_{31}\rangle \equiv \frac{(\hat{\mathcal{L}}_{12}^{++})^{l_{12}} (\hat{\mathcal{L}}_{23}^{++})^{l_{23}} (\hat{\mathcal{L}}_{31}^{++})^{l_{31}}}{(l_{12} + l_{23} + l_{31})!\; l_{12}!\; l_{23}!\; l_{31}!} |0\rangle_x \;, \qquad (6)$$

Action of local loop operators

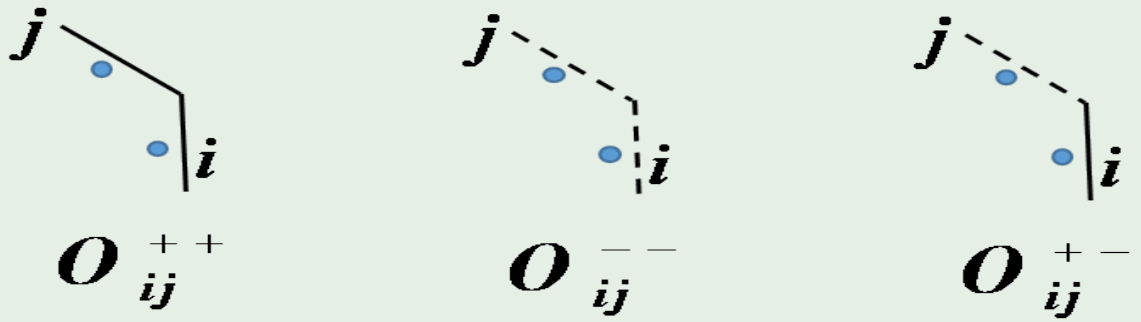
$$\hat{\mathcal{L}}_{ij}^{++}|l_{ij}\rangle = \sqrt{(l_{ij}+1)(l_{12}+l_{23}+l_{31}+2)}|l_{ij}+1\rangle, \tag{7}$$

$$\hat{\mathcal{L}}_{ij}^{--}|l_{ij}\rangle = \sqrt{l_{ij}(l_{12}+l_{23}+l_{31}+1)}|l_{ij}-1\rangle, \tag{8}$$

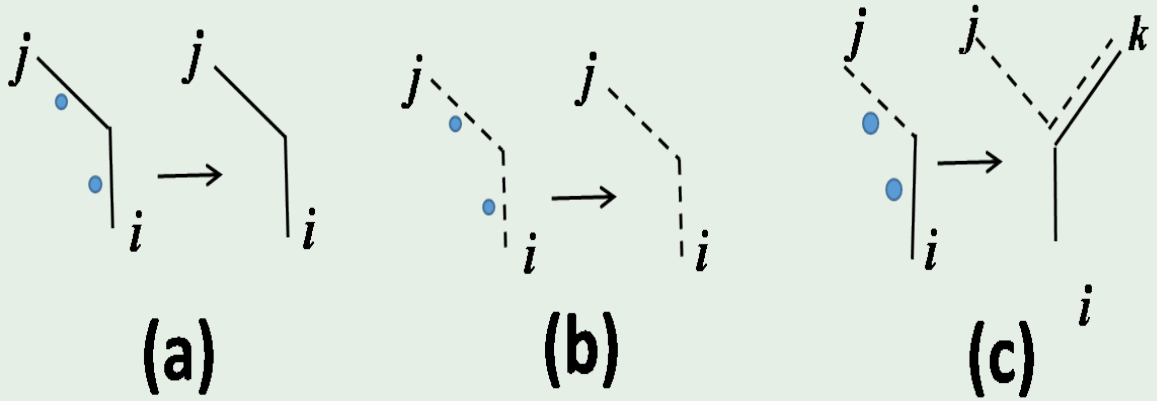
$$\hat{\mathcal{L}}_{ij}^{+-}|l_{ij}\rangle = -\sqrt{(l_{ik}+1)l_{jk}}|l_{jk}-1, l_{ik}+1\rangle. \tag{9}$$

Pictorial representation of loops

Local loops on hexagonal lattice



Action on loop states



Occupation number basis

At each site x:

$$n_1 = l_{12} + l_{31} \quad , \quad n_2 = l_{12} + l_{23} \quad , \quad n_3 = l_{23} + l_{31} \tag{10}$$

or equivalently,

$$\begin{aligned} l_{12} &= \frac{1}{2}(n_1 + n_2 - n_3) \, , \\ l_{23} &= \frac{1}{2}(n_2 + n_3 - n_1) \, , \\ l_{31} &= \frac{1}{2}(n_1 + n_3 - n_2) \end{aligned} \tag{11}$$

Abelian Gauss law

$$n_i(x) = n_{\bar{i}}\,(x + e_i)$$

Counting degrees of freedom

- For x on square lattice:
  6 linking numbers $-$ 2 Abelian Gauss law along two directions $-$ 1 Mandelstam constraint $\Rightarrow$ three physical degrees of freedom.
- For hexagonal lattice, two sites x_1 & x_2 corresponds to actual site x on the square lattice and together should have only three degrees of freedom.
- 2×3 linking numbers $-$ 3 Abelian Gauss law $\Rightarrow$ 3 physical degrees of freedom
- No Mandelstam Constraint!

