Jet Propagation in QGP: An Open Quantum system EFT

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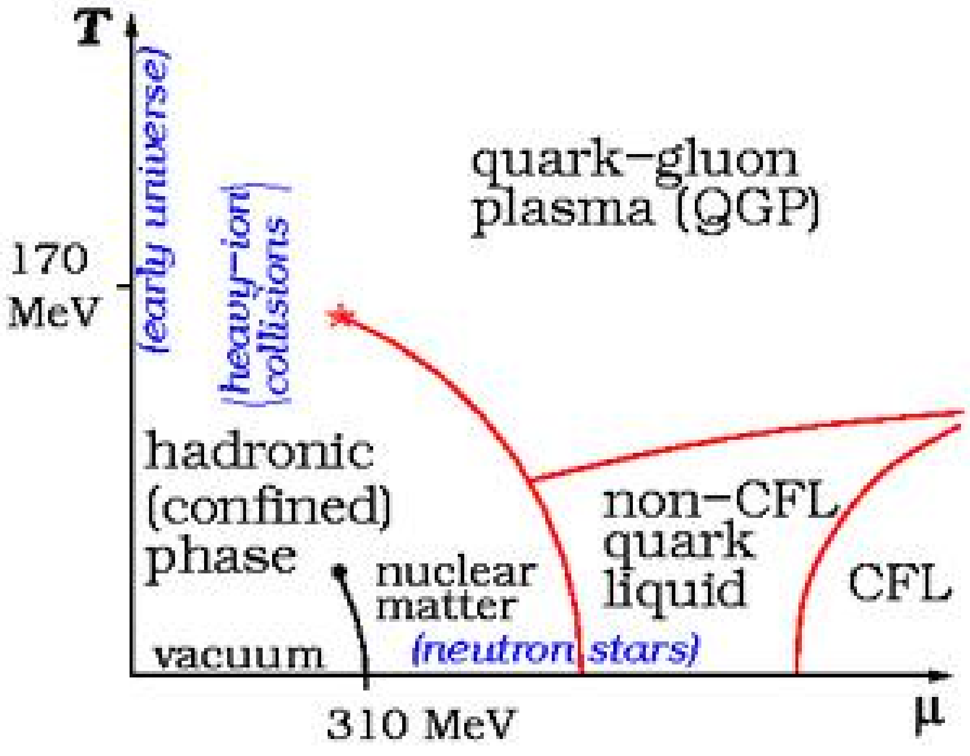
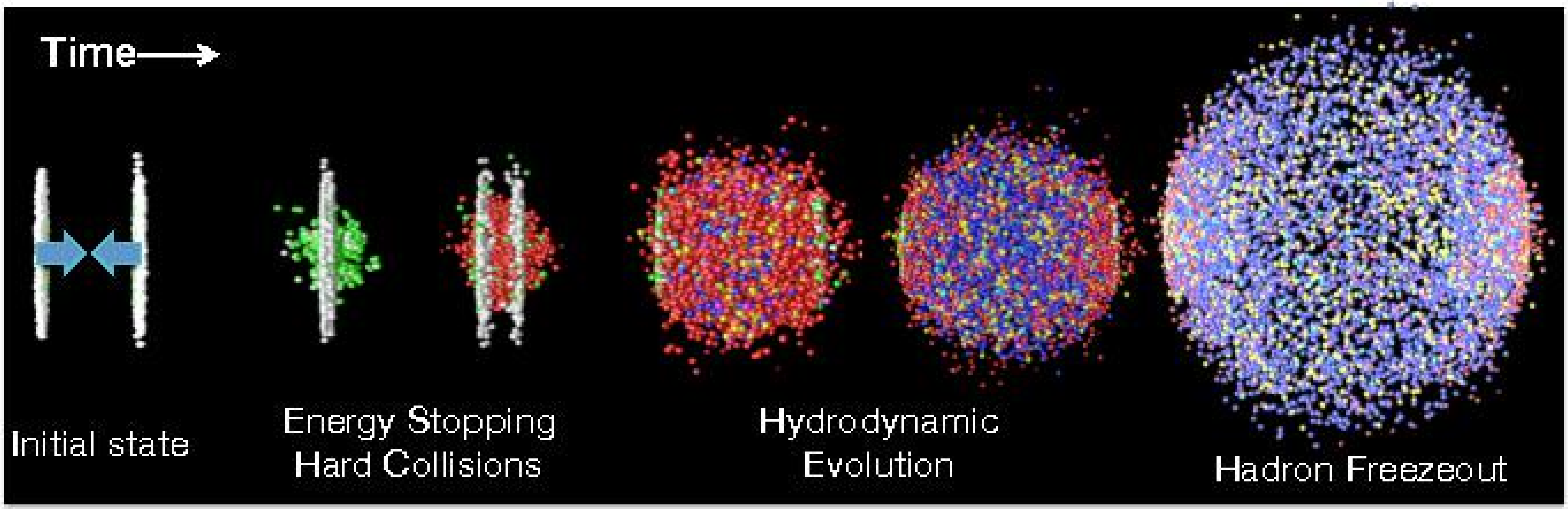


