To the issue of Geodesics and Torsion in Riemannian geometry and Theory of the Gravitation [1]

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Abstract A simple differential analysis of the issue of correspondence between geodesics in gravitation theory of GTR and straights of inertial motion in the Minkowski' space-time discovers that, conventional certification of the **geodesics** in GTR is not compatible with the existence of the **Riemann-Christoffel curvature tensor** (RCT). We show that, a resolution of this crisis consists of a natural **extension of the Christoffels** in the dynamic law to **general connectedness form** including a triadic asymmetric tensor (named the **moderator**). The correspondent Riemann **supertensor** (RST) form, unavoidably annihilating by certification of the moderate geodesics, gives birth to **torsion** (skew-symmetric part of the moderator) and the **gravitensor** (the even-symmetric part); both arrive as a functional of the RCT and become an indispensable integral part of structure of the gravitational field. The **equivalence principle still actual** yet becomes enriched in the content. The **Einstein-Hilbert law** of the metric to matter connection **remains unchanged** at the produced correction of the gravitational dynamics. The produced completion of the connectedness leads also to renormalization of metric in the particle and spin gravitational dynamics. We pay attention to possible implication of torsion in the elementary interactions.

[1] Y. Derbenev, arXiv.org, 2023

Outline

- Introduction
- Elements of GTR and Gravitation Theory
- Questions to certification of the Geodesics
- Curvature Tensor / Geodesics *incompatibility paradox* , and *solution*
- ***Gravitensor*** and ***Torsion*** : integral parties of the upgrade GTR concept
- Renormalization of metric
- *Spin* in GTR
- Summary and Conclusion

Introduction: ***The GTR enterprise***

- ***Problems of the relativistic gravitation theory (after the Newton' theory and special relativity)):***
- Derive a relativistic law of particle motion in a G-field
- Attain a 4-fold covariant extension of the Newton' law of the attraction
- Math.?

- ***Einstein' genius approach:***
- Equivalence principle (start idea)
- Math.: the Riemann-Minkowski geometry objects and methods
- Metric tensor as potential of the G-force
- Particle trajectories in a G-field shall be the Riemann's geodesics
- Find out the ***metric to matter*** connection

Principles and basic constituents of GTR and related Gravitation Theory

- *The Equivalence Principle*

- There are three different but essentially conjugate aspects of the gravitation theory established by A. Einstein.
- ***The first one*** was the eliciting of nature of the *gravitational field,* GF or Christoffel symbols (CS) as associated with the 4-fold gradient of the space-time *metric tensor*.
- ***The second one*** was establishing of the law of particle motion (form of particle *acceleration* in gravitation field), expressed by the rule of *geodesic lines* (*geodesics*, GLs).
- ***The third one*** was establishing of a law connecting metric tensor to the *matter* (substance with the related inner interactions). Such a law in Einstein' concept is expressed in connection of the ***Riemann-Christoffel tensor*** (RCT, *curvature form*) to the ***energy-momentum tensor*** of the matter.

Riemannian space, metric and Christoffels

Riemannian space is a manifold of free variables (coordinates) x^k with introduced tensor w_{lm} :

- $w_{lm} = w_{ml}; \qquad det w_{lm} \neq 0$ (*metric tensor*)

Christoffel symbols There is a well-known symmetric connectedness employed in GTR, Γ^m_{nk}, defined by a condition:

$$\Gamma^m_{nk} \equiv \frac{1}{2} w^{ml}(\partial_n w_{kl} + \partial_k w_{nl} - \partial_l w_{nk}) = \Gamma^m_{kn}$$

Primal CDs of vector functions :

$$\hat{\partial}_k g^m \equiv \partial_k g^m + \Gamma^m_{nk} g^n; \qquad \hat{\partial}_k f_m \equiv \partial_k f_m - \Gamma^n_{mk} f_n \,;$$

- CDs of vectors and tensors are tensors

Euclidian space and geometry

Euclidian space (ES) is a *Riemann* space wherein does exist a frame x^{k_0}(*Cartesian* one) in which metric tensor $E_{l_0 m_0}$ is constant over the space:

$$E_{l_0 m_0} = const \ \left(i.e.\ \partial_{k_0} E_{l_0 m_0} = 0; \qquad \Gamma^{m_0}_{n_0 k_0} = 0.\right).$$

