Novel opportunities for transverse momentum dependent distributions

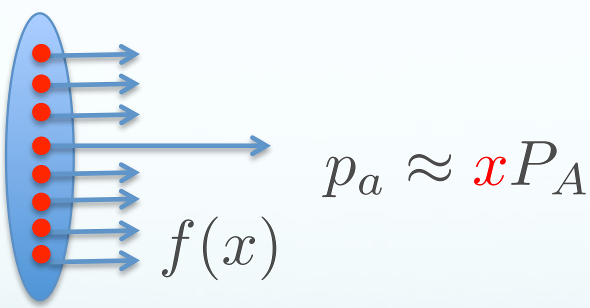
Zhongbo Kang
UCLA

Theory Seminar
Jefferson Lab
July 1, 2019

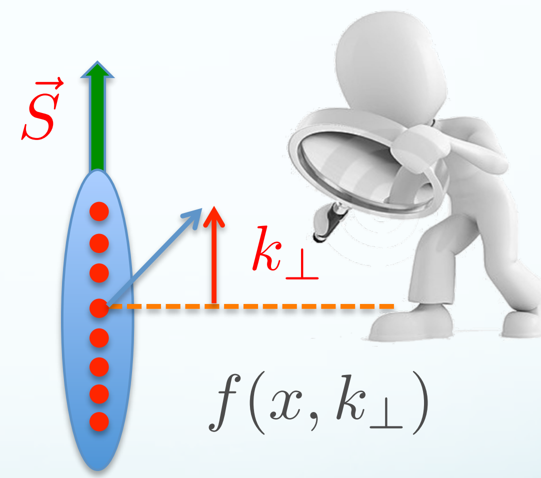
New structure of nucleon

- TMDs provide new structure of nucleon – 3D structure: both longitudinal + transverse momentum dependent structure (confined motion in a nucleon)

Transverse Momentum Dependent parton distribution (TMDs)



