Introduction to Quantum Chromodynamics (QCD)

Lecture 1

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Jefferson Lab
EXPLORING THE NATURE OF MATTER

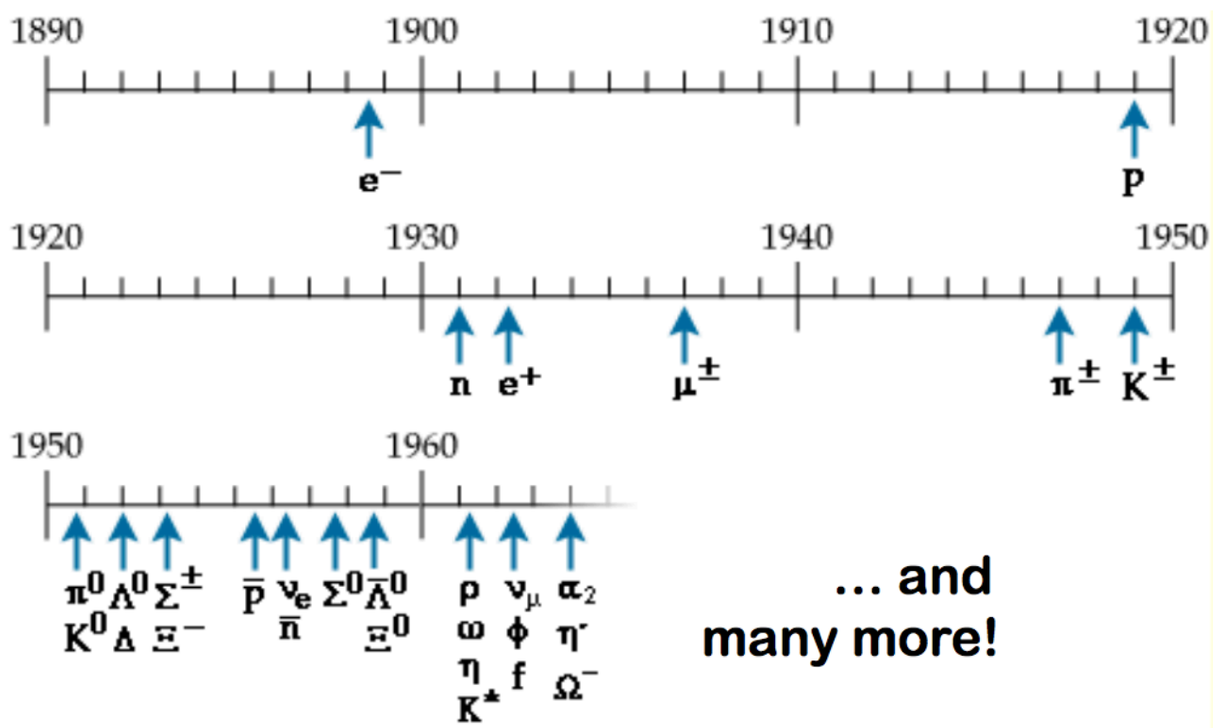
Outlook

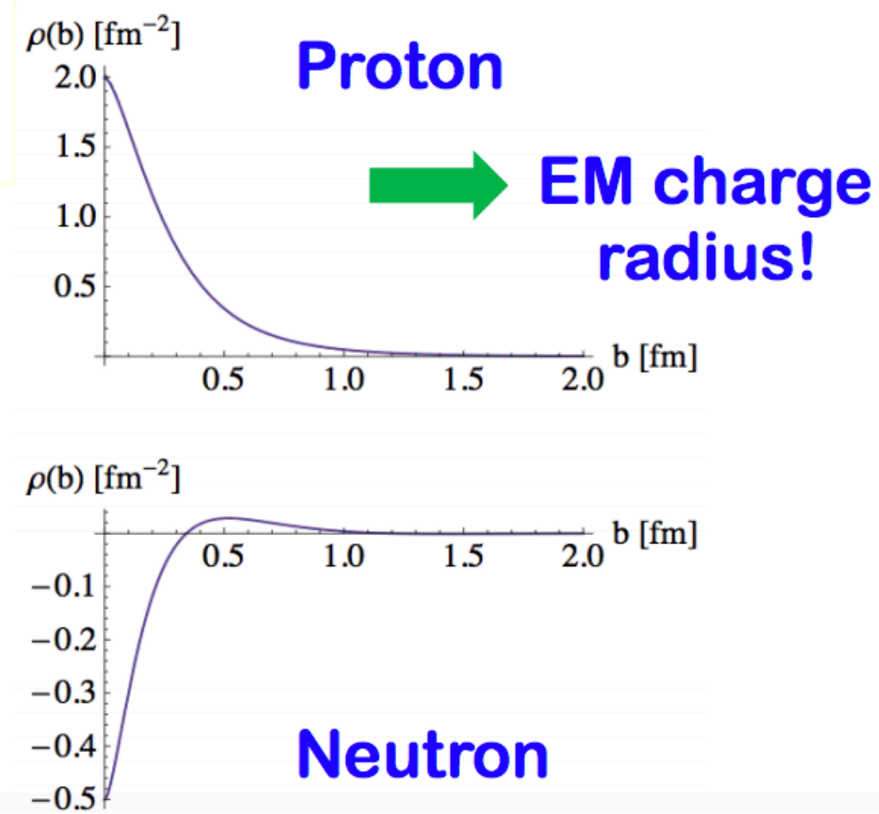
- From hadrons to quarks
- QCD Lagrangian density
- Asymptotic freedom
- Collinear factorization

Motivation:

From hadrons to QCD

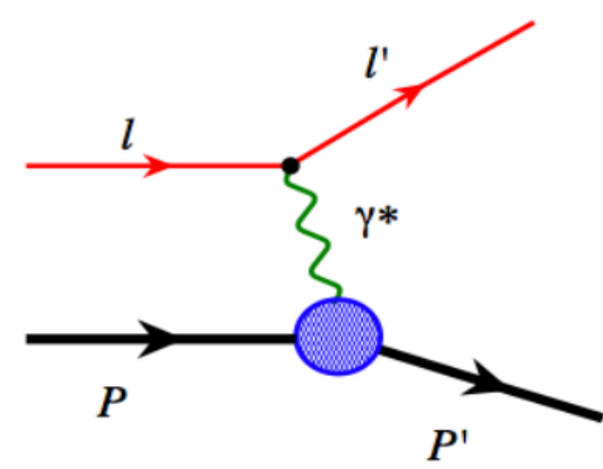
Particle 'explosion'





Before 1950's: protons and neutrons were regarded as fundamental particles

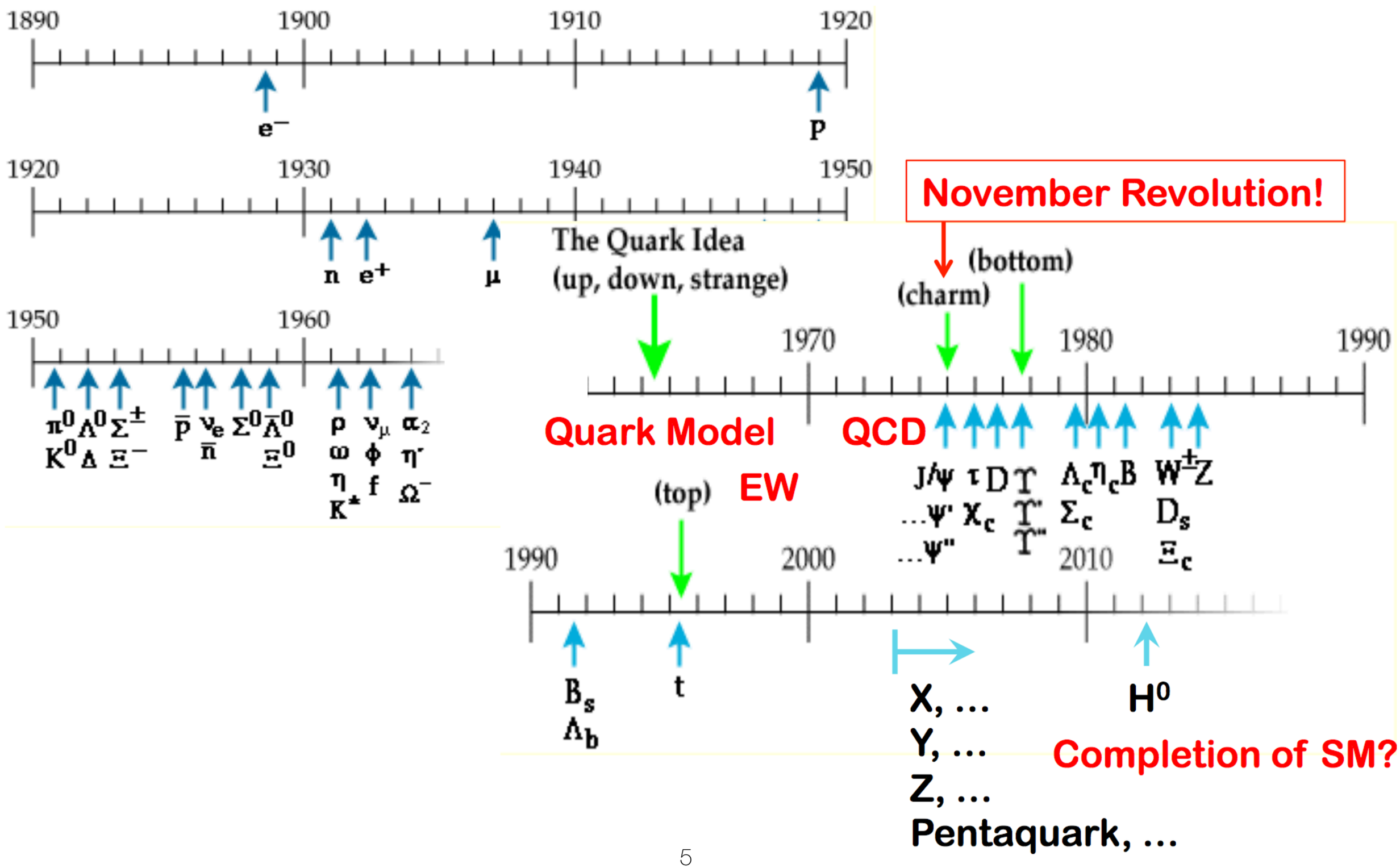
- 1950s-1960: electron-proton scattering experiments reveal a charge density inside protons/neutrons



Robert Hofstadter
Nobel Prize 1961

Nucleons have internal structure!

Particle 'explosion'



The (naïve) quark model

- Nucleons are made by quarks!

  Quark model, Murray Gell-Mann
  Nobel prize 1969

$$\begin{pmatrix}1\\0\\0\end{pmatrix} \quad \begin{pmatrix}0\\1\\0\end{pmatrix} \quad \begin{pmatrix}0\\0\\1\end{pmatrix}$$

- 3 quarks with the same mass. The quarks are eigenstates of the fundamental representation of FLAVOR SU(3)

- SU(3) has $3^2-1=8$ generators (Gell-Mann 3x3 matrices)

$$J_3 = \frac{1}{2}\begin{pmatrix}1 & 0 & 0\\0 & -1 & 0\\0 & 0 & 0\end{pmatrix} \qquad J_8 = \frac{1}{2\sqrt{3}}\begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & -2\end{pmatrix}$$

Isospin: $I_3 \equiv J_3$ Hypercharge: $Y \equiv \frac{2}{\sqrt{3}} J_8$

The (naïve) quark model

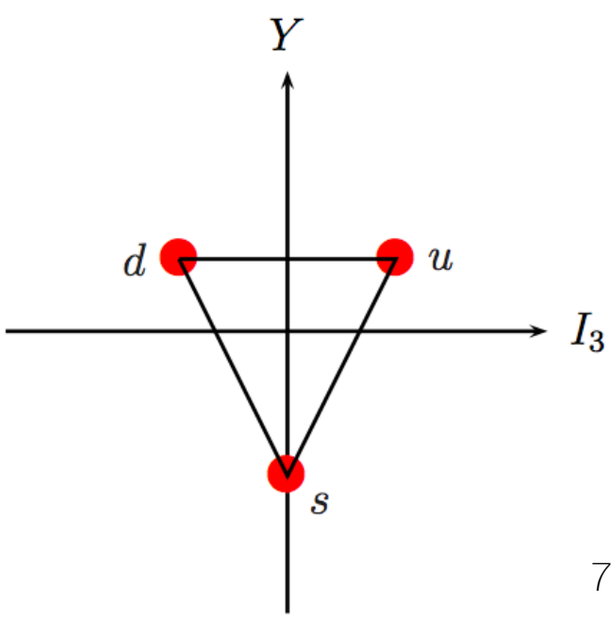
- Quarks and antiquarks eigenstates

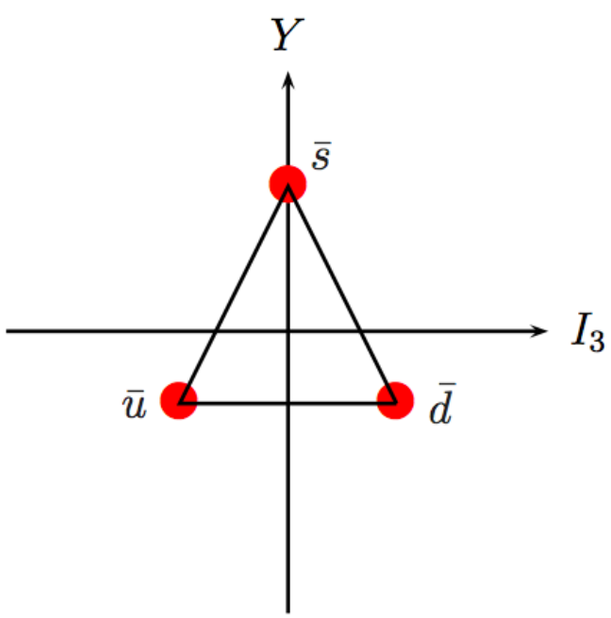
NO COLOR YET

Eigenstates $|I_3, Y\rangle$

$$u = |\frac{1}{2}, \frac{1}{3}\rangle \qquad d = |-\frac{1}{2}, \frac{1}{3}\rangle \qquad s = |0, -\frac{2}{3}\rangle$$

$$\bar{u} = |-\frac{1}{2}, -\frac{1}{3}\rangle \qquad \bar{d} = |\frac{1}{2}, -\frac{1}{3}\rangle \qquad \bar{s} = |0, \frac{2}{3}\rangle$$



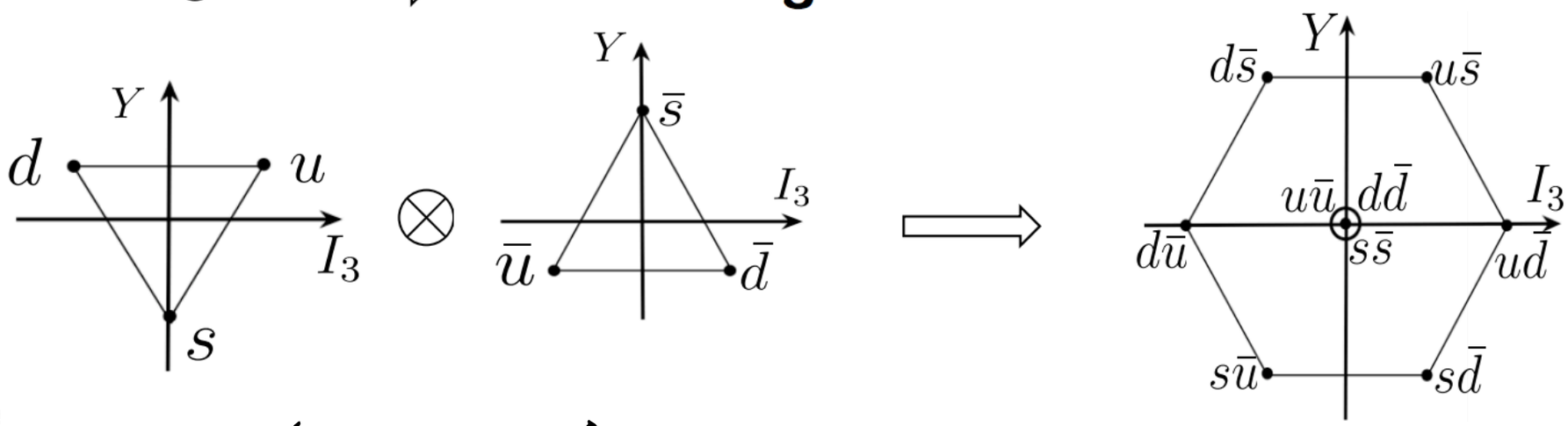


The (naïve) quark model

- Mesons (quark-antiquark states) from Group Theory

$$q(u,d,s) = \mathbf{3}, \quad \bar{q}(\bar{u},d,\bar{s}) = \bar{\mathbf{3}}, \quad \text{of flavor SU(3)}$$

