Confronting Lattice Parton Densities with Global QCD Analysis

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Overview

1. Motivation
   - Why we want to incorporate lattice results in global PDF analysis
2. Theory
   - Status of PDFs on the lattice
   - Mellin transform techniques
3. Combined Global Analysis
   - Initial results for global analysis with lattice (unpolarized and polarized) PDFs
   - Comments on $\bar{d} - \bar{u}$

Motivation

Key Idea

We want to include lattice data in global analysis of unpolarized and polarized PDFs

- Can't directly calculate or measure PDFs from either experiment or lattice
- On which regions of parton fraction x does lattice induce constraints?
- $\bar{d} - \bar{u}$?

PDFs on the lattice

Light cone PDFs can be accessible via quasi-PDFs (Ji 2013), pseudo-PDFs (Radyushkin 2017), good lattice cross sections (Qiu 2018)

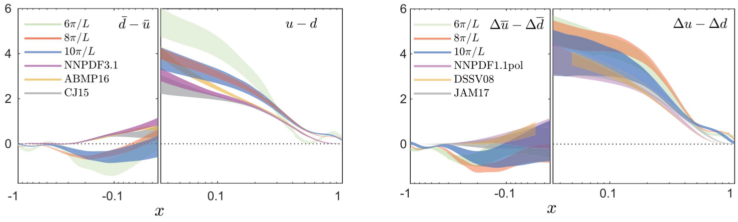
Quasi-PDF:

$$\tilde{f}\left(y, \mu, P_3\right) = \int_{-\infty}^{\infty} \frac{dz}{4\pi}\ e^{-iyP_3 z} \left\langle \mathcal{O} \right\rangle (z)$$

Connection to light cone PDF via perturbative matching procedure:

$$\left\langle \mathcal{O} \right\rangle (z) = -\int_{-\infty}^{\infty} dy\ e^{-iyP_3 z} \int_{-1}^{1} \frac{dx}{|x|}\ C\left(\frac{y}{x}, \frac{\mu}{xP_3}\right)\ f\left(x, \mu\right)$$
$$= \int_{-\infty}^{\infty} dy\ e^{-iyP_3 z} \int_{-\infty}^{\infty} \frac{d\xi}{\xi} \frac{(y/\xi)}{|y/\xi|}\ C\left(\xi, \frac{\mu}{(y/\xi)P_3}\right)\ f\left(\frac{y}{\xi}, \mu\right) \Theta(|\xi| > y)$$

Direct Calculations of PDFs from lattice data



Alexandrou et. al. (2018)

Some features of note:

- Currently $\bar{d}-\bar{u}$ and $\Delta\bar{u}-\Delta\bar{d}$ from lattice is in contradiction with experimental data
- Unpolarized: $\mathcal{R}[\langle\mathcal{O}\rangle(z)]$ -sensitive only to valence dist.
- Polarized: $\mathcal{I}[\langle\mathcal{O}\rangle(z)]$ - sensitive only to valence dist.

Technical details

- We want to solve DGLAP in Mellin space for computational efficiency
- Lattice observables do not have pure convolution structure
- But can still use Mellin space evolved PDFs using the Mellin trick

Mellin transform/inverse:

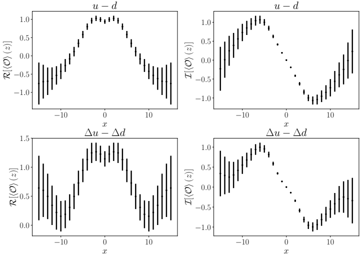
$$f_N = \int_0^\infty dx\ x^{N-1}\ f(x) \qquad\qquad f(x) = \frac{1}{2\pi i}\int dN\ x^{-N}\ f_N$$

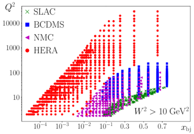
Mellin trick (Vogelsang, Stratmann):

$$\langle \mathcal{O} \rangle (z) = \frac{1}{2\pi i}\int dN\ f_N \left[\int_{-\infty}^{\infty} dy\ F(y,N) e^{-iyP_3 z} \int_{-\infty}^{\infty} \frac{d\xi}{\xi} \frac{(y/\xi)}{|y/\xi|} C\left(\xi, \frac{\mu}{(y/\xi)P_3}\right) \Theta(|\xi| > y) \right]$$

