Quasi-GPDs for quarks : Model results & beyond

Shohini Bhattacharya
Temple University

Theory Seminar
Jefferson Lab
July 15, 2019

Based on:
1) S. Bhattacharya, C. Cocuzza, A. Metz / Phys. Lett. B788, 453 (2019)
2) S. Bhattacharya, C. Cocuzza, A. Metz / arXiv: 1903.05721 (2018)

Outline

- Background : Non-perturbative functions

- Definition of (Quasi-) PDFs & (Quasi-) GPDs

- Analytical results of Quasi-GPDs in Scalar Diquark Model

- Numerical Results
  - Quasi-PDFs
  - Quasi-GPDs
  - Sensitivity of results to model parameters

- Model-independent results
  - Moments of quasi-distributions & Ji's spin-sum rule
  - (Re-) definition of quasi-distributions & ξ -symmetry of quasi-GPDs

- Summary

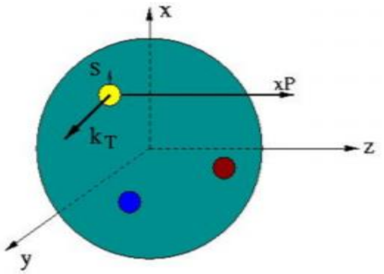
Background : Non-perturbative functions

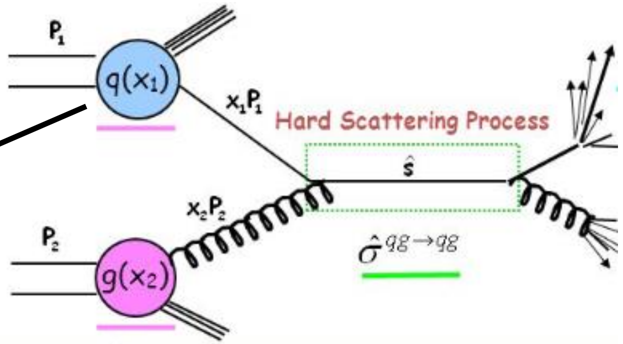
Some objectives :

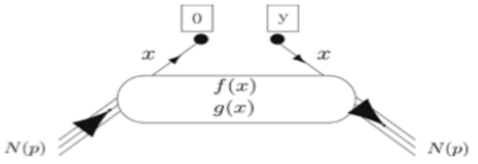
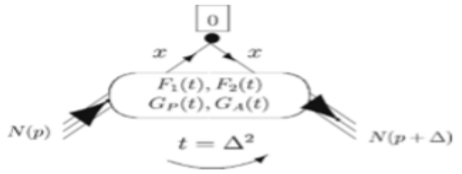
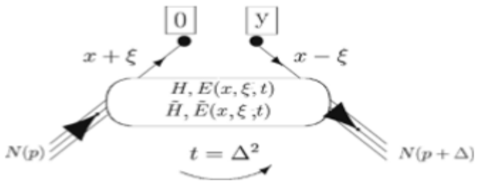
1) How are the partons distributed spatially and in the momentum space inside the nucleon?
2) How do they contribute to the properties of nucleon - **spin**?

Scattering experiments: Inclusive, Exclusive, Semi-inclusive

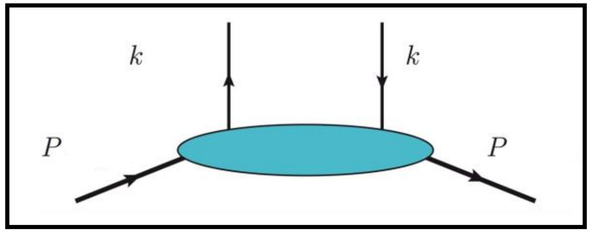
QCD factorization: **Hard part** (point-like, perturbatively calculatable), **Soft part** (non-perturbative QCD)





QCD MATRIX ELEMENT (Diagram)	FEATURES OF THE ELEMENT	PROCESS	NON-PERT. FUNCTIONS
	$\langle p\vert\bar{\psi}_q(0)\mathcal{O}\psi_q(y)\vert p\rangle$ • Non-local • Forward	DIS	**PDFs** $f_1(x)$ $g_1(x)$
	$\langle p'\vert\bar{\psi}_q(0)\mathcal{O}\psi_q(0)\vert p\rangle$ • Local • Off-forward	ELASTIC SCATTERING	**FFs** $F_1(t)$ $F_2(t)$ $G_P(t)$ $G_A(t)$
	$\langle p'\vert\bar{\psi}_q(0)\mathcal{O}\psi_q(y)\vert p\rangle$ • Non-local • Off-forward	EXCLUSIVE SCATTERING	**GPDs** $H(x,\xi,t)$ $\tilde{H}(x,\xi,t)$ $E(x,\xi,t)$ $\tilde{E}(x,\xi,t)$

Definition of (Quasi-) PDFs



$$\lim_{P^3\to\infty} \text{Quasi-PDF} \to \text{Standard PDF}$$

Light-cone (standard) correlator $-1 \le x \le 1$

$$F^{[\Gamma]}(x) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z} \times \langle p|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p\rangle\Big|_{z^+=\vec{z}_\perp=0}$$

- **Time dependence :** $z^0 = \frac{1}{\sqrt{2}}(z^+ + z^-) = \frac{1}{\sqrt{2}}z^-$
- **<u>Cannot</u> be computed on Euclidean lattice**
- **Parameterization :** $\gamma^+ \longrightarrow f_1(x)$
  $\gamma^+\gamma_5 \longrightarrow g_1(x)$
  $i\sigma^{j+}\gamma_5 \longrightarrow h_1(x)$
- **Kinematical variable :**

$$\boxed{x = \frac{k^+}{P^+}}$$

Correlator for quasi-PDFs (Ji, 2013) $-\infty \le x \le \infty$

$$F_{\mathrm{Q}}^{[\Gamma]}(x;P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik\cdot z} \times \langle p|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}_{\mathrm{Q}}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p\rangle\Big|_{z^0=\vec{z}_\perp=0}$$

- **Non-local correlator depending on position** z^3
- **<u>Can</u> be computed on Euclidean lattice**
- **Parameterization :** $\gamma^{0/3} \longrightarrow f_{1,\mathrm{Q}(0/3)}$
  $\gamma^{0/3}\gamma_5 \longrightarrow g_{1,\mathrm{Q}(0/3)}$
  $i\sigma^{j0/3}\gamma_5 \longrightarrow h_{1,\mathrm{Q}(0/3)}$
- **Kinematical variable :**

$x = \frac{k^3}{P^3} \neq \frac{k^+}{P^+}$	P^3

More Concepts

- **Matching**

  a) **UV disparities between quasi & standard treated via matching**

  b) **Matching formula :** (scale-dependence omitted)

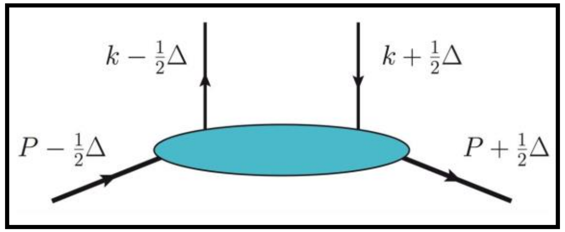
$$f_{1,\mathrm{Q}}(x;P^3) = \int_{-1}^{+1} \frac{dy}{|y|} C\left(\frac{x}{y}\right) f_1(y) + \mathcal{O}\left(\frac{M^2}{(P^3)^2}, \frac{\Lambda^2_{\mathrm{QCD}}}{(P^3)^2}\right)$$

  c) **Matching coefficient C is presently known up to 1-loop order**
  (Xiong, Ji, Zhang, Zhao, 2013/ Stewart, Zhao, 2017/ Izubuchi, Ji, Jin, Stewart, Zhao, 2018/ …)

- **Choice of Dirac matrices**

PDF	Γ	Reason
f_1	γ^0	**Better behaved w.r.t. power corrections** (Radyushkin, 2016) **No mixing under renormalization** (Constantinou, Panagopoulos, 2017)
g_1	$\gamma^3\gamma_5$	**No mixing under renormalization** (Constantinou, Panagopoulos, 2017)
h_1	$i\sigma^{j0}\gamma_5$	**No mixing under renormalization** (Constantinou, Panagopoulos, 2017)

Definition of (Quasi-) GPDs



$$\lim_{P^3\to\infty} \text{Quasi-GPD} \to \text{Standard GPD}$$

Light-cone (standard) correlator $-1 \leq x \leq 1$

$$F^{[\Gamma]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z} \times \langle p';\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p;\lambda\rangle\Big|_{z^+=\vec{z}_\perp=0}$$

- **Time dependence :** $z^0 = \frac{1}{\sqrt{2}}(z^+ + z^-) = \frac{1}{\sqrt{2}} z^-$
- **<u>Cannot</u> be computed on Euclidean lattice**
- **Parameterization :**
  $\gamma^+ \longrightarrow H \quad E$
  $\gamma^+\gamma_5 \longrightarrow \tilde{H} \quad \tilde{E}$
  $i\sigma^{j+}\gamma_5 \longrightarrow H_T \quad E_T \quad \tilde{H}_T \quad \tilde{E}_T$
- **Kinematical variables :**

$x = \frac{k^+}{P^+}$	$\xi = -\frac{\Delta^+}{2P^+}$	$t = \Delta^2 = -\frac{(4\xi^2 M^2 + \vec{\Delta}_\perp^2)}{1-\xi^2}$

Correlator for quasi-GPDs (Ji, 2013) $-\infty \leq x \leq \infty$

$$F_{\mathrm{Q}}^{[\Gamma]}(x,\Delta;\lambda,\lambda';P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik\cdot z} \times \langle p',\lambda'|\bar{\psi}(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}_{\mathrm{Q}}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p,\lambda\rangle\Big|_{z^0=\vec{z}_\perp=0}$$

- **Non-local correlator depending on position** z^3
- **<u>Can</u> be computed on Euclidean lattice**
- **Parameterization :**
  $\gamma^{0/3} \longrightarrow H_{\mathrm{Q}(0/3)} \quad E_{\mathrm{Q}(0/3)}$
  $\gamma^{0/3}\gamma_5 \longrightarrow \tilde{H}_{\mathrm{Q}(0/3)} \quad \tilde{E}_{\mathrm{Q}(0/3)}$
  $i\sigma^{j0/3}\gamma_5 \longrightarrow H_{\mathrm{T,Q}(0/3)} \quad E_{\mathrm{T,Q}(0/3)} \cdots$
- **Kinematical variables :**

$x = \frac{k^3}{P^3} \neq \frac{k^+}{P^+}$	ξ	$\lvert\vec{\Delta}_\perp\rvert$	P^3

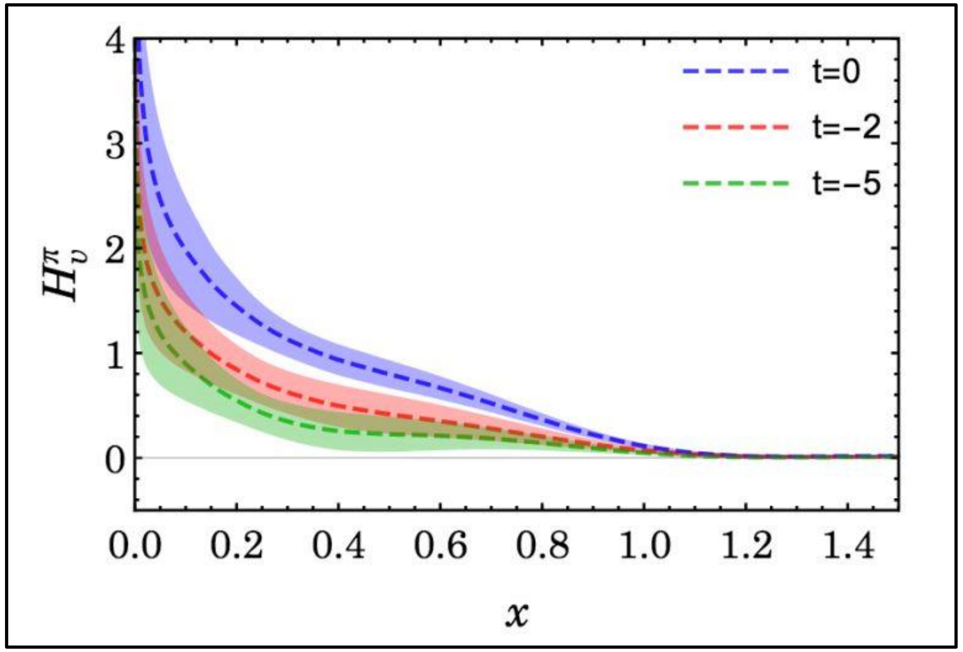
Developments in Quasi-GPD approach

1) **Matching calculations for quasi-GPDs**

(Ji, Schafer, Xiong, Zhang/ Xiang, Zhong, 2015/ Liu, Wang, Xu, Zhang, Zhang, Zhao, Zhao, 2019)

2) **Model calculations** (SB, Cocuzza, Metz, 2018, 2019)

