The role of Chiral Effective Field Theory in the precision era

Jose Manuel Alarcón
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• High demand of calculations from first principles with reliable error estimation.
  • Important to disentangle new physics from theoretical or systematic errors.
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- In the energy regimes of interest, chiral symmetry provides genuine predictions for hadronic interactions on QCD grounds.
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$P = -1$ $P = +1$
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**Introduction**

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**Chiral symmetry is:**
- Global symmetry derivative coupling of the Goldstone bosons.
- Spontaneously broken Constrains the interactions.
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J. M. Alarcón (UCM)

The role of Chiral EFT in the precision era
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• In the energy regimes of interest, chiral symmetry provides genuine predictions for hadronic interactions on QCD grounds.

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- Chiral EFT provides a way to incorporate systematically corrections to the low energy theorems.
- Theoretical progress in the recent years opened new possibilities in the field → Provide hadronic ME and nuclear corrections!
$\pi N$ - scattering
\( \pi N - \text{scattering} \)

- The fundamental purely hadronic interaction involving one nucleon.
$\pi N$ - scattering

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- $\pi N$ gives the long-range part of the 2NF (3fm).
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Heavy Baryon ChPT [Jenkins and Manohar, PLB 255 (1991)]

Infrared Regularization [Becher and Leutwyler, EPJ C9 (1999)]

Extended-On-Mass-Shell [Fuchs, Gegelia, Japaridze and Scherer, PRD68 (2003)]
We used EOMS to study $\pi N$ at low energies up to $O(p^3)$ [Alarcón, Martin Camalich and Oller, Ann. of Phys. 336 (2013)].
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- The low-energy phase shifts are used to determine the LECs. Used to extract valuable phenomenological information
$\pi N$ - scattering

Fits to WI08

$S_{31}$, $P_{31}$, $S_{11}$, $P_{11}$, $P_{33}$, $P_{13}$

$\Delta$-less ChPT, $\Delta$-ChPT

[Alarcón, Martin Camalich and Oller, Ann. of Phys. 336 (2013)]
### Threshold parameters

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### Pion-nucleon coupling \((d_{18})\)

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<td>1.0(2.5)%</td>
<td>2.0(4)%</td>
<td>4.5(7)%</td>
<td>2.1(1)%</td>
<td>0.2(1.0)%</td>
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<tr>
<td>( g_{\pi N} )</td>
<td>13.53(10)</td>
<td>13.00(31)</td>
<td>13.13(5)</td>
<td>13.46(9)</td>
<td>13.15(1)</td>
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<tr>
<td>$a_{q+}$</td>
<td>-1.1(1.0)</td>
<td>-0.12(33)</td>
<td>0.23(20)</td>
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<td>-0.10(12)</td>
<td>0.22(12)</td>
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<tr>
<td>$a_{q+}$</td>
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<td>8.33(44)</td>
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<td>9.2</td>
<td>8.83(5)</td>
<td>7.742(61)</td>
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<tr>
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<td>-8.5(6)</td>
<td>-7.47(22)</td>
<td>-10.0(4)</td>
<td>-8.4</td>
<td>-7.52(16)</td>
</tr>
<tr>
<td>$a_{S_{n1}}$</td>
<td>16.6(1.5)</td>
<td>16.6(9)</td>
<td>15.63(26)</td>
<td>17.5(3)</td>
<td>17.1</td>
<td>15.71(13)</td>
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<tr>
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<td>21.00(20)</td>
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<td>$a_{P_{n3}}$</td>
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<td>-3.159(67)</td>
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### Pion-nucleon coupling ($d_{18}$)

<table>
<thead>
<tr>
<th></th>
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<th>WI08 $\Delta$-ChPT</th>
<th>EM06 $\Delta$-ChPT</th>
<th>KA85 $\Delta$-ChPT</th>
<th>WI08 $\Delta$-ChPT</th>
<th>EM06 $\Delta$-ChPT</th>
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<tbody>
<tr>
<td>$\Delta_{GT}$</td>
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<td>1.0(2.5)%</td>
<td>2.0(4)%</td>
<td>4.5(7)%</td>
<td>2.1(1)%</td>
<td>0.2(1.0)%</td>
</tr>
<tr>
<td>$g_{\pi N}$</td>
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### Sigma-term ($c_1$)

<table>
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<th>WI08 $\Delta$-ChPT</th>
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<tbody>
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<td>$\sigma_{\pi N}$(MeV)</td>
<td>43(5)</td>
<td>59(4)</td>
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### Threshold parameters

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<th>KA85</th>
<th>WI08</th>
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<td>8.33(10)</td>
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<td>5.8(5)</td>
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<tr>
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<td>16.6(9)</td>
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<td>-2.4(3)</td>
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<td>-2.4(3)</td>
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<td>-3.00(32)</td>
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<td>-3.09(8)</td>
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### Pion-nucleon coupling ($d_{18}$)

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<th>KA85</th>
<th>WI08</th>
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<th>KA85</th>
<th>WI08</th>
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<td>$a_{3p1}$</td>
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<tr>
<td>$a_{3p31}$</td>
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<td>9.0(5)</td>
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### Sigma-term ($c_1$)

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<tr>
<td>$\sigma_{\pi N}$ (MeV)</td>
<td>43(5)</td>
<td>59(4)</td>
<td>59(2)</td>
<td>45(8)</td>
<td>64(7)</td>
<td>56(9)</td>
</tr>
</tbody>
</table>

Agreement with the PWA that provides the input. Never achieved before in ChEFT !!!
Agreement with the PWA that provides the input.