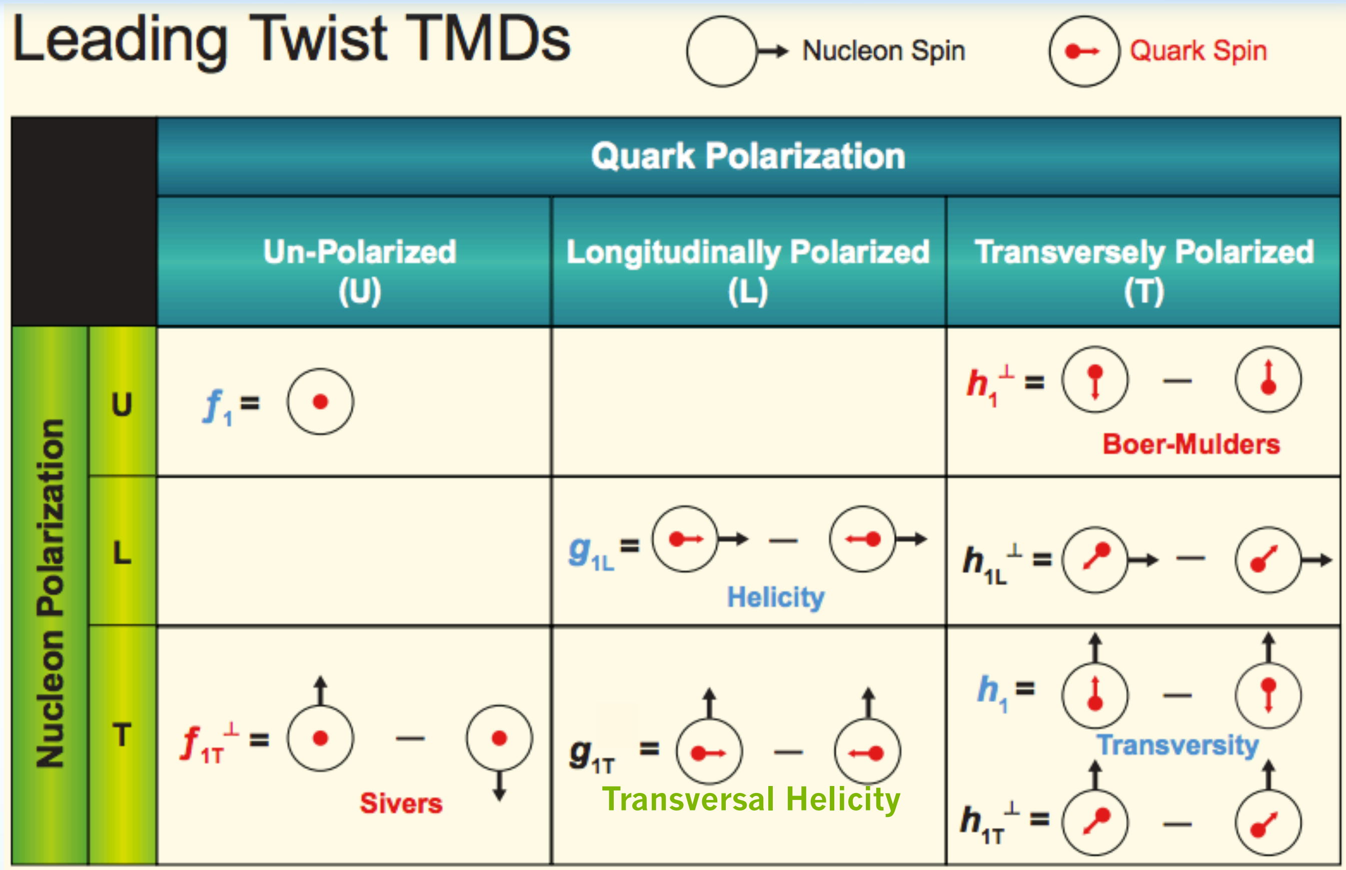
Longitudinal motion only



Longitudinal + transverse motion

TMDs: rich quantum correlations

TMD parton distribution

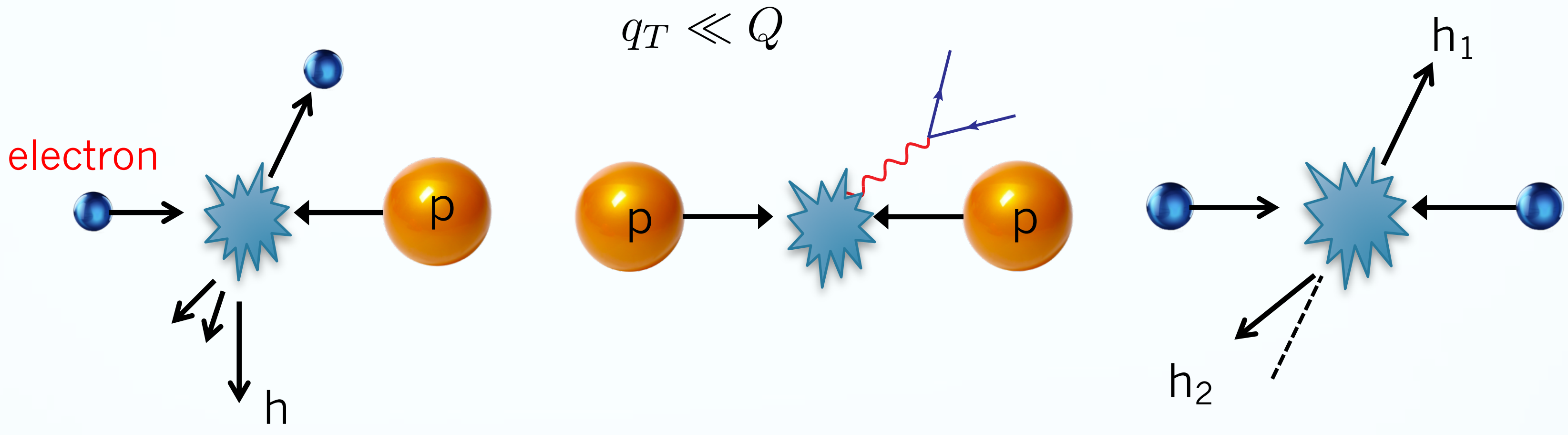


TMD fragmentation function

	Quark Polarization		
	U	L	T
Pion	D_1		$H_1^\perp$ **Collins**

Standard processes to extract TMDs

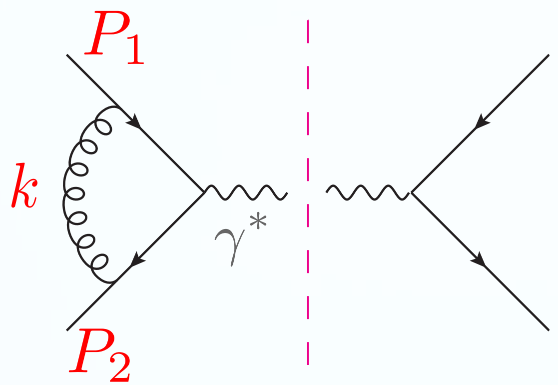
- SIDIS, Drell-Yan, dihadron in e^+e^-

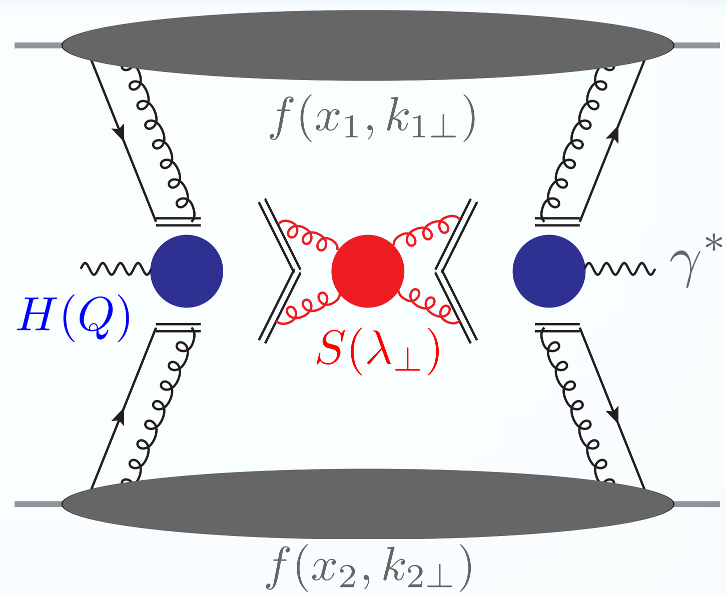


- They have a well-established TMD factorization formalism

TMD factorization in a nut-shell

- Drell-Yan: $p + p \to [\gamma^* \to \ell^+ \ell^-] + X$





Factorization of regions:
(1) k//P_1, (2) k//P_2, (3) k soft, (4) k hard

- Factorized form and mimic "parton model"

$$\frac{d\sigma}{dQ^2 dy d^2 q_\perp} \propto \int d^2 k_{1\perp}\, d^2 k_{2\perp}\, d^2 \lambda_\perp H(Q) f(x_1, k_{1\perp}) f(x_2, k_{2\perp}) S(\lambda_\perp) \delta^2(k_{1\perp} + k_{2\perp} + \lambda_\perp - q_\perp)$$

$$= \int \frac{d^2 b}{(2\pi)^2} e^{i q_\perp \cdot b} H(Q) f(x_1, b) f(x_2, b) S(b)$$

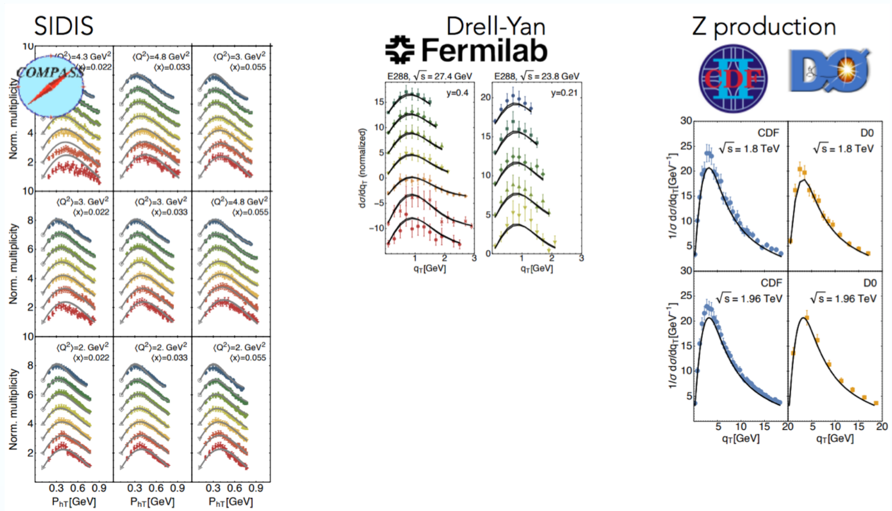
$$F(x, b) = f(x, b)\sqrt{S(b)}$$

$$\boxed{= \int \frac{d^2 b}{(2\pi)^2} e^{i q_\perp \cdot b} H(Q) F(x_1, b) F(x_2, b)}$$

mimic "parton model"

