Current understanding of the proton charge radius

Carl E. Carlson

William & Mary

Cake Seminar–Jefferson Lab
Newport News, VA
18 September 2019

WILLIAM & MARY
CHARTERED 1693

Proton radius

Talk plan:

- Show newest results close to the beginning
- Then proceed
  - Bit of history: ways to measure proton radius
  - Modern times and scattering data
  - Re-analyses of data: controversy
  - Deuteron measurements: lingering problem?
  - Two photon exchange corrections: a bar to the future?
- Closing comments

Proton radius measurements

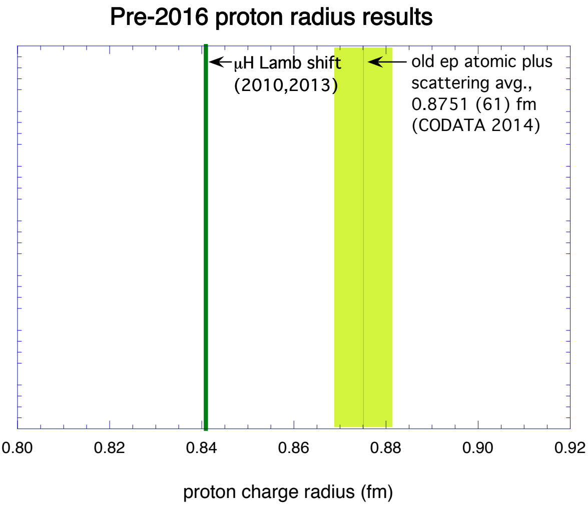
The proton radius has been (to date) measured using:

- electron-proton elastic scattering
- level splittings in traditional hydrogen
- level splittings, specifically the Lamb shift, in muonic hydrogen

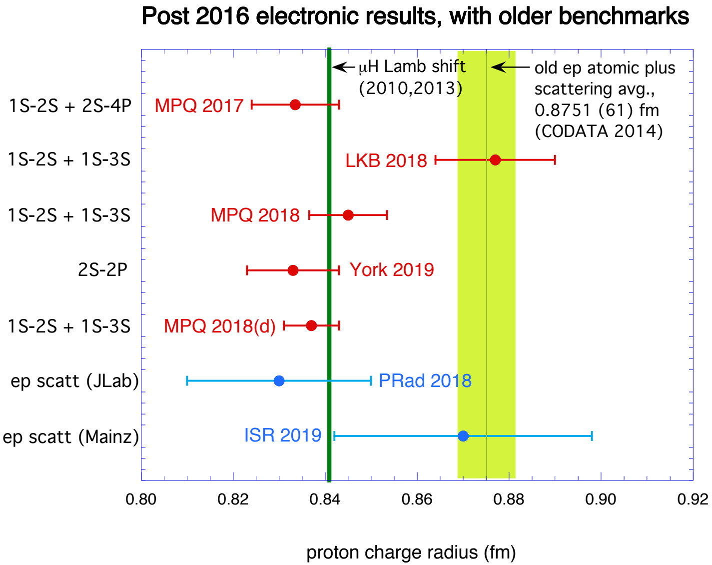
The early results were incompatible, and gave about a 6σ discrepancy, summarized on the next slide. (Early here means before 2016.)

Showing the newest results at the beginning–set up

(Newer experiments coming)



The newest results



References

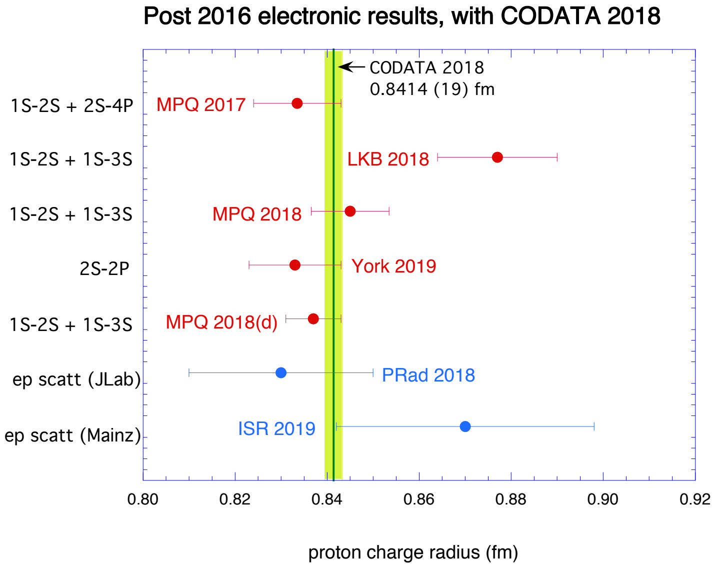
Three results from refereed journals, one from archived preprint, three from public conference talks

