Quark fragmentation as a probe of dynamical mass generation

Alberto Accardi
Hampton U. and Jefferson Lab

Theory Cake Seminar
Jefferson Lab, May 8th, 2019

Based on: Accardi, Signori, arXiv:1903.04458
Accardi, Bacchetta, PLB 773 (2017) 632
+ in progress w/ Bacchetta, Radici, Signori
Overview

- "Inclusive jet" correlator
  - Quarks are not asymptotic states
  - Mass is dynamically generated

- Gauge invariant spectral representation
  - Jet/dressed quark mass

- New FF sum rules
  - Jet/ dressed quark mass is experimentally observable!

- New phenomenology

- Conclusions
Inclusive jet correlator

Inclusive $q \rightarrow X$ “inclusive jet” correlator

$$\Xi_{ij}(k; n_{+}) = \text{Disc} \int \frac{d^4 \xi}{(2\pi)^4} e^{ik_{\perp} \xi} \frac{Tr_{c}}{N_c} \langle \Omega | T W_{(\infty, \xi)}^{n_{+}} \psi_{i}(\xi) \bar{\psi}_{j}(0) W_{(0, \infty)}^{n_{+}} | \Omega \rangle$$

- Partonic picture: gauge invariant dressed quark correlator
  - Quarks are not asymptotic states
  - Note color averaging

- Hadronic picture: “inclusive jet” correlator
  - Hadronization products pass the cut
  - Interpret as (time-ordered) gauge invariant quark-to-jet amplitude
  - No measured hadrons $\rightarrow$ no jet cone / energy

- Can study fragmentation w/o fragments
  - In particular, dynamical mass generation & $\chi$–symmetry breaking

AA, Signori, 1903.04458
Sterman, NPB 281 (‘87)
Gauge invariant spectral representation

First: convolution representation

\[ \Xi_{ij}(k) = \text{Disc} \int d^4p \frac{\text{Tr}_c}{N_c} \langle \Omega | \tilde{S}_{ij}(p) \tilde{W}(k - p) | \Omega \rangle , \]

where

\[ \tilde{S}_{ij}(p) = \int \frac{d^4\xi}{(2\pi)^4} e^{i\xi \cdot p} \mathcal{T} \psi_i(\xi) \bar{\psi}_j(0) , \]
\[ \tilde{W}(k - p) = \int \frac{d^4\xi}{(2\pi)^4} e^{i\xi \cdot (k - p)} \mathcal{T} W(0, \xi) . \]

Invariant decomposition of quark’s bilinear operator:

\[ \tilde{S}_{ij}(p) = \hat{s}_3(p^2) \phi_{ij} + \sqrt{p^2} \hat{s}_1(p^2) \Pi_{ij} + g \cdot f \cdot \Phi \]

“Spectral operators”
gauge fixing term (for axial gauges)
Gauge invariant spectral representation

- Kallen-Lehman representation for Feynman propagator

$$\frac{\text{Tr}_c}{N_c} \langle \Omega | \tilde{S}(p) | \Omega \rangle = \int_{-\infty}^{+\infty} \frac{d\mu^2}{(2\pi)^4} \frac{i}{p^2 - \mu^2 + i\epsilon} \left\{ \varphi \rho_3(\mu^2) + \sqrt{\mu^2} \rho_1(\mu^2) \right\} \theta(\mu^2)$$

\(\rho_{1,3}\) are spectral functions:
- strength of quark-to-multihadron coupling
- color averaging: only colorless final states

- In terms of spectral operators:

$$\text{Disc} \left(2\pi\right)^3 \frac{\text{Tr}_c}{N_c} \langle \Omega | \hat{s}_{1,3}(p^2) | \Omega \rangle = \rho_{1,3}(p^2) \theta(p^2) \theta(p^-)$$
Focus on (l.c.) staple-like Wilson lines
  - But spectral convolution method is general
TMD jet correlator

- Boost the quark at large light-cone momentum
  - (e.g., as it happens in large-$Q$ DIS)

\[ k^- \gg |k_T| \gg k^+ \]

- Integrate out the small momentum component:

  TMD Inclusive jet correlator

  \[
  J_{ij}(k^- , k_T) \equiv \frac{1}{2} \int dk^+ \Xi_{ij}(k),
  \]

  - obtain standard staple
  - time-ordering is automatic
    (we could have started without it in the unintegrated jet correlator definition)
TMD jet correlator in full glory

- Expand in Dirac structures, order in powers of $1/k^-$

$$J(k^-, k_T) = \left\{ \gamma^+ + \frac{M_j}{k^-} + \frac{k_T}{k^-} + \frac{(K_j^2 + T_j^2 + g.f.) + k_T^2}{2(k^-)^2} \right\} \theta(k^-)$$