3) **First exploratory study of unpolarized quasi-GPD for pion on lattice :** (Chen, Lin, Zhang, 2019)



a) **Calculation of H for π^+ for $u_{\text{val.}} - d_{\text{val.}}$**

b) **Calculation for $\xi = 0$ & $m_\pi = 310\,\text{MeV}$**

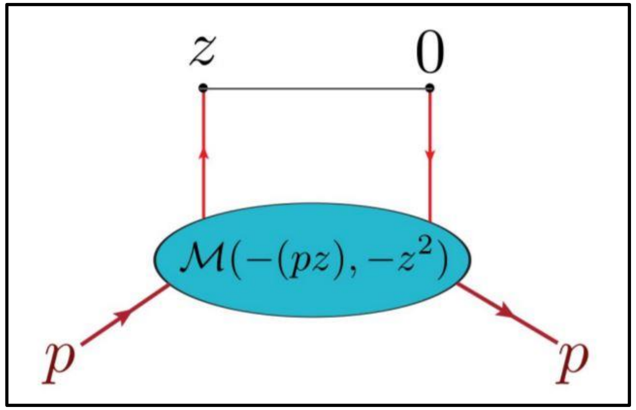
c) **Renormalized in RI-MOM scheme**

d) **Applied matching & meson-mass corrections**

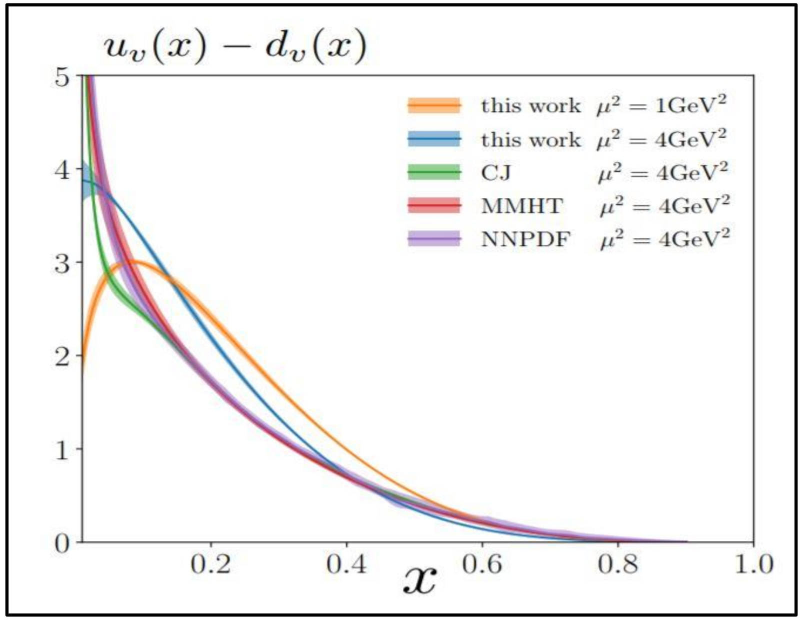
Other approaches

- **Pseudo-distributions** (Radyushkin, 2017)

Example : Unpolarized PDFs



(Orginos et. al, 2017)

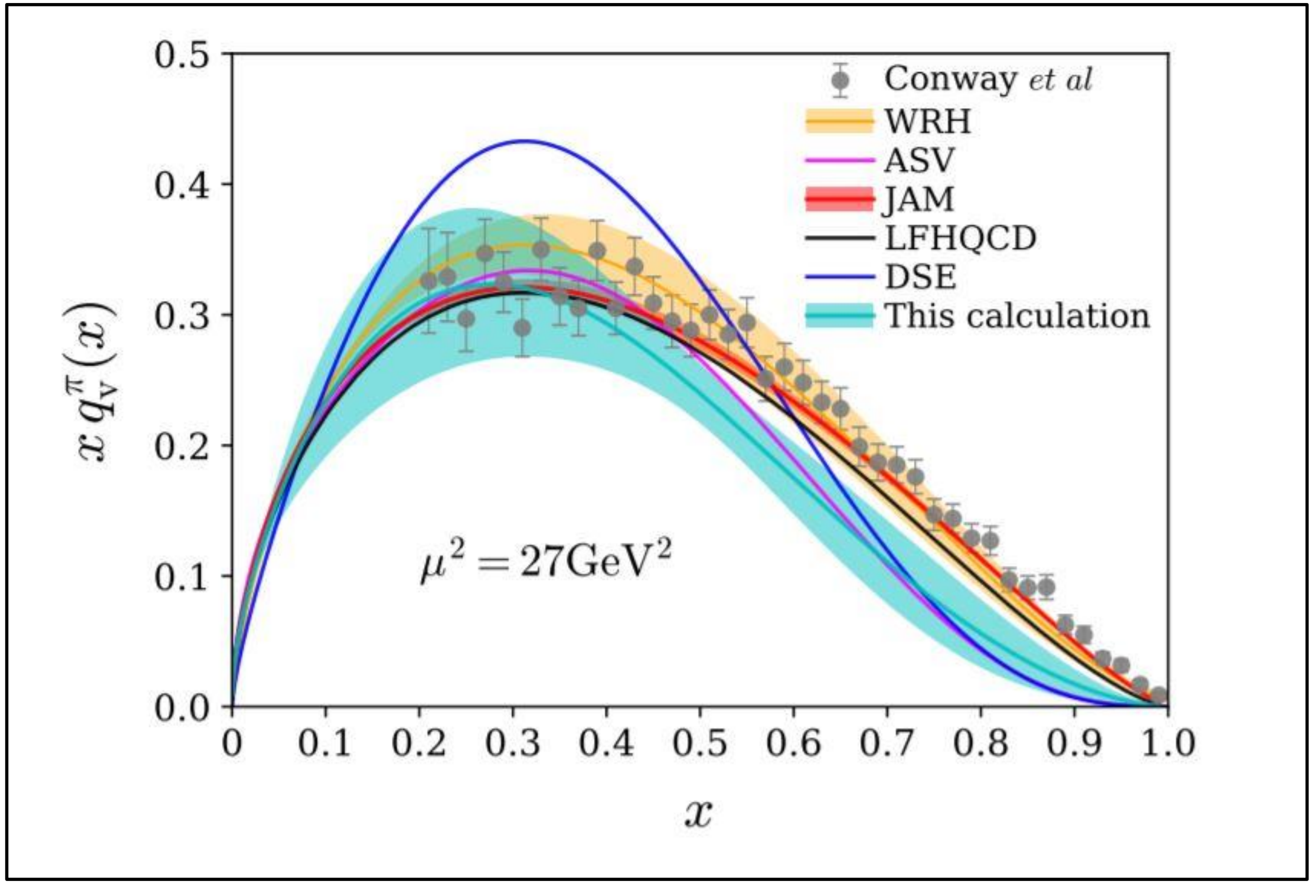


a) **Matrix element of interest :** $M^\alpha(z,p) = \langle p|\bar{\psi}(z)\,\Gamma^\alpha\,\mathcal{W}(z,0)\psi(0)|p\rangle$

b) **Lorentz decomposition :** $M^\alpha(z,p) = 2p^\alpha \boxed{M_p(-z.p, -z^2)} + z^\alpha M_z(-z.p, -z^2)$

c) **Pseudo-PDF :** $P(x, -z^2) = \dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} d\nu M_p(\nu, -z^2)e^{-ix\nu}$

- **<u>Other approaches</u>**



Good lattice cross-section

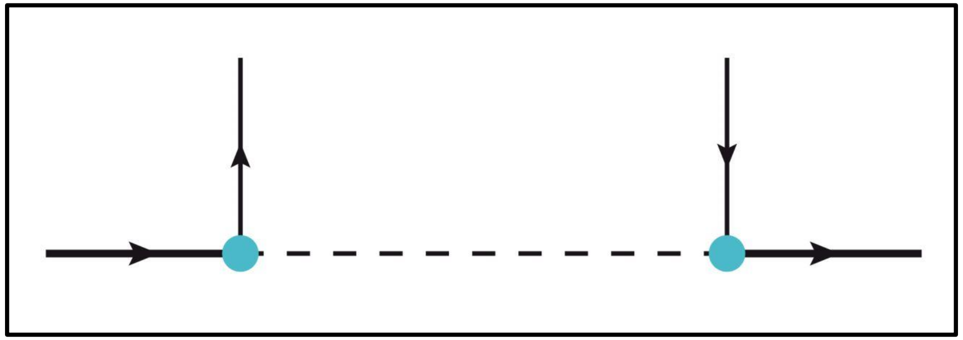
(WM/ JLab, 2019)

Immense progress in this field

Spectator Model

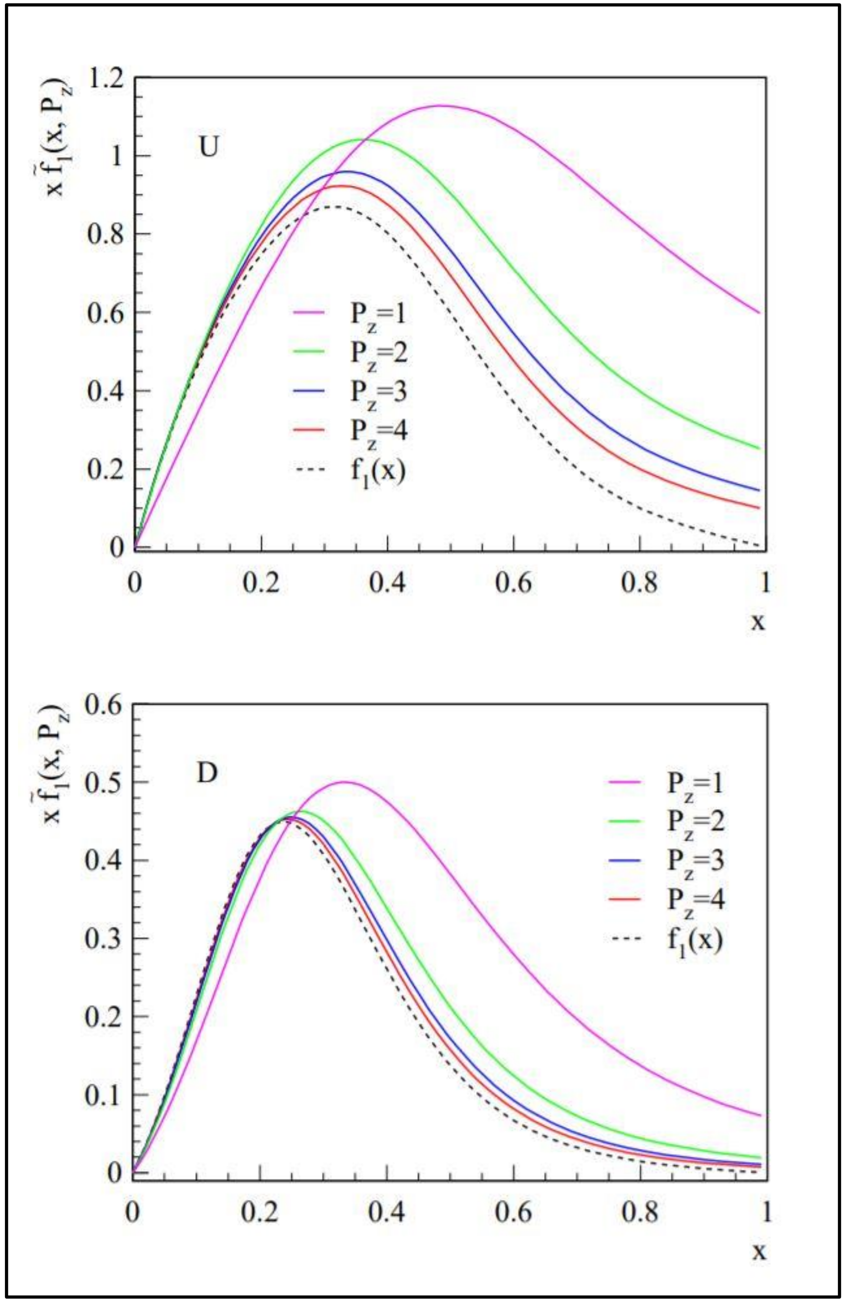
- **Nucleon : Model nucleon as a combination of active quark & spectators as diquarks of spin-0 (or spin-1)**
  (Jakob, Mulders, Rodrigues, 1997)

- **Graphical representation of a quark-quark correlator in scalar diquark model :**



Diquark	Scalar
Vertex	$ig_s\mathbb{1}$
Propagator	$\dfrac{i}{(P-k)^2 - m_s^2 + i\epsilon}$

- **Often form factors used at the vertex**
- **Cut-graph model (diquark on-shell) can be used to compute PDFs, but care must be taken for quasi-PDFs**



- **<u>Spectator model calculation of quasi-PDFs</u> :**
  (Gamberg, Kang, Vitev, Xing, 2014)

- **Worked in cut-graph model**
- **Explored scalar & vector diquarks**
- **Form factor prescription used to regulate UV-divergence**
- **Results shown here are for up & down quarks in proton**
- **For large P^3, quasi-PDFs are close to standard unpolarized f_1 for a wide range of x**
- **Substantial discrepancies between quasi & standard observed for large x**
- **Qualitatively, similar findings for helicity & transversity**

Analytical Results of Quasi-GPDs in Scalar Diquark Model

Correlator for quasi-GPDs :

$$F_{\mathrm{Q}}^{[\Gamma]}(x, \Delta; P^3) = \frac{i\, g^2}{2(2\pi)^4} \int dk^0\, d^2\vec{k}_\perp \frac{\bar{u}(p') \left(\not{k} + \frac{\not{\Delta}}{2} + m_q \right) \Gamma \left(\not{k} - \frac{\not{\Delta}}{2} + m_q \right) u(p)}{D_{\mathrm{GPD}}}$$

$$D_{\mathrm{GPD}} = \left[\left(k + \frac{\Delta}{2} \right)^2 - m_q^2 + i\varepsilon \right] \left[\left(k - \frac{\Delta}{2} \right)^2 - m_q^2 + i\varepsilon \right] \left[(P-k)^2 - m_s^2 + i\varepsilon \right]$$

Quasi-GPDs : Example :

$$H_{\mathrm{Q}(0)}(x, \xi, t; P^3) = \frac{i\, g^2 P^3}{(2\pi)^4} \int dk^0\, d^2\vec{k}_\perp \frac{N_{H(0)}}{D_{\mathrm{GPD}}}$$

$$N_{H(0)} = \delta (k^0)^2 - \frac{2}{P^3} \left[x(P^3)^2 - m_q M - x\frac{t}{4} - \frac{1}{2}\delta\xi t \frac{\vec{k}_\perp \cdot \vec{\Delta}_\perp}{\vec{\Delta}_\perp^2} \right] k^0 + \delta \left[x^2 (P^3)^2 + \vec{k}_\perp^2 + m_q^2 + (1-2x)\frac{t}{4} - \delta\xi t \frac{\vec{k}_\perp \cdot \vec{\Delta}_\perp}{\vec{\Delta}_\perp^2} \right]$$

