Introduction to Quantum Chromodynamics (QCD)

Lecture 2

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May 28, 2019

Jefferson Lab
EXPLORING THE NATURE OF MATTER

Outlook

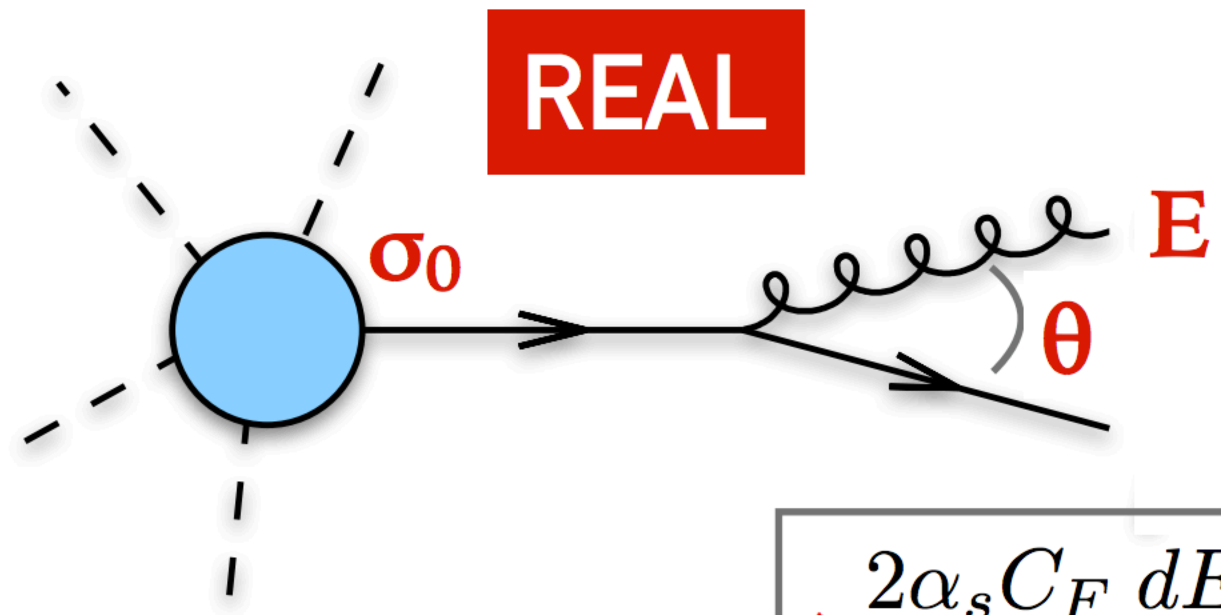
- IR and CO divergences (reminder)

- Fully IRC observables

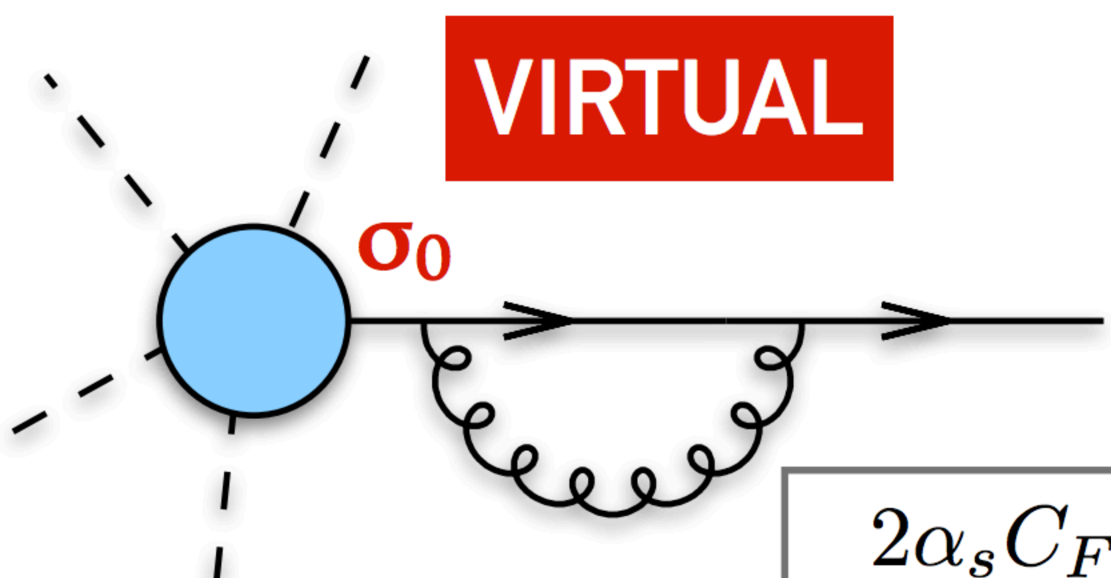
- Deep inelastic scattering (DIS)

- Polarized DIS and the proton spin crisis

IR and CO divergences



$$+\frac{2\alpha_s C_F}{\pi}\frac{dE}{E}\frac{d\theta}{\theta}$$



$$-\frac{2\alpha_s C_F}{\pi}\frac{dE}{E}\frac{d\theta}{\theta}$$

- Divergences appear in real and virtual diagrams
- If you are inclusive, you don't care about emissions of soft/collinear gluons
  real and virtual
  divergences cancel!

IR and CO divergences

Being inclusive = Observables with NO identified hadrons

↓

Purely IRC safe observables

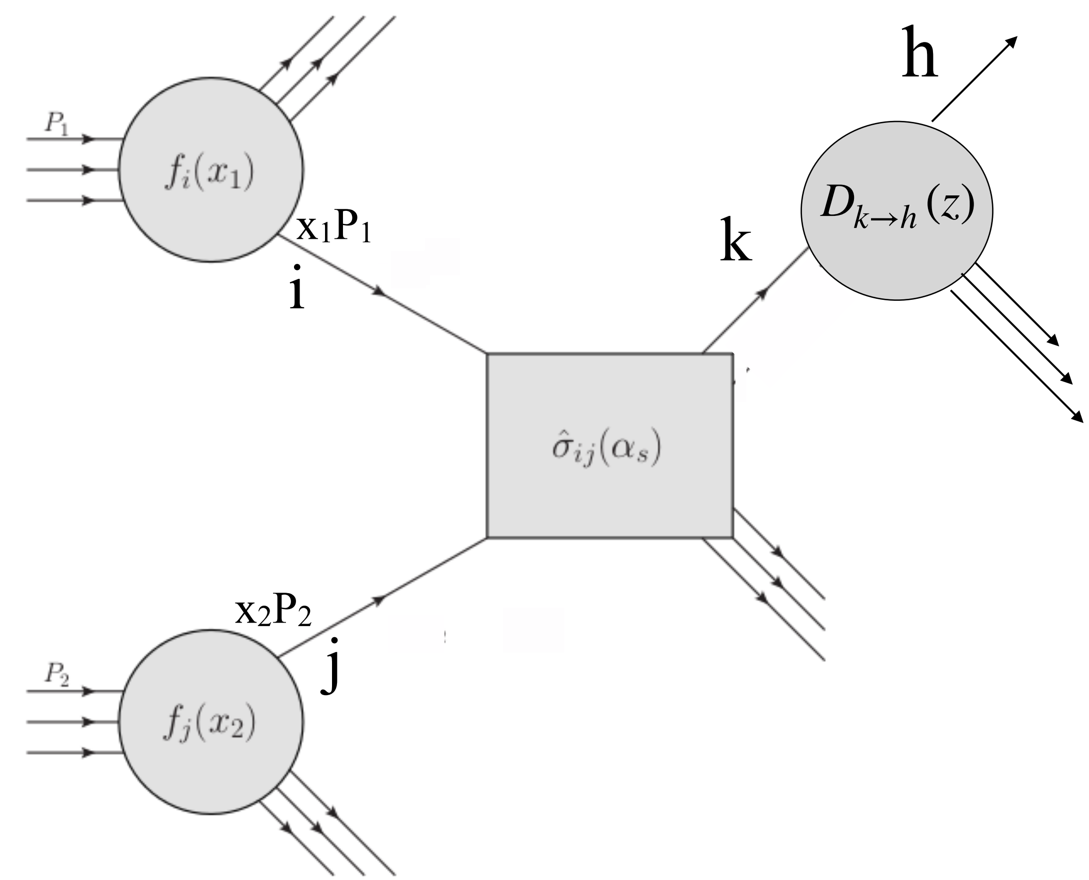
Cross sections with identified hadrons are non-perturbative

↓

Factorization, Effective field theory, Lattice…

How? Collinear Factorization

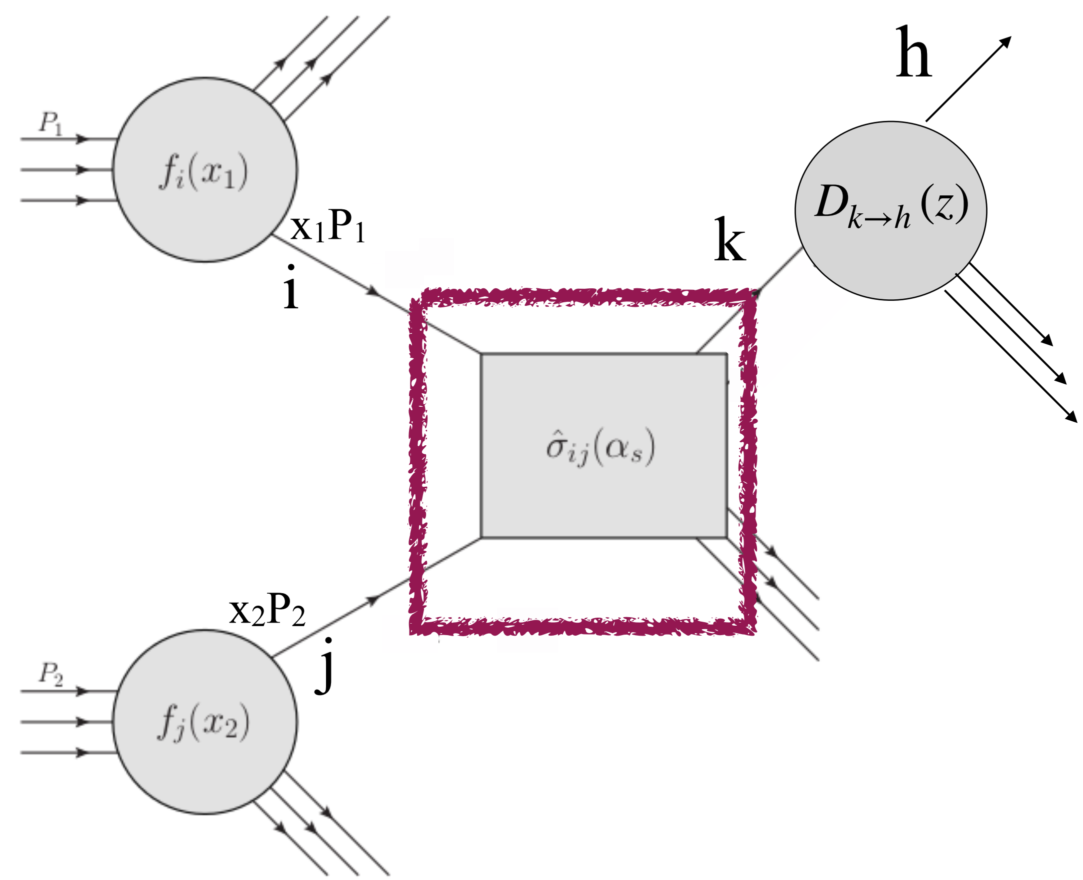
$$d\sigma^{pp\to hX} = \sum_{ijk} f_i(x_1,\mu_f^2) \otimes f_j(x_2,\mu_f^2) \otimes d\sigma_{ij\to k} \otimes D_{k\to h}(z,\mu_f^2) + \mathcal{O}(\mu_f^{-2n})$$



How? Collinear Factorization

pQCD

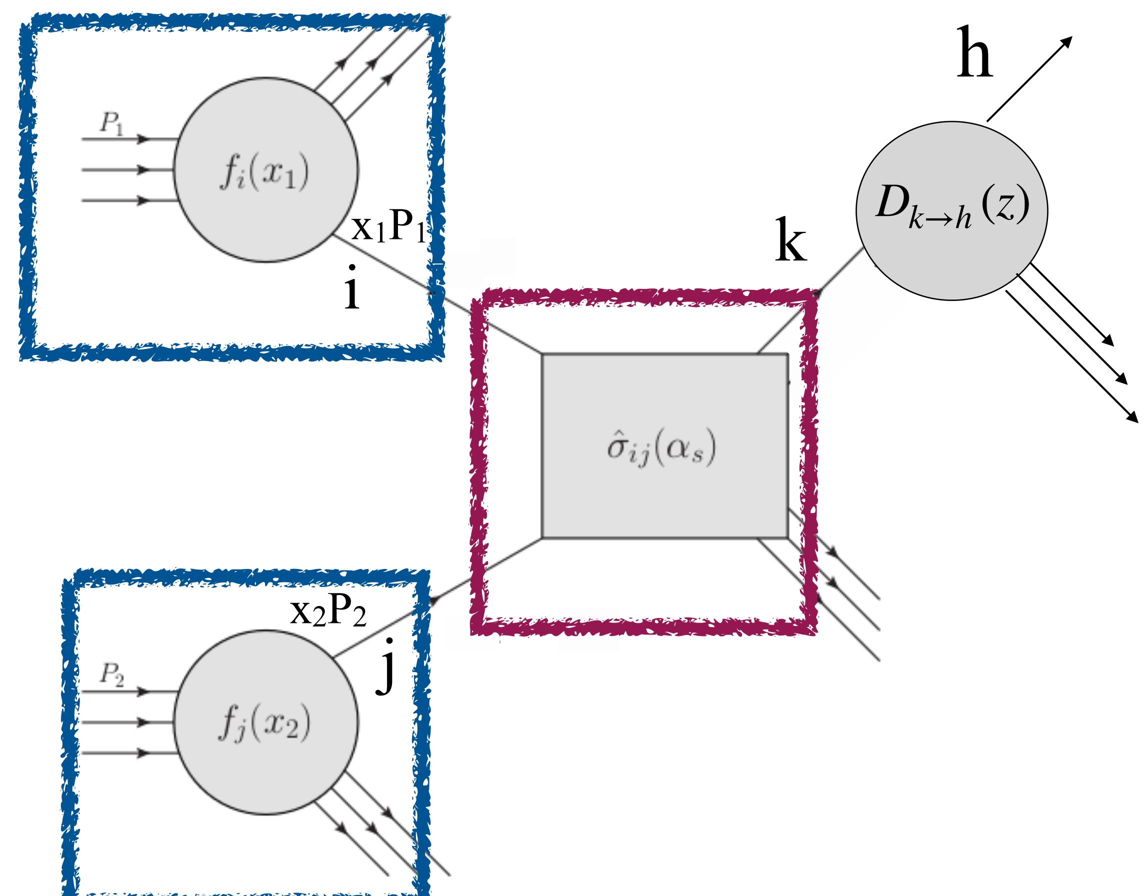
$$d\sigma^{pp\to hX} = \sum_{ijk} f_i(x_1,\mu_f^2) \otimes f_j(x_2,\mu_f^2) \otimes \boxed{d\sigma_{ij\to k}} \otimes D_{k\to h}(z,\mu_f^2) + \mathcal{O}(\mu_f^{-2n})$$



How? Collinear Factorization

PDFs pQCD

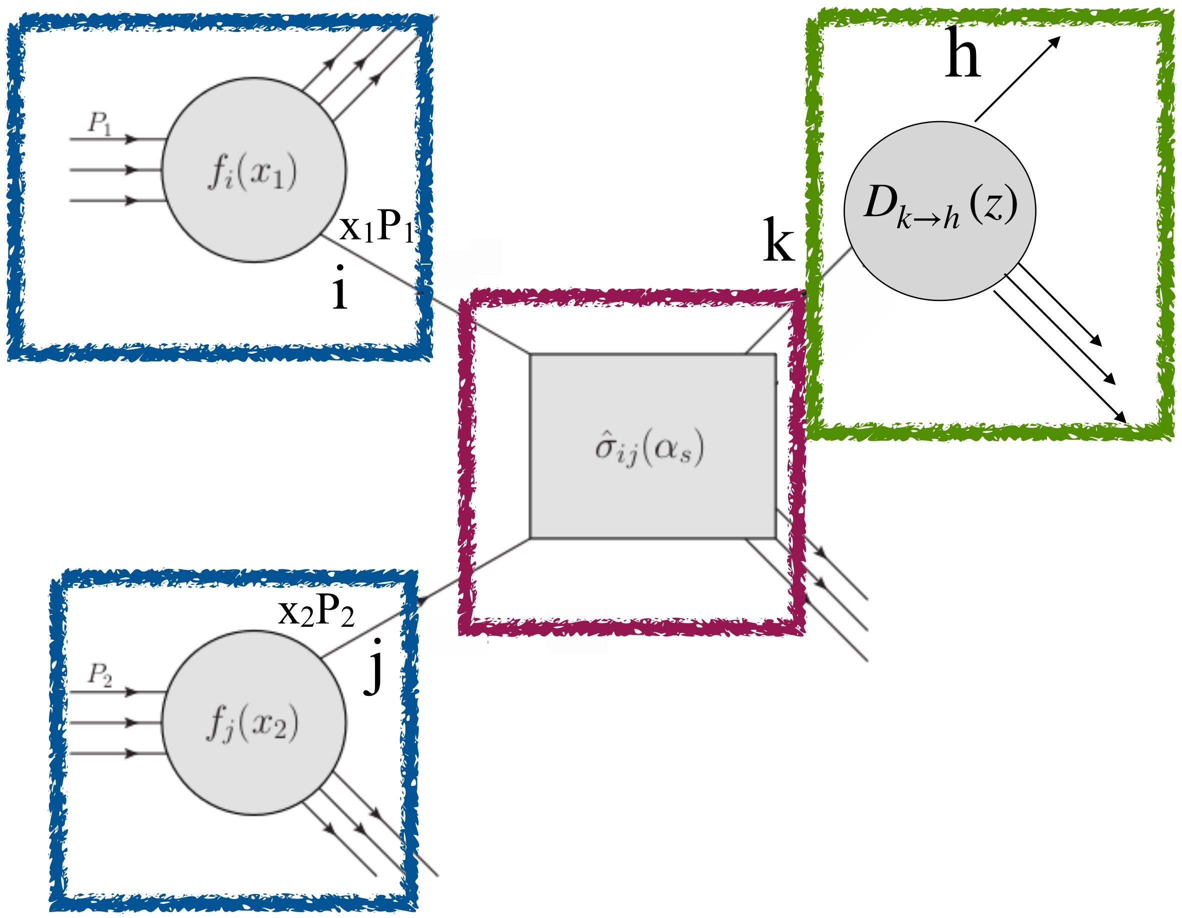
$$d\sigma^{pp\to hX} = \sum_{ijk} \boxed{f_i(x_1,\mu_f^2)} \otimes \boxed{f_j(x_2,\mu_f^2)} \otimes \boxed{d\sigma_{ij\to k}} \otimes D_{k\to h}(z,\mu_f^2) + \mathcal{O}(\mu_f^{-2n})$$



