The ultimate free lunch:
the LF vacuum

Matthias Burkardt

New Mexico State University

February 17, 2020

12 GeV Upgrade
Future Science at Jefferson Lab

Search the JLab Site

Project Information | Scientific Collaboration | Calendar | Public Interest

print versi

- Science Opportunities
- Technical Scope
  - Accelerator
  - Physics Experimental Equipment
  - Civil Construction
- Status
- 12 GeV Updates
- Org Chart
  - IPT
  - Project Organization
- Public Documents
- Adopt-A-Spot

12 GeV Collaboration

- Hall A
- Hall B
- Hall C
- Hall D

Unique Science Opportunities of the 12 GeV Upgrade

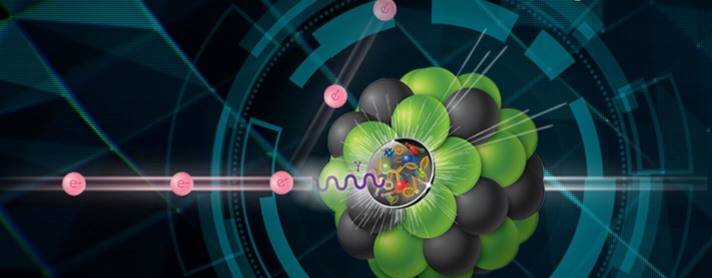
The 12 GeV Upgrade is a unique opportunity for the nuclear physics community to expand its reaches into unknown scientific areas. For the first time, researchers will be able to probe the quark and gluon structure of strongly interacting systems to determine whether QCD (quantum chromodynamics), the theory believed to describe strong interactions, gives a full and complete description of hadronic (3 quark) systems. Jefferson Lab at 12 GeV will make profound contributions to the study of hadronic matter-the matter that makes up everything in the world.

In particular the 12 GeV research program will allow breakthroughs to be launched in five main areas:

- Through the search for exotic mesons, in which gluons are an unavoidable part of the structure, researchers will explore the fascinating and complex vacuum structure of QCD and the nature of confinement.
- Through extremely high precision studies of parity violation, developed in order to study the role of hidden flavors in the nucleon, researchers can explore physics beyond the Standard Model, on an energy scale that cannot be explored ever with the proposed International Linear Collider.
- The combination of luminosity, duty factor and kinematic reach of this machine will far surpass anything available up to th point, allowing the nuclear physics community a previously impossible view of the spin and flavor dependence of the valence parton distributions - the heart of the proton, where its quantum numbers are determined.
- Researchers will be able to take a revolutionary look into the structure of atomic nuclei, exploring how the valence quark structure is modified in a dense nuclear medium. These studies will give the world a far deeper and more fundamental understanding of the structure of atomic nuclei with far-reaching implications for all of nuclear physics and nuclear astrophysics.
- The Generalized Parton Distributions will allow researchers, for the first time, to engage in nuclear tomography, discovering the true three-dimensional structure of the nucleon.

The Electron-Ion Collider

A machine that will unlock the secrets of the strongest force in Nature



The computers and smartphones we use every day depend on what we learned about the atom in the last century. All information technology—and much of our economy today—relies on understanding the electromagnetic force between the atomic nucleus and the electrons that orbit it. The science of that force is well understood but we still know little about the microcosm within the protons and neutrons that make up the atomic nucleus. That's why Brookhaven Lab is building a new machine—an Electron-Ion Collider, or EIC—to look *inside* the nucleus and its protons and neutrons.

The EIC will be a particle accelerator that collides electrons with protons and nuclei to produce snapshots of those particles' internal structure—like a CT scanner for atoms. The electron beam will reveal the arrangement of the quarks and gluons that make up the protons and neutrons of nuclei. The force that holds quarks together, carried by the gluons, is the strongest force in Nature. The EIC will allow us to study this "strong nuclear force" and the role of gluons in the matter within and all around us.

represents the first fruit of more than a decade of effort in this direction.

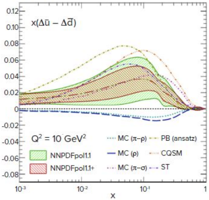


Figure 2.4: The difference between the $\Delta\bar{u}$ and $\Delta\bar{d}$ spin functions as extracted from the NNPDF global analysis. The green (red) band shows the present (final expected) uncertainties from analysis of the RHIC W data set. Various model calculations are also shown.

A Multidimensional View of Nucleon Structure

"With 3D projection, we will be entering a new age. Something which was never technically possible before: a stunning visual experience which 'turbocharges' the viewing." This quotation from film director J. Cameron could just as well describe developments over the last decade or so in hadron physics, in which a multidimensional description of nucleon structure is emerging that is providing profound new insights. Form factors tell us about the distribution of charge and magnetization but contain no direct dynamical information. PDFs allow us to access information on the underlying quarks and their longitudinal momentum but tell us nothing about spatial locations. It has now been established, however, that both form factors and PDFs are special cases of a more general class of distribution functions that merge spatial and dynamic information. Through appropriate measurements, it is becoming possible to construct "pictures" of the nucleon that were never before possible.

3D Spatial Maps of the Nucleon: GPDs

Some of the important new tools for describing hadrons are Generalized Parton Distributions (GPDs). GPDs can be investigated through the analysis of *hard exclusive processes*, processes where the target is probed by high-energy particles and is left intact beyond the production of one or two additional particles. Two processes are recognized as the most powerful processes for accessing GPDs: deeply virtual Compton scattering (DVCS) and deeply virtual meson production (DVMP) where a photon or a meson, respectively, is produced.

One striking way to use GPDs to enhance our understanding of hadronic structure is to use them to construct what we might call 3D spatial maps (see Sidebar 2.2). For a particular value of the momentum fraction *x*, we can construct a spatial map of where the quarks reside. With the JLab 12-GeV Upgrade, the valence quarks will be accurately mapped.