- **Standard (twist-2) GPDs are continuous in entire x range (contrast with twist-3 GPDs** – Aslan et. al, 2018**)**
- **Powers of k^0 do not pose problem - Quasi-GPDs are continuous at $\boxed{x = \pm\xi}$ (even beyond leading-twist)**
- **For $P^3 \to \infty$, we recover ALL 8 leading-twist standard GPDs in SDM**

Numerical Results in Scalar Diquark Model

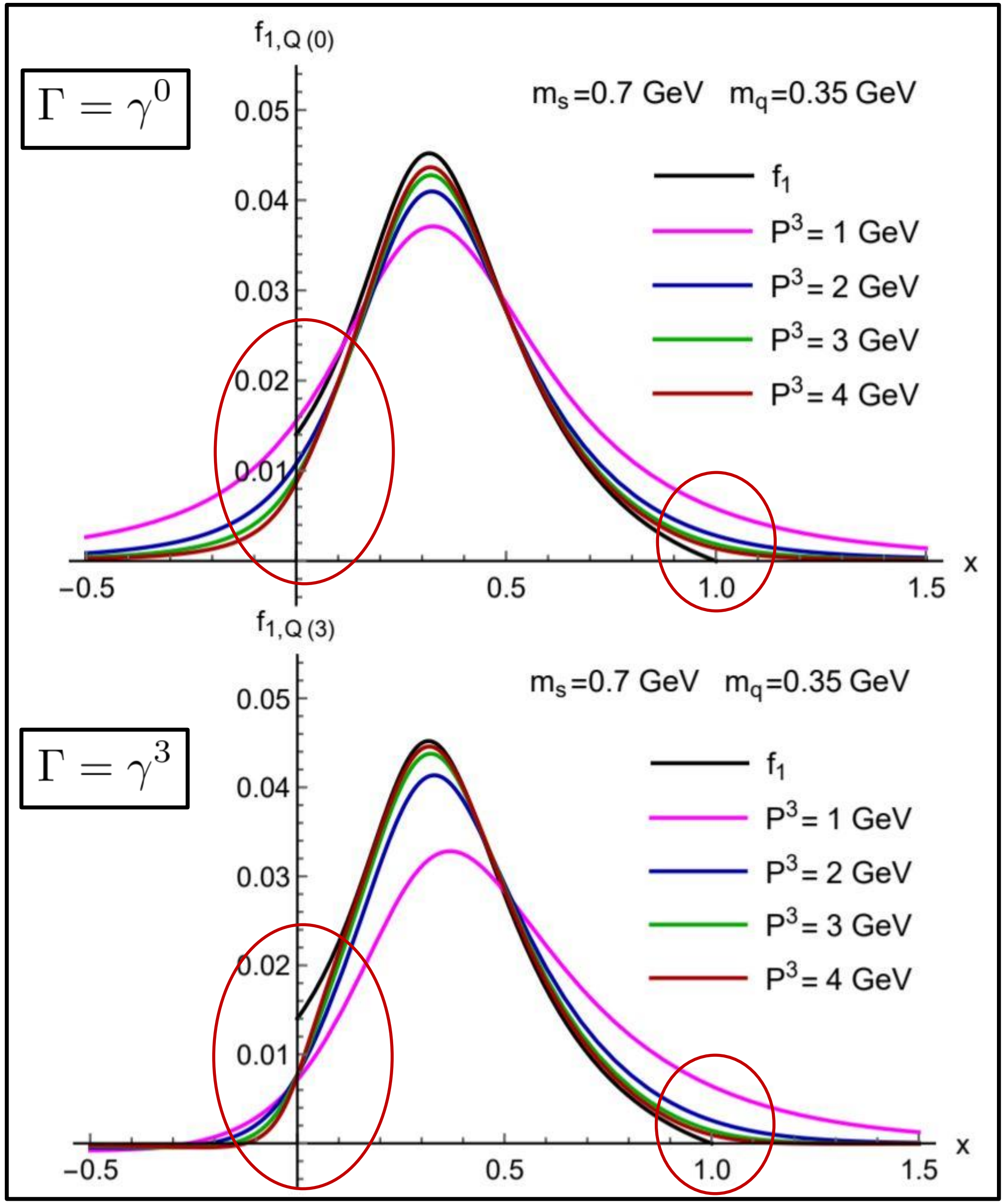
Choice of parameters :

- **Nucleon-Quark-Diquark coupling :** $g = 1$
- **Masses :** **Constraint -** $M < m_q + m_s$ **(** M **- nucleon mass,** m_q **- quark mass,** m_s **- spectator mass)**

  Values assumed - $M = 0.939$ GeV , $m_q = 0.35$ GeV, $m_s = 0.70$ GeV (Gamberg, Kang, Vitev, Xing, 2014)

- **Cut-off for** $|\vec{k}_\perp|$ **integration :** $\Lambda = 1$ GeV
- **Momentum transfer :** $|\vec{\Delta}_\perp| = 0$ GeV

Variations of these parameters do not affect our general results

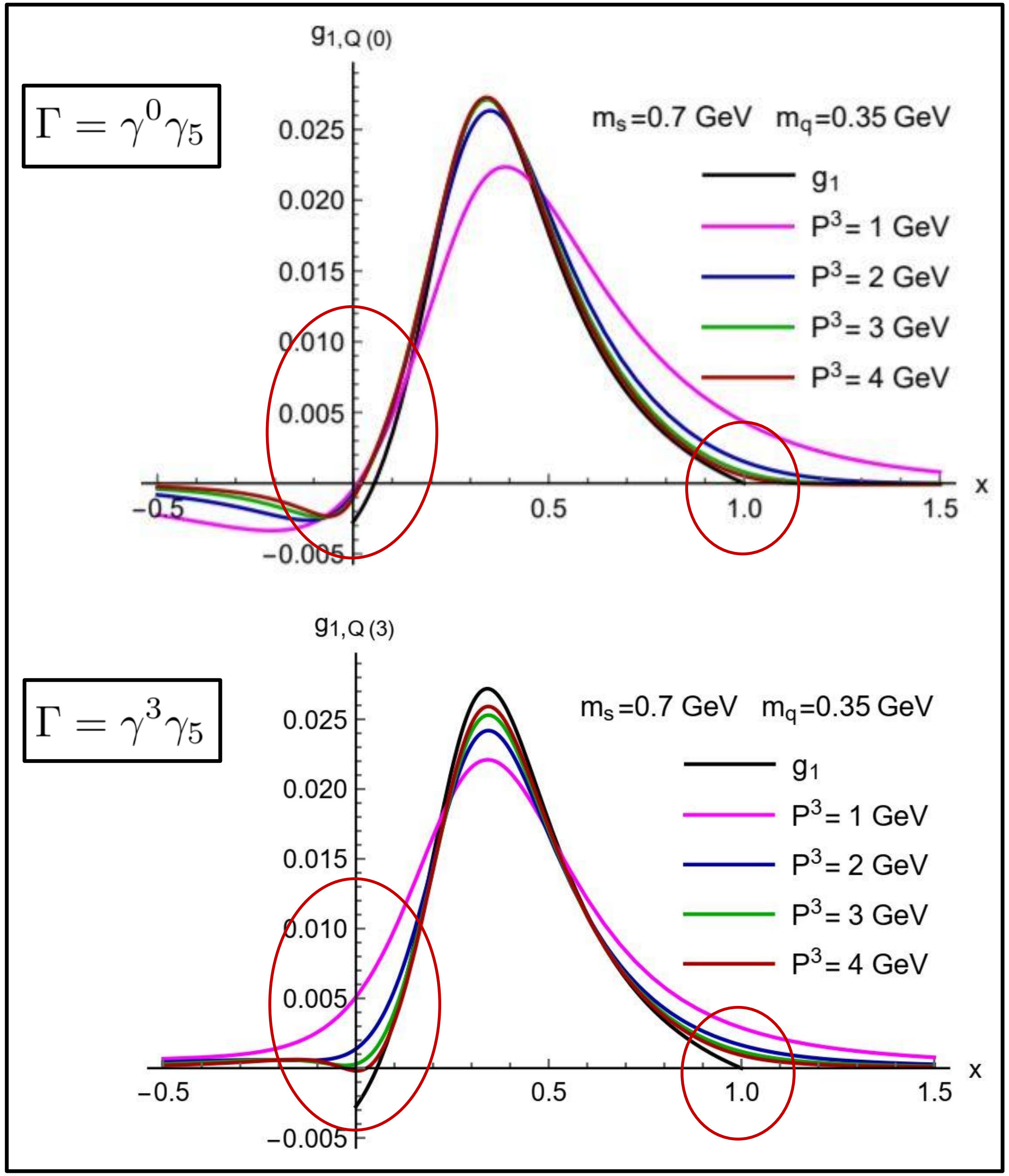


Unpolarized

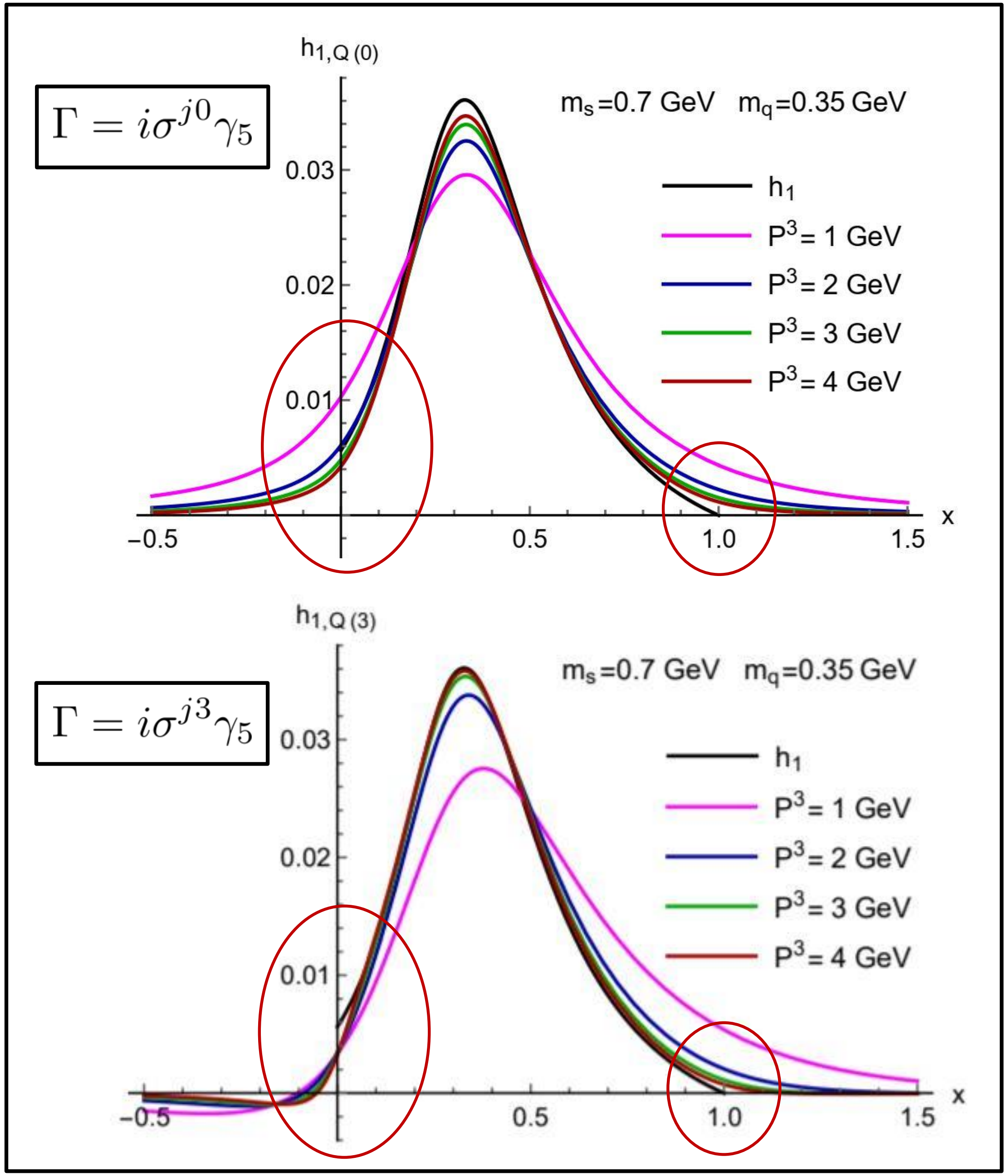
Quasi-PDFs :

- **Consider unpolarized quasi-PDF** $f_{1,\mathrm{Q}(0/3)}$
- **For larger** P^3 **, not much of a difference between** $f_{1,\mathrm{Q}(0)}$ **&** $f_{1,\mathrm{Q}(3)}$
- **For larger** P^3 **, good agreement between quasi and standard PDF for a wide range of** x
- **Considerable discrepancies at small** x **& large** x
- **General features of quasi helicity (**$g_{1,\mathrm{Q}(0/3)}$**) & quasi transversity (** $h_{1,\mathrm{Q}(0/3)}$**) are same as that of** $f_{1,\mathrm{Q}(0/3)}$ (next page)

Helicity



Transversity

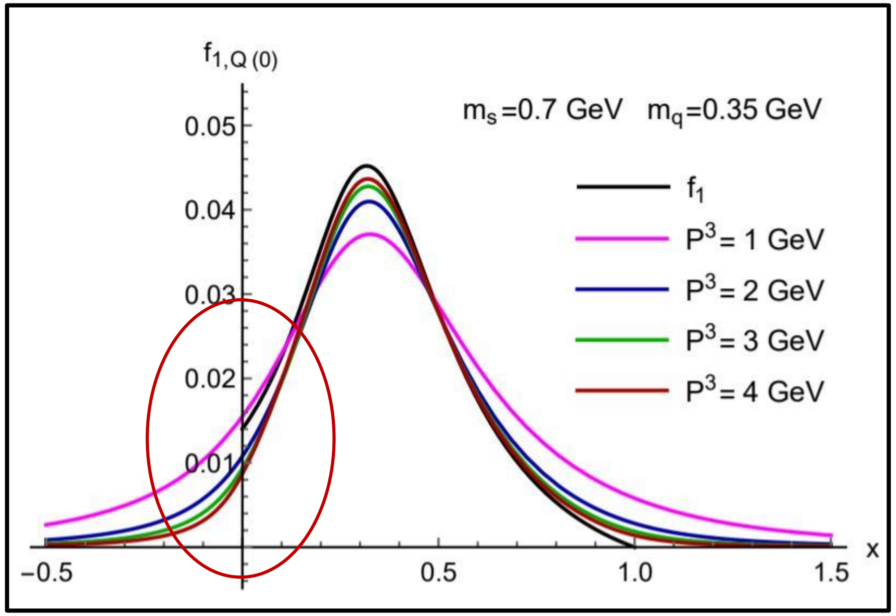


Small - x issue

- **Standard PDFs are discontinuous at $x=0$ (contrast with standard GPDs)**
  Example : $f_1(x<0)=0$

- **Quasi-PDFs are continuous for all x (just like quasi-GPDs)**

- **Quasi-PDFs must agree with standard PDFs & therefore must rapidly change around $x=0$**



What is the value of standard PDFs at $x=0$? Can quasi-PDFs reproduce the discontinuity at $x=0$?