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{8} \oplus \mathbf{1} \quad \Longrightarrow$$ **1 flavor singlet + 8 flavor octet states**



- Physical mesons (L=0, S=0)

Octets

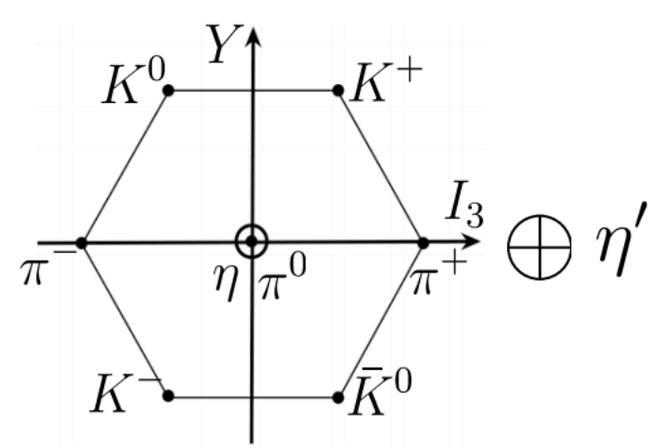
$$A = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \quad \Longrightarrow \quad \pi^0$$

$$B = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}) \quad \Longrightarrow \quad \eta_8$$

Singlet

$$C = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}) \quad \Longrightarrow \quad \eta_1$$

η, η'



Mesons

- Meson states

Spin of quark-antiquark pair $\vec{S} = \vec{s}_q + \vec{s}_{\bar{q}} \quad \vec{S} = 0,1$

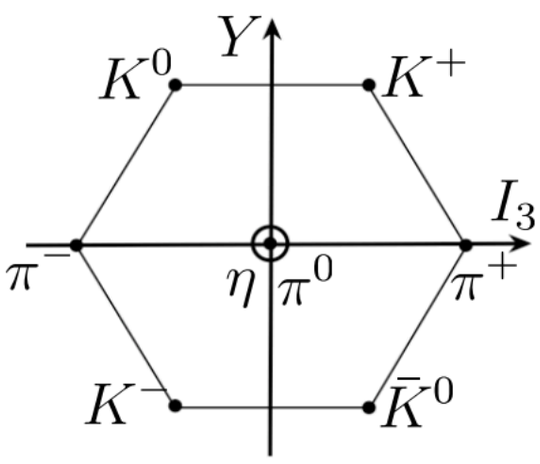
Meson spin $\vec{J} = \vec{L} + \vec{S}$

Parity $P = -(-1)^L$ NO COLOR YET

Charge conjugation $C = -(-1)^{L+S}$

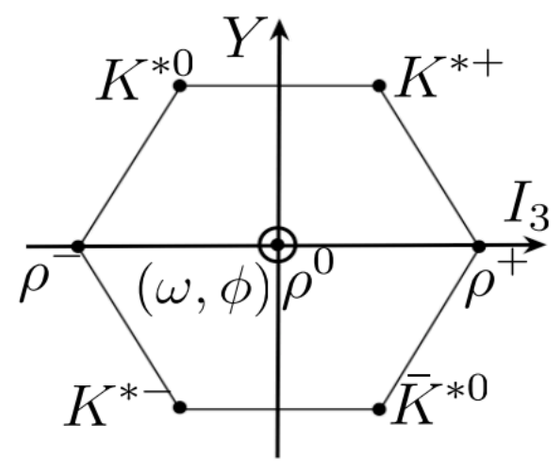
- L=0 states

$J^{PC} = 0^{-+}$: **(Y=S)**



$\oplus\ \eta'$

$J^{PC} = 1^{--}$: **(Y=S)**



$\oplus\ \mathrm{mix}(\omega,\phi)$

Flavor singlet

Flavor octet

Baryons

NO COLOR YET

- Baryons from Group Theory

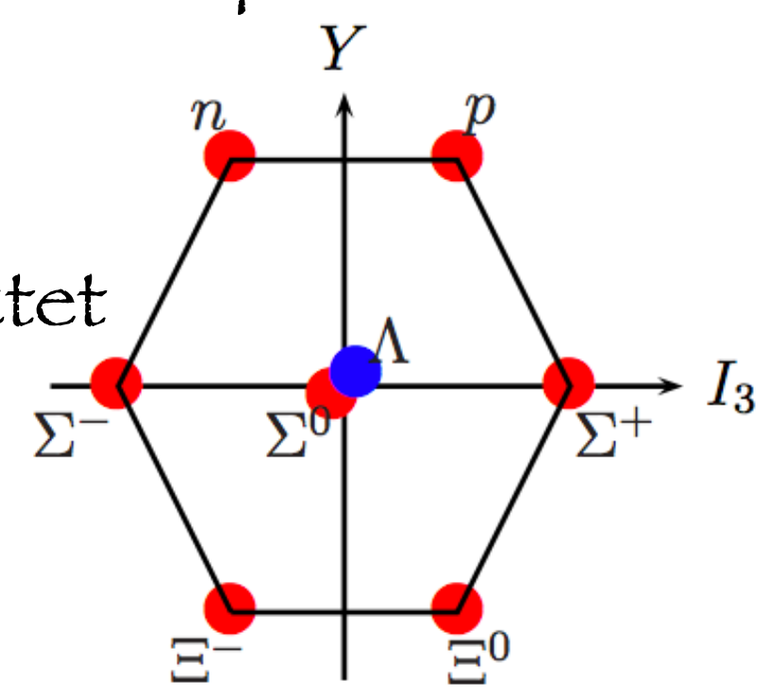
$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10}_S \oplus \mathbf{8}_{M_S} \oplus \mathbf{8}_{M_A} \oplus \mathbf{1}_A$$

S: symmetric in all 3 q, M_S : symmetric in 1 and 2,
M_A : antisymmetric in 1 and 2, A : antisymmetric in all 3

- Baryons spin: 1/2 or 3/2

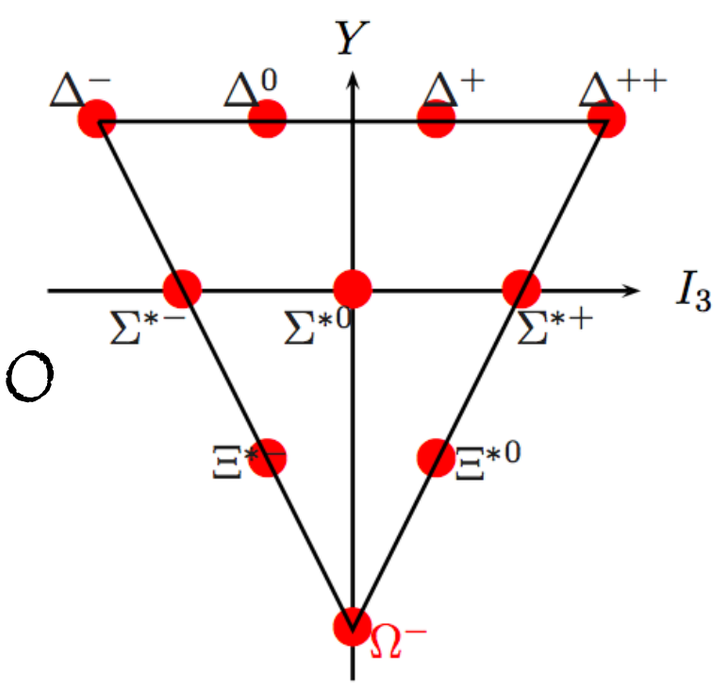
Spin 1/2

Flavor octet



Spin 3/2

Flavor 10



S=0

S=-1

S=-2

S=-3

Baryons

NO COLOR YET

- Baryons from Group Theory

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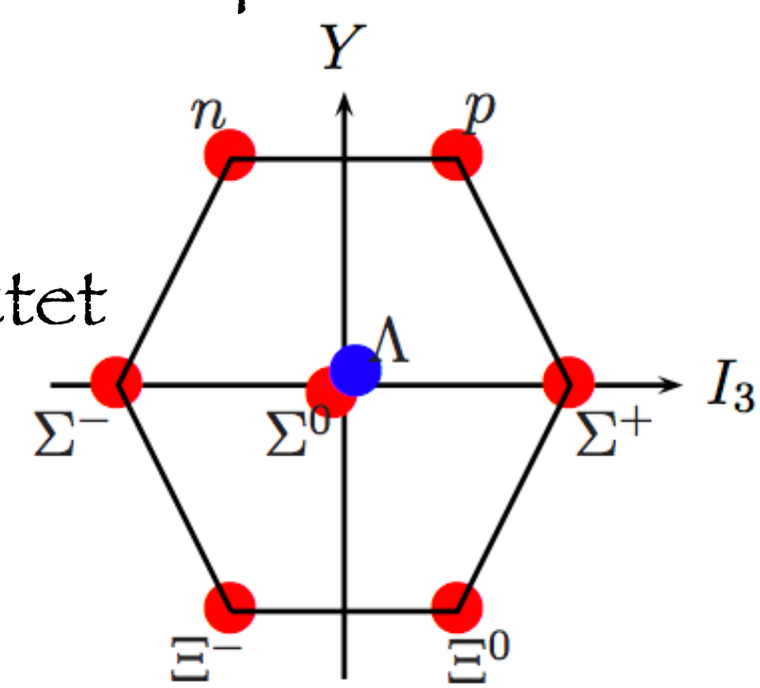
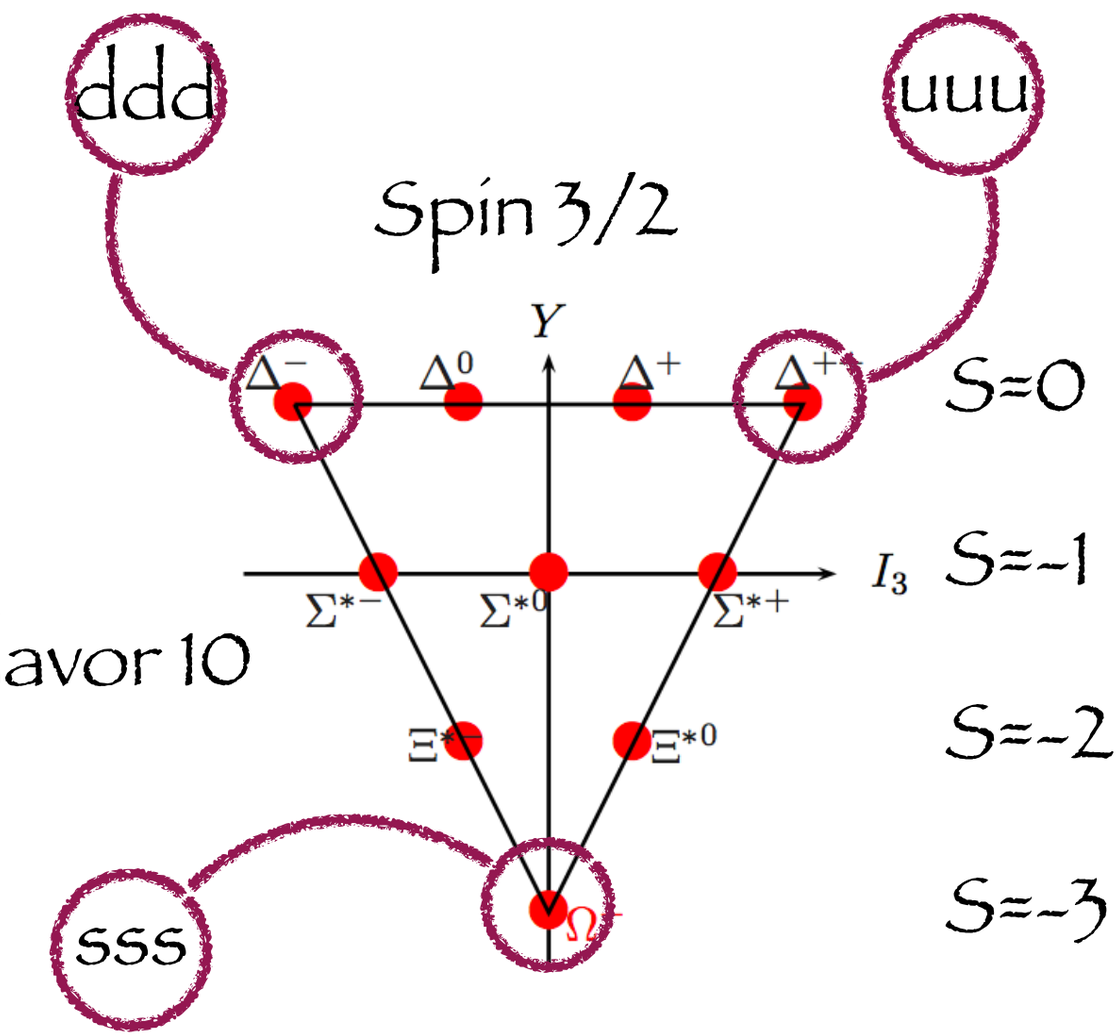
Spin 1/2

Flavor octet



Spin 3/2

Flavor 10



Baryons

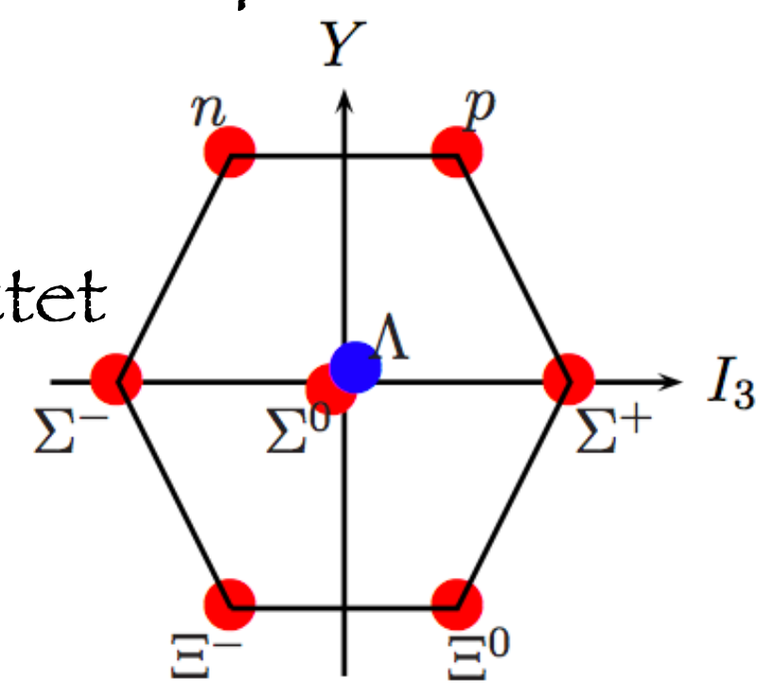
Violation of Pauli exclusive principle

Need of another quantum number: color!