- *Euclidian geometry in Cartesian frame* (τ *is a canonical parameter of the* $4-fold$ *world lines*) *:*

$\partial_{k_0} g^{n_0} = 0;$ (*vectors of straights* g^{n_0} constant in space) $\longrightarrow$ $x^{n_0}(\tau) = g^{n_0}\tau + const$

- *Euclidian geometry in a curved frame:*

Vectors E^{n_0} constant in Cartesian frame become *functions of coordinates* in a *curved* frame:

$$E^{n_0} = const \quad \longrightarrow \quad E^m(x) = A^m_{n_0}(x)\, E^{n_0}$$

- *Forms of metric tensor and Christoffels in a curved frame of ES:*

$$E_{l_0 m_0} \longrightarrow E_{lm}(x) \equiv A^{l_0}_l(x) A^{m_0}_m(x) E_{l_0 m_0}; \qquad \Gamma^m_{nk} \Longrightarrow \Gamma^m_{nk}(E_{pq}) \equiv \bar{\Gamma}^m_{nk} = -A^{n_0}_n \partial_k A^m_{n_0}$$

- *Equation for vectors of straights in a curved frame of ES:* $\partial_k E^m + \bar{\Gamma}^m_{nk} E^n = 0$
- *Equation for straight lines in ES in a curved frame*

$$\frac{d}{d\tau} E^m(x) = \frac{\partial E^m}{\partial x^k}\frac{dx^k}{d\tau} \equiv E^k \partial_k E^m = -\,\bar{\Gamma}^m_{nk} E^n E^k; \qquad \longrightarrow \qquad \frac{d^2 x^m}{d\tau^2} = -\bar{\Gamma}^m_{nk}(x)\frac{dx^n}{d\tau}\frac{dx^k}{d\tau}.$$

Conventional introduction of the geodesics in GTR (1st receipt)

1. Primal covariant extension of the inertia law, based on the reasoning []:

- Effect of the gravitation in whole consists of change of the metric tensor: $w_{mn} = const \quad \longrightarrow \quad w_{mn}(x)$
- Gravitation field (GF) is directly and generally represented by the Christoffel symbols (CS)

- $du^m = 0 \quad (\text{Cartesian frame in ES, no GF})$
- $du^m = -\Gamma^m_{nk} u^n dx^k; \quad \frac{dx^k}{d\tau} = u^k$ (in GF, curved frame)
- Then: $\frac{du^m}{d\tau} + \Gamma^m_{nk} u^n u^k = 0\,; \quad \longrightarrow \quad \frac{d^2 x^m}{d\tau^2} = -\Gamma^m_{nk} \frac{dx^n}{d\tau} \frac{dx^k}{d\tau}.$
- / τ is time integral in the local rest frame of a particle/
- u^k is the 4-vector of particle velocity (4-momentum / rest mass)

Conventional introduction of the geodesics in GTR (2nd receipt)

- Resorting to *Variation Principle* with action like in ES but with changeable metric

  (according to the accepted paradigm:

$$w_{mn}(x) \Longrightarrow gravitation)$$

  results in same geodesic equations:

$$\frac{du^m}{d\tau} = -\Gamma^m_{nk} u^n u^k \ .$$

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Strict Riemannian space (SRS) and geometry

Riemann-Christoffel tensor (RCT)

- In differential geometry and in GTR, the *RCT form* is playing a key role:

$$R^m_{n;kl} \equiv \boldsymbol{R}_{kl} \equiv \partial_k \boldsymbol{\Gamma}_l - \partial_l \boldsymbol{\Gamma}_k + [\boldsymbol{\Gamma}_k , \boldsymbol{\Gamma}_l] = -\boldsymbol{R}_{lk} \,; \qquad \boldsymbol{\Gamma}_k \equiv \Gamma^n_{mk}$$

(symbol [,] means commutator of Γ^m_{nk} and Γ^m_{nl} considered as matrixes on indexes m, n).