- where, using spectral convolution in light-cone gauge, can calculate:

  $$M_j = \int_0^\infty d\mu^2 \mu \rho_1(\mu^2)$$  
  Jet “mass” $\sim$ dressed quark mass 
  $\sim O(100 \text{ MeV})$

  $$K_j^2 = \int_0^\infty d\mu^2 \mu^2 \rho_3(\mu^2)$$  
  Jet’s “virtuality”

  $$T_j^2 \sim \langle \langle \mu_T^2 \rangle \rangle$$  
  Jet’s “transverse size”

- NOTE:
  - Average jet shape (dynamics of hadronization) encoded in TMD jet correlator !
  - Explicit g.f. contributions pushed to twist-4 in light-cone gauge
The jet/quark mass

- Mass associated with **chiral-odd component** of jet amplitude squared:

\[ M_{jet} \sim \frac{\text{Tr}_c}{N_c} \int d k^+ \text{Tr}_D \left( \begin{array}{ccc} & & \\ \rightarrow & \text{Tr}_D & \rightarrow \\ & & \end{array} \right) \times \boxtimes \]
The jet/quark mass

Mass associated with **chiral-odd component** of jet amplitude squared:

\[ M_{jet} \sim \frac{\text{Tr}_c}{N_c} \int dk^+ \left( \begin{array}{c} + \\kappa \\kappa \end{array} \right) \]

- In light cone gauge, it is interpreted as average mass of the color-neutral QCD d.o.f ("hadrons") through cut

\[ M_{jet} = \int_0^\infty d\mu^2 \mu \rho_1(\mu^2) \]
The jet/quark mass

- Mass associated with **chiral-odd component** of jet amplitude squared:

\[ M_{jet} \sim \frac{\text{Tr}_c}{N_c} \int dk^+ \left( \begin{array}{c} + \\ k \\ \end{array} \right) \left( \begin{array}{c} - \\ k \end{array} \right) \]

- In light cone gauge, it is interpreted as average mass of the color-neutral QCD d.o.f ("hadrons") through cut

\[ M_{jet} = \int_0^\infty d\mu^2 \mu \rho_1(\mu^2) \]

- It defines a **mass of a colored-screened dressed quark** which is:
  - **Gauge-invariant**
  - Renormalization-scale dependent (grows with jet energy scale \( k^- \))
  - Calculable theoretically (through spectral functions)
  - Most importantly, **measurable** via a new momentum FF sum rule
Momentum sum rule - operator level

- Extend field-theoretical technique of *Meissner, Metz, Pitonyak, PLB 2010*:

\[ \sum_{h, S_h} \int \frac{d^4 P_h}{(2\pi)^3} \delta(P_h^2 - M_h^2) P_h^\mu \Delta^h(k, P_h, S_h) = k^\mu \Xi^{uncut}(k) \]

- Dressed quark propagator as “average” on-shell four momentum produced by hadronization

- Dirac projections give momentum sum rules for TMD FFs!
Dirac structures

- TMD Fragmentation Functions

\[ \Delta^h(z, P_{h\perp}) = \frac{1}{2} \gamma^+ D_1^h + \frac{M_h}{2 P_h^-} E^h \| + \frac{P_{h\perp}}{2z P_h^-} D^h \| + \text{quark polarized terms} \]

- For the inclusive jet correlator

\[ J(k^-, k_T) = \frac{1}{2} \gamma^+ + \frac{M_j}{2 k^-} \| + \frac{k_T}{2k^-} + \text{higher-twist terms} \]
Mass sum rule

- Projecting the sum rule onto the identity matrix,

\[ M_j = \sum_{h, S_h} \int dz M_h E^h(z) \]

jet/quark mass as average of produced hadron masses weighted by chiral-odd FFs

- Dynamical mass component:

EOM relations:

\[ E = \tilde{E} + \frac{m_q}{M_h} z D_1 \]

neglecting q-g-q correlations

"WW approx."

\[ M_j = m_q \]

full QCD

Dynamical mass!

\[ m_{\text{corr}} = \sum_{h, S_h} \int dz M_h \tilde{E}^h(z) \]

Expect non-zero in \( \chi \)-limit → observable \( \chi \)-symmetry order parameter!
Full set of sum rules