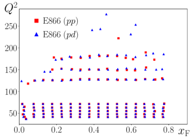
Setup for simultaneous fit

- Unpolarized datasets: DIS, DY, Lattice
- Polarized datasets: DIS, Lattice
- Single fits (LO matching)

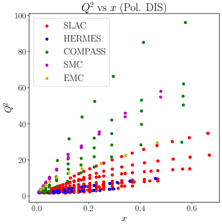




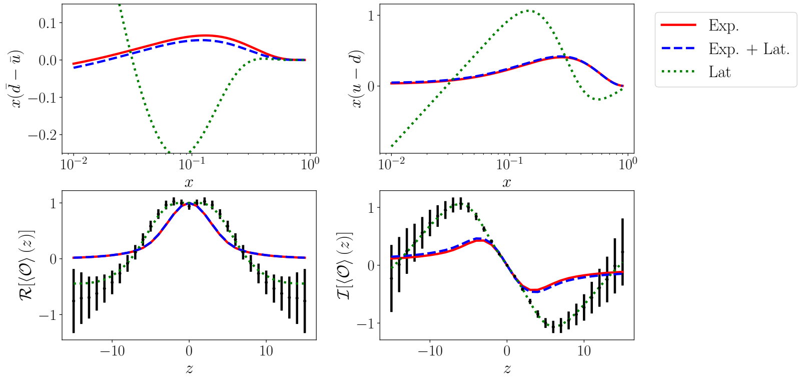
DIS



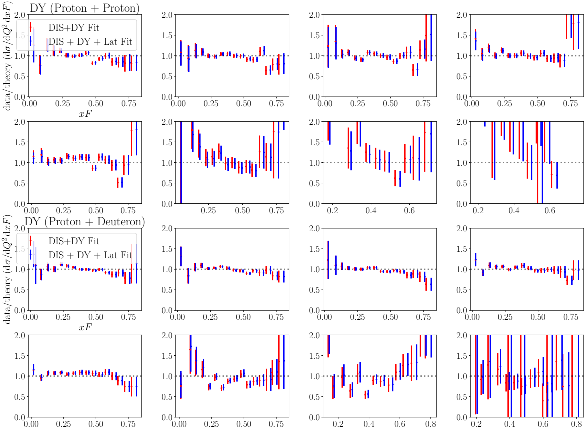
DY



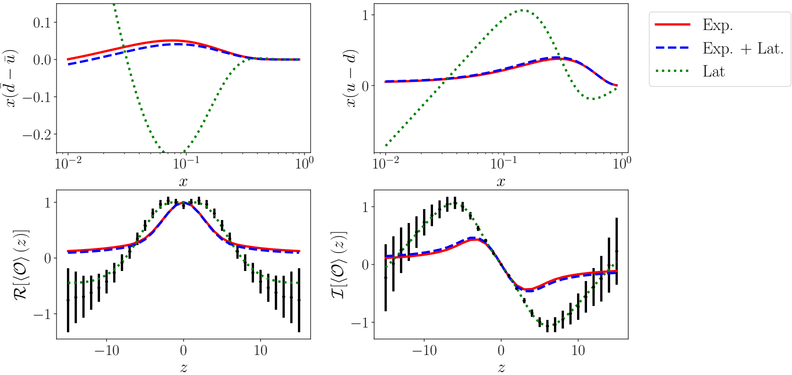
Fits to unpolarized PDFs: DIS + DY + Lattice



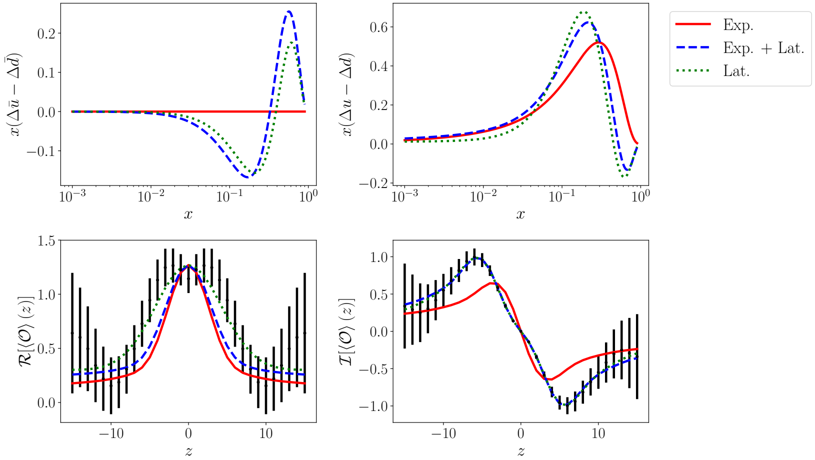
Fits to unpolarized PDFs: DIS + DY + Lattice