GPDs can also be used to evaluate the total angular momentum associated with different types of quarks, using what is known as the Ji Sum Rule. By combining with other existing data, one can directly access *quark orbital angular momentum*. The worldwide DVCS experimental program, including that at Jefferson Lab with a 6-GeV electron beam and at HERMES with 27-GeV electron and positron beams, has already provided constraints (albeit model dependent) on the total angular momentum of the u and d quarks. These constraints can also be compared with calculations from LQCD. Upcoming 12-GeV experiments at JLab and COMPASS-II experiments at CERN will provide dramatically improved precision. A suite of DVCS and DVMP experiments is planned in Hall B with CLAS12; in Hall A with HRS and existing calorimeters; and in Hall C with HMS, the new SHMS, and the Neutral Particle Spectrometer (NPS). These new data will transform the current picture of hadronic structure.

3D Momentum Maps of the Nucleon: TMDs

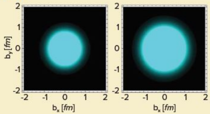
Other important new tools for describing nucleon structure are transverse momentum dependent distribution functions (TMDs). These contain information on both the longitudinal and transverse momentum of the quarks (and gluons) inside a fast moving nucleon. TMDs link the transverse motion of the quarks with their spin and/or the spin of the parent proton and are, thus, sensitive to orbital angular momentum. Experimentally, these functions can be investigated in proton-proton collisions, in inclusive production of lepton pairs in Drell-Yan processes, and in *semi-inclusive deep inelastic scattering* (SIDIS), where one measures the scattered electron and one more meson (typically a pion or kaon) in the DIS process.

Sidebar 2.2: The First 3D Pictures of the Nucleon

A computed tomography (CT) scan can help physicians pinpoint minute cancer tumors, diagnose tiny broken bones, and spot the early signs of osteoporosis. Now physicists are using the principles behind the procedure to peer at the inner workings of the proton. This breakthrough is made possible by a relatively new concept in nuclear physics called generalized parton distributions.

An intense beam of high-energy electrons can be used as a microscope to look inside the proton. The high energies tend to disrupt the proton, so one or more new particles are produced. Physicists often disregarded what happened to the debris and measured only the energy and position of the scattered electron. This method is called inclusive deep inelastic scattering and has revealed the most basic grains of matter, the quarks. However, it has a limitation: it can only give a one-dimensional image of the substructure of the proton because it essentially measures the momentum of the quarks along the direction of the incident electron beam. To provide the three-dimensional (3D) picture, we need instead to measure all the particles in the debris. This way, we can construct a 3D image of the proton as successive spatial quark distributions in planes perpendicular to its motion for slices in the quark's momentum, just like a 3D image of the human body can be built from successive planar views.

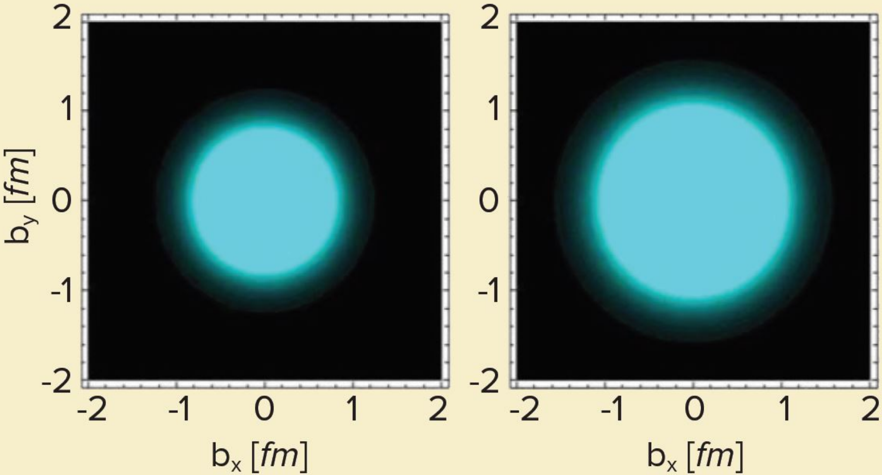
An electron can scatter from a proton in many ways. We are interested in those collisions where a high-energy electron strikes an individual quark inside the proton, giving the quark a very large amount of extra energy. This quark then quickly gets rid of its excess energy, for instance, by emitting a high-energy photon. The quark does not change identity and remains part of the intact target proton. This specific process is called deeply virtual Compton scattering (DVCS). For the experiment to work, the scientists need to measure the speed, position, and energy of the electron that bounced off the quark, of the photon emitted by the quark, and of the reassembled proton. From this information the 3D picture of the proton can be constructed.



The first 3D views of the proton: the spatial charge densities of the proton in a plane (bx, by) positioned at two different values of the quark's longitudinal momentum x: 0.25 (left) and 0.09 (right).

Very recently, using the DVCS data collected with the CLAS detector at JLab and the HERMES detector at DESY/Germany, the first nearly model-independent images of the proton started to appear. The result of this work is illustrated in the figure, where the probabilities for the quarks to reside at various places inside the proton are shown at two different values of its longitudinal momentum x (x = 0.25 left and x = 0.09 right). This is analogous to the "orbital" clouds used to depict the likely position of electrons in various energy levels inside atoms. The first 3D pictures of the proton indicate that when the longitudinal momentum x of the quark decreases, the radius of the proton increases.

The broader implications of these results are that we now have methods to fill in the information needed to extract 3D views of the proton. Physicists worldwide are working toward this goal, and the technique pioneered here will be applied with Jefferson Lab's CEBAF accelerator at 12 GeV for (valence) quarks and, later, with a future EIC for gluons and sea quarks.



The first 3D views of the proton: the spatial charge densities of the proton in a plane (bx, by) positioned at two different values of the quark's longitudinal momentum x: 0.25 (left) and 0.09 (right).

Light Front Wave Functions

LF wave functions of hadrons

- $\psi(x, \mathbf{k}_\perp)$, or $\psi(x_1, x_2, .., \mathbf{k}_{1\perp}, \mathbf{k}_{2\perp})$ with x_i momentum fraction
- defined in terms of light front (LF) correlations
- think of wave function after boost to ∞ momentum frame (IMF)

Galilean subgroup of transverse boosts

- in general, boosts complicated in relativistic theories
- Galilean subgroup of transverse boosts in LF framework

$\hookrightarrow$ many similarities to nonrelativistic wave functions!

