Quark fragmentation as a probe of dynamical mass generation

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Hampton U. and Jefferson Lab

Theory Cake Seminar
Jefferson Lab, May 8th, 2019

Based on: *Accardi, Signori, arXiv:1903.04458*
Accardi, Bacchetta, PLB 773 (2017) 632
+ in progress w/ Bacchetta, Radici, Signori

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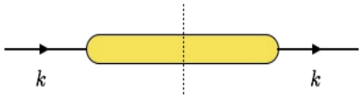
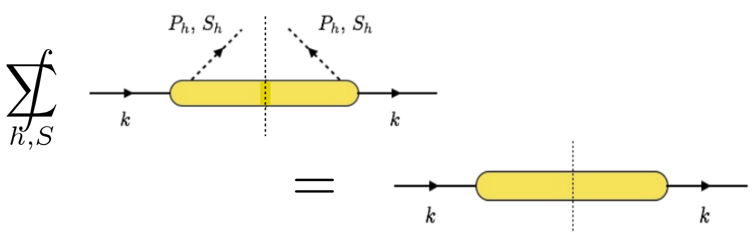
Overview

- **"Inclusive jet" correlator**
  - Quarks are not asymptotic states
  - Mass is dynamically generated



- **Gauge invariant spectral representation**
  - jet/dressed quark mass
- **New FF sum rules**
  - Jet/ dressed quark mass is experimentally observable!



- **New phenomenology**
- **Conclusions**

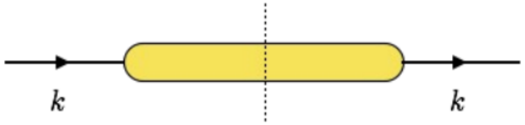
Inclusive jet correlator

AA, Signori, 1903.04458
Sterman, NPB 281 ('87)

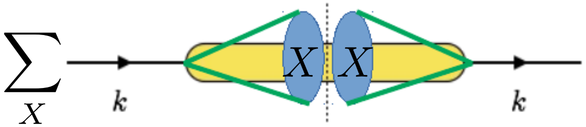
Inclusive $q \longrightarrow X$ "inclusive jet" correlator

$$\Xi_{ij}(k;n_+) = \mathrm{Disc} \int \frac{d^4\xi}{(2\pi)^4} e^{\mathrm{i}k\cdot\xi} \frac{\mathrm{Tr}_c}{N_c} \langle\Omega|\mathcal{T}\, W^{n_+}_{(\infty,\xi)}\, \psi_i(\xi)\overline{\psi}_j(0)\, W^{n_+}_{(0,\infty)}|\Omega\rangle$$

- Partonic picture: **gauge invariant dressed quark correlator**
  - Quarks are not asymptotic states
  - Note color averaging



- Hadronic picture: **"inclusive jet" correlator**
  - Hadronization products pass the cut
  - Interpret as (time-ordered) gauge invariant quark-to-jet amplitude[2]
  - No measured hadrons $\longrightarrow$ no jet cone / energy



- Can study **fragmentation w/o fragments**
  - In particular, dynamical mass generation & χ–symmetry breaking

Gauge invariant spectral representation

AA, Signori, 1903.04458

- First: convolution representation

$$\Xi_{ij}(k) = \text{Disc} \int d^4p \, \frac{\text{Tr}_c}{N_c} \langle \Omega | \widetilde{S}_{ij}(p) \widetilde{W}(k-p) | \Omega \rangle \ ,$$

where

$$\widetilde{S}_{ij}(p) = \int \frac{d^4\xi}{(2\pi)^4} \, e^{\mathrm{i}\xi\cdot p} \, \mathcal{T} \, \psi_i(\xi) \overline{\psi}_j(0) \,,$$

$$\widetilde{W}(k-p) = \int \frac{d^4\xi}{(2\pi)^4} \, e^{\mathrm{i}\xi\cdot(k-p)} \, \mathcal{T} \, W(0,\xi) \,.$$

- Invariant decomposition of quark's bilinear operator:

$$\widetilde{S}_{ij}(p) = \hat{s}_3(p^2) \not{p}_{ij} + \sqrt{p^2} \hat{s}_1(p^2) \mathbb{I}_{ij} \ + g.f. \not{n}$$

"Spectral operators"

gauge fixing term (for axial gauges)