How? Collinear Factorization

PDFs　　pQCD　　FFs

$$d\sigma^{pp\to hX} = \sum_{ijk} \boxed{f_i(x_1,\mu_f^2)} \otimes \boxed{f_j(x_2,\mu_f^2)} \otimes \boxed{d\sigma_{ij\to k}} \otimes \boxed{D_{k\to h}(z,\mu_f^2)} + \mathcal{O}(\mu_f^{-2n})$$



How? Collinear Factorization

PDFs pQCD FFs

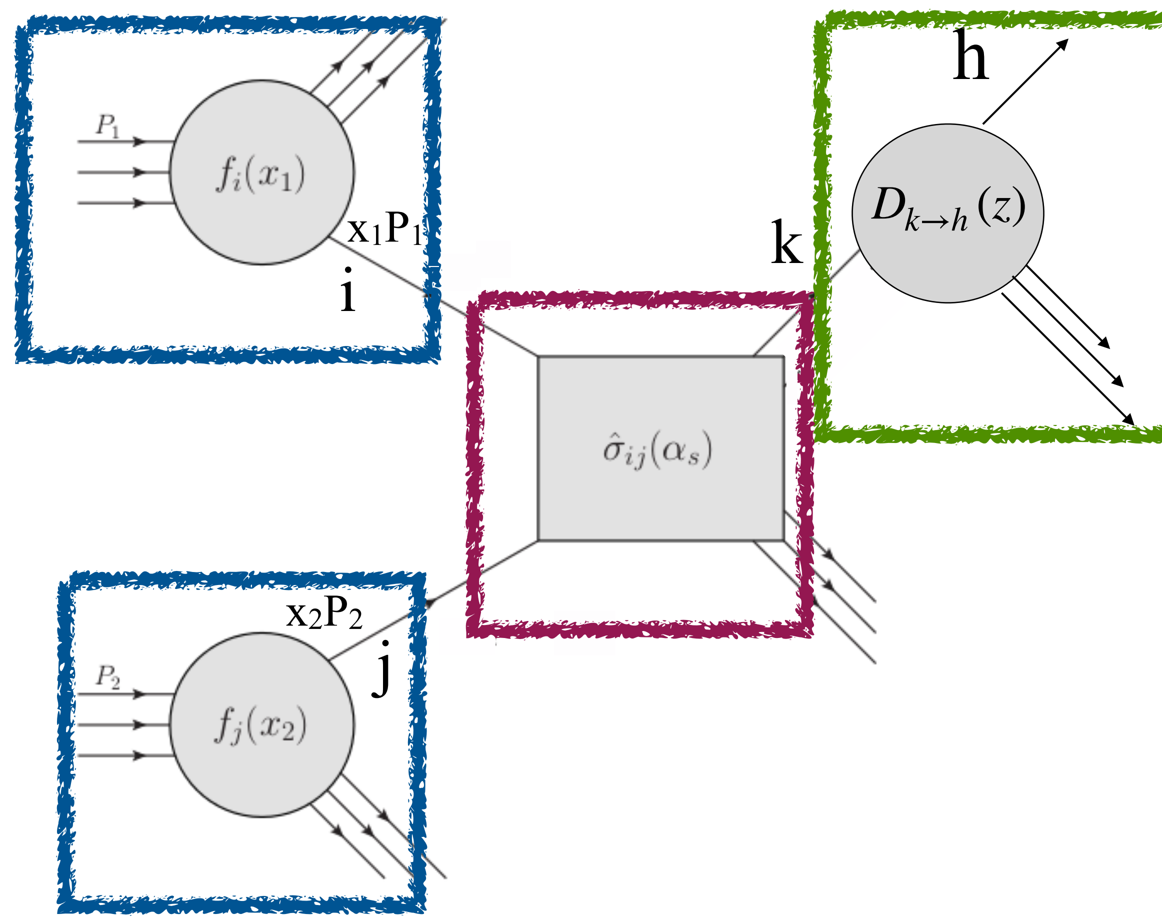
$$d\sigma^{pp\to hX} = \sum_{ijk} \boxed{f_i(x_1,\mu_f^2)} \otimes \boxed{f_j(x_2,\mu_f^2)} \otimes \boxed{d\sigma_{ij\to k}} \otimes \boxed{D_{k\to h}(z,\mu_f^2)} + \mathcal{O}(\mu_f^{-2n})$$

- <u>Non perturbative</u>

  But universal

- Their evolution

  is perturbative



How? Collinear Factorization

PDFs pQCD FFs

$$d\sigma^{pp\to hX} = \sum_{ijk} \boxed{f_i(x_1,\mu_f^2)} \otimes \boxed{f_j(x_2,\mu_f^2)} \otimes \boxed{d\sigma_{ij\to k}} \otimes \boxed{D_{k\to h}(z,\mu_f^2)} + \mathcal{O}(\mu_f^{-2n})$$

- Non perturbative

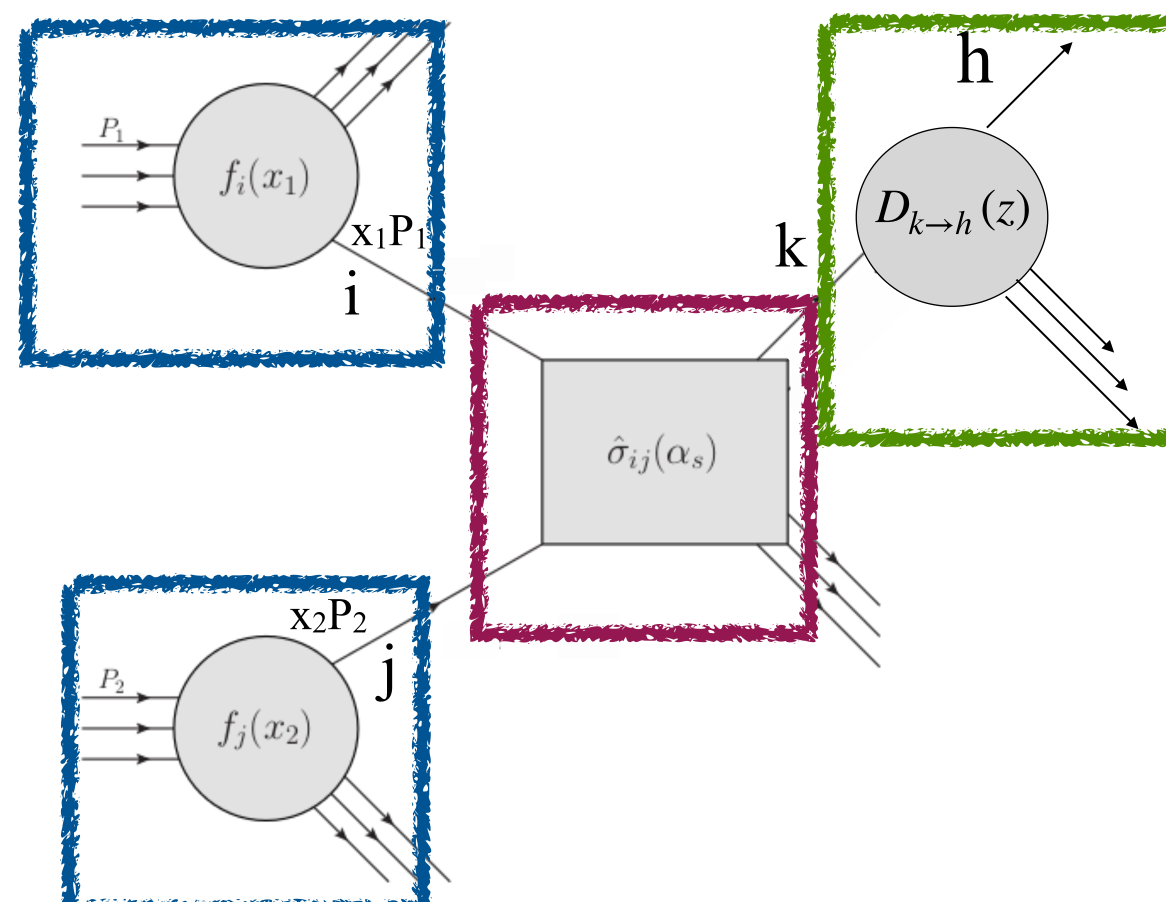
  But universal

- Their evolution

  is perturbative $\longrightarrow$ DGLAP

$$Q^2 \frac{\partial f_i(x,Q^2)}{\partial Q^2} = \sum_j \boxed{P_{ij}} \otimes f_j(x,Q^2)$$

Splitting functions (pQCD)



Fully Infrared observables

Fully Infrared observables:

$e^+e^- \longrightarrow$ hadrons

$e^+e^- \longrightarrow$ hadrons

- Total cross section. Not a specific hadron

$$\boxed{\sigma^{\text{tot}}_{e^+e^-\to\text{hadrons}} = \sigma^{\text{tot}}_{e^+e^-\to\text{partons}}}$$

$$\sigma^{\text{tot}}_{e^+e^-\to\text{hadrons}} = \sum_n P_{e^+e^-\to n} = \sum_n \sum_m P_{e^+e^-\to m} P_{m\to n} = \sum_m P_{e^+e^-\to m} \sum_n P_{m\to n} = \sum_m P_{e^+e^-\to m}$$

$$\sigma^{\text{tot}}_{e^+e^-\to\text{partons}} = \sum_m P_{e^+e^-\to m}$$

$$\sigma^{\text{tot}}_{e^+e^-\to\text{partons}}\left(Q^2\right) = \sum_n \left(\frac{\alpha_s\left(\mu^2\right)}{2\pi}\right)^n \sigma^n\left(Q^2,\mu^2\right)$$

$e^+e^- \longrightarrow$ hadrons

- Total cross section. Not a specific hadron

$$\boxed{\sigma^{\text{tot}}_{e^+e^-\to\text{hadrons}} = \sigma^{\text{tot}}_{e^+e^-\to\text{partons}}}$$

$$\sigma^{\text{tot}}_{e^+e^-\to\text{hadrons}} = \sum_n P_{e^+e^-\to n} = \sum_n \sum_m P_{e^+e^-\to m} P_{m\to n} = \sum_m P_{e^+e^-\to m} \sum_n P_{m\to n} = \sum_m P_{e^+e^-\to m}$$

$$\sigma^{\text{tot}}_{e^+e^-\to\text{partons}} = \sum_m P_{e^+e^-\to m}$$

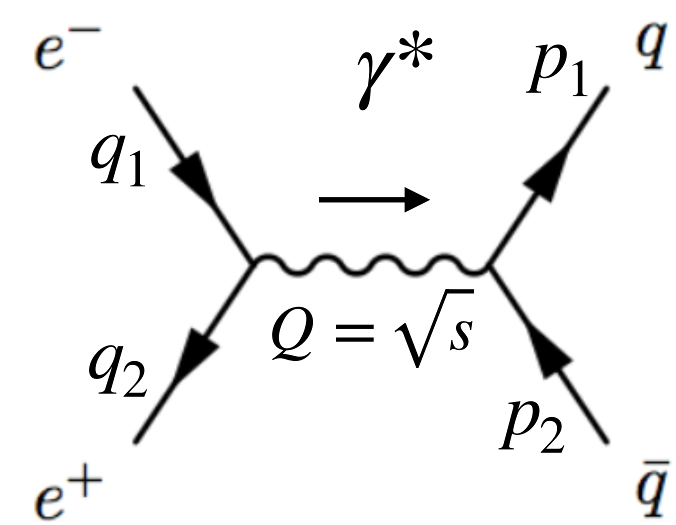
- Partonic cross section computable in pQCD

$$\sigma^{\text{tot}}_{e^+e^-\to\text{partons}}\left(Q^2\right) = \sum_n \left(\frac{\alpha_s\left(\mu^2\right)}{2\pi}\right)^n \sigma^n\left(Q^2,\mu^2\right)$$

$e^+e^- \longrightarrow$ hadrons: LO

- Square amplitude (invariant):

$$\left|M_{e^+e^-\to q\bar{q}}\right|^2 = 2\,e^4 e_Q^2 \frac{N_c}{s^2}\left[(m_q - t)^2 + (m_q - u)^2 + 2m_q^2 s\right]$$



$$s = (q_1 + q_2)^2$$

$$t = (q_1 - p_1)^2$$

$$u = (q_2 - p_1)^2$$

- Cross section at the lowest order:

$$\frac{d\sigma_{e^+e^-\to q\bar{q}}}{dt} = \frac{1}{16\pi Q^2}\left|M_{e^+e^-\to q\bar{q}}\right|^2$$

$$\sigma^0 = \sum_q d\sigma_{e^+e^-\to q\bar{q}} = \sum_q e_q^2 N_c \frac{4\pi\alpha_{em}}{3s}\left(1 + \frac{2m_q^2}{s}\right)\boxed{\sqrt{1 - \frac{4m_q^2}{s}}}$$

Threshold constraint

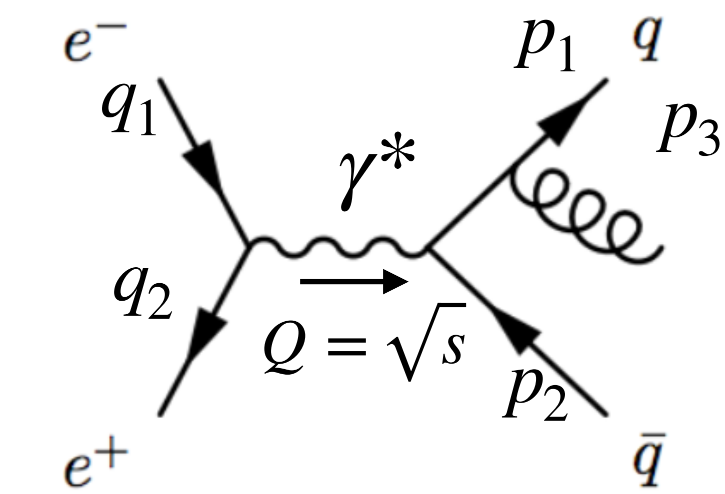
Sensitive to the number of colors

$e^+e^- \longrightarrow$ hadrons: NLO

- Energy fractions of the final state partons

$$x_i = \frac{2E_i}{\sqrt{s}} = \frac{2p_i \cdot Q}{s} \quad \text{with} \quad i = 1,2,3$$

$$2(1-x_1) = x_2 x_3 (1-\cos\theta_{23}), \quad \text{cycl}$$



+ crossing

- Contribution to the cross section

$$\frac{1}{\sigma^0}\frac{d\sigma^{q\bar{q}g}}{dx_1 dx_2} = \frac{\alpha_s}{2\pi} C_F \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$$x_2 = 1 \to \theta_{13} = 0 \quad \text{or} \quad E_3 = E_g = 0$$

$$x_1 = 1 \to \theta_{23} = 0 \quad \text{or} \quad E_3 = E_g = 0$$

Divergent when the gluon is <u>collinear</u> to the quark or antiquark or it is <u>soft</u>

Question: How much is $\sum_i x_i$?