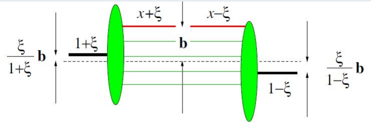
applications

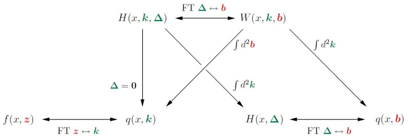
- PDFs: squares of LF wave functions (integrated over redundant momenta)
- form factors: overlap integrals of LF wave functions
- generalized parton distributions (GPDs): overlap integrals of LF wave functions
- transverse momentum dep PDFs (TMDs): squares of LF wave functions (integrated over redundant momenta); addtl. phases!
- Wigner functions: combination of the above

transverse imaging

- deeeply virtual Compton scattering (DVCS) $\rightsquigarrow$ GPDs

$$\frac{\sqrt{1-\zeta}}{1-\frac{\zeta}{2}} H_{(n\to n)}(x,\zeta,t) - \frac{\zeta^2}{4(1-\frac{\zeta}{2})\sqrt{1-\zeta}} E_{(n\to n)}(x,\zeta,t)$$
$$= \sqrt{1-\zeta}^{\,2-n} \sum_{n,\lambda_i} \int \prod_{i=1}^{n} \frac{\mathrm{d}x_i\, \mathrm{d}^2\vec{k}_{\perp i}}{16\pi^3}\, 16\pi^3 \delta\left(1-\sum_{j=1}^{n} x_j\right) \delta^{(2)}\left(\sum_{j=1}^{n} \vec{k}_{\perp j}\right)$$
$$\times\, \delta(x-x_1)\, \psi^{\uparrow\,*}_{(n)}(x'_i, \vec{k}'_{\perp i}, \lambda_i)\, \psi^{\uparrow}_{(n)}(x_i, \vec{k}_{\perp i}, \lambda_i)\ ,$$





Impact parameter space interpretation MB, Phys. Rev. D **62**, 071503 (2000)

- $GPDs(x,0,\Delta_\perp) \stackrel{2dFT}{\longrightarrow} q(x\mathbf{b}_\perp)$
- distribution of quarks relative to center of momentum of nucleon
- probabilistic interpretation!
- possible due to Galilean subgroup of $\perp$ boosts!
- $\perp$ spin effects $\rightsquigarrow \perp$ SSAs MB, Int. J. Mod. Phys. A **18**, 173 (2003)

LF quantization

- take LF Hamiltonian (x^+ evolution - LF time)

$\hookrightarrow$ eigenfunctions = LF wave functions

issues with LF quantization

- renormalization
- gauge invariance
- vacuum

Large momentum effective theory

- lattice gauge theory
- boosting to ∞ momentum
- there are issues with boosting to ∞ momentum ...

- P^+ conservation & P^+ purely kinematical

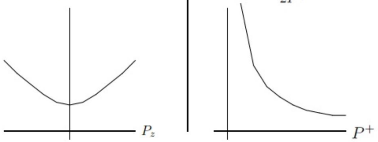
↪ 'empty' or 'trivial' vacuum exact eigenstate of LF Hamiltonian

- nondegenerate state of lowest P^+

↪ also nondegenerate state of lowest P^-

↪ exact ground state of theory

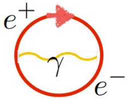
normal coordinates	light-front
free theory	
$P^0 = \sqrt{m^2 + \vec{P}^2}$	$P^- = \frac{m^2 + \vec{P}_\perp^2}{2P^+}$



normal coordinates	light-front
$P^0 = \sum_{\vec{k}} a_{\vec{k}}^\dagger a_{\vec{k}} \sqrt{m^2 + \vec{k}^2}$	$P^- = \sum_{k^+, \vec{k}_\perp} a_{k^+,\vec{k}_\perp}^\dagger a_{k^+,\vec{k}_\perp} \frac{m^2 + \vec{k}_\perp^2}{2k^+}$
vacuum (free theory)	
$a_{\vec{k}}\vert 0\rangle = 0$	$a_{k^+,\vec{k}_\perp}\vert 0\rangle = 0$
vacuum (interacting theory)	
many states with $\vec{P} = 0$ (e.g. $a_{\vec{k}}^\dagger a_{-\vec{k}}^\dagger \vert 0\rangle$)	$k^+ \geq 0$ ↪ only pure zero-mode excitations have $P^+ = 0$
↪ $\vert\tilde{0}>$ very complex	↪ $\vert\tilde{0}>$ can only contain zero-mode excitations

Stan Brodsky:

Instant-Form Vacuum in QED

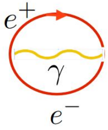


- Loop diagrams of all orders contribute
- Huge vacuum energy: $\rho_\Lambda^{QED} \simeq 10^{120} \rho_\Lambda^{Observed}$
- $\frac{E}{V} = \int \frac{d^3k}{2(2\pi)^3} \sqrt{\vec{k}^2 + m^2}$ Cut off the quadratic divergence at M_{Planck}
- Frame-dependent, acausal
- Divide S-matrix by disconnected vacuum diagrams
- In Contrast: Light-Front Vacuum trivial since plus momenta are positive and conserved: $k^+ = k^0 + k^3 > 0$

Stan Brodsky:

Front-Form Vacuum in QED

$P^+ = 0$



$\sum_i k_i^+ \neq P^+ = 0$

$k_i^+ > 0$

- **All Light-Front Perturbative Vacuum Loop Amplitudes Vanish!**
- **Light-Front Vacuum is trivial since all plus momenta are positive and conserved.**
- **Zero modes (k^+=0) in vacuum allowed in some theories**
- **Zero contribution to Λ from QED LF Vacuum**
- **Instant Form gives same result if one normal-orders.**

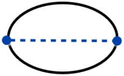
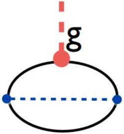
Stan Brodsky:

Front-Form Vacuum (P^μ=0)

All LF propagators have positive k⁺

$$k^+ = k^0 + k^3 \geq 0 \text{ since } |\vec{k}| \leq k^0$$

P⁺ Momentum Conserved

$P^+ = 0$  ***zero !!*** 

zero !!

$$< 0|T^{\mu\nu}|0 >= 0$$

Graviton does not couple to LF vacuum!