In any dimensions

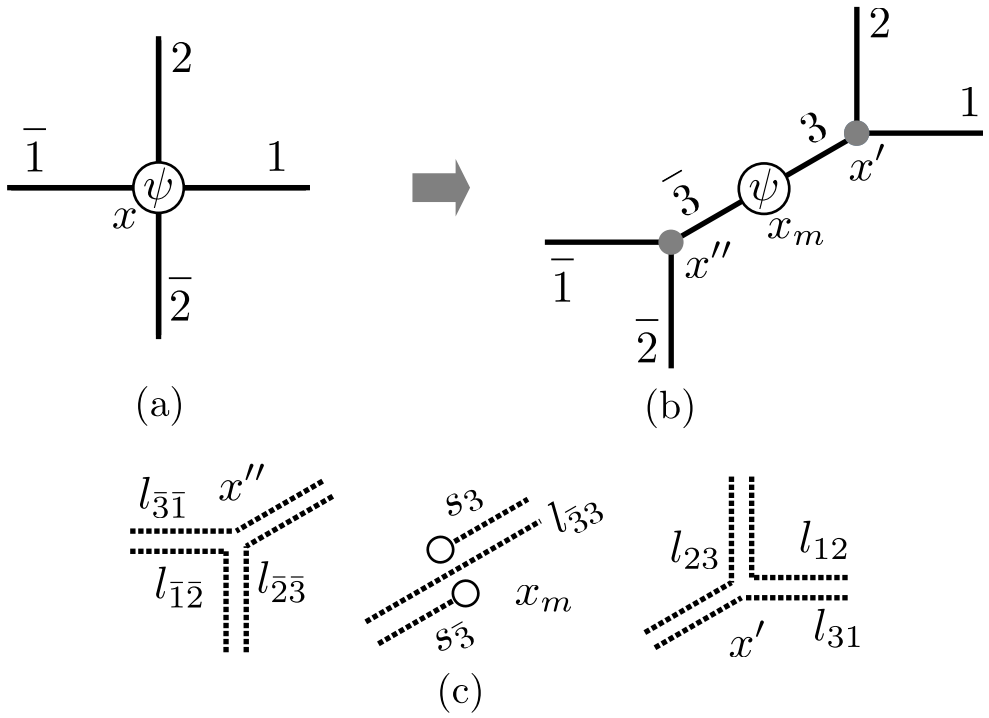
- For arbitrary dimension d, split each lattice site into S three point vertices.
- Total $3S$ number of loop states per original lattice site.
- d Abelian Gauss law per lattice site.
- S virtual sites has introduced $S-1$ number of virtual links in the lattice, each containing one Abelian Gauss law constraint.
- The number of independent loop degrees of freedom per original lattice site counts to

$$\begin{aligned} 3S - d - S + 1 &\equiv 3(d-1) \\ \Rightarrow S &= 2(d-1) \end{aligned}$$

where, $3(d-1)$ is the physical degrees of freedom per lattice site.

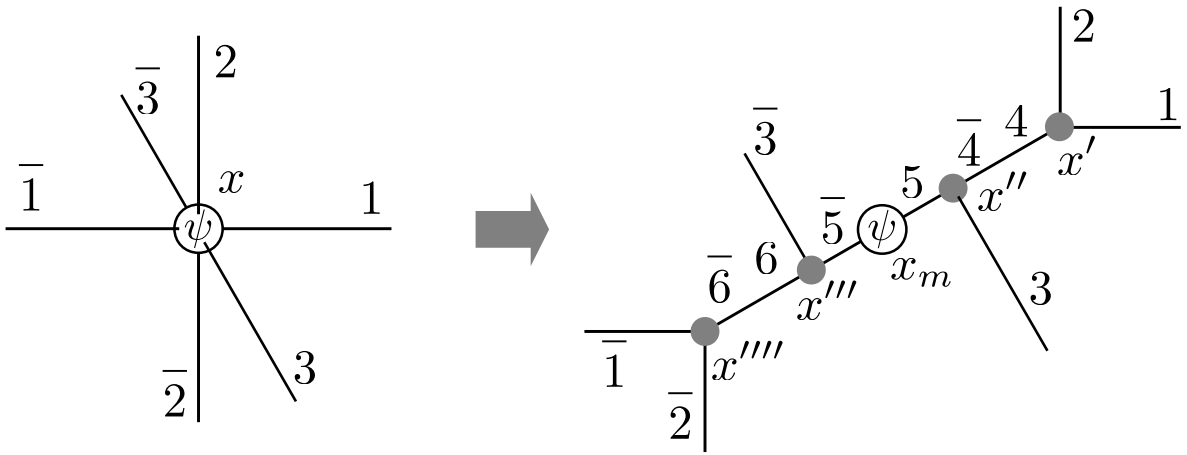
- In 2D, point splitting results in a hexagonal lattice.
- Prepotential formulation.
- On this hexagonal lattice, physical lattice directions are along 1 and 2, and only the electric fields along these two directions contribute to $\hat{H}_E$.
- However, in $\hat{H}_B$, the elementary loops are indeed hexagonal plaquettes.
- The matter field, originally at sites x of the square lattice, now lives on sites x_m, which is at the middle of virtual link along 3.
- We treat the sites x', x'' on the same footing as in pure gauge theory and x_m to be a site of 1D lattice with matter.

Inclusion of Matter in 2d

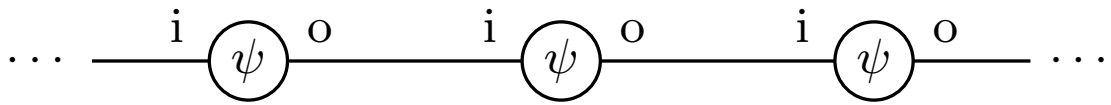


3d lattice with matter

Matter as in 1D + pure gluonic vertices



Matter couples to gauge fields in the same way in all spatial dimention as in 1D



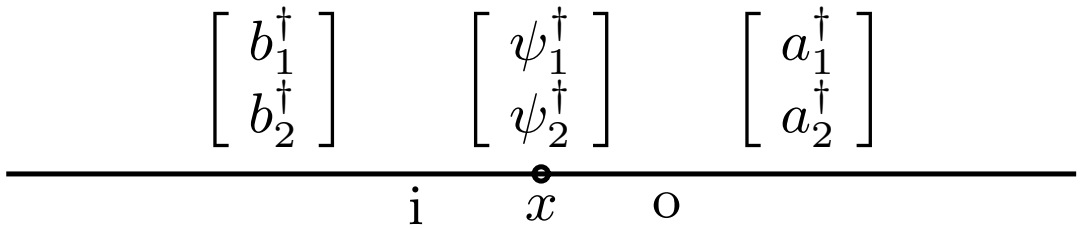
Hamiltonian

$$H = H_E + H_M$$

Gauss Law

$$\hat{G}^{\mathrm{a}}(x) = \hat{E}^{\mathrm{a}}_i(x) + \hat{E}^{\mathrm{a}}_o(x) - \tfrac{1}{2}\hat{\psi}^{\dagger}_{\alpha}(x)\sigma^{\mathrm{a}}{}_{\alpha\beta}\hat{\psi}_{\beta}(x), \qquad (12)$$

Prepotentials in 1D:



$\hat{a}^\dagger_\alpha$ ($\hat{b}^\dagger_\alpha$) is attached to the link along the direction 1 ($\bar{1}$) and a staggered fermion field $\hat{\psi}^\dagger = (\hat{\psi}^\dagger_1, \hat{\psi}^\dagger_2)$ lives on the sites x.

E_i, E_o and $U_{\alpha\beta}$ can be rewritten using prepotentials.