QCD phase diagram

QGP: A soup of free quarks and gluons created in the early universe and recently at Heavy Ion colliders

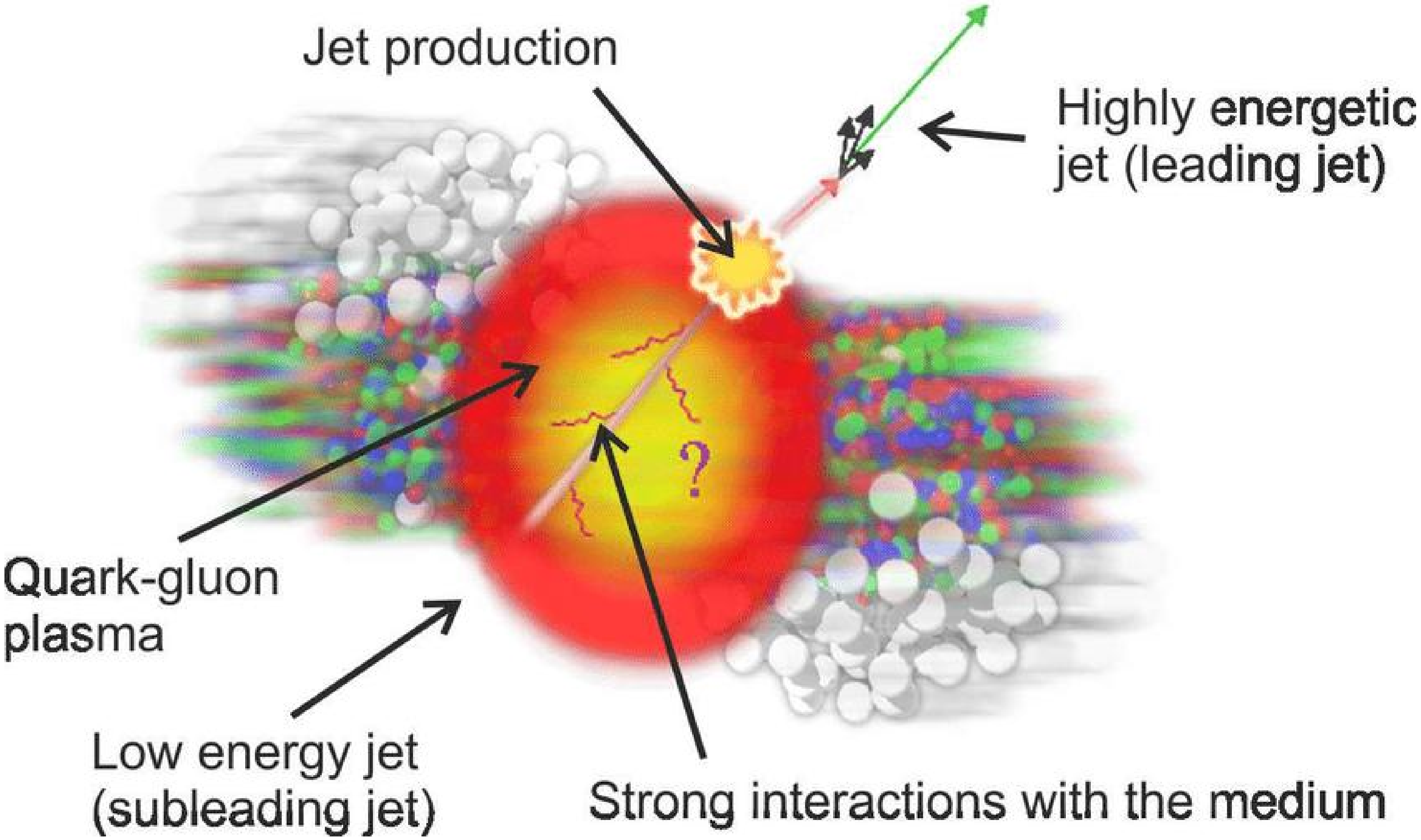
Probing QGP at colliders with jets



Formation of QGP in Heavy Ion collisions

Probing QGP at colliders with jets

- Few events in the QGP background produce energetic partons that evolve into back to back jets



