Current understanding of the proton charge radius

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William & Mary
Cake Seminar–Jefferson Lab
Newport News, VA
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Talk plan:

- Show newest results close to the beginning
- Then proceed
  - Bit of history: ways to measure proton radius
  - Modern times and scattering data
  - Re-analyses of data: controversy
  - Deuteron measurements: lingering problem?
  - Two photon exchange corrections: a bar to the future?
- Closing comments
The proton radius has been (to date) measured using:

- electron-proton elastic scattering
- level splittings in traditional hydrogen
- level splittings, specifically the Lamb shift, in muonic hydrogen

The early results were incompatible, and gave about a $6\sigma$ discrepancy, summarized on the next slide. (Early here means before 2016.)
Pre-2016 proton radius results

- $\mu H$ Lamb shift (2010, 2013)
- Old ep atomic plus scattering avg., 0.8751 (61) fm (CODATA 2014)

(Newer experiments coming)
The newest results

Post 2016 electronic results, with older benchmarks

- 1S-2S + 2S-4P
- 1S-2S + 1S-3S
- 1S-2S + 1S-3S
- 2S-2P

- ep scatt (JLab)
- ep scatt (Mainz)

- MPQ 2017
- MPQ 2018
- MPQ 2018(d)
- LKB 2018
- York 2019
- PRad 2018
- ISR 2019

- old ep atomic plus scattering avg., 0.8751 (61) fm (CODATA 2014)
- µH Lamb shift (2010,2013)

proton charge radius (fm)
Three results from refereed journals, one from archived preprint, three from public conference talks

You can make your own!

CODATA has made their own: CODATA 2018 (available 20 May 2019) has proton radius compatible with muon Lamb shift value. See next slide.

BTW, unweighted average of ≥ 2016 electron-based measurements is 0.842 fm
Newest results, with CODATA 2018 proton radius

Post 2016 electronic results, with CODATA 2018

CODATA 2018
0.8414 (19) fm
How did we get here

Comments on measurements from electron scattering and atomic physics
Elastic electron scattering, $e^- p \rightarrow e^- p$

- There are form factors for electric ($E$) and magnetic ($M$) charge distributions.
- Cross section is given by

$$\frac{d\sigma}{d\Omega} \propto G_E^2(Q^2) + \frac{\tau}{\varepsilon} G_M^2(Q^2)$$

$$[\tau = Q^2/4m_p^2; \quad 1/\varepsilon = 1 + 2(1 + \tau) \tan^2(\theta_e/2)]$$

- Low $Q^2$ is mainly sensitive to $G_E$.
- DEFINE (for historical reasons) charge radius by,

$$R_E^2 = -6 \left( \frac{dG_E}{dQ^2} \right)_{Q^2=0}$$

- From real data, need to extrapolate to $Q^2 = 0$.  

CEC (W&M) proton radius puzzle JLab-2019 10 / 42
Scattering data

- Much data from 20th century, but currently biggest and best data set is Mainz (2010).
- Bernauer et al., PRL 2010 and later articles.
- Low $Q^2$ range, 0.004 to 1 GeV$^2$
- From their eigenanalysis,

$$R_E \text{ or } R_p = 0.879(8) \text{ fm}$$
Proton radius affects atomic energy levels.

\[ E = E_{\text{QED}} + \delta_{\ell 0} \frac{2m_r Z^4 \alpha^4}{3n^3} R_E^2 + E_{\text{TPE}} + \text{very small corrections} \]

- \( E_{\text{TPE}} \) = two photon exchange corrections (calculated: will discuss)
- Accurate measurements of energy splitting and accurate calculation of QED effects allows determination of proton radius.
Just in case: Hydrogen energy levels

- Scale for big splittings is Rydberg, $\text{Ryd} = \frac{1}{2} m_e \alpha^2 \approx 13.6 \text{ eV}$.
- Fine structure and Lamb shift are $\mathcal{O}(\alpha^2 \text{ Ryd})$.
- Hyperfine splitting is $\mathcal{O}(m_e/m_p) \times (\alpha^2 \text{ Ryd})$. 