Ref: Loop, String, and Hadron Dynamics in SU(2) Hamiltonian Lattice Gauge Theories, *IR, Jesse Strylker*, **arXiv: 1912:06133.**

SU(2)-invariant Operators: Loops, Strings and hadrons

Hermitian number operators:

$$\mathcal{N}_a = a^\dagger_\alpha a_\alpha$$
$$\mathcal{N}_b = b^\dagger_\alpha b_\alpha$$
$$\mathcal{N}_\psi = \psi^\dagger_\alpha \psi_\alpha$$

SU(2)-invariant Operators: Loops, Strings and hadrons

Pure gauge operators: Loops

$$\mathcal{L}^{++} = \epsilon_{\alpha\beta} b^\dagger_\alpha a^\dagger_\beta$$
$$\mathcal{L}^{--} = \epsilon_{\alpha\beta} b_\alpha a_\beta = (\mathcal{L}^{++})^\dagger$$
$$\mathcal{L}^{+-} = b^\dagger_\alpha a_\alpha$$
$$\mathcal{L}^{-+} = b_\alpha a^\dagger_\alpha = (\mathcal{L}^{+-})^\dagger$$

SU(2)-invariant Operators: Loops, Strings and hadrons

Gauge-Matter operators: Incoming Strings

$$\mathcal{S}_{\text{in}}^{++} = \epsilon_{\alpha\beta} b_\alpha^\dagger \psi_\beta^\dagger$$
$$\mathcal{S}_{\text{in}}^{--} = \epsilon_{\alpha\beta} b_\alpha \psi_\beta = (\mathcal{S}_{\text{in}}^{++})^\dagger$$
$$\mathcal{S}_{\text{in}}^{+-} = b_\alpha^\dagger \psi_\alpha$$
$$\mathcal{S}_{\text{in}}^{-+} = b_\alpha \psi_\alpha^\dagger = (\mathcal{S}_{\text{in}}^{+-})^\dagger$$

SU(2)-invariant Operators: Loops, Strings and hadrons

Gauge-Matter operators: Outgoing Strings

$$\mathcal{S}^{++}_{\text{out}} = \epsilon_{\alpha\beta}\psi^{\dagger}_{\alpha}a^{\dagger}_{\beta}$$
$$\mathcal{S}^{--}_{\text{out}} = \epsilon_{\alpha\beta}\psi_{\alpha}a_{\beta} = (\mathcal{S}^{++}_{\text{out}})^{\dagger}$$
$$\mathcal{S}^{+-}_{\text{out}} = \psi_{\alpha}a^{\dagger}_{\alpha}$$
$$\mathcal{S}^{-+}_{\text{out}} = \psi^{\dagger}_{\alpha}a_{\alpha} = (\mathcal{S}^{-+}_{\text{out}})^{\dagger}$$

SU(2)-invariant Operators: Loops, Strings and hadrons

Pure matter operators: hadrons

$$\mathcal{B}^{++} = \frac{1}{2!}\epsilon_{\alpha\beta}\psi^{\dagger}_{\alpha}\psi^{\dagger}_{\beta}$$

$$\mathcal{B}^{--} = \frac{1}{2!}\epsilon_{\alpha\beta}\psi_{\alpha}\psi_{\beta} = (\mathcal{B}^{++})^{\dagger}$$

This set of invariants is indeed a complete set; the bosonic operator algebra closes

	$[\cdot,\mathcal{N}_b]$	$[\cdot,\mathcal{N}_a]$	$[\cdot,\mathcal{N}_\psi]$	$[\cdot,\mathcal{L}^{--}]$	$[\cdot,\mathcal{L}^{-+}]$	$[\cdot,\mathcal{L}^{+-}]$	$[\cdot,\mathcal{L}^{++}]$	$[\cdot,\mathcal{B}^{++}]$	$[\cdot,\mathcal{B}^{--}]$
$[\mathcal{N}_b,\cdot]$	0	0	0	$-\mathcal{L}^{--}$	$-\mathcal{L}^{-+}$	$+\mathcal{L}^{+-}$	$+\mathcal{L}^{++}$	0	0
$[\mathcal{N}_a,\cdot]$	0	0	0	$-\mathcal{L}^{--}$	$+\mathcal{L}^{-+}$	$-\mathcal{L}^{+-}$	$+\mathcal{L}^{++}$	0	0
$[\mathcal{N}_\psi,\cdot]$	0	0	0	0	0	0	0	$2\mathcal{B}^{++}$	$-2\mathcal{B}^{--}$
$[\mathcal{L}^{++},\cdot]$	$-\mathcal{L}^{++}$	$-\mathcal{L}^{++}$	0	$-\mathcal{N}_a-\mathcal{N}_b-2$	0	0	0	0	0
$[\mathcal{L}^{+-},\cdot]$	$-\mathcal{L}^{+-}$	$+\mathcal{L}^{+-}$	0	0	$\mathcal{N}_b-\mathcal{N}_a$	0	0	0	0
$[\mathcal{L}^{-+},\cdot]$	$+\mathcal{L}^{-+}$	$-\mathcal{L}^{-+}$	0	0	0	$\mathcal{N}_a-\mathcal{N}_b$	0	0	0
$[\mathcal{L}^{--},\cdot]$	$+\mathcal{L}^{--}$	$+\mathcal{L}^{--}$	0	0	0	0	$\mathcal{N}_a+\mathcal{N}_b+2$	0	0
$[\mathcal{S}_{\text{in}}^{++},\cdot]$	$-\mathcal{S}_{\text{in}}^{++}$	0	$-\mathcal{S}_{\text{in}}^{++}$	$+\mathcal{S}_{\text{out}}^{+-}$	$-\mathcal{S}_{\text{out}}^{++}$	0	0	0	$+\mathcal{S}_{\text{in}}^{+-}$
$[\mathcal{S}_{\text{in}}^{+-},\cdot]$	$-\mathcal{S}_{\text{in}}^{+-}$	0	$+\mathcal{S}_{\text{in}}^{+-}$	$-\mathcal{S}_{\text{out}}^{--}$	$-\mathcal{S}_{\text{out}}^{-+}$	0	0	$+\mathcal{S}_{\text{in}}^{++}$	0
$[\mathcal{S}_{\text{in}}^{-+},\cdot]$	$+\mathcal{S}_{\text{in}}^{-+}$	0	$-\mathcal{S}_{\text{in}}^{-+}$	0	0	$+\mathcal{S}_{\text{out}}^{+-}$	$+\mathcal{S}_{\text{out}}^{++}$	0	$-\mathcal{S}_{\text{in}}^{--}$
$[\mathcal{S}_{\text{in}}^{--},\cdot]$	$+\mathcal{S}_{\text{in}}^{--}$	0	$+\mathcal{S}_{\text{in}}^{--}$	0	0	$+\mathcal{S}_{\text{out}}^{--}$	$-\mathcal{S}_{\text{out}}^{-+}$	$-\mathcal{S}_{\text{in}}^{-+}$	0
$[\mathcal{S}_{\text{out}}^{++},\cdot]$	0	$-\mathcal{S}_{\text{out}}^{++}$	$-\mathcal{S}_{\text{out}}^{++}$	$-\mathcal{S}_{\text{in}}^{-+}$	0	$-\mathcal{S}_{\text{in}}^{++}$	0	0	$+\mathcal{S}_{\text{out}}^{-+}$
$[\mathcal{S}_{\text{out}}^{-+},\cdot]$	0	$-\mathcal{S}_{\text{out}}^{-+}$	$+\mathcal{S}_{\text{out}}^{-+}$	$+\mathcal{S}_{\text{in}}^{--}$	0	$-\mathcal{S}_{\text{in}}^{+-}$	0	$+\mathcal{S}_{\text{out}}^{++}$	0
$[\mathcal{S}_{\text{out}}^{+-},\cdot]$	0	$+\mathcal{S}_{\text{out}}^{+-}$	$-\mathcal{S}_{\text{out}}^{+-}$	0	$+\mathcal{S}_{\text{in}}^{-+}$	0	$-\mathcal{S}_{\text{in}}^{++}$	0	$-\mathcal{S}_{\text{out}}^{--}$
$[\mathcal{S}_{\text{out}}^{--},\cdot]$	0	$+\mathcal{S}_{\text{out}}^{--}$	$+\mathcal{S}_{\text{out}}^{--}$	0	$+\mathcal{S}_{\text{in}}^{--}$	0	$+\mathcal{S}_{\text{in}}^{+-}$	$-\mathcal{S}_{\text{out}}^{+-}$	0
$[\mathcal{B}^{--},\cdot]$	0	0	$2\mathcal{B}^{--}$	0	0	0	0	$1-\mathcal{N}_\psi$	0
$[\mathcal{B}^{++},\cdot]$	0	0	$-2\mathcal{B}^{++}$	0	0	0	0	0	$\mathcal{N}_\psi-1$