Gauge invariant spectral representation

AA, Signori, 1903.04458

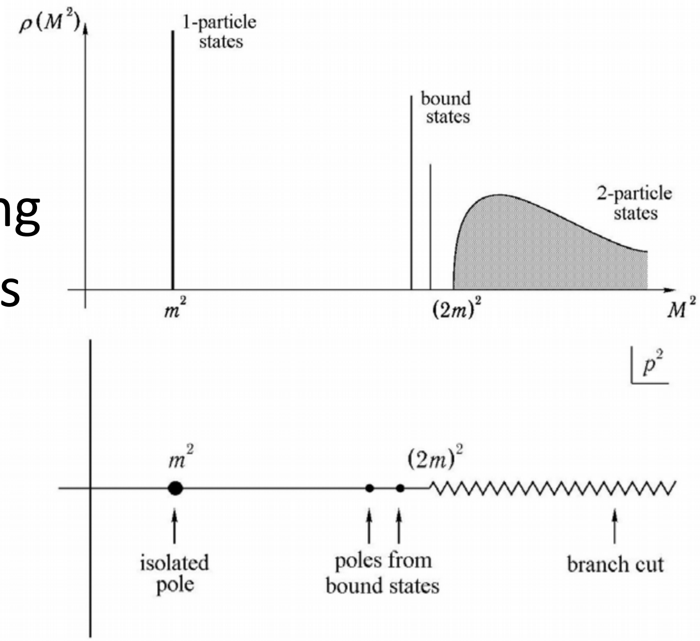
- Kallen-Lehman representation for Feynman propagator

$$\frac{\mathrm{Tr}_c}{N_c}\langle\Omega|\widetilde{S}(p)|\Omega\rangle = \int_{-\infty}^{+\infty}\frac{d\mu^2}{(2\pi)^4}\frac{\mathbf{i}}{p^2-\mu^2+i\epsilon}\Big\{\not p\,\rho_3(\mu^2)+\sqrt{\mu^2}\,\rho_1(\mu^2)\Big\}\theta(\mu^2)$$

***ρ*$_{1,3}$ are spectral functions:**

$\longrightarrow$ strength of quark-to-multihadron coupling

$\longrightarrow$ color averaging: only colorless final states



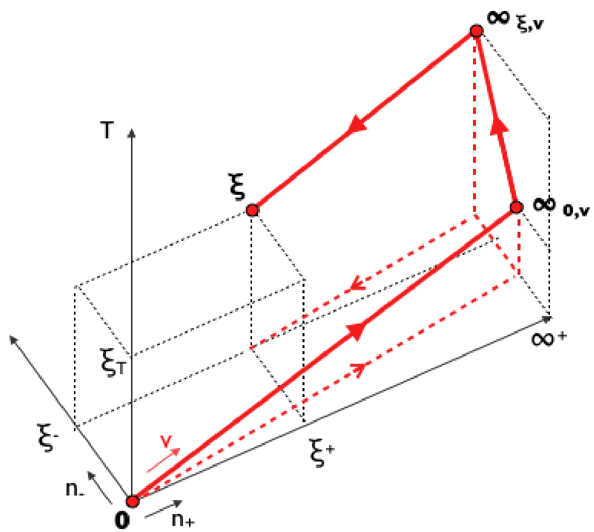
- In terms of **spectral operators**:

$$(2\pi)^3\,\mathrm{Disc}\,\frac{\mathrm{Tr}_c}{N_c}\langle\Omega|\hat{s}_{1,3}(p^2)|\Omega\rangle = \rho_{1,3}(p^2)\,\theta(p^2)\,\theta(p^-)$$

Wilson line structure

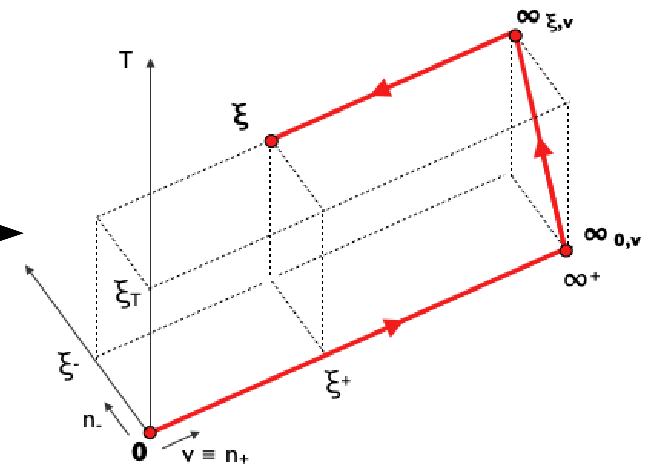
AA, Signori, 1903.04458

- Focus on (l.c.) staple-like Wilson lines
  - But spectral convolution method is general



Generic v

Light-cone $v = n_+$



TMD jet correlator

AA, Signori, 1903.04458

- Boost the quark at large light-cone momentum
  - (e.g., as it happens in large-*Q* DIS)