Ans. 1 $$f_1(x=0)\Big|^{Step\,1:\,\int dk^-}_{Step\,2:\,x=0} = \frac{g^2}{2(2\pi)^3}\int d^2\vec{k}_\perp \frac{1}{(\vec{k}_\perp^2+m_q^2)} \quad \text{or} \quad 0$$

Ans. 2 $$f_1(x=0)\Big|^{Step\,1:\,x=0}_{Step\,2:\,\int dk^-} \xrightarrow{C.P.V.} \frac{g^2}{4(2\pi)^3}\int d^2\vec{k}_\perp \frac{1}{(\vec{k}_\perp^2+m_q^2)}$$

Non-commutativity of $\int dk^-$ & setting $x=0$;
Reliability of ans. 2

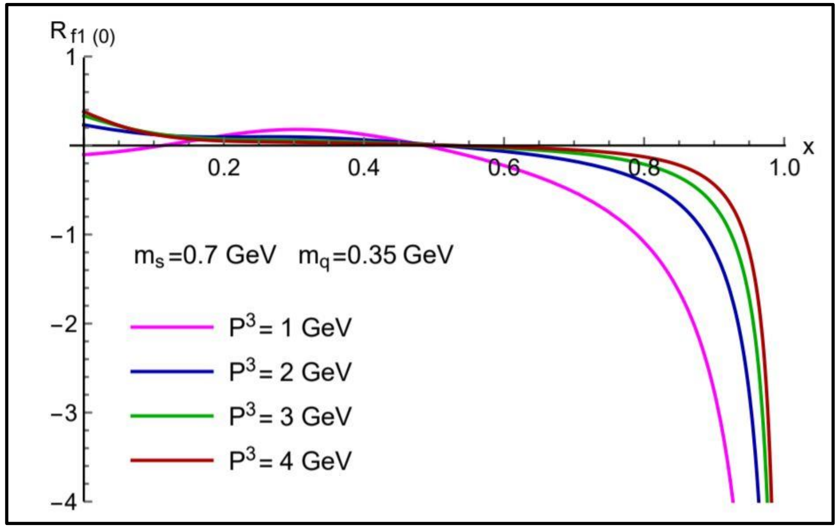
- **Example :** f_1

- $$\boxed{\lim_{P^3\to\infty} f_{1Q(0/3)}(x=0) = \text{Ans. 2}}$$

- $g_{1,Q(0/3)}$ **&** $h_{1,Q(0/3)}$ **analytically reproduces the discontinuity as well**

Large - x issue

- **Observing this issue through – Relative Difference**

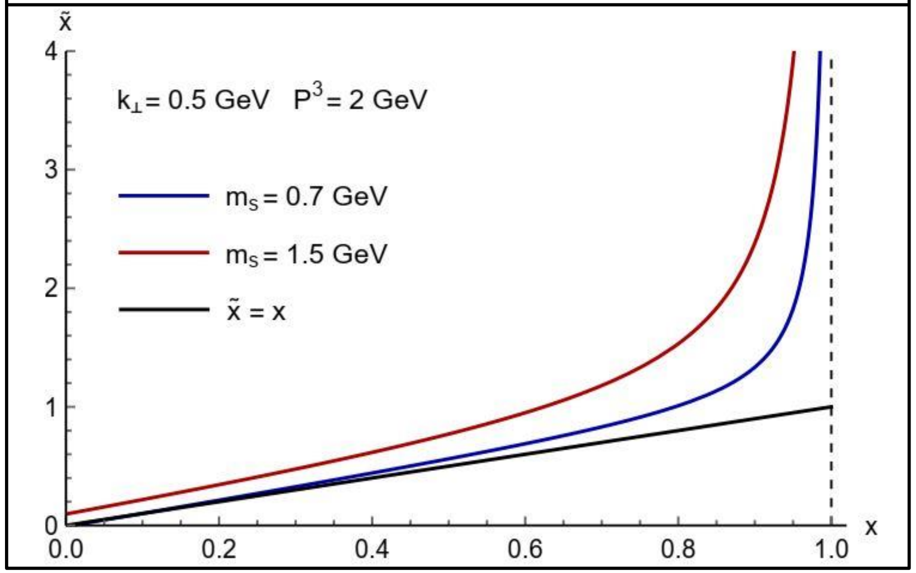


Example :

$$\boxed{R_{f1(0)}(x;P^3) = \frac{f_1(x) - f_{1,\mathrm{Q}(0)}(x;P^3)}{f_1(x)}}$$

- **Cut-graph model analysis :**

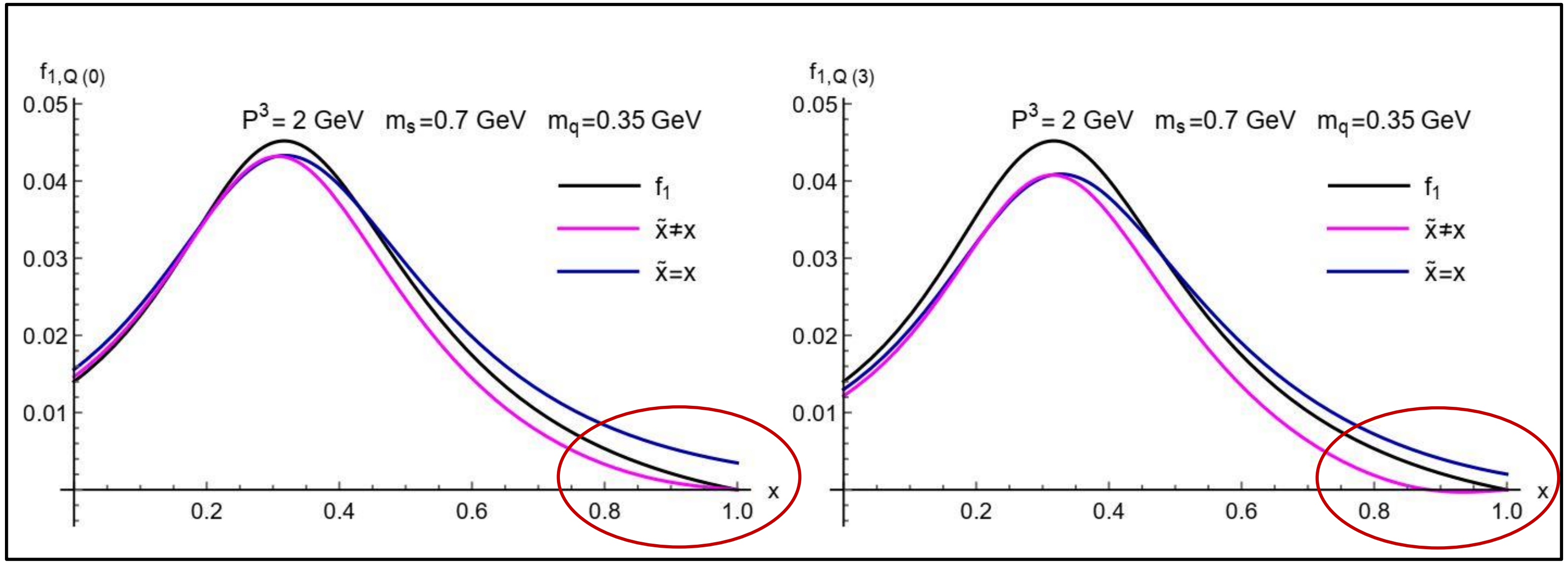
$$\tilde{x} \;=\; x + \frac{1}{4(P^3)^2}\left(\frac{\vec{k}_\perp^2 + m_s^2}{1-x} - (1-x)M^2\right) + \mathcal{O}\left(\frac{1}{(P^3)^4}\right)$$

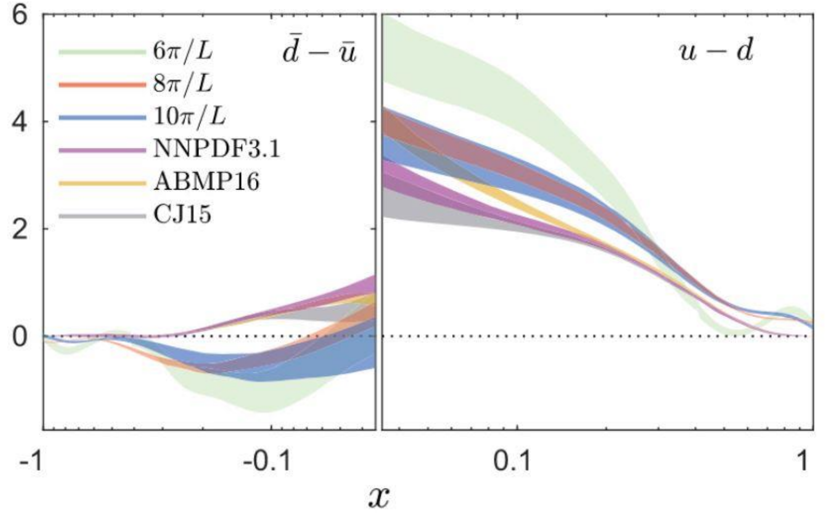


a) **Mismatch in this region between** $\boxed{x = \frac{k^+}{P^+}}$ **&** $\boxed{\tilde{x} = \frac{k^3}{P^3}}$ **for finite** P^3

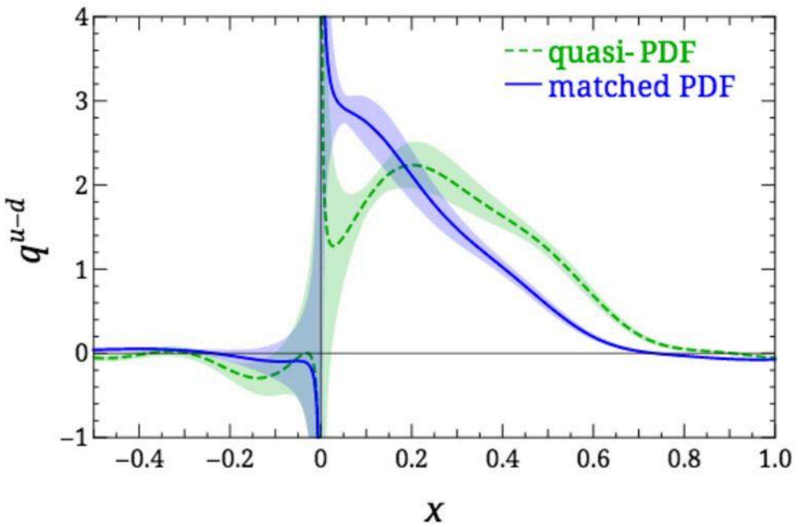
b) **Better results at very large** x **on relating** $\tilde{x}$ **&** x

Example : $f_{1,\mathrm{Q}}$





Alexandrou et al. , 2018

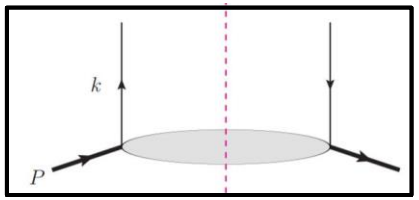


Chen et al. , 2018

Lattice QCD results

- **Results of** $f_{1,\mathrm{Q}(0)}$ **at physical pion mass**
- **Top plot: Largest value of momenta** $P^3 = \dfrac{10\pi}{L} = 1.38\,\mathrm{GeV}$
- **Discrepancies between quasi and standard PDF clearly seen at large** x
- **Large -** x **discrepancies not an artifact of model**
- **Bottom plot:** $P^3 = 2.2\,\mathrm{GeV}$
- **“Matching” may reduce discrepancies at large** x **(?)**