The incoming and outgoing string operators are Fermionic (due to single fermionic content) and satisfy the following anticommutation relations

	$\{\cdot, \mathcal{S}_{\text{in}}^{++}\}$	$\{\cdot, \mathcal{S}_{\text{in}}^{+-}\}$	$\{\cdot, \mathcal{S}_{\text{in}}^{-+}\}$	$\{\cdot, \mathcal{S}_{\text{in}}^{--}\}$	$\{\cdot, \mathcal{S}_{\text{out}}^{++}\}$	$\{\cdot, \mathcal{S}_{\text{out}}^{+-}\}$	$\{\cdot, \mathcal{S}_{\text{out}}^{-+}\}$	$\{\cdot, \mathcal{S}_{\text{out}}^{--}\}$
$\{\mathcal{S}_{\text{in}}^{++}, \cdot\}$	0	0	$2\mathcal{B}^{++}$	$2 + \mathcal{N}_b - \mathcal{N}_\psi$	0	0	$-\mathcal{L}^{++}$	$\mathcal{L}^{+-}$
$\{\mathcal{S}_{\text{in}}^{+-}, \cdot\}$	0	0	$\mathcal{N}_b + \mathcal{N}_\psi$	$2\mathcal{B}^{--}$	$\mathcal{L}^{++}$	$\mathcal{L}^{+-}$	0	0
$\{\mathcal{S}_{\text{in}}^{-+}, \cdot\}$	$2\mathcal{B}^{++}$	$\mathcal{N}_b + \mathcal{N}_\psi$	0	0	0	0	$\mathcal{L}^{-+}$	$\mathcal{L}^{--}$
$\{\mathcal{S}_{\text{in}}^{--}, \cdot\}$	$2 + \mathcal{N}_b - \mathcal{N}_\psi$	$2\mathcal{B}^{--}$	0	0	$\mathcal{L}^{-+}$	$-\mathcal{L}^{--}$	0	0
$\{\mathcal{S}_{\text{out}}^{++}, \cdot\}$	0	$\mathcal{L}^{++}$	0	$\mathcal{L}^{-+}$	0	$2\mathcal{B}^{++}$	0	$2 + \mathcal{N}_a - \mathcal{N}_\psi$
$\{\mathcal{S}_{\text{out}}^{+-}, \cdot\}$	0	$\mathcal{L}^{+-}$	0	$-\mathcal{L}^{--}$	$2\mathcal{B}^{++}$	0	$\mathcal{N}_a + \mathcal{N}_\psi$	0
$\{\mathcal{S}_{\text{out}}^{-+}, \cdot\}$	$-\mathcal{L}^{++}$	0	$\mathcal{L}^{-+}$	0	0	$\mathcal{N}_a + \mathcal{N}_\psi$	0	$2\mathcal{B}^{--}$
$\{\mathcal{S}_{\text{out}}^{--}, \cdot\}$	$\mathcal{L}^{+-}$	0	$\mathcal{L}^{--}$	0	$2 + \mathcal{N}_a - \mathcal{N}_\psi$	0	$2\mathcal{B}^{--}$	0

Loop-String Basis States:

$$|n_l, n_i, n_o\rangle = \frac{||n_l, n_i, n_o\rangle}{\sqrt{n_l!\,(n_l + 1 + (n_i \oplus n_o))!}}\,,$$

where $\oplus$ denotes addition modulo two

$$\begin{aligned}
||n_l, n_i = 0, n_o = 0\rangle &\equiv (\mathcal{L}^{++})^{n_l}\,|0\rangle \\
||n_l, n_i = 0, n_o = 1\rangle &\equiv (\mathcal{L}^{++})^{n_l}\mathcal{S}^{++}_{\text{out}}\,|0\rangle \\
||n_l, n_i = 1, n_o = 0\rangle &\equiv (\mathcal{L}^{++})^{n_l}\mathcal{S}^{++}_{\text{in}}\,|0\rangle \\
||n_l, n_i = 1, n_o = 1\rangle &\equiv (\mathcal{L}^{++})^{n_l}\mathcal{B}^{++}\,|0\rangle
\end{aligned}$$