- Baryons spin: 1/2 or 3/2

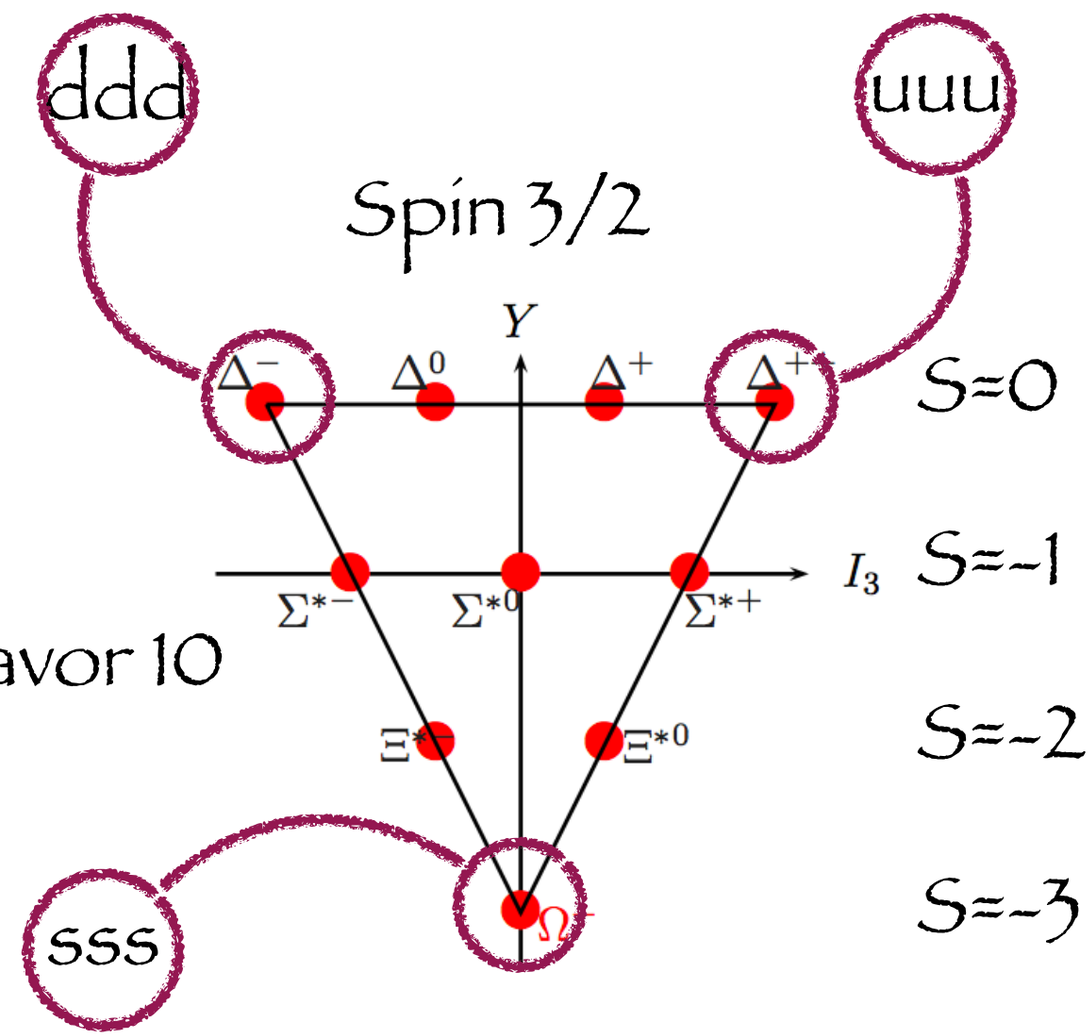
Spin 1/2

Flavor octet



Spin 3/2

Flavor 10



Color

- Color: new quantum number. At least three colors are needed

- 3 quarks' color wave function must be antisymmetric

- Baryon wave function

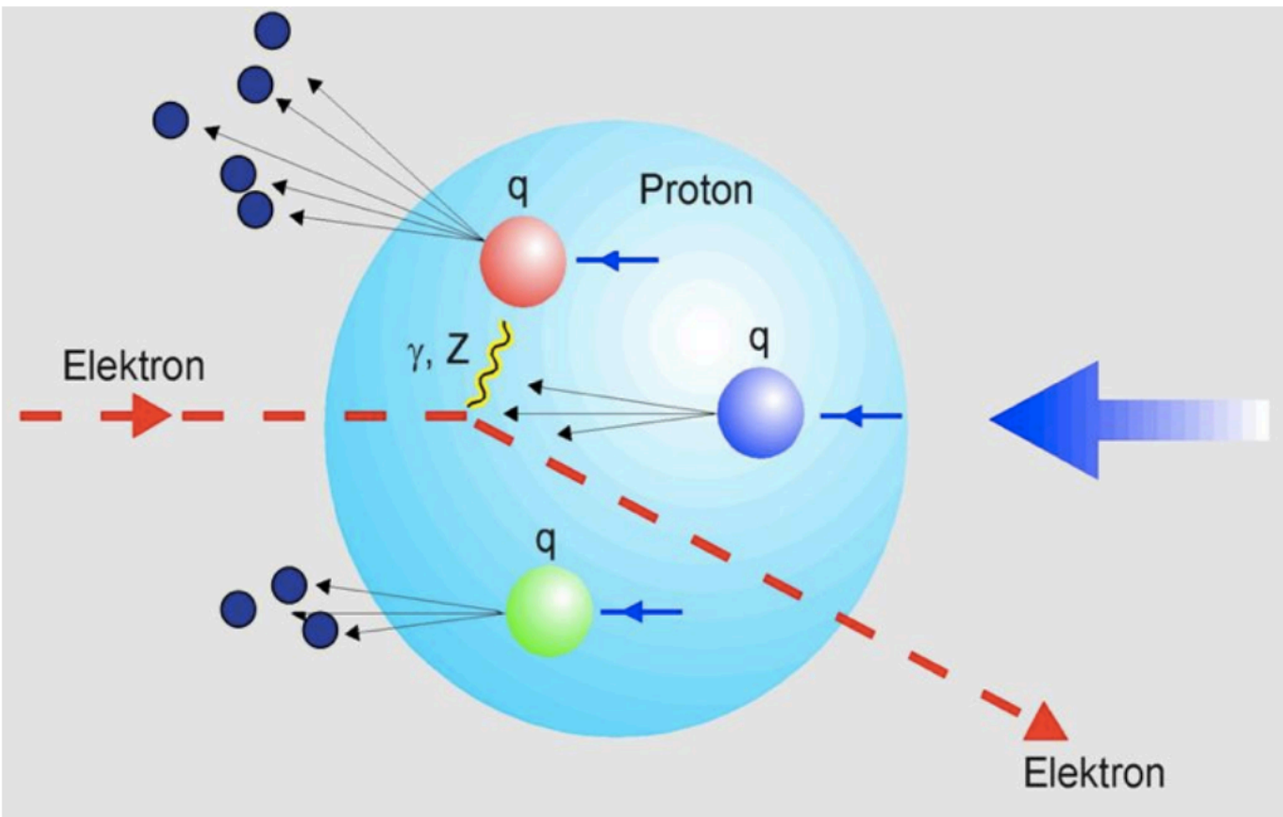
$$\varphi(q_1, q_2, q_3) = \varphi_{\text{spatial}}(x_1, x_2, x_3) \otimes \varphi_{\text{flavor}}(f_1, f_2, f_3) \otimes \varphi_{\text{spin}}(s_1, s_2, s_3) \otimes \varphi_{\text{color}}(c_1, c_2, c_3)$$

Antisymmetric Symmetric Symmetric Symmetric Antisymmetric

Discovery of quarks

- Deep inelastic scattering (DIS)

$$e^-(p^\mu) + h \to e^-(p'^\mu) + X$$



$$Q^2 = (p - p')^2$$

$$1/Q < 1\ \text{fm}$$

SLAC 1968: Discovery of spin-1/2 quarks

Birth of QCD (1973)

Quantum Chromodynamics (QCD)

Theory of quarks, gluons and their interactions

QCD Lagrangian density

QCD Lagrangian density

$$\mathscr{L}_{\mathrm{QCD}} = \bar{\varphi}_i \left(i\gamma^\mu D_\mu - m\,\delta_{ij} \right) \varphi_j - \frac{1}{4} G^A_{\mu\nu} G^{A,\mu\nu}$$

Question: What are the (mass or length) dimensions of the quark and gluon fields? (Hint: the action must be Lorentz invariant)

QCD Lagrangian density

$$\mathscr{L}_{\mathrm{QCD}} = \boxed{\bar{\varphi}_i \left(i\gamma^\mu D_\mu - m\,\delta_{ij} \right) \varphi_j} - \frac{1}{4} G^A_{\mu\nu} G^{A,\mu\nu}$$

Quark term

Quark (3 colors)

Spin 1/2 Dirac fermion

$$\varphi_i = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix}$$

Question: What are the (mass or length) dimensions of the quark and gluon fields? (Hint: the action must be Lorentz invariant)

QCD Lagrangian density

$$\mathscr{L}_{\mathrm{QCD}} = \bar{\varphi}_i \left(i\gamma^\mu D_\mu - m\,\delta_{ij} \right) \varphi_j - \frac{1}{4} G^A_{\mu\nu} G^{A,\mu\nu}$$

Quark term

Gluon term

Quark (3 colors)

Spin 1/2 Dirac fermion

Gluons (color octet)

Spin 1 Vector boson

$$\varphi_i = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix}$$

$$A^C_\mu = A^1_\mu, \ldots A^8_\mu$$

Question: What are the (mass or length) dimensions of the quark and gluon fields? (Hint: the action must be Lorentz invariant)

QCD Lagrangian: quark term

$$\mathscr{L}_q = \bar{\varphi}_i \left(i\gamma^\mu \partial_\mu \delta_{ij} - g_s \gamma^\mu (t^C)_{ij} A^C_\mu - m\,\delta_{ij} \right) \varphi_j$$

- SU(3) local gauge symmetry: 8 generators $t^C_{ij} = t^1_{ij}, \ldots t^8_{ij}$

  corresponding to 8 gluons: $A^C_\mu = A^1_\mu, \ldots A^8_\mu$

  Gell-Mann matrices $t^A = \frac{1}{2}\lambda^A \qquad [t^A, t^B] = i f^{ABC} t^C$

  Structure constants of SU(3)

  (antisymmetric in all indices)

- Gauge invariance $U(x)_{ij} = [e^{i\alpha_a(x) t_a}]_{ij}$

$$\varphi_i(x) \longrightarrow \varphi'_i(x) = U(x)_{ij}\varphi_j(x)$$

$$A_\mu(x) \longrightarrow A'_\mu(x) = U(x)A_\mu(x)U^{-1}(x) + \frac{i}{g}\left[\partial_\mu U(x)\right] U^{-1}(x)$$

QCD Lagrangian: gluon term

$$\mathscr{L}_{\mathrm{g}} = \frac{1}{4} G^{A}_{\mu\nu} G^{A,\mu\nu}$$

Strength field: $G^{A}_{\mu\nu} \equiv \partial_\mu A^{A}_{\nu} - \partial_\nu A^{A}_{\mu} + g_s f^{ABC} A^{B}_{\mu} A^{C}_{\nu}$

QCD Lagrangian: gluon term

$$\mathscr{L}_{\mathrm{g}} = \frac{1}{4} G^A_{\mu\nu} G^{A,\mu\nu}$$

Strength field: $G^A_{\mu\nu} \equiv \partial_\mu A^A_\nu - \partial_\nu A^A_\mu + g_s f^{ABC} A^B_\mu A^C_\nu$

Term needed for gauge local invariance

Responsible of triplet and quartic interactions and asymptotic freedom

QCD Lagrangian density

$$\mathscr{L}_{\mathrm{QCD}} = \mathscr{L}_{\mathrm{q}} + \mathscr{L}_{\mathrm{g}} + \mathscr{L}_{\mathrm{gauge-fixing}} + \mathscr{L}_{\mathrm{ghost}}$$

QCD Lagrangian density

$$\mathscr{L}_{\mathrm{QCD}} = \mathscr{L}_{\mathrm{q}} + \mathscr{L}_{\mathrm{g}} + \boxed{\mathscr{L}_{\mathrm{gauge-fixing}}} + \mathscr{L}_{\mathrm{ghost}}$$

QCD Lagrangian density

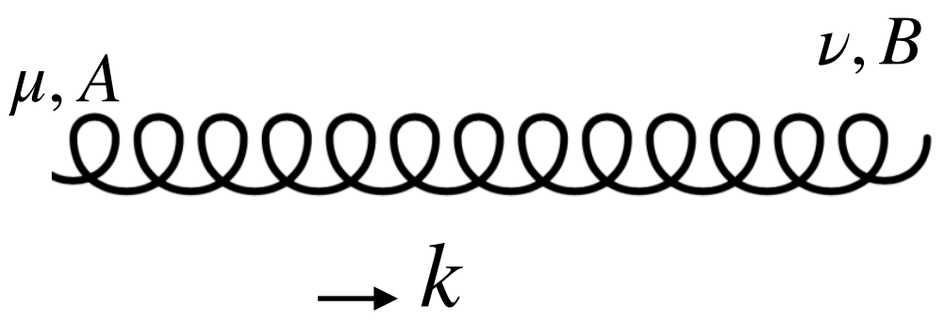
$$\mathscr{L}_{\mathrm{QCD}} = \mathscr{L}_{\mathrm{q}} + \mathscr{L}_{\mathrm{g}} + \boxed{\mathscr{L}_{\mathrm{gauge-fixing}}} + \mathscr{L}_{\mathrm{ghost}}$$

- To define the gluon propagator we need to fix a gauge

$$\mathscr{L}_{\mathrm{gauge-fixing}} = -\frac{1}{2\lambda}(\partial^\mu A^C_\mu)(\partial^\nu A^C_\nu)$$

Covariant or Feynman–'t Hooft gauge: $\lambda = 1$

- Gluon propagator:



$$\frac{\delta^{AB}}{k^2}\left[-g^{\mu\nu} + \left(1 - \frac{1}{\lambda}\right)\frac{k^\mu k^\nu}{k^2}\right]$$

Ghosts in QCD

$$\mathscr{L}_{\mathrm{QCD}} = \mathscr{L}_{\mathrm{q}} + \mathscr{L}_{\mathrm{g}} + \mathscr{L}_{\mathrm{gauge-fixing}} + \mathscr{L}_{\mathrm{ghost}}$$

Ghosts in QCD

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Ghosts in QCD

$$\mathscr{L}_{\text{QCD}} = \mathscr{L}_{\text{q}} + \mathscr{L}_{\text{g}} + \mathscr{L}_{\text{gauge-fixing}} + \boxed{\mathscr{L}_{\text{ghost}}}$$