Never achieved before in ChEFT !!!
### Subthreshold region

Agreement with the PWA that provides the input. Never achieved before in ChEFT !!!

### Agreement with the dispersive results for first time!

Solves the problem found by Becher and Leutwyler!

---

Pion-nucleon coupling ($d_{18}$)

<table>
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<tr>
<th></th>
<th>KA85</th>
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<th>EM06</th>
<th>KA85</th>
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<td>0.00(6)</td>
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### Agreement with the dispersive results for first time!

Solves the problem found by Becher and Leutwyler!

---

**Table 12**

<table>
<thead>
<tr>
<th>$\sigma_{\pi N}$ (MeV)</th>
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<th>EM06</th>
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<td>$\Delta$-ChPT</td>
<td>$\Delta$-ChPT</td>
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<tr>
<td>43(5)</td>
<td>59(4)</td>
<td>59(2)</td>
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</table>

[Alarcón, Martin Camalich and Oller, Ann. of Phys. 336 (2013)]

---

**Table 13**

<table>
<thead>
<tr>
<th>Phase shifts in $0^+$</th>
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<tr>
<td>$\partial \upmu$</td>
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<td>$\varphi$</td>
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<tr>
<td>$\varphi$</td>
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<td>$\varphi$</td>
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</tbody>
</table>

---

**The role of Chiral EFT in the precision era**

J. M. Alarcón (UCM)
The pion-nucleon $\sigma$-term
The pion-nucleon $\sigma$-term

- The sigma-term is defined as

$$\sigma_{\pi N} = \frac{\hat{m}}{2m_N} \langle N | (\bar{u}u + \bar{d}d) | N \rangle$$
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• Fundamental quantity in QCD
The pion-nucleon $\sigma$-term

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$$\sigma_{\pi N} = \frac{\hat{m}}{2m_N} \langle N | (\bar{u}u + \bar{d}d) | N \rangle$$

- Fundamental quantity in QCD \(\rightarrow\) Measures of the strength of explicit chiral symmetry breaking.
The sigma-term is defined as

\[ \sigma_{\pi N} = \frac{\hat{m}}{2m_N} \langle N | (\bar{u}u + \bar{d}d) | N \rangle \]

- Fundamental quantity in QCD measures the strength of explicit chiral symmetry breaking.
- It is important on searches of physics beyond the standard model.
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- Fundamental quantity in QCD Measures of the strength of explicit chiral symmetry breaking.
- It is important on searches of physics beyond the standard model.
  - Dark Matter detection  
The sigma-term is defined as
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\]

• Fundamental quantity in QCD Measures of the strength of explicit chiral symmetry breaking.

• It is important on searches of physics beyond the standard model.
  • CP violation [de Vries, Mereghetti, Walker-Loud, PRC 92 (2015)]
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• Key to understand the origin of the mass of the ordinary matter:
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  \[
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  \]

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  - CP violation [de Vries, Mereghetti, Walker-Loud, PRC 92 (2015)]

- Key to understand the origin of the mass of the ordinary matter:
  \[
  m_N = \frac{1}{2m_N} \langle N | \theta^\mu_\mu | N \rangle = \frac{1}{2m_N} \langle N | \frac{\beta}{2g} G^\mu_\nu G^a_\mu_\nu + \sum_{q=u,d,s} m_q \bar{q}q + \ldots | N \rangle
  \]

“2nd Workshop on The Proton Mass; At the Heart of Most Visible Matter”, ECT*, April 2017, Trento.
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• Tension between the “canonical” value and the updated evaluation:
The pion-nucleon $\sigma$-term

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\[
\sigma \approx 45 \text{ MeV}, \quad \Sigma \approx 60 \text{ MeV}
\]
The pion-nucleon $\sigma$-term

- Tension between the “canonical” value and the updated evaluation:

\[ \sigma \approx 45 \text{ MeV}, \quad \Sigma \approx 60 \text{ MeV} \]

\[ \sigma_{\pi N} = 64 \text{ MeV} \quad \Sigma = 79 \text{ MeV} \]
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- Despite GWU utilizes updated experimental information, the lower value was more common in the literature.
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  - Restoration of chiral symmetry in nuclear matter at lower densities.

---

The pion-nucleon $\Sigma$ term is definitely large: results from a G.W.U. analysis of $\pi N$ scattering data

M.M. Pavan$^a$, R.A. Arndt$^b$, I.I. Strakovskiy$^b$ and R.L. Workman$^b$

$^a$University of Regina
TRIUMF, Vancouver, B.C. V6T-2A3, Canada
$^b$Center for Nuclear Studies, Department of Physics,
The George Washington University, Washington, DC 20052, U.S.A.