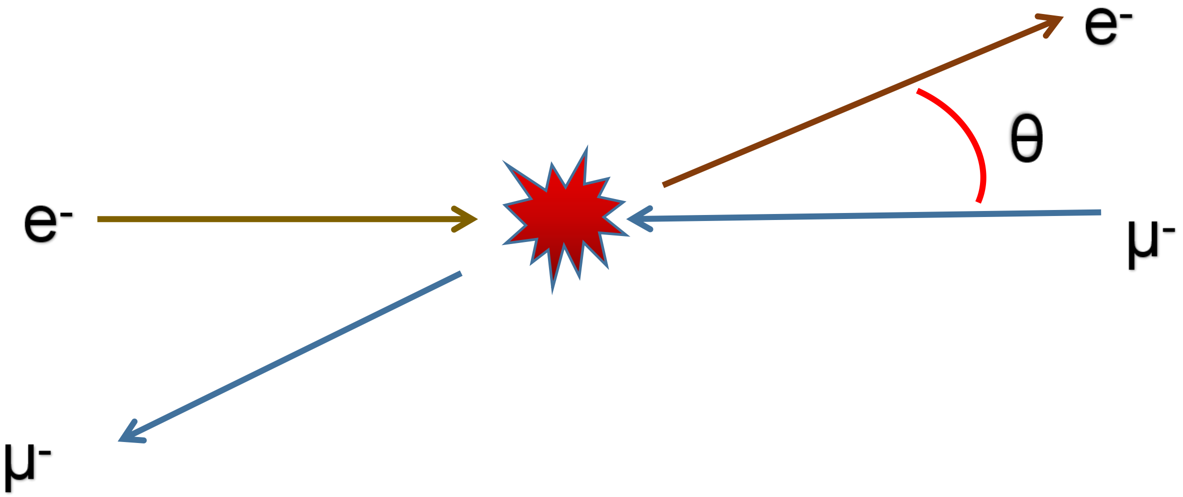
- How is the jet modified as it travels through the medium?

An EFT in the forward scattering regime

What is the dominant regime of interaction of an energetic jet with the QGP?

An EFT in the forward scattering regime

2->2 Forward scattering cross section



$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2E_{cm}\left(1-\cos\theta\right)^2}\left(4+(1+\cos\theta)^2\right)$$

In the limit $\theta \to 0$

$$\frac{d\sigma}{d\Omega} \propto \frac{1}{\theta^4}$$

- Non-integrable t-channel singularity due to infinte range of the Coulomb potential

- Total cross section dominated by the forward scattering regime

An EFT in the forward scattering regime

- Develop an EFT formalism for forward scattering of a jet in QGP with $\lambda = \theta \ll 1$ as the expansion parameter.

- The jet is made up of highly energetic massless partons moving along the light-cone

$$p_c \sim Q(1, \lambda^2, \lambda)$$

- QGP is a thermal bath made of soft partons ($T \ll Q$) that do not have a specific direction of motion.