Word of caution about cut-graph models to describe quasi-distributions



Example : $f_{1,\mathrm{Q}(0)}$

Poles

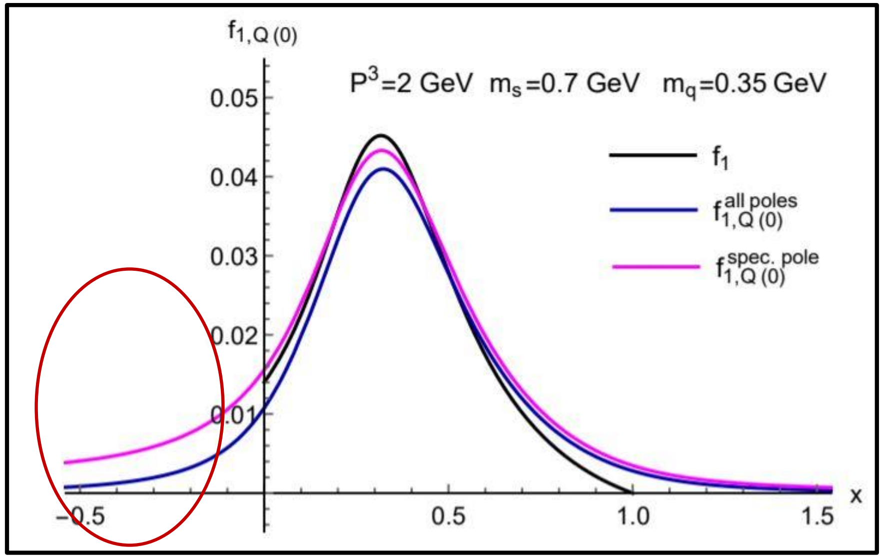
Quark Pole (order 2)

$$k^0_{1\pm} \;=\; \pm\sqrt{x^2(P^3)^2 + \vec{k}_\perp^2 + m_q^2 - i\epsilon}$$

Spectator Pole (order 1)

$$k^0_{3\pm} \;=\; \pm\sqrt{(1-x)^2(P^3)^2 + \vec{k}_\perp^2 + m_s^2 - i\epsilon}$$

$$\boxed{N_{f1(0)} = \lim_{\Delta=0} N_{H(0)}}$$



$$\begin{aligned}
f_{1,\mathrm{Q}(0)}(x;P^3) \;=\; & -\frac{g^2 P^3}{(2\pi)^3}\int d^2\vec{k}_\perp \Bigg[\frac{N_{f1(0)}(k^0_{3-})}{(k^0_{3-}-k^0_{1+})^2\,(k^0_{3-}-k^0_{1-})^2\,(k^0_{3-}-k^0_{3+})} \\
& + \frac{N'_{f1(0)}(k^0_{1-})}{(k^0_{1-}-k^0_{1+})^2\,(k^0_{1-}-k^0_{3+})\,(k^0_{1-}-k^0_{3-})} - \frac{2\,N_{f1(0)}(k^0_{1-})}{(k^0_{1-}-k^0_{1+})^3\,(k^0_{1-}-k^0_{3+})\,(k^0_{1-}-k^0_{3-})} \\
& - \frac{N_{f1(0)}(k^0_{1-})}{(k^0_{1-}-k^0_{1+})^2\,(k^0_{1-}-k^0_{3+})^2\,(k^0_{1-}-k^0_{3-})} - \frac{N_{f1(0)}(k^0_{1-})}{(k^0_{1-}-k^0_{1+})^2\,(k^0_{1-}-k^0_{3+})\,(k^0_{1-}-k^0_{3-})^2}\Bigg]
\end{aligned}$$

Spectator pole contribution

Quark pole contribution

$0 < x < 1$	$x > 1$	$x < 0$
Spectator pole term leading	**All terms power-suppressed**	**First & last term leading; leading powers cancel; quark pole necessary for quasi to approach standard in this region**

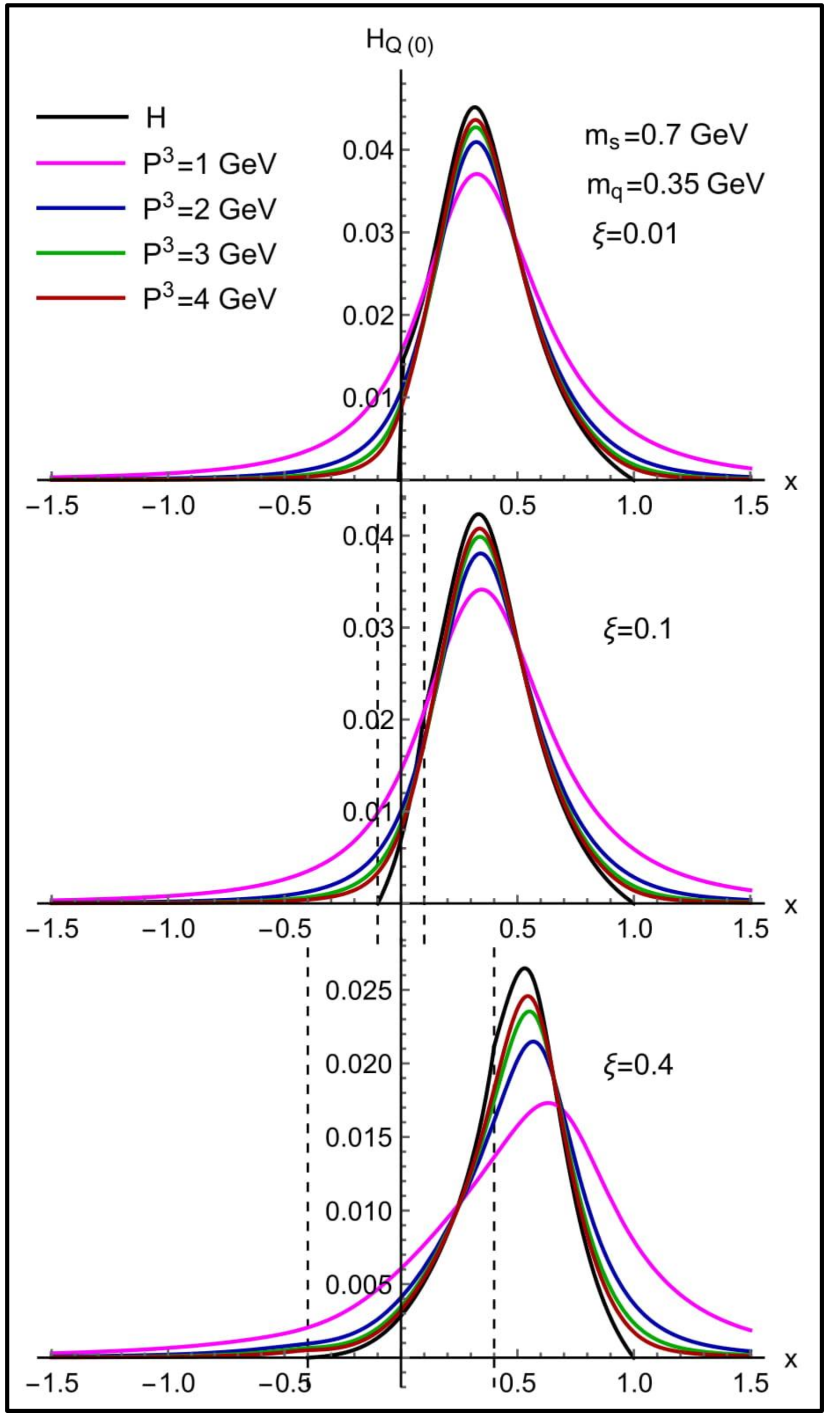
Quasi-GPDs : (specific Dirac structures)

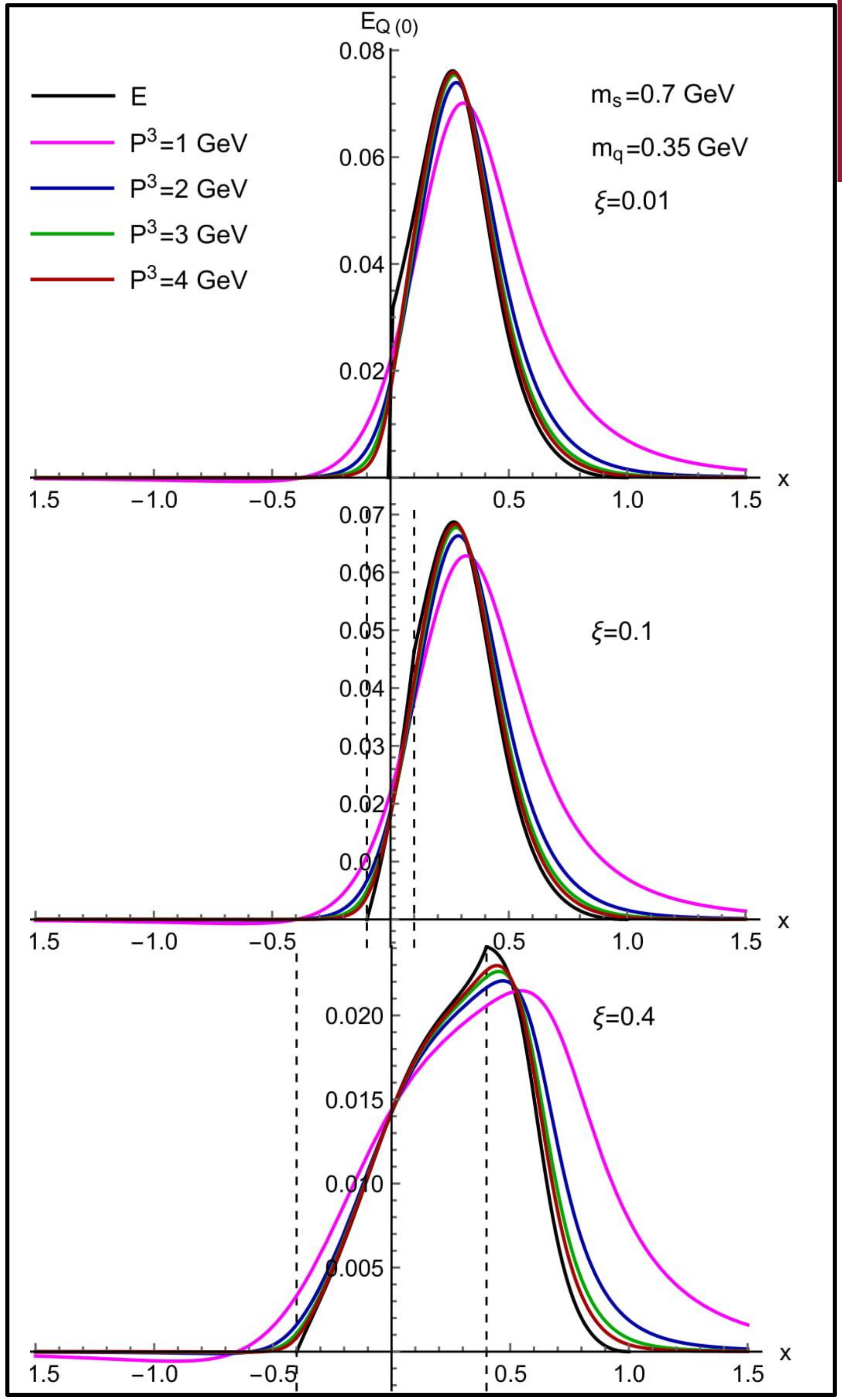
- **Example : Unpolarized quasi-GPDs $H_{\mathrm{Q}(0)}$ & $E_{\mathrm{Q}(0)}$**

- **Qualitatively, similar results for ALL quasi-GPDs**

- **Large - x issue persists for ALL quasi-GPDs**

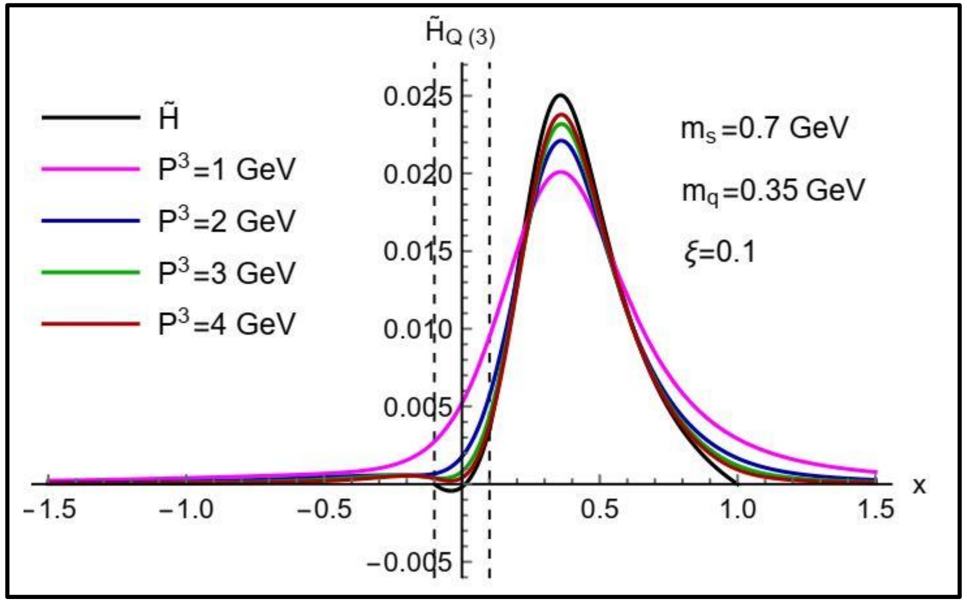
- **Large - x situation worsens if ξ increases (more severe for $\tilde{E}_{\mathrm{Q}}$, $\tilde{E}_{\mathrm{T,Q}}$)**