$||n_l, n_i = 1, n_o = 1\rangle \equiv (\mathcal{L}^{++})^{n_l}\mathcal{B}^{++}\,|0\rangle$, HOW?

$$\begin{aligned}\mathcal{S}_{\text{in}}^{++}\mathcal{S}_{\text{out}}^{++}\,|0\rangle &= \mathcal{S}_{\text{in}}^{++}[\mathcal{S}_{\text{out}}^{-+},\mathcal{B}^{++}]\,|0\rangle \\ &= \mathcal{S}_{\text{in}}^{++}\mathcal{S}_{\text{out}}^{-+}\mathcal{B}^{++}\,|0\rangle \\ &= \{\mathcal{S}_{\text{in}}^{++},\mathcal{S}_{\text{out}}^{-+}\}\mathcal{B}^{++}\,|0\rangle \\ &= -\mathcal{L}^{++}\mathcal{B}^{++}\,|0\rangle\end{aligned}$$

Occupation number basis of prepotentials and matter to loop-string basis

$$\mathcal{N}_i \equiv \frac{1}{2}(\mathcal{N}_\psi + \mathcal{N}_b - \mathcal{N}_a)$$
$$\mathcal{N}_o \equiv \frac{1}{2}(\mathcal{N}_\psi + \mathcal{N}_a - \mathcal{N}_b)$$
$$\mathcal{N}_l \equiv \frac{1}{2}\left[\mathcal{N}_a + \mathcal{N}_b - \mathcal{N}_\psi + \frac{1}{2}\left(\mathcal{N}_\psi^2 - (\mathcal{N}_a - \mathcal{N}_b)^2\right)\right]$$

or equivalently,

$$\mathcal{N}_\psi = \mathcal{N}_i + \mathcal{N}_o \ ,$$
$$\mathcal{N}_a = \mathcal{N}_l + (1 - \mathcal{N}_i)\mathcal{N}_o \ ,$$
$$\mathcal{N}_b = \mathcal{N}_l + (1 - \mathcal{N}_o)\mathcal{N}_i .$$

The Hamiltonian

$\hat{H}_E$ is $\sum_{x,i}(g^2/2)\hat{E}^2_L(x,i)$ (or $\sum_{x,i}(g^2/2)\hat{E}^2_R(x,i)$), which in terms of loop-string quantum number reads as

$$\hat{E}^\alpha_L(x)\hat{E}^\alpha_L(x) = \left[\frac{\mathcal{N}_l(x) + (1-\mathcal{N}_i(x))\mathcal{N}_o(x)}{2}\right] \times \left[\frac{\mathcal{N}_l(x) + (1-\mathcal{N}_i(x))\mathcal{N}_o(x)}{2} + 1\right]$$

$$\hat{E}^\alpha_R(x)\hat{E}^\alpha_R(x) = \left[\frac{\mathcal{N}_l(x) + (1-\mathcal{N}_o(x))\mathcal{N}_i(x)}{2}\right] \times \left[\frac{\mathcal{N}_l(x) + (1-\mathcal{N}_o(x))\mathcal{N}_i(x)}{2} + 1\right]$$

The Hamiltonian

$\hat{H}_M = H_m + H_I$

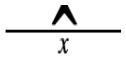
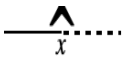
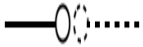
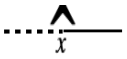
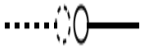
$$H_m = m \sum_x (-)^x (\mathcal{N}_i(x) + \mathcal{N}_o(x))$$

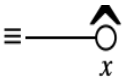
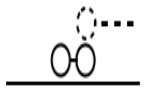
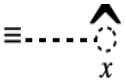
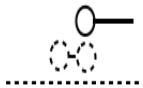
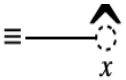
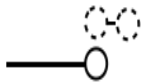
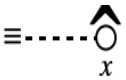
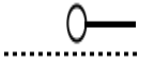
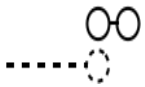
and

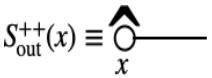
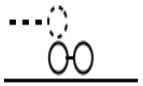
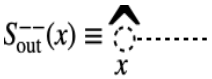
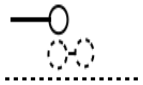
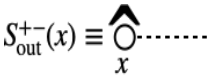
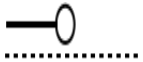
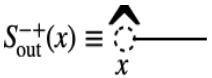
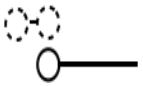
$$H_I = \hat{\psi}^\dagger(x) \hat{U}(x,i) \hat{\psi}(x+e_i) \quad \leftrightarrow \quad \frac{1}{\sqrt{\mathcal{N}_a(x)+1}} \sum_{\sigma=\pm} \sigma \mathcal{S}^{+,\sigma}_{\text{out}}(x) \mathcal{S}^{\sigma,-}_{\text{in}}(x+e_i) \frac{1}{\sqrt{\mathcal{N}_b(x+e_i)+1}}$$

Gauss Law at site x: Equivalent to U(1) theory

$$n_a(x) - n_b(x) = n_o(x) - n_i(x)$$

operator	action (a)	action (b)
$\hat{L}^{++}(x) \equiv$ 		
$\hat{L}^{--}(x) \equiv$ 		
$\hat{L}^{+-}(x) \equiv$ 		
$\hat{L}^{-+}(x) \equiv$ 		

operator	action (a)	action (b)
$S_{\text{in}}^{++}(x) \equiv$ 		
$S_{\text{in}}^{--}(x) \equiv$ 		
$S_{\text{in}}^{+-}(x) \equiv$ 		
$S_{\text{in}}^{-+}(x) \equiv$ 		

operator	action (a)	action (b)
$S^{++}_{\text{out}}(x) \equiv$ 		
$S^{--}_{\text{out}}(x) \equiv$ 		
$S^{+-}_{\text{out}}(x) \equiv$ 		
$S^{-+}_{\text{out}}(x) \equiv$ 		