$$p_s \sim Q(\lambda, \lambda, \lambda)$$

Light-Cone co-ordinates

$$n^\mu \equiv (1,0,0,1) \qquad \bar{n}^\mu \equiv (1,0,0,-1)$$

$$p^\mu \equiv (\bar{n} \bullet p, n \bullet p, \vec{p}_\perp)$$

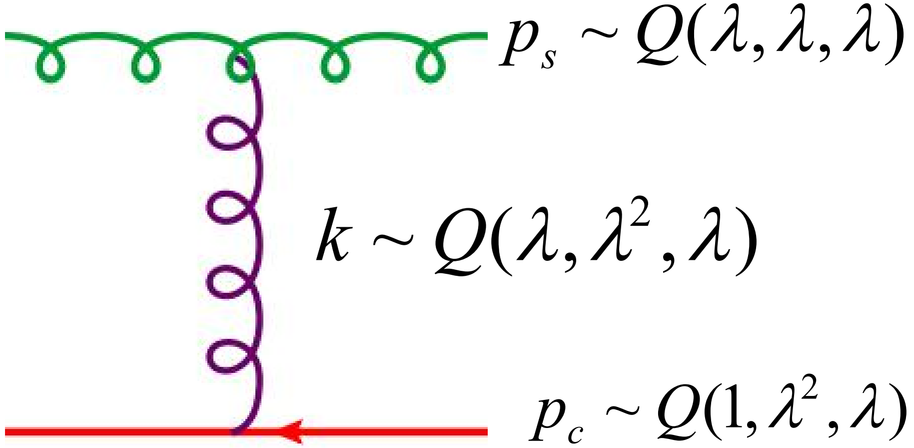
An EFT in the forward scattering regime

- The forward interaction between the Collinear and Soft modes is mediated by the Glauber mode

$$p_G \sim Q(\lambda, \lambda^2, \lambda)$$

- Glauber is a potential (non-propagating) mode

- A Galuber exchange maintains the momentum scaling of Soft and Collinear modes, i.e., it keeps them on-shell.



Glauber propagator

$$\frac{-i}{k^2 - m^2 + i0} \approx \frac{i}{\vec{k}_\perp^{\,2} + m^2 - i0}$$

An EFT in the forward scattering regime

Soft Collinear Effective Theory : An effective QCD Lagrangian at leading power in λ

$$L_{QCD} = L_c + L_s + L_G + O\left(\lambda^2\right)$$

Interactions among Collinear partons

Interactions among soft partons

Effective Soft Collinear interactions mediated by the Glauber mode

An EFT in the forward scattering regime

Effective interaction operator mediated by Glaubers: Quark-Quark interaction

I. Rothstein, I. Stewart,, JHEP 1608 (2016) 025

$$L_G \sim O_{cs}^{qq} = O_n^{q\alpha} \frac{1}{P_\perp^2} O_S^{q\alpha}$$

$$O_n^{q\alpha} = \bar{\chi}_n W_n T^\alpha \frac{\bar{\not{n}}}{2} W_n^+ \chi_n$$

$$O_S^{q\alpha} = \bar{\psi}_s S_n T^\alpha \frac{\not{n}}{2} S_n^+ \psi_s^n$$

$$W_n(x) = P \exp\left[ig \int_{-\infty}^{0} ds \left(\bar{n} \bullet A_n^a(x + s\bar{n}) T^a \right) \right]$$

Collinear Wilson line

$$S_n(x) = P \exp\left[ig \int_{-\infty}^{0} ds \left(n \bullet A_s^a(x + sn) T^a \right) \right]$$

Soft Wilson line

An EFT in the forward scattering regime

How does this EFT apply to our system?

An EFT in the forward scattering regime

Physical scales that describe the system

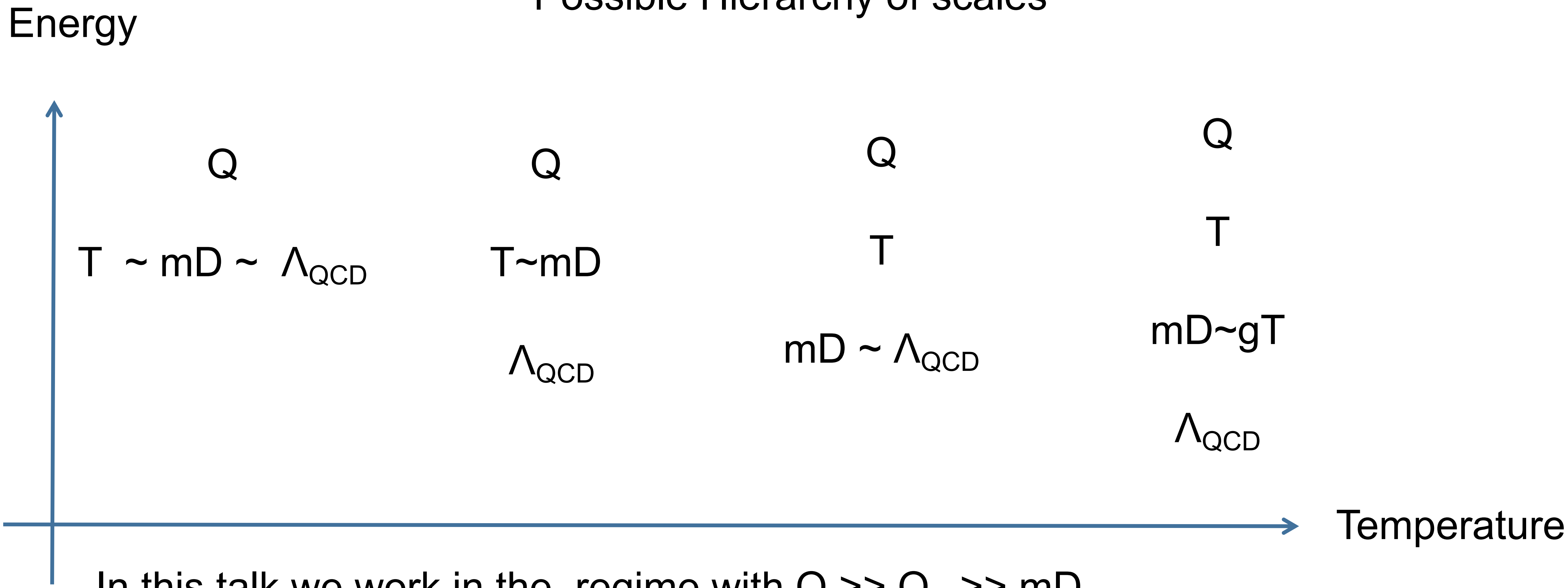
- Hard scale : Energy of the jet Q
- The temperature of the QGP : T
- Parton Mass induced by finite temperature medium : mD ~ gT
- Strong dynamics scale : Λ_{QCD}
- Scale of the measurement : $Q_\perp \geq Q\,\theta = Q\lambda$ can probe any of the IR scales.

