CAN LASER OPTICS RESOLVE THE REMAINING PARTICLE PHYSICS ANGULAR MOMENTUM CONTROVERSY ?

Elliot Leader

Imperial College London

Elementary Particle Physics: why concerned about Angular Momentum (AM) of photon ?

Major challenge: understand internal structure of the proton

Key question: how is the spin of the proton built up from the AM of its quarks and gluons ?

$\Rightarrow$ **need to understand AM of gluons**

a fortiori $\Rightarrow$ **need to understand AM of photons**

What is the problem ?

Two issues:

(I) Question of splitting $J(\text{photon})$ into SPIN and ORBITAL parts.

Inspired the paper of Chen et al, which caused "THE ORIGINAL AM CONTROVERSY"

(II) Exist two fundamentally different versions of J(photon) (Poynting and Canonical)

Which is physically relevant?—“THE REMAINING CONTROVERSY”

Physics Reports, Vol 541, (2014) 163;
Phys Rev D 83, (2011) 096012;
Physics Letters B, Vol 756 (2016) 103.

(I) CLASSICAL ELECTRODYNAMICS

Momentum density (Poynting)

$$\boldsymbol{p}_{\text{poyn}}(x) = \text{poynting vector} = \boldsymbol{E} \times \boldsymbol{B}$$

Angular momentum density (Poynting)

$$\boldsymbol{j}_{\text{poyn}}(x) = \boldsymbol{r} \times (\boldsymbol{E} \times \boldsymbol{B}).$$

Total Poynting AM

$$\boldsymbol{J}_{\text{poyn}} = \int d^3x \, [\boldsymbol{r} \times (\boldsymbol{E} \times \boldsymbol{B})]$$

Has structure of an orbital AM, *i.e.* $\boldsymbol{r} \times \boldsymbol{p}_{\text{poyn}}$, but is the total photon angular momentum.

(Called "Belinfante" by particle physicists. Poynting did not give this expression. I believe Belinfante was the first to do so.)

QUANTUM ELECTRODYNAMICS (QED)TYPE APPROACH

Lagrangian + Noether's theorem ⇒Canonical densities

Momentum density (Canonical)

$$\boldsymbol{p}_{\mathsf{can}}(x) = E^i \boldsymbol{\nabla} A^i$$

Angular momentum density (Canonical)

$$\boldsymbol{j}_{\text{can}}(x) = [\boldsymbol{l}_{\text{can}} + \boldsymbol{s}_{\text{can}}]$$

where the canonical densities are

$$\boldsymbol{s}_{\text{can}} = \boldsymbol{E} \times \boldsymbol{A} \quad \text{and} \quad \boldsymbol{l}_{\text{can}} = E^i(\boldsymbol{x} \times \boldsymbol{\nabla})A^i$$

Angular momentum density (Canonical)

$$\boldsymbol{j}_{\text{can}}(x) = [\boldsymbol{l}_{\text{can}} + \boldsymbol{s}_{\text{can}}]$$

where the canonical densities are

$$\boldsymbol{s}_{\text{can}} = \boldsymbol{E} \times \boldsymbol{A} \quad \text{and} \quad \boldsymbol{l}_{\text{can}} = E^i(\boldsymbol{x} \times \boldsymbol{\nabla})A^i$$

Note: a **spin** plus **orbital** part **but**, clearly, each term is gauge non-invariant.

Total Canonical AM

$$\boldsymbol{J}_{\text{can}} = \int d^3x \, \boldsymbol{j}_{\text{can}}(x)$$

The canonical split into OAM and Spin is **not** gauge invariant.

Key question

CAN J(photon) BE SPLIT INTO SPIN AND ORBITAL PARTS IN A GAUGE-INVARIANT WAY??

Landau and Lifshitz: "Only the total angular momentum of the photon has a meaning."

Akhiezer and Berestetski: "The separation of the total angular momentum of the photon into orbital and spin parts has restricted physical meaning."

Jauch and Rohrlich: 'This separation is, in fact, impossible in a gauge-invariant manner."

etc, etc, etc.