$\sigma_{\pi N} = 64 \text{ MeV} \quad \Sigma = 79 \text{ MeV}$
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  - Restoration of chiral symmetry in nuclear matter at lower densities.
  - Necessary to give a picture fully consistent with phenomenology!
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• However, the scatt. lengths from $\pi$-atoms point to a large $\sigma_{\pi N}$!
However, the scattering lengths from $\pi$-atoms point to a large $\sigma_{\pi N}$!

\[ \sigma_{\pi N} = \Sigma_d - (3.3 \pm 0.2) \text{ MeV} \]

[Gasser, Leutwyler and Sainio, PLB 253 (1991)]
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Solution A: Fit to data of $[P. Y. Bertin et al., NPB 106 (1976)]$

\[ a_{0+}^+ \approx -8 \times 10^{-3} M_\pi^{-1} \quad \rightarrow \quad \Sigma_d = 48 \pm 4 \pm 4 \pm 4 \text{ MeV} \]

Solution B: Fit to data of $[J. S. Frank et al., PRD 28 (1983)]$

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J. M. Alarcón (UCM)
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$\pi\text{-atoms}$ $[Baru, et al. NPA 872 (2011)]$

\[ a_{0+}^+ \approx -1 \times 10^{-3} M_\pi^{-1} \quad \text{Larger } \Sigma_d! \]
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$\pi\pi$-atoms [Baru et al., NPA 872 (2011)]

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Threshold parameters determine $\sigma_{\pi\pi N}$ [Olsson, PLB 482 (2000)]
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$$a_{0+}^+ \approx -10 \times 10^{-3} M_{\pi}^{-1} \quad \rightarrow \quad \Sigma_d = 50 \pm 3 \pm 7 \pm 4 \text{ MeV}$$

$\pi\pi$-atoms [Baru, et al., NPA 872 (2011)]

$$a_{0+}^+ \approx -1 \times 10^{-3} M_{\pi}^{-1} \quad \rightarrow \quad \text{Larger } \Sigma_d!$$

• Threshold parameters determine $\sigma_{\pi N}$ [Olsson, PLB 482 (2000)]

$$\bar{D}^+(0, 2M_{\pi}^2) = 14.5a_{0+}^+ - 5.06(a_{0+}^{(1/2)})^2 - 10.13(a_{0+}^{(3/2)})^2 - 5.55C(+) - 0.06a_{1-}^+ + 5.70a_{1+}^+ - (0.08 \pm 0.03)$$
The pion-nucleon $\sigma$-term

- However, the scattering lengths from $\pi\pi$-atoms point to a large $\sigma_{\pi N}$!

![Diagram showing pion-nucleon scattering lengths](Gasser, Leutwyler and Sainio, PLB 253 (1991))

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\[ a_{0+}^+ = 3.5(2.6) \times 10^{-3} M_\pi^{-1} \quad \text{[Gashi, et al., NPA 778 (2006)]} \]

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The pion-nucleon $\sigma$-term

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In order to recover $\sigma_{\pi N} = 45 \text{ MeV}$ one needs $a_{0+}^+ \sim -9 \times 10^{-3} M_{\pi}^{-1}$
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The pion-nucleon $\sigma$-term
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\(a_{0+}\):

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<td>(a_{0+})</td>
<td>(10^{-3} M_{\pi}^{-1})</td>
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The pion-nucleon $\sigma$-term

Convergence

$O(p^2)$

$O(p^3)$

$\sigma_{\pi N} = 78(4)$ MeV $- 19$ MeV $= 59(7)$ MeV

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- **Convergence**

\[ \mathcal{O}(p^2) \quad \mathcal{O}(p^3) \quad \mathcal{O}(p^{7/2}) \]

- $\mathcal{O}(p^2)$
  - $78(4)$ MeV

- $\mathcal{O}(p^3)$
  - $-19$ MeV

- $\mathcal{O}(p^{7/2})$
  - $-6$ MeV

\[ \sigma_{\pi N} = 78(4) \text{ LO} \quad -19 \text{ NLO} \quad (6) \text{ N}\text{LO} \text{ MeV} = 59 \pm 4(\text{stat.}) \pm 6(\text{sys.}) \text{ MeV} = 59(7) \text{ MeV} \]
The pion-nucleon $\sigma$-term

- **Convergence**

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<td></td>
<td>1</td>
<td>−19 MeV</td>
</tr>
<tr>
<td>$O(p^{7/2})$</td>
<td></td>
<td>1</td>
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$$\sigma_{\pi N} = 78(4) \text{ MeV} ^{\text{LO}} - 19(6) \text{ MeV} ^{\text{NLO}} - 6 \text{ MeV} ^{\text{N^2LO}} = 59(7) \text{ MeV}$$
The pion-nucleon $\sigma$-term

- **Convergence**

\[
\begin{align*}
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\[\sigma_{\pi N} = 78(4)_{\text{LO}} - 19(6)_{\text{NLO}} \text{ MeV} = 59\pm4(\text{stat.})\pm6(\text{sys.}) \text{ MeV} = 59(7) \text{ MeV}\]