IR scales

In this talk, $Q_\perp \sim T$

An EFT in the forward scattering regime

Possible Hierarchy of scales



In this talk we work in the regime with $Q \gg Q_{\perp} \gg m_D$

An EFT in the forward scattering regime

How do we solve for the evolution of a jet as it traverses a region of the QGP?

- Treat the jet as an open quantum system interacting with an environment (via Glaubers)
- Only keep track of the degrees of freedom of the jet.
- Write an evolution eqution for the reduced density matrix of the jet.
- Compute observables on the time evolved density matrix.

Open Quantum systems

- Lindblad equation for Open Quantum systems :

G. Schaller, *Open Quantum systems far from equilibrium,* Lecture Notes in Physics 881.

A master evolution equation for a system density matrix local in time that preserves trace, self- adjointedness and positivity of the density matrix.

$$\partial_t \rho = -i\left[H_{eff}, \rho\right] + \sum_\alpha \left(L_\alpha \rho L_\alpha^+ - \frac{1}{2}\left\{L_\alpha^+ L_\alpha, \rho\right\}\right)$$

Effective Hamiltonian yields unitary evolution

Non-unitary/Dissipative evolution

L_α :Lindblad operator that encodes the interaction between the system and environment

Open Quantum systems

A perturbative derivation of the Lindblad equation for a high temperature QGP: Weak coupling and short times.

$$H_I = \sum_{\alpha} O_C^{\alpha} \otimes O_S^{\alpha}$$

Effective glauber mediated interaction operator

$$\rho_C(t) = \rho_C(0) - i\sum_{ab}\sigma_{ab}(t)\left[L_{ab}, \rho_C(0)\right] + \sum_{a,b,c,d}\gamma_{ab,cd}(t)\left[L_{ab}\rho_C(0)L_{cd}^{+} - \frac{1}{2}\left\{L_{cd}^{+}L_{ab}, \rho_C(0)\right\}\right] + O\left(H_I^3\right)$$

Unitary evolution

Non-Unitary/Dissipative evolution

$$L_{ab} = |\, a\rangle\langle b\,|$$

The Lindblad step operator

Open Quantum systems

Unitary evolution

$$\rho_C(t) = \rho_C(0) - i\sum_{ab}\sigma_{ab}(t)\left[L_{ab}, \rho_C(0)\right] + \sum_{a,b,c,d}\gamma_{ab,cd}(t)\left[L_{ab}\rho_C(0)L_{cd}^{+} - \frac{1}{2}\left\{L_{cd}^{+}L_{ab}, \rho_C(0)\right\}\right] + O\left(H_I^3\right)$$

$$\sigma_{ab}(t) = -\frac{i}{2}\sum_{\alpha,\beta}\int_0^t dt_1\int_0^t dt_2 C_{\alpha\beta}(t_1,t_2)\,\mathrm{sgn}(t_1-t_2)\left\langle a \mid O_C^{\alpha}(t_1)O_C^{\beta}(t_2) \mid b\right\rangle$$

$$C_{\alpha\beta}(t_1,t_2) = Tr_S\left[O_S^{\alpha}(t_1)O_S^{\beta}(t_2)\rho_S\right]$$