The ORIGINAL AM CONTROVERSY

Chen, Lu, Sun, Wang and Goldman: 2008

"We address and solve the long-standing gauge-invariance problem of the nucleon spin structure. Explicitly gauge-invariant spin and orbital angular momentum operators of photons and gluons are obtained. **THIS WAS PREVIOUSLY THOUGHT TO BE AN IMPOSSIBLE TASK** ...."

THE CHEN et al PROCEDURE

Introduce fields called "pure" and "physical", but **identical** to $\boldsymbol{A}_{\|}$ and $\boldsymbol{A}_{\perp}$, à la Helmholz, with

$$\boldsymbol{A} = \boldsymbol{A}_{\|} + \boldsymbol{A}_{\perp}$$

where

$$\boldsymbol{\nabla} \times \boldsymbol{A}_{\|} = \boldsymbol{0}, \quad \text{and} \quad \boldsymbol{\nabla} \cdot \boldsymbol{A}_{\perp} = 0$$

THE CHEN et al PROCEDURE

Introduce fields called "pure" and "physical", but **identical** to $\boldsymbol{A}_{||}$ and $\boldsymbol{A}_{\perp}$, à la Helmholz, with

$$\boldsymbol{A} = \boldsymbol{A}_{||} + \boldsymbol{A}_{\perp}$$

where

$$\boldsymbol{\nabla} \times \boldsymbol{A}_{||} = \boldsymbol{0}, \quad \text{and} \quad \boldsymbol{\nabla} \cdot \boldsymbol{A}_{\perp} = 0$$

Chen et al then obtain

$$\boldsymbol{J}_{\text{chen}} = \underbrace{\int d^3x \, \boldsymbol{E} \times \boldsymbol{A}_{\perp}}_{\boldsymbol{S}_{\text{chen}}} + \underbrace{\int d^3x \, E^i (\boldsymbol{x} \times \boldsymbol{\nabla}) A^i_{\perp}}_{\boldsymbol{L}_{\text{chen}}}$$

and since $\boldsymbol{A}_{\perp}$ and $\boldsymbol{E}$ are unaffected by gauge transformations, they claim to achieve the impossible.

But the **Chen et al** operators are exactly the same as those discussed in the textbook of **Cohen-Tannoudji, Dupont-Roc and Grynberg (1989)** and studied in detail by **van Enk and Nienhuis (1994)**, who state:

But the **Chen et al** operators are exactly the same as those discussed in the textbook of **Cohen-Tannoudji, Dupont-Roc and Grynberg (1989)** and studied in detail by **van Enk and Nienhuis (1994)**, who state:

"Therefore we may write

$$[S_i, S_j] = 0. \quad !!!!!!!!!!!!!$$

This implies that $\boldsymbol{S}$ does **NOT** generate rotations of the polarization of the field"

Moreover van Enk and Nienhuis show that

$\boldsymbol{L}$ and $\boldsymbol{S}$ do NOT commute

They state: "Thus $\boldsymbol{S}$ ($\equiv \boldsymbol{S}_{\text{chen},\,z}$) **CANNOT** be interpreted as spin angular momentum. .... this result does not seem to have been noticed before."

Consequently, van Enk-Nienhuis write "**spin**" and "**orbital angular momentum"** in inverted commas.

Amazing!!! No Particle Theorist realised that the

Original Particle Physics AM Controversy

2008

HAD BEEN RESOLVED

van Enk-Nienhuis

1994

So Chen et al are wrong and, **strictly speaking**

spin	$\longrightarrow$	**“spin”**
orbital AM	$\longrightarrow$	**“orbital AM”**

So Chen et al are wrong and, **strictly speaking**

spin $\rightarrow$ **“spin”**

orbital AM $\rightarrow$ **“orbital AM”**

But does this matter?

[Note that $\boldsymbol{J}_{\text{chen}}$ is now called Gauge Invariant Canonical (**gican**) in Particle Physics]

Reasons why it does NOT matter

a) In general $\boldsymbol{L}_{\text{gican}}$ does not commute with $\boldsymbol{S}_{\text{gican}}$, but

$$[L_{\text{gican},\,z}\,, S_{\text{gican},\,z}] = 0$$

so $L_{\text{gican},\,z}$ and $S_{\text{gican},\,z}$ can be measured simultaneously, even at a quantum level.