Extremely active phenomenology

- Examples: Pavia, Torino, EIKV, KSPY, DEMS, SV…

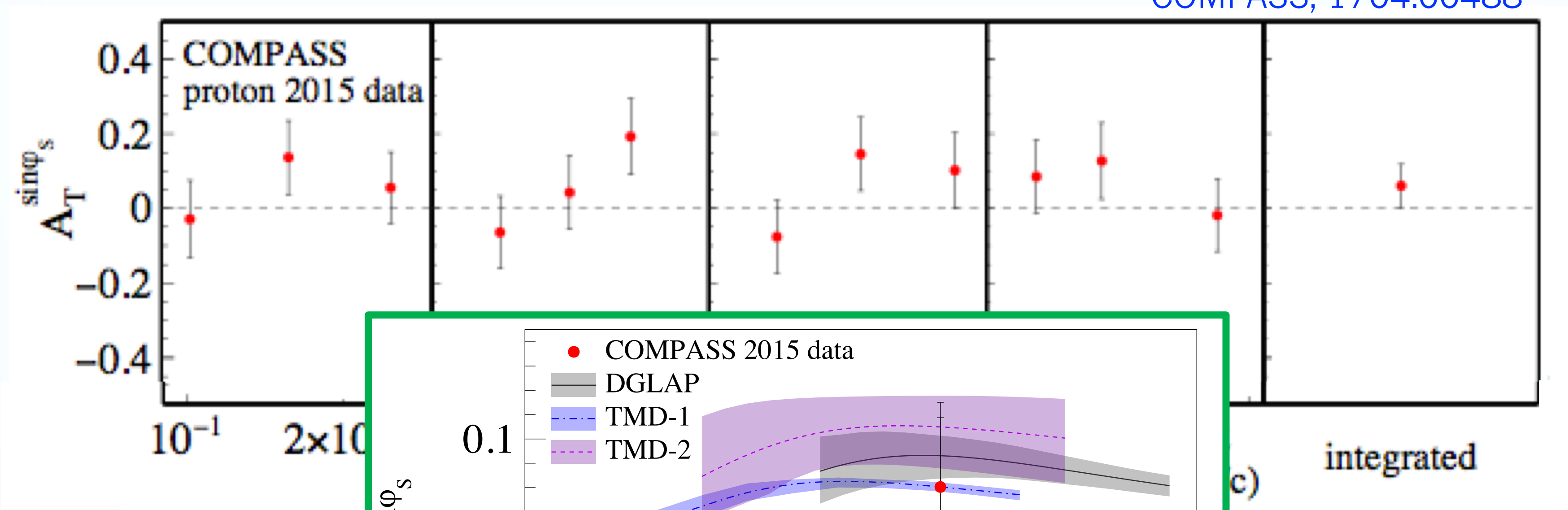


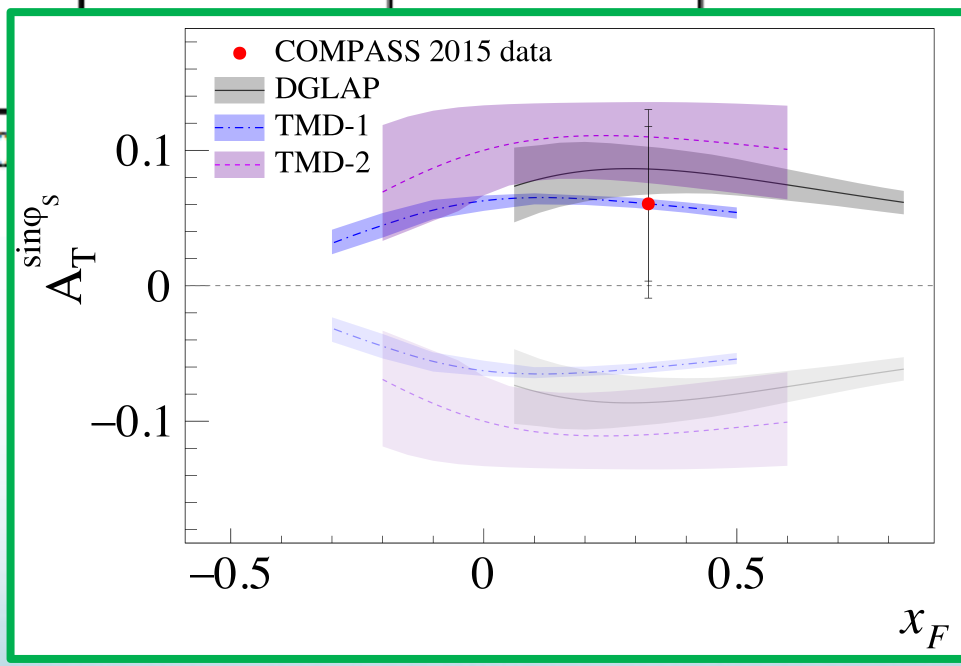
	Framework	W+Y	HERMES	COMPASS	DY	Z production	N of points
KN 2006 hep-ph/0506225	LO-NLL	W	✘	✘	✔	✔	98
QZ 2001 hep-ph/0506225	NLO-NLL	W+Y	✘	✘	✔	✔	28 (?)
RESBOS resbos@msu	NLO-NNLL	W+Y	✘	✘	✔	✔	>100 (?)
Pavia 2013 arXiv:1309.3507	LO	W	✔	✘	✘	✘	1538
Torino 2014 arXiv:1312.6261	LO	W	✔ (separately)	✔ (separately)	✘	✘	576 (H) 6284 (C)
DEMS 2014 arXiv:1407.3311	NLO-NNLL	W	✘	✘	✔	✔	223
EIKV 2014 arXiv:1401.5078	LO-NLL	W	1 (x,Q^2) bin	1 (x,Q^2) bin	✔	✔	500 (?)
SIYY 2014 arXiv:1406.3073	NLO-NLL	W+Y	✘	✔	✔	✔	200 (?)
Pavia 2017 arXiv:1703.10157	LO-NLL	W	✔	✔	✔	✔	8059
SV 2017 arXiv:1706.01473	NNLO-NNLL	W	✘	✘	✔	✔	309
BSV 2019 arXiv:1902.08474	NNLO-NNLL	W	✘	✘	✔	✔	457