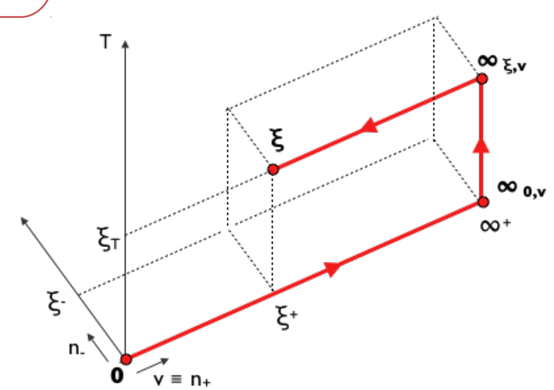
$$k^- \gg |\mathbf{k_T}| \gg k^+$$

- Integrate out the small momentum component:

TMD Inclusive jet correlator

$$\boxed{J_{ij}(k^-, \boldsymbol{k}_T) \equiv \frac{1}{2} \int dk^+ \, \Xi_{ij}(k) \,,}$$

  - obtain standard staple
  - time-ordering is automatic
    (we could have started without it in
    the unintegrated jet correlator definition)



TMD jet correlator in full glory

AA, Signori, 1903.04458
+ work in progress

- Expand in Dirac structures, order in powers of $1/k^-$

$$J(k^-, k_T) = \left\{ \gamma^+ + \frac{M_j}{k^-} + \frac{\not{k}_T}{k^-} + \frac{(K_j^2 + T_j^2 + g.f.) + k_T^2}{2(k^-)^2} \not{n}_- \right\} \theta(k^-)$$

- where, using spectral convolution in light-cone gauge, can calculate:

$$M_j = \int_0^\infty d\mu^2\, \mu\, \rho_1(\mu^2)$$

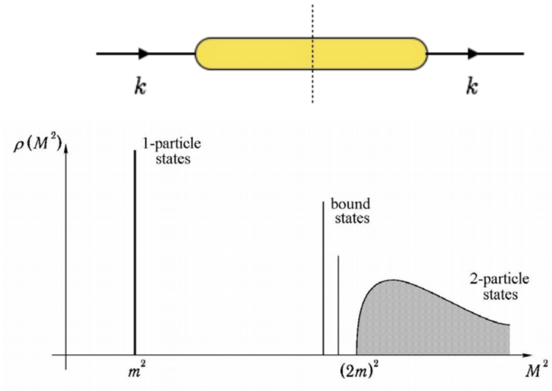
Jet "mass" ~ dressed quark mass ~ O(100 MeV)

$$K_j^2 = \int_0^\infty d\mu^2\, \mu^2\, \rho_3(\mu^2)$$

Jet's "virtuality"

$$T_j^2 \sim \langle\langle \mu_T^2 \rangle\rangle$$

Jet's "transverse size"



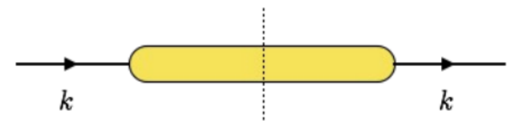
- **NOTE**:
  - Average jet shape (dynamics of hadronization) encoded in TMD jet correlator !!
  - Explicit g.f. contributions pushed to twist-4 in light-cone gauge

The jet/quark mass

AA, Signori, 1903.04458

- Mass associated with **chiral-odd component** of jet amplitude squared:

$$M_{jet} \sim \frac{\mathrm{Tr_c}}{N_c} \int dk^+ \, \mathrm{Tr_D} \Bigg($$



$$\times \, \mathbb{I} \Bigg)$$

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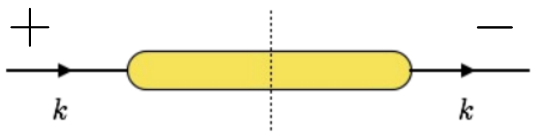

$$\Bigg)$$

  - In light cone gauge, it is interpreted as average mass of the color-neutral QCD d.o.f (“hadrons”) through cut

$$M_{jet} = \int_0^\infty d\mu^2 \, \mu \, \rho_1(\mu^2)$$

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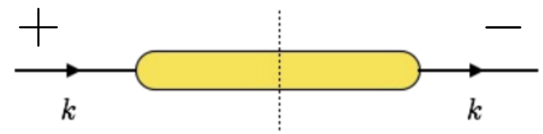

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$$M_{jet} = \int_0^\infty d\mu^2 \, \mu \, \rho_1(\mu^2)$$