- P^+ conservation & P^+ purely kinematical

$\hookrightarrow$ 'empty' or 'trivial' vacuum exact eigenstate of LF Hamiltonian

- nondegenerate state of lowest P^+

$\hookrightarrow$ also nondegenerate state of lowest P^-

$\hookrightarrow$ exact ground state of theory

issues with this result

- Higgs mechanism
- QCD vacuum:
  - lattice: $\langle 0|\bar{q}q|0\rangle \neq 0$
  - Gell-Mann, Oakes, Renner $f_\pi^2 m_\pi^2 = (m_u + m_d)\langle 0|\bar{q}q|0\rangle \neq 0$

possible resolutions

- $\langle 0|\bar{q}q|0\rangle \neq 0$ fake news!

$\hookrightarrow$ GOR made it up!

- P^+ conservation & P^+ purely kinematical

↪ 'empty' or 'trivial' vacuum exact eigenstate of LF Hamiltonian

- nondegenerate state of lowest P^+

↪ also nondegenerate state of lowest P^-

↪ exact ground state of theory

issues with this result

- Higgs mechanism
- QCD vacuum:
  - lattice: $\langle 0|\bar{q}q|0\rangle \neq 0$
  - Gell-Mann, Oakes, Renner $f_\pi^2 m_\pi^2 = (m_u + m_d)\langle 0|\bar{q}q|0\rangle \neq 0$

possible resolutions

- $\langle 0|\bar{q}q|0\rangle \neq 0$ fake news!

↪ GOR made it up!

- or: LF formalism is fake!

↪ Dirac (& 'do-nothing coneheads') made it up!

- P^+ conservation & P^+ purely kinematical

$\hookrightarrow$ 'empty' or 'trivial' vacuum exact eigenstate of LF Hamiltonian

- nondegenerate state of lowest P^+

$\hookrightarrow$ also nondegenerate state of lowest P^-

$\hookrightarrow$ exact ground state of theory

issues with this result

- Higgs mechanism
- QCD vacuum:
  - lattice: $\langle 0|\bar{q}q|0\rangle \neq 0$
  - Gell-Mann, Oakes, Renner $f_\pi^2 m_\pi^2 = (m_u + m_d)\langle 0|\bar{q}q|0\rangle \neq 0$

possible resolutions

- $\langle 0|\bar{q}q|0\rangle \neq 0$ fake news!

$\hookrightarrow$ GOR made it up!

- or: LF formalism is fake!

$\hookrightarrow$ Dirac (& 'do-nothing coneheads') made it up!

- maybe there is a 3^{rd} option ...

SSB in $1+1$ dimensions?

- no spontaneous symmetry breaking (SSB) in 1+1 (S.Coleman)
- however not valid for $N_C \longrightarrow \infty$ as Hartree-Fock approx. becomes exact

$\hookrightarrow$ SSB possible

't Hooft model

- $QCD_{1+1}(N_C \longrightarrow \infty)$
- LF quantization & gauge

$$M_n^2 \phi_n(x) = \left(\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} \right) \phi_n(x) + \frac{g^2 C_F}{\pi} \int_0^1 dy \frac{\phi_n(x) - \phi_n(y)}{(x-y)^2}$$

- M^2 meson mass; x $(1-x)$ momentum fraction carried by q $(\bar{q})$
- trivial vacuum, lowest Fock sector for meson exact as $N_C \to \infty$
- infinite 'tower' of solutions
- lowest meson state $M_\pi^2 \propto m_q$

$\hookrightarrow$ hint that $\langle 0|\bar{q}q|0\rangle \neq 0$

- meson spectrum confirmed by Li, Wilets, Birse in ET/BS (1986)

't Hooft model: $QCD_{1+1}(N_C \longrightarrow \infty)$

$$M_n^2 \phi_n(x) = \left(\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} \right) \phi_n(x) + \frac{g^2 C_F}{\pi} \int_0^1 dy \frac{\phi_n(x) - \phi_n(y)}{(x-y)^2}$$

- lowest meson state $M_\pi^2 \propto m_q$

$\hookrightarrow$ hint that $\langle 0|\bar{q}q|0\rangle \neq 0$

Zhitnitzky PLB 165B (1985) 405, Sov.JNP 43, 999; 44, 139 (1984)

- GMOR: $\lim_{m_q \to 0} \langle 0|\bar{q}q|0\rangle = -\frac{N_C}{\sqrt{12}} \sqrt{\frac{g^2 C_F}{\pi}}$
- confirmed by ET calculation: M. Li, PRD34 (1986) 3888
- nonperturbative analytic expression for $\langle 0|\bar{q}q|0\rangle$ valid for all m_q: MB&N.Uraltsev, PRD 63 (2001) 014004

free lunch?

- Solving LF wave functions from diagonalizing LF Hamiltonian based on trivial vacuum yields same rusults (incl. condensate numbers - using GMOR) as complicated ET calculation!!!!!

't Hooft model: $QCD_{1+1}(N_C \longrightarrow \infty)$

$$M_n^2 \phi_n(x) = \left(\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} \right) \phi_n(x) + \frac{g^2 C_F}{\pi} \int_0^1 dy \frac{\phi_n(x) - \phi_n(y)}{(x-y)^2}$$

- lowest meson state $M_\pi^2 \propto m_q$

$\hookrightarrow$ hint that $\langle 0|\bar{q}q|0\rangle \neq 0$

Zhitnitzky PLB 165B (1985) 405, Sov.JNP 43, 999; 44, 139 (1984)

- GMOR: $\lim_{m_q \to 0} \langle 0|\bar{q}q|0\rangle = -\frac{N_C}{\sqrt{12}} \sqrt{\frac{g^2 C_F}{\pi}}$
- confirmed by ET calculation: M. Li, PRD34 (1986) 3888
- nonperturbative analytic expression for $\langle 0|\bar{q}q|0\rangle$ valid for all m_q: MB&N.Uraltsev, PRD 63 (2001) 014004

free lunch?