$e^+e^- \longrightarrow$ hadrons: NLO

- Dimensional regularization: $\varepsilon = \frac{1}{2}(4-n)$

Real $\quad \sigma^{q\bar{q}g} = \sigma^0 \frac{\alpha_s}{2\pi} H(\varepsilon) \left[\frac{2}{\varepsilon^2} + \frac{3}{\varepsilon} + \frac{19}{2}\right]$

$$H(\varepsilon) = \frac{3\,(1-\varepsilon)^2}{(3-2\varepsilon)\,\Gamma\,(2-2\varepsilon)} = 1 + \mathcal{O}(\varepsilon)$$

Virtual $\quad \sigma^{q\bar{q}(g)} = \sigma^0 \frac{\alpha_s}{2\pi} H(\varepsilon) \left[-\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} - 8\right]$

NLO $\quad \sigma^{\mathrm{NLO}} = \sigma^{q\bar{q}g} + \sigma^{q\bar{q}(g)} = \sigma^0 \left[\frac{\alpha_s}{\pi} + \mathcal{O}(\varepsilon)\right] \quad$ NO ε dependence!

$$\sigma^{\mathrm{tot}} = \sigma^0 \left[1 + \frac{\alpha_s}{\pi} + \mathcal{O}\left(\alpha_s^2\right)\right]$$

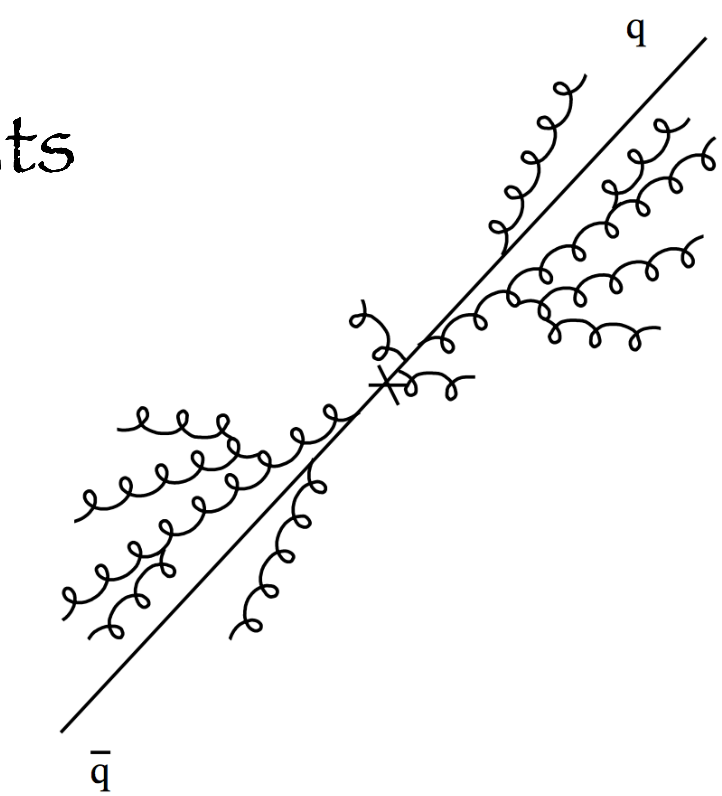
<u>IRC safe.</u> Independent of the choice of the IRC regularization

Fully Infrared observables:

$e^+e^- \longrightarrow$ jets

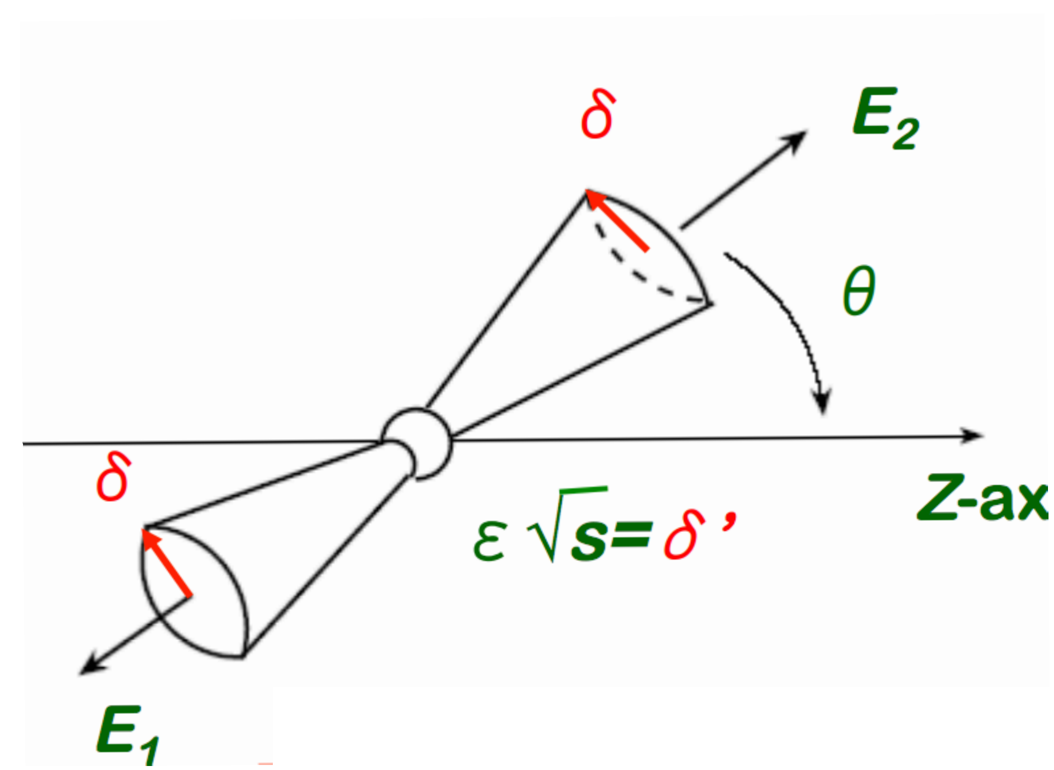
Jets

- Jets: same invariant amplitud + phase constraints



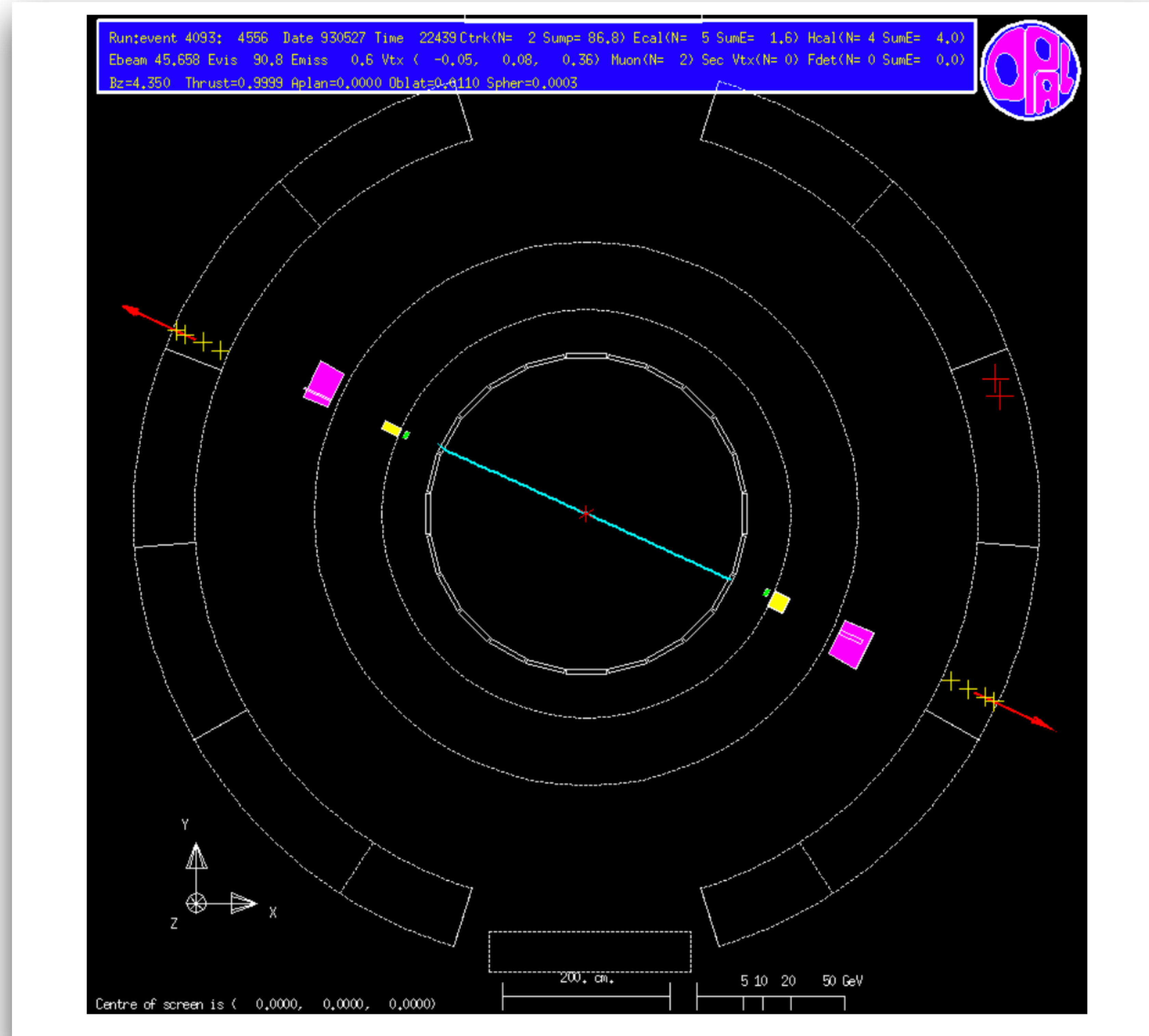
- Soft/collinear gluon emissions do not change the direction of the leading parton (IRC safe)
- Insensitive to hadronization
- Many jet algorithms
- Sterman-Weinberg jets

An event has 2 jets if at least $(1-\varepsilon)$ of the event energy is contained into two cones of half-angle δ