Definitely not to scale:
Requirements for calculation

- **QED**
  \[
  E_{\text{QED}} = \frac{1}{2} m_r \alpha^2 \left[ 1 + \ldots + O\left(\frac{\alpha}{2\pi}\right)^3 + O\left(\frac{\alpha}{2\pi}\right)^4 + \ldots \right]
  \]

- **leading proton size correction**
  \[
  \Delta E_{\text{proton size}} = \frac{1}{2} m_r \alpha^2 \cdot \frac{4\alpha^2}{3n^3} \cdot (m_r R_E)^2
  \]
  \[
  = 6.7 \times 10^{-6} \quad 6 \times 10^{-11}
  \]
  for \( R_E = 1 \) fm and \( n = 2 \).

- **Hence need** \( O(\alpha/2\pi)^4 \) corrections. First available about year 2000.
Now can get proton radius from atomic splitting. As of early 2016:

- Crucial observation: $R_p$ from electron scattering and from electronic hydrogen agreed.

![Graph showing proton charge radius (fm) vs. proton radius puzzle](image)
Crucial: why in atomic physics do we use the derivative of $G_E$ to define the proton radius? Why not, for example, derivative of $F_1$?

Answer by doing the relativistic perturbation theory calculation for proton size effect on atoms.

Indeed find effect $\propto G'_E(Q^2)|_{Q^2=0}$

Since atomic results measures $G'_E(0)$, quote $R_p = R_E$, to match.
Can do analogous measurements with muonic atoms.

Muons weigh $200 \times$ what electron does. Muons orbit $200 \times$ closer. Proton looks $200 \times$ bigger and proton size effects are magnified.

Opportunity to obtain more accurate proton radius, despite short muon lifetime.

Done by CREMA specifically for the $2S-2P$ splitting (Lamb shift)

Obtained

$$R_p = 0.84087(39) \text{ fm}$$
Repeat

\[ R_p = 0.84087(39) \text{ fm} \]

- Uncertainty limit ca. 20X better than old electronic results.

Current box:

<table>
<thead>
<tr>
<th>(fm)</th>
<th>atomic</th>
<th>scattering</th>
</tr>
</thead>
<tbody>
<tr>
<td>electron</td>
<td>0.8759 (77)</td>
<td>0.879(8)</td>
</tr>
<tr>
<td>muon</td>
<td>0.84087 (39)</td>
<td>no data yet</td>
</tr>
</tbody>
</table>
Modern times and scattering data

Two thrusts:

1. new experiments, some just finished, some coming
2. reanalysis of old data
NEW:

PRad (JLab) does electron scattering down to $Q^2 = 0.0002$ GeV$^2$. Mentioned earlier: $R_E = 0.831 \pm$ ca. 2%.

Initial state radiation experiment at Mainz. Published 2017, republished with better understanding of systematics 2019. Second run to come.

FUTURE:

New experiment at Mainz, in Hall A2, observing final proton in TPC MUSE, Muon scattering experiment at PSI will do both muon (first time, at this accuracy) and electron scattering, down to 0.002 GeV$^2$. Expect relative error between $e$ and $\mu$ output radii about 0.7%. “Production run” started (?) July this year.

Proton radius from $\mu$ scattering at COMPASS, using a TPC to see the final proton
Discussions of new fits to old data

From a long time ago:

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**Physics Seminar**

Dr. Douglas Higinbotham

Jefferson Laboratory

Why the proton radius is smaller in Virginia

Abstract:

Recent Muonic hydrogen Lamb shift measurements have determined the proton's charge radius to be 0.84 fm, a result systematically different from the CODATA value of 0.88 fm from atomic hydrogen Lamb shift and recent electron scattering results. I will review the history of the electron results, starting from the 1965 review article by Ham et al. with its 0.81 fm standard dipole radius, and track the evolution of the proton charge radius up to the recent 0.88 fm results from Mainz. I will then discuss why groups in Virginia (JLab, UVA, and W&M) are extracting a radius from the electron scattering data close to the Muonic result. I will also show how PRad will hopefully settle the issue.

Friday, May 13, 2016

11:00 am

CEBAF Auditorium
But not limited to one locale

And still continuing

A few references (apologies . . .)