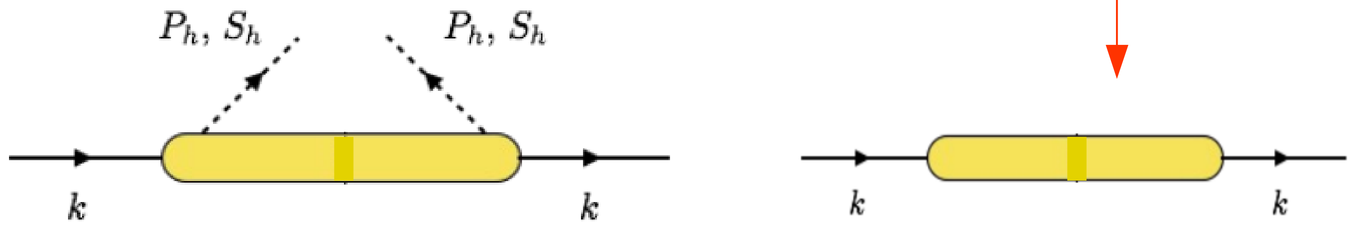
- It defines a **mass of a colored-screened dressed quark** which is:
  - **Gauge-invariant**
  - Renormalization-scale dependent (grows with jet energy scale k^-)
  - Calculable theoretically (through spectral functions)
  - **Most importantly, measurable via a new momentum FF sum rule**

Momentum sum rule - operator level

AA, Signori, 1903.04458
+ work in progress

- Extend field-theoretical technique of *Meissner, Metz, Pitonyak, PLB 2010* :

$$\sum_{h,\,S_h} \int \frac{d^4 P_h}{(2\pi)^3}\, \delta(P_h^2 - M_h^2) P_h^\mu\, \Delta^h(k, P_h, S_h) = k^\mu\, \Xi^{uncut}(k)$$



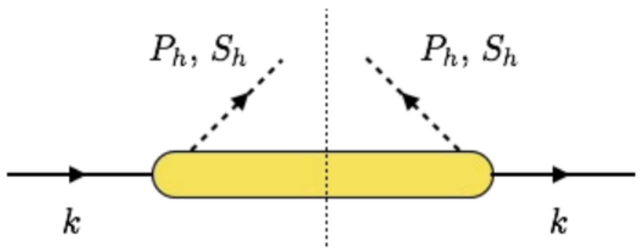
– **Dressed quark propagator as “average” on-shell four momentum produced by hadronization**

– **Dirac projections give momentum sum rules for TMD FFs !**

Dirac structures

AA, Signori, 1903.04458

- TMD Fragmentation Functions



$$\Delta^h(z, P_{h\perp} = \frac{1}{2}\gamma^+ D_1^h + \frac{M_h}{2P_h^-} E^h \mathbb{I} + \frac{\not{P}_{h\perp}}{2zP_h^-} D^{\perp h} \quad + \text{quark polarized terms}$$

- For the inclusive jet correlator

$$J(k^-, k_T) = \frac{1}{2}\gamma^+ + \frac{M_j}{2k^-}\mathbb{I} + \frac{\not{k}_T}{2k^-} \qquad + \text{higher-twist terms}$$

Mass sum rule

AA, Signori, 1903.04458

- Projecting the sum rule onto the identity matrix,

$$M_j = \sum_{h,S_h} \int dz M_h E^h(z)$$

jet/quark mass as
average of produced hadron masses
weighted by chiral-odd FFs

- **Dynamical mass component:**

EOM relations:

$$E = \tilde{E} + \frac{m_q}{M_h} z\, D_1$$

neglecting
q-g-q correlations

"WW approx."

$$M_j = m_q$$

full QCD

$$M_j \equiv m_q + m^{corr}$$

Dynamical mass!

$$\boxed{m^{corr} = \sum_{h,S_h} \int dz M_h \tilde{E}^h(z)}$$

Expect non-zero in χ-limit ⟶ **observable χ-symmetry order parameter!**

Full set of sum rules

AA, Signori, PoS(DIS2018)
+ in progress

- Sum rules for quarks into unpolarized hadrons, up to twist-3
  - (only thing missing for twist-4: full FF-TMD analysis)

$$\sum_{h\,S_h}\int dz\, z D_1^h(z) = 1$$

Collins-Soper

NEW $$\sum_{h\,S_h}\int dz\, M_h E^h(z) = M_j$$

NEW $$\sum_{h\,S_h}\int dz\, M_h \tilde{E}^h(z) = M_j - m_{q0} = m_q^{corr}$$

NEW $$\sum_{h\,S_h}\int dz\, M_h H^h(z) = 0$$

$$\sum_{h\,S_h}\int dz\, M_h \tilde{H}^h(z) = 0$$

Diehl-Sapeta

fully dynamical quantities

$$\sum_{h\,S_h}\int dz\, z M_h H_1^{\perp\,(1)\,h}(z) = 0$$

Schaefer-Teryaev

NEW $$\sum_{h\,S_h}\int dz\, M_h^2 D^{\perp\,(1)\,h}(z) = 0$$

NEW $$\sum_{h\,S_h}\int dz\, M_h^2 \tilde{D}^{\perp\,(1)\,h}(z) = -\frac{1}{2}\langle P_\perp^2/z\rangle$$