As known, RCT can be elicited by exposition from the 2nd CD of vector functions :

- $(\hat{\partial}_k \hat{\partial}_l - \hat{\partial}_l \hat{\partial}_k) G^m \Longrightarrow R^m_{n;kl} G^n; \quad and\ (or): \quad (\hat{\partial}_k \hat{\partial}_l - \hat{\partial}_l \hat{\partial}_k) S_m \Longrightarrow -R^n_{m;kl} S_n\,,$

  under the following conditions ***(our accent !):***

$$\hat{\partial}_k G^m \equiv \partial_k G^m + \Gamma^m_{nk} G^n \neq 0; \qquad \hat{\partial}_k S_m \equiv \partial_k S_m - \Gamma^n_{mk} S_n \neq 0$$

- Here G^m and S_m are the contra-and co-variant vector functions of arbitrary (free) direction. They should be recognized as associated with group of basic elements of the SRS geometry. In the context of consideration of role they play in exposition of the RCT (), we call them the *mediators.*
- Form $\boldsymbol{R}_{kl}$ arrives a tensor, once transformation law () for $\boldsymbol{\Gamma}_k$ is assumed or stated, and vice versa.

RCT /CS criterions

- If $\boldsymbol{R}_{kl}$= 0, then CS $\boldsymbol{\Gamma}_k$ can be turned to zero overall the manifold (case of the ES)
- In ES, $\boldsymbol{\Gamma}_k$ may not be zero (curved frame !), but $\boldsymbol{R}_{kl} \equiv 0$ (proved elementary)
- When $\boldsymbol{R}_{kl} \neq 0,$ the CS cannot be turned to zero over a region of the manifold

Metric to matter connection /Einstein-Hilbert equation/

- Relativistic concept of the gravitation field.
- Concept of the gravitation field (GF) introduced by A. Einstein and D. Hilbert in 1915 consists of a statement of the metric tensor w_{ik} coupling to the energy – momentum tensor of the matter M_{ik} (our notation):

- $R_{ik} + \frac{1}{2} w_{ik} R = \kappa M_{ik}$.
- Here: $R_{ik} \equiv R^{l}_{i;kl} = R_{ki}$; $R \equiv w^{ik} R_{ik}$; (19)
- and κ is the gravitation constant. This equation substantiates metric tensor w_{ik} as function of the coordinates, connecting it to the matter.

- The built up the gravitation concept (the GF theory + the dynamic law in form of the geodesics) looks a complete and consistent system.
- However, an undiscovered devil appears living there silently.

Questions to the *Conventional introduction of the geodesics in GTR (1st receipt)*

1. Primal covariant extension of the inertia law, based on the reasoning []:

- Effect of the gravitation in whole consists of change of the metric tensor: $w_{mn} = const \quad \longrightarrow \quad w_{mn}(x)$
- Gravitation field (GF) is directly and generally represented by the Christoffel symbols (CS)

- $du^m = 0 \quad (\text{Cartesian frame in ES, no GF})$
- $du^m = -\Gamma^m_{nk} u^n dx^k; \quad \frac{dx^k}{d\tau} = u^k$ (in GF, curved frame)
- Then: $\frac{du^m}{d\tau} + \Gamma^m_{nk} u^n u^k = 0\,; \quad \longrightarrow \quad \frac{d^2x^m}{d\tau^2} = -\Gamma^m_{nk} \frac{dx^n}{d\tau}\frac{dx^k}{d\tau}.$
- ***Missed question***: How about a curved frame in ES ??
- ***Note 1***: in a curved frame of ES metric and particle velocity become functions of the coordinates x^k, so it appears that (proved elementary, as shown above): $\partial_k u^m + \Gamma^m_{nk} u^n = 0.$
- ***Note 2*** ***:*** Differential law for vector function must be written in covariant derivatives. However, replacement ordinary derivative with the PCD itself cannot constitute a new law corresponding the Riemannian geometry of GTR ; tensor addition is inquired in order to correspond existence of the RCT, as shown above.