Reasons why it does NOT matter

a) In general $\boldsymbol{L}_{\text{gican}}$ does not commute with $\boldsymbol{S}_{\text{gican}}$, but

$$[L_{\text{gican},\,z}\,, S_{\text{gican},\,z}] = 0$$

so $L_{\text{gican},\,z}$ and $S_{\text{gican},\,z}$ can be measured simultaneously, even at a quantum level.

b) Although the eigenvalues of $S_{\text{gican},\,z}$ are continuous, in general, for **Paraxial Fields they are approximately integer multiples of $\hbar$**

Reasons why it does NOT matter

a) In general $\boldsymbol{L}_{\text{gican}}$ does not commute with $\boldsymbol{S}_{\text{gican}}$, but

$$[L_{\text{gican},\,z}\,, S_{\text{gican},\,z}] = 0$$

so $L_{\text{gican},\,z}$ and $S_{\text{gican},\,z}$ can be measured simultaneously, even at a quantum level.

b) Although the eigenvalues of $S_{\text{gican},\,z}$ are continuous, in general, for **Paraxial Fields they are approximately integer multiples of** $\hbar$

c) For a paraxial photon absorbed by an atom the photon's $S_{\text{gican},\,z}$ is transferred, approximately, to the **internal** AM of the atom.

Hence, for paraxial fields, $J_{\textsf{gican},\,z}$, $L_{\textsf{gican},\,z}$ and $S_{\textsf{gican},\,z}$ seem to function as perfectly good physical angular momenta

Hence, for paraxial fields, $J_{\text{gican},z}$, $L_{\text{gican},z}$ and $S_{\text{gican},z}$ seem to function as perfectly good physical angular momenta

So there are two versions of AM on the market: J_{gican} and J_{poyn}

(II) THE REMAINING CONTROVERSY

Is J_{gican} different from J_{poyn} ??

And if so, which is physically relevant?

Since, for the total AM,

$$\boldsymbol{J}_{\text{poyn}} = \boldsymbol{J}_{\text{gican}} + \text{surface term}.$$

it is usually said that IF fields vanish at infinity the surface term vanishes so that

$$\boldsymbol{J}_{\text{poyn}} = \boldsymbol{J}_{\text{gican}}$$

Fine for classical fields, but

QUANTUM FIELDS are OPERATORS

What does it mean to say OPERATORS vanish at infinity?

Conclude: as OPERATORS

$$\boldsymbol{J}_{\mathsf{poyn}} \neq \boldsymbol{J}_{\mathsf{gican}}.$$

BUT KEY POINT

Even for CLASSICAL fields: their DENSITIES are different

BUT KEY POINT

Even for CLASSICAL fields: their DENSITIES are different

Hence

A Laser Optics measurement sensitive to the AM DENSITY could settle the issue!

Approach via Laser Optics experiments which measure the transfer of AM from the field to a particle

Beautiful experiments in the 1990s

[He et al, PRL 75 (1995); Frese et al, PR A54 (1996); Simpson et al, Opt. Lett. 22 (1997)]

which demonstrated the transfer, used particles whose dimensions were **comparable** to the beam diameter.

Approach via Laser Optics experiments which measure the transfer of AM from the field to a particle

Beautiful experiments in the 1990s

[He et al PRL 75 (1995); Frese et al PR A54 (1996); Simpson et al Opt. Lett. 22 (1997)]

which demonstrated the transfer, used particles whose dimensions were **comparable** to the beam diameter.

Hence were sensitive only to **TOTAL** $\boldsymbol{J}$, so could not distinguish between $\boldsymbol{J}_{\text{poyn}}$ and $\boldsymbol{J}_{\text{gican}}$.

Ground-breaking experiments involving transfer of AM from the field to a SMALL particle

[O'Neil et al, PRL 88 (2002); Garcés-Chávez et al, PRL 91 (2003)]

Here the variation of the field with ρ i.e. distance from the beam axis, is important.

and the reaction of the particle is sensitive to the momentum **density** and AM **density**.