NEW $$\sum_{h\,S_h}\int dz\, M_h^2 G^{\perp\,(1)\,h}(z) = 0$$

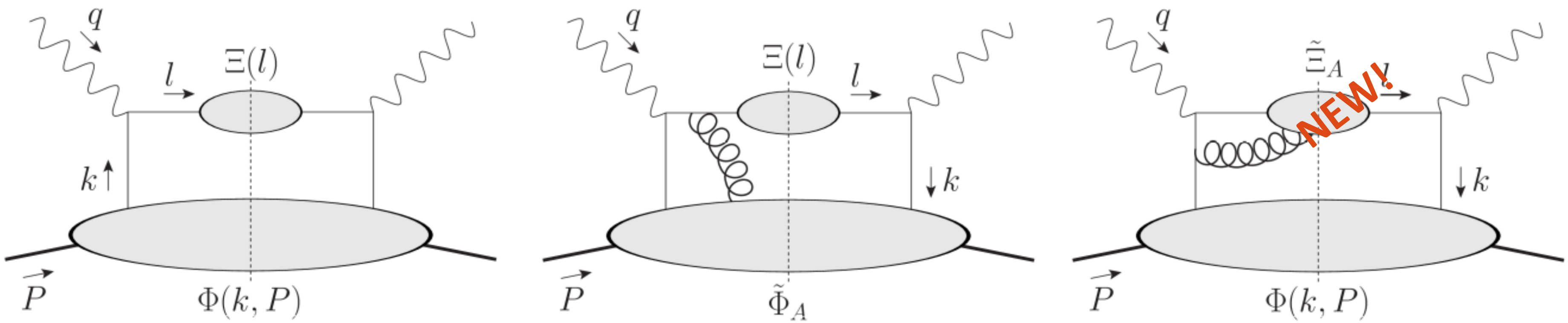
NEW $$\sum_{h\,S_h}\int dz\, M_h^2 \tilde{G}^{\perp\,(1)\,h}(z) = 0\;.$$

Some phenomenology

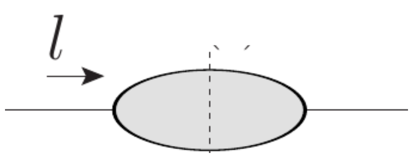
Inclusive DIS with jet correlators

AA, Bacchetta, PLB 773 ('17) 632

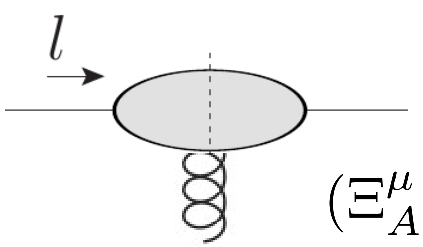
- At large x, limited available invariant mass W^2 → jet-like final state



Jet correlators: → non-asymptotic quark states / dressed quarks



$$\Xi_{ij}(l,n_+) = F.T.\langle\Omega|\,W^{n_+}_{(+\infty,\xi)}\,\psi_i(\xi)\bar{\psi}_j(0)\,W^{n_+}_{(0,+\infty)}\,|\Omega\rangle$$

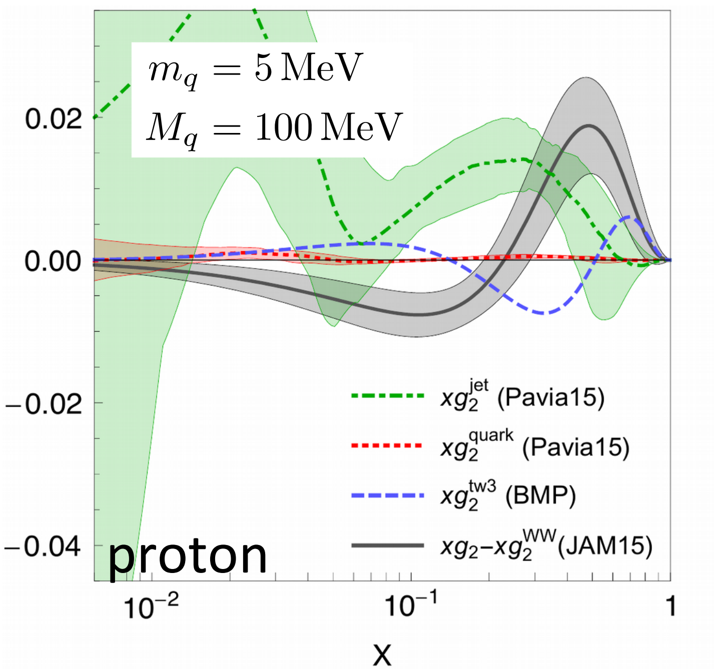


$$(\Xi^\mu_A)_{ij} = F.T.\,\langle\Omega|\,W^{n_+}_{(+\infty,\xi)}\,gA^\mu(\xi)\;\psi_i(\xi)\;\bar{\psi}_j(0)\,W^{n_+}_{(0,+\infty)}|\Omega\rangle$$