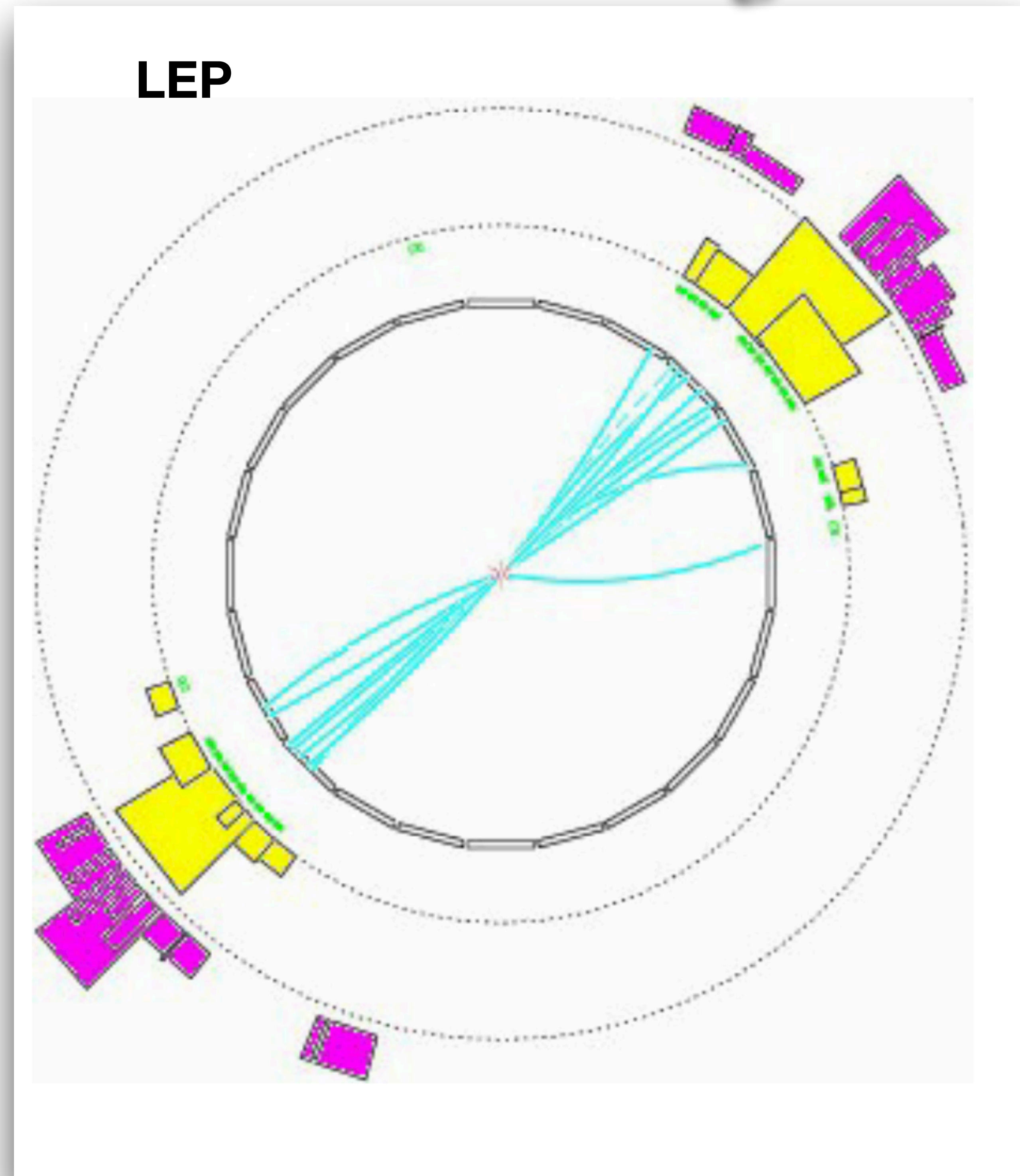


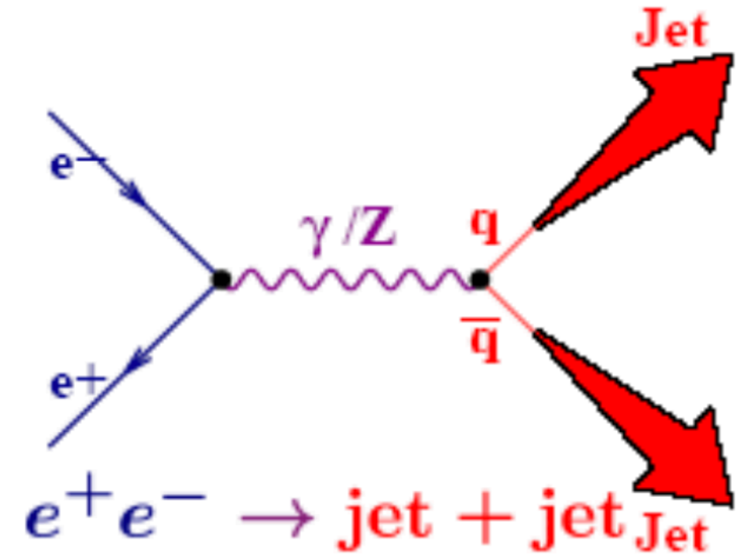
Typical $e^+e^- \to \mu^+\mu^-$

OPAL



A clean 2-jet event

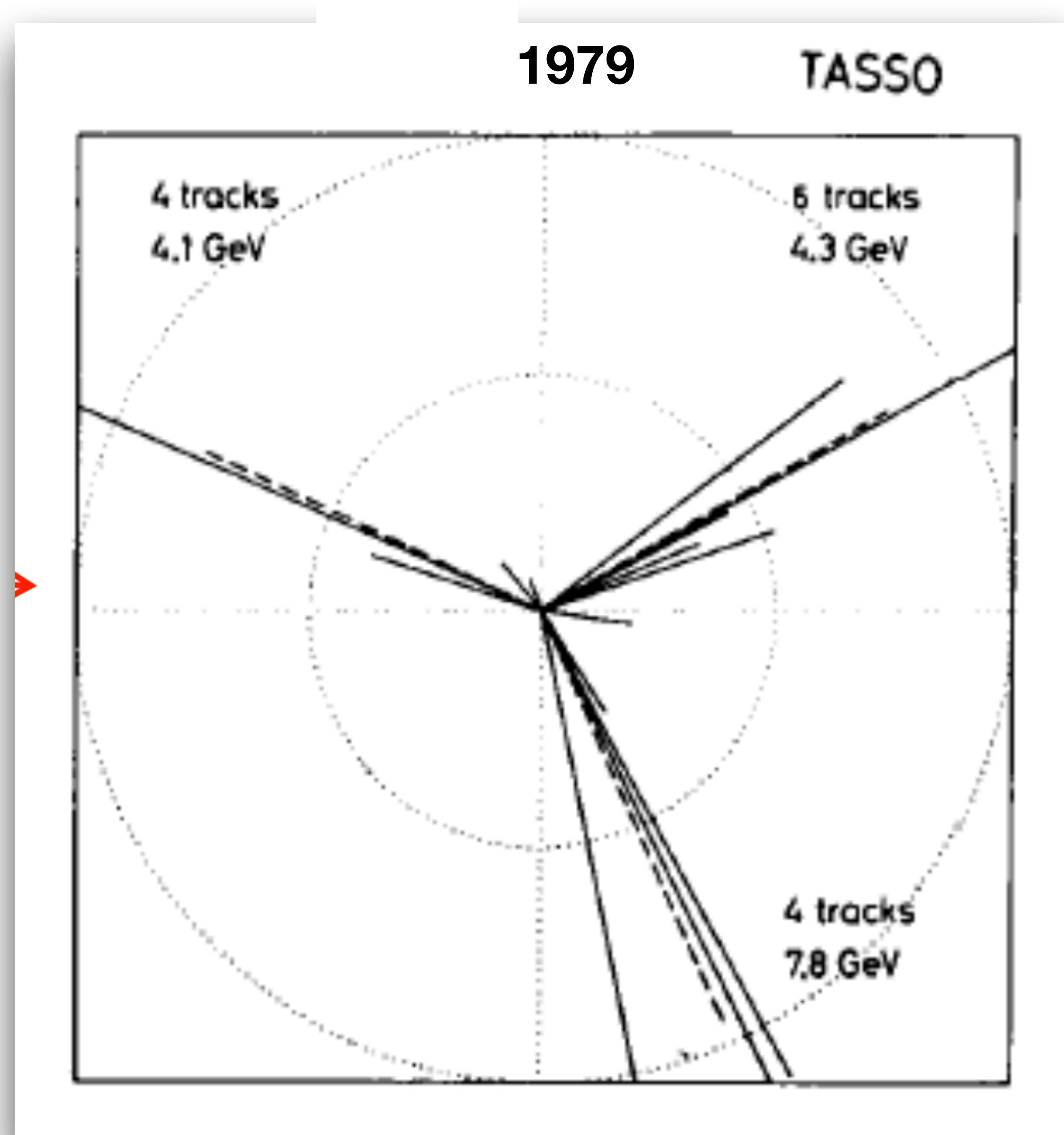




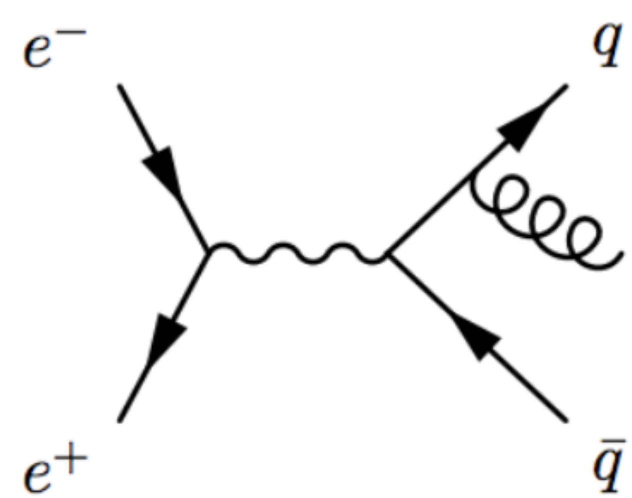
- 2 well-collimated jets

- Almost all the energy contained in two cones

Discovery of a gluon jet

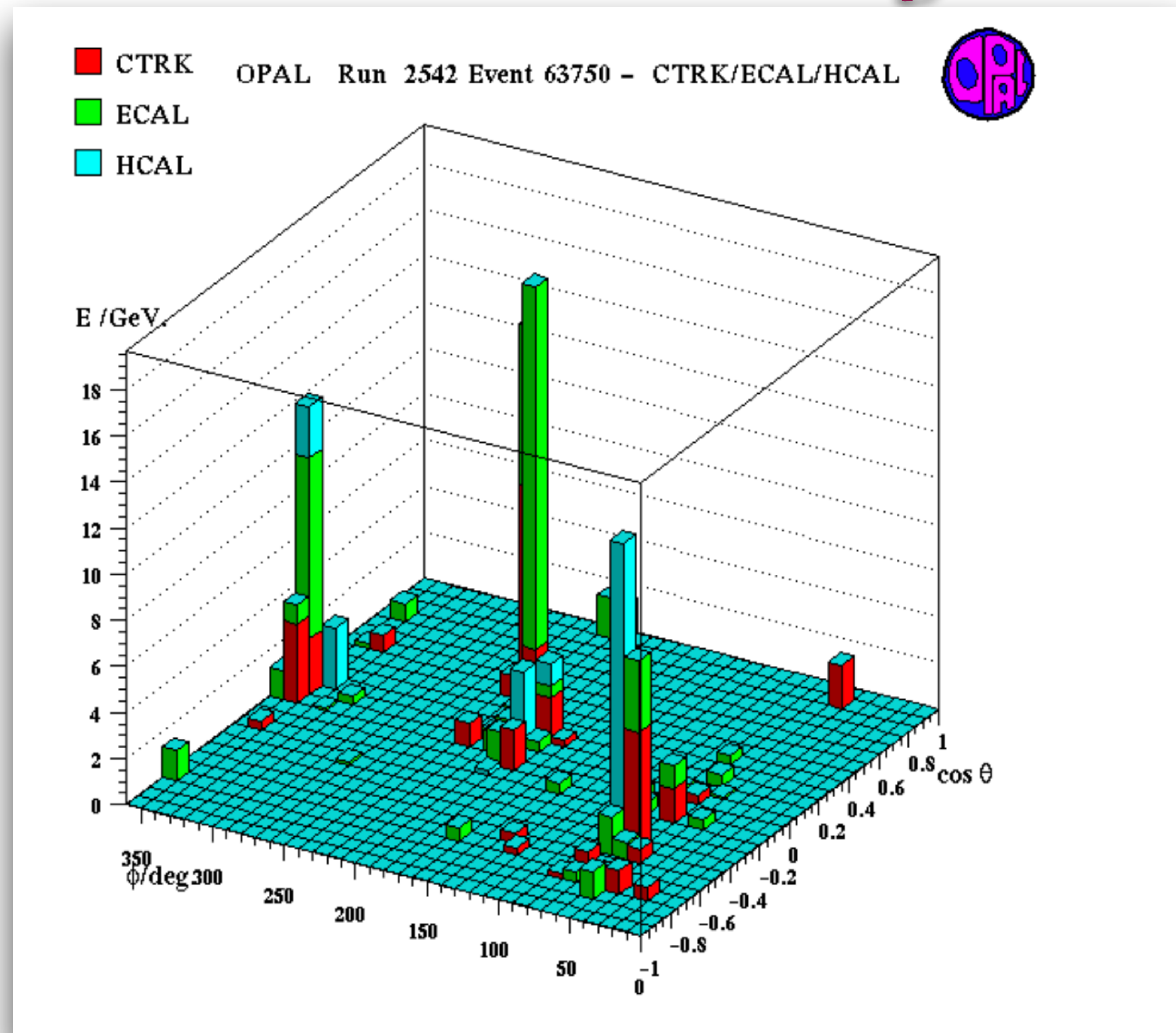


- First 3-jet event



Sterman-Weinberg procedure becomes complicated for multi-jet events

Typical $e^+e^- \longrightarrow$ 3-jets



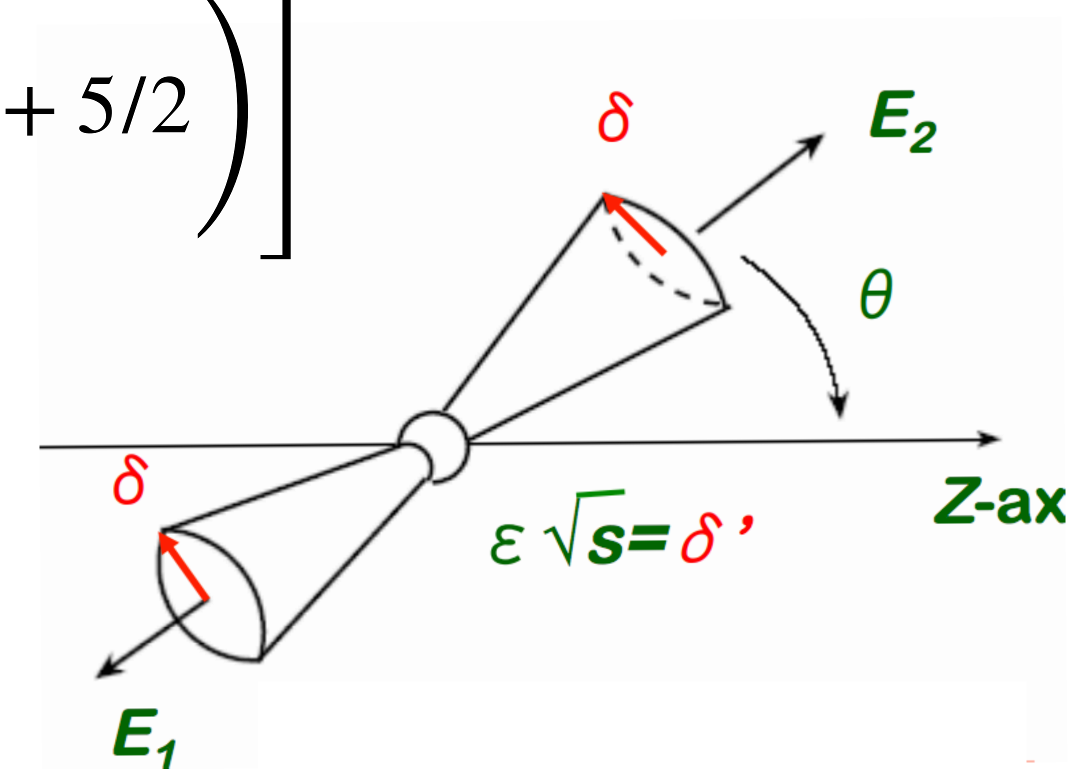
2-jet cross section in e⁺e⁻

- 2-jet in QCD

$$\sigma_{2-\text{jet}} = \sigma_{q\bar{q}}(1 + c_1\alpha_s + c_2\alpha_s^2) \qquad \text{with } c_1, c_2 \sim 1$$

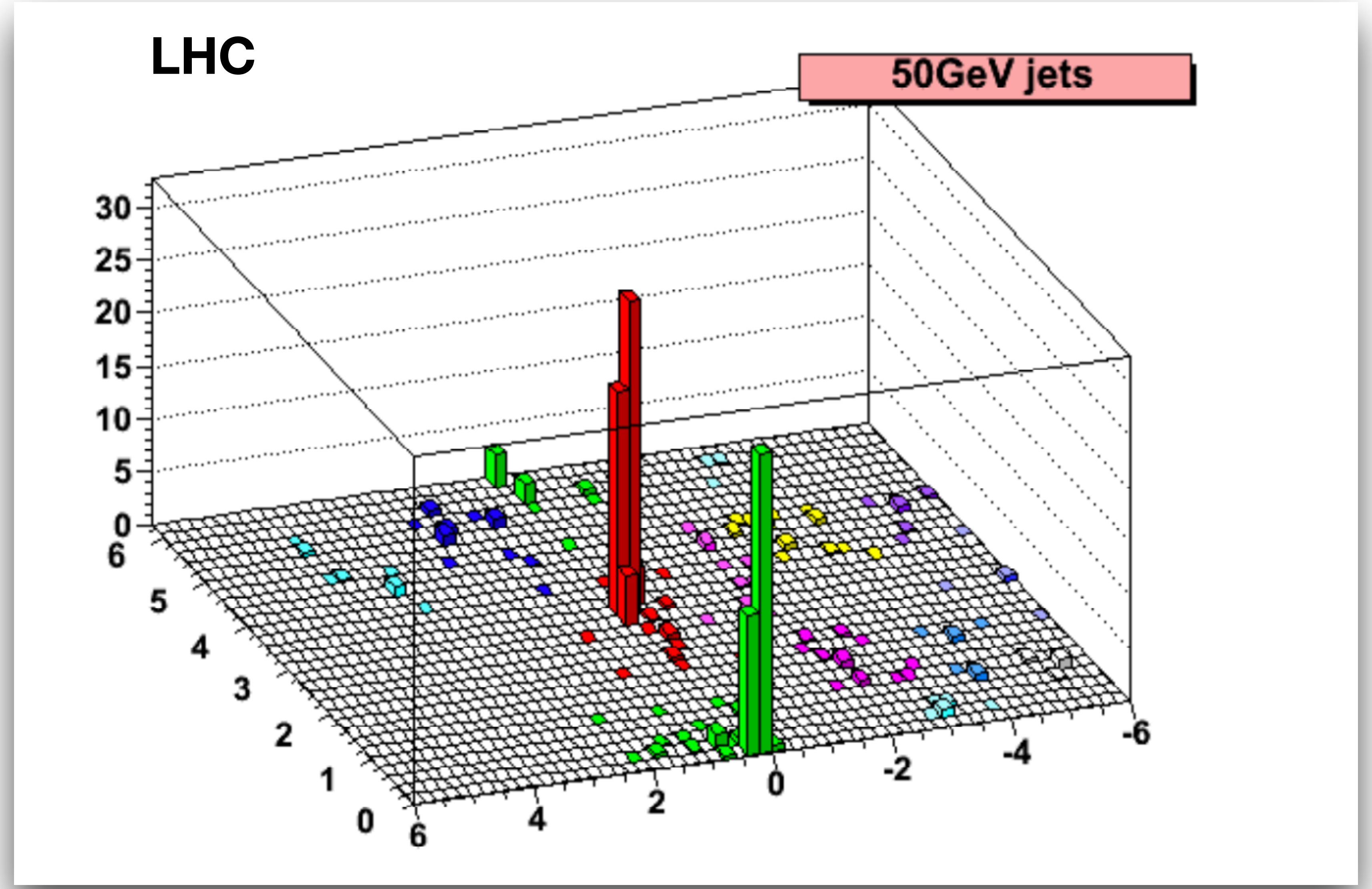
- Sterman-Weinberg jets

$$\sigma_{2-\text{jet}} = \sigma_{q\bar{q}} \left[1 - C_F \frac{\alpha_s}{\pi} \left(\ln\delta \ln\delta' + 3\ln\delta + \frac{\pi^2}{3} + 5/2 \right) \right]$$



Jets nowadays

pp⟶jets at the LHC



Around 300-400 particles

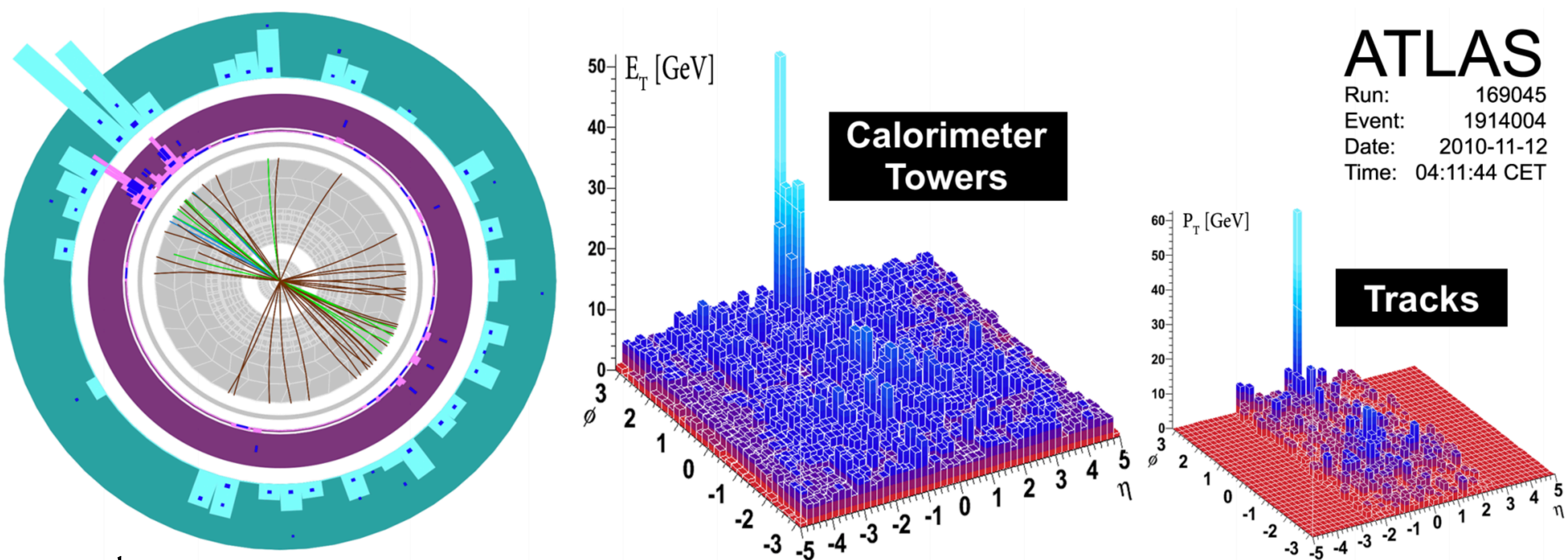
Some jet algorithms

SEQUENTIAL RECOMBINATION ALGORITHMS

- Define: $d_{ij} = \min\left(p_{T_i}^{2n}, p_{T_j}^{2n}\right)\frac{\Delta_{ij}^2}{R}$ $\quad \Delta_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$
  $d_{iB} = p_{T_i}^{2n}$
- Find: $\min(d_{ij}, d_{iB})$
  - If d_{iB}, remove i
  - If d_{ij}, combine i and j
- Go on until exhausting the list
- n=1 k_T, n=-1 anti-k_T, n=0 C/A

Jets in Heavy Ion collisions

Jet quenching



arXiv:1011.6182 [hep-ex]

Highly asymmetric dijet event

See Abhijit Majumder lectures

What happens if we are NOT inclusive?