Sivers sign change between SIDIS and DY

- First experimental hint on the sign change in Drell-Yan

COMPASS, 1704.00488

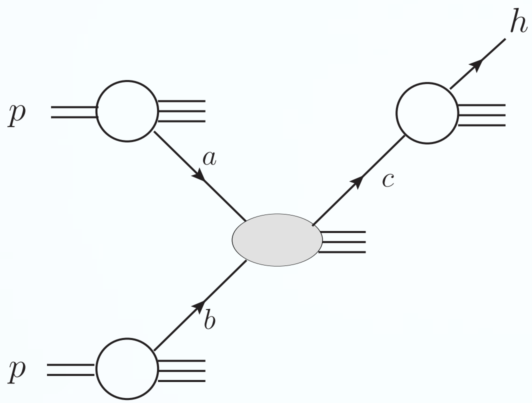


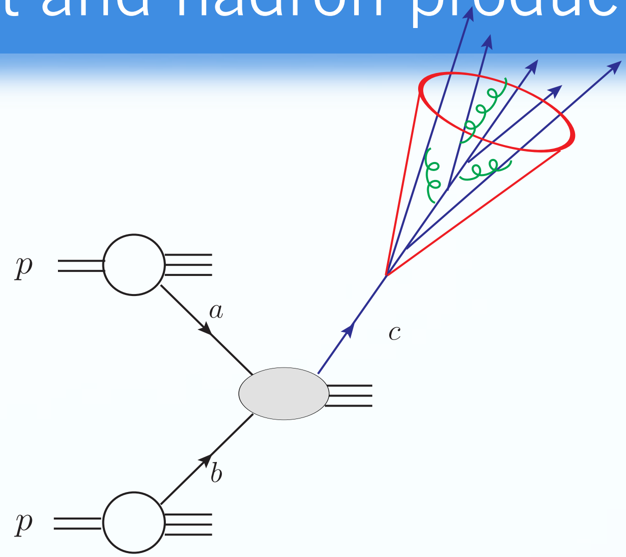


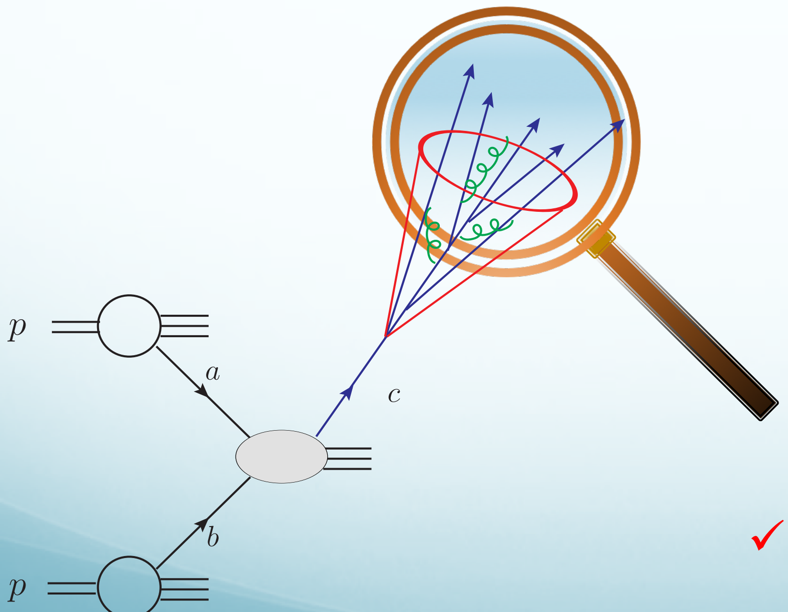
How to move forward

- Constraining ourselves ONLY to these three processes would limit the productivity of ourselves
- It is opportune time to explore other opportunities
- What are they?

A unified framework for jet and hadron production







$$\frac{d\sigma^{pp\to hX}}{dp_T d\eta} \sim f_a \otimes f_b \otimes H_{ab\to c} \otimes D_c^h(z,\mu)$$

$$\frac{d\sigma^{pp\to \mathrm{jet}X}}{dp_T d\eta} \sim f_a \otimes f_b \otimes H_{ab\to c} \otimes J_c(z, p_T R, \mu)$$

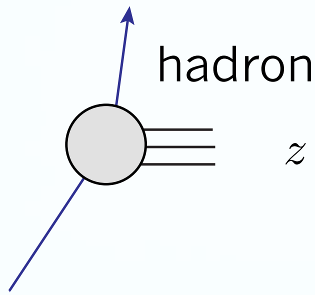
$$\frac{d\sigma^{pp\to \mathrm{jet}(\tau)X}}{dp_T d\eta} \sim f_a \otimes f_b \otimes H_{ab\to c} \otimes \mathcal{G}_c(z, p_T R, \tau, \mu)$$

- ✓ Same hard functions, telling us the quark and gluon jet ratios order by order in pQCD

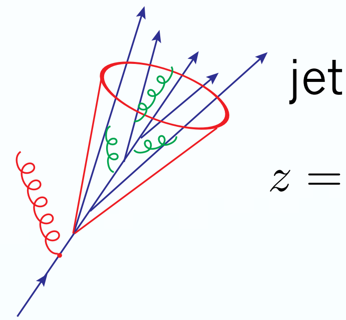
Kang, Ringer, Vitev, 1606.06732, 1606.07063, Dai, Kim, Leibovich, 1606.07411, see also, Kaufmann, Mukherjee, Vogelsang, 1506.01415

What are these jet functions?