g2 structure function revisited

- Integrating SIDIS, and using EOM, Lorentz Invariance Relations:

$$g_2(x_B) - g_2^{WW}(x_B) = \frac{1}{2}\sum_a e_a^2 \Bigg(g_2^{q,\mathrm{tw3}}(x_B) + \overbrace{\frac{m_q}{M}\left(\frac{h_1^q}{x}\right)^{\star}(x_B)}^{\equiv\, g_2^{quark}} + \overbrace{\boxed{\frac{M_j - m_q}{M}\frac{h_1^q(x_B)}{x_B}}}^{\equiv\, g_2^{jet}} \Bigg)$$



Consequences:

- h1 accessible in inclusive DIS
  $\leftrightarrow$ Potentially large signal
- Burkardt-Cottingham sum rule broken

$$\int_0^1 g_2(x) = (M_j - m_q)\int_0^1 dx\,\frac{h_1(x)}{x}$$

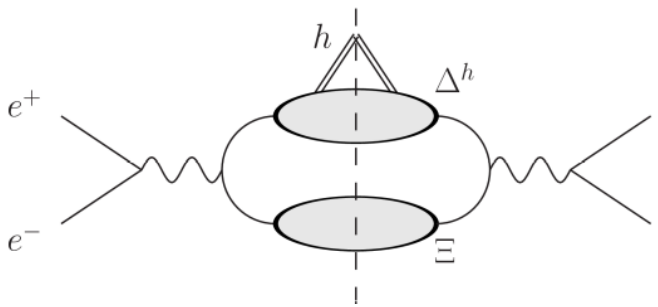
- ETL: novel way to measure tensor charge

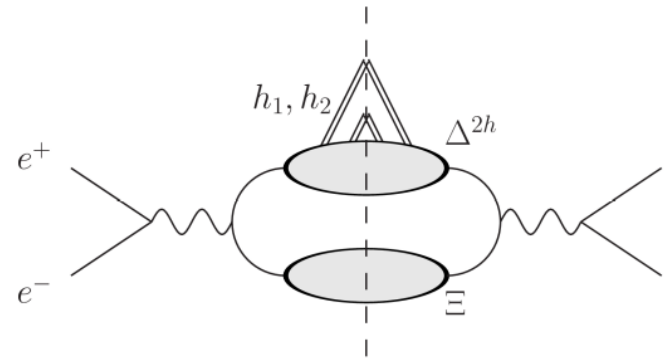
$$\int_0^1 x g_2^{q-\bar{q}}(x) = 2\,(M_j - m_q)\int_0^1 dx\, h_1^{q-\bar{q}}(x)$$

Measuring the jet correlator

Accardi, Bacchetta, Signori, Radici, in progress

- Jet mass $\mathbf{M_{jet}}$ **can be measured in polarized e⁺ + e⁻ :**





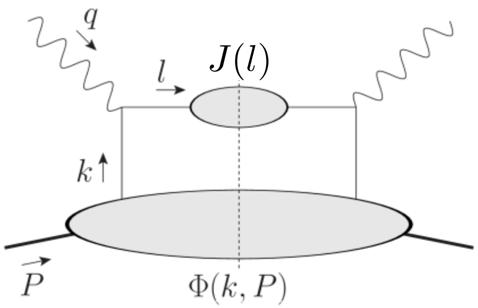
  - Needs **LT asymmetry** in semi-inclusive **Lambda** production

$$\frac{d\sigma^L(e^+e^- \to \text{jet}\, h\, X)}{d\Omega dz}$$
$$= \frac{3\alpha^2}{Q^2} \lambda_e \sum_a e_a^2 \left\{ \frac{C(y)}{2} \lambda_h G_1 + D(y) |\boldsymbol{S}_T| \cos(\phi_S) \frac{2M_h}{Q} \left(\frac{G_T}{z} + \frac{M_q - m_q}{M_h} H_1 \right) \right\}$$

  - Similarly a **LU asymmetry** in unpolarized **dihadron** production

χ-odd phenomenology at large x

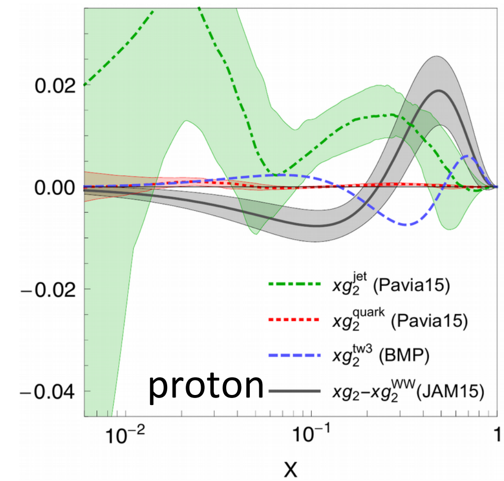
AA, Bacchetta, PLB 2017
AA, Signori, PoS(DIS2018)

