“Hidden” properties of composite systems in and out of the conformal window

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Jlab Seminar
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"Hidden" properties of composite systems

- QCD / hadron physics
- Beyond- Standard Model
  composite Higgs

“Hidden” properties of composite systems

- non spectral observables like
- running coupling,
- anomalous dimension
- (renormalization factors)

- QCD / hadron physics
- Beyond- Standard Model composite Higgs

“Hidden” properties of composite systems in and out of the conformal window

Composite Higgs models that describe BSM
are viable near the conformal window

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are viable near the conformal window

Collaborators:
Oliver Witzel, Claudio Rebbi,
Ethan Neil, Andrea Carosso
LSD Collaboration

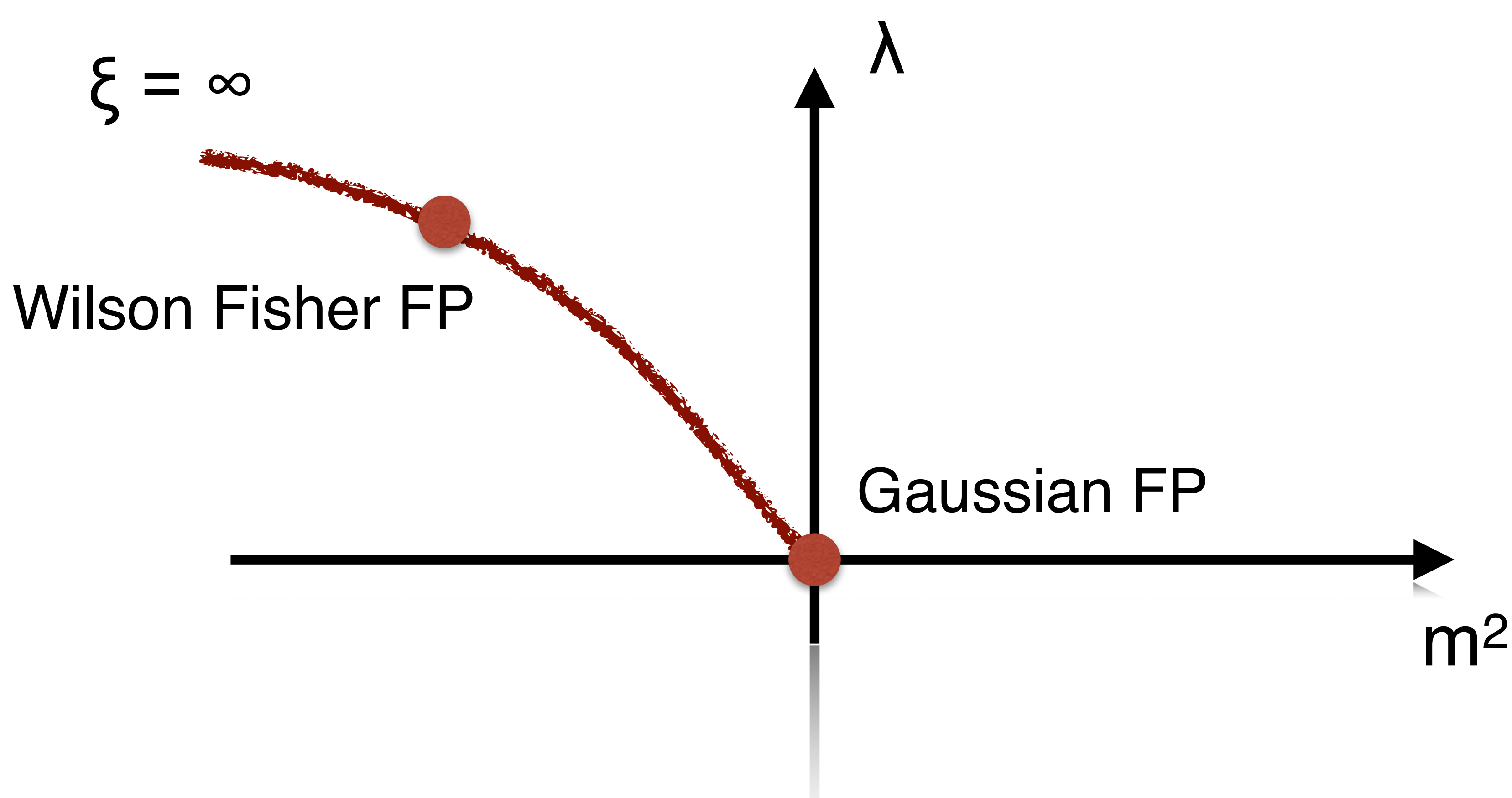
Conformal systems

are scale invariant :

describe the same physics at scale μ and $s\mu$

- QCD is not scale invariant
- 3D φ^4 (Ising) model at criticality is conformal :

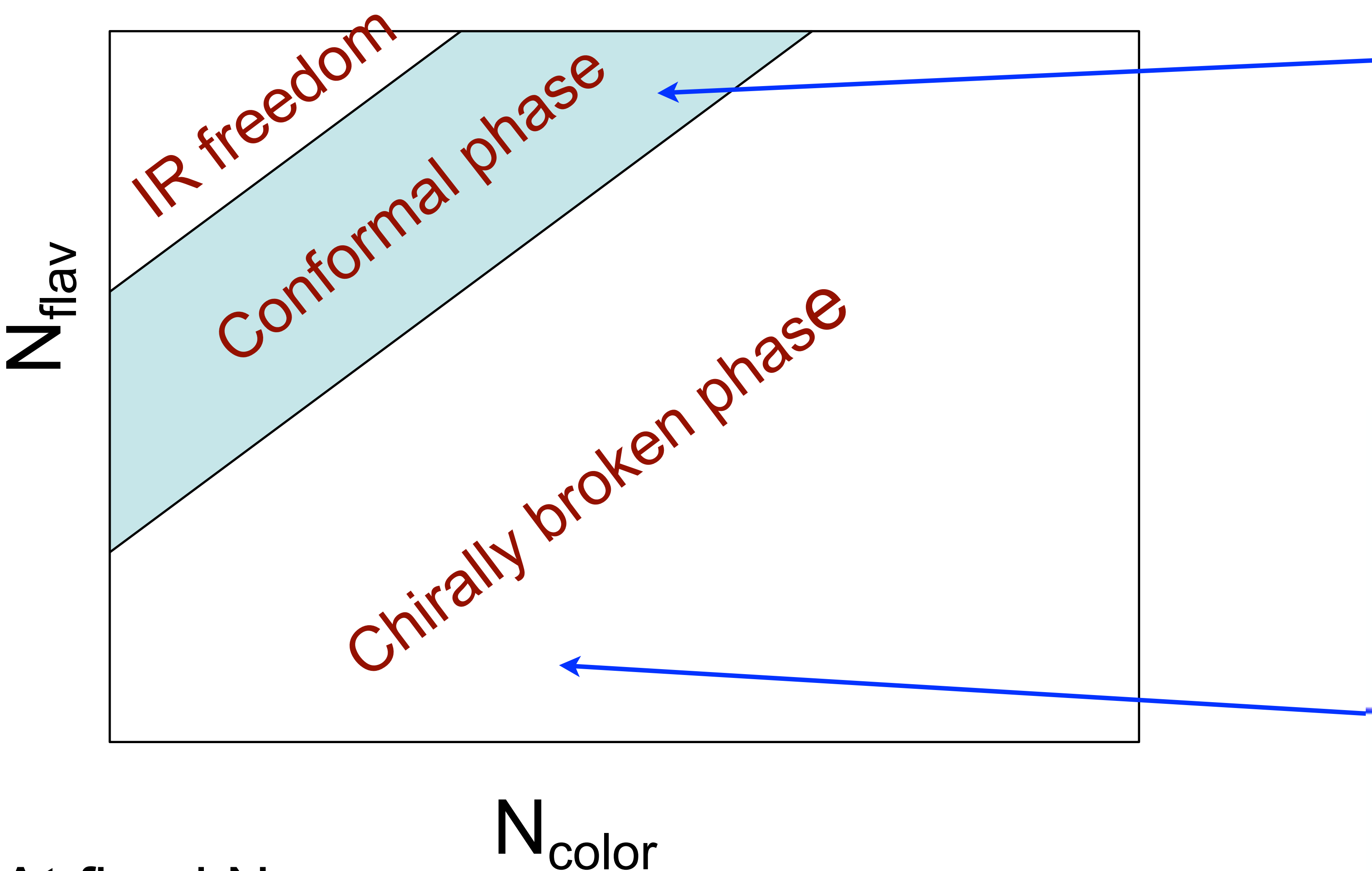
  correlation length $\xi = \infty$, all length scales are identical

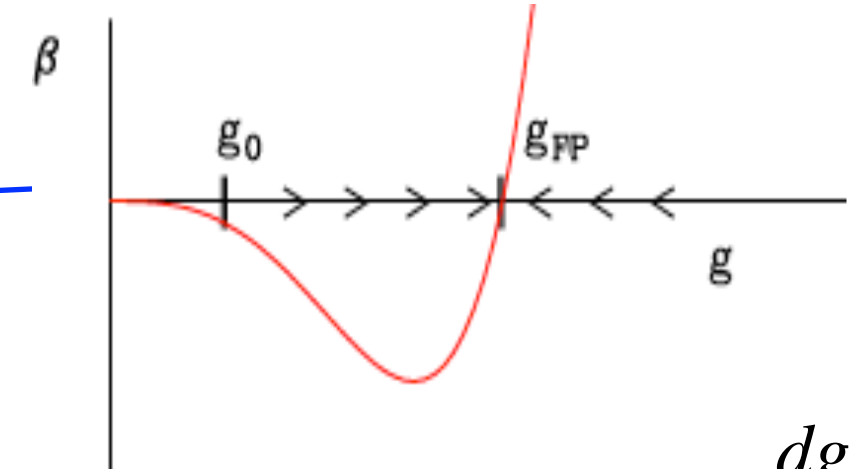


In mathematical physics, the **conformal symmetry** of spacetime is expressed by an extension of the Poincaré group. The extension includes special conformal transformations and dilations.

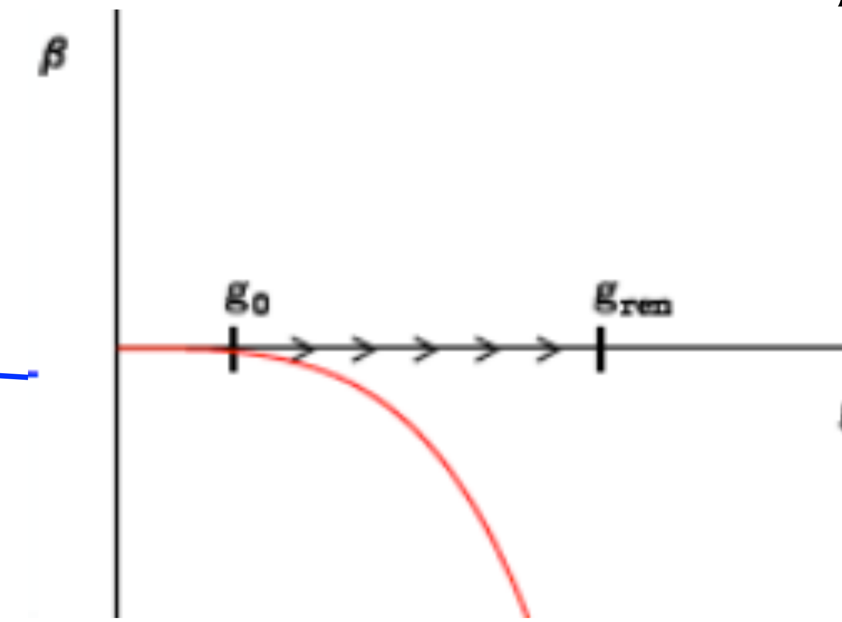
Conformal systems - 4D

- 4D SU(N_c) gauge with N_f massless fermions (in some rep)





$$\beta = \mu \frac{dg^2}{d\mu}$$



At fixed N_c :

- small N_f : chirally broken, QCD-like
- $N_f^* < N_f < N_f^{(IF)}$: conformal at a new FP (IRFP)
- $N_f^{(IF)} < N_f$: IR free