- Solving LF wave functions from diagonalizing LF Hamiltonian based on trivial vacuum yields same rusults (incl. condensate numbers - using GMOR) as complicated ET calculation!!!!!
- Does that mean the vacuum is trivial or that it is not trivial?!?

explicit LF calculations MB, F.Lenz, M.Thies, PRD 65 (2002) 125002

- vacuum condensate $\langle 0|\bar{q}(0)q(0)|0\rangle$ ill-defined
- employ point-splitting in LF time x^+, i.e. $\langle 0|\bar{q}(0)q(0)|0\rangle \longrightarrow \langle 0|\bar{q}(0)Wq(\varepsilon)|0\rangle$ with $\varepsilon^2 \neq 0 \quad \Rightarrow \quad \varepsilon^+ \neq 0$
- W Wilson line gauge link
- same as heavy-light correlator: for straight Wilson line, W represents a 'static' heavy quark

$\hookrightarrow$ relate $\langle 0|\bar{q}(\varepsilon)Wq(0)|0\rangle$ to properties of heavy-light mesons (calculated using LF quantization: masses, decay constants)

- reproduced $\langle 0|\bar{q}(0)q(0)|0\rangle$ from GMOR (Zhitnitsky)
- take $\varepsilon^\pm \to 0$ (subtract free-field divergence)
- condensate only from zero-modes $k^+ \longrightarrow 0$

implications for LF vacuum (QCD_{1+1} only)

- condensates (properly regularized) nonzero
- don't affect hadron structure/dynamics in QCD_{1+1}

$\hookrightarrow$ fine to pretend that vacuum is trivial (QCD_{1+1})

Implication for LF Vacuum

implications for LF vacuum QCD_{1+1}

- condensates (properly regularized) nonzero
- don't affect hadron structure/dynamics in QCD_{1+1}

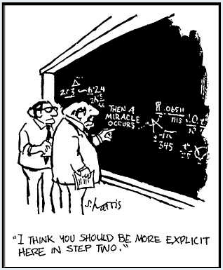
$\hookrightarrow$ fine to pretend that vacuum is trivial (QCD_{1+1})

physics behind this fairy tale:

- zero-modes high-energy degrees of freedom

$\hookrightarrow$ parton degrees not enough energy to excite zero-mode sector

$\hookrightarrow$ LF Hamiltonian works like effective Hamiltonian



"I THINK YOU SHOULD BE MORE EXPLICIT HERE IN STEP TWO."

implications for LF vacuum in general

- Is it fine to pretend that vacuum is trivial in more complicated theories, such as ϕ^n, QCD_{3+1}, ...?

implications for LF vacuum QCD_{1+1}

- condensates (properly regularized) nonzero
- don't affect hadron structure/dynamics in QCD_{1+1}

$\hookrightarrow$ fine to pretend that vacuum is trivial (QCD_{1+1})

implications for LF vacuum in general

- Is it fine to pretend that vacuum is trivial in more complicated theories, such as ϕ^n, QCD_{3+1}, ...?
- unfortunately not!

implications for LF vacuum QCD_{1+1}

- condensates (properly regularized) nonzero
- don't affect hadron structure/dynamics in QCD_{1+1}

$\hookrightarrow$ fine to pretend that vacuum is trivial (QCD_{1+1})

implications for LF vacuum in general

- Is it fine to pretend that vacuum is trivial in more complicated theories, such as ϕ^n, QCD_{3+1}, ...?
- unfortunately not!
- can this be 'fixed'?

implications for LF vacuum QCD_{1+1}

- condensates (properly regularized) <u>nonzero</u>
- don't affect hadron structure/dynamics in QCD_{1+1}

$\hookrightarrow$ fine to pretend that vacuum is trivial (QCD_{1+1})

implications for LF vacuum in general

- Is it fine to pretend that vacuum is trivial in more complicated theories, such as ϕ^n, QCD_{3+1}, ...?
- unfortunately not!
- can this be 'fixed'?
- maybe!

implications for LF vacuum QCD_{1+1}

- condensates (properly regularized) nonzero
- don't affect hadron structure/dynamics in QCD_{1+1}

$\hookrightarrow$ fine to pretend that vacuum is trivial (QCD_{1+1})

implications for LF vacuum in general

- Is it fine to pretend that vacuum is trivial in more complicated theories, such as ϕ^n, QCD_{3+1}, ...?
- unfortunately not!
- can this be 'fixed'?
- maybe!
- can it be fixed by introducing a single zero mode?

implications for LF vacuum QCD_{1+1}

- condensates (properly regularized) nonzero
- don't affect hadron structure/dynamics in QCD_{1+1}

$\hookrightarrow$ fine to pretend that vacuum is trivial (QCD_{1+1})

implications for LF vacuum in general

- Is it fine to pretend that vacuum is trivial in more complicated theories, such as ϕ^n, QCD_{3+1}, ...?
- unfortunately not!
- can this be 'fixed'?
- maybe!
- can it be fixed by introducing a single zero mode?
- no! Need ∞ many modes in infinitesimal vicinity of $k^+ = 0$

vacuum correlator

$$\Pi(p^2) = \int d^2x \; e^{ip\cdot x} \langle 0|T \tfrac{1}{2}\phi^2(x) \tfrac{1}{2}\phi^2(0)|0\rangle_{\text{conn.}}.$$

naive treatment

$$\Pi(0) = -\frac{1}{8\pi^2} \int dk^+ dk^- \frac{1}{\left[2k^+k^- - m^2 + i\varepsilon\right]^2}$$

- $k^+ > 0$ close k^- contour in upper comples plane $\longrightarrow 0$
- $k^+ < 0$ close k^- contour in lower comples plane $\longrightarrow 0$