- They are usually referred to as “semi-inclusive jet function”



$$z = \frac{p_{\rm h}^{+}}{p_{\rm parton}^{+}}$$



$$z = \frac{p_{\rm jet}^{+}}{p_{\rm parton}^{+}}$$

- They follow DGLAP evolution equation
  - All jet substructures are contained in these functions

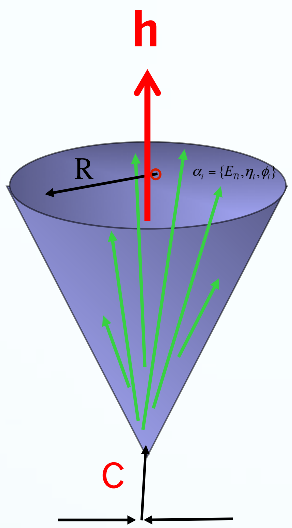
$$\mu\frac{d}{d\mu}D_i^h(z,\mu) = \sum_j P_{ji}\otimes D_j^h(z,\mu)$$

$$\mu\frac{d}{d\mu}J_i(z,p_T R,\mu) = \sum_j P_{ji}\otimes J_j(z,p_T R,\mu)$$

$$\mu\frac{d}{d\mu}\mathcal{G}_i(z,p_T R,\tau,\mu) = \sum_j P_{ji}\otimes \mathcal{G}_j(z,p_T R,\tau,\mu)$$

Jet fragmentation function

- First produce a jet, and then further look for a hadron inside the jet



$$F(z_h, p_T) = \frac{d\sigma^h}{dy\,dp_T\,dz_h} \Big/ \frac{d\sigma}{dy\,dp_T}$$

$$z_h = p_T^h / p_T$$

$$z = p_T / p_T^c$$

Kang, Ringer, Vitev, JHEP 2016

- Just like the single inclusive jet production, we have
  - Semi-inclusive fragmenting jet function

$$\frac{d\sigma}{dy\,dp_T\,dz_h} \propto \sum_{a,b,c} f_a \otimes f_b \otimes H_{ab\to c} \otimes \mathcal{G}_c^h(z, z_h, R, \mu)$$

Two DGLAPs

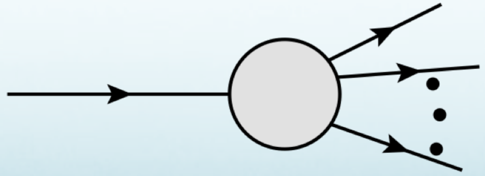
- Again DGLAP evolution: evolution is for variable z

$$\mu \frac{d}{d\mu} \mathcal{G}_i^h(z, z_h, \mu) = \frac{\alpha_s(\mu)}{\pi} \sum_j \int_z^1 \frac{dz'}{z'} P_{ji}\left(\frac{z}{z'}\right) \mathcal{G}_j^h(z', z_h, \mu)$$

- Relation to standard FFs: relevant to variable z_h

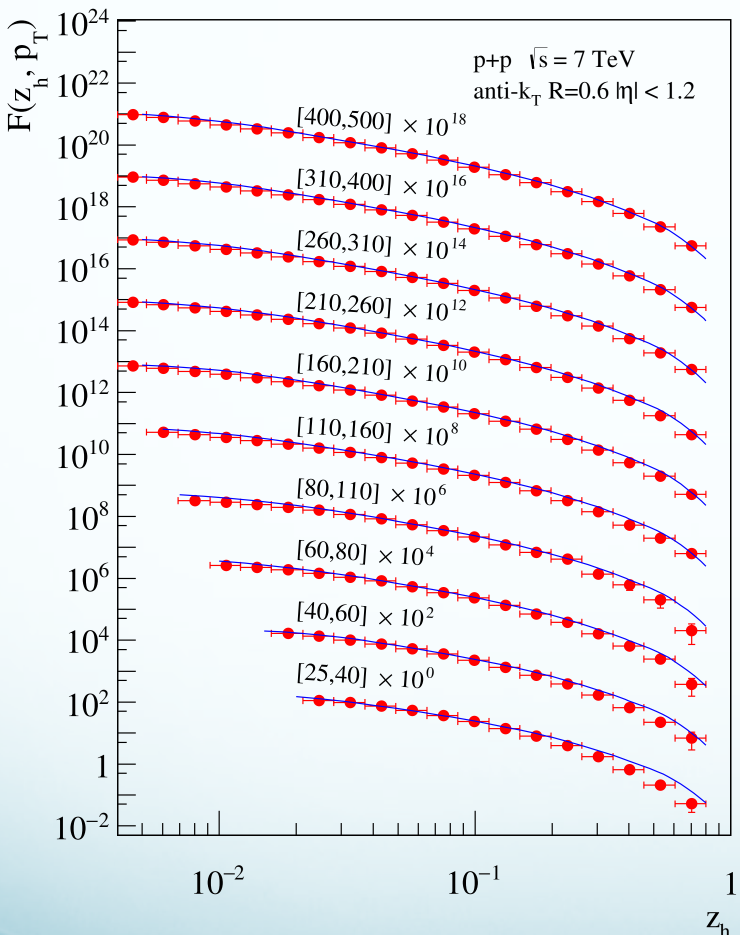
$$\mathcal{G}_i^h(z, z_h, \mu) = \sum_j \int_{z_h}^1 \frac{dz_h'}{z_h'} \mathcal{J}_{ij}(z, z_h', \mu) D_j^h\left(\frac{z_h}{z_h'}, \mu\right)$$

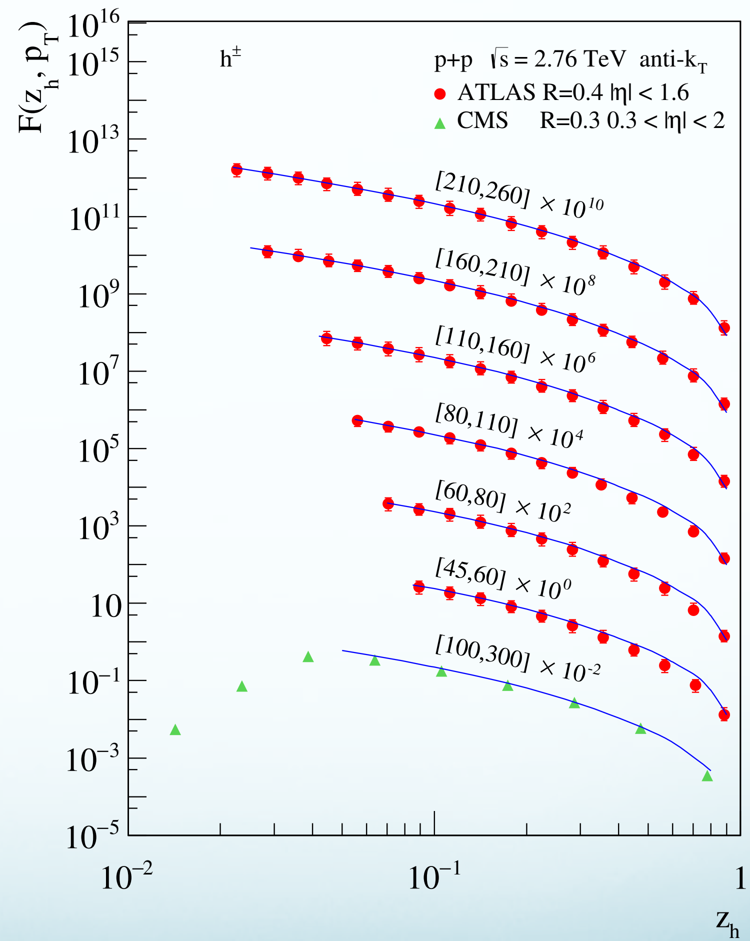
- Fragmentation function: probability for a quark/gluon converted itself into a hadron