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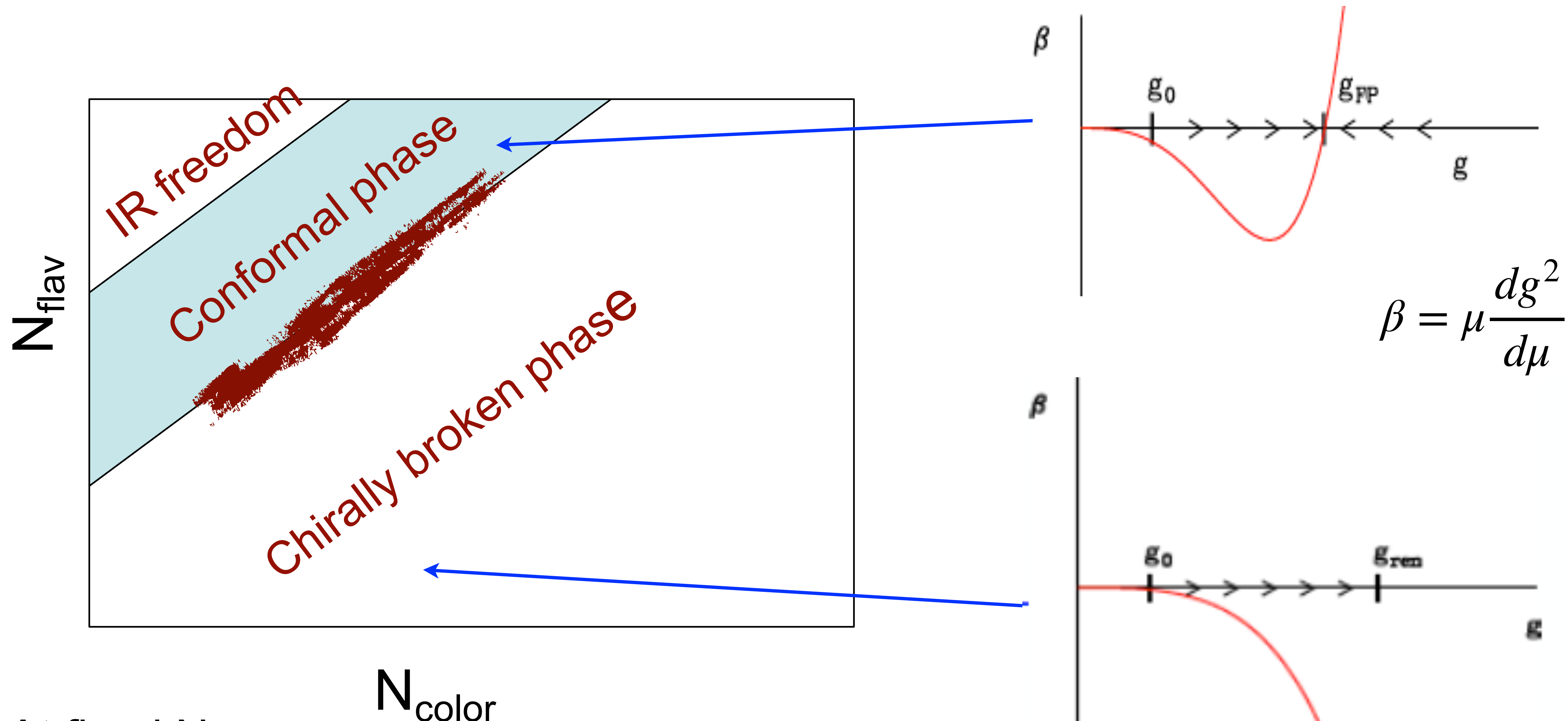

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Running coupling

RG analysis predicts the running coupling:

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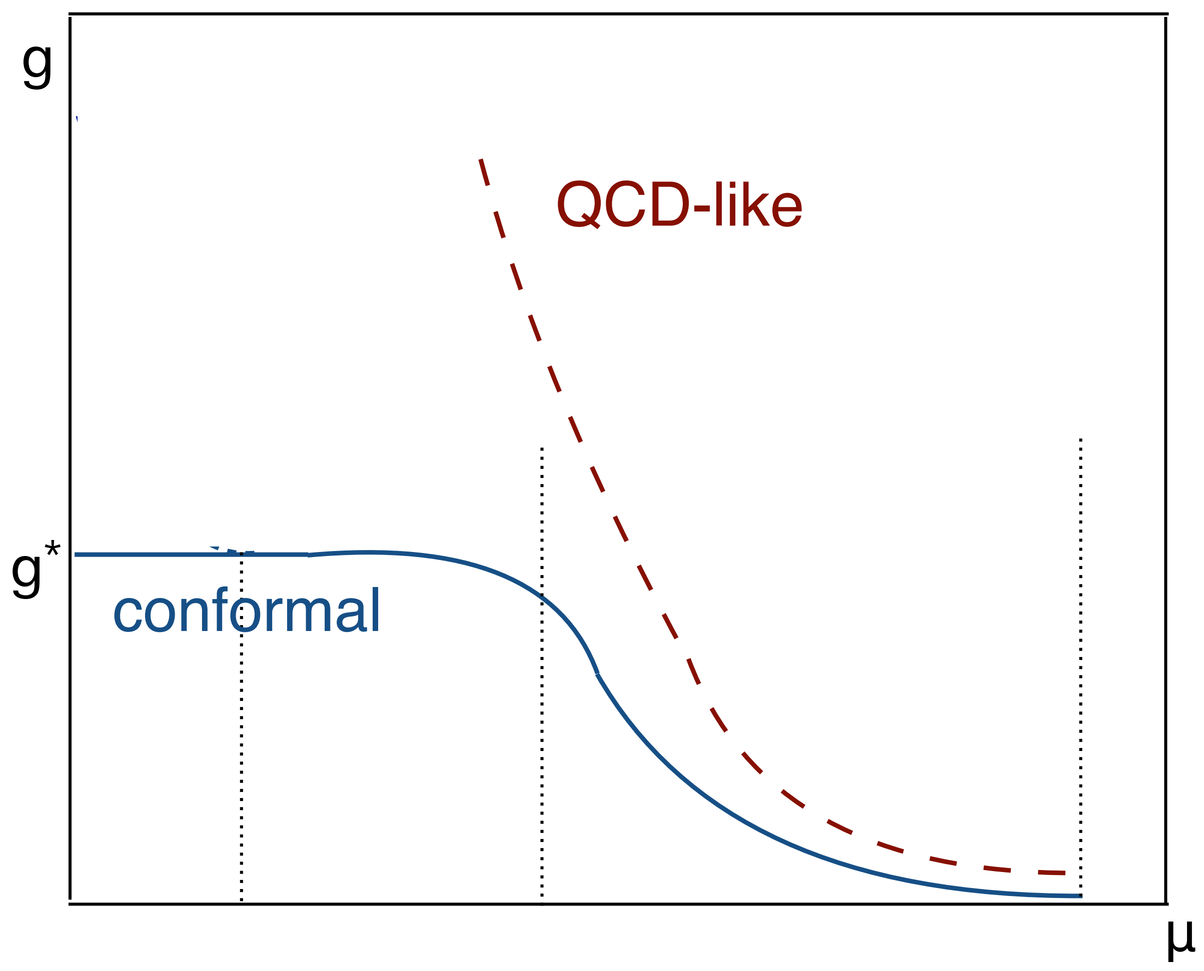

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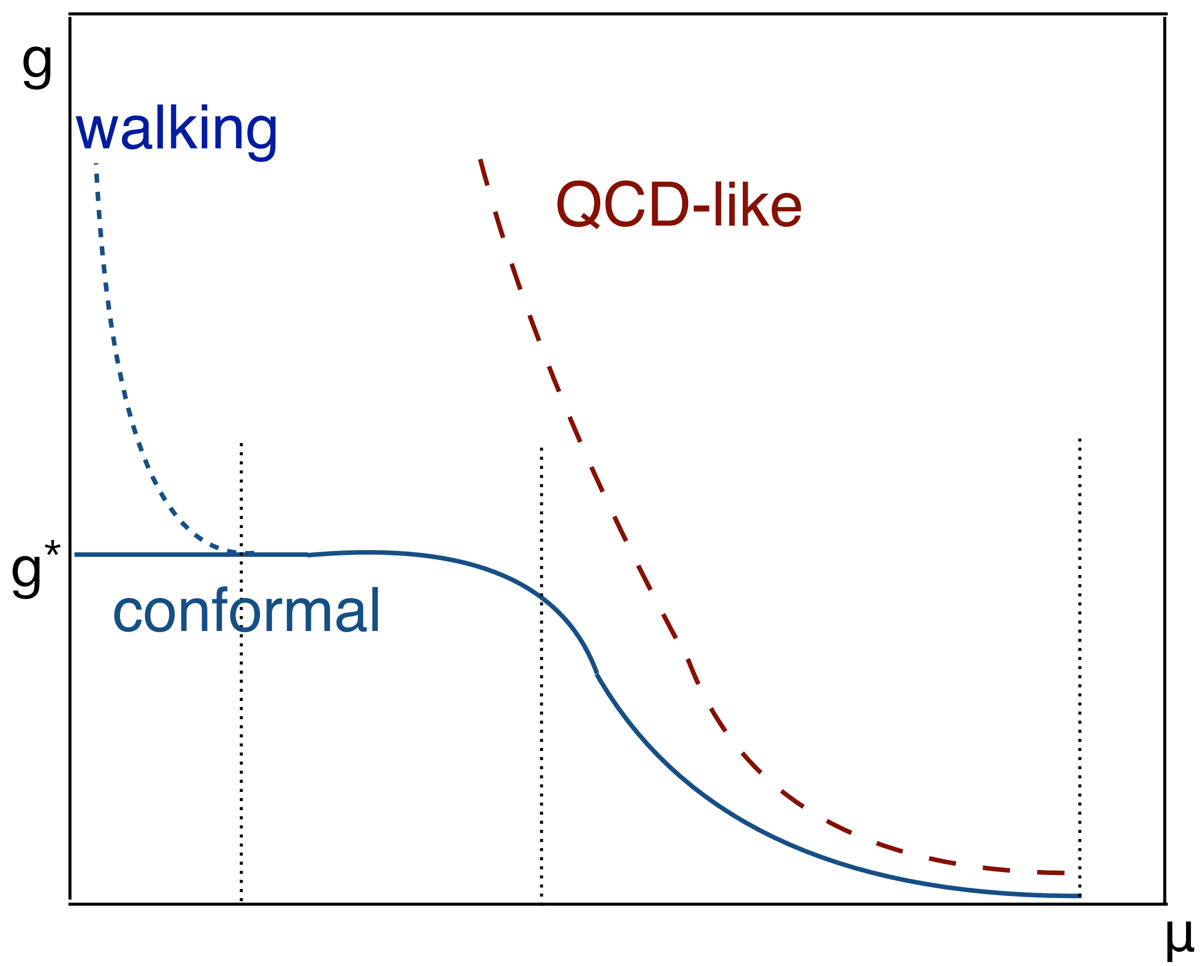

Walking systems are conformal - like at high energies but chirally broken in the IR "near conformal"

Walking systems exhibit
- large scale separation
- governed by the IRFP in the UV

Beyond SM: Composite Higgs models

Start with Higgsless, massless SM $\longrightarrow$ Full SM

$$\mathcal{L}_{SM\,0} \rightarrow \mathcal{L}_{SM}$$

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$$\mathcal{L}_{SD} + \mathcal{L}_{SM\,0} + \mathcal{L}_{int} \quad \rightarrow \quad \mathcal{L}_{SM} + \ldots$$

Full SM + additional states from strong dynamics $\mathcal{L}_{SD}$

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The construction ideally will

- predict the 125GeV Higgs
- give mass to the SM gauge fields
- give mass to the SM fermions :
  (4-fermion interaction or partial compositness?)

$\Big\}\ \mathcal{L}_{SD}$

- give mass to $\mathcal{L}_{SD}$ fermions: $\mathcal{L}_{UV}$ sector

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$$\mathcal{L}_{UV} \rightarrow \mathcal{L}_{SD} + \mathcal{L}_{SM\,0} + \mathcal{L}_{int} \rightarrow \mathcal{L}_{SM} + \ldots$$

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Near the conformal window

Conformal, near-conformal, walking systems are important

- Interesting QFTs on their own right
- Naturally exhibit walking — large scale separation
- UV is governed by the conformal FP — non-QCD properties

- both scale separation and large anomalous dimensions are needed for viable BSM models

Characterization of (near-) conformal systems

- zero of the β function
  - RG/ step scaling studies can predict it
  - harder than it sounds but doable
- anomalous dimensions of composite operators
  - scalar and baryon operators drive SM fermion mass generation typically large anomalous dimensions are required
- bound state spectrum
  - masses of new states - how predictive is the model?
  - is there any light state? (0++ ?)

Walking (accidental or tunable) makes these simulations hard

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Requires renormalization group study

Gradient flow : a new approach

Luscher Comm.Math Phys 293, 899 (2010)

GF is a continuous smoothing that removes short distance fluctuations

For scalar model :

$$\partial_t \phi_t = -(\partial_\phi S(\phi_t))\phi_t\,, \quad \phi_{t=0} = \phi$$

free flow : $\Phi(p) = \mathcal{N}\; e^{-p^2/t}\phi(p)$

Luscher JHEP 04 123 (2013)

Gauge flow: $\partial_t V_t = -(\partial S_{\mathrm{W}}[V_t])V_t\,, \quad V_0 = U$

Fermions evolve on the gauge background: $\partial_t \chi_t = \Delta[V_t]\chi_t\,, \quad \chi_0 = \psi$

(The flow action does not have to match the system’s)

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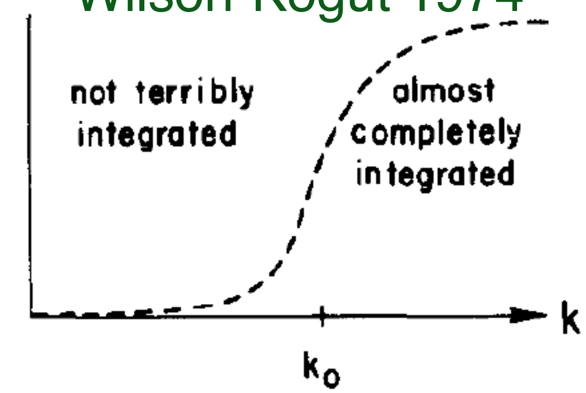
Wilson-Kogut 1974



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Is gradient flow an RG transformation ?