$\hookrightarrow$ $\Pi(0) = 0$!???

correct treatment

$$\Pi(0) = -\frac{1}{8\pi^2} \int dk^+ dk^- \frac{1}{\left[2k^+k^- - m^2 + i\varepsilon\right]} \frac{1}{\left[2(k^+ - p^+)(k^- - p^-) - m^2 + i\varepsilon\right]}$$

- evaluate for $p^+ > 0$: contribution only from $0 < k^+ < p^+$

$\hookrightarrow$ representation of $\delta(k^+)$ as $p^+ \to 0$

P.Mannheim, P. Lowdon, S.J.Brodsky, PLB 797 (2019) 134916

$\langle\Omega|\phi^2|\Omega\rangle$

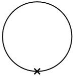
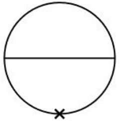


FIG. 1: Connected $\langle\Omega|\phi(0)\phi(0)|\Omega\rangle$

$$
\begin{aligned}
D(x^\mu,\text{instant}) &= \frac{1}{(2\pi)^4}\int dp_0 dp_1 dp_2 dp_3 \frac{e^{-i(p_0x^0+p_1x^1+p_2x^2+p_3x^3)}}{(p_0)^2-(p_1)^2-(p_2)^2-(p_3)^2-m^2+i\epsilon},\\
D(x^\mu,\text{front}) &= \frac{2}{(2\pi)^4}\int dp_+ dp_1 dp_2 dp_- \frac{e^{-i(p_+x^++p_1x^1+p_2x^2+p_-x^-)}}{4p_+p_- -(p_1)^2-(p_2)^2-m^2+i\epsilon},\\
D(x^\mu=0,\text{instant}) &= \frac{1}{(2\pi)^4}\int dp_0 dp_1 dp_2 dp_3 \frac{1}{(p_0)^2-(p_1)^2-(p_2)^2-(p_3)^2-m^2+i\epsilon},\\
D(x^\mu=0,\text{front}) &= \frac{2}{(2\pi)^4}\int dp_+ dp_1 dp_2 dp_- \frac{1}{4p_+p_- -(p_1)^2-(p_2)^2-m^2+i\epsilon}.
\end{aligned}
$$



- naively vanishing
- nonzero when point-splitting is applied
- issue repeats itself at higher orders

- 'The concerns raised in this paper thus carry over to dressed light front vacuum graphs as well and cannot be ignored.'
- 'Since in analog to $\langle\Omega|\phi^2|\Omega\rangle$ the light front circle at infinity contribution to $\langle\Omega|\bar{\psi}\psi|\Omega\rangle$ is nonzero, in the light front the circle at infinity contributes to the cosmological constant.'
- 'It is this circle at infinity contribution that is then paramount in the light front vacuum sector, to thus make the off-shell Feynman diagram approach with its non-zero value for light front vacuum graphs the correct one.'

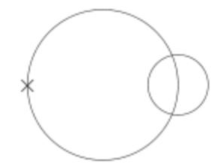
$\langle 0|\phi^2|0\rangle$

- LF: no particles popping out of vacuum ($\to$SJB)

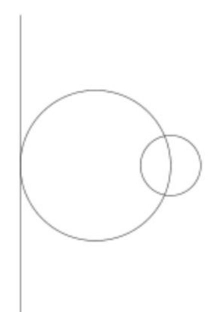
$\hookrightarrow$ LF: no contribution to $\langle 0|\phi^2|0\rangle$ beyond 1 loop

- cov. calc.: contribution to $\langle 0|\phi^2|0\rangle$ to all orders!
- discrepancy!
- relevant since corresponding tadpoles contribute to self-energy!

example for diagram that contributes to $\langle 0|\phi^2|0\rangle$, but cannot be generated by LF Hamiltonian



example for contri to self-energy, that cannot be generated by H_{LF}



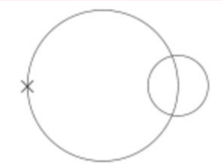
$\langle 0|\phi^2|0\rangle$

- LF: no particles popping out of vacuum (→SJB)

$\hookrightarrow$ LF: no contribution to $\langle 0|\phi^2|0\rangle$ beyond 1 loop

- cov. calc.: contribution to $\langle 0|\phi^2|0\rangle$ to all orders!
- discrepancy!
- relevant since corresponding tadpoles contribute to self-energy!

example for diagram that contributes to $\langle 0|\phi^2|0\rangle$, but cannot be generated by LF Hamiltonian



$\int dk^- \frac{\Pi(k^2)}{(k^2-m^2+i\varepsilon)^n}$

- issue arises for all integrals of above type!
- $\Pi(k^2)$ same pole structure as $\frac{1}{k^2-m^2+i\varepsilon}$

$\hookrightarrow$ $\delta(k^+)$ S.-J. Chang & S.-K. Ma, PR 180 (1969) 1506; T.-M. Yan,PRD 7 (1973) 1780

key integral (see e.g. Peskin & Schröder)

$$I_n \equiv \int \frac{d^4k}{(2\pi)^4} \frac{1}{\left(k^2 - \mathcal{M}^2 + i\varepsilon\right)^n} = \frac{c_n}{\left(\mathcal{M}^2\right)^{n-2}} \neq 0$$

- $\int dk^- \frac{1}{\left(2k^+k^- - k_\perp^2 - \mathcal{M}^2 + i\varepsilon\right)^n} = 0$ for $k^+ \neq 0$

$\hookrightarrow$ $\int dk^- \frac{1}{\left(2k^+k^- - k_\perp^2 - \mathcal{M}^2 + i\varepsilon\right)^n} \sim \delta(k^+) \frac{1}{(\mathcal{M}^2)^{n-1}}$

- pure zero-mode contribution!
- when 'going slightly away' from LF, zero-mode contribution arises from ∞ number of modes in vicinity of $k^+ = 0$

bad news

- LF calc. misses whole class of diagrams:
  generalized tadpoles
- improper treatment of zero modes