- **Summarizing …**
The pion-nucleon $\sigma$-term

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  \[
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  \]

  \[
  \begin{array}{c}
  2 \\
  78(4) \text{ MeV} \\
  \end{array}
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  Modern $\pi N$ scattering data
The pion-nucleon $\sigma$-term

• Convergence

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\sigma_{\pi N} = \begin{cases} 
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The strangeness content of the nucleon

- The strangeness content of the nucleon is related to the sigma-term through

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\sigma_0 \equiv \frac{\hat{m}}{2m_N} \langle N | \bar{u}u + \bar{d}d - 2\bar{s}s | N \rangle
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<tr>
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- Compatible with modern experimental information.
- \( \sigma_s \) Compatible with LQCD.
Nucleon Polarizabilities & Lamb shift
Nucleon Polarizabilities & the Proton Radius Puzzle

- Nucleon Polarizabilities encode the response of the nucleon under electromagnetic probes.
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Show up in the theoretical prediction ( $\mathcal{O}(\alpha_{em}^5)$ ) of the proton radius through the Lamb shift $\Delta E_{2P-2S}$. 
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T^{\mu\nu}(P, q) = -\left( g^{\mu\nu} + \frac{q^{\mu}q^{\nu}}{q^2} \right) T_1(\nu^2, Q^2) + \frac{1}{M_p^2} \left( P^{\mu} - \frac{P \cdot q}{q^2} q^{\mu} \right) \left( P^{\nu} - \frac{P \cdot q}{q^2} q^{\nu} \right) T_2(\nu^2, Q^2)
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\[
\Delta E^{(pol)}_{2S} \approx \frac{\alpha_{\text{em}}^2}{\pi^2} \phi_n^2 \int_0^\infty \frac{dQ}{Q^2} w(\tau e) \left[ T_1^{(NB)}(0,Q^2) - T_2^{(NB)}(0,Q^2) \right] \begin{align*}
T_1^{(NB)} &= 4\pi Q^2 \beta_{M1}(Q^2) + \ldots \\
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\end{align*}
\]
The main contribution to the polarizabilities comes from the low $Q^2$ region.
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Important to reduce contributions from $Q^2 > \Lambda_{\chi_{SB}}^2$. 
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- Chiral EFT provides predictions of the leading contribution.
- Important to reduce contributions from \( Q^2 > \Lambda_{\chi SB}^2 \).

\[
\Delta E_{2S}^{(pol)} \approx \frac{\alpha_{em}}{\pi} \Phi_n^2 \int_0^{Q_{max}} \frac{dQ}{Q^2} w(\tau_{\ell}) \left[ T_1^{(NB)}(0, Q^2) - T_2^{(NB)}(0, Q^2) \right]
\]

\[
w(\tau_{\ell}) = \sqrt{1 + \tau_{\ell}} - \sqrt{\tau_{\ell}}
\]

\[
\tau_{\ell} = \frac{Q^2}{4m_{\ell}^2}
\]

\[\Delta E_{2S}^{(pol)} \text{ (\mu eV)}\]

\( Q_{max}^2 \text{ (GeV}^2\) \)

\[\{\text{Within the uncertainty of the calculation}\]

\[\{\text{Too large contribution from } Q^2 > \Lambda_{\chi SB}^2\}

[Alarcón, Lensky, Pascalutsa, EPJ C 74 (2014).]

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**Lamb shift**

- The relativistic structure is important to agree with phenomenological determinations of $\Delta E_{2S}^{(pol)}$.

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Form Factors & Proton Radius
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- Important to understand and solve the “Proton Radius Puzzle”.
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Form Factors with ChEFT

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![Diagram showing Feynman diagrams for pion transitions](image)

![Graph showing imaginary part of electromagnetic form factor](image)
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Higher order calculations become necessary Unpractical
\begin{align*}
\langle N(p', s')|J_\mu(0)|N(p, s)\rangle &= \bar{u}(p', s')[\gamma_\mu F_1(t) + \frac{i\sigma_{\mu\nu} q^\nu}{2m_N} F_2(t)] u(p, s) \\
J_\mu(x) &= \sum_{q=u,d,\ldots} e_q \bar{q}(x) \gamma_\mu q(x)
\end{align*}
Form factors and their analytic structure

\[ \langle N(p',s')|J_\mu(0)|N(p,s)\rangle = \bar{u}(p',s')[\gamma_\mu F_1(t) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_N}F_2(t)]u(p,s) \]

\[ J_\mu(x) = \sum_{q=u,d,...} e_q \bar{q}(x)\gamma_\mu q(x) \]

\[ G_E(t) = F_1(t) + \frac{t}{4m_N^2} F_2(t) \]

\[ G_M(t) = F_1(t) + F_2(t) \]

\[ G_{E,M}^{V,S} = \frac{1}{2}(G_{E,M}^p \mp G_{E,M}^n) \]
Form factors and their analytic structure

\[
\langle N(p', s')|J_\mu(0)|N(p, s)\rangle = \bar{u}(p', s') \left[ \gamma_\mu F_1(t) + \frac{i\sigma^{\mu\nu}q^\nu}{2m_N} F_2(t) \right] u(p, s)
\]

\[ G_E(t) = F_1(t) + \frac{t}{4m_N^2} F_2(t) \]