It is not.

GF misses two important attributes of an RG transformation:

- there is no rescaling $\Lambda_{cut} \to \Lambda_{cut}/b$ or coarse graining
- linear transformation does not have the correct normalization (wave function renormalization or η exponent $Z_\phi = b^{-\eta/2}$)

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Both issues can be solved

Real-space GF-RG:

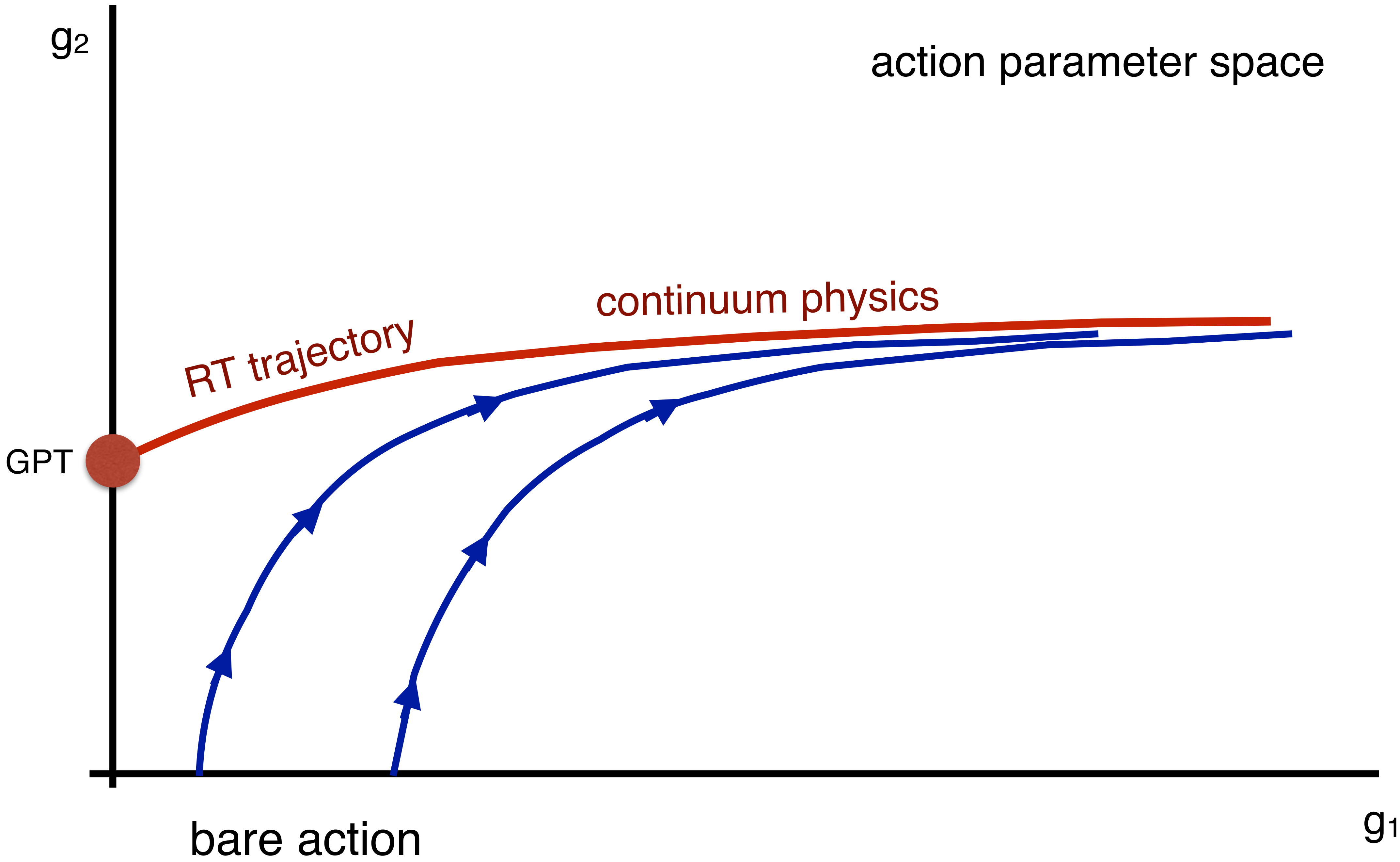
- GF defines the blocked variables
- coarse graining is done during measurement
- wave function renormalization (η exponent) calculated from an operator that does not have anomalous dimension (vector)

-

- A. Carosso, A.H, E. Neil, PRL 121 (2018)

RG flow

Only GF-RG has fixed points, not GF



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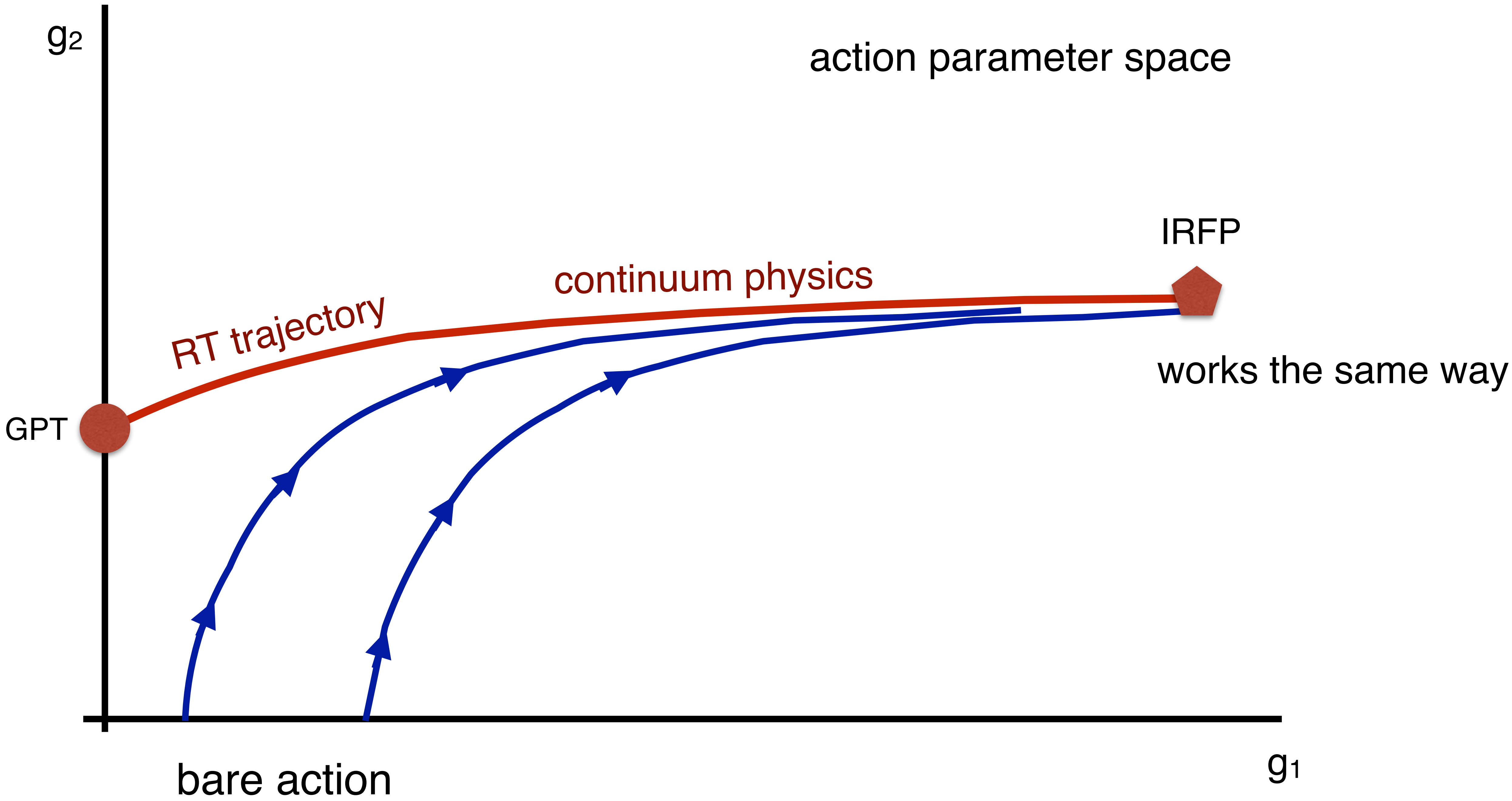

GF as RG

Along the RT all cut-off effects are removed.
RG with scale change b predicts S(g,m) → S(g',m')

$$\langle \mathcal{O}(0)\mathcal{O}(x_0)\rangle_{g,m} = b^{-2\Delta_O}\langle \mathcal{O}(0)\mathcal{O}(x_b = x_0/b)\rangle_{g',m'}$$

RG

$$\langle \mathcal{O}(0)\mathcal{O}(x_b)\rangle_{g',m'} = \langle \mathcal{O}_b(0)\mathcal{O}_b(x_b)\rangle_{g,m}$$

MCRG

$$\langle O_b(\Phi_b(0))O_b(\Phi_b(x_b))\rangle_{g,m} = b^{-\eta}\langle O(\phi_t(0))O(\phi_t(x_b))\rangle_{g,m}$$

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$$\langle O_b(\Phi_b(0))O_b(\Phi_b(x_b))\rangle_{g,m} = b^{-\eta}\langle O(\phi_t(0))O(\phi_t(x_b))\rangle_{g,m}$$ GF

Ratio of flowed & unflowed correlators predict the anomalous dimension

$$\frac{\langle O_t(0)O_t(x_0)\rangle}{\langle O(0)O(x_0)\rangle} = b^{2\Delta_O - 2n_O\Delta_\phi}$$

$$x_0 \gg b$$

$$\Delta_O = d_O + \gamma_O$$

$$\Delta_\phi = d_\phi + \eta/2$$

Anomalous dimensions

Calculate η by an operator that does not have an anomalous dimension:

— vector or axial charge (A(x))

The super-ratio

$$R(t,x_0)=\frac{\langle O_t(0)O_t(x_0)\rangle}{\langle O(0)O(x_0)\rangle}\left(\frac{\langle A(0)A(x_0)\rangle}{\langle A_t(0)A_t(x_0)\rangle}\right)^{n_O/n_A}=b^{\gamma_O}$$

independent of $x_0 \gg b$ and predicts γ

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independent of $x_0 >> b$ and predicts γ

- t and b are still independent!
  - Natural choice (required asymptotically) : $b^2 \sim t$
- it is advantageous to flow only the source, not the sink
- γ is universal at the FP only : set fermion mass to zero
- t has to be large enough, and

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- it is advantageous to flow only the source, not the sink
- γ is universal at the FP only : set fermion mass to zero
- t has to be large enough, and $x_0 \gg \sqrt{8t}$

β function and running coupling

- RG of single operator : $\langle \mathcal{O} \rangle_S = b^{-\Delta_O} \langle \mathcal{O} \rangle_{S'}$
- operators with no scaling dimension ($\Delta_O = 0$) define a running coupling
- $\mathcal{O} = b^4 E \propto t^2 E$ has no anomalous dimension ($\Delta_O = 0$)
- Luscher defined gradient flow running coupling

$$g_c^2(L) = \frac{128\pi^2}{3(N^2-1)} \frac{1}{C(c,L)} \langle t^2 E(t) \rangle$$

RG β function:

- Finite volume step scaling function β_2

$$\beta_{c,s}(g_c^2; L) = \frac{g_c^2(sL; a) - g_c^2(L; a)}{\log(s^2)}$$

- Continuous β function

$$\beta(g^2) = 2t \frac{dg^2}{dt}$$

Some recent results

The β function

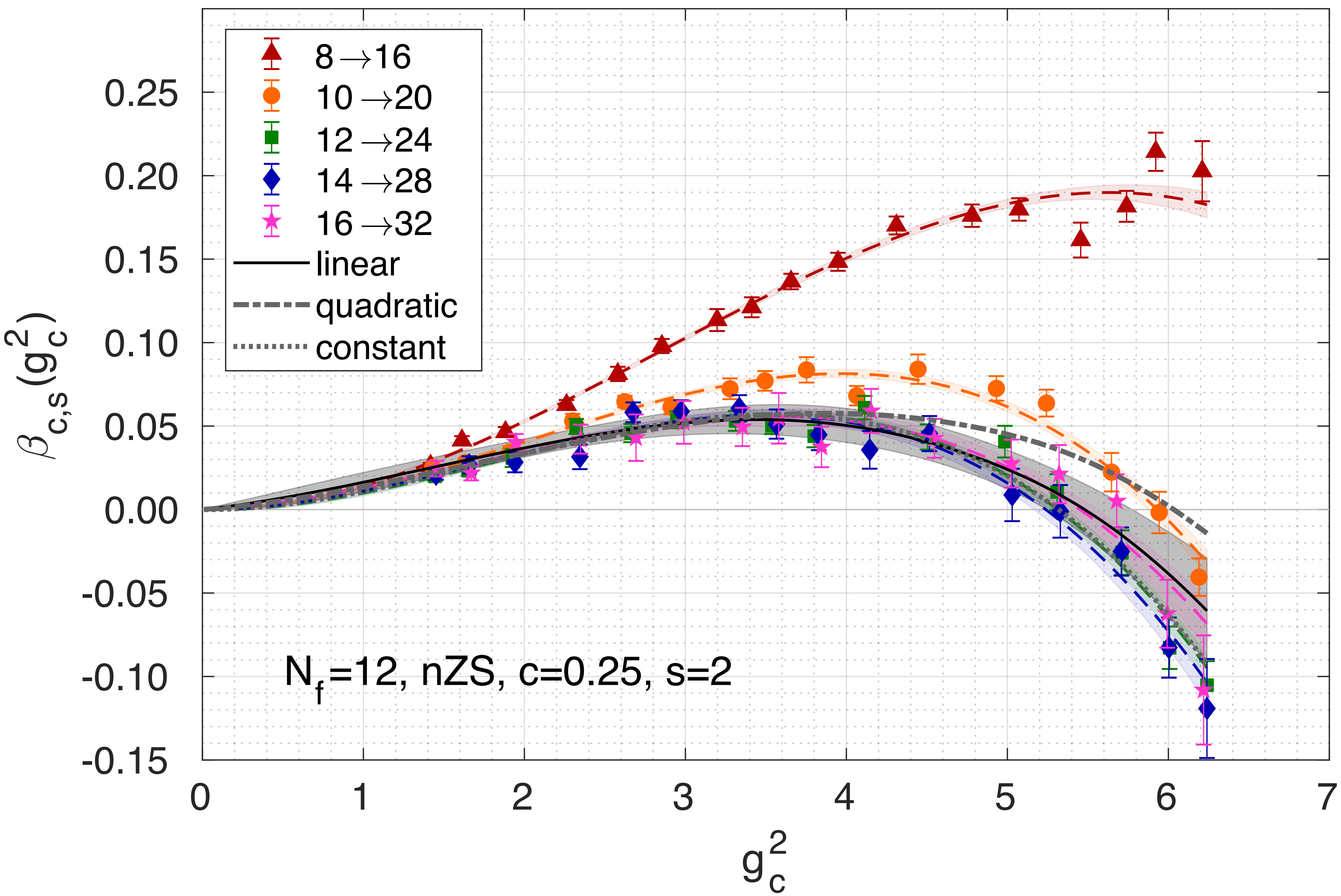
Shows the opening of the conformal window

- Numerically difficult in near-conformal systems
- Some results are controversial
- I believe a better fermion action (domain wall) will resolve the issues
- Recent results: N_f=12 fundamental fermions is conformal
  N_f=10 appears conformal
  (N_f=8 is expected too be chirally broken)

N_f=12 fundamental

Step scaling function (~ - β function) with domain wall fermions
Detailed study of systematics with flow, operator, fits, etc

AH,Rebbi, Witzel



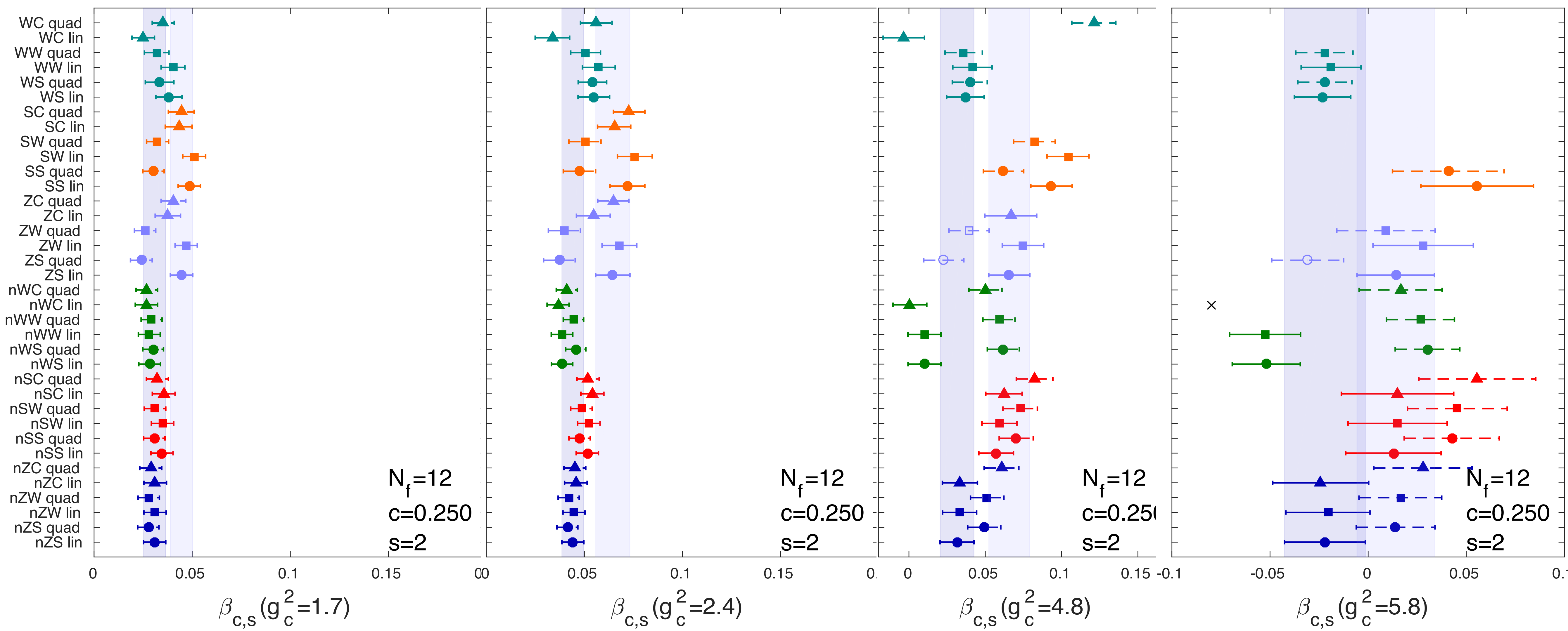
Data points: raw data

shaded band: continuum extrapolation

N_f=12 fundamental - systematics

AH,Rebbi, Witzel

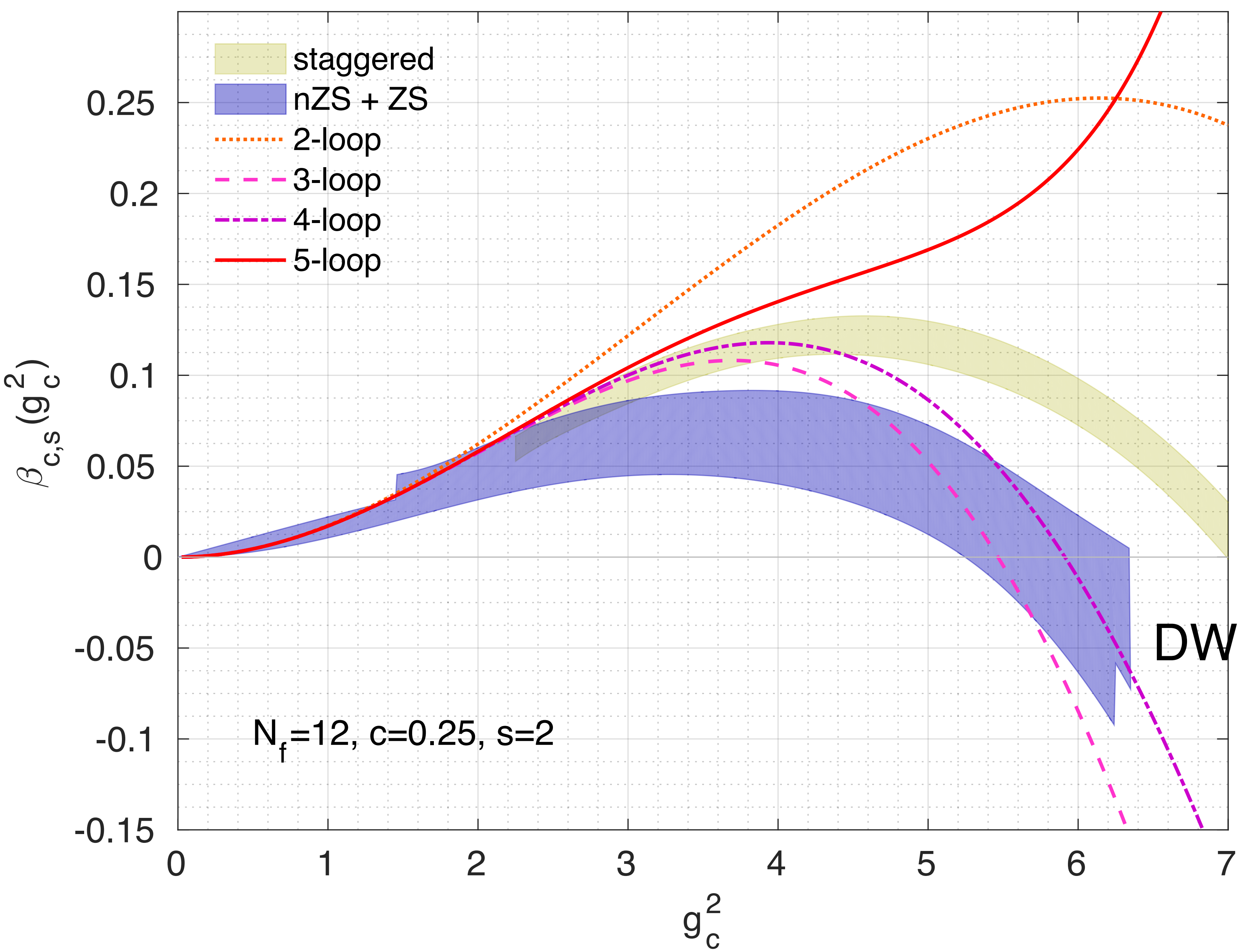
3 different flows, 3 different operators, optional tree-level improvement,
2 continuum extrapolations
Do the 36 predictions agree? → measure of systematical errors



(c=0.25 GF scheme)

N_f=12 fundamental

Continuum extrapolated result implies IRFP at $g^2 = 5.5$
(the value is scheme dependent, the existence universal)

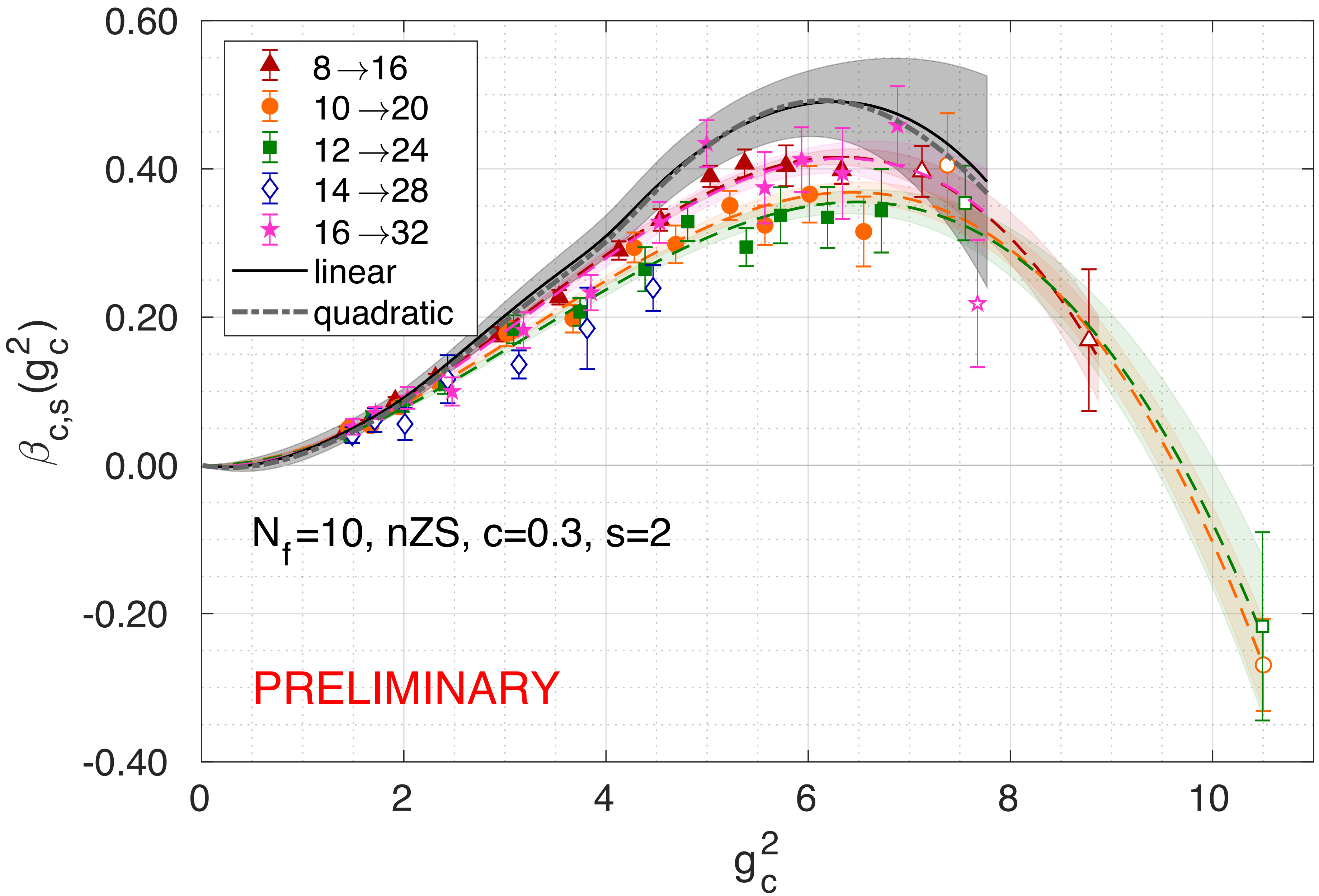


(c=0.25 GF scheme)