1. MPQ 2017: Axel Beyer et al., Science, **358**, pp. 79-85 (2017).
2. LBK 2018: Hélène Fleurbaey et al., PRL, **120**, 183001 (2018).
3. MPQ 2018: Arthur Matveev, talk, PSAS 2018, Vienna.
4. York 2019: Bezginov et al., Science **365**, 10071012 (2019).
5. MPQ 2018(d): Arthur Matveev, talk, Proton Rad Conf 2018, Mainz.
6. PRad 2018: Chao Gu, talk, APS-JPS 2018 joint meeting, Hawaii. Also: Haiyan Gao, talk, GHP-APS mtg., Denver, April 2019.
7. ISR 2019: Mihovilovič et al., arXiv: 1905.11182 [nucl-ex].

Mini-summary

- You can make your own!
- CODATA has made their own: CODATA 2018 (available 20 May 2019) has proton radius compatible with muon Lamb shift value. See next slide.
- BTW, unweighted average of $\geq$ 2016 electron-based measurements is 0.842 fm

Newest results, with CODATA 2018 proton radius



How did we get here

Comments on measurements from electron scattering and atomic physics

Elastic electron scattering, $e^- p \to e^- p$

- There are form factors for electric (E) and magnetic (M) charge distributions.
- Cross section is given by

$$\frac{d\sigma}{d\Omega} \propto G_E^2(Q^2) + \frac{\tau}{\varepsilon} G_M^2(Q^2)$$

$$\left[\tau = Q^2/4m_p^2\ ; \quad 1/\varepsilon = 1 + 2(1+\tau)\tan^2(\theta_e/2)\right]$$

- Low Q^2 is mainly sensitive to G_E.
- DEFINE (for historical reasons) charge radius by,

$$\boxed{R_E^2 = -6\left(dG_E/dQ^2\right)_{Q^2=0}}$$

- From real data, need to extrapolate to $Q^2 = 0$.

Scattering data

- Much data from 20th century, but currently biggest and best data set is Mainz (2010).

- Bernauer et al., PRL 2010 and later articles.
- Low Q^2 range, 0.004 to 1 GeV^2
- From their eigenanalysis,

$$R_E \text{ or } R_p = 0.879(8)\,\text{fm}$$

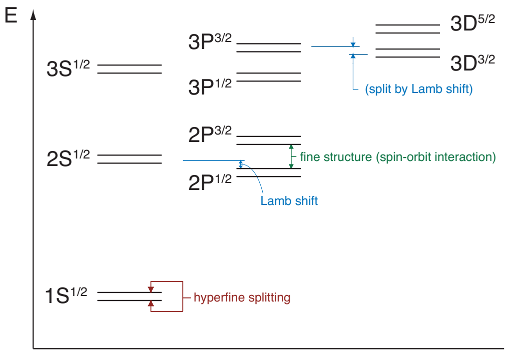
Atomic energy splitting measurements

- Proton radius affects atomic energy levels.

$$E = E_{\mathrm{QED}} + \delta_{\ell 0} \frac{2 m_r^3 Z^4 \alpha^4}{3 n^3} R_E^2 + E_{\mathrm{TPE}} + \text{very small corrections}$$

- $E_{\mathrm{TPE}} =$ two photon exchange corrections (calculated: will discuss)
- Accurate measurements of energy splitting and accurate calculation of QED effects allows determination of proton radius.

Just in case: Hydrogen energy levels



Definitely not to scale:

- Scale for big splittings is Rydberg, $\text{Ryd} = \frac{1}{2} m_e \alpha^2 \approx 13.6$ eV.
- Fine structure and Lamb shift are $\mathcal{O}(\alpha^2 \, \text{Ryd})$.
- Hyperfine splitting is $\mathcal{O}(m_e/m_p) \times (\alpha^2 \, \text{Ryd})$.