Some interesting phenomenology

- Works pretty well in comparison with experimental data



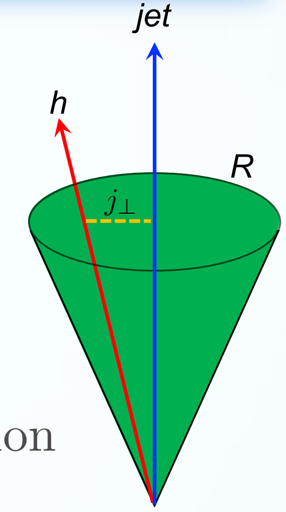


Kang, Ringer, Vitev, arXiv:1606.07063

TMD hadron distribution inside the jet?

- Definition

$$F(z_h, j_\perp; p_T) = \frac{d\sigma^h}{dp_T d\eta dz_h d^2 j_\perp} \Big/ \frac{d\sigma}{dp_T d\eta}$$



$$z_h = p_T^{\mathrm{h}} / p_T^{\mathrm{jet}}$$

$j_\perp$: hadron transverse momentum with respect to the jet direction

- Factorization formalism

Kang, Liu, Ringer, Xing, 1705.08443

$$\frac{d\sigma}{dp_T d\eta dz_h d^2 j_\perp} \propto \sum_{a,b,c} f_a \otimes f_b \otimes H_{ab\to c} \otimes \mathcal{G}_c^h(z, z_h, \omega_J R, j_\perp, \mu)$$

- Related to transverse momentum dependent (TMD) fragmenting function

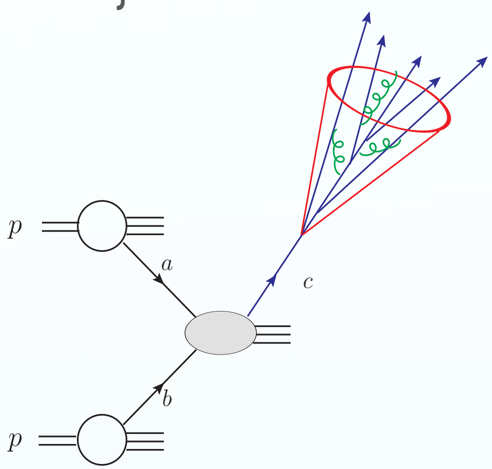
$$\mathcal{G}_c^h(z, z_h, \omega_J R, \boldsymbol{j}_\perp, \mu) = \mathcal{H}_{c\to i}(z, \omega_J R, \mu) \int d^2\boldsymbol{k}_\perp d^2\boldsymbol{\lambda}_\perp \delta^2\left(z_h \boldsymbol{\lambda}_\perp + \boldsymbol{k}_\perp - \boldsymbol{j}_\perp\right)$$
$$\times D_{h/i}(z_h, \boldsymbol{k}_\perp, \mu, \nu) S_i(\boldsymbol{\lambda}_\perp, \mu, \nu R)$$

Characteristics: hadron in the jet

- Soft radiation has to happen inside the jet
  - Only the soft radiation inside the jet can change the hadron transverse momentum with respect to the jet axis
- Restricts soft radiation to be within the jet
  - Cuts half of the rapidity divergence

$$\int_0^\infty \frac{dy}{y} \Rightarrow \int_0^{\tan^2\frac{R}{2}} \frac{dy}{y}$$

$$y \sim \frac{\ell^+}{\ell^-}$$

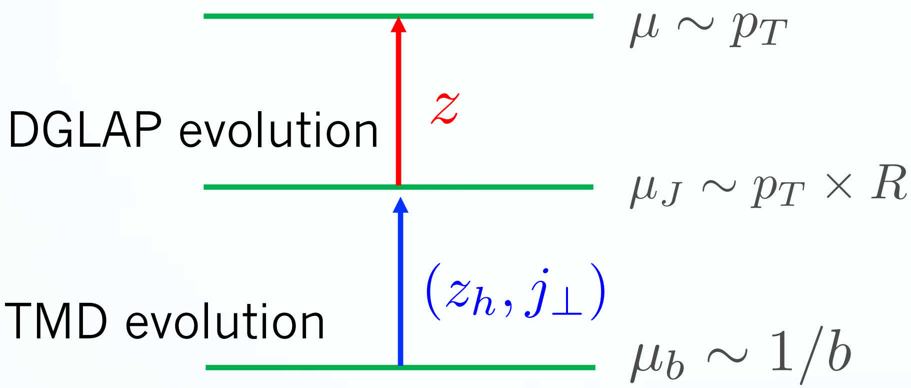


- Rapidity divergence cancel between restricted "soft factor" and TMD FFs
  - At least up to this order, the combined evolution is the same as the usual TMD evolution in SIDIS, e+e-; justify the use of same TMD evolution here

$$\sqrt{S(b)}D_c^h(z_h, b)_{\mathrm{e^+e^-}} \Rightarrow S(b,R)D_c^h(z_h,b)_{\mathrm{pp}}$$

TMD + DGLAP evolution

- Evolution structure



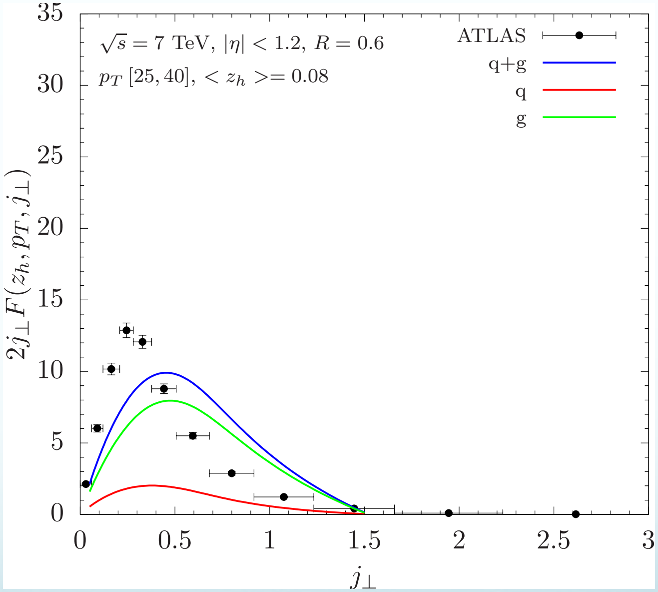
Resum ln(R)