AH,Rebbi, Witzel

N_f=10 fundamental

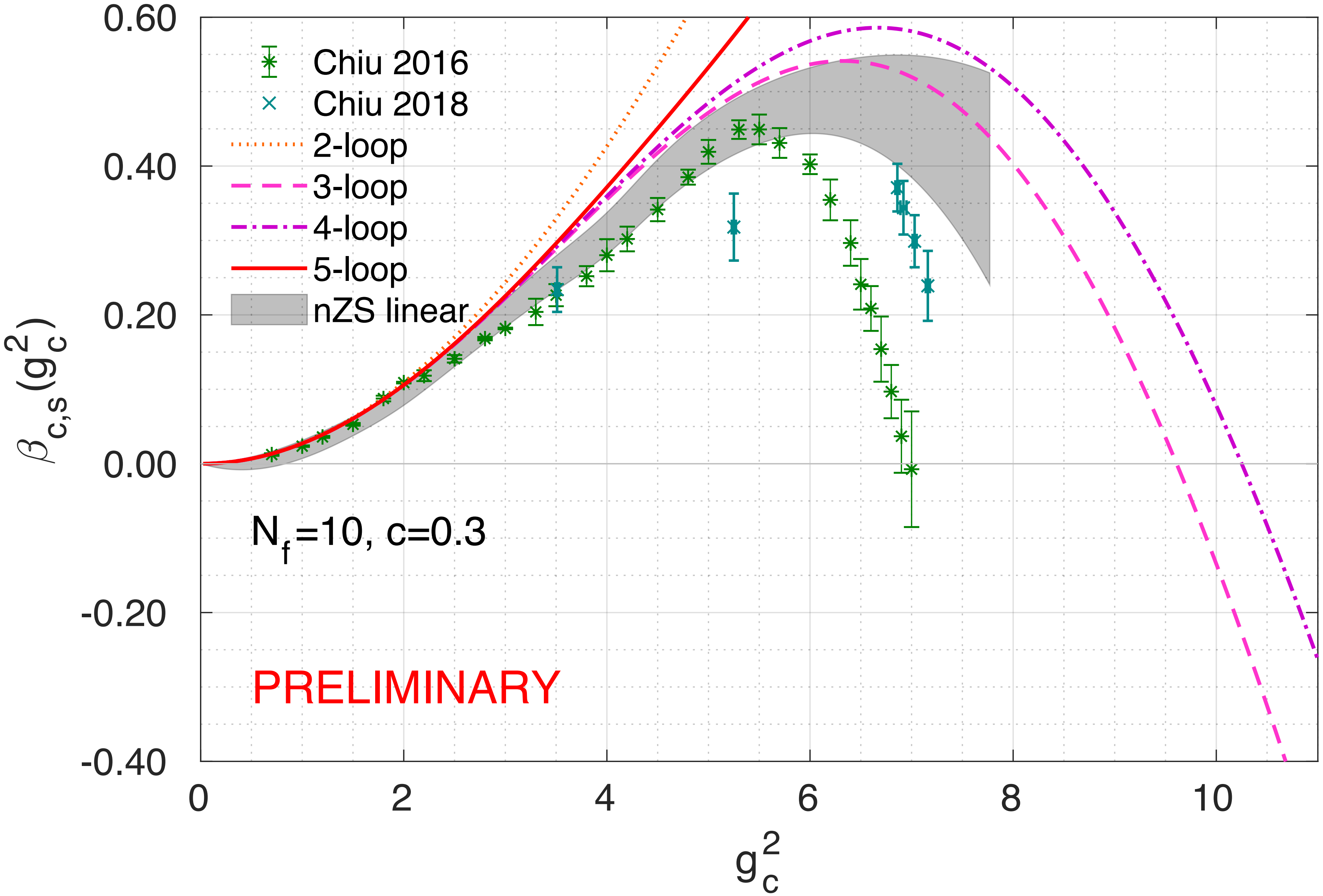
Step scaling function, domain wall fermions
Needs more data but indication for IRFP around $g^2 \sim 10$



AH,Rebbi, Witzel

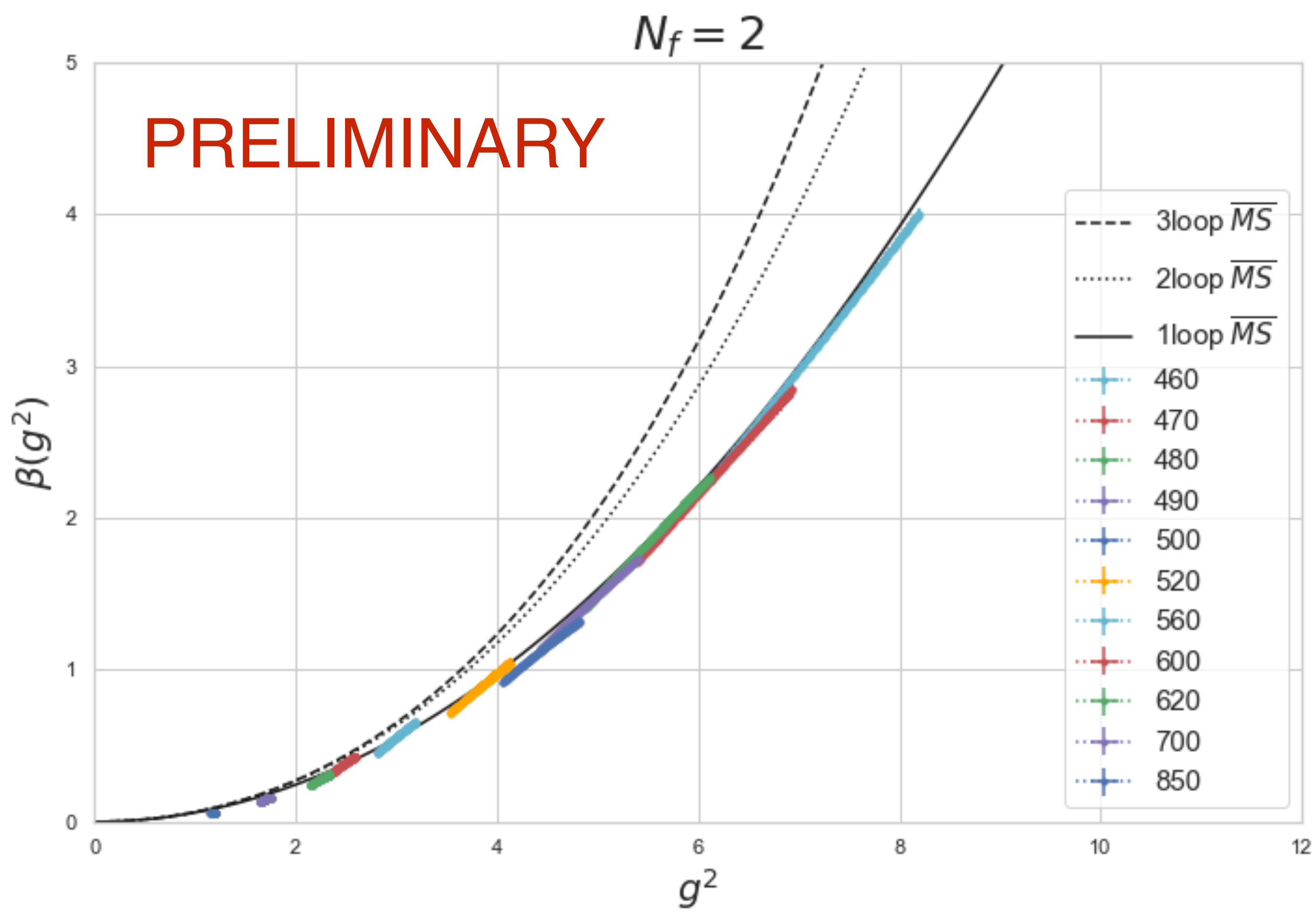
N_f=10 fundamental

Compare to Chiu, DW results



If N_f=10 is conformal, it is perfect for 2+8 or 4+6 mass-split system

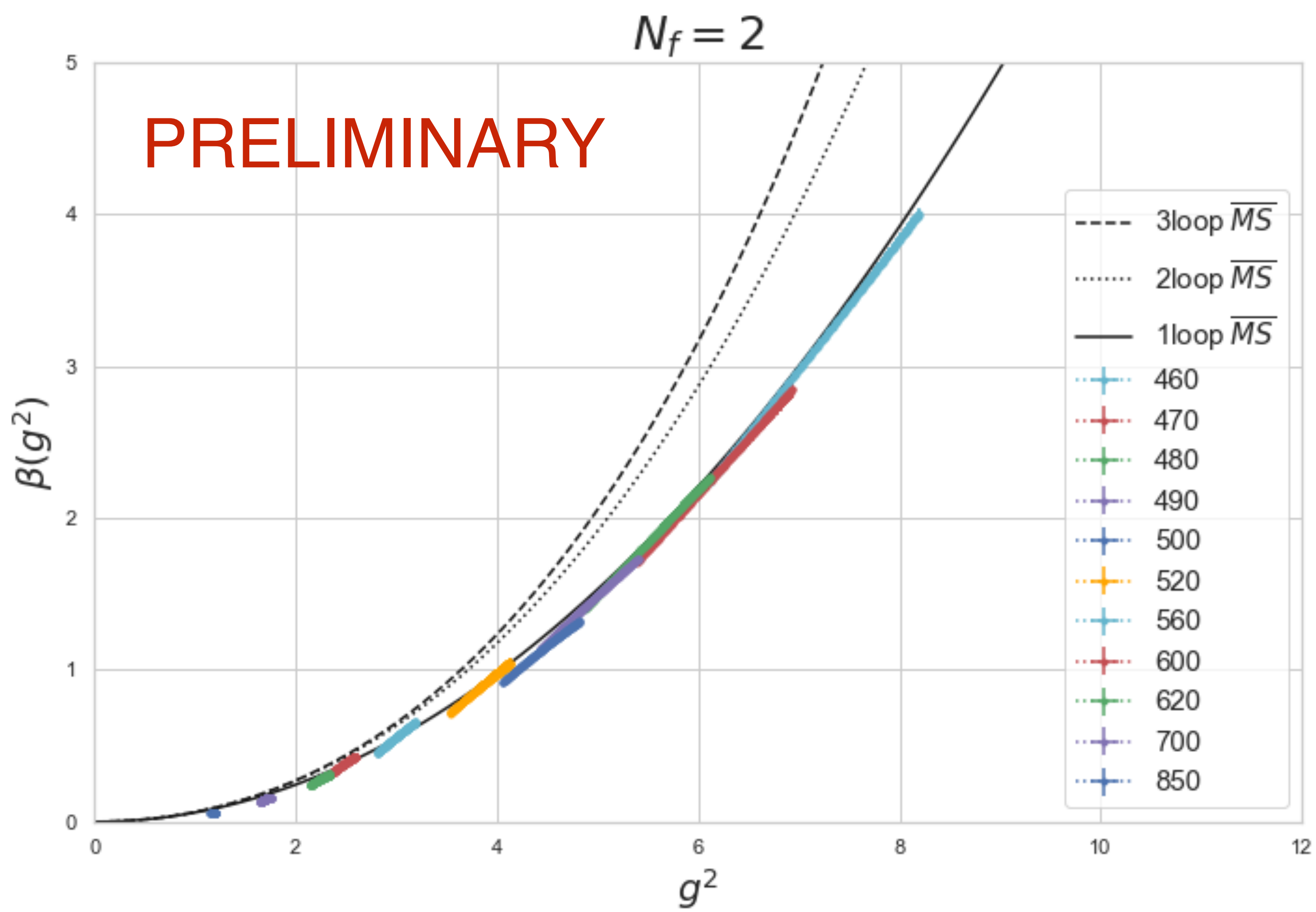
N_f=2 continuous β function



The continuous beta function is predicted by chaining together several bare gauge coupling runs

In the GF scheme the β function follows the 1-loop perturbative prediction (Consistent with observation of Alpha collaboration with 3 flavors)