- Sum rules for quarks into unpolarized hadrons, up to twist-3
  - (only thing missing for twist-4: full FF-TMD analysis)

\[ \sum_{h \text{, } S_h} \int dzzD_{1}^{h}(z) = 1 \quad \text{(Collins-Soper)} \]
\[ \sum_{h \text{, } S_h} \int dzM_{h}E^{h}(z) = M_{j} \]
\[ \sum_{h \text{, } S_h} \int dzM_{h}H^{h}(z) = 0 \]
\[ \sum_{h \text{, } S_h} \int dzzM_{h}H_{1}^{\perp (1)}^{h}(z) = 0 \quad \text{(Schaefer-Teryaev)} \]
\[ \sum_{h \text{, } S_h} \int dzM_{h}^{2}D_{1}^{\perp (1)}^{h}(z) = 0 \]
\[ \sum_{h \text{, } S_h} \int dzM_{h}^{2}G_{1}^{\perp (1)}^{h}(z) = 0 \]

\[ \sum_{h \text{, } S_h} \int dzM_{h}^{2} \tilde{E}^{h}(z) = M_{j} - m_{q0} = m_{q}^{\text{corr}} \]
\[ \sum_{h \text{, } S_h} \int dzM_{h}^{2} \tilde{H}^{h}(z) = 0 \quad \text{(Diehl-Sapeta)} \]
\[ \sum_{h \text{, } S_h} \int dzM_{h}^{2} \tilde{D}_{1}^{\perp (1)}^{h}(z) = -\frac{1}{2} \langle P_{\perp}^{2} / z \rangle \]
\[ \sum_{h \text{, } S_h} \int dzM_{h}^{2} \tilde{G}_{1}^{\perp (1)}^{h}(z) = 0 . \]
Some phenomenology
Inclusive DIS with jet correlators

- At large $x$, limited available invariant mass $W^2 \to$ jet-like final state

Jet correlators: $\to$ non-asymptotic quark states / dressed quarks

\[ \Xi_{ij}(l, n_+) = F.T. \langle \Omega | W_{(+\infty, \xi)}^{n+} \psi_i(\xi) \bar{\psi}_j(0) W_{(0, +\infty)}^{n+} | \Omega \rangle \]

\[ (\Xi^\mu_A)_{ij} = F.T. \langle \Omega | W_{(+\infty, \xi)}^{n+} gA^\mu(\xi) \psi_i(\xi) \bar{\psi}_j(0) W_{(0, +\infty)}^{n+} | \Omega \rangle \]
g2 structure function revisited

- Integrating SIDIS, and using EOM, Lorentz Invariance Relations:

\[ g_2(x_B) - g_2^{WW}(x_B) = \frac{1}{2} \sum_a e_a^2 \left( g_2^{q, tw3}(x_B) + \frac{m_q}{M} \left( \frac{h_1^q}{x} \right)^* (x_B) + \frac{M_j - m_q}{M} \frac{h_1^q(x_B)}{x_B} \right) \]

\[ \equiv g_2^{\text{quark}} \]
\[ \equiv g_2^{\text{jet}} \]

Consequences:

- h1 accessible in inclusive DIS
  \[ \leftrightarrow \] Potentially large signal

- Burkardt-Cottingham sum rule broken

\[ \int_0^1 g_2(x) = (M_j - m_q) \int_0^1 dx \frac{h_1(x)}{x} \]

- ETL: novel way to measure tensor charge

\[ \int_0^1 x g_2^{q-\bar{q}}(x) = 2 (M_j - m_q) \int_0^1 dx h_1^{q-\bar{q}}(x) \]
Measuring the jet correlator

Jet mass $M_{\text{jet}}$ can be measured in polarized $e^+ + e^-$:

- Needs LT asymmetry in semi-inclusive Lambda production

$$\frac{d\sigma^L(e^+e^- \rightarrow \text{jet } h \ X)}{d\Omega dz}$$

$$= \frac{3\alpha^2}{Q^2} \lambda_e \sum \epsilon_\alpha^2 \left\{ \frac{C(y)}{2} \lambda_h G_1 + D(y) |S_T| \cos(\phi_S) \frac{2M_h}{Q} \left( \frac{G_T}{z} + \frac{M_q - m_q}{M_h} H_1 \right) \right\}$$

- Similarly a LU asymmetry in unpolarized dihadron production
**χ-odd phenomenology at large x**

*AA, Bacchetta, Melnitchouk, Schlegel, 2009*

\[ g_2(x_B) = \text{“usual”} + \frac{m^{corr}}{M} \frac{h_1^q(x_B)}{x_B} \]

\[ \int_0^1 dx g_2(x) = m^{corr} \int_0^1 dx \frac{h_1(x)}{x} \neq 0 \]

*AA, Bacchetta, PLB 2017
AA, Signori, PoS(DIS2018)*

... and more: the door is now open...
Conclusions
Conclusions

- We can quantitatively connect quark fragmentation to the dynamical generation of mass
  - **Gauge invariant definition for dressed quark mass,** $M_j$
  - The dynamical component $m_{corr} = M_j - m_q$ is recognized as an **observable order parameter for $\chi$-symmetry breaking**

$$m_{corr} = \sum_{h,S_h} \int dz M_h \tilde{E}^h(z)$$

- **Novel phenomenology:**
  - Transversity in g2, same side di-hadrons, ...