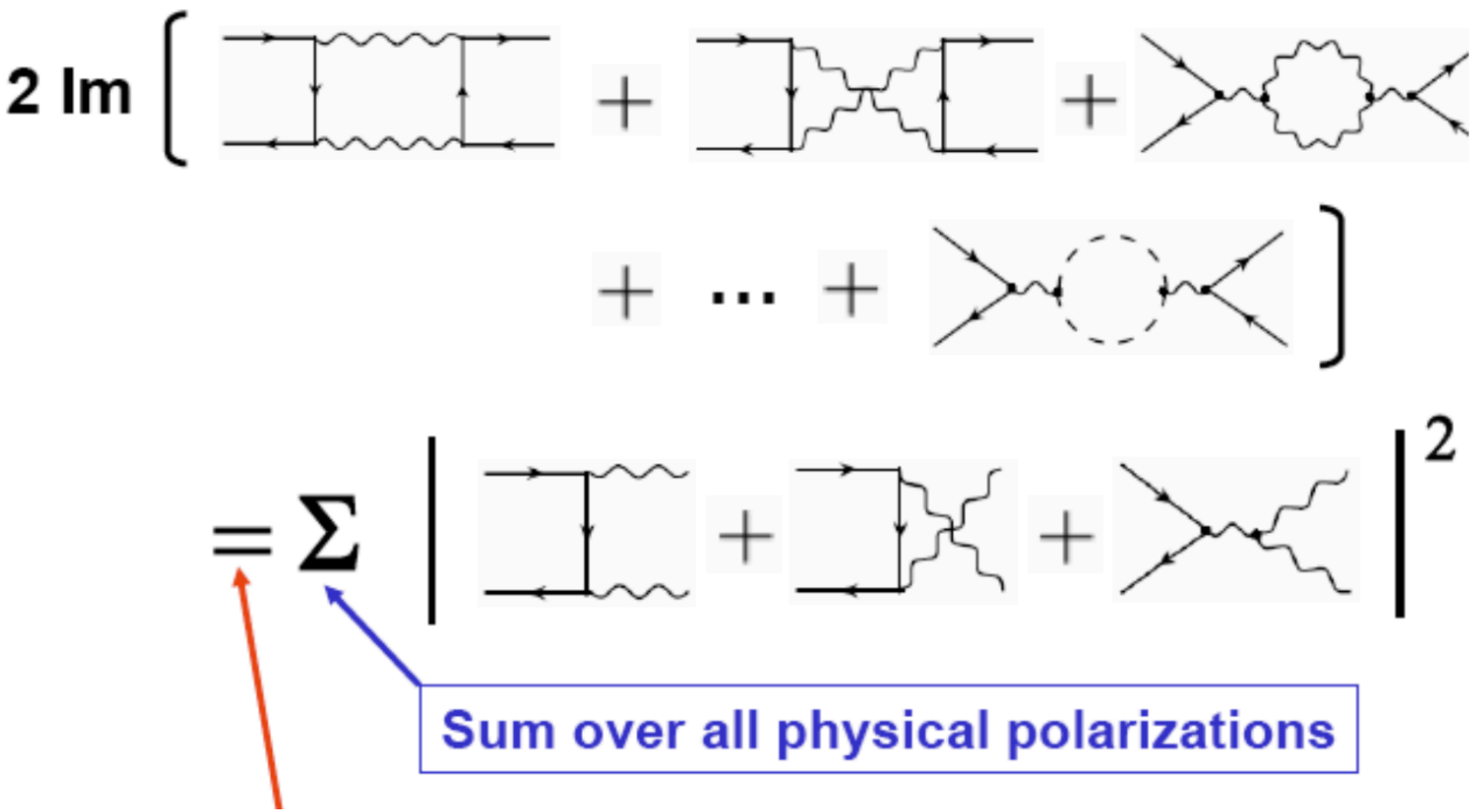
- To preserve unitarity:

$$\mathscr{L}_{\text{ghost}} = \partial_\mu \eta_A^\dagger \left(D^\mu_{AB} \eta^B \right) = \left(\partial_\mu \eta_A^\dagger \right) \left(\partial^\mu \eta_B - g_s f^{ABC} A^{\mu B} \eta^C \right)$$

Ghosts

Question: Do ghosts interact with the gauge fields in QED?

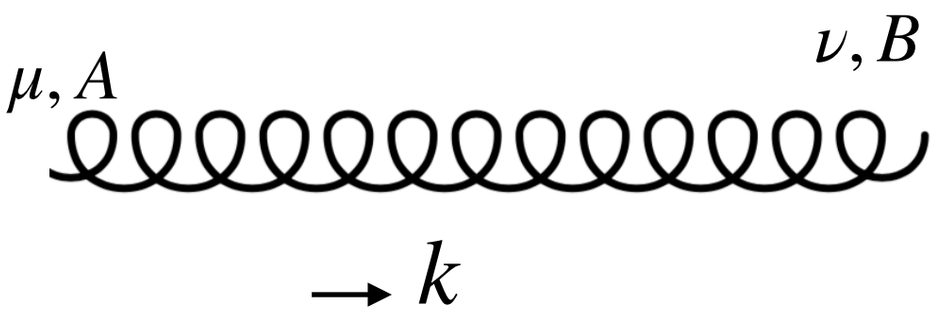
Ghosts in QCD



Question: Do ghosts interact with the gauge fields in QED?

Feynman rules

- Propagators

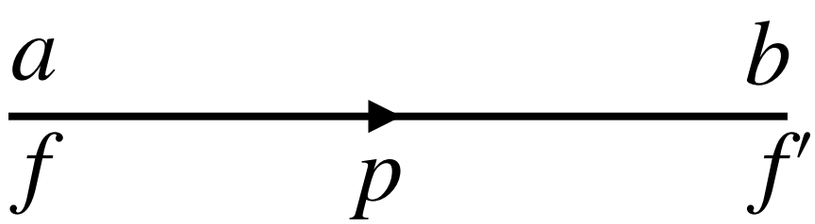


$$\frac{\delta^{AB}}{k^2}\left[-g^{\mu\nu}+\left(1-\frac{1}{\lambda}\right)\frac{k^\mu k^\nu}{k^2}\right]$$

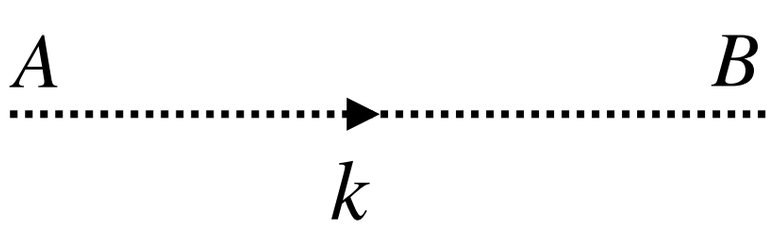
Light cone gauge:

$$\frac{\delta^{AB}}{k^2}\left[-g^{\mu\nu}+\frac{k^\mu n^\nu + k^\nu n^\mu}{k\cdot n}\right]$$

$$n\cdot A = 0 \quad n^2 = 0$$



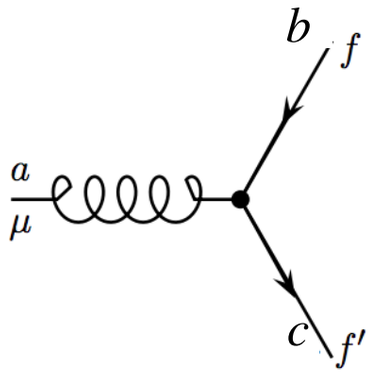
$$\delta^{ab}\delta^{ff'}\frac{i}{\gamma p - m}$$



$$\delta^{AB}\frac{i}{k^2}$$

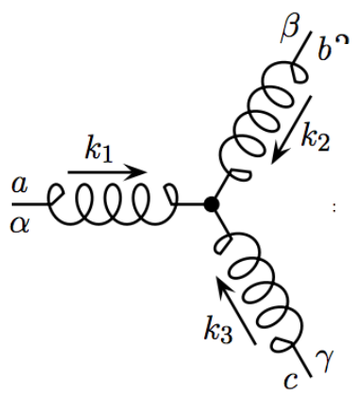
Feynman rules: interactions

$$-g_s\bar{\varphi}_b\gamma^\mu(t^A)_{bc}A^A_\mu\varphi_c$$



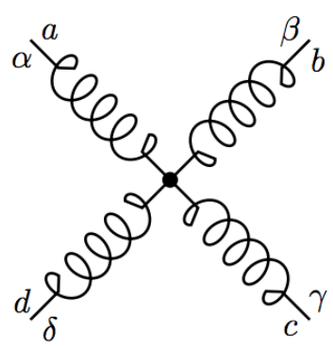
$$-ig_s(t^A)_{bc}\delta^f_{f'}$$

$$\frac{g_s}{2}f^{ABC}(\partial^\mu A^{\nu A}-\partial^\nu A^{\mu A})A^B_\mu A^C_\nu$$



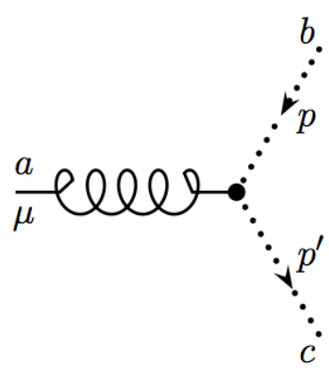
$$g_s f^{ABC}\left[(k_1-k_2)^\gamma g^{\alpha\beta}+(k_2-k_3)^\alpha g^{\beta\gamma}+(k_3-k_1)^\beta g^{\gamma\alpha}\right]$$

$$-\frac{g_s^2}{4}f^{ABC}f^{AB'C'}A^B_\mu A^C_\nu A^{\mu B'}A^{\nu C'}$$



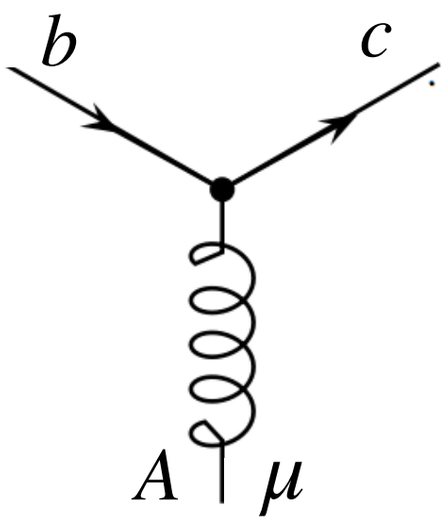
$$-ig_s^2 f^{EAC}f^{EBD}\left[g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\delta}g^{\beta\gamma}\right]$$
$$-ig_s^2 f^{EAD}f^{EBD}\left[g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\gamma}g^{\beta\delta}\right]$$
$$-ig_s^2 f^{EAB}f^{ECD}\left[g^{\alpha\gamma}g^{\beta\delta}-g^{\alpha\delta}g^{\beta\gamma}\right]$$

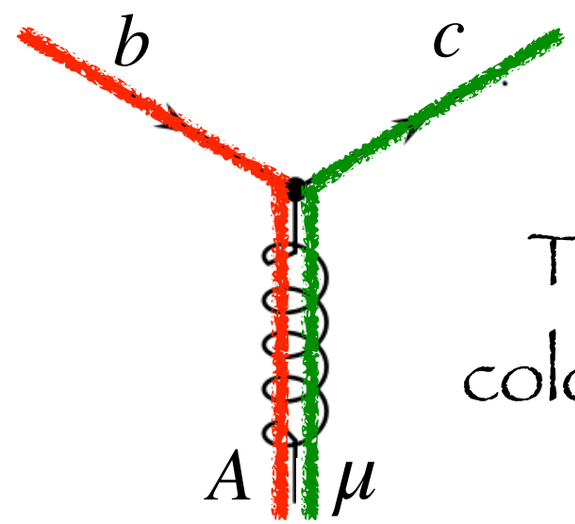
$$\partial_\mu\eta^\dagger_B\left(g_s f^{BAC}A^{\mu A}\right)\eta^C$$



$$-g_s f^{ABC}p'^\mu$$

What do they physically mean?





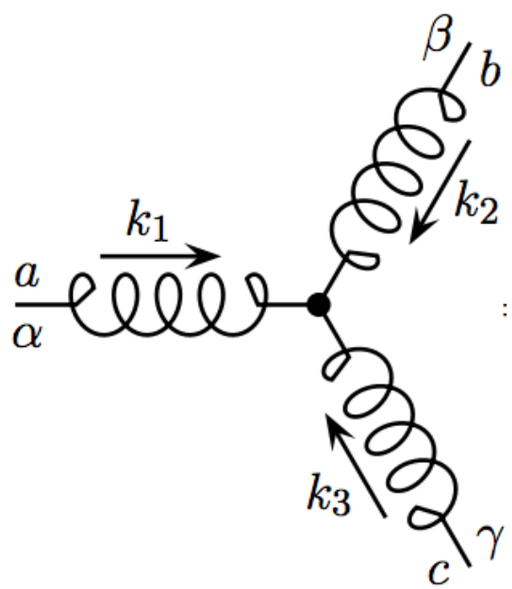
The gluon carries color and anti-colour

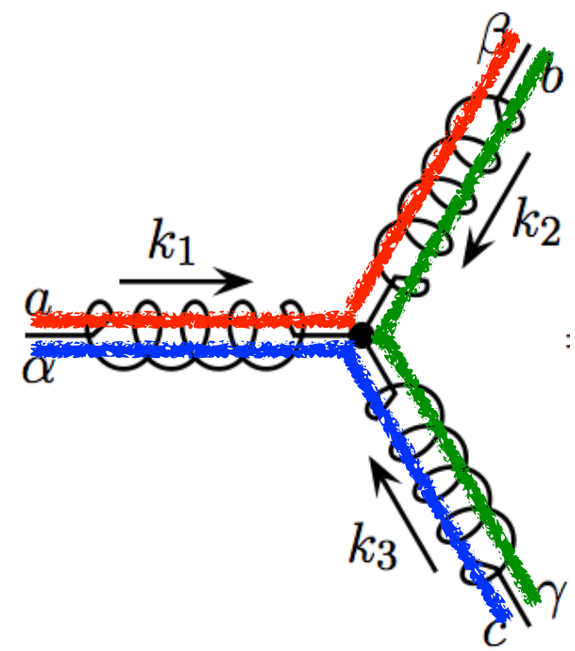
$$-g_s \bar{\varphi}_b \gamma^\mu (t^A)_{bc} A^A_\mu \varphi_c$$

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}}_{\bar{\varphi}_b} \underbrace{\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{(t^1)_{bc}} \underbrace{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}_{\varphi_c}$$

A gluon emission changes the quark color

What do they physically mean?





$$\frac{g_s}{2} f^{ABC}(\partial^\mu A^{\nu A} - \partial^\nu A^{\mu A}) A^B_\mu A^C_\nu$$

A gluon emission also changes the gluon colors

The coupling constant

Renormalization

- In (QED,QCD…) diagrams beyond LO ultraviolet (UV) divergences appear

- Renormalization is needed to remove the UV diagrams

- Renormalization introduces a new energy scale μ

- Physical observables (R) cannot depend on $\mu \Longrightarrow$ Renormalization group equation

$$\mu^2 \frac{d}{d\mu^2} R\left(Q^2/\mu^2, g_s(\mu)\right) = 0 \Longrightarrow R(Q) = R_1 \alpha_s(\mu) + R_2 \alpha_s^2(\mu^2) + \ldots$$

$$\alpha_s = \frac{g_s^2(\mu)}{4\pi}$$

Running coupling

- All (QED, QCD, EW) couplings run: they depend on the momentum scale (Q^2) of the process

$$Q^2 \frac{\partial \alpha_s}{\partial Q^2} = \beta(\alpha_s) \quad \beta(\alpha_s) = \alpha_s^2 (b_0 + b_1 \alpha_s + b_2 \alpha_s^2 + \dots)$$

$$b_0 = -\frac{11 C_A - 2 n_f}{12\pi} \quad b_1 = -\frac{153 - 19 n_f}{24\pi^2} \qquad b_0 < 0 \quad \text{for} \quad n_f \leq 16$$

$(C_A = 3)$

- Solve: $$Q^2 \frac{\partial \alpha_s}{\partial Q^2} = b_0 \alpha_s^2 \implies \alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 - b_0\, \alpha_s(\mu^2) \ln\left(\frac{Q^2}{\mu^2}\right)} \xrightarrow[(b_0 < 0)]{Q^2 \to \infty} 0$$

Asymptotic freedom!