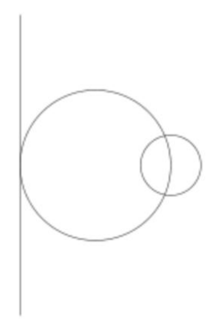
good news MB, PRD (1993)

- all of the missed diagrams only contribute constants

$\hookrightarrow$ can be taken care of by renormalization

$\hookrightarrow$ $m^2_{eff} = m^2 + \lambda\langle 0|\phi^2|0\rangle$

example for contri to self-energy, that cannot be generated by H_{LF}



determining m^2_{eff}

- only match physical quantities during renorm.
- determine $\lambda\langle 0|\phi^2|0\rangle$ by point-splitting in LF time & inserting complete set of states (MB, S.Chabysheva, J.Hiller, PRD (2016))

bad news

- LF calc. misses whole class of diagrams:
  generalized tadpoles
- improper treatment of zero modes

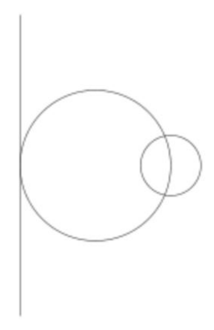
good news MB, PRD (1993)

- all of the missed diagrams only contribute constants

$\hookrightarrow$ can be taken care of by renormalization

$\hookrightarrow$ $m^2_{eff} = m^2 + \lambda \langle 0|\phi^2|0\rangle$

example for contri to self-energy, that cannot be generated by H_{LF}



effective LF Hamiltonian P^-_{eff}

- zero-modes high-energy (k^-) degrees of freedom

$\hookrightarrow$ plausible that 'integrating out' zero modes leads to $P^- \longrightarrow P^-_{eff}$

- by construction, P^-_{eff} containe no zero-mode degrees of freedom!

no tadpoles!?

- naively tadpole issue absent
- k^- from Dirac numerators can cancel one propagator:
  $k^- = p^- - \frac{(p_\perp - k_\perp)^2 + \lambda^2}{2(p^+ - k^+)} - \frac{(p-k)^2 - \lambda^2}{2(p^+ - k^+)}$
  - cancels one denominator
  - 'canonical term' (incl. instantaneous)

$\hookrightarrow$ self-energies contain pieces with same pole structure as generalized tadpoles

$\hookrightarrow$ condensates matter!

- renormalization can fix it...! (e.g. vertex mass $\neq$ kin. mass)

self-energies

$$\Sigma \sim \int \frac{d^4k}{(2\pi)^4} \frac{\not{k} + m}{k^2 - m^2 + i\varepsilon} \frac{1}{(p-k)^2 - \lambda^2 + i\varepsilon}$$

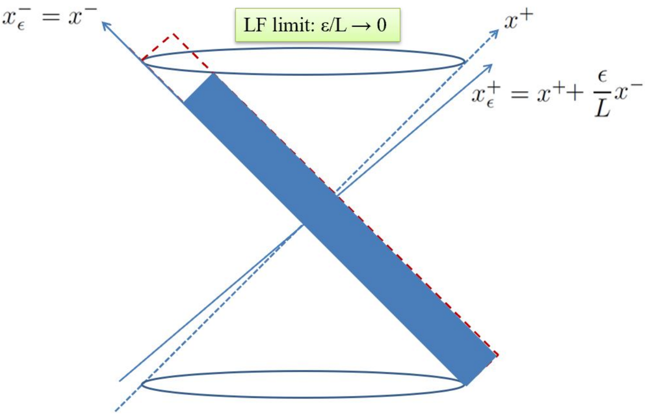
vertices

$$\Sigma \sim \int \frac{d^4k}{(2\pi)^4} \frac{\not{k} - \frac{\not{\Delta}}{2} + m}{\left(k - \frac{\Delta}{2}\right)^2 - m^2 + i\varepsilon} \Gamma \frac{\not{k} + \frac{\not{\Delta}}{2} + m}{\left(k + \frac{\Delta}{2}\right)^2 - m^2 + i\varepsilon} \frac{1}{(p-k)^2 - \lambda^2 + i\varepsilon}$$

ε coordinates

ε-COORDINATES

Provide a controlled and well-defined approach to the LF



Lenz, Thies, Yazaki and Levit
Hamiltonian formulation of two-dimensional gauge theories on the light cone (1991).

ε coordinates

Order of operations and Infinitely many zero modes

Example: Simple tadpole with a mass insertion in 1+1 dimensions.

$$\mathcal{I} = \int \frac{d^2k}{(2\pi)^2} \frac{1}{(k^2 - m^2 + i\epsilon)^2} = \frac{i}{4\pi m^2}$$

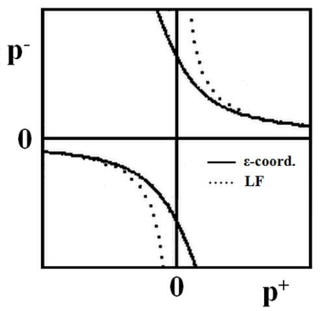
$$\mathcal{I}_\epsilon = \frac{1}{L} \sum \int \frac{dk^-}{2\pi} \frac{1}{\left(\frac{2\epsilon}{L} k^{-2} + 2k^+k^- - m^2 + i\epsilon\right)^2} = \frac{i}{4\sqrt{2\epsilon L}} \sum \left[\frac{(2\pi n)^2}{2\epsilon L} + m^2\right]^{-3/2}$$

1^{st} operation	2^{nd} operation	Results in
LF limit ($\varepsilon \to 0$, L fixed)	Continuum Limit ($L \to \infty$, ε/L fixed)	Divergent contribution from the zero mode ✗
Continuum Limit ($L \to \infty$, ε/L fixed)	LF limit ($\varepsilon \to 0$, L fixed)	Infinitely many zero modes contribute ✓

Order of operations reveal that there is not only one zero mode but infinitely many.