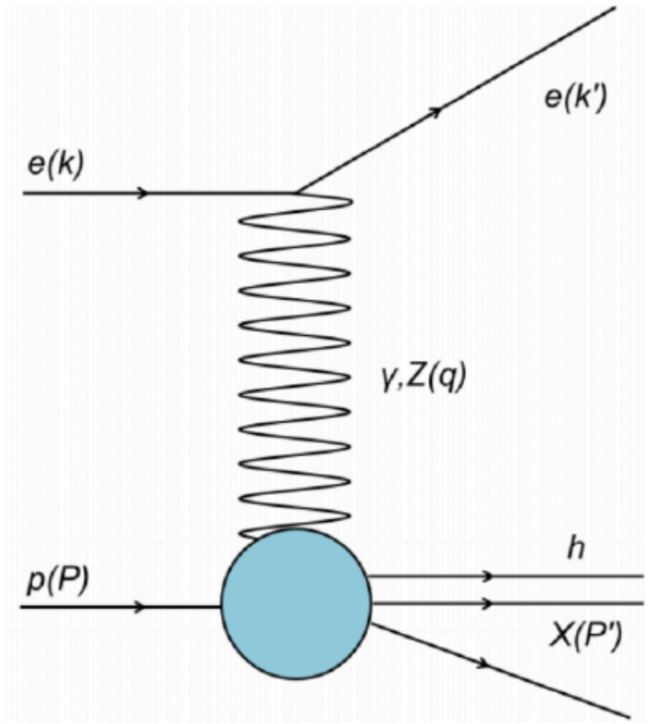
Exclusive processes

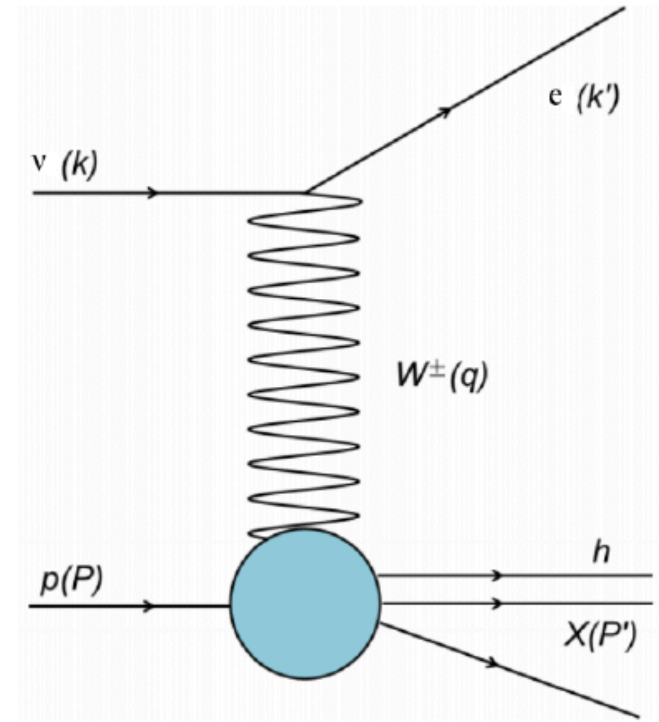
- How to test QCD in a reaction with identified hadrons?
- Hadronic scale is NON-perturbative
- Solution: Factorization
    - Isolate the calculable dynamics in terms of q and g (partonic cross sections
    - Quark and gluons are connect to hadrons via universal collinear distributions

Provide information about the partonic structure of hadrons

Deep Inelastic Scattering
(DIS)

DIS



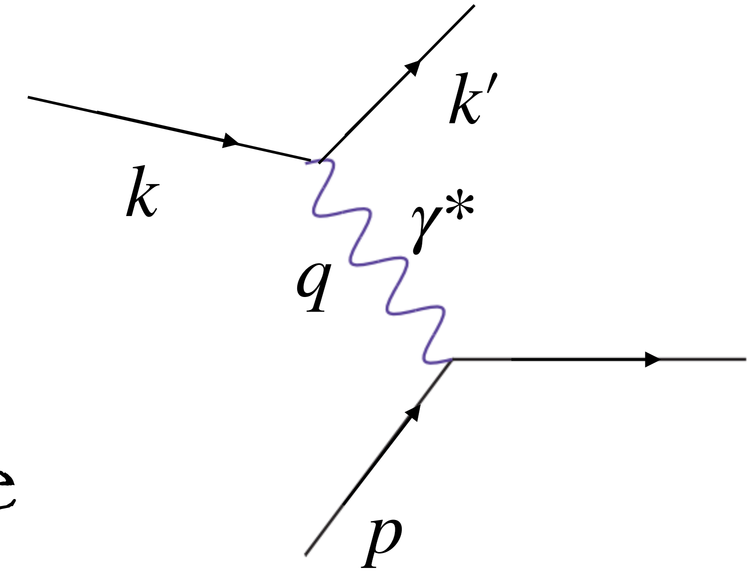


- DIS kinematic variables

$$Q^2 \equiv -q^2 \qquad x \equiv \frac{Q^2}{2p \cdot q} \qquad y \equiv \frac{p \cdot q}{p \cdot k}$$

- Q^2: gauge boson virtuality. Transverse resolution at which the proton structure is probed
- x: fraction of longitudinal momentum from the proton carried by the interacting quark
- y: momentum fraction lost by the electron (in proton rest frame)

DIS at LO



At LO, considering only the photon exchange

- Scattering amplitude: $$M = -\bar{u}(k')ie_q\gamma_\mu u(k)\frac{i}{q^2}g^{\mu\nu}\langle X|j_\nu(0)|P\rangle$$

$$\frac{d\sigma}{dxdQ^2} \propto |M|^2 \propto L_{\mu\nu}W^{\mu\nu}$$

Leptonic tensor (QED)

$$L_{\mu\nu} = 4e^2\left(k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu}k\cdot k'\right)$$

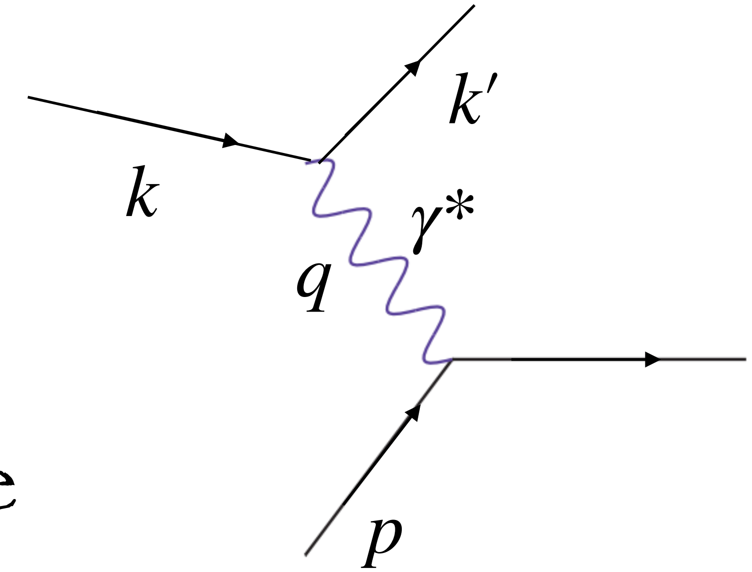
- Hadronic tensor:

$$W_{\mu\nu} = F_1\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) + \frac{F_2}{p\cdot q}\left(p_\mu - \frac{p\cdot q\, q_\mu}{q^2}\right)\left(p_\nu - \frac{p\cdot q\, q_\nu}{q^2}\right)$$
$$+ \frac{ig_1}{p\cdot q}\epsilon_{\mu\nu\lambda\sigma}q^\lambda s^\sigma + \frac{ig_2}{(p\cdot q)^2}\epsilon_{\mu\nu\lambda\sigma}q^\lambda\left(p\cdot q\, s^\sigma - s\cdot q\, p^\sigma\right)$$

Structure functions

Polarized lepton and target

DIS at LO



At LO, considering only the photon exchange

- EM Cross section at LO:

$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha_{em}^2}{xQ^4}\left[\left(1 - y + \frac{y^2}{2}\right) F_2^{em}(x,Q^2) - \frac{y^2}{2} F_L^{em}(x,Q^2)\right]$$

$$F_L^{em}(x,Q^2) = F_2^{em}(x,Q^2)\left(1 + \frac{4x^2M^2}{Q^2}\right) - 2xF_1^{em}(x,Q^2)$$

- In the Bjorken limit: $Q^2 \to \infty$, x finite

$$F_2(x) = 2xF_1(x)$$

Callan-Gross relation

Bjorken scaling

Quarks: spin 1/2

DIS and PDFs

- Beyond LO, the structure functions are not IRC safe
- Divergences (except collinear ones) cancel when performing dimensional regularization

$$F_2^{em}(x,Q^2) = \sum_i x e_i^2 \Bigg[f_i(x) + \frac{\alpha_s}{2\pi}\int_0^1 \frac{d\xi}{\xi} \sum_j \left(P_{ij}\left(\frac{x}{\xi}\right) \log\frac{Q^2}{\kappa^2} + W_{ij}(x) \right) f_j(\xi) + \mathcal{O}(\alpha_s^2) \Bigg] .$$

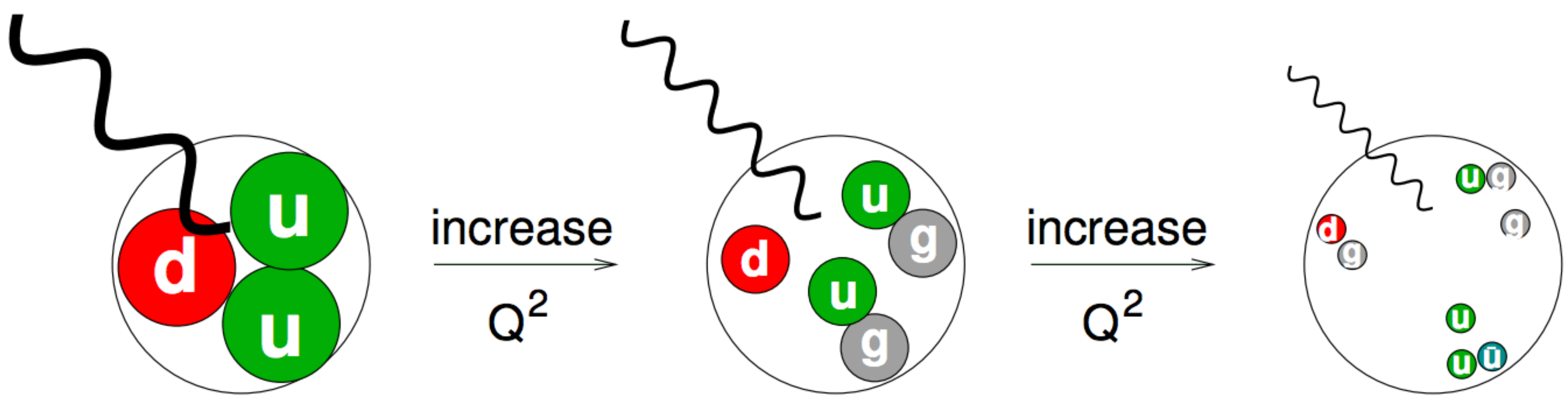
- Collinear divergences are absorbed into the parton distribution functions (PDFs) at the factorization scale μ_F

- PDFs are non-perturbative. But their evolution with respect to The factorization scale is perturbative (DGLAP)

DGLAP equation

- PDFs are non-perturbative. But universal
- Their evolution is perturbative: DGLAP evolution equation

$$\mu_F^2 \frac{\partial F_2(x_B, \mu^2)}{\partial \mu_F^2} = 0 \implies \boxed{\mu_F^2 \frac{\partial f_i(x, \mu_F^2)}{\partial \mu_F^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} P_{ij}\left(\frac{x}{z}\right) f_j\left(z, \mu_F^2\right)}$$



Dokshitzer-Gribov-Lipatov-Altarelli-Parisi

DGLAP equation

- DGLAP

$$\mu_F^2 \frac{\partial f_i(x,\mu_F^2)}{\partial \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z} \boxed{P_{ij}\left(\frac{x}{z}\right)} f_j\left(z,\mu_F^2\right)$$

Splitting functions (pQCD)

- Splitting functions at LO:

$$P_{qq} = C_F \frac{1+x^2}{(1-x)_+} + 2\delta(1-x) \quad ,$$

$$P_{qg} = \frac{1}{2}\left[x^2 + (1-x)^2\right] \quad ,$$

$$P_{gq} = C_F\left[\frac{1+(1-x)^2}{x}\right] \quad ,$$

$$P_{gg} = 2N_c\left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x)\right] + \left(\frac{11N_c - 2n_f}{6}\right)\delta(1-x)$$

$$\int_0^1 \frac{f(x)}{(1-x)_+}\,dx = \int_0^1 \frac{f(x)-f(1)}{1-x}\,dx$$

Known up to NNLO, NNNLO some limits known (work in progress)

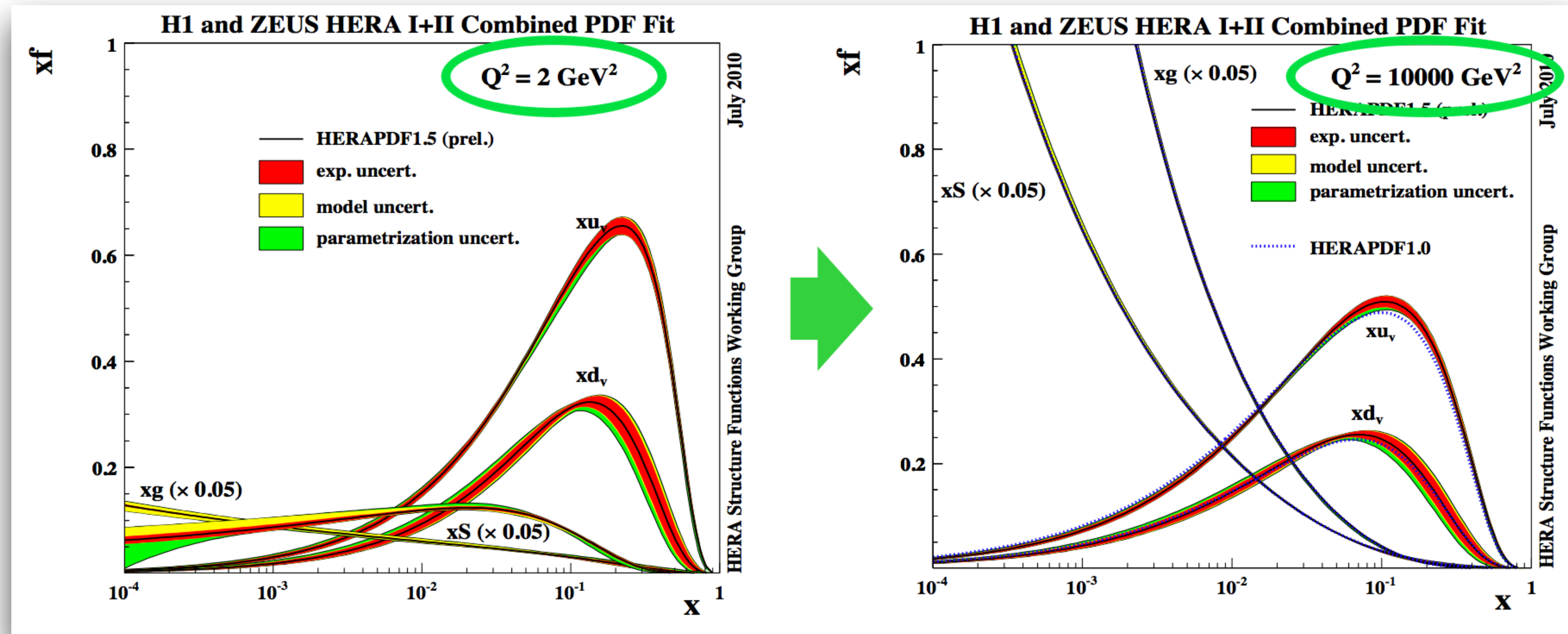
Global analyses

- PDFs are non perturbative. But universal
- They can be extracted from experiments: DIS, DY, SIDIS, jets, W/Z…