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Space-like region ( \( t < 0 \) )
Form factors and their analytic structure

\[ \langle N(p', s')|J_\mu(0)|N(p, s)\rangle = \bar{u}(p', s')[\gamma_\mu F_1(t) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_N} F_2(t)] u(p, s) \]

\[ J_\mu(x) = \sum_{q=u,d,...} e_q \bar{q}(x) \gamma_\mu q(x) \]

\[ G_E(t) = F_1(t) + \frac{t}{4m_N^2} F_2(t) \]

\[ G_M(t) = F_1(t) + F_2(t) \]

\[ G_{E,M}^{V,S} = \frac{1}{2}(G_{E,M}^p \mp G_{E,M}^n) \]

Space-like region (\( t < 0 \))

Time-like region (\( t > 0 \))

\[ t = -Q^2 \]
From unitarity + analyticity

$$\text{Im} G_{E,M} \propto \sum_h \int d\Pi_h \ M(\gamma^* \rightarrow h) M(h \rightarrow \bar{N}N)$$
Form factors and their analytic structure

- From unitarity + analyticity

\[ \text{Im} G_{E,M} \propto \sum_h \int d\Pi_h \, M(\gamma^* \to h)M(h \to \bar{N}N) \]
From unitarity + analyticity

$$\text{Im}G_{E,M} \propto \sum_h \int d\Pi_h \ M(\gamma^* \to h)M(h \to \bar{N}NN)$$
Form factors and their analytic structure

- From unitarity + analyticity

$$\text{Im} G_{E,M} \propto \sum_h \int d\Pi_h \, M(\gamma^* \rightarrow h) M(h \rightarrow \tilde{N}N)$$

 Isovector

 Isoscalar

 Isovector

 Isovector

 ...
From unitarity + analyticity

\[ \text{Im} G_{E,M} \propto \sum \int d\Pi_h \ M(\gamma^* \to h)M(h \to \bar{N}N) \]

Diagram:
- Isovector
- Isoscalar
- Isovector
From unitarity + analyticity

\[ \text{Im} G_{E,M} \propto \sum_h \int d\Pi_h \, M(\gamma^* \to h)M(h \to \bar{N}N) \]

\[ \text{Im} G_{V}^{E}(t) = \frac{k_{cm}^{3}}{m_{N}\sqrt{t}} F_{\pi}^{*}(t)f_{+}^{1}(t) \]

\[ \text{Im} G_{V}^{M}(t) = \frac{k_{cm}^{3}}{\sqrt{2t}} F_{\pi}^{*}(t)f_{-}^{1}(t) \]
Form factors and their analytic structure

• From unitarity + analyticity

\[ \text{Im} G_{E,M} \propto \sum_h \int d\Pi_h \, M(\gamma^* \rightarrow h) M(h \rightarrow \bar{N}N) \]

\[ \text{Im} G^V_E(t) = \frac{k^3_{cm}}{m_N \sqrt{t}} F^*_\pi(t) f^1_+(t) \]

\[ \text{Im} G^V_M(t) = \frac{k^3_{cm}}{\sqrt{2t}} F^*_\pi(t) f^1_-(t) \]

\[ \text{Im} G^V_{E,M}(t) = \frac{k^3_{cm}}{\{m_N, \sqrt{2}\} \sqrt{t}} F^*_\pi(t) f^1_\pm(t) \]
Form factors and their analytic structure

- From unitarity + analyticity

\[ \text{Im} G_{E,M} \propto \sum_h \int d\Pi_h \; M(\gamma^* \rightarrow h) M(h \rightarrow \bar{N}N) \]

\[ \text{Im} G_V^E(t) = \frac{k_\text{cm}^3}{m_N \sqrt{t}} F_\pi^*(t) f_+^1(t) \]

\[ \text{Im} G_V^M(t) = \frac{k_\text{cm}^3}{\sqrt{2t}} F_\pi^*(t) f_-^1(t) \]

Non-Perturbative
Form factors and their analytic structure

- From unitarity + analyticity

$$\text{Im} G_{E,M} \propto \sum_h \int d\Pi_h \ M(\gamma^* \rightarrow h) M(h \rightarrow \bar{N}N)$$

\[ \gamma^* \rightarrow \pi N \rightarrow \pi N \]

\[ \gamma^* \rightarrow F_\pi \rightarrow f_\pm \]

\[ \gamma^* \rightarrow \pi \rightarrow \pi \]

\[ \gamma^* \rightarrow \pi \rightarrow \pi \]

\[ \gamma^* \rightarrow \pi \rightarrow \pi \]

Isovector

Isoscalar

Isoscalar

Non-Perturbative

$$\text{Im} G_{E,M}^V(t) = \frac{k_{cm}^3}{m_N \sqrt{2} \sqrt{t}} F_\pi^*(t) f_\pm^1(t)$$

$$\text{Im} G_{E,M}^V(t) = \frac{k_{cm}^3}{\sqrt{2}t} F_\pi^*(t) f_\pm^1(t)$$
Form factors and their analytic structure