Evolve TMD FFs from $\boldsymbol{\mu}_b$ to pT*R

- TMD FFs thus are related to the usual TMD FFs in SIDIS at scale pT*R
- Thus hadron TMD distribution inside the jet could be used to test the universality of TMD FFs from SIDIS, e+e- processes

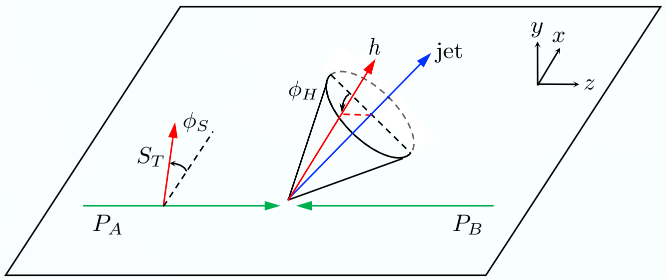
Problem in comparison with LHC data

- Currently the LHC data integrate over entire z_h region: [0,1]
  - Fragmentation function is only constrained for z > 0.05
  - At both small *z* and large *z*, there are logarithm of ln(*z*) or ln(*1-z*), which has to be resumed to have a better convergence
- Inclusive jet is more sensitive to gluon TMD fragmentation functions
- What about polarized case?
- What about quark TMD FFs?
  - Photon+jet
  - Z+jet



Collins asymmetry in p+p

- It can be studied through the azimuthal distribution of hadrons inside a jet in p+p collisions



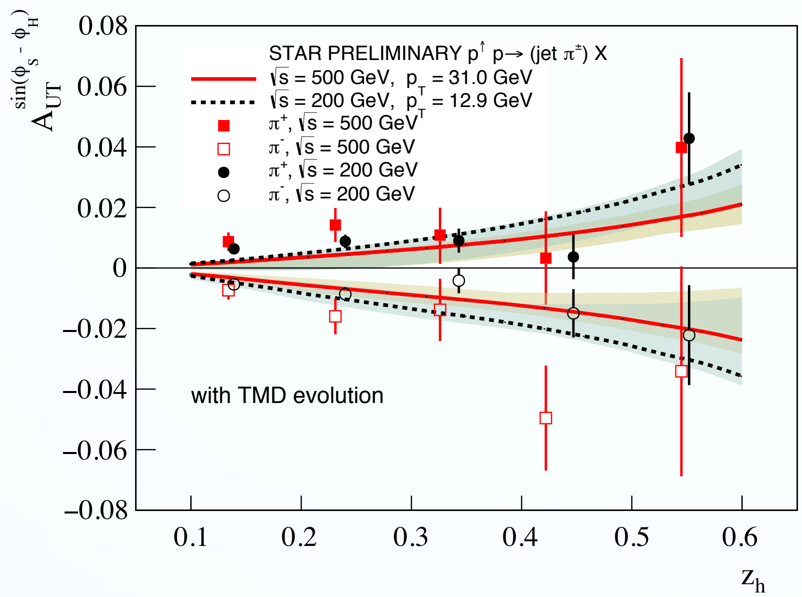
$$p^{\uparrow}\left[\vec{S}_{\perp}(\phi_S)\right] + p \to [\text{jet}\, h(\phi_H)] + X$$

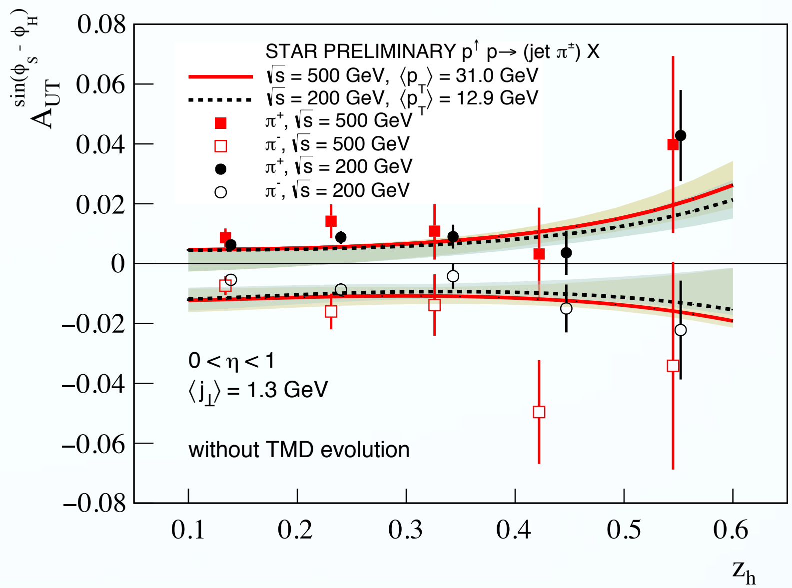
$$\frac{d\sigma}{dy d^2 p_{\perp}^{\text{jet}} dz d^2 j_T} = F_{UU} + \sin(\phi_S - \phi_H) F_{UT}^{\sin(\phi_S - \phi_H)}$$

$$F_{UT}^{\sin(\phi_S - \phi_H)} \propto h_1^a(x_1) \otimes f_{b/B}(x_2) \otimes \frac{j_T}{z M_h} H_1^{\perp c}(z, j_T^2) \otimes H_{ab\to c}^{\text{Collins}}(\hat{s}, \hat{t}, \hat{u})$$

- Such an asymmetry has been measured by STAR at RHIC
  - Could be used to test the universality of the Collins functions