$$
\begin{aligned}
\mathcal{L}^{++} &= \Lambda^{+}\sqrt{\mathcal{N}_l(\mathcal{N}_l+1)+(\mathcal{N}_i\oplus\mathcal{N}_o)+2} \\
\mathcal{L}^{--} &= \Lambda^{-}\sqrt{\mathcal{N}_l(\mathcal{N}_l-1)+(\mathcal{N}_i\oplus\mathcal{N}_o)+2} \\
\mathcal{L}^{+-} &= \chi_i^{\dagger}\,\chi_o \\
\mathcal{L}^{-+} &= -\chi_i\,\chi_o^{\dagger} \\
\mathcal{S}_{\text{in}}^{++} &= \chi_i^{\dagger}\,(\Lambda^{+})^{\mathcal{N}_o}\quad\sqrt{\mathcal{N}_l+2-\mathcal{N}_o} \\
\mathcal{S}_{\text{in}}^{--} &= \chi_i\,(\Lambda^{-})^{\mathcal{N}_o}\quad\sqrt{\mathcal{N}_l+2(1-\mathcal{N}_o)} \\
\mathcal{S}_{\text{out}}^{++} &= \chi_o^{\dagger}\,(\Lambda^{+})^{\mathcal{N}_i}\quad\sqrt{\mathcal{N}_l+2-\mathcal{N}_i} \\
\mathcal{S}_{\text{out}}^{--} &= \chi_o\,(\Lambda^{-})^{\mathcal{N}_i}\quad\sqrt{\mathcal{N}_l+2(1-\mathcal{N}_i)} \\
\mathcal{S}_{\text{in}}^{-+} &= -\chi_o^{\dagger}\,(\Lambda^{-})^{1-\mathcal{N}_i}\sqrt{\mathcal{N}_l+2\mathcal{N}_i} \\
\mathcal{S}_{\text{in}}^{+-} &= -\chi_o\,(\Lambda^{+})^{1-\mathcal{N}_i}\sqrt{\mathcal{N}_l+1+\mathcal{N}_i} \\
\mathcal{S}_{\text{out}}^{+-} &= \chi_i^{\dagger}\,(\Lambda^{-})^{1-\mathcal{N}_o}\sqrt{\mathcal{N}_l+2\mathcal{N}_o} \\
\mathcal{S}_{\text{out}}^{-+} &= \chi_i\,(\Lambda^{+})^{1-\mathcal{N}_o}\sqrt{\mathcal{N}_l+1+\mathcal{N}_o} \\
\mathcal{B}^{++} &= \chi_i^{\dagger}\chi_o^{\dagger}
\end{aligned}
$$

Global Loop-string Hilbert space on 1D lattice: local loop-string Hilbert space + Abelian Gauss Law

Abelian Gauss law

$$n_l(x) + n_o(x)[1 - n_i(x)] = n_l(x+1) + n_i(x+1)[1 - n_o(x+1)]$$

or,

$$n_b(x) = n_a(x+1)$$

Wigner-Jordan transform in one dimension

- We have expressed physical matter degrees of freedom in terms of the excitations of fermionic modes $\chi_i(x)$, $\chi_o(x)$ for $x = 0, \ldots, L_x - 1$.

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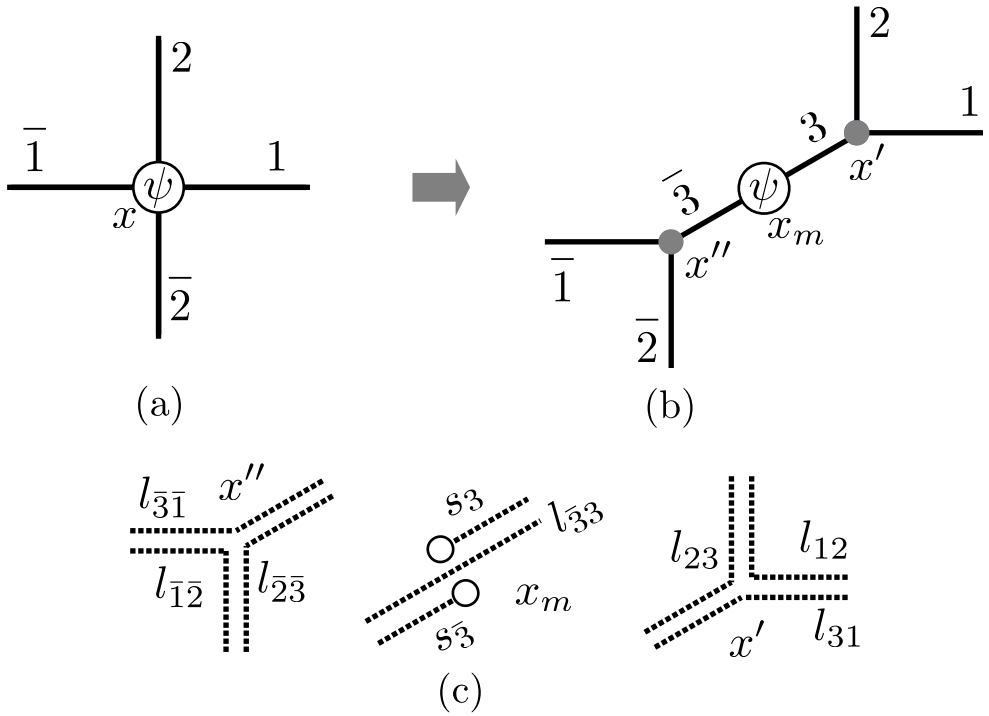
- Assuming open boundary conditions, all fermionic couplings are nearest-neighbor.
- All couplings can be of the form $\sigma_k^{\pm}\sigma_{k+1}^{\mp}$:

$$\chi_i^\dagger(x)\chi_i(x+1) \to -\sigma_x^-\sigma_{x+1}^+ \quad , \quad \chi_o^\dagger(x)\chi_o(x+1) \quad \to -\sigma_{L_x+x}^-\sigma_{L_x+x+}^+$$