• From unitarity + analyticity

\[
\text{Im} G_{E,M} \propto \sum_h \int d\Pi_h \, M(\gamma^* \to h) M(h \to \bar{N}N)
\]

\[
\text{Im} G_V(t) = \frac{k_{\text{cm}}^3}{m_N \sqrt{t}} F_{\pi}^*(t) f_+^1(t)
\]

\[
\text{Im} G_M(t) = \frac{k_{\text{cm}}^3}{\sqrt{2t}} F_{\pi}^*(t) f_-^1(t)
\]

Non-Perturbative

[Frazer and Fulco, Phys. Rev. 117, 1609 (1960)]
From unitarity + analyticity

\[
\text{Im} G_{E,M} \propto \sum_h \int d\Pi_h \, M(\gamma^* \to h) M(h \to \bar{N}N)
\]

\[
\text{Im} G^V_{E,M}(t) = \frac{k_{cm}^3}{m_N \sqrt{t}} F^*_\pi(t) f_\pm(t)
\]

\[
\text{Im} G^V_{E,M}(t) = \frac{k_{cm}^3}{\sqrt{2}t} F^*_\pi(t) f_\pm(t)
\]

[Frazer and Fulco, Phys. Rev. 117, 1609 (1960)]
Form factors and their analytic structure

- From unitarity + analyticity

\[
\text{Im} G_{E,M} \propto \sum_h \int d\Pi_h \, M(\gamma^* \to h) M(h \to \bar{N}N)
\]

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\text{Im} G_V(t) = \frac{k_{cm}^3}{m_N \sqrt{t}} F_\pi^*(t) f_\pm^1(t)
\]

\[
\text{Im} G_M(t) = \frac{k_{cm}^3}{\sqrt{2t}} F_\pi^*(t) f_\mp^1(t)
\]

\[
\text{Im} G_{E,M}^V(t) = \frac{k_{cm}^3}{\{m_N, \sqrt{2}\} \sqrt{t}} F_\pi^*(t) f_\pm^1(t)
\]

\[
\text{Im} G_{E,M}^M(t) = \frac{k_{cm}^3}{\{m_N, \sqrt{2}\} \sqrt{t}} |F_\pi(t)|^2 \frac{f_\pm^1(t)}{F_\pi(t)} J_\pm^1
\]
$$\text{Im}G_E^V(t) = \frac{k_{cm}^3}{m_N \sqrt{t}} |F_\pi(t)|^2 J_+^1(t)$$

$$\text{Im}G_M^V(t) = \frac{k_{cm}^3}{\sqrt{2t}} |F_\pi(t)|^2 J_-^1(t)$$
\[
\text{Im} G^V_E(t) = \frac{\kappa^3_{cm}}{m_N \sqrt{t}} |F_\pi(t)|^2 J^1_+(t) \quad \text{ChEFT}
\]
\[
\text{Im} G^V_M(t) = \frac{\kappa^3_{cm}}{\sqrt{2t}} |F_\pi(t)|^2 J^1_-(t) \quad \text{ChEFT}
\]
$$\text{Im} G^V_E(t) = \frac{k^3_{cm}}{m_N \sqrt{t}} |F_\pi(t)|^2 J^1_+(t)$$

Experiment

ChEFT

$$\text{Im} G^V_M(t) = \frac{k^3_{cm}}{\sqrt{2t}} |F_\pi(t)|^2 J^1_-(t)$$

Experiment

ChEFT
\[ \text{Im} G_V^E(t) = \frac{k^3_{cm}}{m_N \sqrt{t}} |F_\pi(t)|^2 J_+^1(t) \]  
ChEFT

\[ \text{Im} G_V^M(t) = \frac{k^3_{cm}}{\sqrt{2t}} |F_\pi(t)|^2 J_1^1(t) \]  
ChEFT

\[ \text{Im} G_V^E(t) = \frac{k^3_{cm}}{m_N \sqrt{t}} |F_\pi(t)|^2 J_+^1(t) \]  
ChEFT

\[ \text{Im} G_V^M(t) = \frac{k^3_{cm}}{\sqrt{2t}} |F_\pi(t)|^2 J_1^1(t) \]  
ChEFT


J. M. Alarcón (UCM)  
The role of Chiral EFT in the precision era  
26/33
To compute the EM form factors of proton and neutron, we need the isoscalar component as well.
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One cannot apply the same approach as in the isovector case.
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We parametrize the isoscalar spectral function through the $\omega$ exchange in the narrow with approximation $+ \text{higher mass pole } P_s$. 

**DIxEFT**
To compute the EM form factors of proton and neutron, we need the isoscalar component as well.

One cannot apply the same approach as in the isovector case.

We parametrize the isoscalar spectral function through the $\omega$ exchange in the narrow with approximation + higher mass pole $P_S$.

\[
\text{Im} G^{S, E,M}_{E,M} = -\pi \sum_{V=\omega, P_S} a^{E,M}_i \delta(t - M_i^2)
\]
To compute the EM form factors of proton and neutron, we need the isoscalar component as well.

One cannot apply the same approach as in the isovector case.

We parametrize the isoscalar spectral function through the $\omega$ exchange in the narrow with approximation + higher mass pole $P_S$.