Correlator in the environment

System correlator

Treat the QGP environment as a canonical ensemble at temperature T and compute correlators using Thermal field theory

Transport equation for a jet in a medium

- Start with an initial state which is a energetic quark produced in a hard collision
- Ignore vacuum evolution as well as medium induced splitting->
  The jet consists of a single collinear parton being kicked about as it travels
  through the medium.
- Measure the distribution of transverse momentum($Q_\perp$)gained by this parton over a time t.

$$\langle Q|\,\rho_S(t)\,|Q\rangle = \langle Q|\,\rho_S(0)\,|Q\rangle - t\times p\times\langle Q|\,\rho_S(0)\,|Q\rangle + t\int \tilde{d}q\times\tilde{p}(Q,q)\langle q\,|\,\rho_S(0)\,|\,q\rangle$$

Evolution of the density matrix over a small time step

Transport equation for a jet in a medium

$$\mathrm{p} = \frac{2\alpha_s^2 N_f C_F T_F}{\pi^3} \int \frac{|k_\perp|\,\mathrm{d}|k_\perp|}{|k_\perp|^4}\,\mathrm{d}\phi_k \int \mathrm{d}|p_\perp|\,\mathrm{d}p^-\,\mathrm{d}\phi\,\frac{|p_\perp|^3}{(p^-)^2} n_F\Big(\frac{(p^-)^2+|p_\perp|^2}{2p^-}\Big)$$
$$\times\left[1 - n_F\Big(\frac{(p^-)^2(|p_\perp|^2+|k_\perp|^2+2|p_\perp||k_\perp|\cos\phi)+|p_\perp|^4}{2|p_\perp|^2 p^-}\Big)\right],$$

$$\langle Q|\rho_S(t)|Q\rangle = \langle Q|\rho_S(0)|Q\rangle - t\times p\times\langle Q|\rho_S(0)|Q\rangle + t\int \tilde{d}q\times\tilde{p}(Q,q)\langle q|\rho_S(0)|q\rangle$$

In the limit t->0, which is the Markovian approximation, this yields a semi-classical transport equation for the jet

$$\partial_t F(Q,t) = -p\times F(Q,t) + \int \tilde{d}q\times\tilde{p}(Q,q)F(q,t)$$

Markovian Approximation

$$\partial_t F(Q,t) = -p \times F(Q,t) + \int \tilde{d}q \times \tilde{p}(Q,q) F(q,t)$$

- We assume that between two interactions, the environment loses any memory of interaction with the system.

- This holds as long as the time scales over which coherence is lost in the environment (t_e)is much smaller than the time scale of the interaction with system (t_I)

- For a thermal bath at temperature T, $t_e \sim 1/T$

- $t_I \sim 1/p \sim 1/(T\,\alpha_s)$

- So for a weak coupling regime, the Markovian approximation holds.

Transport equation for a jet in a medium

The Glauber interaction connects the system and the bath only via transverse momentum exchange

$$\partial_t F\left(Q^-,\vec{Q}_\perp,t\right) = -p \times F\left(Q^-,\vec{Q}_\perp,t\right) + \int d^2\vec{k}_\perp \times \tilde{p}(\vec{k}_\perp) F\left(Q^-,\vec{Q}_\perp + \vec{k}_\perp,t\right)$$

This equation can be solved by going to impact parameter space.

$$F(Q^-,\vec{Q}_\perp,t) = \int d^2\vec{r}_\perp e^{-i\vec{r}_\perp \bullet \vec{Q}_\perp} e^{[-p+\tilde{p}(-\vec{r}_\perp)]t} F(Q^-,\vec{r}_\perp,t=0)$$

For an initial state with zero transverse momentum

$$F(Q^-,\vec{r}_\perp,t=0) \equiv F(Q^-,t=0)$$

Transport equation for a jet in a medium

Preservation of trace for the reduced density matrix

$$F(Q^-,\vec{Q}_\perp,t) = F(Q^-,t=0)\int d^2\vec{r}_\perp e^{-i\vec{r}_\perp \bullet \vec{Q}_\perp} e^{[-p+\tilde{p}(-\vec{r}_\perp)]t}$$