General concept of these experiments

(a) Tiny particle trapped in a ring of radius ρ in, for example, a Bessel beam

(b) Particle spins about its CM driven by the spin AM absorbed

General concept of these experiments

(a) Tiny particle trapped in a ring of radius ρ in, for example, a Bessel beam

(b) Particle spins about its CM driven by the spin AM absorbed

(c) Particle rotates in the ring driven by the azimuthal force, proportional to the orbital AM of the beam.

(d) Because of viscous drag and torque there results limiting angular velocities for the rotation and the spin.

Hence, in principle, the local Orbital and Spin densities can be measured as a function of ρ.

What is expected for **poyn** and **gican** densities?

Take the standard form for a **paraxial field**

$$\boldsymbol{E}(\boldsymbol{r}) = \Big(u(\boldsymbol{r}),\, v(\boldsymbol{r}),\, \frac{-i}{k}\big(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\big)\Big) e^{i(kz-\omega t)}$$

and take

$$v(\boldsymbol{r}) = i\sigma\, u(\boldsymbol{r}) \quad \text{with} \quad \sigma = \pm 1$$

corresponding, approximately, to R or L circular polarization.

Consider a typical field, with an azimuthal mode index l, with the form, in cylindrical coordinates, $(\rho,\ \phi,\ z)$,

$$u(\rho, \phi, z) = f(\rho, z)e^{il\phi}.$$

The **densities**, for the cycle averages, per unit power, modulo $\frac{\epsilon_0}{\omega}$, are:

$$\langle l_{\text{poyn, z}} \rangle \approx \langle l_{\text{gican, z}} \rangle \approx l|u|^2$$

and

$$\langle s_{\text{poyn, z}} \rangle \approx \underbrace{-\frac{\sigma}{2}\rho\frac{\partial|u|^2}{\partial\rho}}_{\text{Allen et al}} \qquad \langle s_{\text{gican, z}} \rangle \approx \sigma|u|^2$$

Note that Allen et al, in their foundation paper on optical OAM, used the Poynting form for $\boldsymbol{J}$.

One finds for the **densities**, for the cycle averages, per unit power, modulo $\frac{\epsilon_0}{\omega}$,