<table>
<thead>
<tr>
<th>minimalist (small radius)</th>
<th>more expansive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meissner et al. (2015)</td>
<td>original Mainz (Bernauer et al.)</td>
</tr>
<tr>
<td>Horbatsch &amp; Hessels (2016)</td>
<td>Hill &amp; Paz</td>
</tr>
<tr>
<td>Alarcón, Higinbotham, et al. (2019)</td>
<td></td>
</tr>
</tbody>
</table>

To repeat the last on the left: Alarcón, Higinbotham, Weiss, Ye, Phys.Rev. C99 (2019) no.4, 044303 (about a proton radius extraction by combining dispersion analysis and chiral EFT)
Basic viewpoint that leads to small results: Charge radius requires extrapolation to $Q^2 = 0$. Fits with lots of parameters tend to be less smooth outside data region. Fits to full data set generally require lots of parameters. For charge radius, better to fit to narrower, low $Q^2$ region of data. Have fewer parameters, less "wiggly" functions, and more faith in extrapolations.

slope gives $R_E = 0.84 (1\%)$ fm
But still unsettled: fitters obtaining larger radii have not recanted

In fact, may consult “Avoiding common pitfalls and misconceptions in extractions of the proton radius,” 1606.02159

Truly exciting: if larger radius from electrons is correct, then need explanation of difference between electron and muon interactions: we are into beyond the standard model physics (BSM).

Hope: Studies are proceeding with serious testing on pseudodata and with analysis of reliability and robustness of fit procedures, and may lead to some criteria for agreement.
Also exciting: The 1S to 2S splitting in both hydrogen and deuterium can be measured to 15 figures! (The 2S is metastable, hence narrow, leaving no fuzziness as to where it is.)

Only things that cannot be well calculated in difference are the radius terms. Hence get very accurate radius difference (called “isotope shift”):

\[ R_d^2 - R_p^2 = 3.820\,07 (65) \, \text{fm}^2 \]

\[ \therefore \text{If you know the deuteron radius to 4 figures after the decimal point, you can obtain the proton radius to that accuracy.} \]

Used by MPQ 2018 in figure seen earlier.
merely a reminder

Post 2016 electronic results, with older benchmarks

- 1S-2S + 2S-4P: MPQ 2017
- 1S-2S + 1S-3S: LKB 2018, York 2019, MPQ 2018, MPQ 2018(d)
- 2S-2P: MPQ 2018
- ep scatt (JLab): PRad 2018
- ep scatt (Mainz): ISR 2019

- μH Lamb shift: (2010, 2013)
- old ep atomic plus scattering avg., 0.8751 (61) fm (CODATA 2014)
If the electronically measured radii for the proton come down, is there any lingering problem?

Maybe . . .

CREMA has also measured the deuteron radius,

\[ R_d = 2.12562 (78) \text{ fm} \]

Using the muonic hydrogen value and at the isotope shift, get

\[ R_d = 2.12771 (22) \text{ fm} \]

which is \(2.6\sigma\) higher.
Two Photon Exchange (TPE): Dispersive calculation

- Need the box diagram with two photons
- Some calculate by noting putting the intermediate states on shell
  (a) gives the Imaginary part of the whole diagram, and
  (b) means each half of the diagram is an amplitude for a real
  scattering process, and hence can be gotten from scattering data.

\[
\begin{array}{c}
\mu(k) \\
\vdots \\
q \\
^3\text{He}(p) \\
\vdots \\
q \\
\mu(k) \\
\end{array}
\]

- What matters is the lower vertex, so can use electron scattering data.
- Mostly need low $Q^2$, low energy data
- Reconstruct whole diagram using dispersion relations.
- Something of a problem: One of the Compton amplitudes requires a
  subtracted dispersion relation, with a subtraction term that is not
  experimentally measured and must be estimated. We believe that can be
  done with sufficient accuracy.
Begin with the proton

- Theory for Lamb shift splitting, with numbers for proton,

\[
\Delta E_{L}^{\text{theo}} = \Delta E_{\text{QED}} - \frac{m_r Z^4 \alpha^4}{12} R_p^2 - \Delta E_{\text{TPE}} \\
= 206.0336(15) - 5.2275(10)R_p^2 + 0.0332(20) \\
\text{(units are meV and fm)}
\]

- TPE number from Birse and McGovern, following CEC and Vdh; ongoing consideration using other techniques
- Faith,

\[
\Delta E_{L}^{\text{theo}} = \Delta E_{L}^{\text{expt}} = 202.3706(23) \text{ meV}
\]

- Solve,

\[
R_p = 0.84087(39) \text{ fm} \quad [0.038%]
\]

- If the TPE were perfect,

\[
R_p = 0.84087(32) \text{ fm}
\]

- Conclude: for the proton theorists have done their job. Uncertainty in TPE not dominant.
Jump to other light nuclei: e.g., $^4\text{He}$