See Pavel Nadolsky lectures

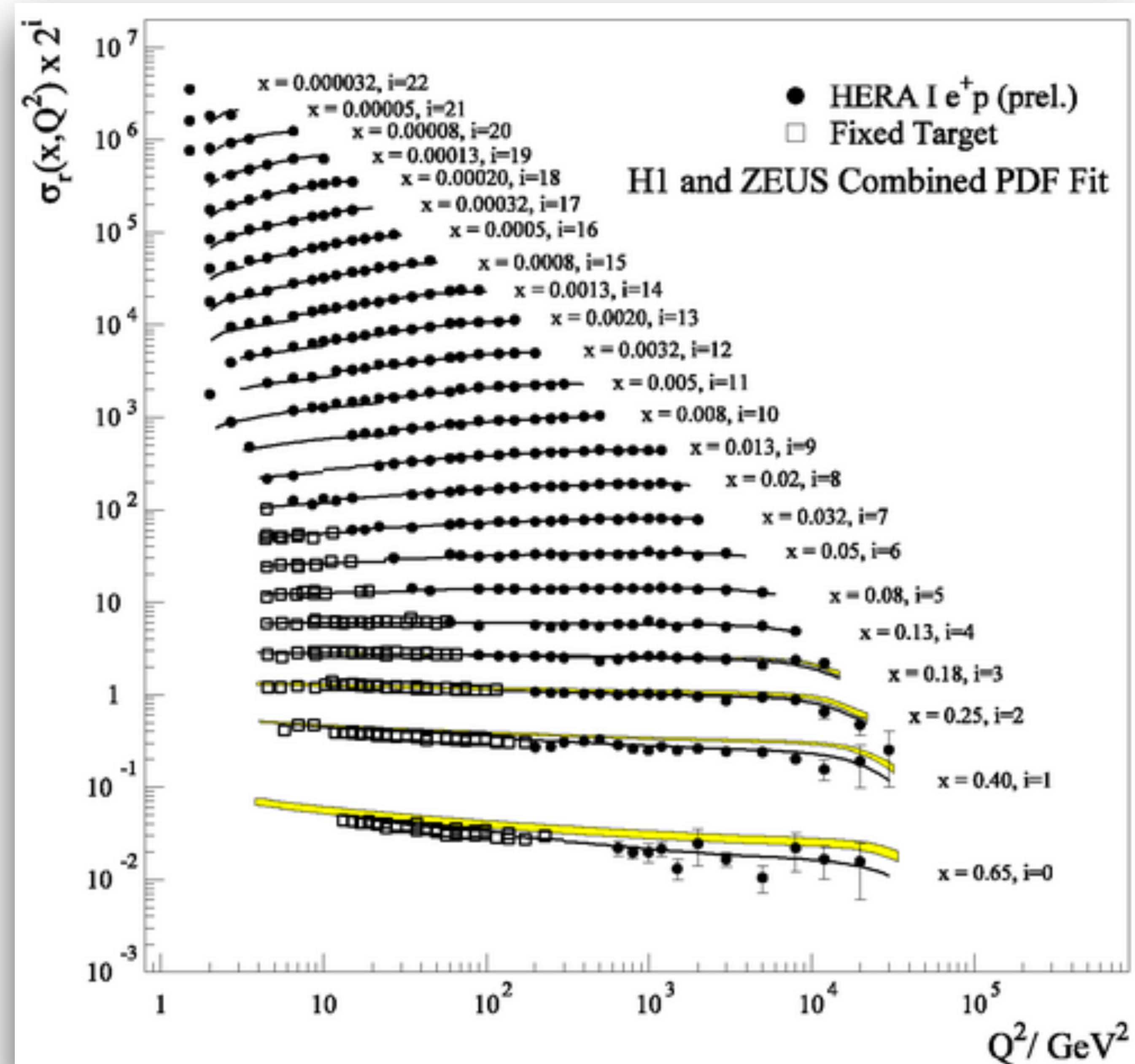
- How? Global analysis
  - Parametrize your PDFs at the initial scale
  - Evolve to the scale of the experiments with DGLAP
  - Fit to the data

PDFs from DIS



Q^2 dependence is pQCD

Scaling violations



Lines: predicted scale dependence from pQCD

Polarized DIS and the proton spin crisis

The proton spin crisis

- Originally all the proton spin was thought to be carried by its valence quarks
    - 2 valence quarks with spin parallel to proton spin and 1 valence quark with spin anti-parallel
    - Sea quarks and gluons arranged to have spin 0
    - Angular momentum assumed to be zero
- 1987 EMC experiment at CERN:

PLB 206, 364 (1988)

Overall spin coming from quarks compatible with 0 (large uncertainties)

How do we obtain info on the spin content of the proton?

Polarized DIS

- Lepton beam and target nucleon polarized in the longitudinal direction

- Asymmetries

$$A_{||} = \frac{d\sigma^{\rightrightarrows} - d\sigma^{\leftrightarrows}}{\boxed{d\sigma^{\rightrightarrows} + d\sigma^{\leftrightarrows}}}$$

2 times the spin averaged cross section

$$\frac{d^2\sigma^{\uparrow\downarrow}}{dxdy} - \frac{d^2\sigma^{\uparrow\uparrow}}{dxdy} = \frac{8\pi\alpha^2 ME}{Q^4}\left[\left(2y - y^2 - \frac{Mxy^2}{E}\right)2xg_1(x,Q^2) - \frac{4M}{E}x^2y\boxed{g_2(x,Q^2)}\right]$$

Suppressed

- Polarized structure function:

$$A_{||}(x,Q^2) \sim \frac{g_1(x,Q^2)}{F_1(x,Q^2)}$$

Polarized DIS

- The polarized structure function g_1:

$$
\begin{aligned}
g_1(x,Q^2) &= \tfrac{1}{2}\sum_q e_q^2 \int_x^1 \frac{d\xi}{\xi} \boxed{\left[\Delta q(x/\xi,Q^2)+\Delta\bar{q}(x/\xi,Q^2)\right]} \\
&\times \left\{\delta(1-\xi)+\frac{\alpha_S(Q^2)}{2\pi}\Delta C_q(\xi)+\ldots\right\} \\
&+ \left(\sum_q e_q^2\right)\int_x^1 \frac{d\xi}{\xi}\boxed{\Delta g(x/\xi,Q^2)}\left\{\frac{\alpha_S(Q^2)}{2\pi}\Delta C_g(\xi)+\ldots\right\}
\end{aligned}
$$

pQCD

- Helicity distributions (ΔPDFs): $\Delta q(x,Q^2) = q^{\uparrow}(x,Q^2) - q^{\downarrow}(x,Q^2)$

Spin parallel
to the proton's one

Spin anti-parallel
to the proton's one

$q = q^{\uparrow} + q^{\downarrow}$ Usual (spin averages) PDFs

Spin

See Barbara Pasquini lectures

- The first moment of a polarized PDF gives the net spin carried by that helicity distribution

SU(3) singlet: $$\Delta\Sigma = \int dx \left(\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}\right)(x, Q^2)$$

Gluon spin $$\Delta G = \int dx\, \Delta g\,(x, Q^2)$$

- Proton spin

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + \boxed{L_q + L_g}$$

Orbital angular momentum

Determination of Δq

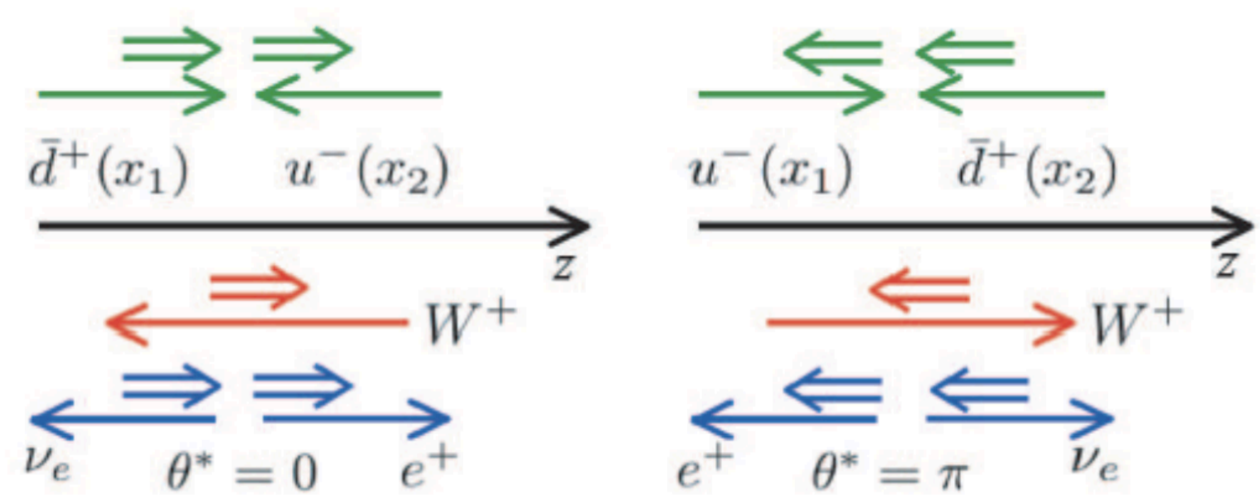
- Polarized DIS in only sensitive to the sum of quark and antiquark helicities. How do we separate quarks from antiquarks?

W production at polarized pp collisions at RHIC

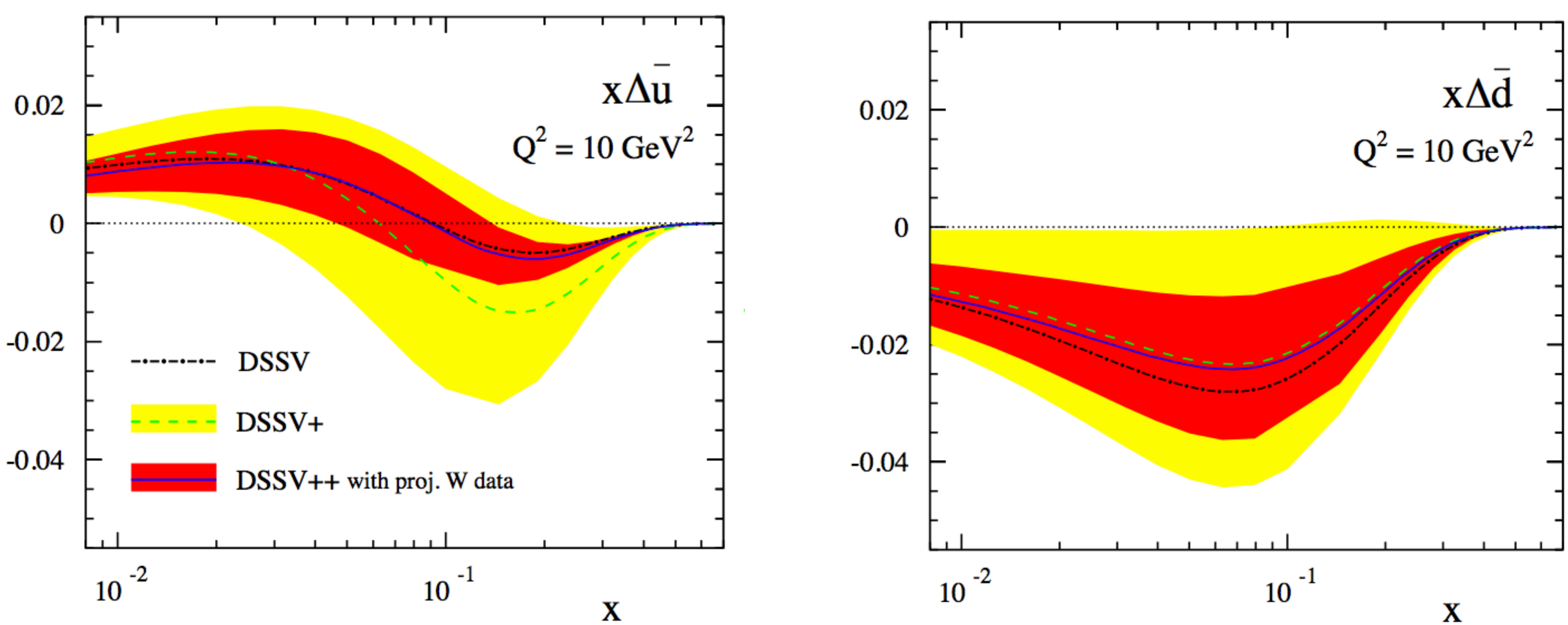
- W's are left-handed. They select left-handed quarks and right-handed antiquarks

Forward W+ (backward e+) $A_L^{W^+} \approx -\frac{\Delta u(x_1)}{u(x_1)} < 0$

Backward W+ (forward e+) $A_L^{W^+} \approx -\frac{\Delta \bar{d}(x_2)}{\bar{d}(x_2)} < 0$

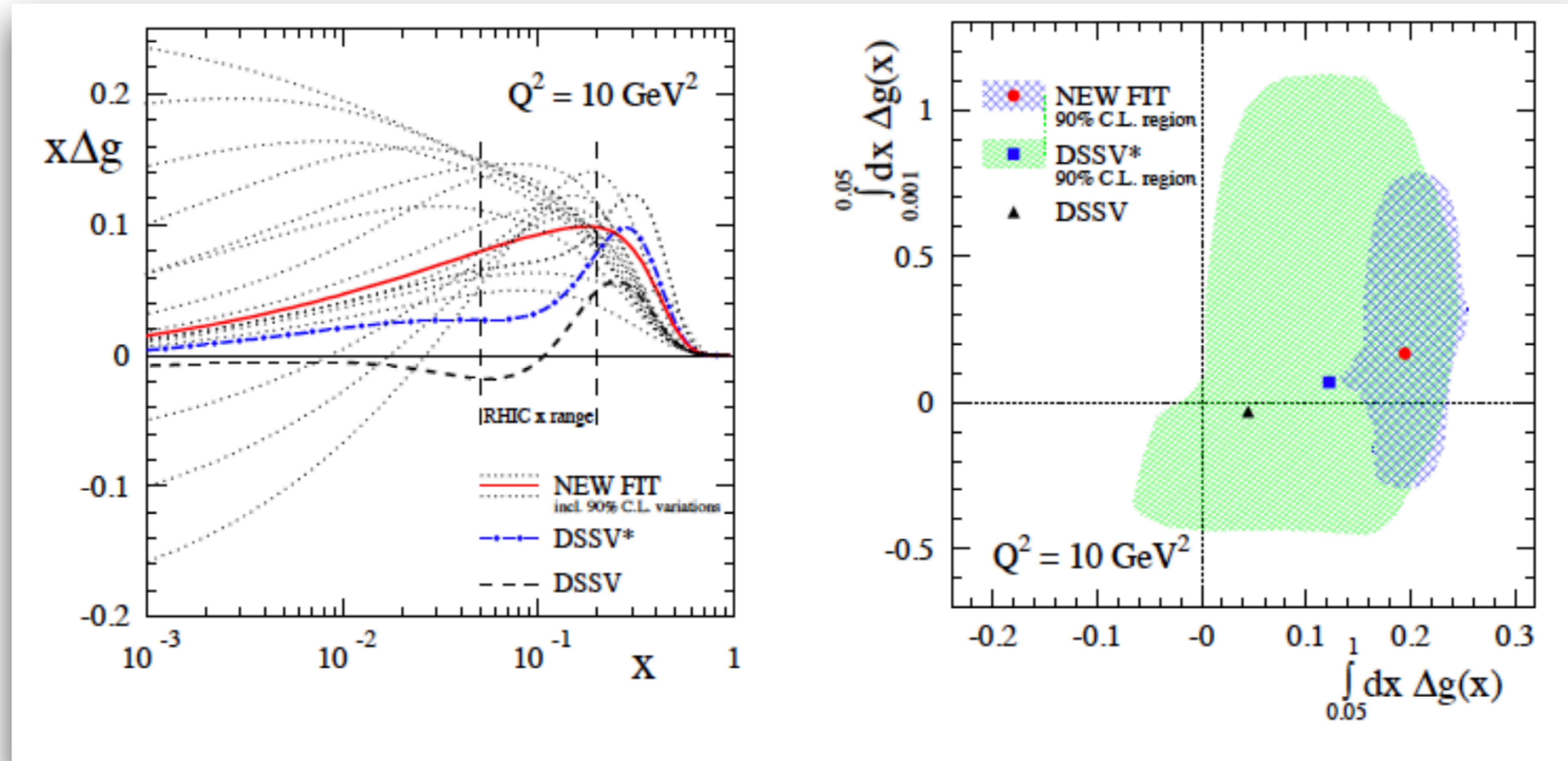


Sea quark polarization



DSSV global analysis

De Florian, Sassot, Stratmann and Vogelsang, Phys. Rev. Lett. 113, 012001 (2014)



Red line: RHIC data on pion and jets production from 2009 $\Delta G = \int dx\, \Delta g \sim 0.20$

The proton spin crisis

Nowadays

See Barbara Pasquini lectures

- Proton spin:

$$\boxed{\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g}$$

- Quark helicity:

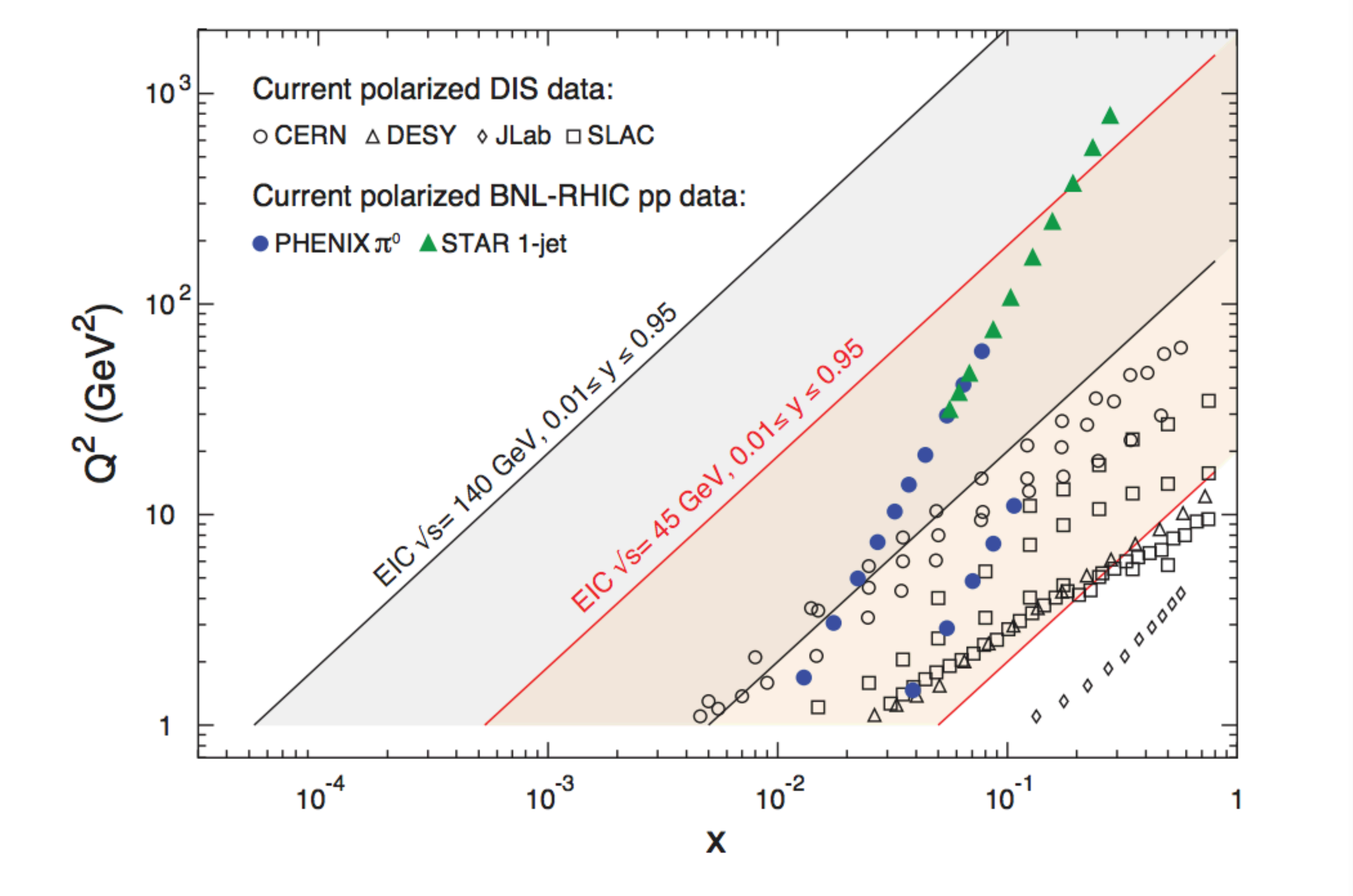
$$\Delta\Sigma = \int dx \left(\Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} + \Delta s + \Delta \bar{s}\right) \sim 0.30$$

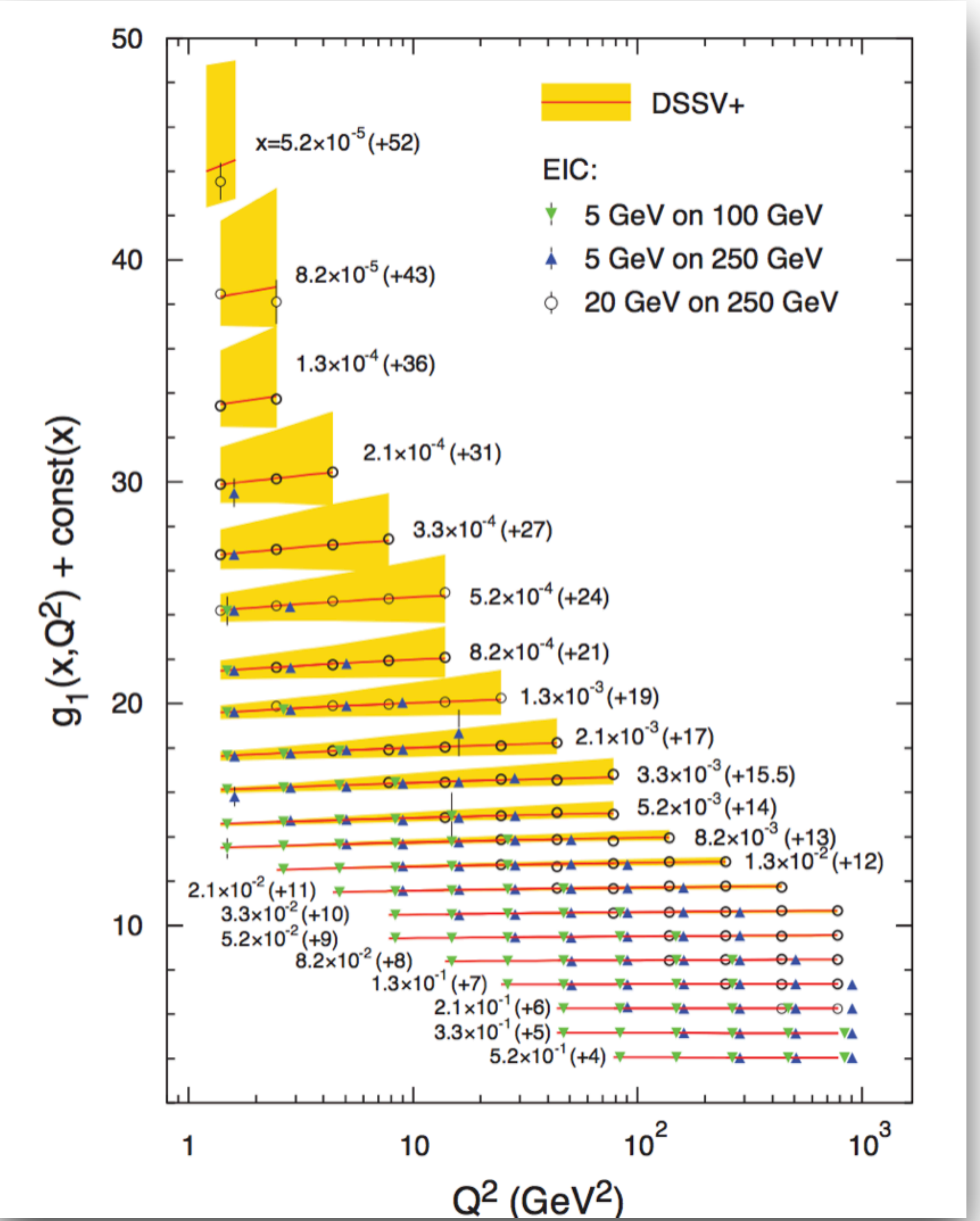
- Gluon helicity

$$\Delta G = \int dx\, \Delta g \sim 0.20$$

- Orbital angular momentum of quarks and gluons: little known

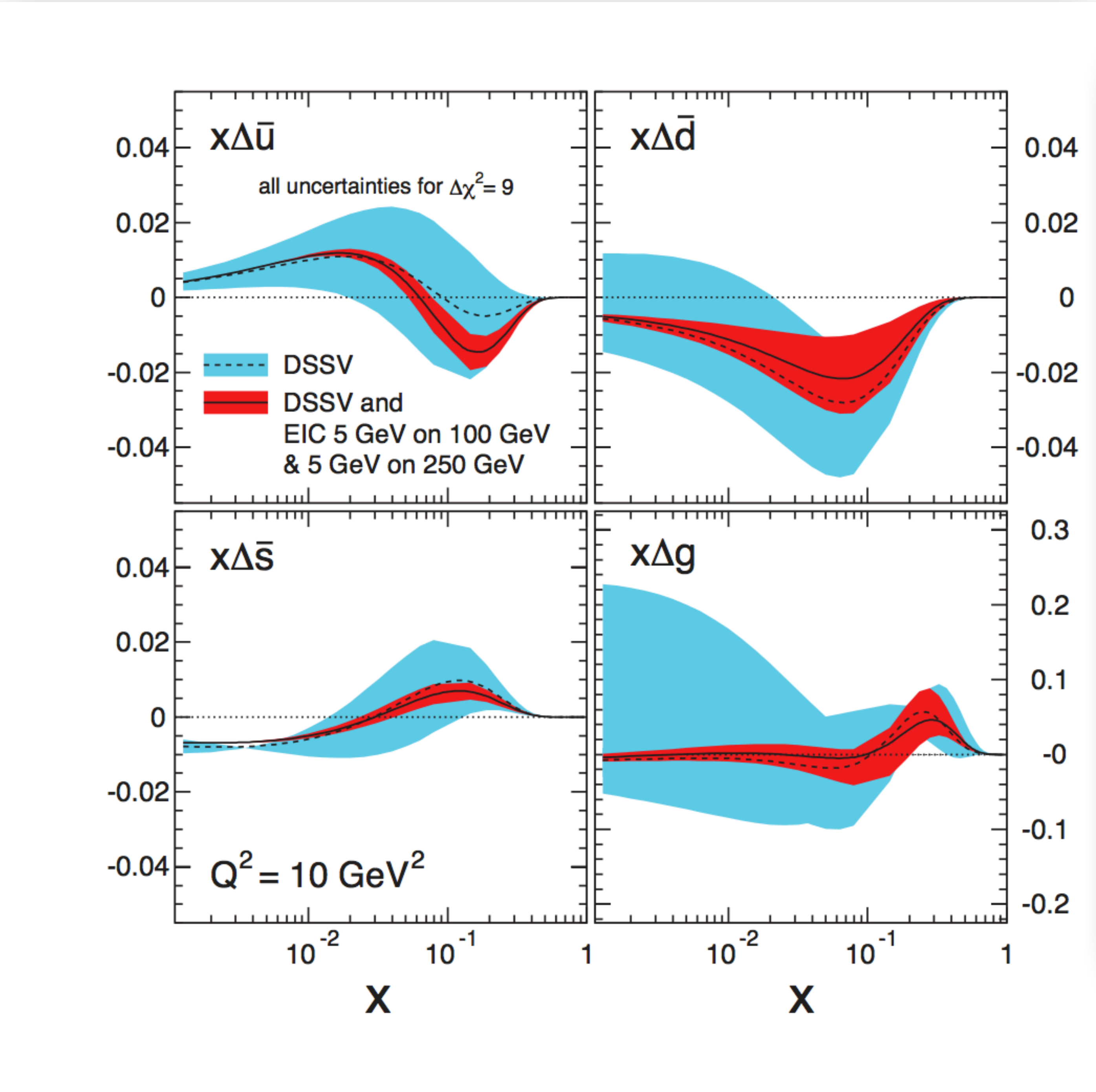
The future: the EIC

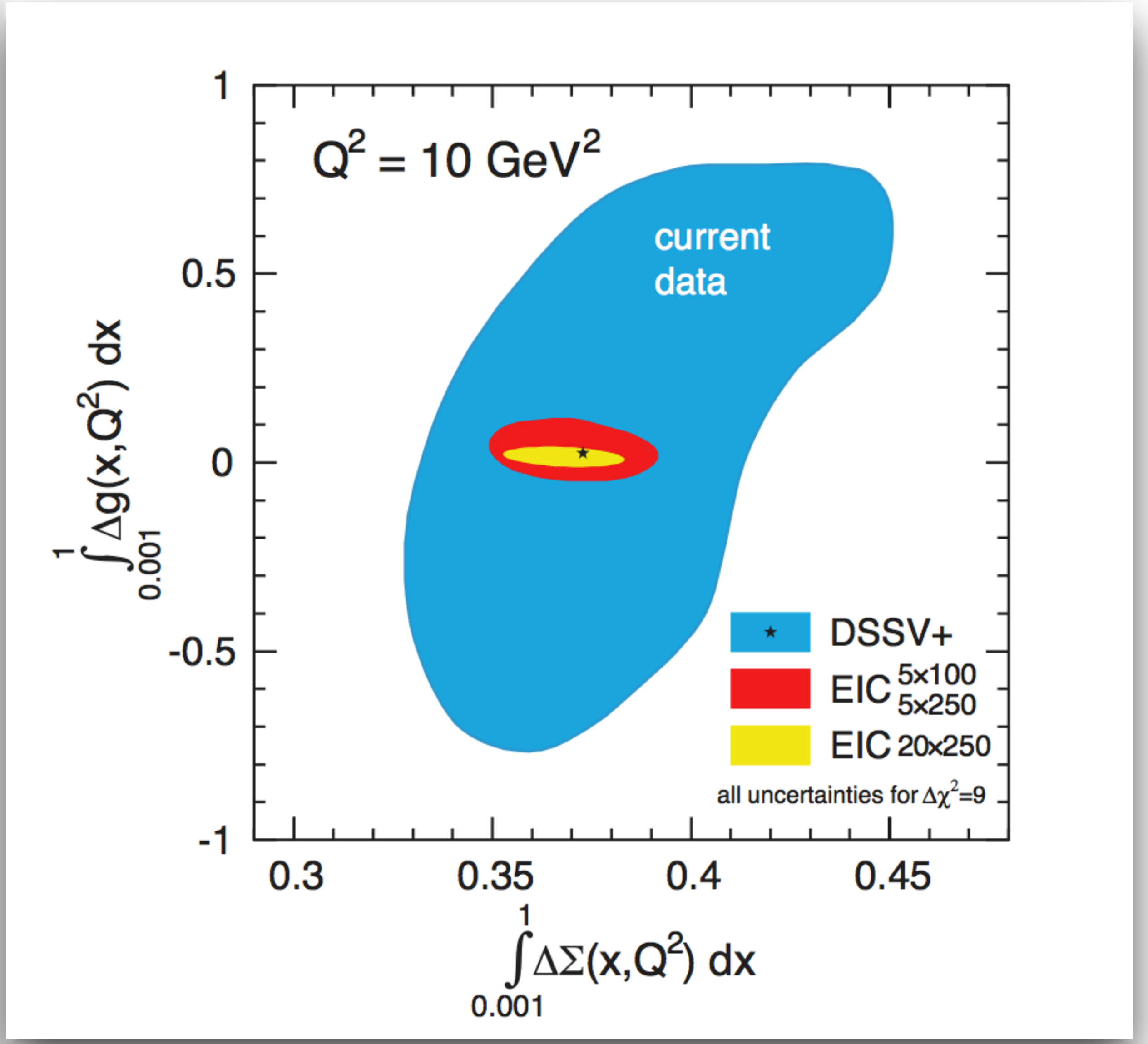




Plots from the EIC White paper

The future: the EIC





Plots from the EIC White paper

Summary

- QCD is very successful predicting high energy experimental data
- QCD factorization is the tool to computed cross sections with identified hadrons
- We do not know much yet about hadron structure
- TMDS, GPDS give insight on the 3D hadron structure. The EIC will make them more accessible!

See Barbara Pasquini lectures

Thanks!