Requirements for calculation

- QED

$$E_{\text{QED}} = \frac{1}{2} m_r \alpha^2 \left[1 + \ldots + \underbrace{\mathcal{O}\left(\frac{\alpha}{2\pi}\right)^3}_{1.6\times 10^{-9}} + \underbrace{\mathcal{O}\left(\frac{\alpha}{2\pi}\right)^4}_{1.8\times 10^{-12}} + \ldots \right]$$

- leading proton size correction

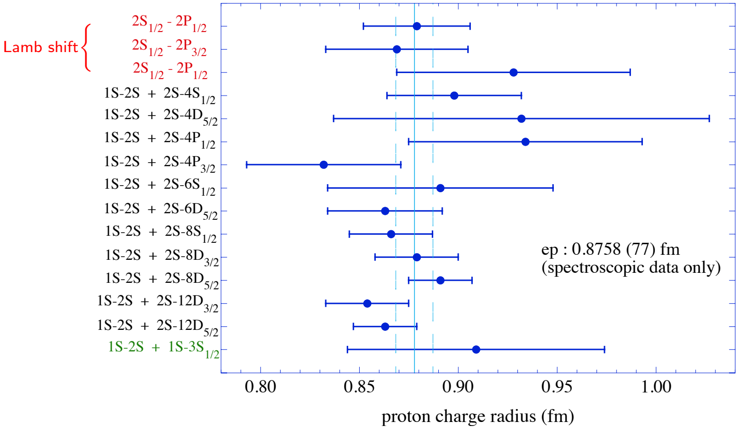
$$\Delta E_{\text{proton size}} = \frac{1}{2} m_r \alpha^2 \cdot \underbrace{\frac{4\alpha^2}{3n^3} \cdot \underbrace{(m_r R_E)^2}_{6.7\times 10^{-6}}}_{6\times 10^{-11}}$$

for $R_E = 1$ fm and $n = 2$.

- Hence need $\mathcal{O}(\alpha/2\pi)^4$ corrections. First available about year 2000.

Old version of plot shown earlier

Now can get proton radius from atomic splitting. As of early 2016:



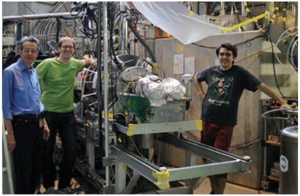
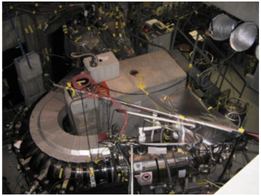
- Crucial observation:
  R_p from electron scattering and from electronic hydrogen agreed.

Comment regarding R_E and atomic physics

- Crucial: why in atomic physics do we use the derivative of G_E to define the proton radius? Why not, for example, derivative of F_1?
- Answer by doing the relativistic perturbation theory calculation for proton size effect on atoms.
- Indeed find effect $\propto G'_E(Q^2)\big|_{Q^2=0}$
- Since atomic results measures $G'_E(0)$, quote $R_p = R_E$, to match.

Muonic atoms energy splitting (2010)

- Can do analogous measurements with muonic atoms.
- Muons weigh 200× what electron does. Muons orbit 200× closer. Proton looks 200× bigger and proton size effects are magnified.
- Opportunity to obtain more accurate proton radius, despite short muon lifetime.


- Done by CREMA specifically for the $2S$-$2P$ splitting (Lamb shift)
- Obtained

$$\boxed{R_p = 0.84087(39)\,\text{fm}}$$

Conflict!

Repeat

$$\boxed{R_p = 0.84087(39)\,\text{fm}}$$

- Uncertainty limit ca. 20X better than old electronic results.