We fix the couplings by imposing the charge and radii sum rules:

$$\text{Im} G_{E,M}^S(t) = -\pi \sum_{V=\omega,P_S} \alpha_{i,E,M}^S \delta(t - M_i^2)$$

$$G_{E,M}^S(0) = \frac{1}{\pi} \int_{4M_i^2}^{\infty} dt' \frac{\text{Im} G_{E,M}^S(t')}{t'}$$

$$\langle r_{E,M}^2 \rangle^S = \frac{6}{\pi} \int_{4M_i^2}^{\infty} dt' \frac{\text{Im} G_{E,M}^S(t')}{t'^2}$$
Reconstructing the form factors with

\[ G_{E,M}^{p,n}(t) = \frac{1}{\pi} \int_{4M^2}^{\infty} dt' \frac{\text{Im} G_{E,M}^{p,n}(t')}{t' - t - i0^+} \]
Reconstructing the form factors with

\[ G_{E,M}^{p,n}(t) = \frac{1}{\pi} \int_{4M^2}^{\infty} dt' \frac{\text{Im} G_{E,M}^{p,n}(t')}{t' - t - i0^+} \]
\[ \chi^2(r_E^p) \equiv N^{-1} \sum_{\text{bins}} \frac{(\text{thy}_i - \text{fit}_i)^2}{(\Delta \text{thy}_i)^2 + (\Delta \text{fit}_i)^2} \]

\{ \text{thy}_i \equiv G_E^p(Q_i^2) \ [\text{DIxEFT, given } r_E^p], \\
\text{fit}_i \equiv G_E^p(Q_i^2) \ [\text{global fit, given } r_E^p] \}

**Figure**: Reduced \( \chi^2 \) as a function of the proton radius \( r_E^p \) for different \( Q_{\text{max}} \) values.

\[ r_E^p = 0.844(7) \text{ fm} \]

Summary and Conclusions
Summary and Conclusions

• Chiral EFT is a useful tool to investigate hadronic processes at low energies from first principles.
• It provided important hadronic input for searches of physics beyond the standard model:
  • Dark Matter searches: $\sigma_{\pi N}$, t-dependence of the scalar FF ($D\chi$EFT).
  • Proton Radius Puzzle: $\Delta E_{2P-2S}$, moments of the EM FF ($D\chi$EFT), Proton radius from $e^- p$ agrees with $\mu H \rightarrow r_E^p = 0.844(7)$ fm
• Insights into the origin of mass:

<table>
<thead>
<tr>
<th>$m_p$</th>
<th>$59(7)$ MeV</th>
<th>$16(80)$ MeV</th>
<th>$860(87)$ MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>%</td>
<td>$6.3(7)$%</td>
<td>$1.7(8.5)$%</td>
<td>$92.0(9.3)$%</td>
</tr>
</tbody>
</table>

• Prominent role in the solution of current and future challenges in hadron and nuclear physics.
FIN
Spares
Fits to PWAs
Fits to PWAs

Fits to KA85

\[ S_{31} \]

\[ S_{11} \]

\[ P_{31} \]

\[ P_{11} \]

\[ P_{33} \]

\[ P_{13} \]

\[ \Delta \text{-less ChPT} \]

\[ \Delta \text{-ChPT} \]

[Alarcón, Martin Camalich and Oller, Ann. of Phys. 336 (2013)]

J. M. Alarcón (JLab)

The role of Chiral EFT in the precision era
Fits to PWAs

Fits to EM06

\[ S_{31} \quad S_{11} \]
\[ P_{31} \quad P_{11} \]
\[ P_{33} \quad P_{13} \]

---

Δ-less ChPT

Δ-ChPT

[Alarcón, Martin Camalich and Oller, Ann. of Phys. 336 (2013)]
Consequences of $\sigma_{\pi N}$ for nuclear matter
Consequences of $\sigma_{\pi N}$ for nuclear matter

\[
\langle \Omega | \bar{q}q | \Omega \rangle = \langle 0 | \bar{q}q | 0 \rangle \left( 1 - \frac{\sigma_{\pi N}}{M_\pi^2 f_\pi^2} \rho + \ldots \right)
\]

- Restoration of chiral symmetry requires a zero temporal component of $f$

\[
f_t = f_\pi \left\{ 1 + \frac{2\rho}{f^2} \left( c_2 + c_3 - \frac{g_A^2}{8m_N} \right) \right\}
\]
- This plot is for $m_0 = 750$ MeV, which is equivalent to fix $b_0$.
- Gasser points out that the natural choice is $\Lambda = 1$ GeV because corresponds to the axial vector form factor fit given by Sehgal [Sehgal, “Proceedings of the International Conference on High Energy Physics”].
- He finally takes $\Lambda = 700$ MeV because for $\Lambda = 1$ GeV the mass shift of the nucleon due to massless pions is $-200$ MeV while for $\Lambda = 700$ MeV is $-90$ MeV.