Inclusion of Matter in 2d

Matter as in 1D + pure gluonic vertices



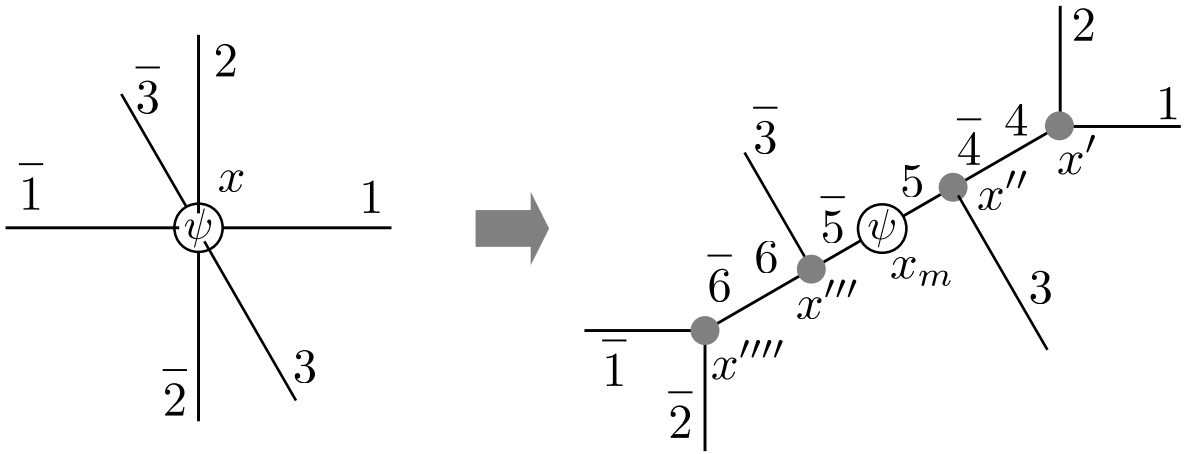
(i) The two sites x', x'' have only loop states $|l_{12}, l_{23}, l_{31}\rangle_{x'/x''}$, being treated identically as in pure gauge theory.
(ii) The third virtual site x_m along the $3-\bar{3}$ direction contains both local loop and string states $|l_{3\bar{3}}, s_3, s_{\bar{3}}\rangle$, being structurally identical to a site with matter in 1D.
(iii) The Abelian Gauss laws along the three directions of the hexagonal lattice are

$$n_1(x) = n_{\bar{1}}(x + e_1),\ n_2(x) = n_{\bar{2}}(x + e_2)\ , \tag{21}$$

$$n_3(x) + s_{\bar{3}} = n_{\bar{3}}(x + e_3) + s_3\ , \tag{22}$$

3d lattice with matter

Matter as in 1D + pure gluonic vertices



The modified Abelian Gauss laws on the 3D lattice are

$$n_i(x) = n_{\bar{i}}(x + e_i) , \qquad (i = 1, 2, 3, 4, 6) \tag{23}$$

$$n_5(x) + s_{\bar{5}} = n_{\bar{5}}(x + e_5) + s_5 . \tag{24}$$

Operator Factorization at pure Gluonic vertices

$$
\begin{aligned}
\mathcal{L}_{ij}^{++} &= \hat{\Lambda}_{ij}^{+}\sqrt{(\mathcal{N}_{ij}+1)(\mathcal{N}_{t}+1)} \\
\mathcal{L}_{ij}^{--} &= \hat{\Lambda}_{ij}^{-}\sqrt{\mathcal{N}_{ij}\mathcal{N}_{t}} \\
\mathcal{L}_{ij}^{+-} &= -\hat{\Lambda}_{ki}^{+}\hat{\Lambda}_{jk}^{-}\sqrt{(\mathcal{N}_{ki}+1)\mathcal{N}_{jk}} \\
\mathcal{L}_{ij}^{-+} &= -\hat{\Lambda}_{ki}^{-}\hat{\Lambda}_{jk}^{+}\sqrt{\mathcal{N}_{ki}(\mathcal{N}_{jk}+1)} \\
ijk &= 123,\ 231,\ \text{or}\ 312
\end{aligned}
$$

Advantages of using this framework

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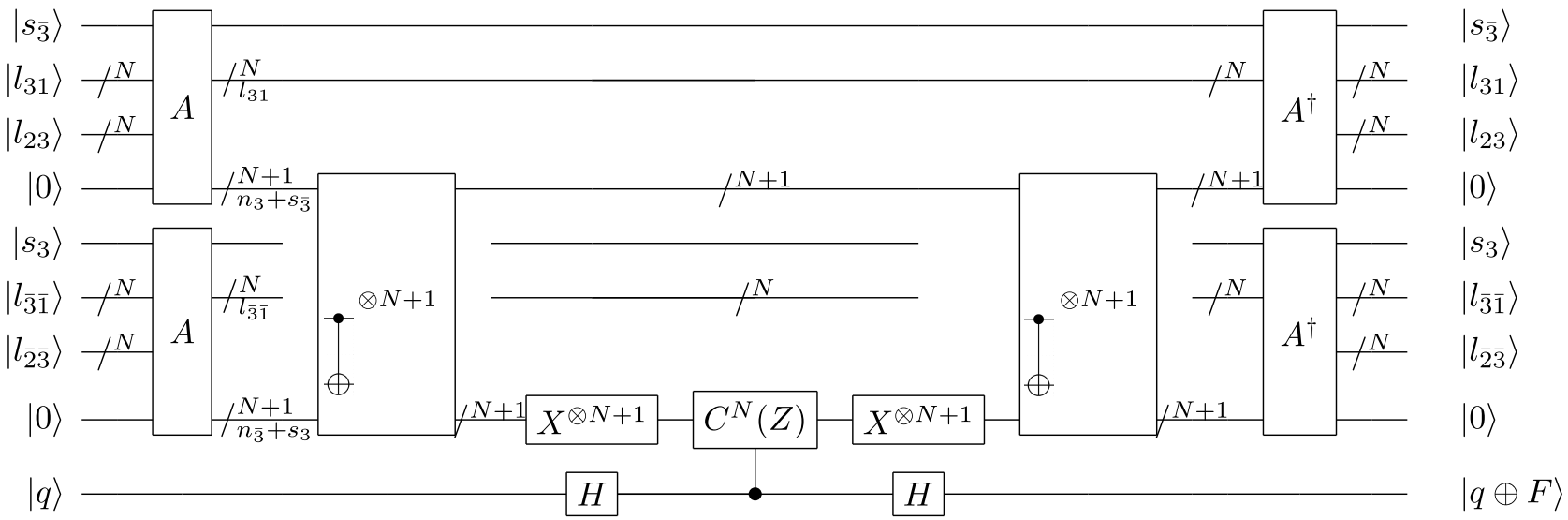
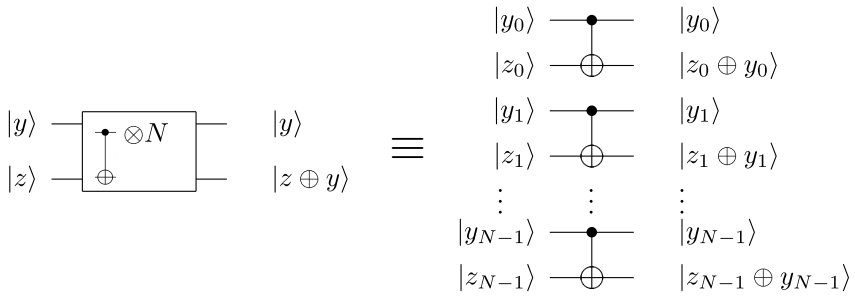
- This formalism is completely geometric and free from using Clebsch Gordon coefficients specific to SU(2), and hence is generalizable to SU(3).
- Constructing quantum simulator for QCD may not be so far.