  Current box:

(fm)	atomic	scattering
electron	0.8759 (77)	0.879(8)
muon	0.84087 (39)	no data yet

Modern times and scattering data

Two thrusts:

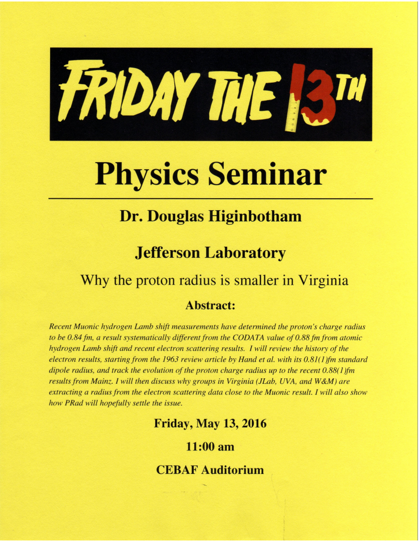
1. new experiments, some just finished, some coming
2. reanalysis of old data

New and future scattering experiments

- NEW:
- PRad (JLab) does electron scattering down to $Q^2 = 0.0002$ GeV^2. Mentioned earlier: $R_E = 0.831 \pm$ ca. 2%.
- Initial state radiation experiment at Mainz. Published 2017, republished with better understanding of systematics 2019. Second run to come.
- FUTURE:
- New experiment at Mainz, in Hall A2, observing final proton in TPC
- MUSE, Muon scattering experiment at PSI will do both muon (first time, at this accuracy) and electron scattering, down to 0.002 GeV^2. Expect relative error between e and μ output radii about 0.7%. "Production run" started (?) July this year.
- Proton radius from μ scattering at COMPASS, using a TPC to see the final proton

Discussions of new fits to old data

From a long time ago:



FRIDAY THE 13TH

Physics Seminar

Dr. Douglas Higinbotham

Jefferson Laboratory

Why the proton radius is smaller in Virginia

Abstract:

Recent Muonic hydrogen Lamb shift measurements have determined the proton's charge radius to be 0.84 fm, a result systematically different from the CODATA value of 0.88 fm from atomic hydrogen Lamb shift and recent electron scattering results. I will review the history of the electron results, starting from the 1963 review article by Hand et al. with its 0.81(1)fm standard dipole radius, and track the evolution of the proton charge radius up to the recent 0.88(1)fm results from Mainz. I will then discuss why groups in Virginia (JLab, UVA, and W&M) are extracting a radius from the electron scattering data close to the Muonic result. I will also show how PRad will hopefully settle the issue.

Friday, May 13, 2016

11:00 am

CEBAF Auditorium

But not limited to one locale

And still continuing

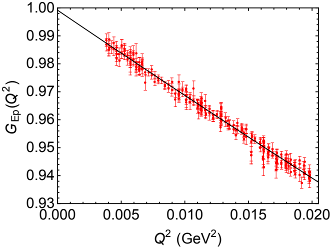
A few references (apologies ...)

minimalist (small radius)	more expansive
Meissner et al. (2015)	original Mainz (Bernauer et al.)
Horbatsch & Hessels (2016)	Hill & Paz
Higinbotham et al. (2016)	Graczyk & Juszczak (2014)
Griffioen et al. (2016)	Arrington & Sick (2015)
Yan, Higinbotham, et al. (2018)	Lee, Arrington, & Hill (2015)
Hayward & Griffioen (2018)	Ye, Arrington, Hill, and Lee (2018)
Alarcón, Higinbotham, et al. (2019)	

To repeat the last on the left: Alarcón, Higinbotham, Weiss, Ye, Phys.Rev. C99 (2019) no.4, 044303 (about a proton radius extraction by combining dispersion analysis and chiral EFT)

One plot

- Basic viewpoint that leads to small results: Charge radius requires extrapolation to $Q^2 = 0$. Fits with lots of parameters tend to be less smooth outside data region. Fits to full data set generally require lots of parameters. For charge radius, better to fit to narrower, low Q^2 region of data. Have fewer parameters, less "wiggly" functions, and more faith in extrapolations.



slope gives
$R_E = 0.84\,(1\%)$ fm

Minimalist fits to real data, with bias analysis

- But still unsettled: fitters obtaining larger radii have not recanted
- In fact, may consult "Avoiding common pitfalls and misconceptions in extractions of the proton radius," 1606.02159
- Truly exciting: if larger radius from electrons is correct, then need explanation of difference between electron and muon interactions: we are into beyond the standard model physics (BSM).
- Hope: Studies are proceeding with serious testing on pseudodata and with analysis of reliability and robustness of fit procedures, and may lead to some criteria for agreement.