Questions to the *Conventional introduction of the geodesics in GTR (2nd receipt)*

"2. Resorting to *Variation Principle* with action like in ES but with changeable metric

(according to the accepted paradigm: $w(x) \equiv gravitation)$ results immediately in

same geodesic equations: $\frac{du^m}{d\tau} + \Gamma^m_{nk} u^n u^k = 0$. "

- By the way, taking into account relations: $\frac{du^m}{d\tau} = \frac{\partial u^m}{\partial x^k}\frac{dx^k}{d\tau} \equiv u^k \partial_k u^m$, one can disclose () to equations for particular PCD:

$$\frac{du^m}{d\tau} + \Gamma^m_{nk} u^n u^k = (\partial_k\, u^m + \Gamma^m_{nk} u^n) u^k \equiv u^k \hat{\partial}_k u^m = 0: \quad \longrightarrow \quad \hat{\partial}_k u^m = 0.$$

- Natural and universal algebraic reduction of these equations is acceptance of zero PCD of vector function $u^m(x)$, i.e.: $\hat{\partial}_k u^m = 0$. However, it leaves us with equations for straights in ES written in terms of a curved frame, as shown below (crisis ?...).

Connectedness dilemma, and potential release

Dilemma in whole:

- Option A : $\partial_k u^m + \Gamma^m_{nk} u^n = 0$; in this case $R^m_{n;kl} \Longrightarrow 0$ (proven in the next slide)
- Option B: $\partial_k u^m + \Gamma^m_{nk} u^n \neq 0$;

  here , for our autonomic vector u^m we can assume:

$$\partial_k u^m + \Gamma^m_{nk} u^n = -\mathrm{T}^m_{nk} u^n;$$

here T^m_{nk} is a triadic tensor, generally asymmetric. Below we will find out connection of this tensor to the RCT $R^m_{n;kl}$.

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Mediator theorem 1 of a Strict Riemann Space

- Remind the *mediator issue* : when applying the exposition procedure to recognize the RCT, we have to mean that there should exist vector functions G^m *and* S_m *(mediators)* with a non-zero PCD*:*

$$\hat{\partial}_k G^m \neq 0; \quad \hat{\partial}_k S_m \neq 0.$$

Question: is the opposite case (i.e. existence of VF with zero PCD) compatible with existence of the RCT ?

Mediator theorem 1 *:* ***SRS geometry excludes existence of vector functions with zero PCD***

Namely: $\hat{\partial}_k g^m = 0$ *and (or)* $\hat{\partial}_k f_m = 0$ $results\ in$: $\boldsymbol{R}_{kl} \equiv \partial_k \boldsymbol{\Gamma}_l - \partial_l \boldsymbol{\Gamma}_k + [\boldsymbol{\Gamma}_k\,, \boldsymbol{\Gamma}_l] = 0.$

Proof Let us consider the alternate second ordinary derivative of $g^m(x)$, a subject of equation ().

Since $\partial_k \partial_l g^m - \partial_l \partial_k{}^m \equiv 0,$ it follows immediately from vector equation (51), that:

$$\partial_k(\Gamma^m_{nl} g^n) - \partial_l(\Gamma^m_{nk} g^n) = 0.$$

Opening the derivatives and using, again, equation (), we find:

$$R^m_{n;kl} g^n = 0.$$

Similar with respect of f_m: $R^n_{m;kl} f_n = 0.$

- These equations can be referred to arbitrary direction of vectors $g^n$and f_m at arbitrary point and overall matrices $\boldsymbol{R}_{kl}$, so we have to conclude:

$$\boldsymbol{R}_{kl} = \partial_k \boldsymbol{\Gamma}_l - \partial_l \boldsymbol{\Gamma}_k + [\boldsymbol{\Gamma}_k\,, \boldsymbol{\Gamma}_l] = 0.$$

- Note that, the above pointed indication of conditions (), () of existing VFs of a non-zeroth PCD corresponds to the shown incompatibility of RCT with existence of the zeroth PCD VFs.