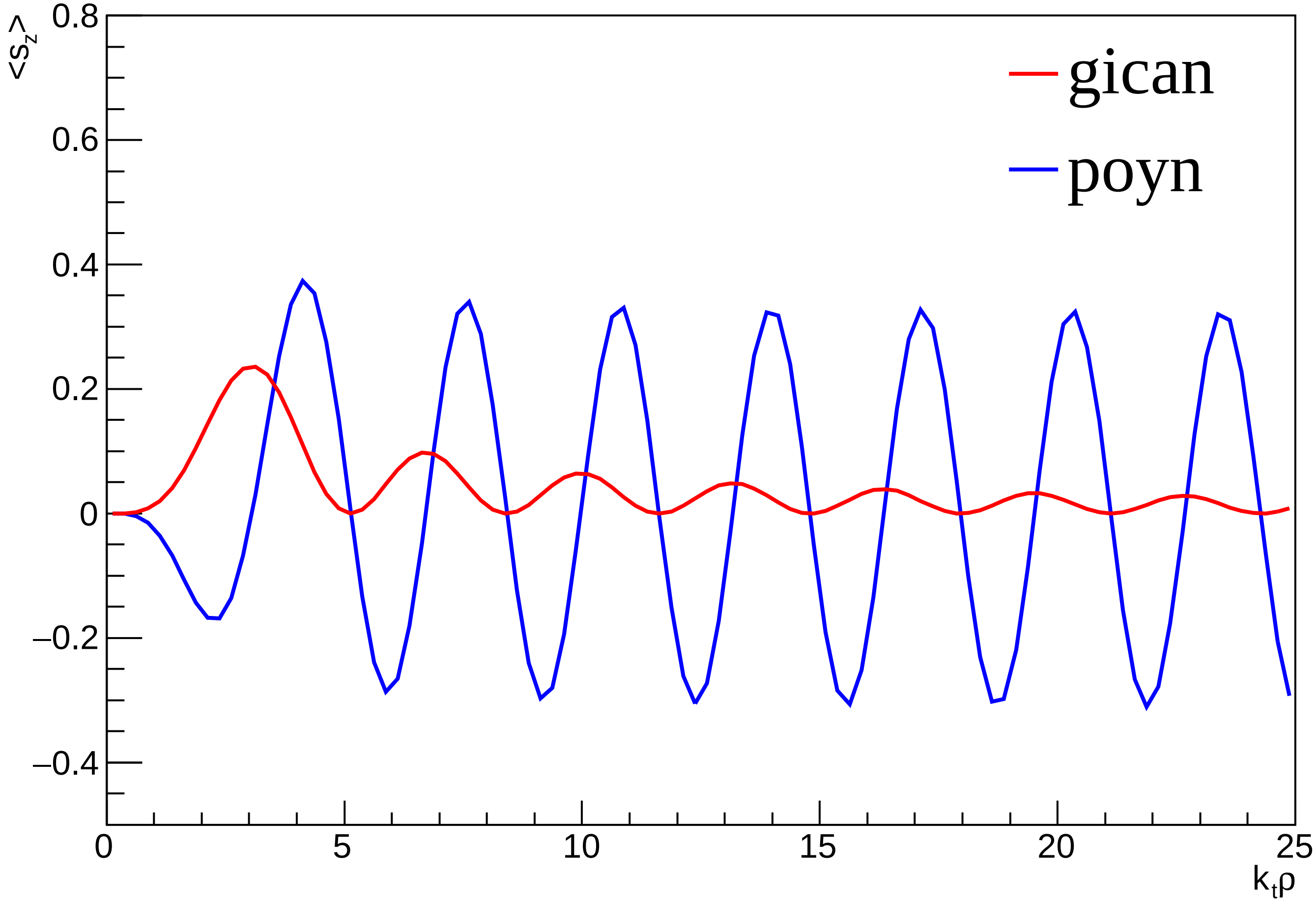
$$\langle l_{\text{poyn, z}} \rangle \approx \langle l_{\text{gican, z}} \rangle \approx l|u|^2$$

and

$$\langle s_{\text{poyn, z}} \rangle \approx \underbrace{-\frac{\sigma}{2}\rho\frac{\partial|u|^2}{\partial\rho}}_{\text{Allen et al}} \qquad \langle s_{\text{gican, z}} \rangle \approx \sigma|u|^2$$

Note that Allen et al, in their foundation paper on optical OAM, used the Poynting form for $\boldsymbol{J}$.

Compare $\langle s_{\text{poyn, z}} \rangle$ and $\langle s_{\text{gican, z}} \rangle$ as function of ρ for a $J_2(k_t\rho)$ paraxial Bessel beam



<s_z>
0.8
0.6
0.4
0.2
0
−0.2
−0.4
0
5
10
15
20
25
k_tρ
gican
poyn

Very different. Excellent!

Very different. Excellent!

A problem, however,

integrated across a "bright ring"

$$\int d\rho\,\rho\,\langle s_{\mathsf{poyn,\,z}}\rangle = \int d\rho\,\rho\,\langle s_{\mathsf{gican,\,z}}\rangle$$

Very different. Excellent!

A problem, however,

integrated across a "bright ring"

$$\int d\rho \, \rho \, \langle s_{\mathsf{poyn,\,z}} \rangle = \int d\rho \, \rho \, \langle s_{\mathsf{gican,\,z}} \rangle$$

Similar problem for Laguerre-Gaussian beam with radial mode index $p > 1$.

Is a definitive experiment possible??

Any experimental evidence, at present, in favour of one or other?

Bliokh and Nori Review (2015): several experiments favour **gican**

Chen and Chen (2012, unpublished) claim that the Ghai et al (2009) paper on the shift of diffraction fringes in single slit diffraction of beams with a phase singularity favours **gican**

Any experimental evidence, at present, in favour of one or other?