ε coordinates

F.Aslan, Lightcone2019



vacuum in ε coordinates

- negative momenta, i.e. nontrivial vacuum possible
- nontrivial structure localized around $k^+ = 0$ for ε small

$\hookrightarrow$ ∞ many modes near $k^+ = 0$ to describe condensates correctly

- LF P^- as effective Hamiltonian (integrate out zero modes)
- similarities to quasi PDFs (IMF)

renormalization

zero-modes essential for renormalization (rotational invariance)

MB & A. Langnau, PRD 44 (1991) 3857; A.Langnau & MB, PRD 47 (1993) 3452

higher twist sum-rules

- $\delta(x)$ contributions to twist-3 PDFs

$\hookrightarrow$ not probed in DIS

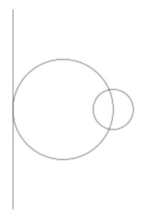
$\hookrightarrow$ apparent 'violations' of twist-3 sum rules & Lorentz invariance relations (σ-term sum rule, Burkhardt-Cottingham sum rule, ...)

F. Aslan & MB, "Singularities in Twist-3 Quark Distributions," PRD **101** (2020) 016010

$J = 0$ fixed poles

- diagrams that result in $\delta(x)$ contributions to PDFs also result in ν-independent contributions to Compton amplitude

$\hookrightarrow$ $J = 0$ fixed poles S. J. Brodsky, F. E. Close and J. F. Gunion, "Compton Scattering And Fixed Poles In Parton Field Theoretic Models," Phys. Rev. D **5**, 1384 (1972).



F.Aslan, PhD thesis 2019

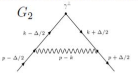
- twist 3 GPDs contain discontinuities at $x = \pm\xi$
- ERBL region can become representation for $\delta(x)$ as $\xi \to 0$

$\hookrightarrow$ violation of sum rules for twist 3 GPDs! (DIS cannot measure $x = 0$)

How do the discontinuities behave as ξ→0 ?

G_2 and $\widetilde{G}_2$ in Quark Target Model

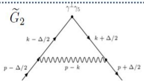
G_2



$$ig^2 4p^+ \frac{(1+x)}{(1-x)} \int \frac{d^2k_\perp dk^-}{(2\pi)^4} \frac{1}{[(k+\frac{\Delta}{2})^2 - m^2 + i\epsilon][(k-\frac{\Delta}{2})^2 - m^2 + i\epsilon]}$$

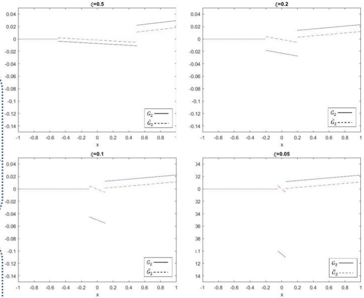
$$\lim_{\xi\to 0} \frac{-g^2}{(2\pi)^2} \frac{(1+x)}{\xi(1-x)} \ln\Lambda_\perp \to \delta(x)$$

$\widetilde{G}_2$



$$ig^2 4p^+ \frac{(x+\xi^2)}{(1-x)} \int \frac{d^2k_\perp dk^-}{(2\pi)^4} \frac{1}{[(k+\frac{\Delta}{2})^2 - m^2 + i\epsilon][(k-\frac{\Delta}{2})^2 - m^2 + i\epsilon]}$$

$$-\frac{g^2}{(2\pi)^2} \frac{(x+\xi^2)}{\xi(1-x)} \ln\Lambda_\perp.$$



Twist-3 GPD	Discontinuities as $\xi \to 0$
G_2	Divergent
$\widetilde{G}_2$	Finite

The ERBL region becomes a representation of δ(x)

18

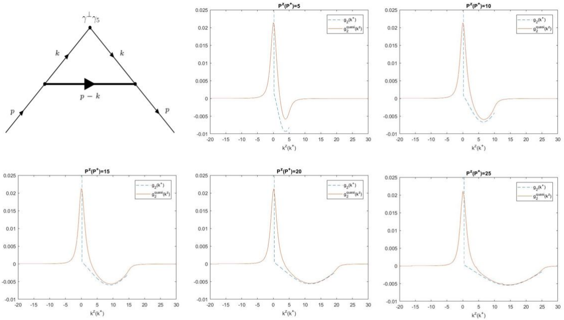
Higher-Twist Sum Rules

F.Aslan, PhD thesis 2019

- $\delta(x)$ from non-scaling wave function component in IMF!

$\hookrightarrow$ important for quasi PDFs/LMET!

$g_2\,(k^+)$ and $g_2^{\text{Quasi}}\,(k^z)$ in SDM



There is a momentum component in the nucleon state which does not scale as the nucleon is boosted to the infinite momentum frame.

Aslan, Burkardt- Lorentz Invariance of Twist-3 Quark Distributions

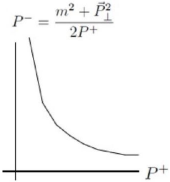
27

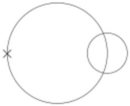
Summary

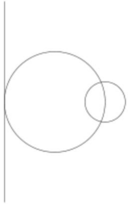
- naively LF vacuum trivial
- apparent contradiction with pheno & lattice
- regularization (point splitting in ε^+) yields nonzero condensates
- consistent with covariant in QCD_{1+1} & ϕ^n
- $P^- \longrightarrow P^-_{eff}$ embodies effect of zero modes on non-zero modes

↪ 'vacuum condensate' contributions essential for equivalence of LF with ET field theory

- zero modes also contribute $\delta(x)$ in twist 3 PDFs
- non-scaling contribution for quasi-PDFs!
- QCD evolution does <u>not remove</u> $\delta(x)$ – QCD evolution <u>contributes to</u> $\delta(x)$







- zero-mode contribution to matrix elements of $T^{\mu\nu}$ (F.Aslan, MB, X.Ji)

$\hookrightarrow$ origin of nucleon mass?

- connection between vacuum condensates and $\delta(x)$ in PDFs
- connection between vacuum condensates and P^-_{eff} in QCD
- relation to subtractions in dispersion relations

$\hookrightarrow$ implications for D-term, ERBL region,...