- Interested for similar reasons: want to find radius discrepancy
- Compare radius from electron scattering to radius from $\mu$ Lamb shift
- From electron scattering $R_\alpha = 1.681(4) \text{ fm} \ [0.25\%]$
- If this is the right radius, can calculate the $^4\text{He}$ finite size energy shift. The 0.25\% uncertainty becomes an predicted energy shift uncertainty

$$\delta E_{fs}^{^4\text{He}} = 1.42 \text{ meV}$$

- We and nuclear theorists using entirely different method calculate for the TPE,

<table>
<thead>
<tr>
<th>how</th>
<th>who</th>
<th>$\Delta E_{\text{TPE}}$ (meV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nuclear potentials</td>
<td>Hernandez et al. (2016)</td>
<td>-9.58(38)</td>
</tr>
<tr>
<td>Dispersion theory</td>
<td>CEC, Gorchtein, Vanderhaeghen</td>
<td>-12.23(xx)</td>
</tr>
</tbody>
</table>
Conflict! (BTW, we were in good agreement for $^3$He)

With a split-the-difference overall error bar,

uncertainty ($E_{\text{TPE}}$) $\approx 1.5$ meV

The muonic Lamb shift measurement cannot beat the electron radius scattering measurement because of the two-photon correction uncertainties. Ugh.

R. Pohl: “You are killing our experiment,”
Remarkable: After 9 years, the problem shows signs of being settled.

Interesting: little discussion of the correctness of the $\mu$-H Lamb shift data.

Radius results from electron scattering currently mixed, both experimentally (PRad vs. Mainz) and in reanalyses. More experiments coming.

Most recent ordinary hydrogen measurements of radius agree with results of level splitting in $\mu$-hydrogen.

Either

- The puzzle isn’t a puzzle: The electron based radius measurements are reducing to the muonic value.
  - The scattering analysis is under discussion, and more data coming
  - The newer spectroscopy measurements are giving the smaller radius.
- Those who insist on a large radius from electrons and a smaller one from muons have to be all in on a BSM explanation of the puzzle.
For us in the field

- Possibility the problem is settled
- Some mop-ups:
  - Resolve conflicts in the analysis of the full set of electron scattering data
  - Resolve the remaining deuteron conflict
  - Improve the $^4$He TPE calculation
Beyond the end
History

- Averages from the Committee on Data for Science and Technology (CODATA)
- There have been 9 CODATA reports.

<table>
<thead>
<tr>
<th>Year</th>
<th>Proton radius (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2018</td>
<td>0.8414(19)</td>
</tr>
<tr>
<td>2014</td>
<td>0.8751(61) mostly atomic</td>
</tr>
<tr>
<td>2010</td>
<td>0.8775(51) &quot;</td>
</tr>
<tr>
<td>2006</td>
<td>0.8768(69) &quot;</td>
</tr>
<tr>
<td>2002</td>
<td>0.8750(68) &quot;</td>
</tr>
<tr>
<td>1998</td>
<td>0.8545(120) election scattering</td>
</tr>
<tr>
<td>1986</td>
<td>– no $R_E$ quoted</td>
</tr>
<tr>
<td>1973</td>
<td>–</td>
</tr>
<tr>
<td>1969</td>
<td>0.805(11) electron scattering</td>
</tr>
</tbody>
</table>

(Only for 2002 and later is the proton radius among the constants CODATA provided recommended values for.)

- What happened in or about year 2000?
Re the 2S-4P splitting measurement

- “MPQ 2017” announced at proton radius workshop June 2016

- Data heard around the world,

\[ R_p(2S-4P) = 0.8297(91) \text{ fm} \]

- Now have proton radius puzzle for ordinary hydrogen all by itself!
Two Photon Exchange (TPE)

- One of the “other corrections”: not the biggest term, but the biggest source of uncertainty. E.g.,

  \[ \mu(k) \quad q \downarrow \quad 3^\text{He}(p) \quad \quad 3^\text{He}(p) \quad \mu(k) \quad q \uparrow \]

- Blob is off shell proton or any higher state. Makes calculation hard.
- How good are we?
- How good do we have to be?
Some calculate by noting putting the intermediate states on shell (a) gives the Imaginary part of the whole diagram, and (b) means each half of the diagram is an amplitude for a real scattering process, and hence can be gotten from scattering data.