Completed/ongoing projects

Necassary tool for state preparation: *IR, Stryker'18*

- We construct an oracle for checking the Abelian Gauss law constraints along a link.
- The same circuit can actually be used for all possible links in any dimension.
- These routines are likely to be useful in digital simulations because non-gauge invariant errors can easily arise from the Trotter approximation to $e^{-it\hat{H}}$ or from quantum noise.


$|s_{\bar{3}}\rangle$
$|l_{31}\rangle$
$|l_{23}\rangle$
$|0\rangle$
$|s_3\rangle$
$|l_{\bar{3}\bar{1}}\rangle$
$|l_{\bar{2}\bar{3}}\rangle$
$|0\rangle$
$|q\rangle$
A
A
$\otimes N+1$
$X^{\otimes N+1}$
$C^N(Z)$
$X^{\otimes N+1}$
H
H
$A^\dagger$
$A^\dagger$
$\otimes N+1$
$|s_{\bar{3}}\rangle$
$|l_{31}\rangle$
$|l_{23}\rangle$
$|0\rangle$
$|s_3\rangle$
$|l_{\bar{3}\bar{1}}\rangle$
$|l_{\bar{2}\bar{3}}\rangle$
$|0\rangle$
$|q \oplus F\rangle$


$|y\rangle$
$|z\rangle$
$\otimes N$
$|y\rangle$
$|z \oplus y\rangle$
$\equiv$
$|y_0\rangle$ $|y_0\rangle$
$|z_0\rangle$ $|z_0 \oplus y_0\rangle$
$|y_1\rangle$ $|y_1\rangle$
$|z_1\rangle$ $|z_1 \oplus y_1\rangle$
$|y_{N-1}\rangle$ $|y_{N-1}\rangle$
$|z_{N-1}\rangle$ $|z_{N-1} \oplus y_{N-1}\rangle$

- An analogous construction using the conventional group representation states is much less straightforward because different components of the non-Abelian Gauss law operator are not simultaneously diagonalizable.

- An analogous construction using the conventional group representation states is much less straightforward because different components of the non-Abelian Gauss law operator are not simultaneously diagonalizable.
- The present SU(2) physicality oracle valid in any dimension is actually simpler and cheaper than the Abelian Gauss Law Oracle (Stryker'18) for 3 (or more) dimenions.

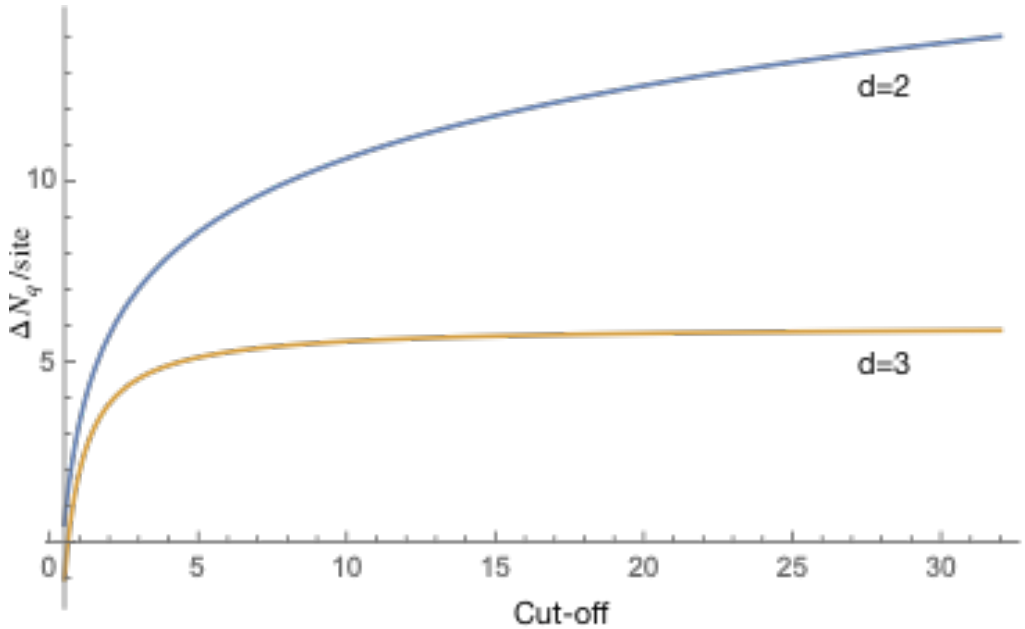
Trotterization of SU(2) Hamiltonian: Work in progress

- Utilizes the loop-string operators and factorization into normalized ladder operators discussed before.
- Utilizes the trotterization technique for Schwinger model Hamiltonian developed in INT-ORNL collaboration, that is yet to be communicated.

- In 1 spatial dimension, the loop-string-hadron model is mapped directly to a spin system and is free from any cut-off dependence with open boundary condition. Work is in progress in this direction.
- Comparative study of resource requirement for different Hamiltonian frameworks available in literature is under progress.

KS to LSB: Gain in qubits

Before imposing Abelian Gauss law



$\Delta N_q =$ no. of qubits required in (KS $-$ LSB)

Summary: some general features of this formalism

- non-Abelian gauge redundancy is absent: attain a higher cutoff on the physical Hilbert space than when working with all the redundant gauge degrees of freedom with same number of qubits.
- Abelian Gauss law constraints are checked using the physicality oracles for SU(2), in any dimension.
- Dynamics within infinite towers of states rather than multiplets of varying dimensions: natural truncation scheme $l_{ij} = 2^{n_q}$.
- Operating on towers of states more closely resembles U(1) gauge theory, so it is conceivable that other algorithms developed for Abelian theories can also be ported over to SU(2).

Summary: some general features of this formalism

Drawbacks:

- point splitting technique increases the number of links to be simulated and that plaquette operators must deal with more links.

Hope, that this drawback is outweighed by the simpler action of individual link operators in a plaquette.
Our construction nonetheless stands to more directly benefit from any progress made in algorithms for implementing U(1) plaquette operators.

THANK YOU