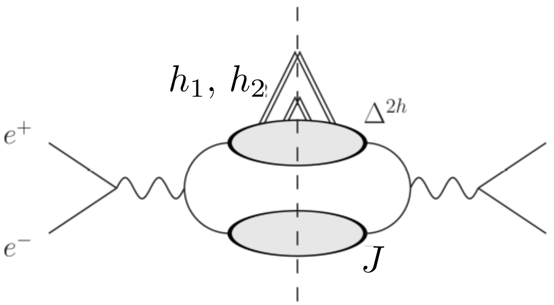


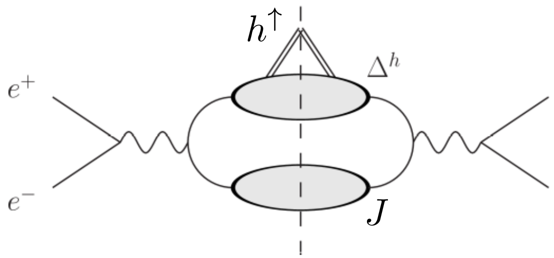
AA, Bacchetta, Melnitchouk, Schlegel, 2009

$$g_2(x_B) = \text{“usual”} + \frac{m^{corr}}{M}\,\frac{h_1^q(x_B)}{x_B}$$

$$\int_0^1 dx\, g_2(x) = m^{corr} \int_0^1 dx\, \frac{h_1(x)}{x} \neq 0$$







AA, Bacchetta, Radici, Signori, in progress

... and more: the door is now open...

Conclusions

Conclusions

- We can quantitatively connect quark fragmentation to the dynamical generation of mass
  - **Gauge invariant definition for dressed quark mass,** M_j
  - The dynamical component $m^{corr} = M_j - m_q$ is recognized as an **observable order parameter for χ-simmetry breaking**

$$m^{corr} = \sum_{h,S_h} \int dz M_h \tilde{E}^h(z)$$

Practical exp. recipe:
– measure $\widetilde{E}$, obtain m^{corr}
– flavor decomposition, too!

- **Novel phenomenology:**
  - Transversity in g2, same side di-hadrons, …

- **New sum rules:** guidance for future fits

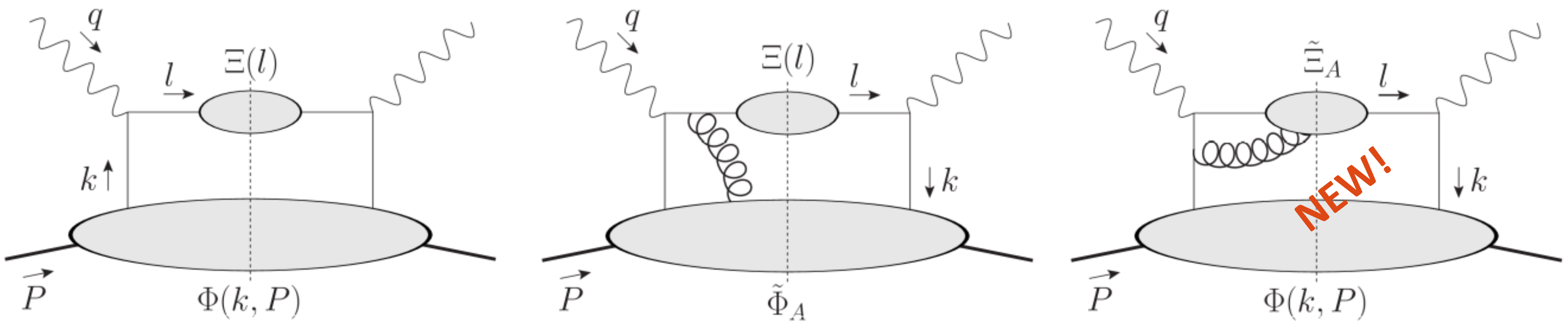
- **New spectral convolution technique** for treating Wilson lines