Backup

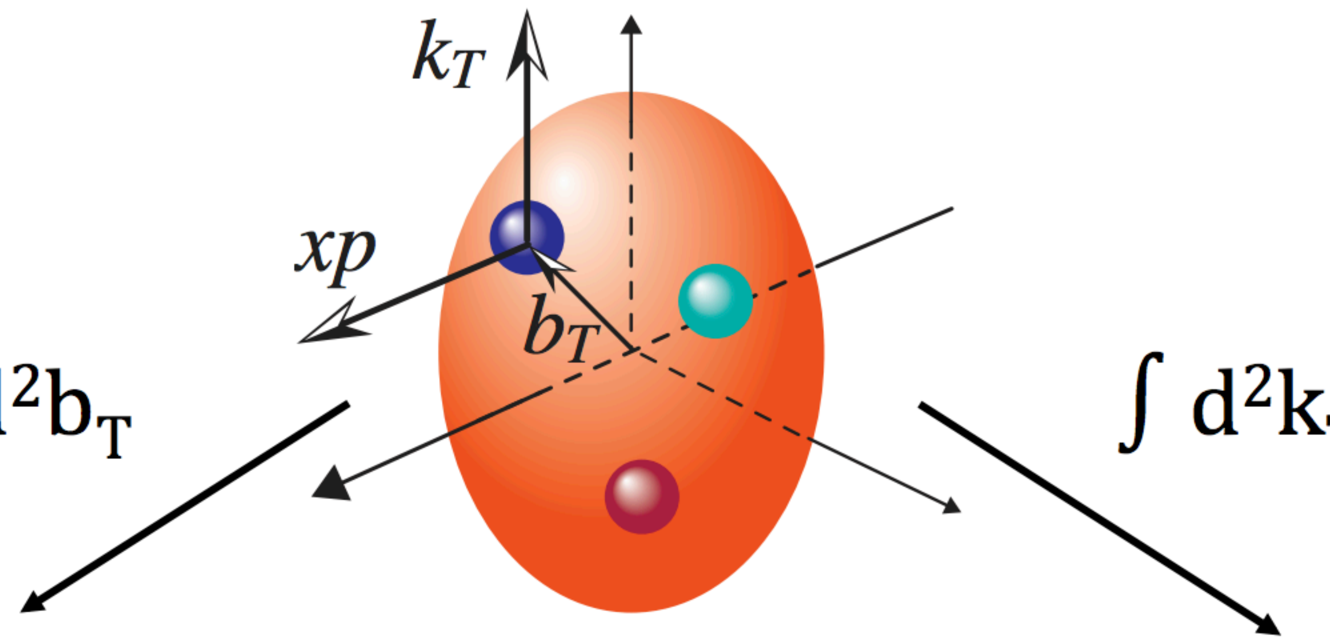
3D proton structure

3D partonic structure

Momentum Space

TMDs

Confined motion



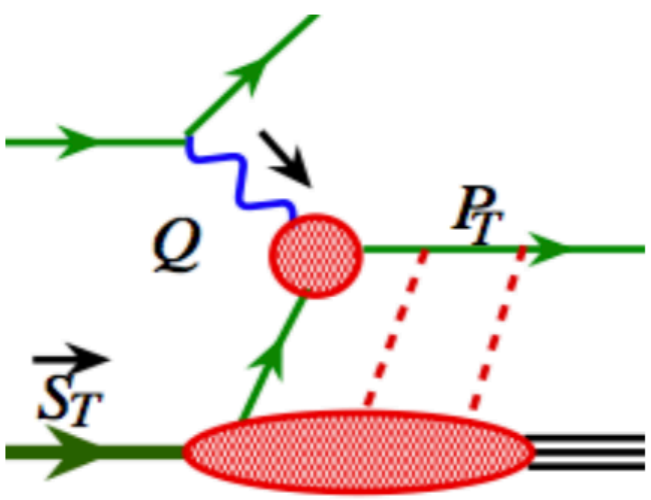
$\int d^2b_T$ $\int d^2k_T$

Coordinate Space

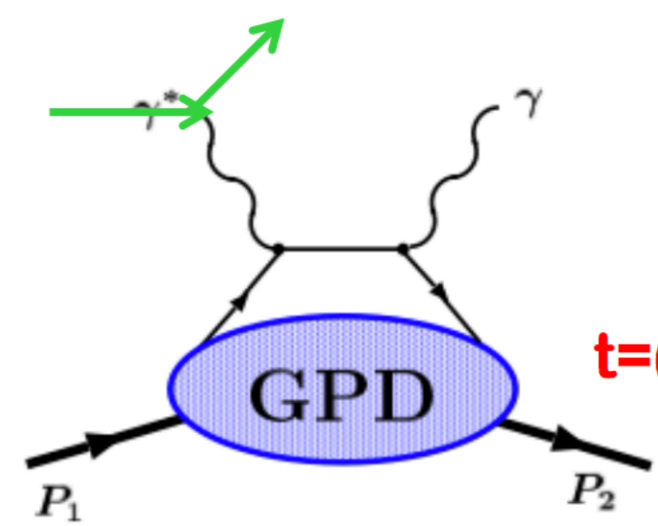
GPDs

Spatial distribution

$f(x,k_T)$ **Two-scales observables** $f(x,b_T)$



SIDIS: $Q \gg P_T$

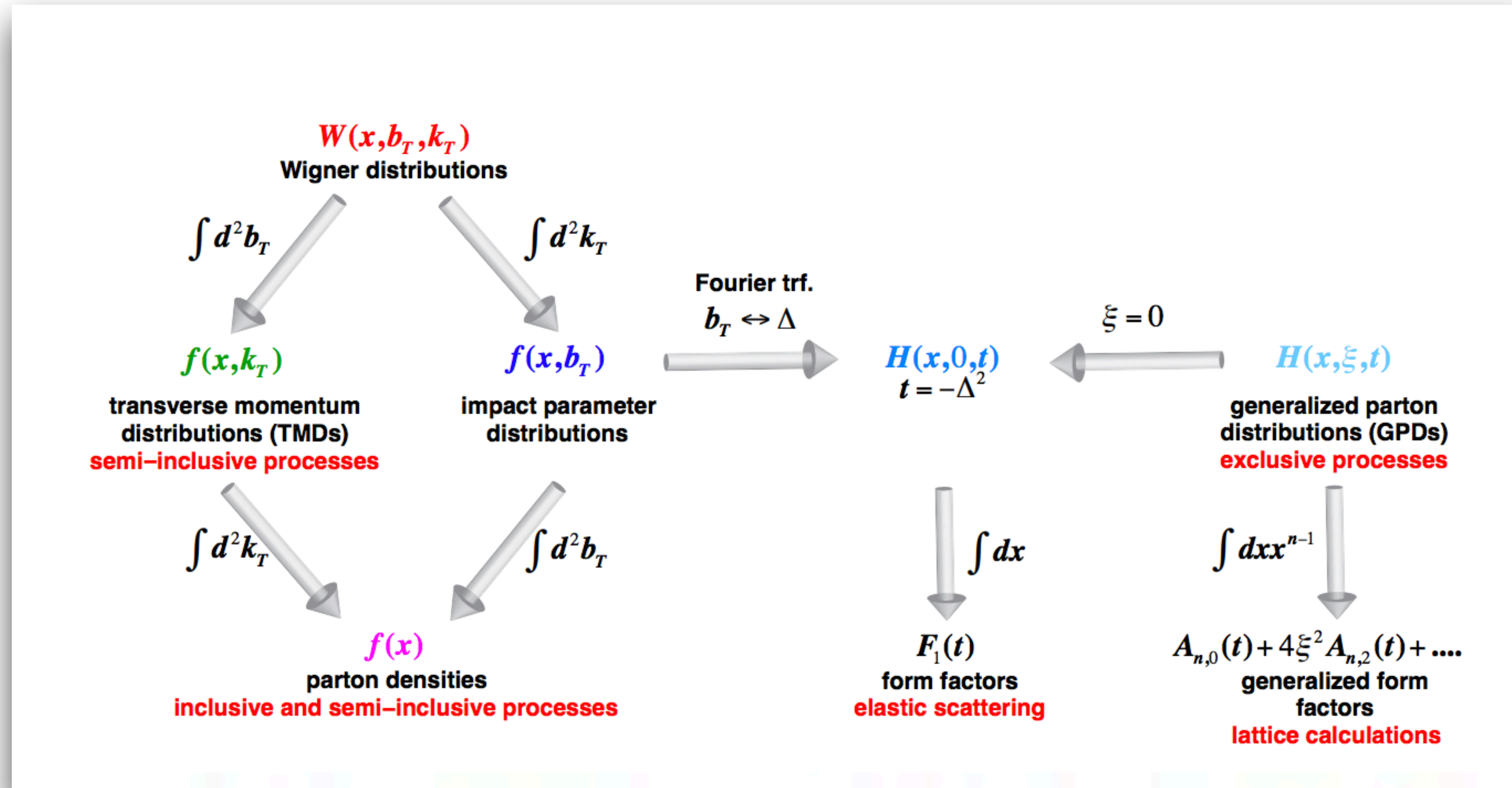


+

$t=(p_1-p_2)^2$

DVCS: $Q^2 \gg |t|$

Relations

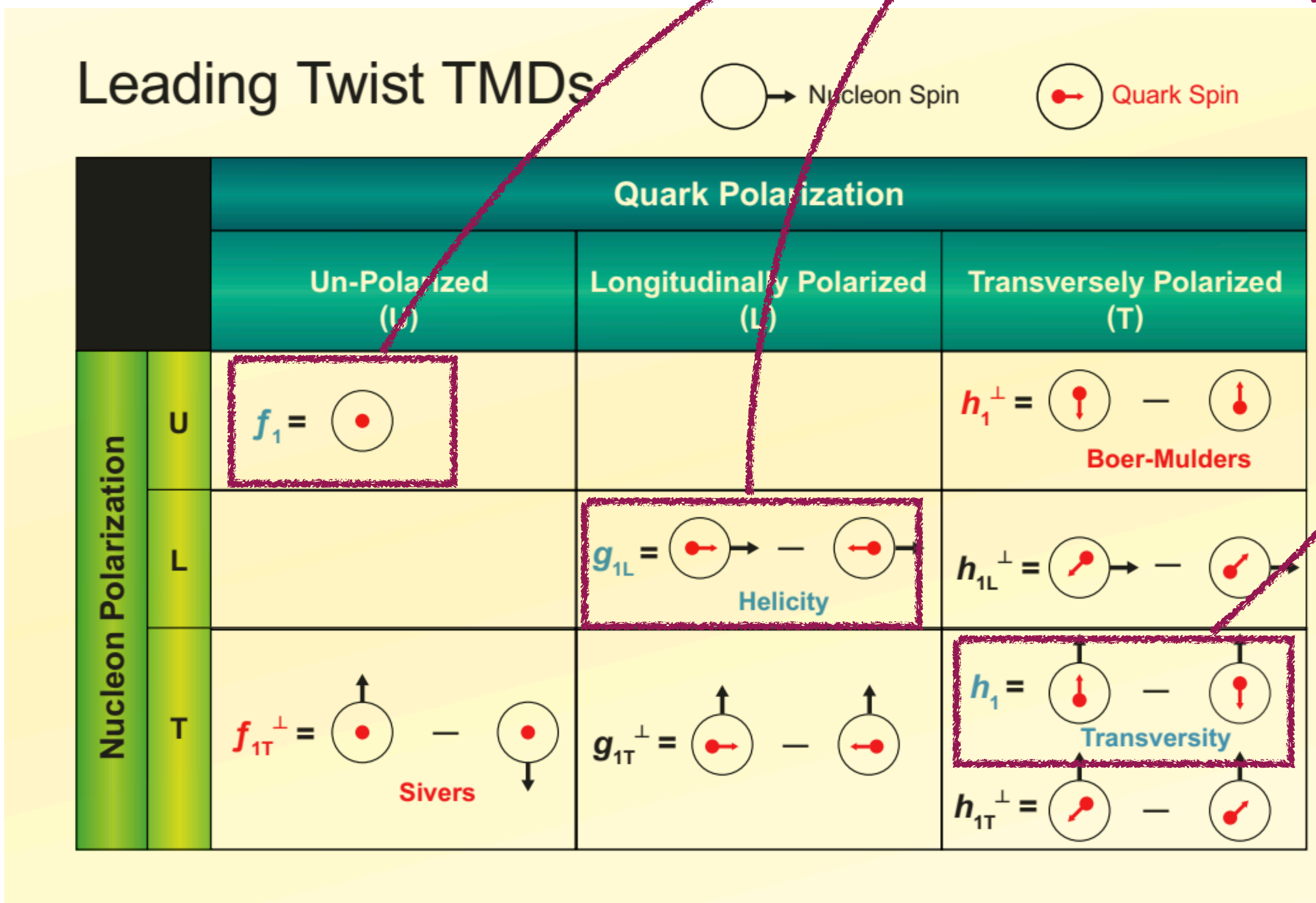


$W(x, b_T, k_T)$
Wigner distributions
$\int d^2 b_T$
$\int d^2 k_T$
$f(x, k_T)$
transverse momentum distributions (TMDs)
semi–inclusive processes
$f(x, b_T)$
impact parameter distributions
Fourier trf.
$b_T \leftrightarrow \Delta$
$H(x, 0, t)$
$t = -\Delta^2$
$\xi = 0$
$H(x, \xi, t)$
generalized parton distributions (GPDs)
exclusive processes
$\int d^2 k_T$
$\int d^2 b_T$
$\int dx$
$\int dx x^{n-1}$
$f(x)$
parton densities
inclusive and semi–inclusive processes
$F_1(t)$
form factors
elastic scattering
$A_{n,0}(t) + 4\xi^2 A_{n,2}(t) + \ldots$
generalized form factors
lattice calculations

TMDs

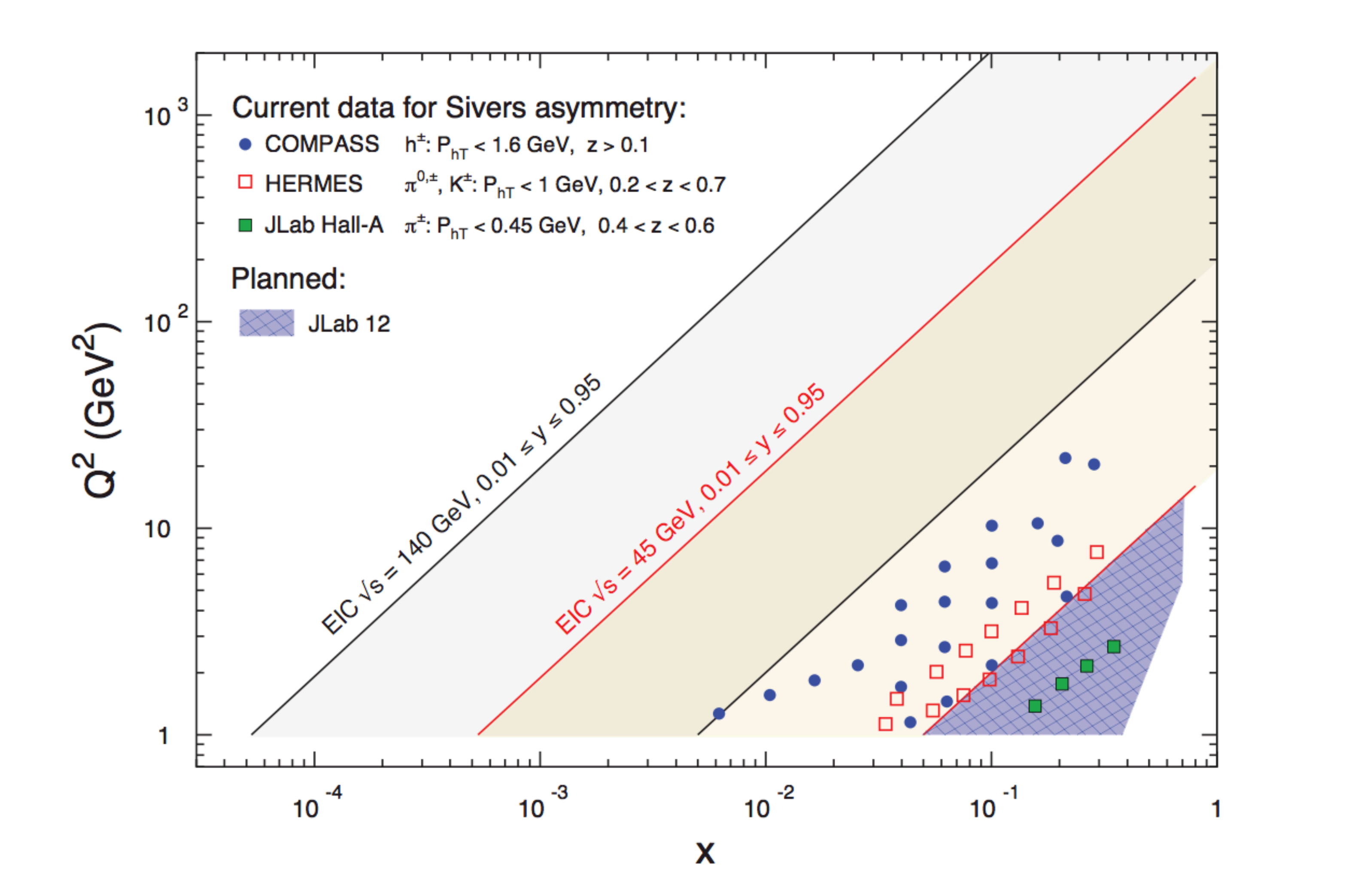
Quark TMDs

Natural extension of their associate collinear distributions

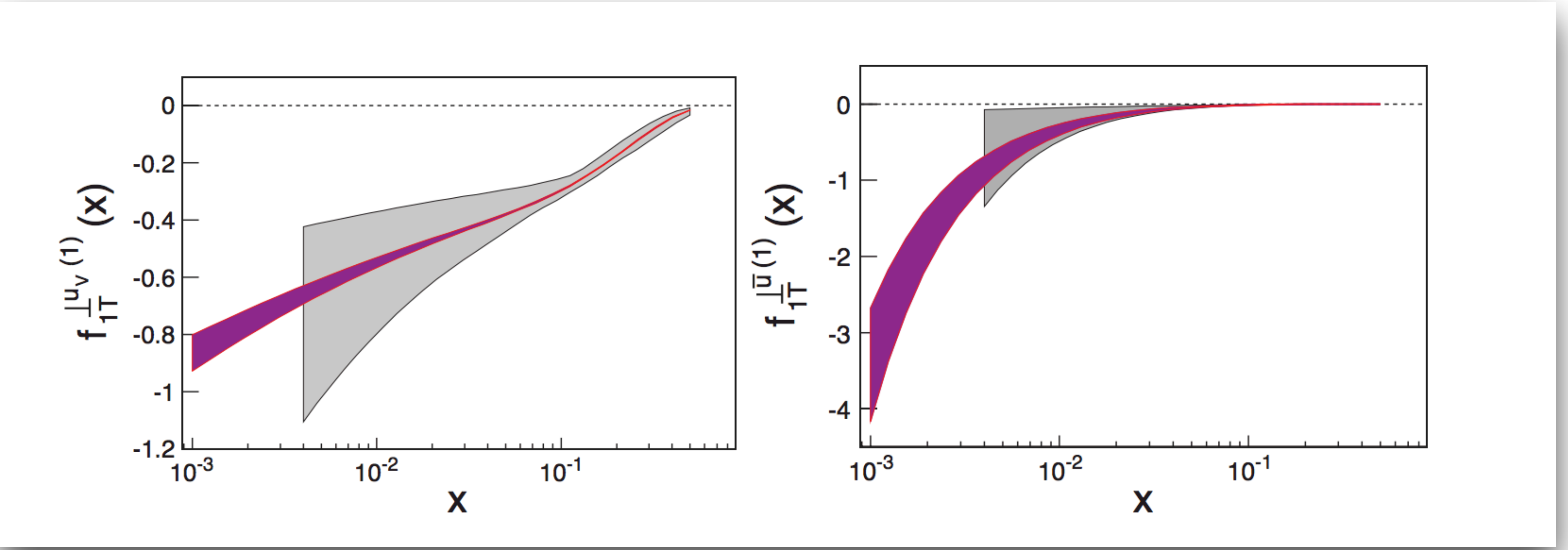


Similar for gluons

TMDS at the EIC



TMDS at the EIC



EIC with c.o.m energy of 45 GeV and luminosity of 10 fb^{-1}

GPDs at the EIC