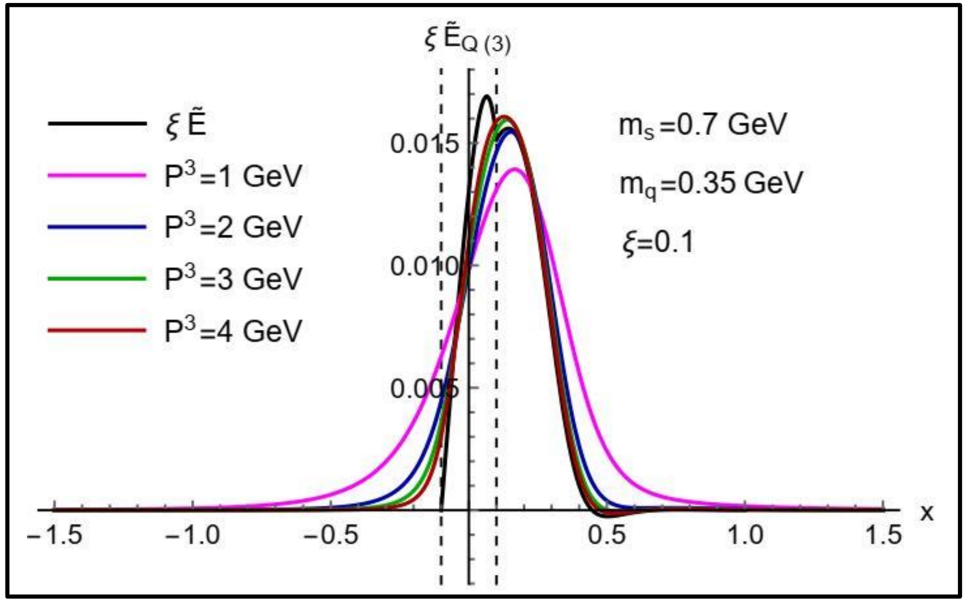




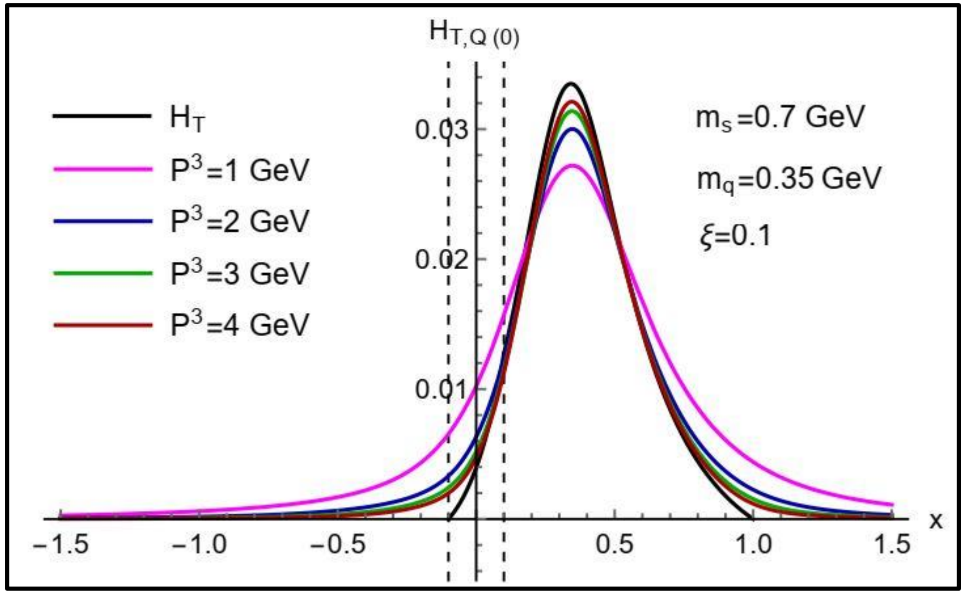
Polarized Quasi-GPDs :

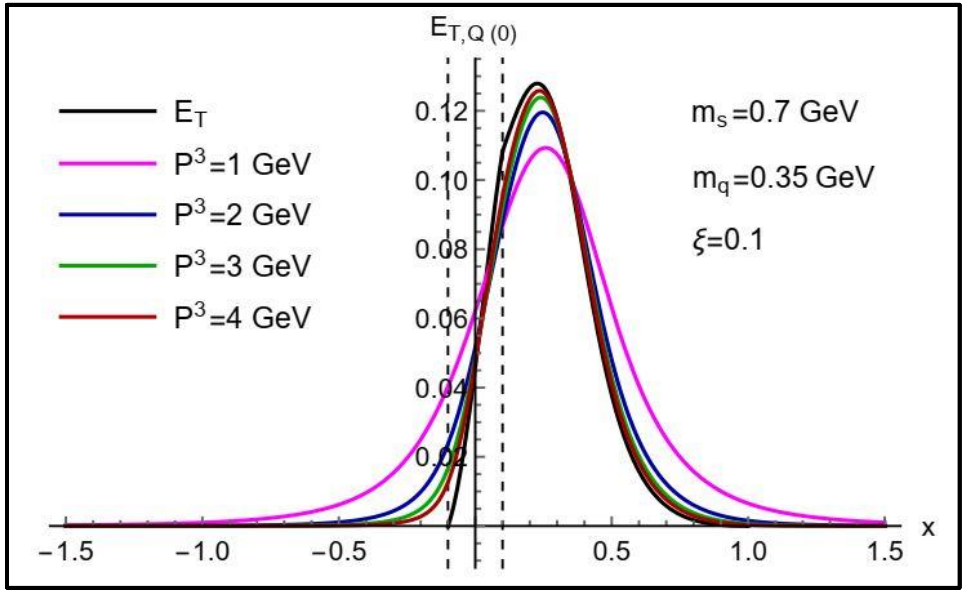
- **Plots for Longitudinally Polarized Quasi-GPDs :**

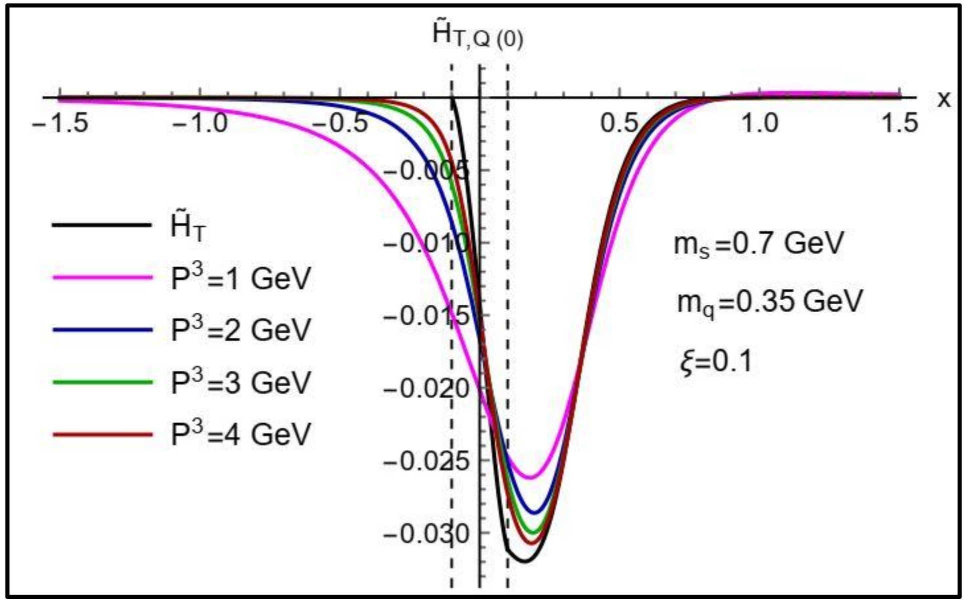


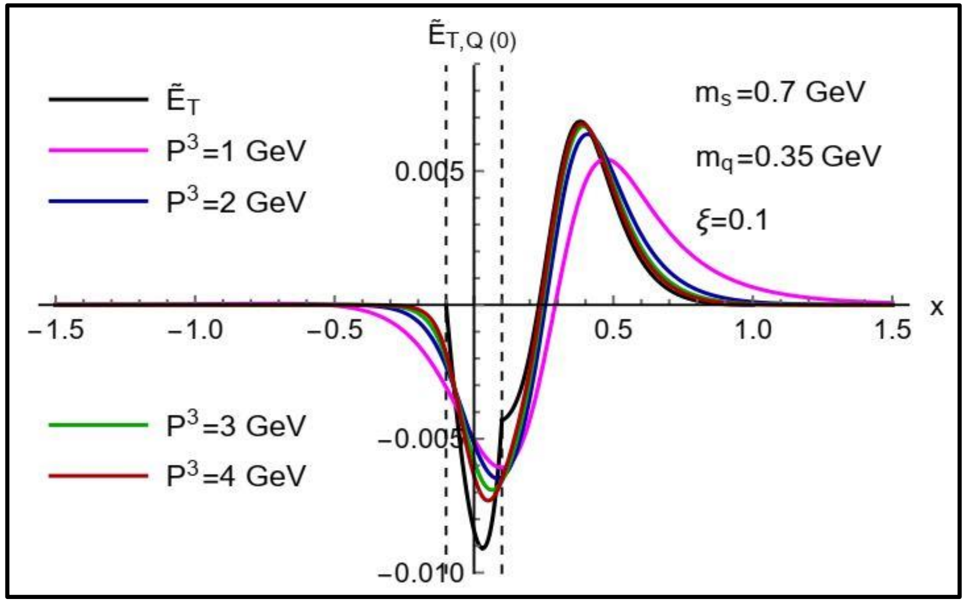


- **Plots for <u>Transversely Polarized</u> Quasi-GPDs :**





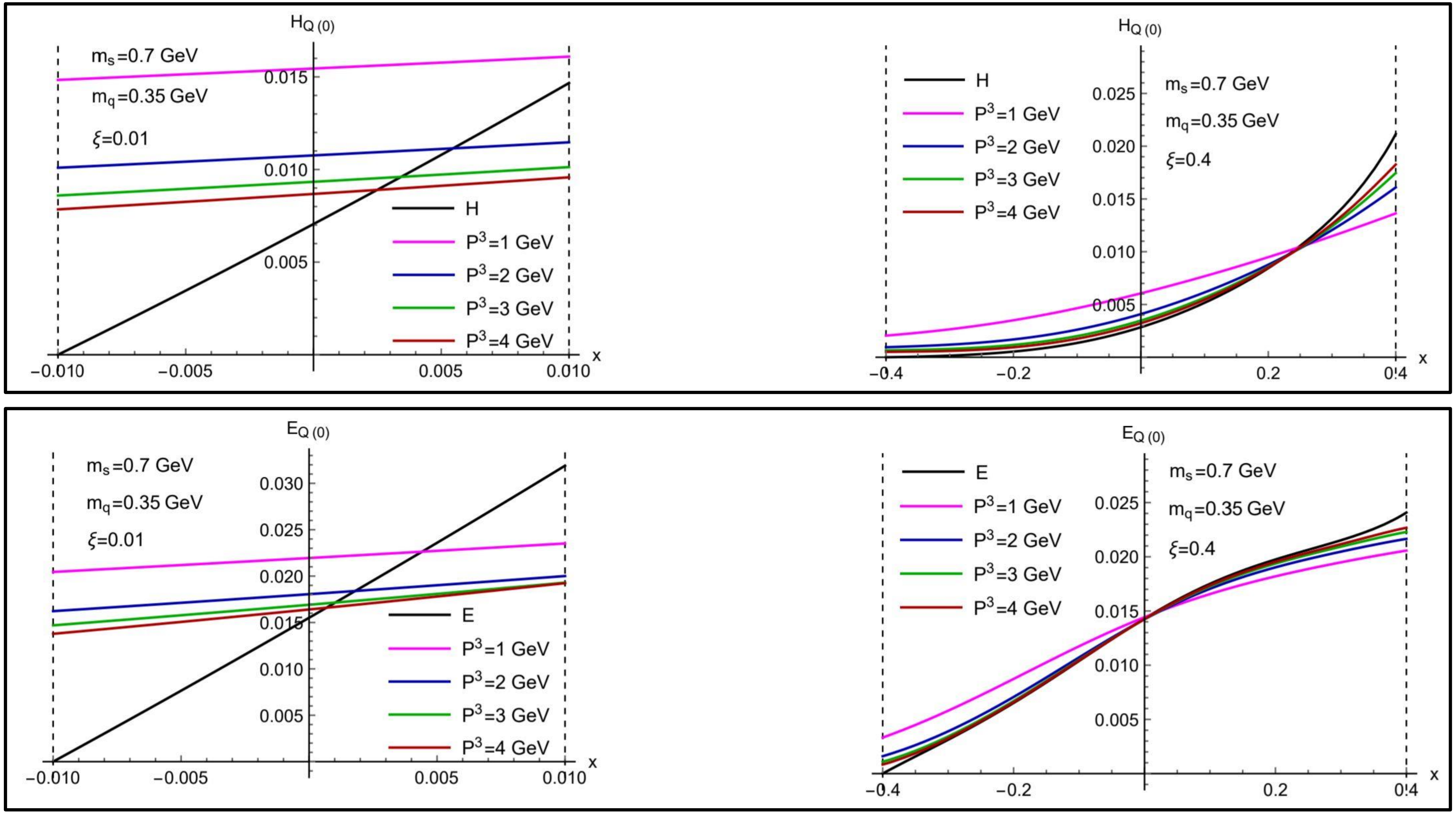




Quasi-GPDs in ERBL region

Example : Unpolarized quasi-GPDs

$\xi = 0.01$



$\xi = 0.4$

- **Large discrepancies between ALL quasi-GPDs & standard distributions in ERBL region if ξ is small (compare with region around $x = 0$ for PDFs)**