- What matters is the lower vertex, so can use electron scattering data.
- Mostly need low $Q^2$, low energy data
- Reconstruct whole diagram using dispersion relations.
Begin with the proton

- Theory for Lamb shift splitting, with numbers for proton,

\[ \Delta E_{L}^{\text{theo}} = \Delta E_{\text{QED}} - \frac{m_r Z^4 \alpha^4}{12} R_p^2 + \Delta E_{\text{TPE}} \]

\[ = 206.0336(15) - 5.2275(10) R_p^2 + 0.0332(20) \]

(units are meV and fm)

- Faith,

\[ \Delta E_{L}^{\text{theo}} = \Delta E_{L}^{\text{expt}} = 202.3706(23) \text{ meV} \]

- Solve,

\[ R_p = 0.84087(39) \text{ fm} \quad [0.038\%] \]

- IF THE TPE WERE PERFECT,

\[ R_p = 0.84087(32) \text{ fm} \]

- Conclude: for the proton theorists have done their job. Uncertainty in TPE not dominant.
Trouble: the deuteron is loosely bound, a little energy turns it into other states. Proton remains just a proton until there is enough energy to make a pion.

Theory with numbers for deuteron is now,

$$\Delta E_{L}^{\text{theo}} = 228.7766(10) - 6.1103(3)R_{d}^{2} + \Delta E_{TPE}$$

and there are now two ways to obtain the TPE,

<table>
<thead>
<tr>
<th>how</th>
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<th>$\Delta E_{TPE}$ (meV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nuclear potentials</td>
<td>Hernandez et al.</td>
<td>1.6900(200)</td>
</tr>
<tr>
<td>Nuclear potentials</td>
<td>Pachucki-Wienczek</td>
<td>1.7170(200)</td>
</tr>
<tr>
<td>Dispersion theory</td>
<td>Carlson et al.</td>
<td>2.0100(7400)</td>
</tr>
<tr>
<td>Summary</td>
<td>Krauth et al.</td>
<td>1.7096(200)</td>
</tr>
</tbody>
</table>

Work out, with $\Delta E_{L}^{\text{expt}} = 202.8785(34)$ meV

$$R_{d} = 2.12562(78) \text{ fm}$$

If TPE be perfect,

$$R_{d} = 2.12562(15) \text{ fm}$$
For dispersion theorists, better case than the deuteron because the binding is stronger, the thresholds are higher, and there is data near the thresholds, which is the important region for this calculation.

With $^3\text{He}$ numbers,

$$\Delta E_{L}^{\text{theo}} = 1644.4643(150) - 103.5184(98)R_T^2 + \Delta E_{\text{TPE}}$$

and for the TPE,

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Nuclear potentials</td>
<td>Hernandez et al. (2016)</td>
<td>15.46(39)</td>
</tr>
<tr>
<td>Dispersion theory</td>
<td>CEC, Gorchtein, Vanderhaeghen</td>
<td>15.14(49)</td>
</tr>
<tr>
<td>Summary</td>
<td>Franke et al.</td>
<td>15.30(52)</td>
</tr>
</tbody>
</table>
comparison will be to current electron scattering data for $R_T$

direct electron scattering on $^3$He: $R_T = 1.973(14)$ fm

can do somewhat better using $^4$He data, $R_\alpha = 1.681(4)$ and isotope shift, except that:

<table>
<thead>
<tr>
<th>Group</th>
<th>$R_T^2 - R_\alpha^2$ (fm$^2$)</th>
<th>$R_T$ (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cancio Pastor et al. (2012)</td>
<td>1.074(4)</td>
<td>1.975(4)</td>
</tr>
<tr>
<td>Shiner et al. (1995)</td>
<td>1.066(4)</td>
<td>1.973(4)</td>
</tr>
<tr>
<td>van Rooij et al. (2011)</td>
<td>1.028(11)</td>
<td>1.963(6)</td>
</tr>
</tbody>
</table>

subsumption

1.968(11)

How well will the $\mu$-$^3$He Lamb shift do? Use the result given for $\Delta E_{\text{TPE}}$ and work out the anticipated uncertainty:

$$R_T = 1.96\text{xxx}(13) \text{ fm}$$

Uncertainty about 8× smaller than that from $e^-$ scattering. (Although, (13) → (2) if TPE were perfect.)

Still, if no BSM, will easily separate results from different isotope shift measurements.