The Complete CD (CCD) and Riemann' supertensor form (RST)

As known, applying procedure of the alternate second CCD to vectors V^n and U_m, ones obtain:

$$(\nabla_k \nabla_l - \nabla_l \nabla_k) V^m = \mathrm{G}^m_{n;kl} V^n + 2\overline{\mathrm{T}}^n_{kl} \nabla_n V^m \,; \qquad (\nabla_k \nabla_l - \nabla_l \nabla_k) U_m = -\mathrm{G}^n_{m;kl} U_n + 2\overline{\mathrm{T}}^n_{kl} \nabla_n U_m \,; \qquad \overline{\mathrm{T}}^n_{kl} = \frac{1}{2}(\mathrm{G}^m_{kl} - \mathrm{G}^m_{lk})$$

here: $\mathrm{G}^m_{n;kl} \equiv \mathbf{G}_{kl} \equiv \partial_k \mathbf{G}_l - \partial_l \mathbf{G}_k + [\mathbf{G}_k \,, \mathbf{G}_l] = -\mathbf{G}_{lk}$

- In the context of general characterization of form $\mathbf{G}_{kl}$, we call it the ***Riemann supertensor form*** **(RST)**
- It is straightforward to derive a relation between two forms, RSF and RCT:

$$\mathbf{G}_{kl} = \mathbf{R}_{kl} + \mathbf{T}_{kl};$$

here $\mathbf{T}_{kl}$ is tensor form structured on tensor-moderator $\mathbf{T}_k$:

$$\mathbf{T}_{kl} \equiv \hat{\partial}_k \mathbf{T}_l - \hat{\partial}_l \mathbf{T}_k + [\mathbf{T}_k \,, \mathbf{T}_l] \,.$$

- Certification of a non-zero form $\mathbf{G}_{kl}$ would require absence of vectors with zero CCD. But it would be a case contrary to establishment of GVs G^m and S_m whose CCDs are zero :

$$\nabla_k G^m \equiv \partial_k G^m + \widehat{\mathrm{G}}^m_{nk} G^n = 0; \qquad \nabla_k S_m \equiv \partial_k S_m - \widehat{\mathrm{G}}^n_{mk} S_n = 0; \qquad \widehat{\mathrm{G}}^n_{mk} \equiv \Gamma^m_{nk} + \mathrm{T}^m_{nk}$$

- We have to preclude that, $\mathbf{G}_{kl} \Longrightarrow 0$; (proven next below).

Mediator theorem 2 : *Annihilation of the Riemann supertensor form*

- ***Theorem 2*:** RST form annihilates due to zero CCD of the Geodesic Vectors

  Namely, it follows from zeroth CCD of geodesic vectors that:

$$\partial_k \partial_l G^m = -\partial_k (\mathrm{G}^m_{nl} G^n); \quad \text{hence, } \partial_k (\mathrm{G}^m_{nl} G^n) - \partial_l (\mathrm{G}^m_{nk} G^n) = 0,$$

and, in a complete correspondence with the above consideration with zeroth PCD, we have to conclude:

$$\mathbf{G}_{kl} \equiv \partial_k \mathbf{G}_l - \partial_l \mathbf{G}_k + [\mathbf{G}_k , \mathbf{G}_l] = 0 ,$$

i.e. :

$$\mathbf{R}_{kl} + \mathbf{T}_{kl} = 0.$$

Tensor $\mathbf{T}_{kl}$ can be called *curvature anti-tensor (CAT)*.

- It should be stressed that, annihilation of tensor form $\mathbf{G}_{kl}$ appears an intrinsic attribute of the Riemannian geometry with its culture of the geodesics.

- However, there is a crucially constructive internal content in the nullification of the RST. Namely, the RCT now becomes a driver for tensor-moderator $\mathbf{T}_k$:

$$\mathbf{G}_{kl} = \mathbf{R}_{kl} + \mathbf{T}_{kl} = 0; \quad \longrightarrow \quad \mathbf{T}_{kl} = -\mathbf{R}_{kl} \,,$$

i.e.:

$$\hat{\partial}_k \mathbf{T}_l - \hat{\partial}_l \mathbf{T}_k + [\mathbf{T}_k \,, \mathbf{T}_l] = -\mathbf{R}_{kl} \,,$$

or:

$$\partial_k \mathbf{T}_l - \partial_l \mathbf{T}_k + [\boldsymbol{\Gamma}_k \,, \mathbf{T}_l] - [\mathbf{T}_k \,, \boldsymbol{\Gamma}_l] + [\mathbf{T}_k \,, \mathbf{T}_l] = -\mathbf{R}_{kl} \,.$$

- Investigation of this equation towards setting a solution etc goes beyond the scope of this report.
- What can be outlined immediately, is that, there is no indication of that, either one of the two symmetry party of tensor T_{nk}^{m}, *torsion* or *g-tensor*, could be canceled to zero.
- Thinking generally, one may consider that, asymmetry of the tensor-moderator T_{nk}^{m} on the covariant indexes n and k is due to the distinction in the nature of their implication in equations of the Tensor-moderator connection to RCT.