- **New sum rules:** guidance for future fits

- **New spectral convolution technique** for treating Wilson lines

- **Theory to-do:** renormalization of J, connection to OPE, ...

**Practical exp. recipe:**
- measure $\tilde{E}$, obtain $m_{corr}$
- flavor decomposition, too!
Backup
Inclusive DIS with jet correlators

Jet correlators: $\rightarrow$ non-asymptotic quark states / dressed quarks

$$\Xi_{ij}(l, n_+) = F.T. \langle \Omega | W_{(+\infty, \xi)}^{n_+} \psi_i(\xi) \bar{\psi}_j(0) W_{(0, +\infty)}^{n_+} | \Omega \rangle$$

$$\left(\Xi^\mu_A\right)_{ij} = F.T. \langle \Omega | W_{(+\infty, \xi)}^{n_+} g A^\mu(\xi) \psi_i(\xi) \bar{\psi}_j(0) W_{(0, +\infty)}^{n_+} | \Omega \rangle$$
g2 structure function revisited

Integrating SIDIS, and using EOM, Lorentz Invariance Relations:

\[
g_2(x_B) - g_2^{WW}(x_B) = \frac{1}{2} \sum \sigma e(a)^2 \left( g^{q,tw3}_2(x_B) + \frac{m_q}{M} \left( \frac{h_1^q}{x} \right)^* (x_B) + \left( \frac{M_j - m_q}{M} \frac{h_1^q(x_B)}{x_B} \right) \right)
\]

Consequences:

- h1 accessible in inclusive DIS
  ↔ Potentially large signal
- Burkardt-Cottingham sum rule broken
  \[
  \int_0^1 g_2(x) = (M_j - m_q) \int_0^1 dx \frac{h_1(x)}{x}
  \]
- ETL: novel way to measure tensor charge
  \[
  \int_0^1 x g_2^{q,q}(x) = 2 (M_j - m_q) \int_0^1 dx h_1^{q-q}(x)
  \]
Measuring the jet correlator

Accardi, Bacchetta, Signori, Radici, in progress

- Jet mass $M_{\text{jet}}$ can be measured in polarized $e^+ + e^-$:

\[
\frac{d\sigma^L(e^+e^- \rightarrow \text{jet } h X)}{d\Omega dz} = \frac{3\alpha^2}{Q^2} \lambda_e \sum_a e_a^2 \left\{ \frac{C(y)}{2} \lambda_h G_1 + D(y) \left| S_T \right| \cos(\phi_S) \frac{2M_h}{Q} \left( \frac{G_T}{z} + \frac{M_q - m_q}{M_h} H_1 \right) \right\}
\]

- Needs LT asymmetry in semi-inclusive Lambda production

- Similarly a LU asymmetry in unpolarized dihadron production
Where can we measure jet correlators?

- Can we get a (polarized) e+ e- collider at JLab / BNL?
  - At JLab12? EIC + positron beam?

- Are existing facilities enough?

<table>
<thead>
<tr>
<th></th>
<th>BEPC</th>
<th>super KEKB</th>
<th>ILC</th>
<th>JLab/BNL</th>
</tr>
</thead>
<tbody>
<tr>
<td>E beam [GeV]</td>
<td>1.9</td>
<td>4 (e^-) 7 (e^-)</td>
<td>250</td>
<td>?</td>
</tr>
<tr>
<td>√s [GeV]</td>
<td>3 – 5</td>
<td>10</td>
<td>500</td>
<td>?</td>
</tr>
<tr>
<td>polarization</td>
<td>?</td>
<td>maybe</td>
<td>80% e^- 60% e^+</td>
<td>YES!</td>
</tr>
</tbody>
</table>

- What else?
A new “universal” fits

- Chiral-odd collinear sector across processes:

\[ (\text{Di})e^+e^- \]

- **DIS**
  - \( M_{\text{jet}} h_1 \)

- **(Di)SIDIS**
  - \( H_1^< \otimes h_1 \)

\[ M_{\text{jet}} H_1^< \quad H_1^< \otimes H_1^< \]