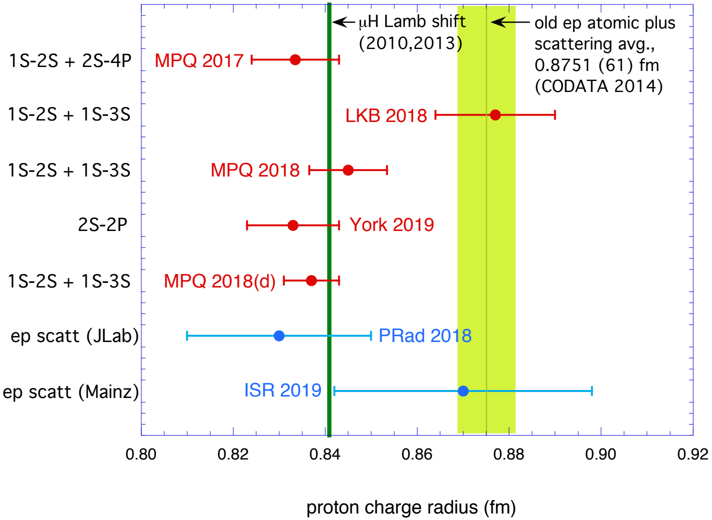
Proton results from deuteron measurements

- Also exciting: The $1S$ to $2S$ splitting in both hydrogen and deuterium can be measured to 15 figures! (The $2S$ is metastable, hence narrow, leaving no fuzziness as to where it is.)
- Only things that cannot be well calculated in difference are the radius terms. Hence get very accurate radius difference (called "isotope shift"):

$$\boxed{R_d^2 - R_p^2 = 3.820\,07\,(65)\ \text{fm}^2}$$

- $\therefore$ If you know the deuteron radius to 4 figures after the decimal point, you can obtains the proton radius to that accuracy.
- Used by MPQ 2018 in figure seen earlier.

Post 2016 electronic results, with older benchmarks



Proton results from deuteron measurements II

- If the electronically measured radii for the proton come down, is there any lingering problem?
- Maybe . . .
- CREMA has also measured the deuteron radius,

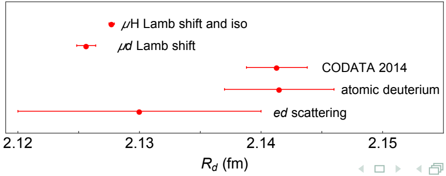
$$R_d = 2.125\,62\,(78)\ \text{fm}$$

- Using the muonic hydrogen value and at the isotope shift, get

$$R_d = 2.127\,71\,(22)\ \text{fm}$$

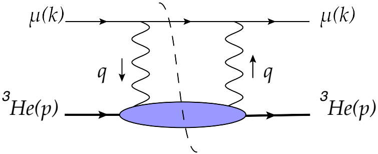
which is $2.6\,\sigma$ higher.

Worried?



Two Photon Exchange (TPE): Dispersive calculation

- Need the box diagram with two photons
- Some calculate by noting putting the intermediate states on shell
  (a) gives the Imaginary part of the whole diagram, and
  (b) means each half of the diagram is an amplitude for a real scattering process, and hence can be gotten from scattering data.



- What matters is the lower vertex, so can use electron scattering data.
- Mostly need low Q^2, low energy data
- Reconstruct whole diagram using dispersion relations.
- Something of a problem: One of the Compton amplitudes requires a subtracted dispersion relation, with a subtraction term that is not experimentally measured and must be estimated. We believe that can be done with sufficient accuracy.

Begin with the proton

- Theory for Lamb shift splitting, with numbers for proton,

$$\Delta E_L^{\text{theo}} = \Delta E_{\text{QED}} - \frac{m_r^3 Z^4 \alpha^4}{12} R_p^2 - \Delta E_{\text{TPE}}$$
$$= 206.0336(15) - 5.2275(10) R_p^2 + 0.0332(20)$$

(units are meV and fm)

- TPE number from Birse and McGovern, following CEC and Vdh; ongoing consideration using other techniques
- Faith,

$$\Delta E_L^{\text{theo}} = \Delta E_L^{\text{expt}} = 202.3706(23)\,\text{meV}$$

- Solve,

$$R_p = 0.84087(39)\,\text{fm} \qquad [0.038\%]$$

- If the TPE were perfect,

$$R_p = 0.84087(32)\,\text{fm}$$

- Conclude: for the proton theorists have done their job. Uncertainty in TPE not dominant.