Geodesic Invariant and renormalization of Metric

- *Conventional vector norm:* $\boldsymbol{u}^2 = w_{mn} u^m u^n = u_m u^m = const$ *(Euclidian space)*

- $\partial_k u^m = -G^m_{nk} u^n; \quad \partial_k u_m = G^n_{mk} u_n; \quad \longrightarrow \quad \partial_k (u^m u_m) = 0; \quad \boldsymbol{u}^2 = u^m u_m = inv = const$

- $u_m = \widehat{w}_{mn} u^n; \quad \nabla_k u_m = u^n \nabla_k \widehat{w}_{mn} = 0; \quad \longrightarrow \quad \nabla_k \widehat{w}_{mn} = 0;$
- $\widehat{w}_{mn} = w_{mn} + \lambda_{mn}$
- $\nabla_k \lambda_{lm} = -\nabla_k w_{lm} \equiv \mathrm{T}^n_{lk} w_{mn} + \mathrm{T}^n_{mk} w_{ln}; \quad \lambda_{lm} = \lambda_{ml}$
- $\hat{\partial}_k \lambda_{lm} - \mathrm{T}^n_{lk} \lambda_{mn} - \mathrm{T}^n_{mk} \lambda_{ln} = \mathrm{T}^n_{lk} w_{mn} + \mathrm{T}^n_{mk} w_{ln}$

Spin as a 4-vector. Spin Invariant and precession in Gravitation Field

4-vector of Spin:

$$\nabla_k S_m \equiv \partial_k S_m - \widehat{\mathrm{G}}^n_{mk} S_n = 0; \qquad \widehat{\mathrm{G}}^n_{mk} \equiv \Gamma^m_{nk} + \mathrm{T}^m_{nk}$$

- Scalar product of two 4-vectors is not only a transformation invariant, but also the dynamic invariant:

$$u^m S_m = inv; \qquad \partial_k (u^m S_m) = 0, \qquad i.e.: \quad u^m S_m = const$$

- Furthermore, two 4-vectors should be defined orthogonal, to avoid mixing their conceptual designations:

$$u^m S_m = 0.$$

Spin invariant

- $S^m = \widehat{w}^{mn} S_m; \qquad S^m S_m = \widehat{w}_{mn} S^m S^n = inv = const$
- $\widehat{w}_{mn} u^m S^n = 0$
- $\partial_k S^m = -\widehat{\mathrm{G}}^m_{nk} S^n = -(\Gamma^m_{nk} + \mathrm{T}^m_{nk}) S^n$

- ***Spin precession in GF*** along the geodesics:

$$\frac{dS^m}{d\tau} = u^k\, \partial_k S^m = -\widehat{\mathrm{G}}^m_{nk} u^k S^n = -\frac{1}{2}\left[\overline{\overline{\mathrm{G}}}{}^m_{nk}(u^k S^n + u^u S^k) + \overline{\mathrm{T}}^m_{nk}(u^k S^n - u^n S^k)\right]$$

The update gravitational dynamics

- *Gravitation field:*

$$GF = -\,\widehat{\mathrm{G}}^{m}_{nk} \equiv -(\Gamma^{m}_{nk} + \mathrm{T}^{m}_{nk})\ ;$$