2014 Nobel prize: Gross, Politzer & Wilczek

Asymptotic freedom

- At high scales, the coupling becomes small
  Interaction is weak: quarks and gluons are almost free

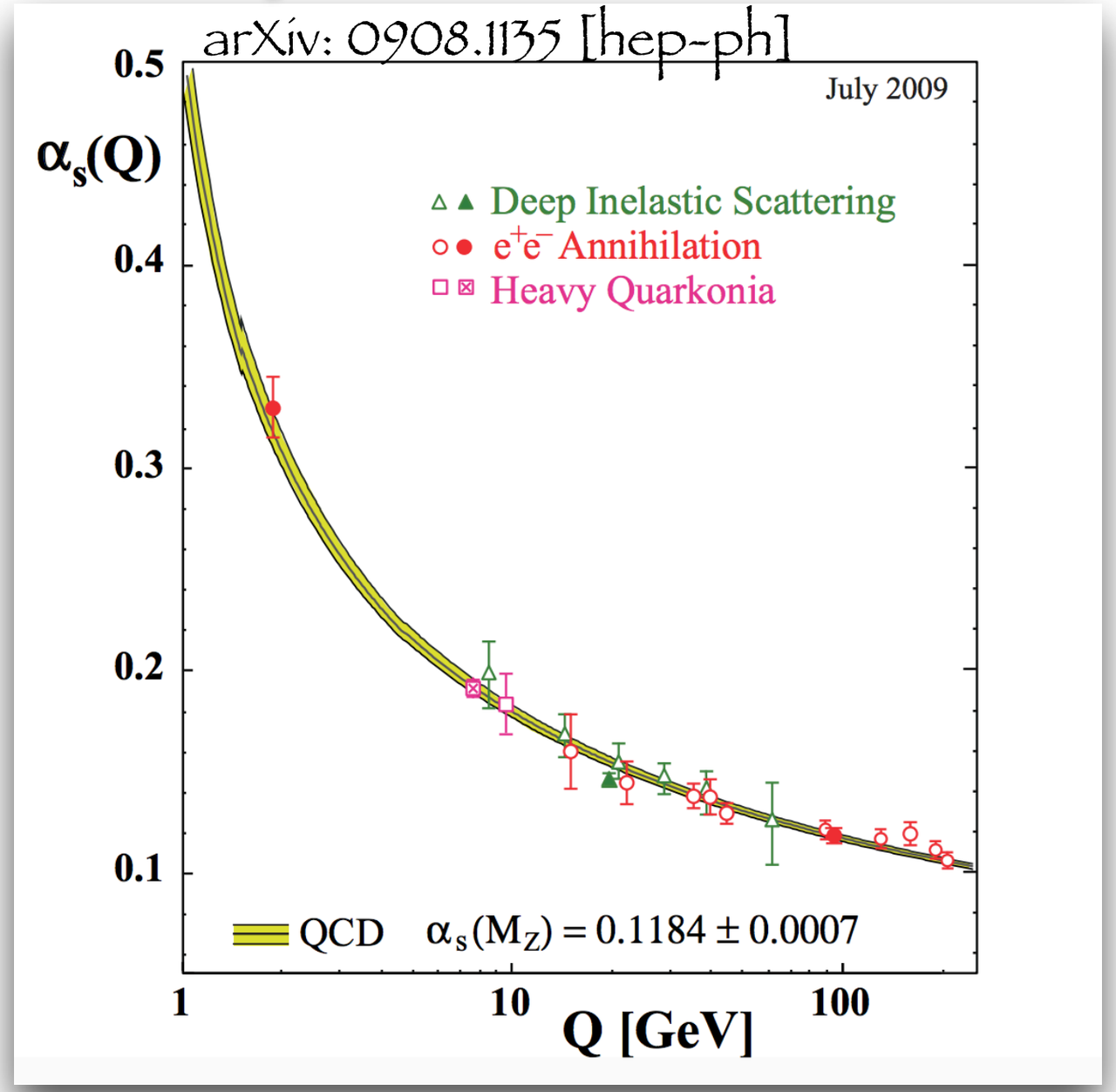
- At low scales, the coupling becomes strong
  Interaction is strong - quarks and gluons confined into hadrons
  Perturbation theory fails

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 - b_0\,\alpha_s(\mu^2)\,\ln\left(\frac{Q^2}{\mu^2}\right)} \overset{(b_0 < 0)}{=} \frac{1}{-b_0\,\ln\left(\frac{Q^2}{\Lambda^2_{\rm QCD}}\right)} \qquad \text{(LO)}$$

$\Lambda_{QCD} \approx 200\,\text{MeV}$ Fundamental scale of QCD at which the coupling blows up

- pQCD valid for $Q^2 \gg \Lambda_{\rm QCD}$

Strong coupling determination



- Asymptotic freedom → Collider phenomenology
  pQCD calculations under control

Quark masses

- Running quark mass equation: mass anomalous dimension

$$Q^2 \frac{\partial m}{\partial Q^2} = -\gamma_m(\alpha_s) m(Q^2) \qquad \gamma_m(\alpha_s) > 0$$

$$m(Q^2) \overset{Q^2 \to \infty}{\Longrightarrow} 0$$

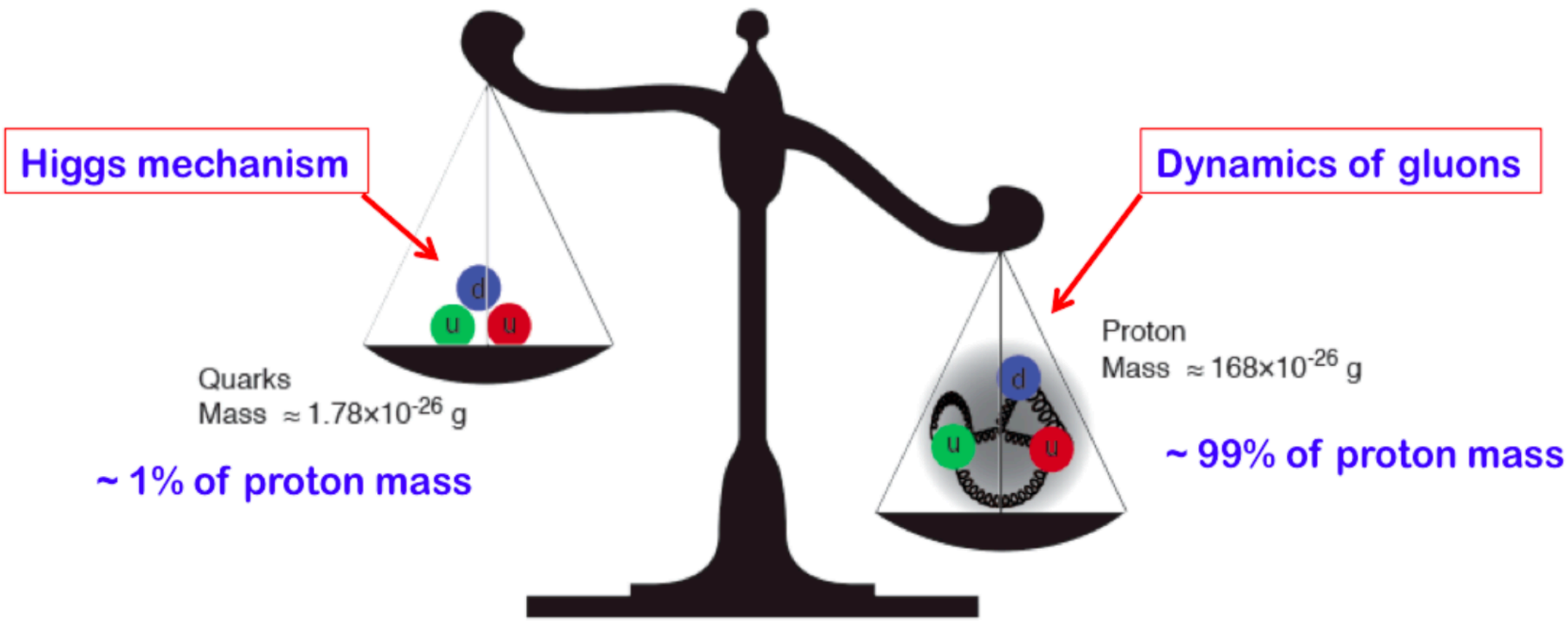
- Light quark masses: $m_f(Q^2) \ll \Lambda_{\rm QCD}$ for f=u, d and even s

QCD perturbative theory $(Q^2 \gg \Lambda_{\rm QCD})$
is effectively a massless theory

Note: The Higgs mechanism is irrelevant for hadron masses, they are (mostly) dynamically generated

Hadron masses

- The Higgs mechanism is irrelevant for hadron masses, they are (mostly) dynamically generated

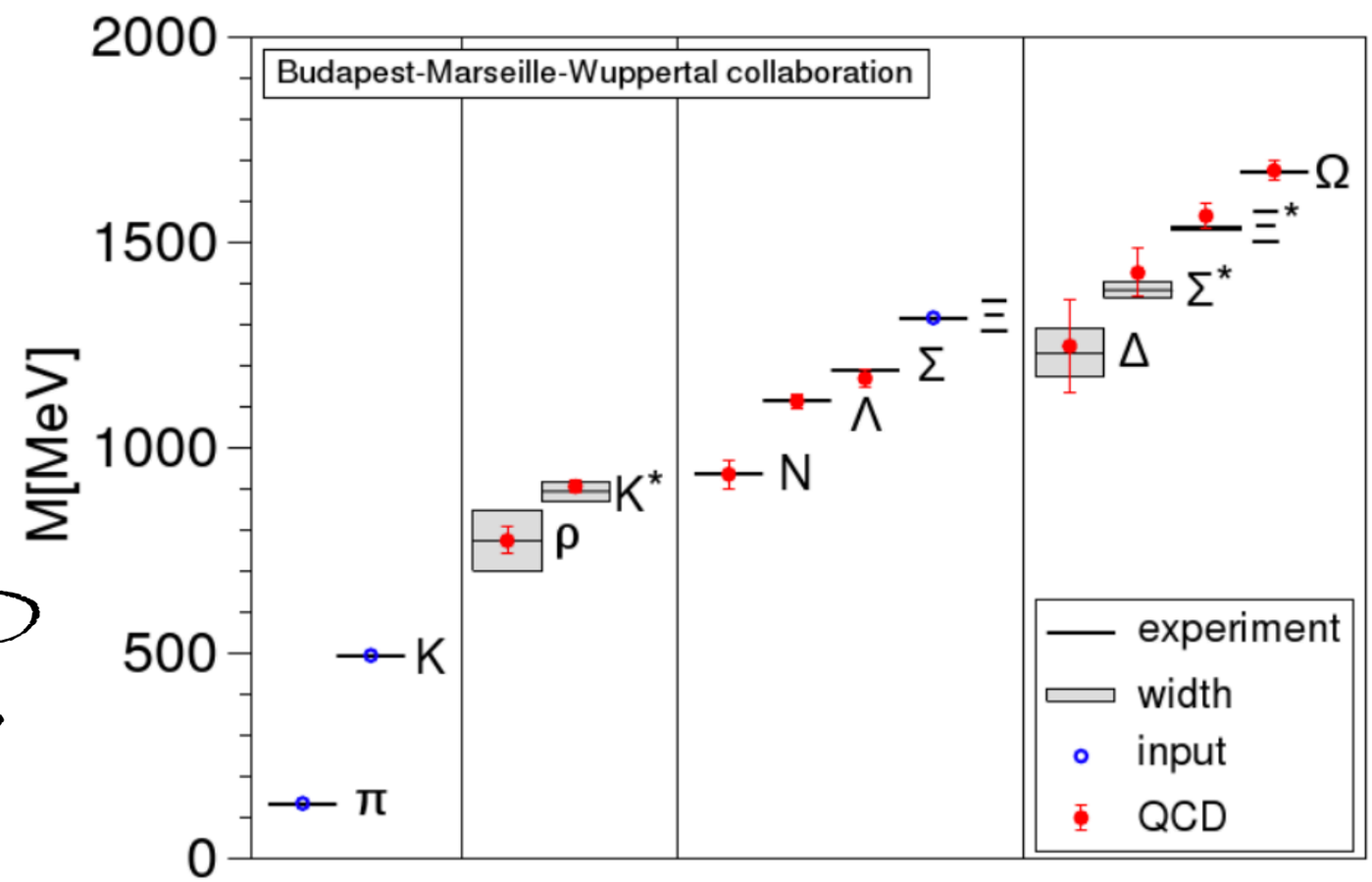


"Mass without mass!"

Lattice hadron masses

- The hadron mass spectrum is a great success of Lattice QCD

How does QCD generate this?



Durr et al '08

QCD is the right theory for strong interactions!

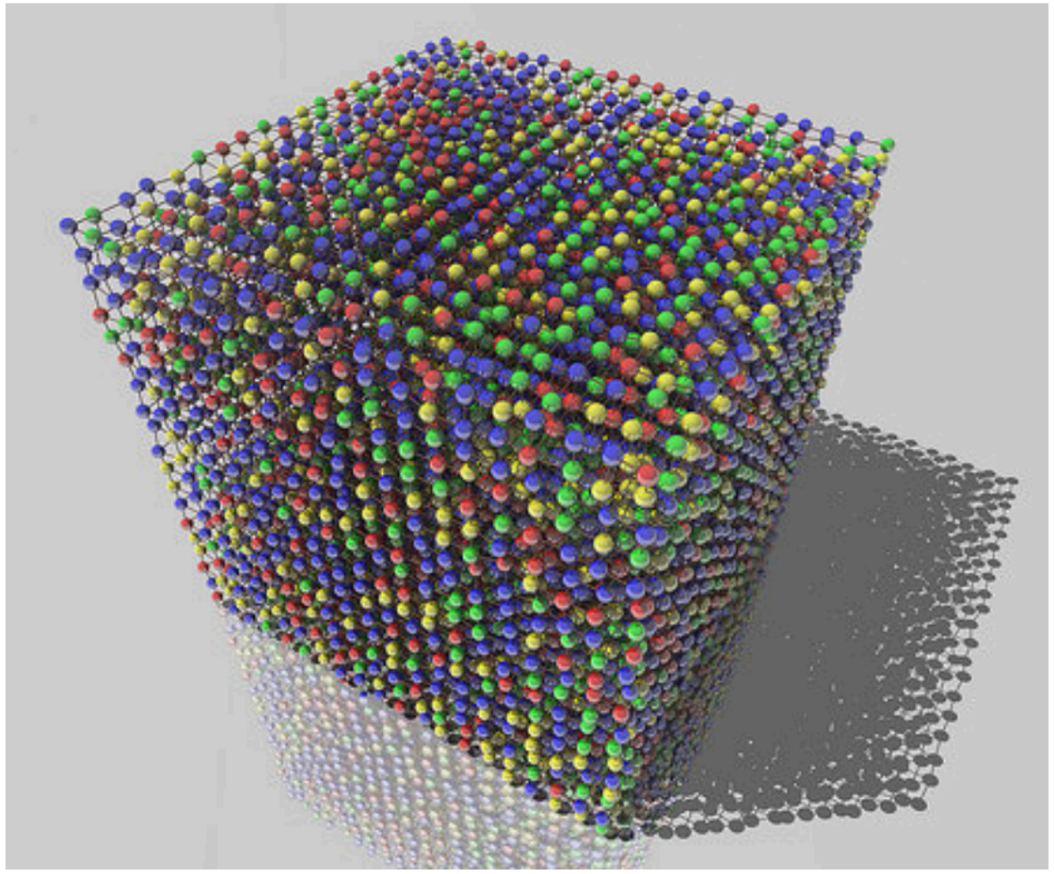
Lattice QCD

See Martha Constantinou lectures

- Non-perturbative approach to solve QCD

- Discrete. Momentum cut-off 1/a

Allows us to study non-perturbative phenomena



- Put all the quarks and gluon fields on a 4D-lattice (with imaginary time)

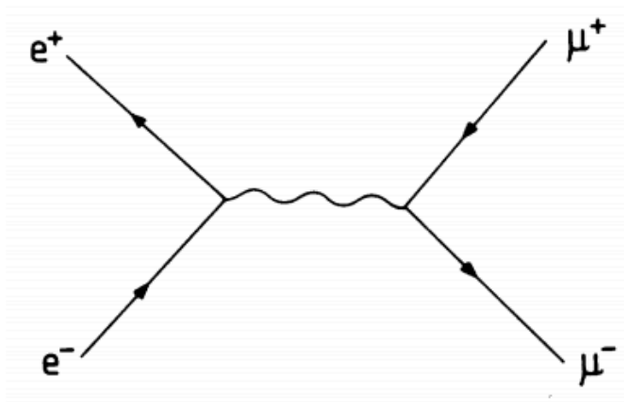
- Use MC sampling to figure out which field configurations are most likely

How many colors are there?