N_f=2 continuous β function



- AH, Witzel - in prep

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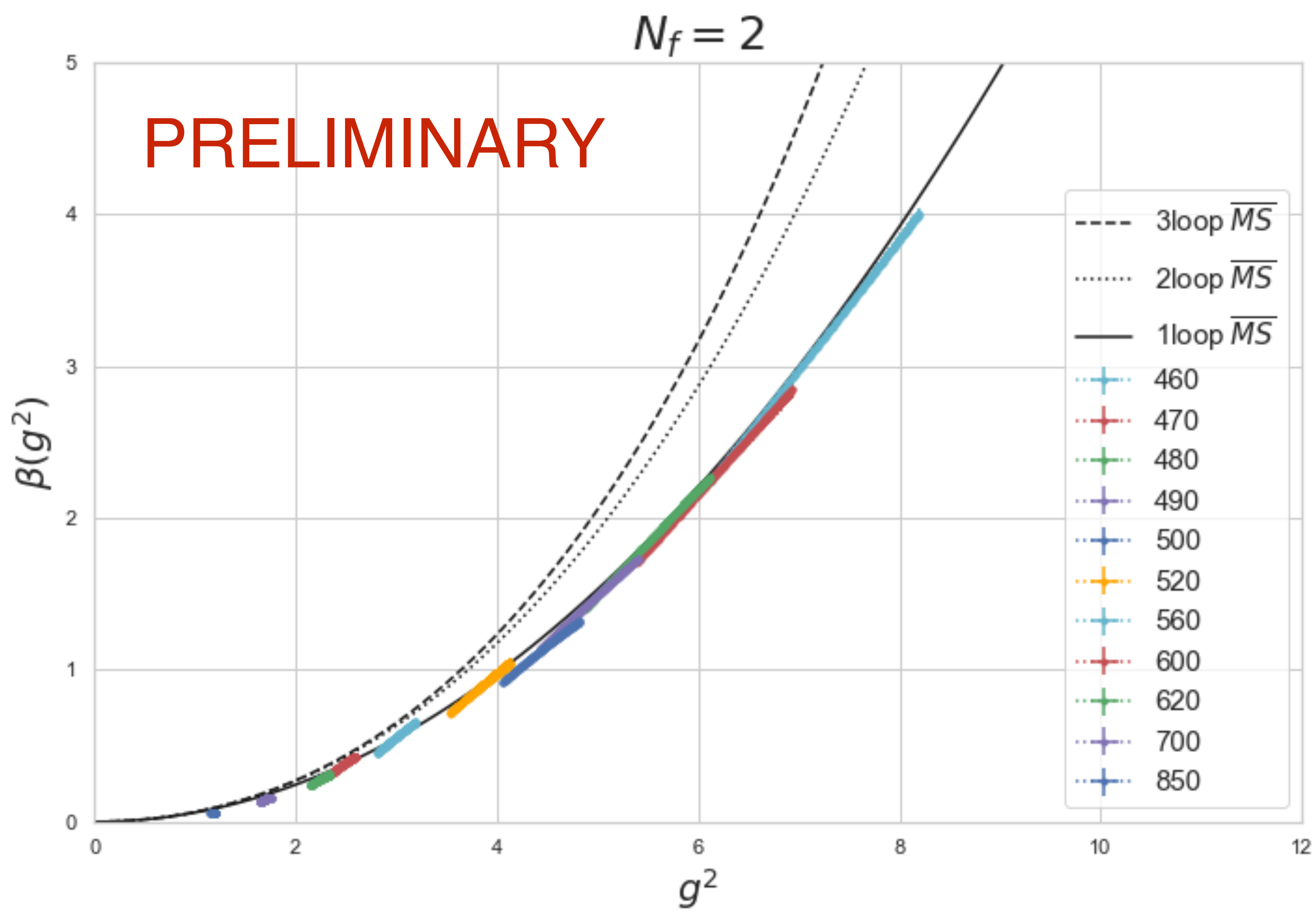

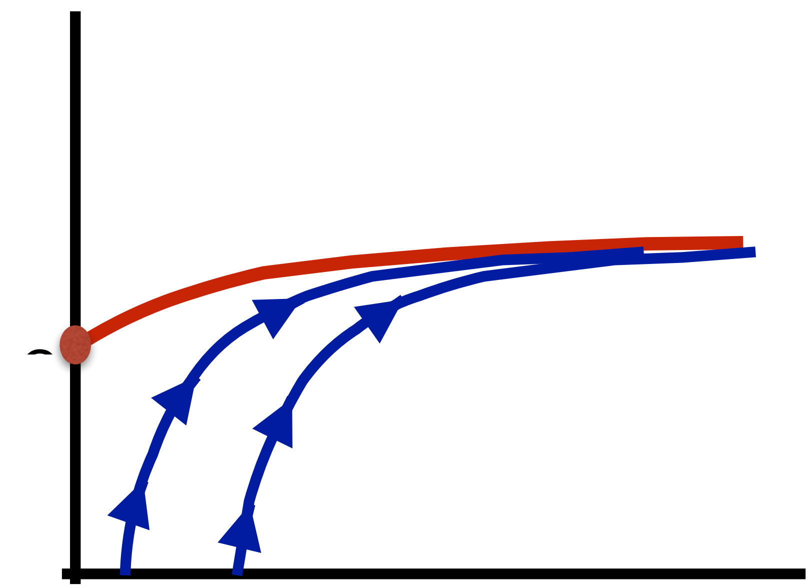

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Anomalous dimensions

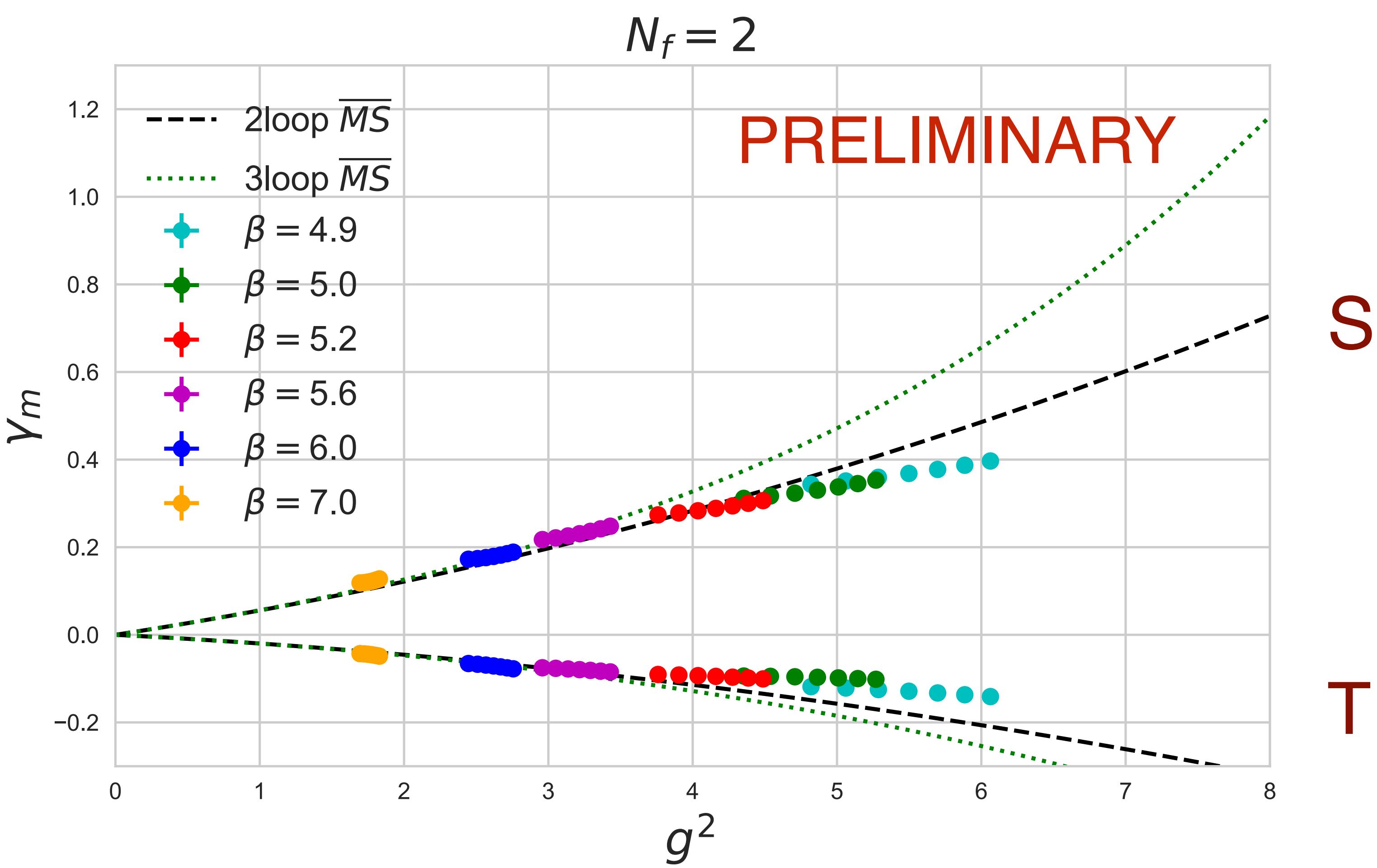
Anomalous dimension of conformal operators characterize the IRFP
Most pheno models require large anomalous dimensions to be consistent with measurements

Real-space GF-RG can predict any operator anomalous dimension
Works in QCD as well

N_f=2 running anomalous dimension

In the GF scheme both the β function and anomalous dimensions follow the 1-loop perturbative prediction

N_f=2 running anomalous dimension



Proton is straightforward

Isosinglet tensor is more difficult but doable

- AH, Witzel - in prep

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N_f=12 anomalous dimension

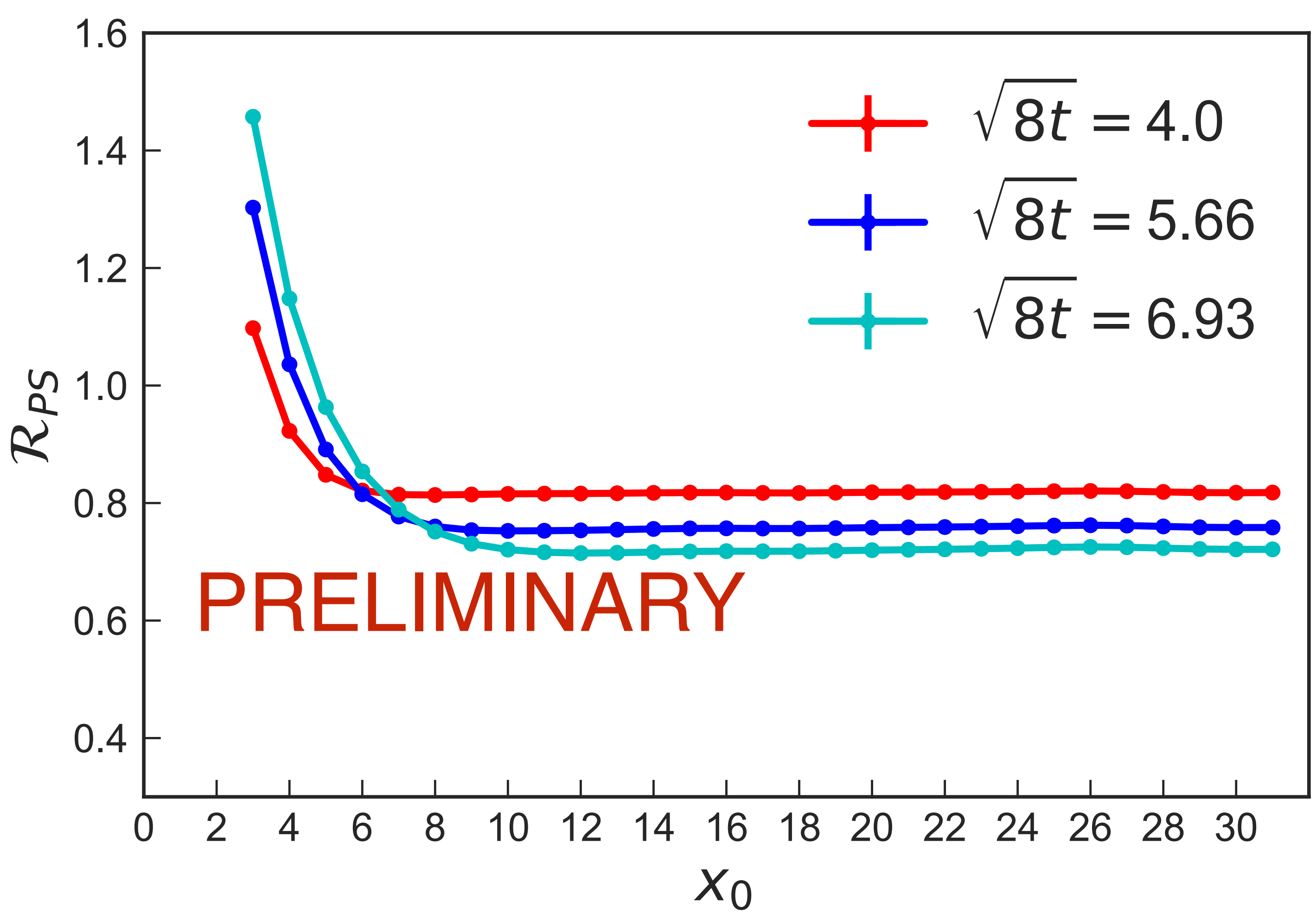
Domain wall

Super-ratio

$$\mathcal{R}_t^O(x_0) = \frac{\langle O(0)O_t(x_0)\rangle}{\langle O(0)O(x_0)\rangle}\left(\frac{\langle A(0)A(x_0)\rangle}{\langle A(0)A_t(x_0)\rangle}\right)^{n_O/n_A} = t^{\gamma_O}$$