Jump to other light nuclei: e.g., ^{4}He

- Interested for similar reasons: want to find radius discrepancy
- Compare radius from electron scattering to radius from μ Lamb shift
- From electron scattering $R_\alpha = 1.681(4)$ fm [0.25%]
- If this is the right radius, can calculate the ^{4}He finite size energy shift. The 0.25% uncertainty becomes an predicted energy shift uncertainty

$$\delta E_{\rm fs}^{\alpha} \overset{^4\rm He}{=} 1.42 \text{ meV}$$

- We and nuclear theorists using entirely different method calculate for the TPE,

how	*who*	$\Delta E_{\rm TPE}$ (meV)
Nuclear potentials	Hernandez *et al.* (2016)	-9.58(38)
Dispersion theory	CEC, Gorchtein, Vanderhaeghen	-12.23(xx)

^{4}He — Are we good enough?

- Conflict! (BTW, we were in good agreement for ^{3}He)
- With a split-the-difference overall error bar,

$$\text{uncertainty}\ (E_{\text{TPE}}) \approx 1.5\ \text{meV}$$

- The muonic Lamb shift measurement cannot beat the electron radius scattering measurement because of the two-photon correction uncertainties. Ugh.
- R. Pohl: "You are killing our experiment,"

Ending

- Remarkable: After 9 years, the problem shows signs of being settled.
- Interesting: little discussion of the correctness of the μ-H Lamb shift data.
- Radius results from electron scattering currently mixed, both experimentally (PRad vs. Mainz) and in reanalyses. More experiments coming.
- Most recent ordinary hydrogen measurements of radius agree with results of level splitting in μ-hydrogen.
- Either
  - The puzzle isnt a puzzle: The electron based radius measurements are reducing to the muonic value.
    - The scattering analysis is under discussion, and more data coming
    - The newer spectroscopy measurements are giving the smaller radius.
  - Those who insist on a large radius from electrons and a smaller one from muons have to be all in on a BSM explanation of the puzzle.

For us in the field

- Possibility the problem is settled
- Some mop-ups:
  - Resolve conflicts in the analysis of the full set of electron scattering data
  - Resolve the remaining deuteron conflict
  - Improve the ^{4}He TPE calculation

Extras

Beyond the end

History

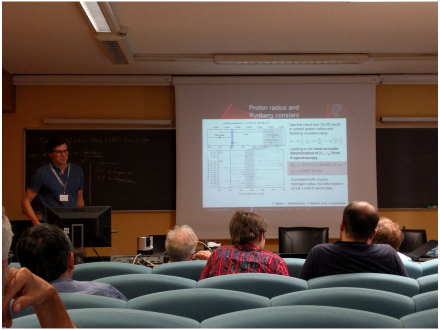
- Averages from the Committee on Data for Science and Technology (CODATA)
- There have been 9 CODATA reports.

Year	Proton radius (fm)	
2018	0.8414(19)	
2014	0.8751(61)	mostly atomic
2010	0.8775(51)	"
2006	0.8768(69)	"
2002	0.8750(68)	"
1998	0.8545(120)	election scattering
1986	–	no R_E quoted
1973	–	"
1969	0.805(11)	electron scattering

- (Only for 2002 and later is the proton radius among the constants CODATA provided recommended values for.)
- What happened in or about year 2000?

Re the $2S$-$4P$ splitting measurement

- "MPQ 2017" announced at proton radius workshop June 2016



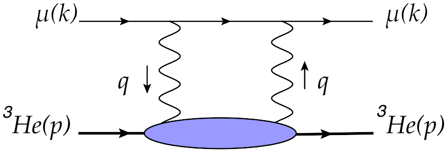
- Data heard around the world,

$$R_p(2S\text{-}4P) = 0.8297(91)\,\text{fm}$$

- Now have proton radius puzzle for ordinary hydrogen all by itself!