Integrating over all diagonal matrix elements of the density matrix

$$\int d^2\vec{Q}_\perp F(Q^-,\vec{Q}_\perp,t) = F(Q^-,t=0)\int d^2\vec{r}_\perp \delta^2(\vec{r}_\perp) e^{[-p+\tilde{p}(-\vec{r}_\perp)]t}$$

$$= F(Q^-,t=0) e^{[-p+\tilde{p}(-\vec{r}_\perp=0)]t}$$

We can explicitly check that $p = \tilde{p}(-\vec{r}_\perp = 0)$ so that trace is preserved

Transverse momentum broadening

$$G(Q^-,\vec{Q}_\perp,t) \equiv \frac{F(Q^-,\vec{Q}_\perp,t)}{F(Q^-,t=0)} = \beta^2 \int d^2\vec{r}_\perp e^{-i\vec{r}_\perp \bullet \vec{Q}_\perp} e^{[-p+\tilde{p}(-\vec{r}_\perp)]t}$$

$Q_\perp$, t are expressed in units of β.

G describes the transverse momentum distribution of the jet as a function of time and temperature.

Infrared Safety

Transverse momentum broadening

Infrared Safety of the solution

$$G(Q^-,\vec{Q}_\perp,t)=\beta^2\int d^2\vec{r}_\perp e^{-i\vec{r}_\perp\bullet\vec{Q}_\perp}e^{[-p+\widetilde{p}(-\vec{r}_\perp)]t}$$

$$-p+\widetilde{p}(-\vec{r}_\perp)=\int d^2\vec{k}_\perp\left[e^{i\vec{r}_\perp\bullet\vec{k}_\perp}-1\right]\widetilde{p}(\vec{k}_\perp)$$

$$\widetilde{p}(\vec{k}_\perp)|_{k_\perp\to 0}\sim\frac{1}{k_\perp^4}\propto\frac{1}{\theta^4}$$ so that $$-p+\widetilde{p}(-\vec{r}_\perp)\overset{k_\perp\to 0}{\to}\int\frac{d^2\vec{k}_\perp}{k_\perp^2}$$

So the result appears to be IR divergent.

Transverse momentum broadening

Separate out the result at zero transverse momentum: Add and subtract r independent Sudakov factor.

$$G(Q^-,\vec{Q}_\perp,t) = \beta^2 \int d^2\vec{r}_\perp e^{-i\vec{r}_\perp \bullet \vec{Q}_\perp} \left(e^{[-p+\tilde{p}(-\vec{r}_\perp)]t} + e^{[-p]t} - e^{[-p]t} \right)$$

$$G(Q^-,\vec{Q}_\perp,t) = \delta^2\left(\vec{Q}_\perp\right)\beta^2 e^{[-p]t} + \beta^2 \int d^2\vec{r}_\perp e^{-i\vec{r}_\perp \bullet \vec{Q}_\perp} \left(e^{[-p+\tilde{p}(-\vec{r}_\perp)]t} - e^{[-p]t} \right)$$

Result for non-zero $Q_\perp$

Is this result IR safe?

A naive argument would suggest that the measurement $Q_\perp$ >> mD would cut-off any IR singularities.

Transverse momentum broadening

Lets check the result to first few orders

$$G(Q^-,\vec{Q}_\perp,t) \sim \int d^2\vec{r}_\perp e^{-i\vec{r}_\perp \bullet \vec{Q}_\perp}\left(e^{[-p+\tilde{p}(-\vec{r}_\perp)]t} - e^{[-p]t}\right)$$

$$G^{(0)}(Q_\perp,t) = 0$$

$$G^{(1)}(Q_\perp,t) \sim \frac{\alpha_s^2 t}{Q_\perp^4}$$

The measurement cuts-off the IR divergence

$$G^{(2)}(Q_\perp,t) \sim \frac{\alpha_s^4 t^2}{Q_\perp^6}\int \frac{d^2k_\perp}{k_\perp^2}$$