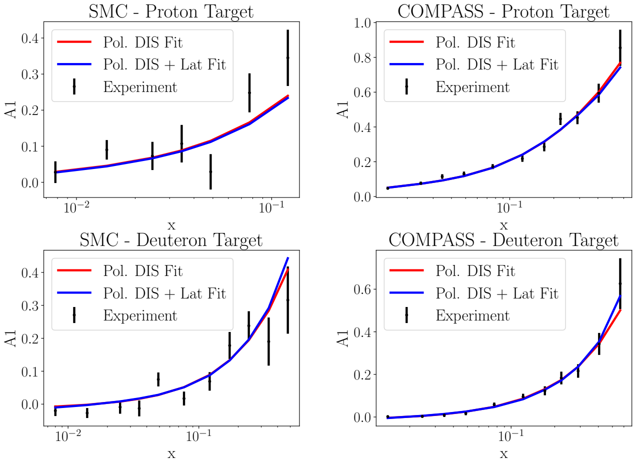
Fits to unpolarized PDFs: DIS + Lattice



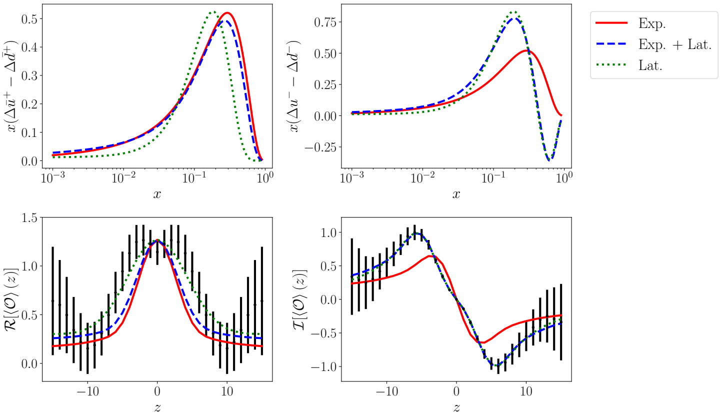
Fits to polarized PDFs: DIS + Lattice



Fits to polarized PDFs: Comparison to experiment



Fits to polarized PDFs: Effect on valence distribution (DIS + Lattice)



Summary and Outlook

- We are incorporating lattice data into global analysis fits
- Initial results suggest a tension between experimental and lattice PDFs
- Exception: Polarized imaginary lattice observable: better constrains valence polarized PDFs than polarized DIS alone
- We will add NLO matching and use Monte Carlo analysis to identify precisely which lattice data points are consistent with phenomenological results

Backup Slides

Choosing the Mellin contour: f_N

- Contour must be to the right of poles; "close" for numerical convergence
- Can get f_N analytically for a particular choice of functional form

$$f(x) = Mx^a(1-x)^b \;\rightarrow\; f_N = M\frac{\Gamma(N+a)\Gamma(b+1)}{\Gamma(N+a+b+1)}$$

Sum rules for PDFs constrain a:

$$\int_0^1 dx\; u_v(x) = 2 \quad \int_0^1 dx\; d_v(x) = 1 \quad \int_0^1 dx\; x\sum_{q,\bar{q},g} f(x) = 1$$

- Valence quark PDFs $\rightarrow a > -1$
- Sea quark/gluon PDFs $\rightarrow a > -2$

Sensitivity of lattice observables to valence distributions

$$q(x) = \frac{2P_3}{4\pi}\int_{-\infty}^{\infty} dz e^{-izxP_3}(R(z) + iI(z))$$
$$= \frac{P_3}{\pi}\int_0^{\infty} dz[\cos(zxP_3)R(z) + \sin(zxP_3)I(z)]$$

Similarly

$$q(-x) = \frac{P_3}{\pi}\int_0^{\infty} dz[\cos(zxP_3)R(z) - \sin(zxP_3)I(z)]$$

For unpolarized: $\bar{q}(x) = -q(-x)$ so
$q(x) + q(-x) = q_v(x) \ \propto \ R(z)$
For polarized: $\bar{q}(x) = q(-x)$ so $q(x) - q(-x) = q_v(x) \ \propto \ I(z)$