Two Photon Exchange (TPE)

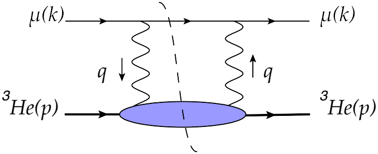
- One of the "other corrections": not the biggest term, but the biggest source of uncertainty. E.g.,



- Blob is off shell proton or any higher state. Makes calculation hard.
- How good are we?
- How good do we have to be?

Dispersive calculation

- Some calculate by noting putting the intermediate states on shell
  (a) gives the Imaginary part of the whole diagram, and
  (b) means each half of the diagram is an amplitude for a real scattering process, and hence can be gotten from scattering data.



- What matters is the lower vertex, so can use electron scattering data.
- Mostly need low Q^2, low energy data
- Reconstruct whole diagram using dispersion relations.

Begin with the proton

- Theory for Lamb shift splitting, with numbers for proton,

$$\Delta E_L^{\text{theo}} = \Delta E_{\text{QED}} - \frac{m_r^3 Z^4 \alpha^4}{12} R_p^2 + \Delta E_{\text{TPE}}$$
$$= 206.0336(15) - 5.2275(10) R_p^2 + 0.0332(20)$$

(units are meV and fm)

- Faith,

$$\Delta E_L^{\text{theo}} = \Delta E_L^{\text{expt}} = 202.3706(23)\,\text{meV}$$

- Solve,

$$R_p = 0.84087(39)\,\text{fm} \qquad [0.038\%]$$

- IF THE TPE WERE PERFECT,

$$R_p = 0.84087(32)\,\text{fm}$$

- Conclude: for the proton theorists have done their job.
Uncertainty in TPE not dominant.

Deuteron

- Trouble: the deuteron is loosely bound, a little energy turns it into other states. Proton remains just a proton until there is enough energy to make a pion.
- Theory with numbers for deuteron is now,

$$\Delta E_L^{\text{theo}} = 228.7766(10) - 6.1103(3) R_d^2 + \Delta E_{\text{TPE}}$$

- and there are now two ways to obtain the TPE,

how	*who*	ΔE_{TPE} (meV)
Nuclear potentials	Hernandez *et al.*	1.6900(200)
Nuclear potentials	Pachucki-Wienczek	1.7170(200)
Dispersion theory	Carlson *et al.*	2.0100(7400)
Summary	Krauth *et al.*	1.7096(200)

- Work out, with $\Delta E_L^{\text{expt}} = 202.8785(34)$ meV

$$R_d = 2.12562(78)\,\text{fm}$$

- If TPE be perfect,

$$R_d = 2.12562(15)\,\text{fm}$$

^{3}He

- For dispersion theorists, better case than the deuteron because the binding is stronger, the thresholds are higher, and there is data near the thresholds, which is the important region for this calculation.
- With ^{3}He numbers,

$$\Delta E_L^{\text{theo}} = 1644.4643(150) - 103.5184(98) R_T^2 + \Delta E_{\text{TPE}}$$

- and for the TPE,

how	*who*	ΔE_{TPE} (meV)
Nuclear potentials	Hernandez *et al.* (2016)	15.46(39)
Dispersion theory	CEC, Gorchtein, Vanderhaeghen	15.14(49)
Summary	Franke *et al.*	15.30(52)

^{3}He — How good do we have to be?

- comparison will be to current electron scattering data for R_T
- direct electron scattering on ^{3}He: $R_T = 1.973(14)$ fm
- can do somewhat better using ^{4}He data, $R_\alpha = 1.681(4)$ and isotope shift, except that:

group	$R_T^2 - R_\alpha^2$ (fm^2)	R_T (fm)
Cancio Pastor *et al.* (2012)	1.074(4)	1.975(4)
Shiner *et al.* (1995)	1.066(4)	1.973(4)
van Rooij *et al.* (2011)	1.028(11)	1.963(6)
subsumption		*1.968(11)*

- How well will the μ-^{3}He Lamb shift do? Use the result given for ΔE_{TPE} and work out the anticipated uncertainty:

$$R_T = 1.96xxx(13)\,\text{fm}$$

- Uncertainty about $8\times$ smaller than that from e^- scattering. (Although, (13) $\rightarrow$ (2) if TPE were perfect.)
- Still, if no BSM, will easily separate results from different isotope shift measurements.