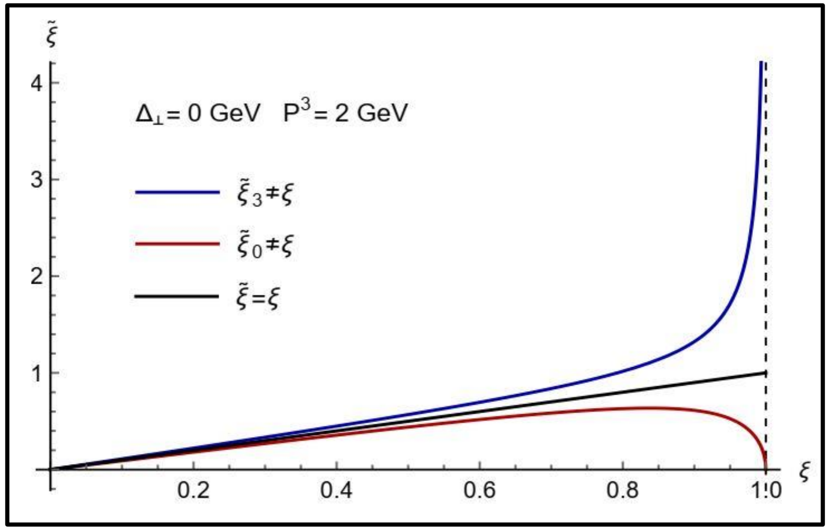
- **Lattice QCD calculations might be promising if ξ is larger**

Exploring different skewness variables

- **Standard & quasi-skewness variables :**
  Model-independent relations

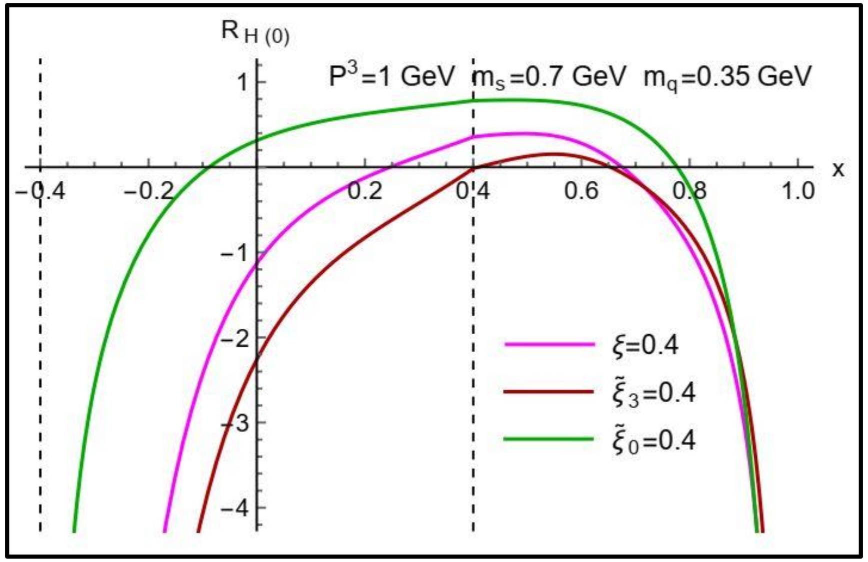
$\xi = -\dfrac{\Delta^+}{2P^+}$	$\tilde{\xi}_3 = -\dfrac{\Delta^3}{2P^3} = \delta\xi$	$\tilde{\xi}_0 = -\dfrac{\Delta^0}{2P^0} = \dfrac{\xi}{\delta}$

$$\delta = \sqrt{1 + \frac{M^2 - t/4}{(P^3)^2}}$$

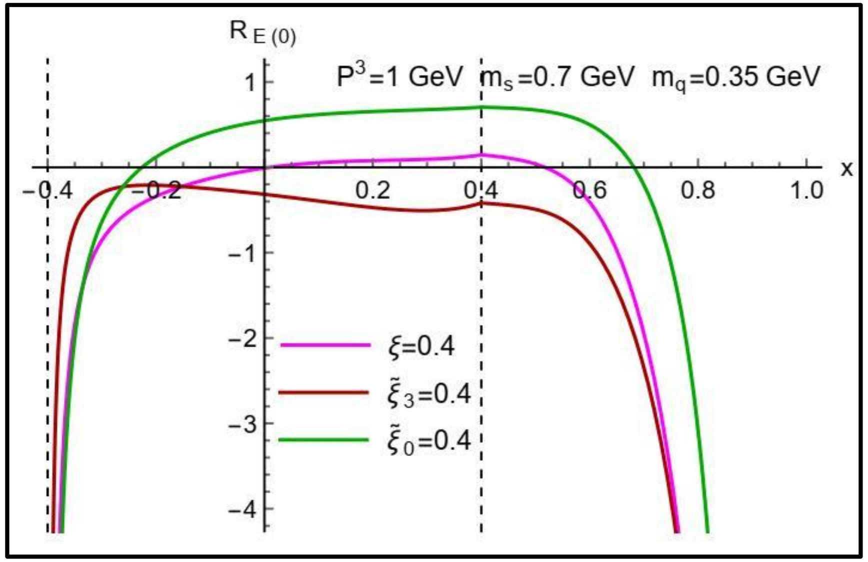


- **Significant discrepancies between variables for finite P^3**

- **<u>Comparative study of quasi-GPDs as functions of ξ vs. quasi-GPDs as functions of $\tilde{\xi}_3$ & $\tilde{\xi}_0$</u> :**



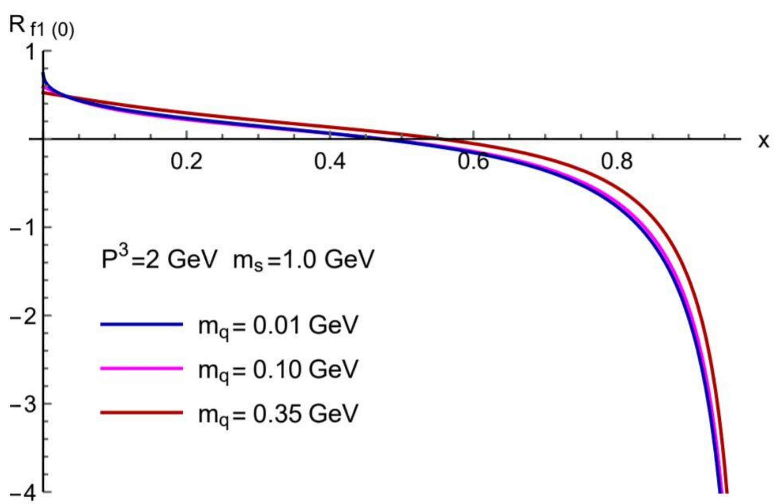
<u>Trend for majority</u>



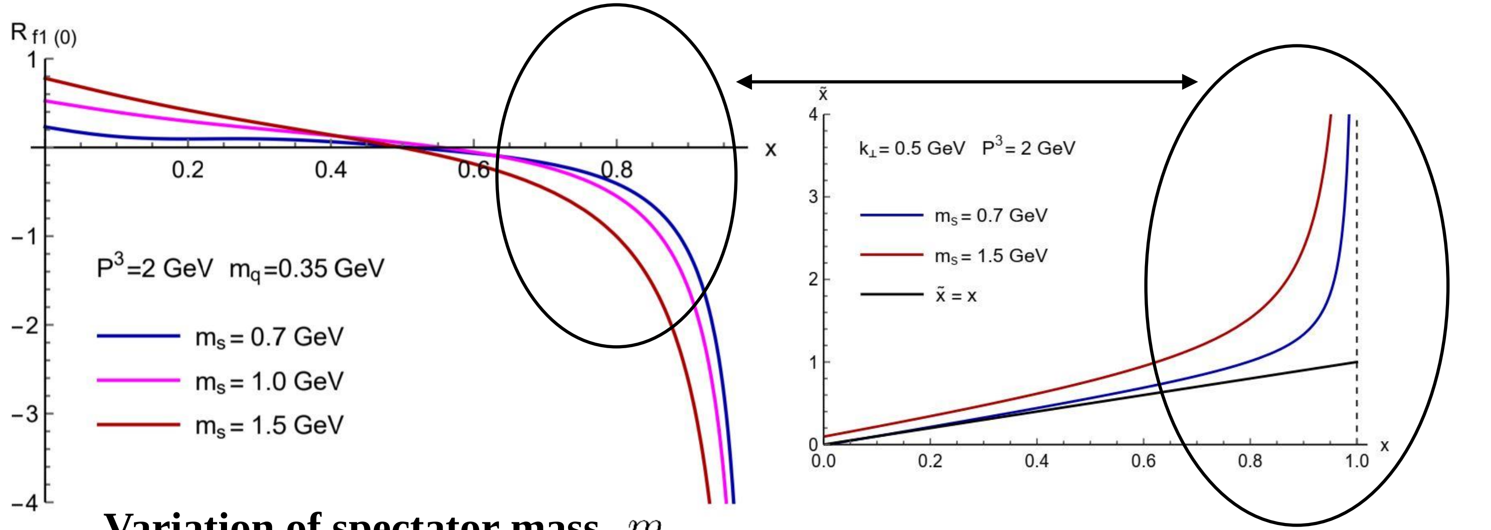
<u>Trend for outliers like</u> : $E_{\mathrm{Q}(0)}$ $\tilde{E}_{\mathrm{Q}(0/3)}$ $E_{\mathrm{T,Q}(0)}$

Sensitivity of numerical results to model parameters :

a) Variation of masses :



Variation of quark mass m_q



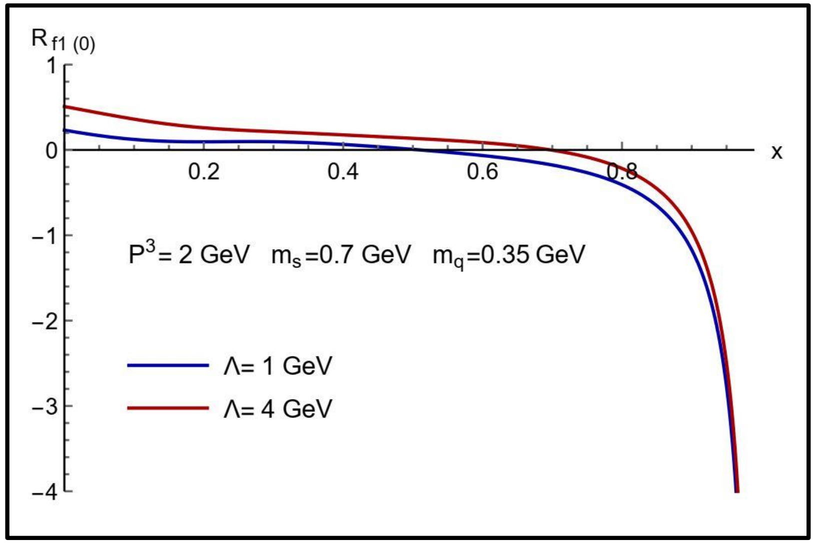
Variation of spectator mass m_s

- **Mild sensitivity to variation in** m_q **for ALL PDFs & GPDs (no pattern for ERBL)**

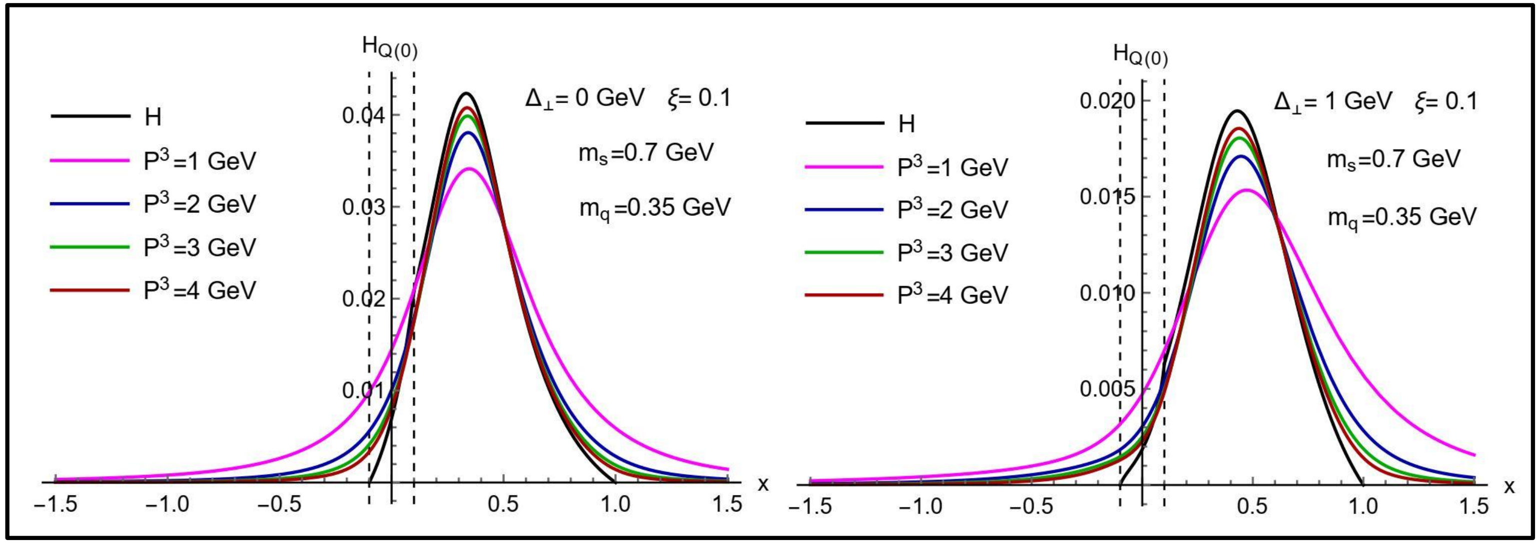
- **Greater sensitivity to variation in** m_s **for ALL PDFs & GPDs (no pattern for ERBL);** **smaller values of** m_s **is "optimal"**

Sensitivity of numerical results to model parameters :

b) **Variation of cut-off (Λ) :**



c) **Variation of transverse momentum transfer** ($|\vec{\Delta}_\perp|$) :



- **Mild impact of variations in Λ**
- **Mild impact of variations in $|\vec{\Delta}_\perp|$ on R.D. for ALL GPDs x**
- **General features are robust**