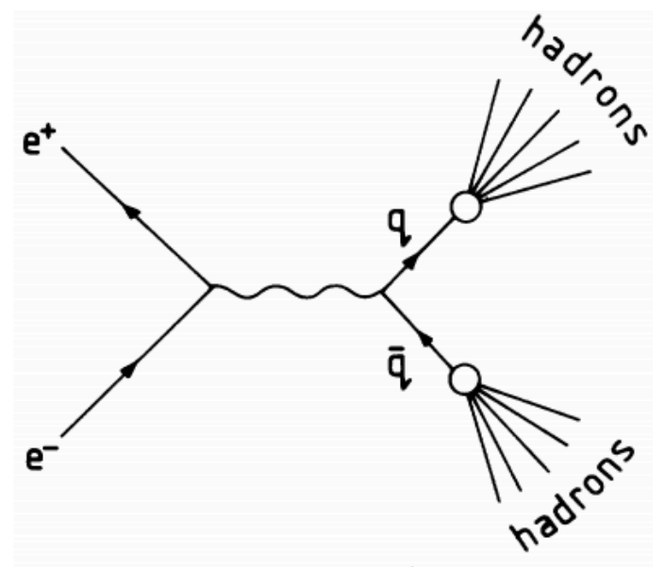
Evidence of color

- Compare the cross sections of:

$$e^+e^- \to \mu^+\mu^-$$



$$e^+e^- \to q + \bar{q}$$



- Without color*:

$$R = \frac{\sigma(e^+e^- \to \mu^+\mu^-)}{\sum_q \sigma(e^+e^- \to q\bar{q})} = \sum_i^{n_f} q_i^2$$

*Omitting hadronization

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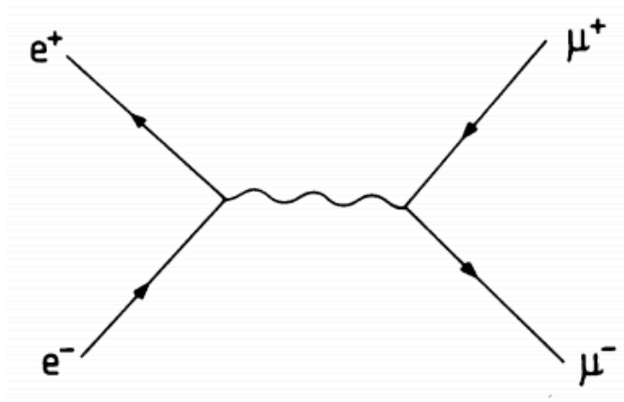

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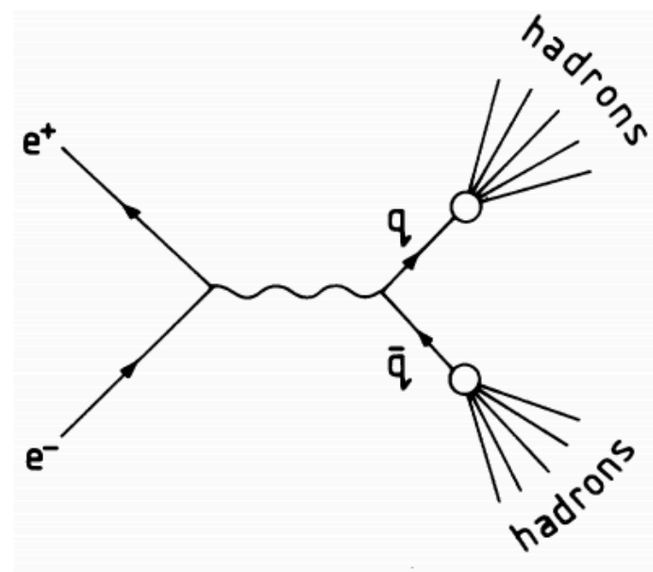
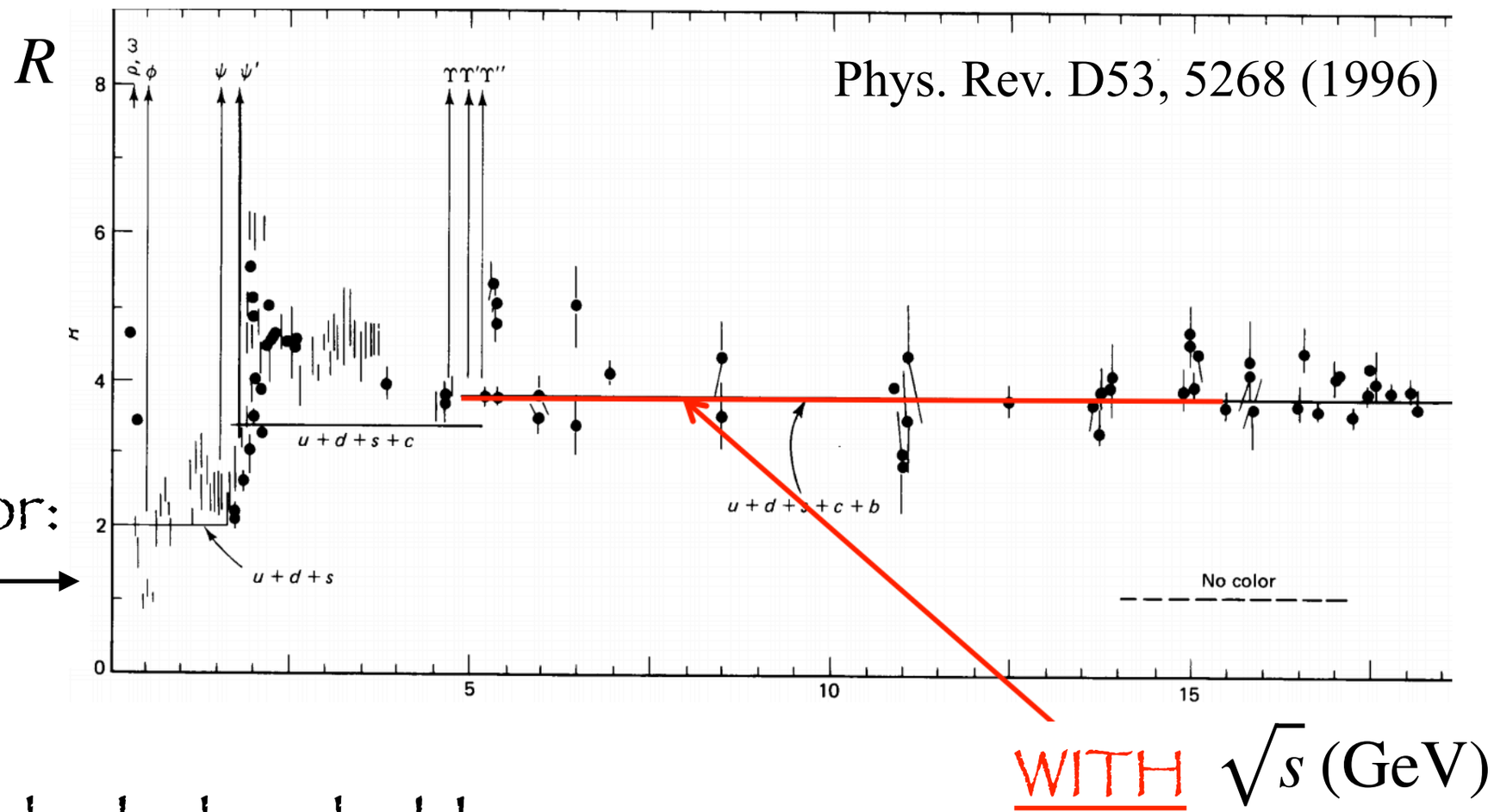


- <u>WITH COLOR:*</u>:

$$R = \frac{\sigma(e^+e^- \to \mu^+\mu^-)}{\sum_q \sigma(e^+e^- \to q\bar{q})} = 3\sum_i^{n_f} q_i^2$$

*Omitting hadronization

Evidence of color



Without color: 11/9

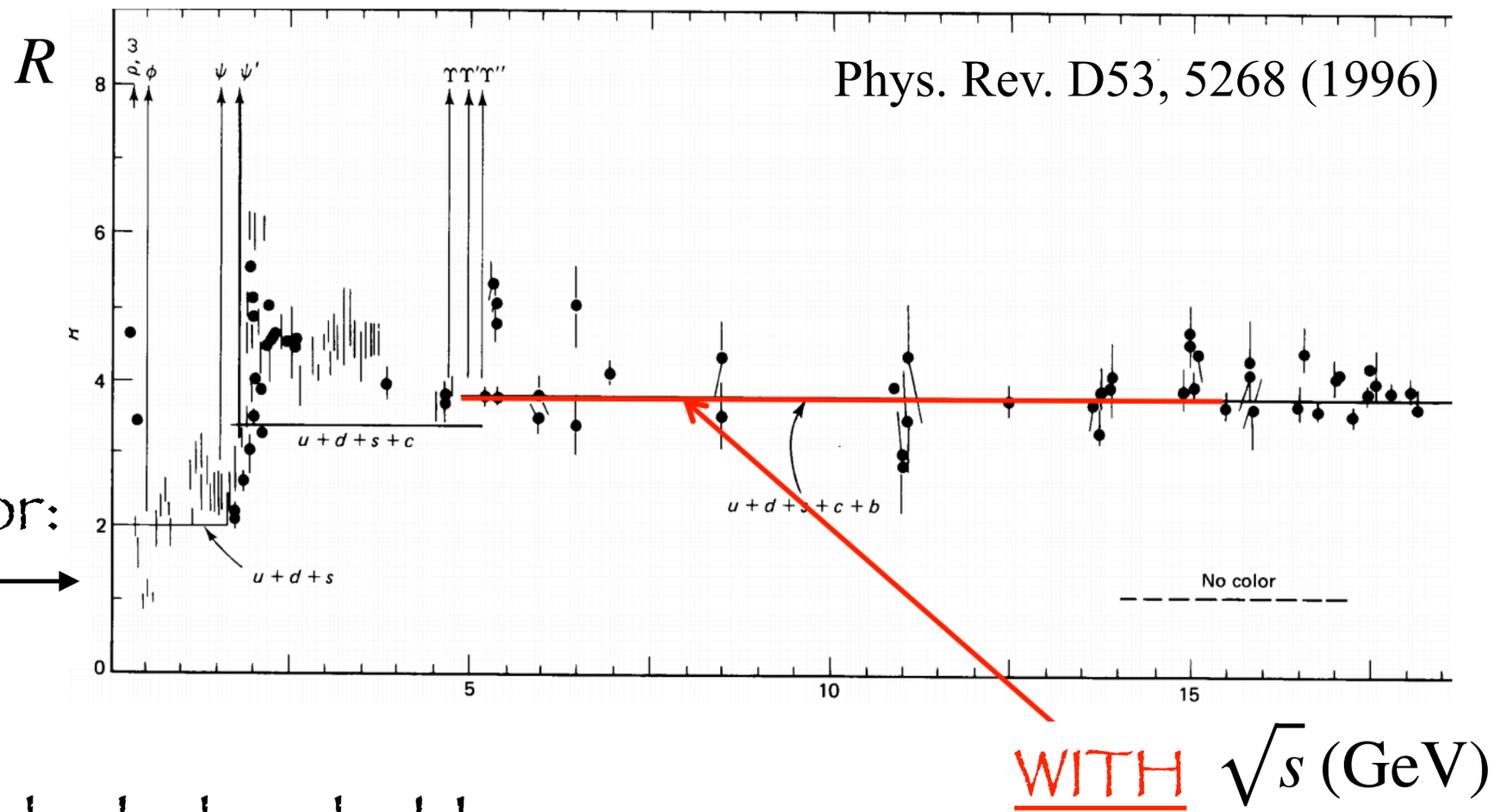
- Above the b threshold:

Without color:

$$R = \left[\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2\right] = 11/9 \approx 1.2$$

u d s c b

Evidence of color



Without color:

11/9

WITH COLOR

- Above the b threshold:

WITH COLOR

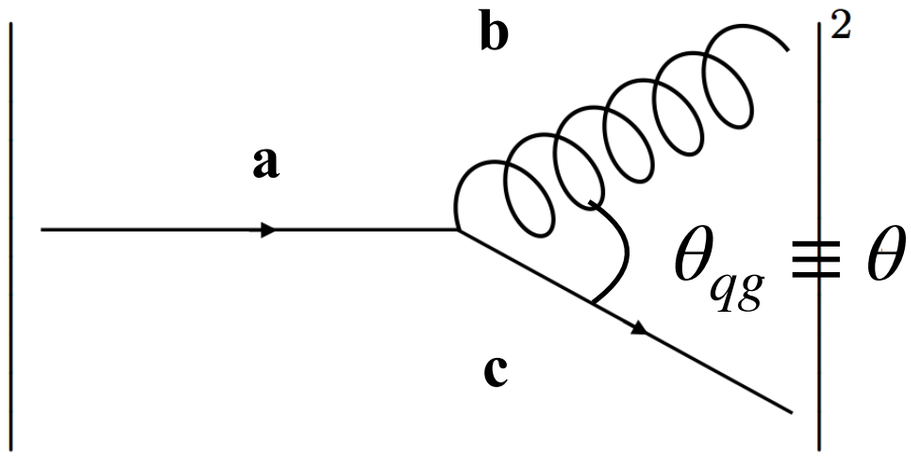
$$R = 3 \cdot \left[\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 \right] = 3 \cdot 11/9 \approx 3.67$$

u d s c b

IR and collinear divergences

The building block of QCD

$P(q \to qg) =$



Taking the final state parsons as massless:

$$\mathbb{P}^{\mu}_{a} = (E_a, 0, 0, \overrightarrow{p}_a)$$

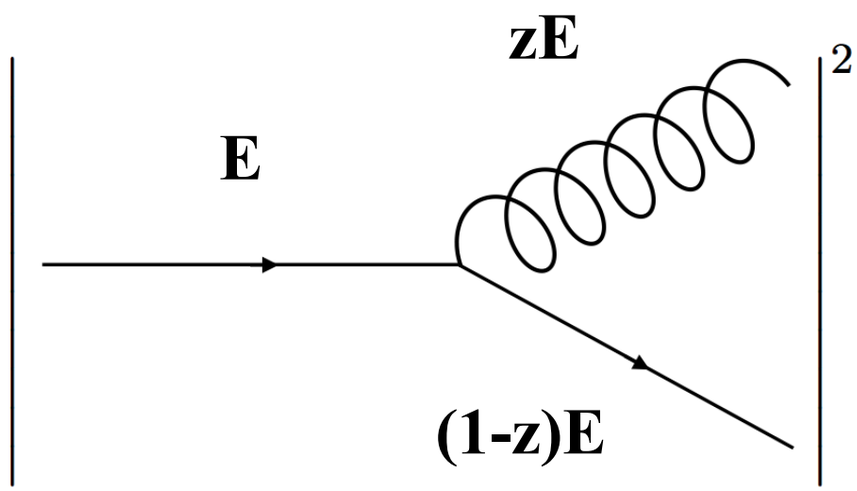
$$\mathbb{P}^{\mu}_{b} = (E_b, 0, E_b \sin\theta_1, E_b \cos\theta_1)$$

$$\mathbb{P}^{\mu}_{c} = (E_c, 0, -E_c \sin\theta_2, E_c \cos\theta_2)$$

$$M^2 \equiv \left(\mathbb{P}^{\mu}_{b} + \mathbb{P}^{\mu}_{c}\right)^2 = 2\,\mathbb{P}^{\mu}_{b}\,\mathbb{P}^{c}_{\mu}$$

$$M^2 = E_b E_c \left(1 + \sin\theta_1 \sin\theta_2 - \cos\theta_1 \cos\theta_2\right) = 2 E_b E_c (1 - \cos\theta)$$