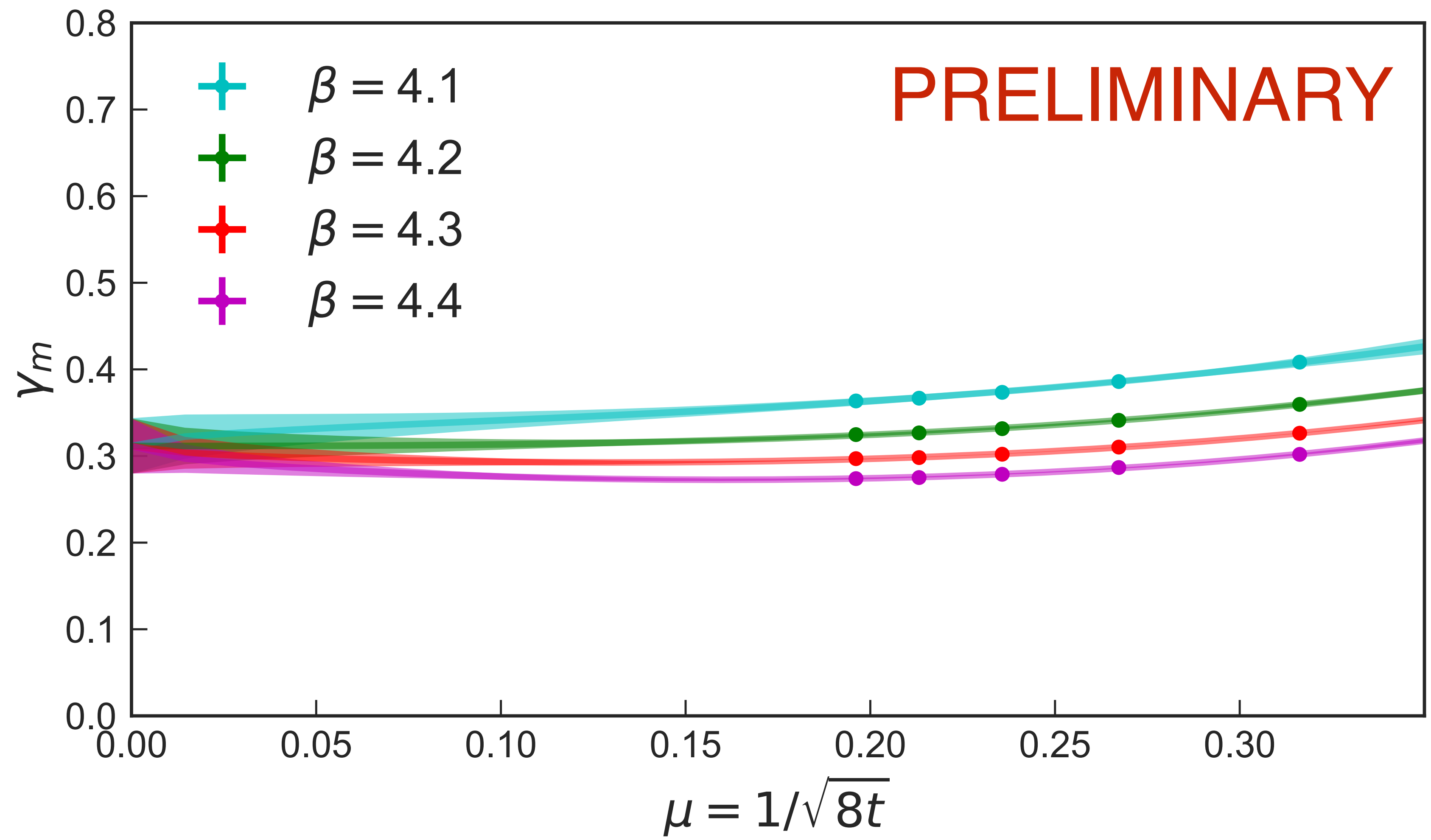
has no x_0 dependence if $x_0 >> b$

pseudoscalar



flow time dependence of the plateau gives anomalous dimension

N_f=12 anomalous dimension, pseudo scalar:



Domain wall

$\gamma_m = 0.31(3)$, $t \to \infty$

(needs finite volume extrapolation)

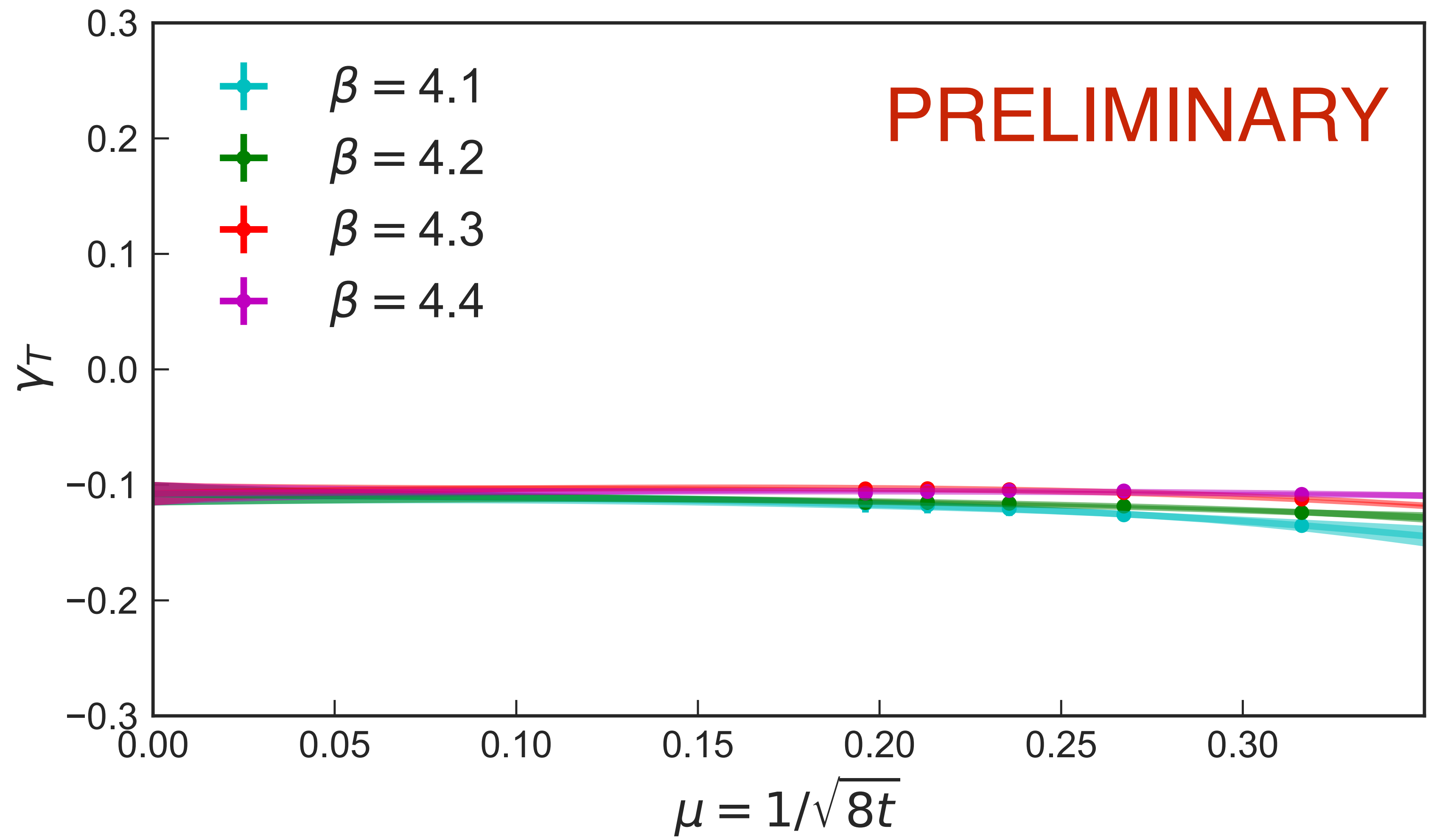
Staggered result

$\gamma_m = 0.24(3)$

extrapolate to t → ∞ :

$$\gamma_m(\beta,t) = \gamma_0 + c_\beta t^{\alpha_1} + d_\beta t^{\alpha_2}$$

N_f=12 tensor:



Domain wall

$\gamma_T = -0.11(1)$, $t \to \infty$

First non-perturbative determination of γ_T

extrapolate to $t \to \infty$:

$$\gamma_m(\beta,t) = \gamma_0 + c_\beta t^{\alpha_1} + d_\beta t^{\alpha_2}$$

Walking systems

Walking systems

Walking systems

Classic realization: dilaton-like Higgs

Take N_f just below the conformal window

- Numerical results suggest light 0^{++}
  ex: SU(3) with 8 fundamental flavors (LSD) ;
  SU(3) with 2 sextet flavors (LatHC);

Mass-split systems: both dilation and pNGB Higgs

Take N_f above the conformal window

- Split the masses: $N_f = N_\ell + N_h$

  N_h flavors are massive, $m_\ell \ll m_h \ll \Lambda_{\text{cut-off}}$ $\rightarrow$ decouple in the IR

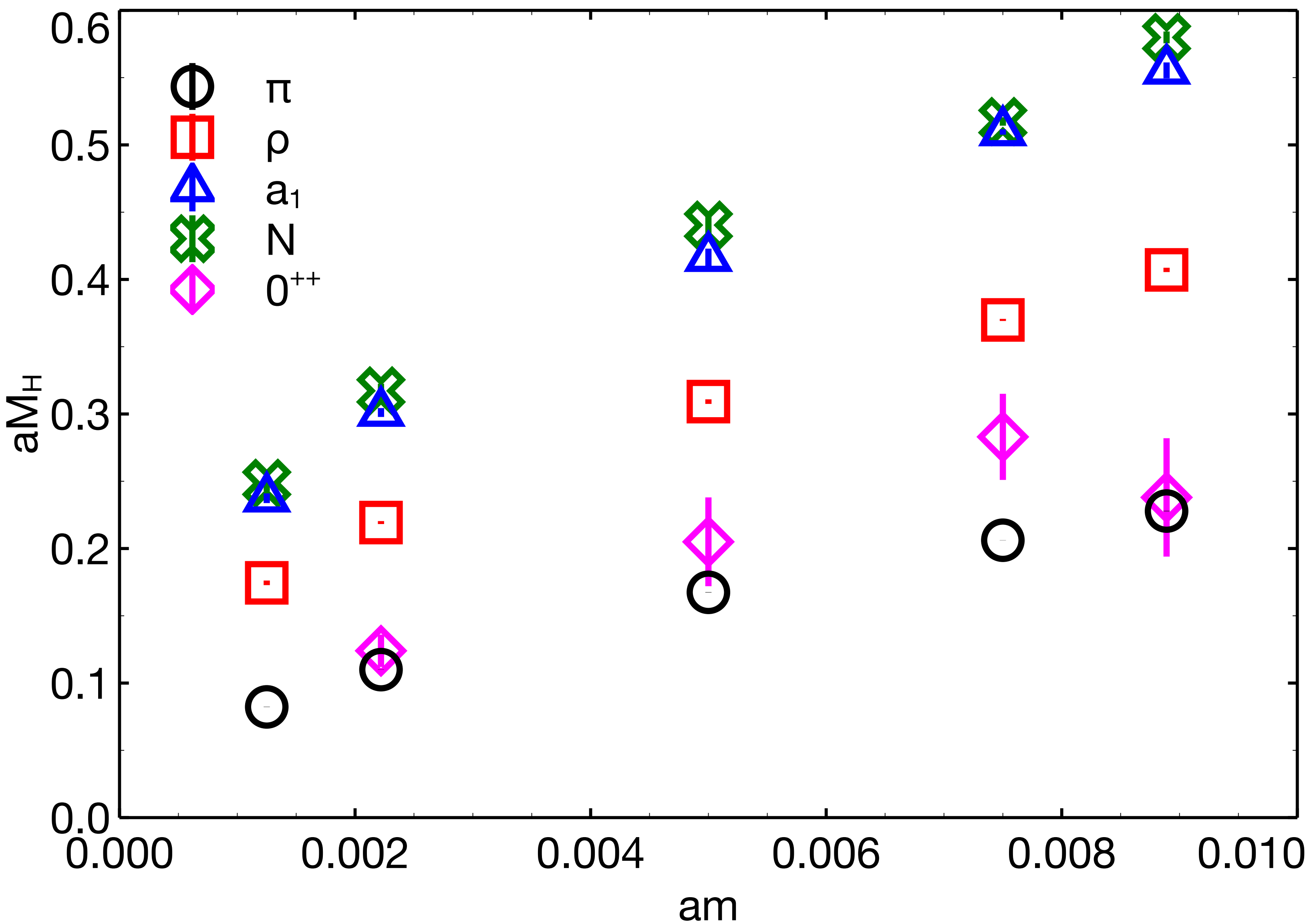
  N_ℓ (= 2 - 4) flavors are massless, $m_\ell = 0 \rightarrow$ chirally broken

(Brower, AH,Rebbi, Witzel and LSD Coll)

Emerging light 0^{++} is a property of near-conformal systems that is not seen in QCD.