Comparison with HB

<table>
<thead>
<tr>
<th></th>
<th>Octet</th>
<th>$\mathcal{O}(p^3)$</th>
<th>Octet + Decuplet</th>
<th>$\mathcal{O}(p^3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HB</td>
<td>Cov.</td>
<td>HB</td>
<td>Cov.</td>
</tr>
<tr>
<td>$\sigma_0$ (MeV)</td>
<td>58(23)</td>
<td>46(8)</td>
<td>89(23)</td>
<td>58(8)</td>
</tr>
</tbody>
</table>
Subthreshold region
Subthreshold region

- The disagreement found in [Becher and Leutwyler, JHEP (2001)] is related to the disagreement in the subthreshold expansion.

\[ T(\nu, t) = \bar{u} \left( D(\nu, t) - \frac{1}{4m_N} B(\nu, t)[q, q'] \right) u \]

\[ \bar{D}^+(\nu, t) = d_{00}^+ + d_{01}^+ t + d_{10}^+ \nu^2 + d_{02}^+ t^2 + \ldots \]

\[ \bar{D}^- (\nu, t) = d_{00}^- \nu + d_{01}^- \nu t + d_{10}^- \nu^3 + \ldots \]

\[ \bar{B}^+(\nu, t) = b_{00}^+ \nu + \ldots \]

\[ \bar{B}^- (\nu, t) = b_{00}^- + \ldots \]

<table>
<thead>
<tr>
<th>KA85</th>
<th>WI08</th>
<th>EM06</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ-ChPT</td>
<td>Δ-ChPT</td>
<td>Δ-ChPT</td>
</tr>
<tr>
<td>( d_{00}^+ (M_{\pi}) )</td>
<td>-2.02(41)</td>
<td>-1.65(28)</td>
</tr>
<tr>
<td>( d_{01}^+ (M_{\pi}) )</td>
<td>1.73(19)</td>
<td>1.70(18)</td>
</tr>
<tr>
<td>( d_{10}^+ (M_{\pi}) )</td>
<td>1.81(16)</td>
<td>1.60(18)</td>
</tr>
<tr>
<td>( d_{00}^- (M_{\pi}) )</td>
<td>0.02(16)</td>
<td>0.021(6)</td>
</tr>
<tr>
<td>( b_{00}^+ (M_{\pi}) )</td>
<td>-6.5(2.4)</td>
<td>-7.4(2.3)</td>
</tr>
<tr>
<td>( b_{00}^- (M_{\pi}) )</td>
<td>1.81(24)</td>
<td>1.68(16)</td>
</tr>
<tr>
<td>( d_{00}^- (M_{\pi}) )</td>
<td>-0.17(6)</td>
<td>-0.20(5)</td>
</tr>
<tr>
<td>( d_{10}^- (M_{\pi}) )</td>
<td>-0.35(10)</td>
<td>-0.33(10)</td>
</tr>
<tr>
<td>( b_{00}^+ (M_{\pi}) )</td>
<td>17(7)</td>
<td>17(7)</td>
</tr>
</tbody>
</table>

\[ \Sigma = f_\pi^2 (d_{00}^+ + 2M_{\pi}^2 d_{01}^+ + \ldots) \]

\[ \Sigma_d = \frac{f_\pi^2 (4M_{\pi}^4 d_{02}^+ + \ldots)}{\Delta_d} \]

\[ \Delta_d - \Delta_{\sigma} = -3.3(2) \text{ MeV (disp.)} \]

\[ \Delta^{(3)}_d - \Delta^{(3)}_{\sigma} = -3.5(2.0) \text{ MeV (O(p^3) ChEFT)} \]

[Alarcón, Martin Camalich and Oller, Ann. of Phys. 336 (2013)]

Agreement with the dispersive results!

- CD theorem: \( \Sigma \equiv \int f_\pi^2 \tilde{D}^+(0, 2M_{\pi}^2) = \sigma(t = 2M_{\pi}^2) + \Delta_R = \sigma_{\pi N} + \Delta_{\sigma} + \Delta_R \)

\[ \Delta_{\sigma} = \sigma_{\pi N} = \Sigma_d + \Delta_D - \Delta_{\sigma} - \Delta_R \]

\[ \Delta_D - \Delta_{\sigma} = -3.3(2) \text{ MeV (disp.)} \]

\[ \Delta^{(3)}_D - \Delta^{(3)}_{\sigma} = -3.5(2.0) \text{ MeV (O(p^3) ChEFT)} \]

\[ (\text{Underestimated in } \sim 10 \text{ MeV}) \]

\[ (\text{Remains small}) \]

Underestimated in \( \sim 10 \) MeV as well!
The sigma-term puzzle
The sigma-term puzzle

- Phenomenological extractions rely on two different sources:
  - \( \pi N \)-scattering data
    - Inconsistent data base \((\pi^\pm N \rightarrow \pi^\pm N \text{ vs CEX reactions})\)
    - Coulomb [Tromborg, Waldenstrom and Overbo, PRD 15 (1977)].
  - \( \pi \)-atom spectroscopy
    - Experimental uncertainties negligible compared to theoretical error relating \((\epsilon, \Gamma)\) to \(a^\pm\).
    - \(\pi D\) scattering, isospin violation, Coulomb…

What can be done?

- Analysis of the \(\pi N\) world data base.
- Reanalysis of Coulomb corrections.
- Reanalysis of extraction of SL through \(\epsilon\) and \(\Gamma\).