$P(q \to qg) =$



$z, \theta << 1$:

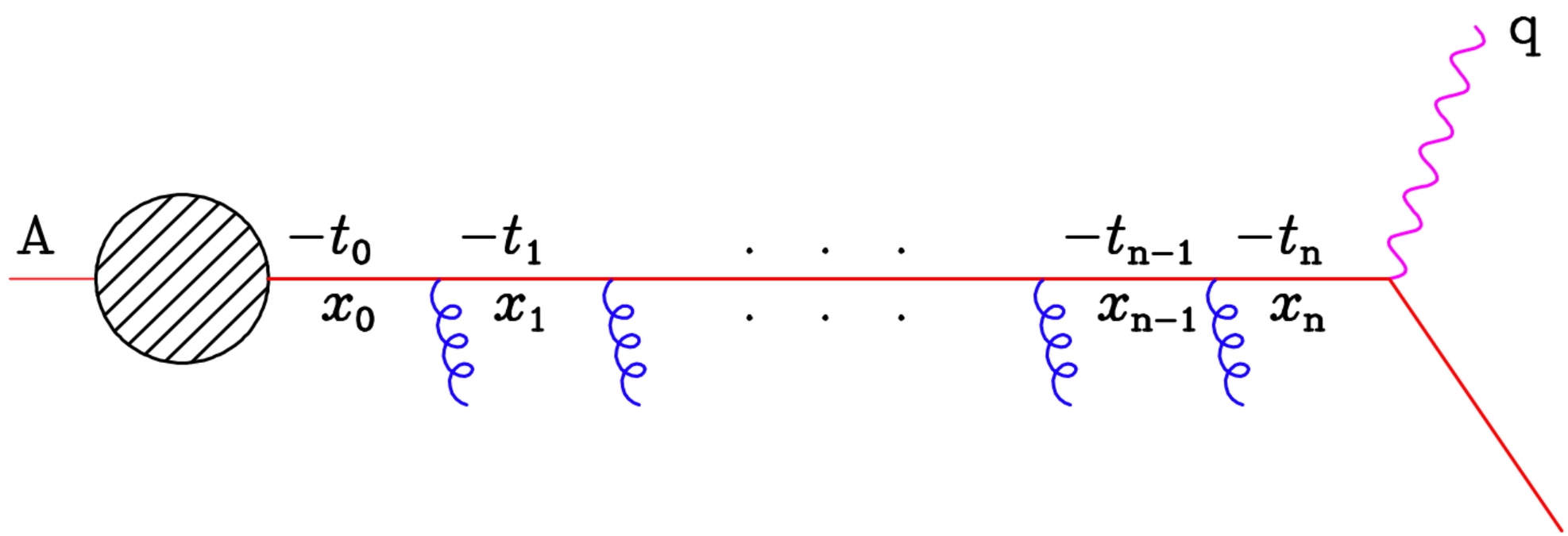
$$dP(z,\theta) \propto \alpha_s C_F \frac{dz}{z} \frac{d\theta^2}{\theta^2}$$

Collinear and soft divergent

States with no gluons, with collinear gluons or with really soft gluons are indistinguishable

- Divergences need to be resumed: renormalization (DGLAP, DLA…)

$$\alpha_s \log \frac{Q^2}{Q_0^2} \sim 1$$



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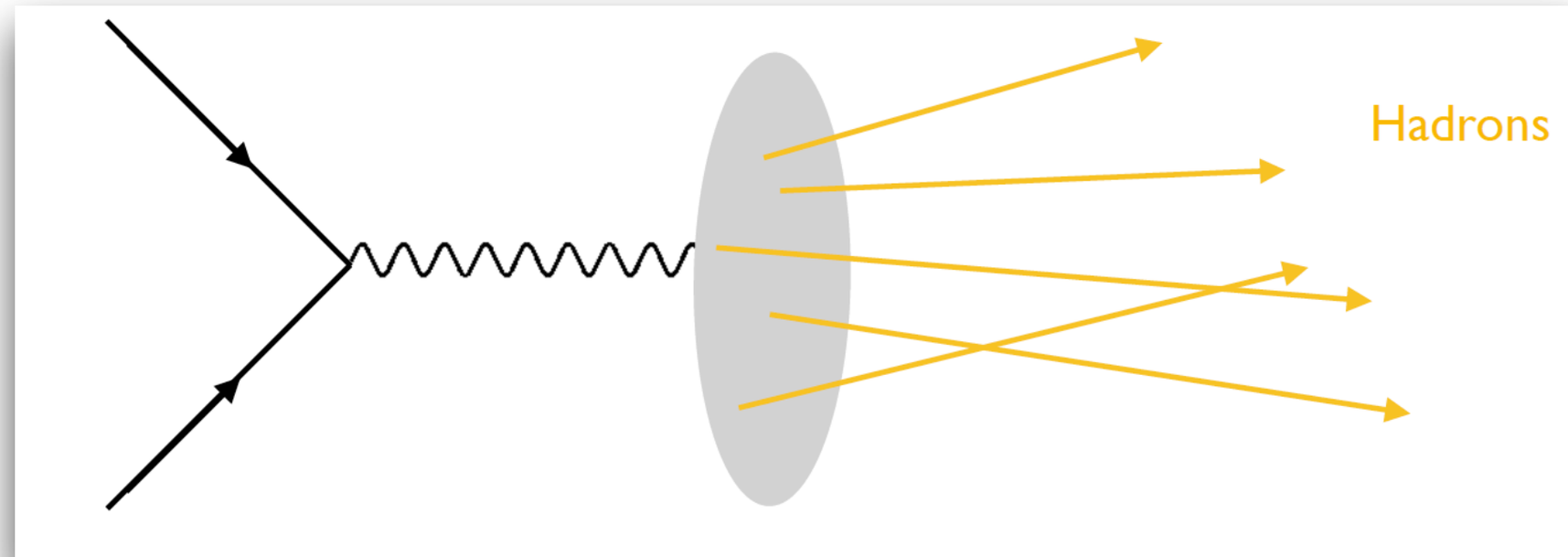
How? IR safe observables

- IR safe observables

- Simple example: $e^+e^- \longrightarrow$ hadrons



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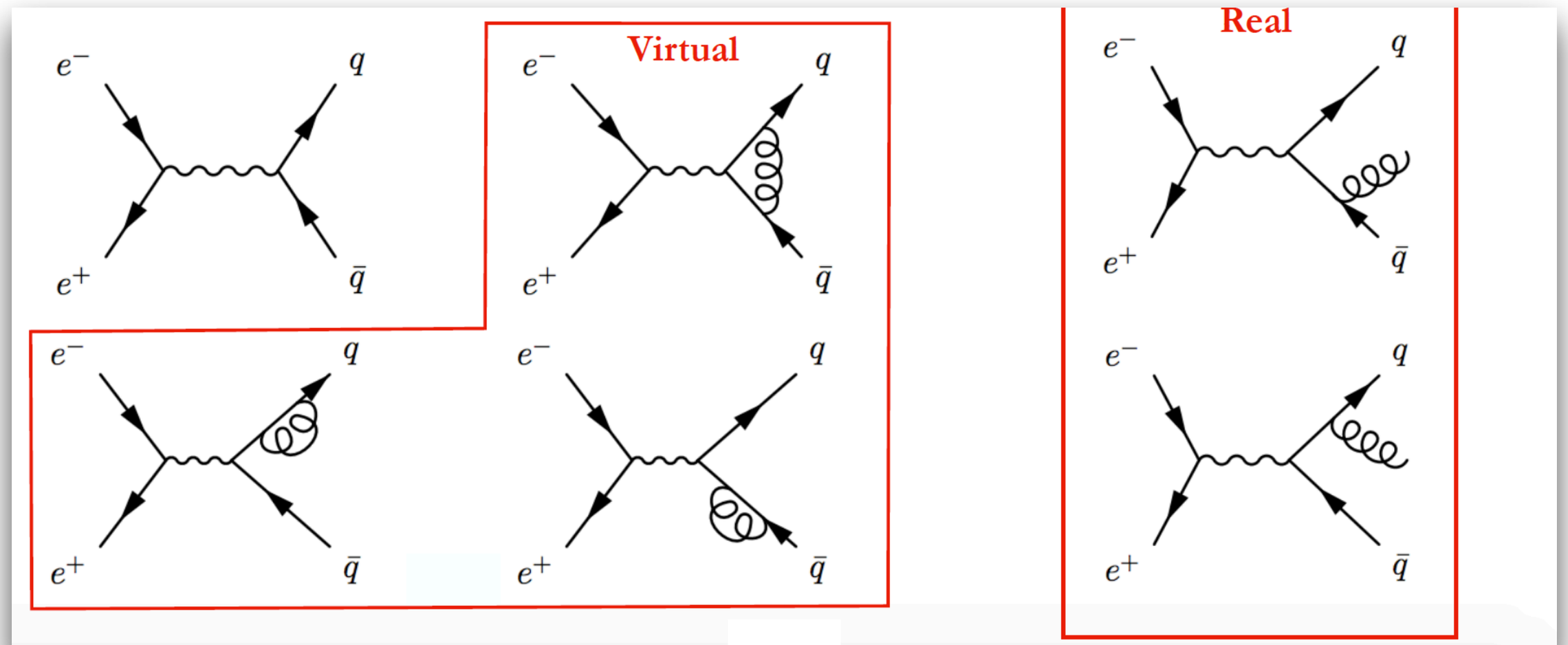
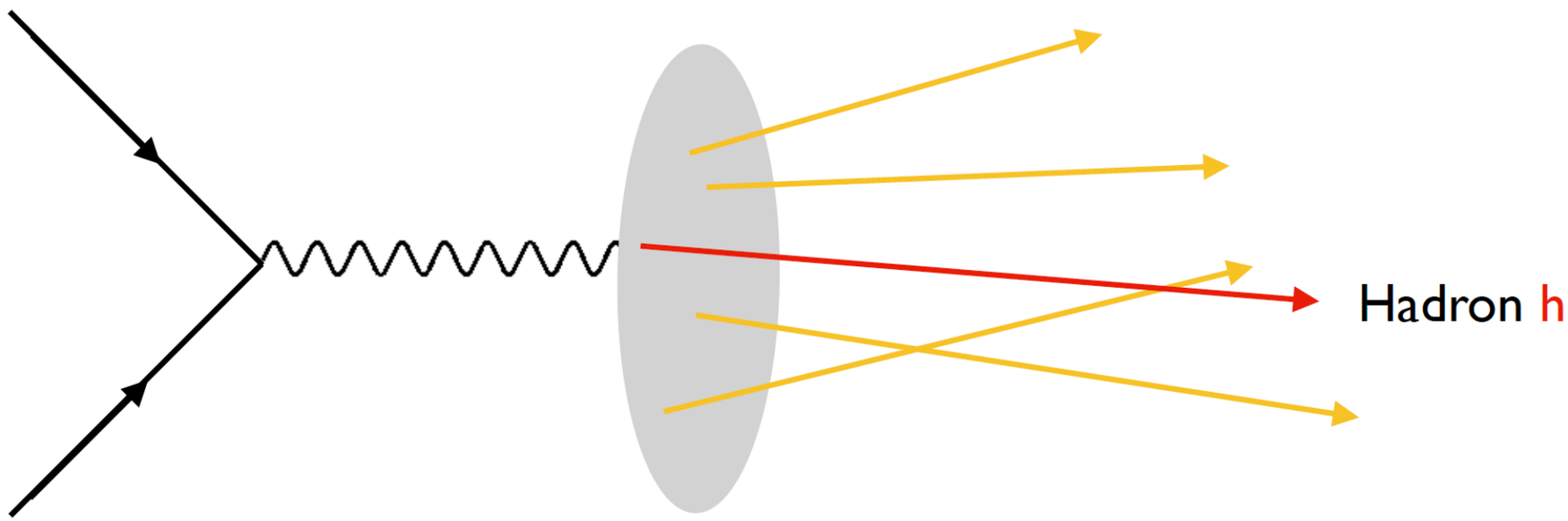


How?

- pQCD provides a full perturbative calculation in terms of partons

- But experiments can only see hadrons

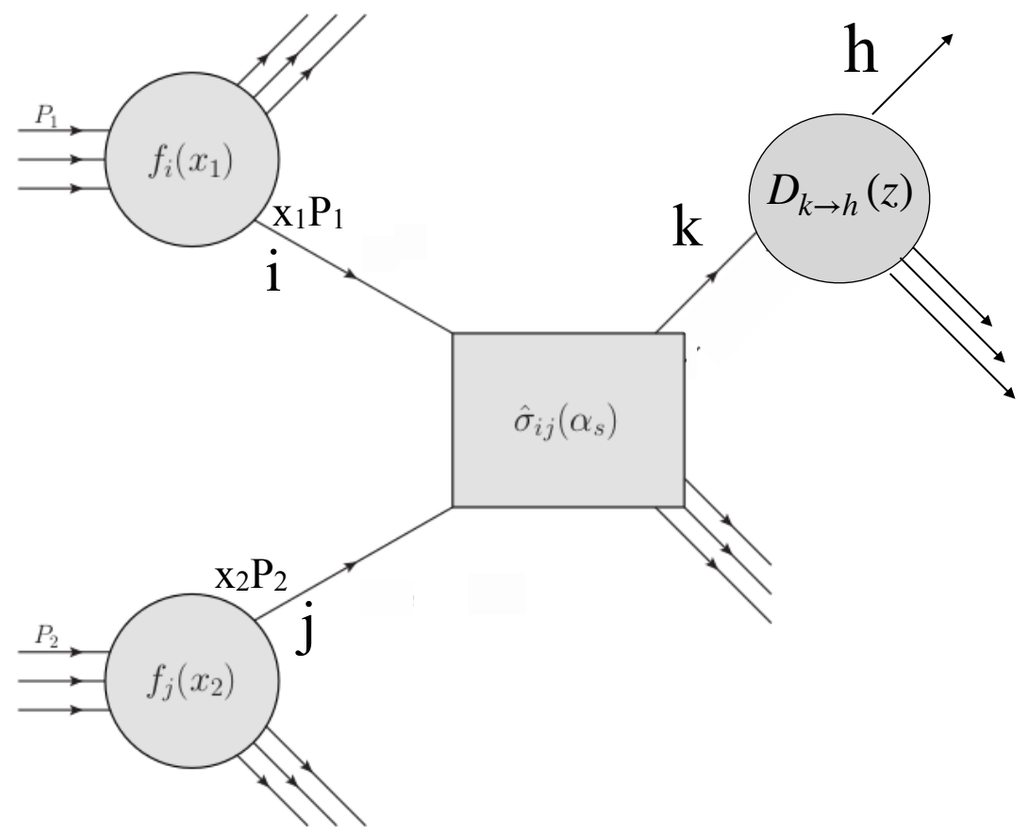
i.e. $e^+e^- \to h + X$



- Sensitivity to non-perturbative physics

How? Collinear Factorization

$$d\sigma^{p\,p\to h\,X} = \sum_{ijk} f_i(x_1,\mu_f^2) \otimes f_j(x_2,\mu_f^2) \otimes d\sigma_{ij\to k} \otimes D_{k\to h}(z,\mu_f^2) + \mathcal{O}(\mu_f^{-2n})$$