- *Equation* (44) *for tensor-moderator* T^{m}_{nk} appears a complementary to equations for metric tensor (33) and geodesics (46), (47) in the presented adjusted gravitation concept of the GTR:

$$\hat{\partial}_k \mathbf{T}_l - \hat{\partial}_l \mathbf{T}_k + [\mathbf{T}_k\,, \mathbf{T}_l] = -\mathbf{R}_{kl}\ ; \qquad \mathbf{T}_k = \overline{\mathbf{T}}_k + \overline{\overline{\mathbf{T}}}_k; \qquad \overline{\mathbf{T}}_k \ \text{ - torsion}$$

- *Law for tangent vectors* $u^m(x)$ *of the geodesics:*

$$\partial_k u^m + \Gamma^{m}_{nk} u^n = -\mathrm{T}^{m}_{nk} u^n.$$

- *Gravitational 4-acceleration (GA), and geodesic equations:*

$$\frac{du^m}{d\tau} = -\left(\Gamma^{m}_{nk} + \overline{\overline{\mathrm{T}}}{}^{m}_{nk}\right) u^n u^k; \qquad \frac{d^2x^m}{d\tau^2} = -\left(\Gamma^{m}_{nk} + \overline{\overline{\mathrm{T}}}{}^{m}_{nk}\right) \frac{dx^n}{d\tau}\frac{dx^k}{d\tau}$$

- / We have to note the following: disposition of torsion effect in the orbital motion still not totally clarified (subject of analysis in terms of a specified frame of the coordinates) /
- Equations for the 4-vector of Spin

Summary

- 1. The elementary differential analysis of the basic establishment of GTR as theory of the gravitation conducted in the present paper has led to a conclusion that, the conventional definition of the geodesics in GTR is incompatible with the existence of the Riemann-Christoffel tensor.
- 2. Proposed resolution of the pointed contradiction consists of a natural extension of the Christoffel symbols in definition of the geodesics to a complete form of the connectedness incorporating asymmetric triadic tensor as a *moderator* term.
- 3. Similar analysis of the geodesic law, managed with the enriched connectedness, allows one to derive equation connecting the tensor-moderator to the Riemann-Christoffel tensor, in this way closing the pointed problem of inconsistence in GTR.
- 4. The tensor-moderator with its two parts, *torsion* and *gravitensor* (the even-symmetric part, contributing in the gravitational force) appears an indispensable integral structural element of the upgrade dynamical concept of the GTR.
- 5. We underline that, occurrence of the *gravitensor* as a symmetric tensor addition to Christoffels in the gravitational force does not abolish the *equivalence principle* posed by A. Einstein in foundation of the General Theory of the Relativity.
- 6. In the context of consideration of the phenomenon of torsion in Riemannian geometry, we have to distinct between the *gravitation field*

  and *gravitation force*
- 7. Change in the gravitational dynamic law of GTR considered in this paper does not touch the form of the Einstein-Hilbert equation that connects metric tensor to the matter.
- 8. The derived equations for the 4-vector of Spin include effect of the tensor-moderator
- 9. The elicited contradiction in formulation of the GTR concerns not only concept of the gravitation dynamic law of the GTR, but equally the consistence of the geometrical conception itself as a mathematical base in the treats and applications when ones restrict the certification of the *connectedness* object by the Christoffel symbols. Namely, we state that, a consistent certification of the geodesics can be realized only with implication of the tensor-moderator in dynamics of the tangent vectors of the geodesic lines. Tensor-moderator with its two party, torsion and gravitensor, arrives an integral element of Riemannian geometry certified with the existence of the curvature tensor. A uniqueness of this disposition consists of an essential circumstance that, tensor-moderator appears driven by the curvature tensor.

Conclusion

- The pointed out an inconsistence in the dynamical aspects of the gravitation concept of the GTR, and the described natural way to reconstruct certification of the geodesics and gravitational dynamics, in our understanding, may serve as a sign of that, the gravitation concept of GTR in the existing conventional formulation is essentially incomplete, thus suggesting further going explorations of general relativity and gravitation in principles and applications.
- In particular and, perhaps, first of all, the elicited presence of torsion in structure of the upgrade gravitational dynamics may lead potentially to investigations of possible effects of torsion' interference with the electrodynamics and forces of the strong and weak interactions.
- In this note we do not touch questions of a confrontation of the GTR dynamical concept with the astronomical observations; the related analysis deserves a separate consideration.

Thank you for your attention!