- **Theory to-do:** renormalization of J, connection to OPE, …

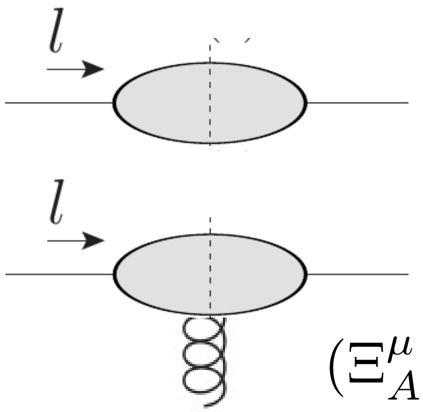
Backup

Inclusive DIS with jet correlators

AA, Bacchetta, PLB 773 ('17) 632



Jet correlators: → non-asymptotic quark states / dressed quarks



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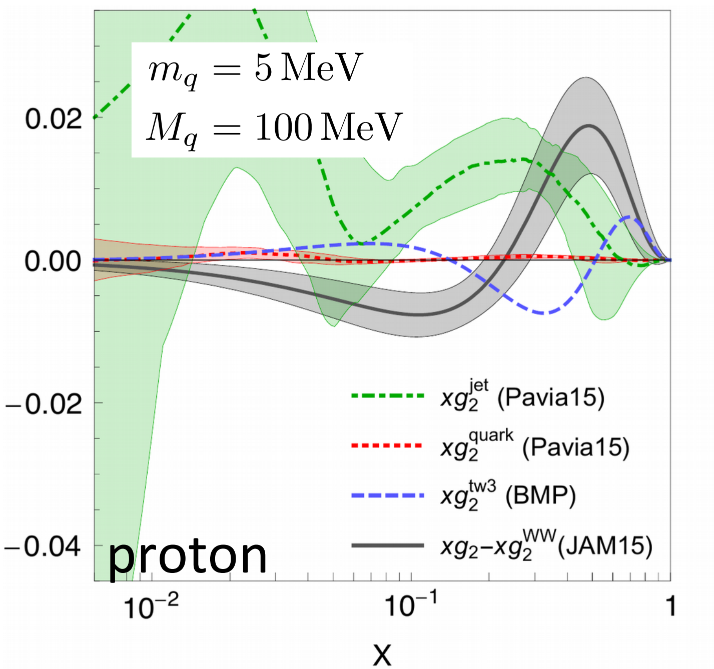

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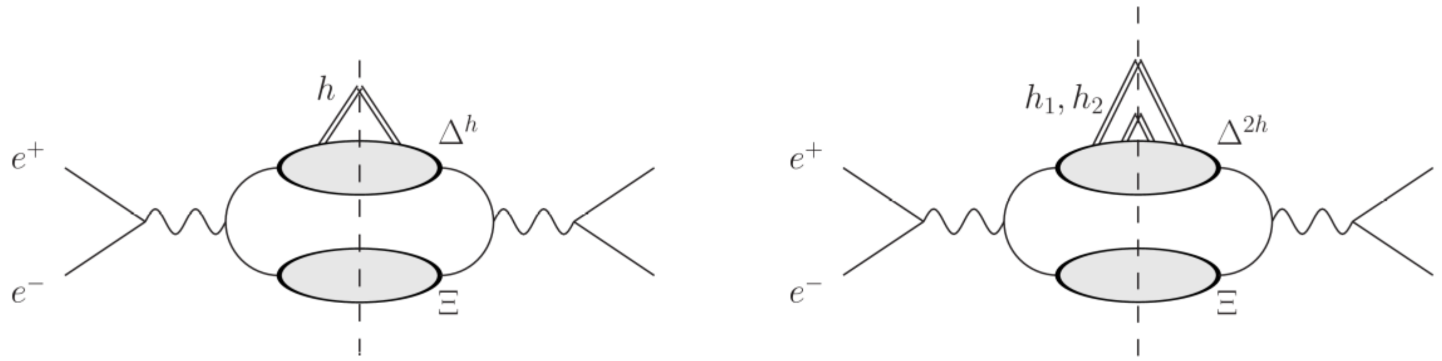
Measuring the jet correlator

Accardi, Bacchetta, Signori, Radici, in progress

- Jet mass $\mathbf{M_{jet}}$ **can be measured in polarized $e^+ + e^-$:**



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  - Similarly a **LU asymmetry** in unpolarized **dihadron** production

Where can we measure jet correlators?

- Can we get a (polarized) **e+ e- collider at JLab / BNL?**
  - At JLab12 ? EIC + positron beam ?
- Are **existing facilities** enough?

	BEPC	super KEKB	ILC	JLab/BNL
E beam [GeV]	1.9	4 (e^{-}) 7 (e^{-})	250	?
√s [GeV]	3 – 5	10	500	?
polarization	?	maybe	80% e^{-} 60% e^{+}	YES!

- What else?

A new "universal" fits

- **Chiral-odd collinear sector across processes:**

$M_{jet}\, H_1^{\sphericalangle}$ polarized

$H_1^{\sphericalangle} \otimes H_1^{\sphericalangle}$ unpolarized

(Di)e^+e^-

DIS

$M_{jet}\, h_1$

(Di)SIDIS

$H_1^{\sphericalangle} \otimes h_1$