A logarithmic singularity persists at this order

At $O(t^n)$ we expect a singularity of the form $(\alpha_s^n)^2 \log^{n-1}$

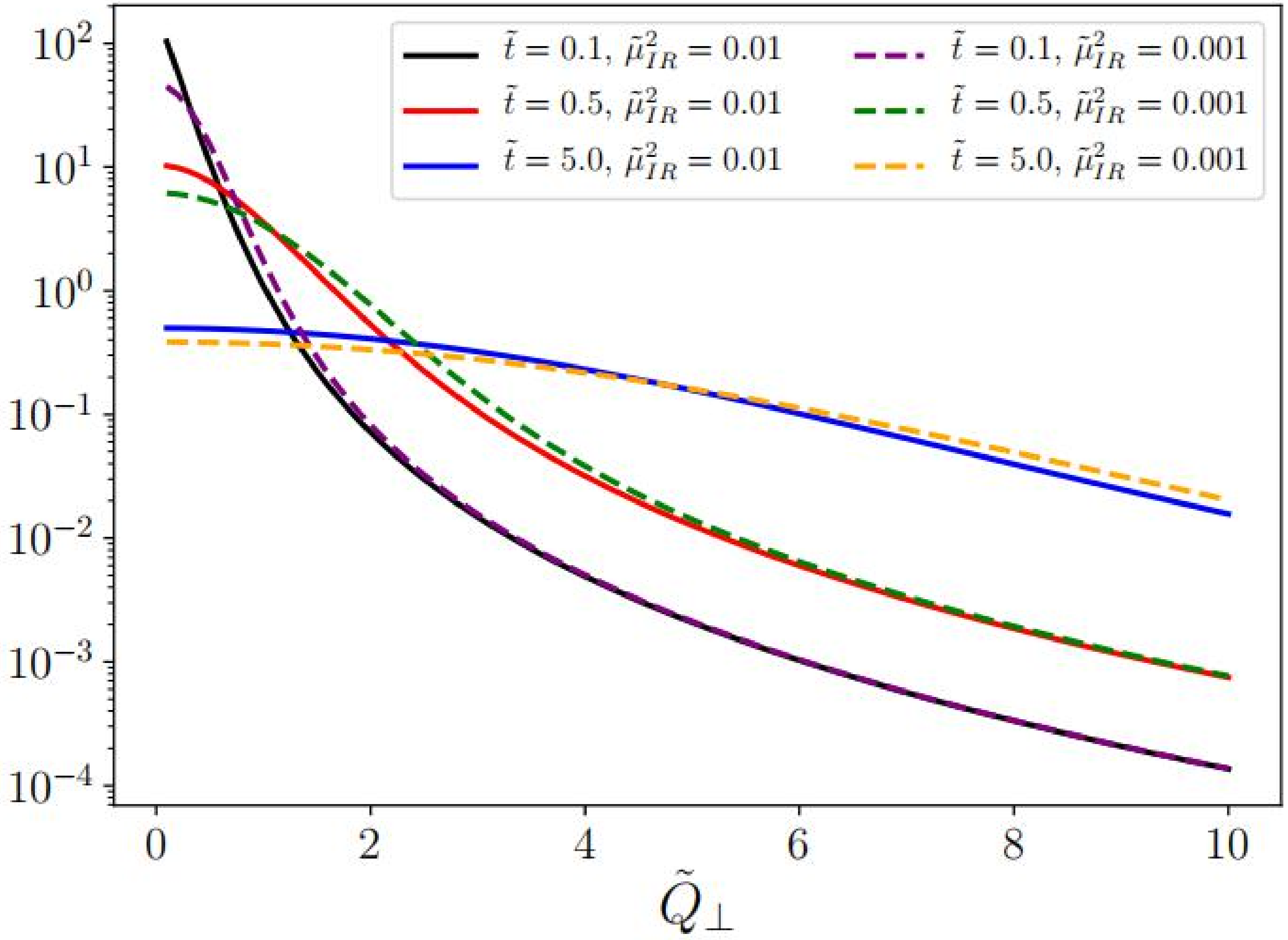
IR singularity

How do we interpret this singularity?

- The reason for the singualrity is the use of the Markovian approximation.
- Coherence is lost between separate interactions in a multiple interaction event (high order in t)
- We only demand that the total transverse momentum accumulated be $Q_{\perp}$
- Hence we can allow arbitrarily small k to be exchanged in intermediate events
- IR singularity must then be regulated by finite parton mass mD

Transverse momentum broadening

Numerical results : Preliminary results



Summary and future directions

- A forward scattering EFT formalism for an energetic jet interacting with a QGP medium.
- An open quantum system approach deriving evolution equation of the jet density matrix in the Markovian approximation
- A leading order solution for Transverse momentum broadening

To do list

- Include vacuum evolution along with medium induced radiative corrections.
- Extend formalism for jets initiated by heavy quarks -> sPHENIX
- Compute more involved jet substructure observables, Non-perturbative effects

THANKS