Bliokh and Nori Review (2015): several experiments favour **gican**

Chen and Chen (2012, unpublished) claim that the Ghai et al (2009) paper on the shift of diffraction fringes in single slit diffraction of beams with a phase singularity favours **gican**

But the evidence is not totally convincing.

Some theoretical arguments against $J_{\mathbf{poyn}}$

A CLASSICAL ARGUMENT AGAINST J_{poyn}

Circularly polarized plane wave propagating along OZ normalized to one photon per unit volume

Find:

$$\boldsymbol{J}_{\text{gican},\, z} \text{ per photon} = \pm\hbar$$

as expected intuitively!

A CLASSICAL ARGUMENT AGAINST J_{poyn}

Circularly polarized plane wave propagating along OZ normalized to one photon per unit volume

Find:

$$\boldsymbol{J}_{\text{gican},\, z} \text{ per photon} = \pm\hbar$$

as expected intuitively!

BUT

$$\boldsymbol{J}_{\text{poyn},\, z} \text{ per photon} = 0$$

Strictly speaking the ONLY "spin-like" quantum number for a massless particle is its HELICITY

$$\mathcal{H} \equiv \boldsymbol{J} \cdot \boldsymbol{P} / |\boldsymbol{P}|$$

Strictly speaking the ONLY "spin-like" quantum number for a massless particle is its HELICITY

$$\mathcal{H} \equiv \boldsymbol{J} \cdot \boldsymbol{P}/|\boldsymbol{P}|$$

For a photon:

Eigenvalues of $\mathcal{H}$ are $\pm\,\hbar$

How do the different versions compare?

Acting on a photon state with momentum $\boldsymbol{k}$:

$$\boldsymbol{\mathcal{H}}_{\text{can}}|\,\boldsymbol{k}\,\rangle = \boldsymbol{\mathcal{H}}_{\text{gican}}|\,\boldsymbol{k}\,\rangle = \pm\hbar|\,\boldsymbol{k}\,\rangle$$

($\boldsymbol{\mathcal{H}}_{\text{can}}$ **is** gauge invariant)

How do the different versions compare?

Acting on a photon state with momentum $\boldsymbol{k}$:

$$\mathcal{H}_{\text{can}} |\, \boldsymbol{k}\, \rangle = \mathcal{H}_{\text{gican}} |\, \boldsymbol{k}\, \rangle = \pm\hbar |\, \boldsymbol{k}\, \rangle$$

($\mathcal{H}_{\text{can}}$ **is** gauge invariant)

but

$$\mathcal{H}_{\text{poyn}} |\, \boldsymbol{k}\, \rangle = 0$$

This is a second reason to be suspicious about $\boldsymbol{J}_{\text{poyn}}$

SUMMARY

- The original photon AM controversy: the Chen et al claim that $\boldsymbol{J}$(photon) **can** be split, in a **gauge invariant** way, into genuine Orbital AM and Spin AM parts, is **INCORRECT**.

SUMMARY

- The original photon AM controversy: the Chen et al claim that $\boldsymbol{J}$(photon) **can** be split, in a **gauge invariant** way, into genuine Orbital AM and Spin AM parts is **INCORRECT**.

- Nevertheless, in paraxial approximation the "Orbital" and "Spin" parts are measurable, important characteristics of a laser beam and play a key role in the transfer of physical AM from a laser beam to particles.

- There remains the controversy as to whether the Poynting formula or the Gauge Invariant Canonical formula for the AM correctly describes the AM in a photon beam.

- There remains the controversy as to whether the Poynting formula or the Gauge Invariant Canonical formula for the AM correctly describes the AM in a photon beam.

- On basis of classical force and torque arguments for simple cases I am convinced that the Gauge Invariant Canonical formula for the AM is the correct one.

- There remains the controversy as to whether the Poynting formula or the Gauge Invariant Canonical formula for the AM correctly describes the AM in a photon beam.

- On basis of classical force and torque arguments for simple cases I am convinced that the Gauge Invariant Canonical formula for the AM is the correct one.

- A tantalizing open question: can one design an experiment to give a definitive answer??