Moments of quasi-distributions

Link between Quasi-GPDs & Form Factors :

$$(P^3)^{n+1}\int_{-\infty}^{\infty} dx\, x^n \int_{-\infty}^{\infty}\frac{dz^3}{2\pi}e^{ixP^3z^3}\langle p',\lambda'|\bar{\psi}^q(-\tfrac{z^3}{2})\Gamma\mathcal{W}_{\mathrm{Q}}(-\tfrac{z}{2},\tfrac{z}{2})\psi^q(\tfrac{z^3}{2})|p,\lambda\rangle\Big|_{z^0=\vec{z}_\perp=0} \qquad n=0,1$$

$$= \langle p',\lambda'|\bar{\psi}^q(0)\Gamma(\tfrac{i}{2}\overleftrightarrow{D}^3)^n\psi^q(0)|p,\lambda\rangle$$

First moment : $n=0$ $\boxed{\Gamma=\gamma^0}$ $\boxed{\Gamma=\gamma^3}$

$\boxed{\Gamma=\gamma^\mu}$

$$\int_{-1}^{1} dx\, H^q(x,\xi,t) = \int_{-\infty}^{\infty} dx \frac{1}{\delta} H^q_{\mathrm{Q}(0)}(x,\xi,t;P^3) = \int_{-\infty}^{\infty} dx\, H^q_{\mathrm{Q}(3)}(x,\xi,t;P^3) = F_1^q(t)$$

Dirac F.F.

$$\int_{-1}^{1} dx\, E^q(x,\xi,t) = \int_{-\infty}^{\infty} dx \frac{1}{\delta} E^q_{\mathrm{Q}(0)}(x,\xi,t;P^3) = \int_{-\infty}^{\infty} dx\, E^q_{\mathrm{Q}(3)}(x,\xi,t;P^3) = F_2^q(t)$$

Pauli F.F.

$$\delta = \sqrt{1+\frac{M^2-t/4}{(P^3)^2}}$$

- **Re-define (half) the quasi-GPDs by $\frac{1}{\delta}$ to restore P^3- independence of lowest moments**

First moment : $n = 0$

$\Gamma = \gamma^\mu \gamma_5$

$\Gamma = \gamma^0 \gamma_5$ $\quad$ $\Gamma = \gamma^3 \gamma_5$

$$\int_{-1}^{1} dx\, \widetilde{H}^q(x,\xi,t) = \int_{-\infty}^{\infty} dx\, \widetilde{H}^q_{\mathrm{Q}(0)}(x,\xi,t;P^3) = \int_{-\infty}^{\infty} dx\, \frac{1}{\delta} \widetilde{H}^q_{\mathrm{Q}(3)}(x,\xi,t;P^3) = G^q_A(t)$$

Axial-vector F.F.

$$\int_{-1}^{1} dx\, \widetilde{E}^q(x,\xi,t) = \int_{-\infty}^{\infty} dx\, \widetilde{E}^q_{\mathrm{Q}(0)}(x,\xi,t;P^3) = \int_{-\infty}^{\infty} dx\, \frac{1}{\delta} \widetilde{E}^q_{\mathrm{Q}(3)}(x,\xi,t;P^3) = G^q_P(t)$$

Pseudo-scalar F.F.

$$\delta = \sqrt{1 + \frac{M^2 - t/4}{(P^3)^2}}$$

$\Gamma = i\sigma^{\mu\nu} \gamma_5$

$\Gamma = i\sigma^{j0} \gamma_5$ $\quad$ $\Gamma = i\sigma^{j3} \gamma_5$

$$\int_{-1}^{1} dx\, H^q_T(x,\xi,t) = \int_{-\infty}^{\infty} dx\, \frac{1}{\delta} H^q_{\mathrm{T,Q}(0)}(x,\xi,t;P^3) = \int_{-\infty}^{\infty} dx\, H^q_{\mathrm{T,Q}(3)}(x,\xi,t;P^3) = F^q_{1,T}(t)$$

$$\int_{-1}^{1} dx\, E^q_T(x,\xi,t) = \int_{-\infty}^{\infty} dx\, \frac{1}{\delta} E^q_{\mathrm{T,Q}(0)}(x,\xi,t;P^3) = \int_{-\infty}^{\infty} dx\, E^q_{\mathrm{T,Q}(3)}(x,\xi,t;P^3) = 2F^q_{2,T}(t)$$

$$\int_{-1}^{1} dx\, \widetilde{H}^q_T(x,\xi,t) = \int_{-\infty}^{\infty} dx\, \frac{1}{\delta} \widetilde{H}^q_{\mathrm{T,Q}(0)}(x,\xi,t;P^3) = \int_{-\infty}^{\infty} dx\, \widetilde{H}^q_{\mathrm{T,Q}(3)}(x,\xi,t;P^3) = F^q_{3,T}(t)$$

F.F. of local tensor current

- **First moments of quasi-PDFs can be extracted from these relations**

Second moment : $n = 1$

- **Focus on local vector operator :** $\bar{\psi}^q(0)\gamma^\mu\psi^q(0)$
- **Ji's spin-sum rule :** $\int_{-1}^{1} dx\, x\, \big(H^q(x,\xi,t) + E^q(x,\xi,t)\big) = A^q(t) + B^q(t)$
- **Relation between quasi-GPDs & Ji's spin-sum rule :**

$$\begin{aligned}
\int_{-\infty}^{\infty} dx\, x\, \frac{1}{\delta}\left(H^q_{\mathrm{Q}(0)}(x,\xi,t;P^3) + E^q_{\mathrm{Q}(0)}(x,\xi,t;P^3)\right) &= \frac{1}{2}(1+\delta^2)\big(A^q(t)+B^q(t)\big) - \frac{1}{2}(1-\delta^2)D^q(t) \\
\int_{-\infty}^{\infty} dx\, x\, \left(H^q_{\mathrm{Q}(3)}(x,\xi,t;P^3) + E^q_{\mathrm{Q}(3)}(x,\xi,t;P^3)\right) &= A^q(t)+B^q(t)
\end{aligned}$$

$$\delta = \sqrt{1 + \frac{M^2 - t/4}{(P^3)^2}}$$

a) **Higher-twist contamination in second moment of** $H_{\mathrm{Q}(0)} + E_{\mathrm{Q}(0)}$ **; also F.F. from anti-symmetric EMT,** D **, contributes at finite** P^3

b) **Second moment of** $H_{\mathrm{Q}(3)} + E_{\mathrm{Q}(3)}$ **is directly related to total angular momentum of quarks**

- **Second moment relations for PDFs :**

$$\int_{-1}^{1} dx\, x\, f_1(x) = A^q(0)$$

$$\int_{-\infty}^{\infty} dx\, x\, \frac{1}{\delta_0} f_{1,\mathrm{Q}(0)}(x; P^3) = A^q(0)$$

$$\int_{-\infty}^{\infty} dx\, x\, f_{1,\mathrm{Q}(3)}(x; P^3) = A^q(0) - \frac{M^2}{(P^3)^2} \bar{C}^q(0)$$

a) **Second moment of** $f_{1,\mathrm{Q}(0)}$ **is** P^3**- independent only if divided by** δ_0

$$\delta_0 = \delta(\mathrm{t}=0) = \sqrt{1 + \frac{M^2}{(P^3)^2}}$$

b) **Second moment of** $f_{1,\mathrm{Q}(3)}$ **is** P^3**- dependent involving F.F.** $\bar{C}$

- **Our model calculations agree with all moment relations**
- P^3 **-dependence of moments either absent or calculable; moments may assist in studying systematic errors in lattice QCD results**

(Re-) definition of quasi-distributions

- **Moment analysis suggests preferred definition of quasi-PDFs & quasi-GPDs**

Example : $\tilde{f}_{1,\mathrm{Q}(0)} \equiv \frac{1}{\delta_0} f_{1,\mathrm{Q}(0)}$ $\quad \tilde{g}_{1,\mathrm{Q}(3)} \equiv \frac{1}{\delta_0} g_{1,\mathrm{Q}(3)}$ $\quad \tilde{h}_{1,\mathrm{Q}(0)} \equiv \frac{1}{\delta_0} h_{1,\mathrm{Q}(0)}$ $\quad \delta_0 = \sqrt{1 + \frac{M^2}{(P^3)^2}}$

- **Both definitions work since their difference is power-suppressed**

ξ - symmetry of quasi-GPDs

- **Behavior of standard GPDs under** $\boxed{\xi \to -\xi}$ **(Hermiticity & Time-Reversal) :**

For All twist-2 GPDs : $\boxed{X(x, -\xi, t) = +X(x, \xi, t)}$

Exception : $\boxed{\widetilde{E}_T(x, -\xi, t) = -\widetilde{E}_T(x, \xi, t)}$

- **Quasi-GPDs have the same behavior under** $\xi \to -\xi$

- **Negative** ξ **values can be used in lattice QCD calculations**

Summary

Quasi-GPDs in a scalar diquark model

- **For $P^3 \to \infty$, all our expressions for quasi-GPDs agree with 8 leading-twist standard GPDs**
- **For finite P^3, large discrepancies between quasi & standard GPDs at large x**
- **For finite P^3 & large ξ , good agreement between quasi & standard GPDs in ERBL region**
- **Explored higher-twist effects associated with x & ξ**
- **Model-independent analysis of moments of quasi-distributions**
- **Moment analysis suggests a preferred definition of quasi-distributions**
- **Moments might help to study systematic uncertainties in lattice QCD**
- **Quasi-GPDs & standard GPDs have the same ξ-symmetry**

Back-Up Slides

Standard GPD parameterization equations : (Meissner, Metz, Goeke, 2007)

$$
\begin{aligned}
F^{[\gamma^+]}(x,\Delta;\lambda,\lambda') &= \frac{1}{2P^+}\bar{u}(p',\lambda')\left[\gamma^+ H(x,\xi,t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M}E(x,\xi,t)\right]u(p,\lambda) \\
F^{[\gamma^+\gamma_5]}(x,\Delta;\lambda,\lambda') &= \frac{1}{2P^+}\bar{u}(p',\lambda')\left[\gamma^+\gamma_5 \widetilde{H}(x,\xi,t) + \frac{\Delta^+\gamma_5}{2M}\widetilde{E}(x,\xi,t)\right]u(p,\lambda) \\
F^{[i\sigma^{j+}\gamma_5]}(x,\Delta;\lambda,\lambda') &= -\frac{i\varepsilon^{-+ij}}{2P^+}\bar{u}(p',\lambda')\left[i\sigma^{+i}H_T(x,\xi,t) + \frac{\gamma^+\Delta_\perp^i - \Delta^+\gamma_\perp^i}{2M}E_T(x,\xi,t)\right. \\
&\quad \left. + \frac{P^+\Delta_\perp^i}{M^2}\widetilde{H}_T(x,\xi,t) - \frac{P^+\gamma_\perp^i}{M}\widetilde{E}_T(x,\xi,t)\right]u(p,\lambda)
\end{aligned}
$$

Quasi-GPD parameterization equations :

$$
\begin{aligned}
F^{[\gamma^0]}(x,\Delta;\lambda,\lambda';P^3) &= \frac{1}{2P^0}\bar{u}(p',\lambda')\left[\gamma^0 H_{\mathrm{Q}(0)}(x,\xi,t;P^3) + \frac{i\sigma^{0\mu}\Delta_\mu}{2M}E_{\mathrm{Q}(0)}(x,\xi,t;P^3)\right]u(p,\lambda) \\
F^{[\gamma^3\gamma_5]}(x,\Delta;\lambda,\lambda';P^3) &= \frac{1}{2P^0}\bar{u}(p',\lambda')\left[\gamma^3\gamma_5 \widetilde{H}_{\mathrm{Q}(3)}(x,\xi,t;P^3) + \frac{\Delta^3\gamma_5}{2M}\widetilde{E}_{\mathrm{Q}(3)}(x,\xi,t;P^3)\right]u(p,\lambda) \\
F^{[i\sigma^{j0}\gamma_5]}(x,\Delta;\lambda,\lambda';P^3) &= -\frac{i\varepsilon^{03ij}}{2P^0}\bar{u}(p',\lambda')\left[i\sigma^{3i}H_{\mathrm{T,Q}(0)}(x,\xi,t;P^3) + \frac{\gamma^3\Delta_\perp^i - \Delta^3\gamma_\perp^i}{2M}E_{\mathrm{T,Q}(0)}(x,\xi,t;P^3)\right. \\
&\quad \left. + \frac{P^3\Delta_\perp^i}{M^2}\widetilde{H}_{\mathrm{T,Q}(0)}(x,\xi,t;P^3) - \frac{P^3\gamma_\perp^i}{M}\widetilde{E}_{\mathrm{T,Q}(0)}(x,\xi,t;P^3)\right]u(p,\lambda)
\end{aligned}
$$

Parameterizations consistent with forward limit

Most general local EMT : (C. Lorcé, H. Moutarde, A. P. Trawiński, 2019)

$$\boxed{\begin{aligned}\langle p',\lambda'|T^{\mu\nu,q}(0)|p,\lambda\rangle &= \bar{u}(p',\lambda')\bigg[\frac{P^\mu P^\nu}{M}A^q(t)+\frac{\Delta^\mu\Delta^\nu-g^{\mu\nu}\Delta^2}{M}C^q(t)+Mg^{\mu\nu}\bar{C}^q(t)\\ &\quad+\frac{P^{\{\mu}i\sigma^{\nu\}\alpha}\Delta_\alpha}{4M}\big(A^q(t)+B^q(t)\big)+\frac{P^{[\mu}i\sigma^{\nu]\alpha}\Delta_\alpha}{4M}D^q(t)\bigg]u(p,\lambda)\end{aligned}}$$

$$T^{\mu\nu,q}(0)=\bar{\psi}^q(0)\gamma^\mu\tfrac{i}{2}\overleftrightarrow{D}^\nu\psi^q(0)$$