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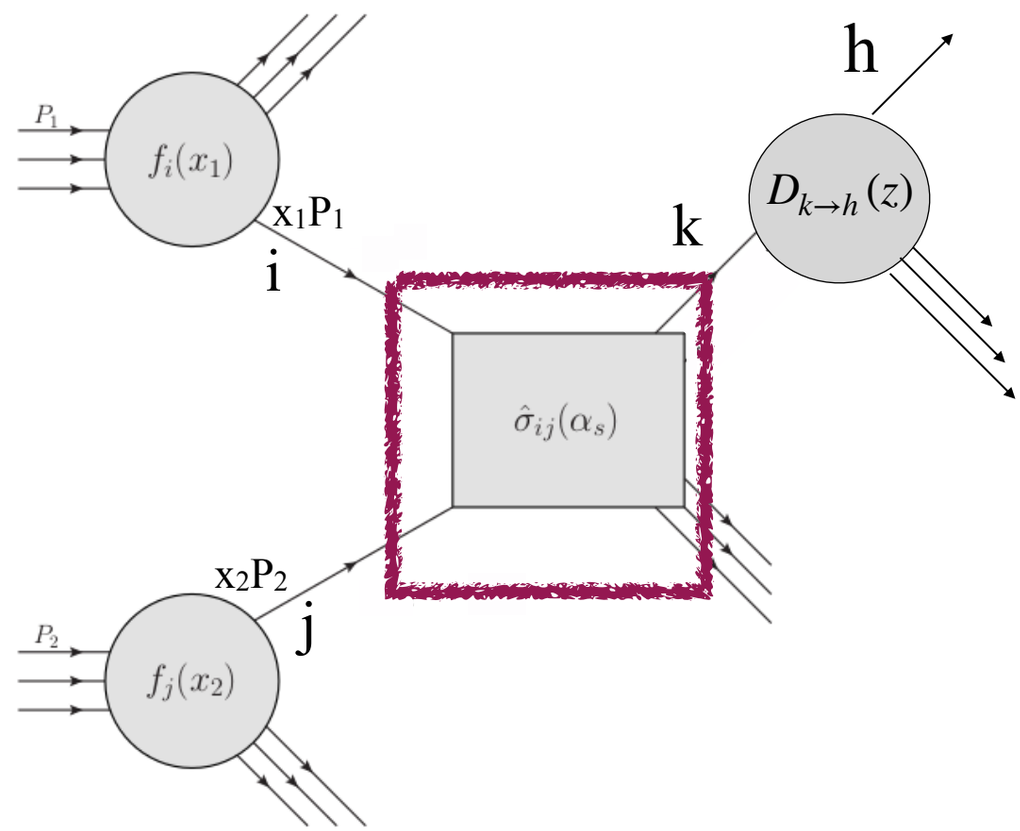
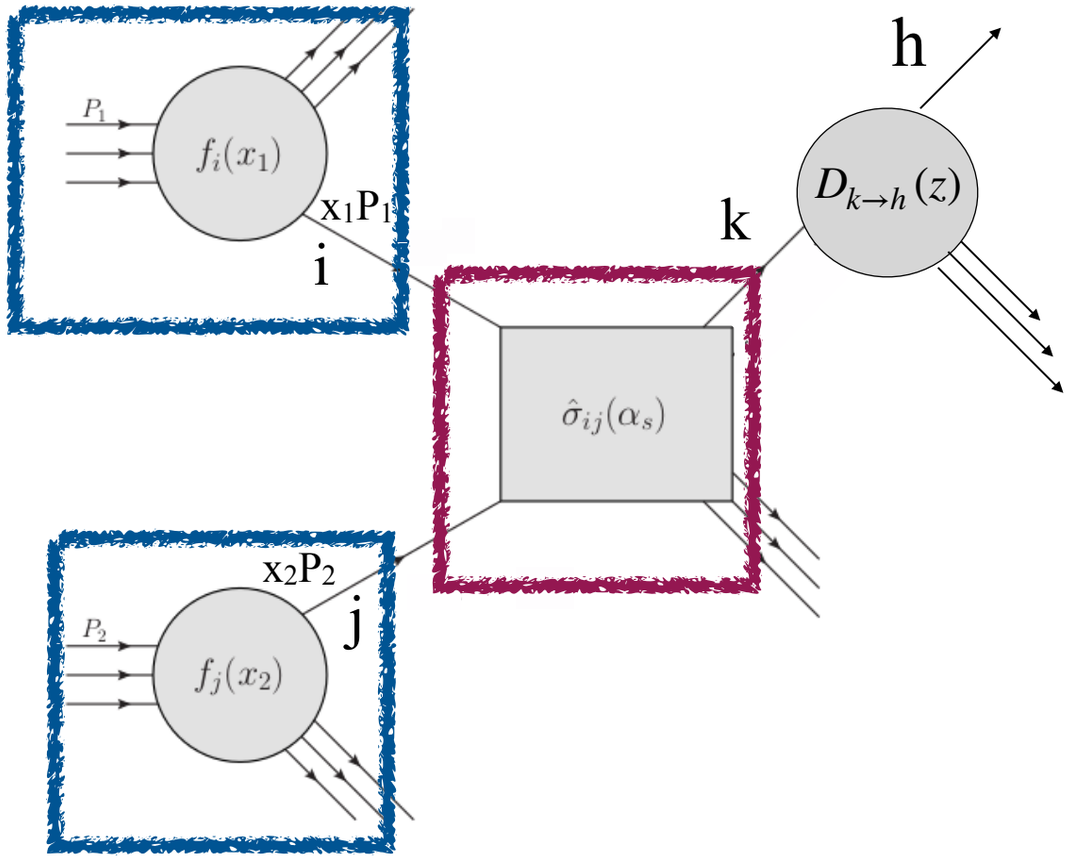


How? Collinear Factorization

PDFs pQCD

$$d\sigma^{p\,p\to h\,X} = \sum_{ijk} \boxed{f_i(x_1,\mu_f^2)} \otimes \boxed{f_j(x_2,\mu_f^2)} \otimes \boxed{d\sigma_{ij\to k}} \otimes D_{k\to h}(z,\mu_f^2) + \mathcal{O}(\mu_f^{-2n})$$



How? Collinear Factorization

PDFs pQCD FFs

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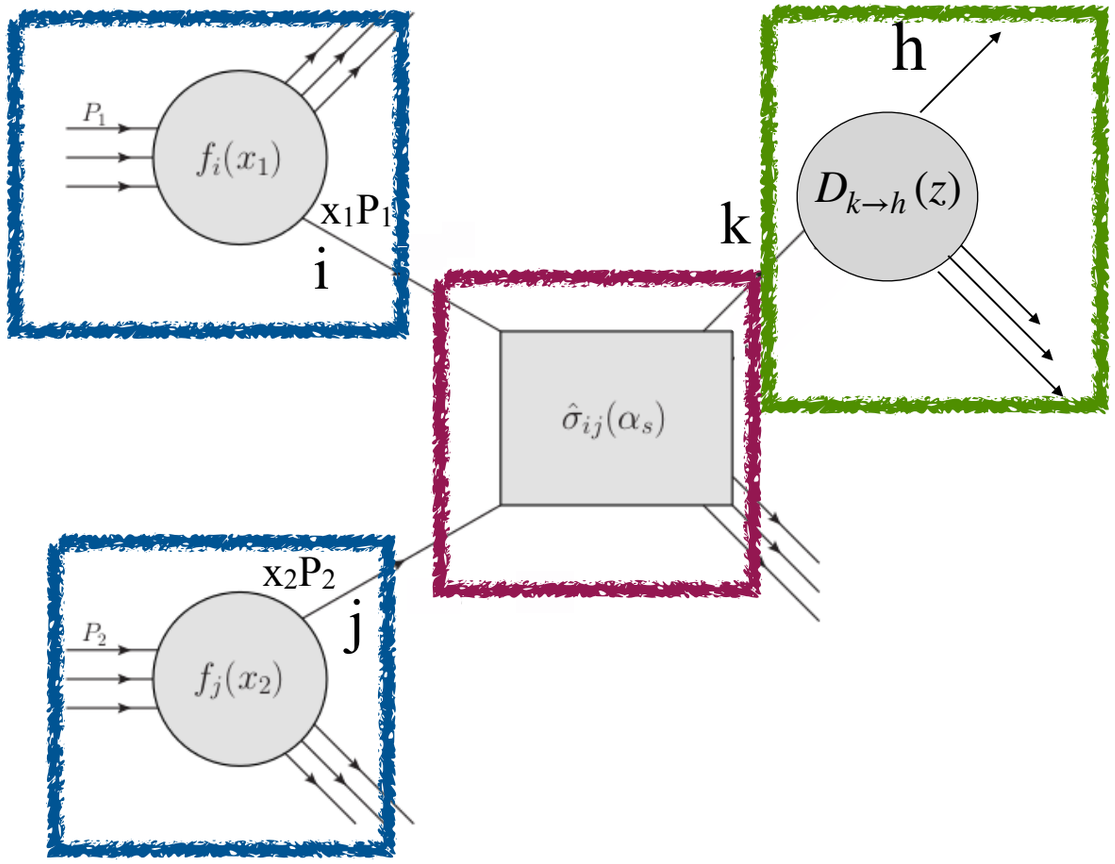

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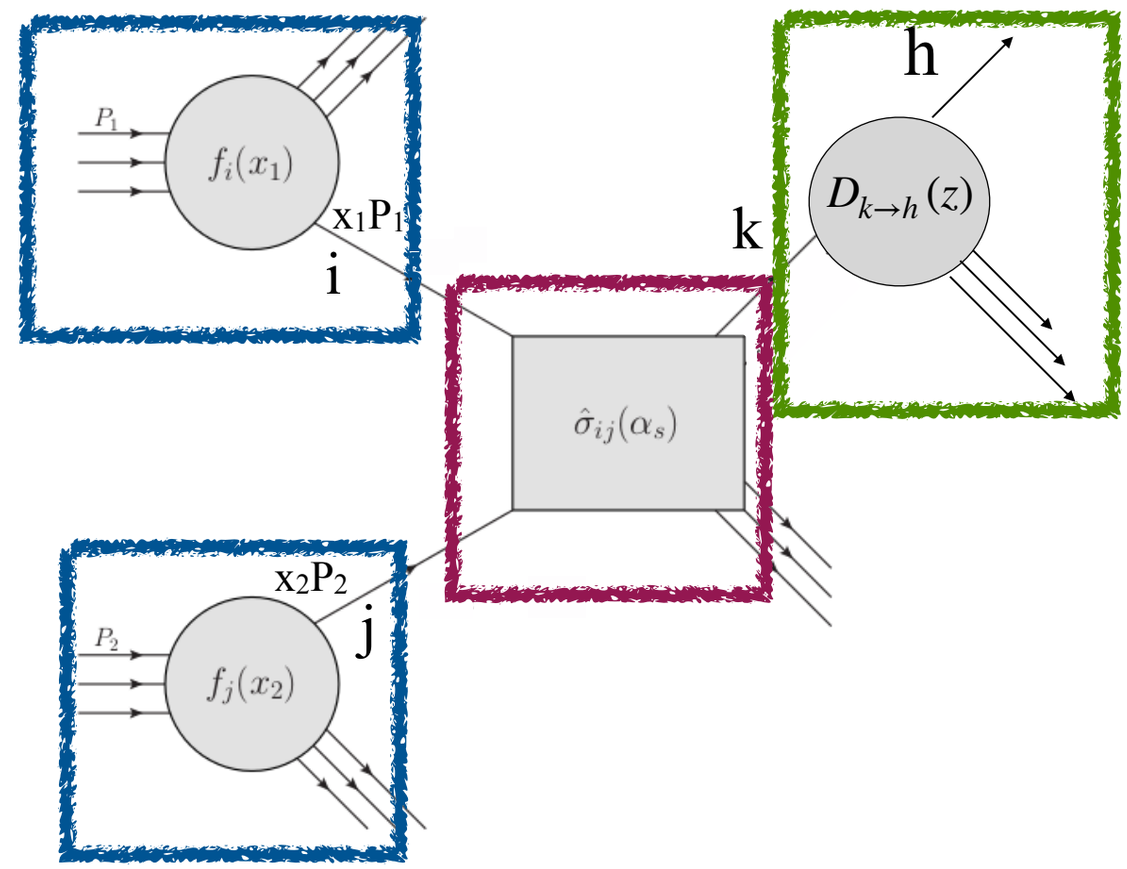
- <u>Non perturbative</u>
  But universal
- Their evolution is perturbative



How? Collinear Factorization

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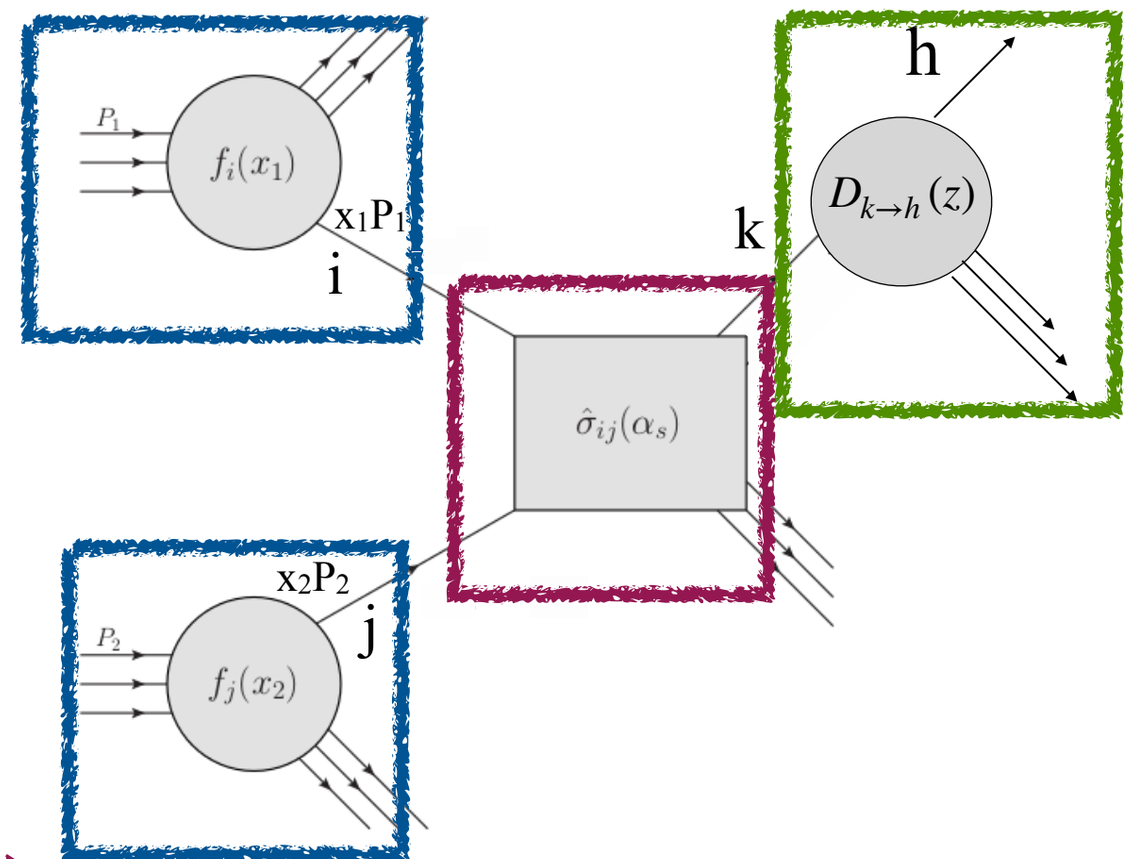
- <u>Non perturbative</u>
  But universal

- Their evolution is perturbative → DGLAP

$$Q^2 \frac{\partial f_i(x,Q^2)}{\partial Q^2} = \sum_j P_{ij} \otimes f_j(x,Q^2)$$

Splitting functions (pQCD)



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Backup

DGLAP

- Splitting functions at LO:

$$
\begin{aligned}
P_{\mathrm{qq}} &= C_F \frac{1+x^2}{(1-x)_+} + 2\delta(1-x) \quad , \\
P_{\mathrm{qg}} &= \frac{1}{2}\left[x^2 + (1-x)^2\right] \quad , \\
P_{\mathrm{gq}} &= C_F \left[\frac{1+(1-x)^2}{x}\right] \quad , \\
P_{\mathrm{gg}} &= 2N_c \left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x)\right] + \left(\frac{11N_c - 2n_f}{6}\right)\delta(1-x) \quad ,
\end{aligned}
$$

DGLAP

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- Splitting functions at LO:

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