N_f=8 fundamental flavors - dilation Higgs candidate

Expected to be below the conformal window - close enough ?
The 0++ is light - still degenerate with the pion
— good technicolor candidate



LSD collaboration
(PhysRevD.99.014509)

Mass-split systems

"Foolproof" realization: mass-split models

Take N_f above the conformal window

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N_h flavors are massive, $m_\ell \ll m_h \ll \Lambda_{\text{cut-off}}$ → decouple in the IR

N_ℓ (= 2 - 4) flavors are massless, $m_\ell = 0$ → spont. chirally broken

- large scale separation controlled by m_h
- chirally broken in the IR
- conformal in the UV → hyperscaling :
  dimensionless ratios are universal functions of m_ℓ/m_h

$$M_{H_1}/M_{H_2} = \Phi_H(m_\ell/m_h),$$

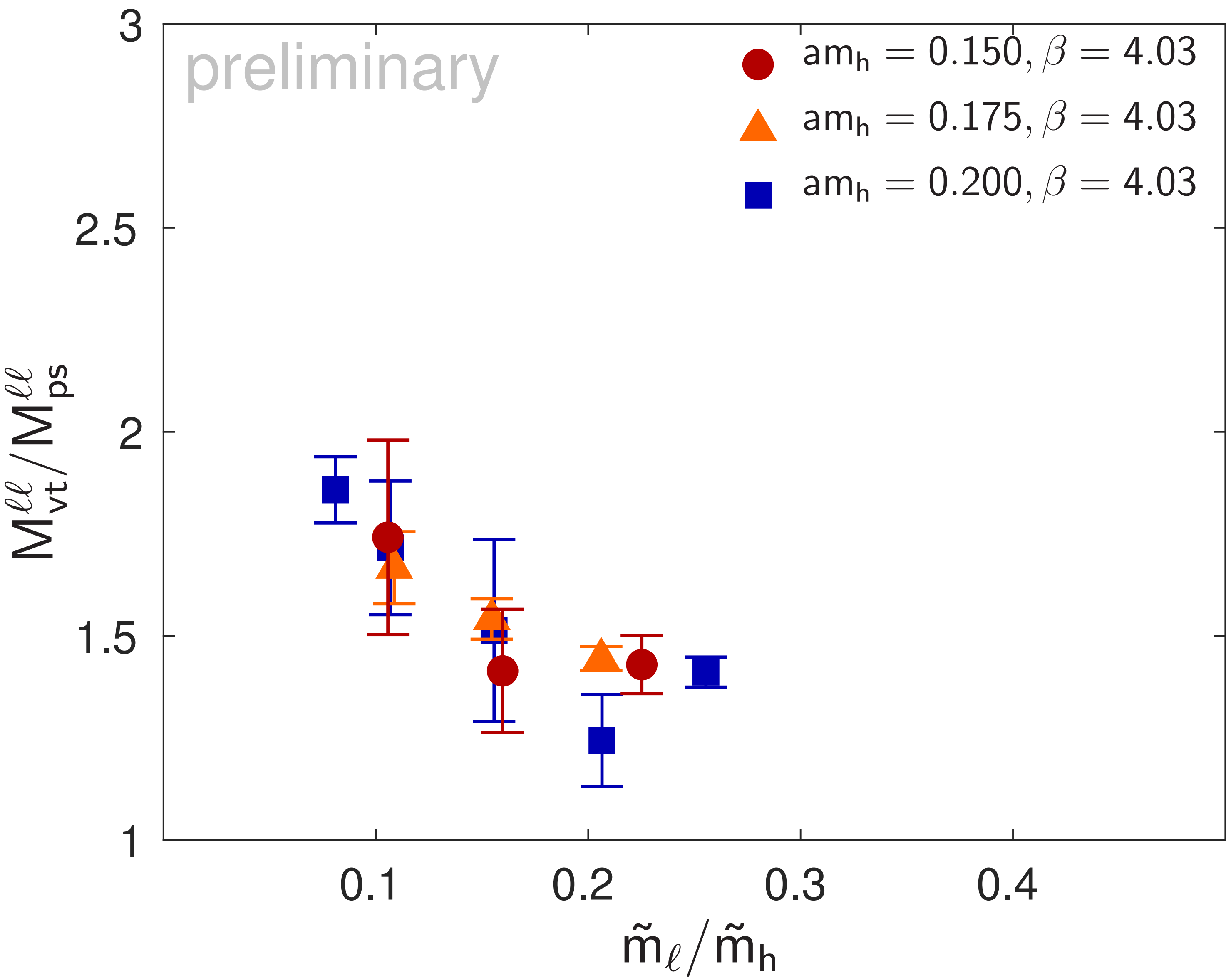
$$M_{H_1}/F_\pi = \tilde{\Phi}_H(m_\ell/m_h)$$

In the $m_\ell=0$ chiral limit the spectrum is fully predictable in terms of F_π

spectrum

N_f=4+6 mass split system

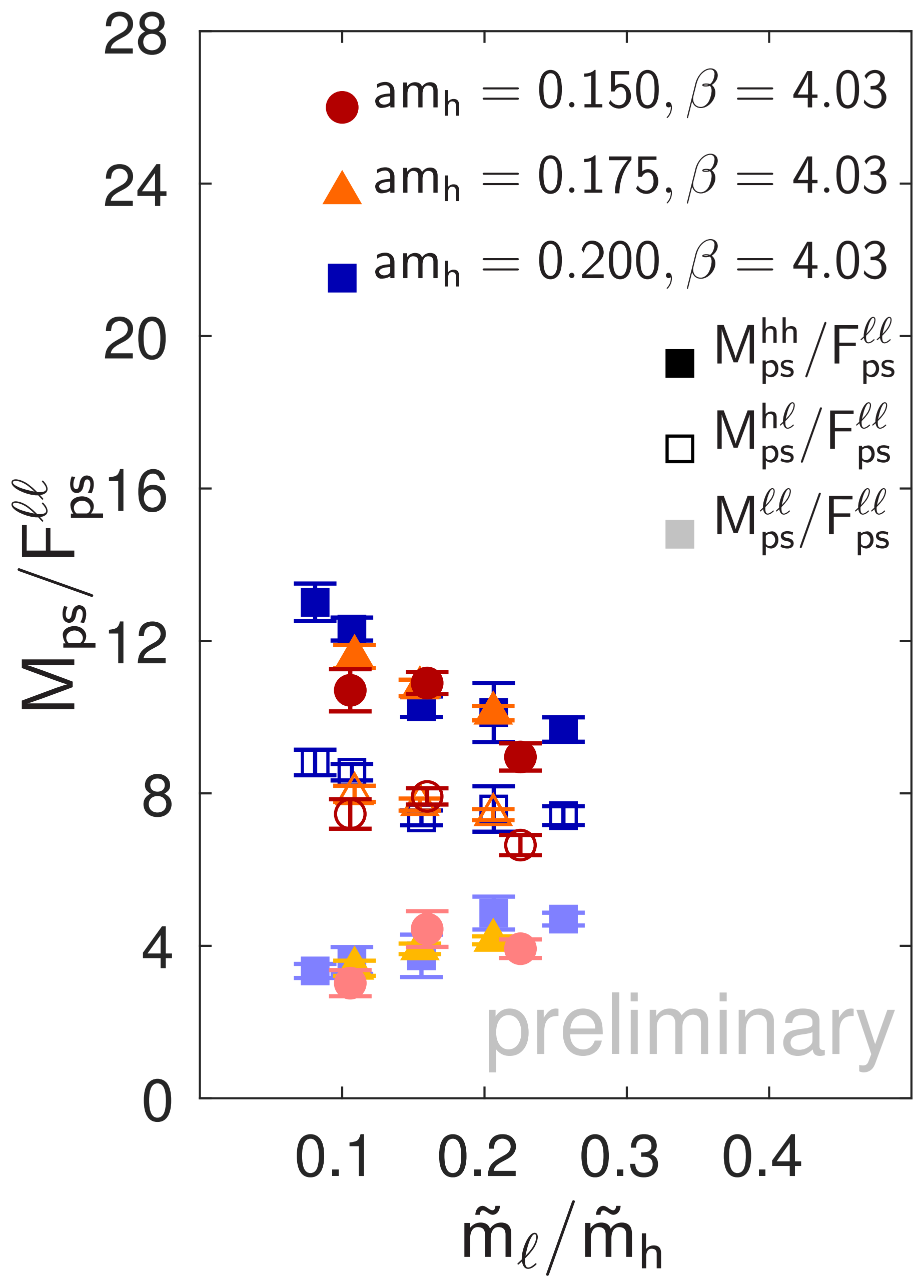
N_f=10 is conformal and close to the conformal window

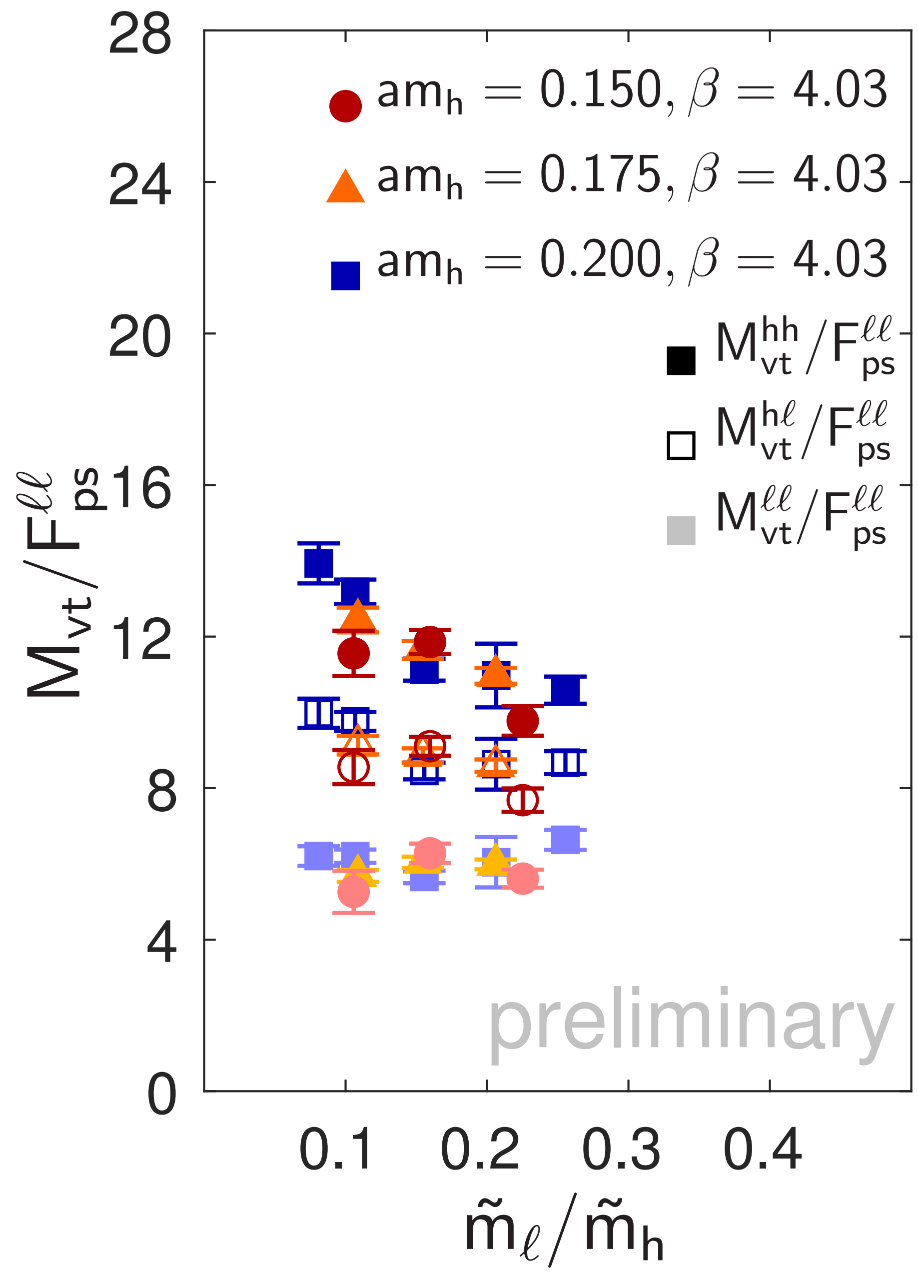


- M_v/M_{ps} ratio is independent of m_h
- system is chirally broken

N_f=4+6 mass split system

Even more interesting: heavy-light and heavy-heavy masses are also independent of m_h — very much NOT QCD-like





N_f=4+6 mass split system

Fun model:

- Tunable walking
- Anomalous dimensions from IRFP
- Hyper scaling —
  in the chiral limit the spectrum has no free parameters
  (besides the scale)
- Indications for light 0++

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  (besides the scale)
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Embedding the SM to 4 flavors:

- Caccapaglia, Ma : possible with natural DM candidate
- Vecchi: partial composite fermion mass generation is possible

What if the BSM is composite, but not 4-flavors?

- General properties still apply!

Near-conformal systems: the worlds unknown…

- They are not QCD-like : new properties, new applications
- Composite BSM candidates
- Lattice methods can determine many properties:
    - spectrum, scattering, decay, etc : just like in QCD
    - RG beta function
    - anomalous dimensions of composite operators

- These systems especially benefit from domain wall simulations, the cost is compensated by improved scaling properties

EXTRA SLIDES

EXTRAS