Calculated Collins azimuthal asymmetry



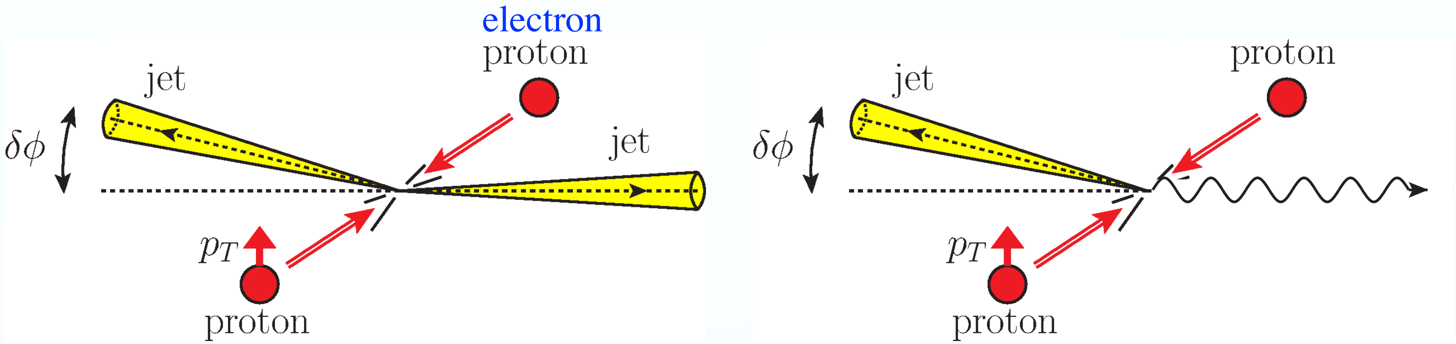


- *Universality of Collins function between e+p, e+e, and p+p*
- *Test TMD evolution*

Kang, Prokudin, Ringer, Yuan, 1707.00913, PLB

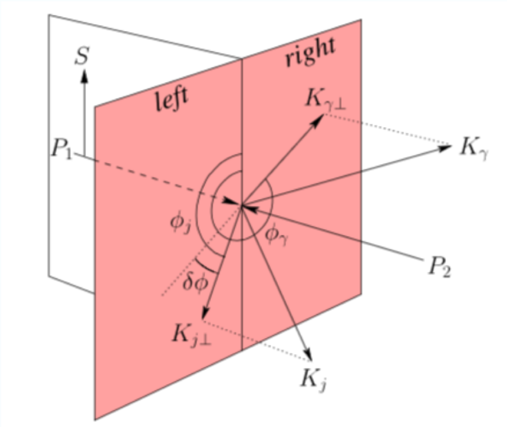
Vector-boson tagged jets or dijets

- Back-to-back two particle/jet production in p+p/e+p collisions
  - jets as a novel probe: many other interesting ideas



- What is the status?
  - Theory – proposed in the past: Boer-Vogelsang 04, Qiu-Vogelsang-Yuan 07 (dijet), Bacchetta-Bomhof-D'Alesio-Mulders-Murgia 07 (photon+jet)
  - Experiment – measurements available in the past at RHIC: STAR 08 (dijet), PHENIX has a proposal for photon+jet

Early study on photon+jet

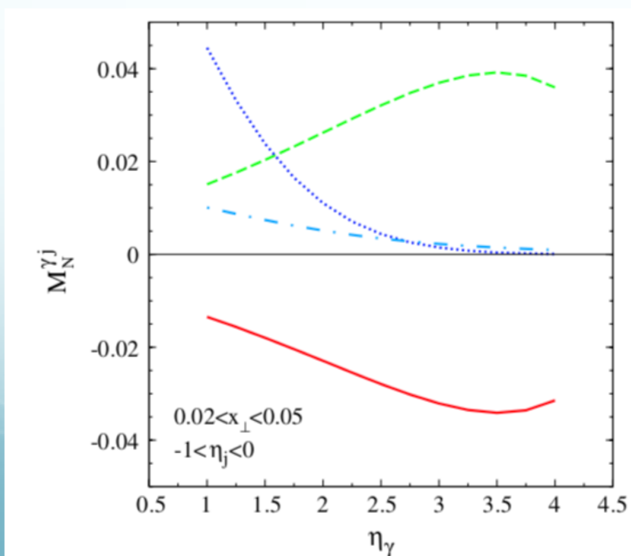


PRL **99**, 212002 (2007) PHYSICAL REVIEW LETTERS week ending 23 NOVEMBER 2007

Sivers Single-Spin Asymmetry in Photon-Jet Production

Alessandro Bacchetta,[1] Cedran Bomhof,[2] Umberto D'Alesio,[3] Piet J. Mulders,[2] and Francesco Murgia[3]
[1]*Theory Group, Deutsches Elektronen-Synchroton DESY, 22603 Hamburg, Germany*
[2]*Department of Physics and Astronomy, Vrije Universiteit Amsterdam, 1081 HV Amsterdam, The Netherlands*
[3]*INFN, Sezione di Cagliari and Dipartimento di Fisica, Università di Cagliari, 09042 Monserrato, Italy*
(Received 19 March 2007; published 21 November 2007)

$$\delta\phi = \phi_j - \phi_\gamma - \pi$$



$$M_N^{\gamma j}(\eta_\gamma, \eta_j, x_\perp) = \frac{\int d\phi_j d\phi_\gamma \frac{2|\boldsymbol{K}_{\gamma\perp}|}{M} \sin(\delta\phi)\cos(\phi_\gamma) \frac{d\sigma}{d\phi_j d\phi_\gamma}}{\int d\phi_j d\phi_\gamma \frac{d\sigma}{d\phi_j d\phi_\gamma}}$$

FIG. 5 (color online). Prediction for the azimuthal moment $M_N^{\gamma j}$ at $\sqrt{s} = 200$ GeV, as a function of η_γ, integrated over $-1 \leq \eta_j \leq 0$ and $0.02 \leq x_\perp \leq 0.05$. Solid line: using gluonic-pole cross sections. Dashed line: using standard partonic cross sections. Dotted line: maximum contribution from the gluon Sivers function (absolute value). Dot-dashed line: maximum contribution from the Boer-Mulders function (absolute value).

Issues: TMD factorization breaking

- TMD factorization breaking for dijet and photon+jet in pp collisions

  Collins-Qiu 07, Yuan-Vogelsang 07, Rogers-Mulder 10, …

- After this, many experimental efforts have been discouraged

Legitimate concerns and what's needed

- Experimentalists' general concern
  - Since no TMD factorization formalism any more, if I performed a measurement (which takes lots of efforts), how am I going to interpret my results?
  - What theory to compare with?
- Recently there are experimental measurements pointing to probe factorization breaking
  - Is there factorization breaking?
  - If there is, is it small or large?
  - How do you assess?
- All these concerns are due to the fact that we do NOT have a theoretical framework for TMDs in these processes
  - Such a TMD framework is urgently needed

Opportunity, NOT a failure

PHYSICAL REVIEW D **75,** 114014 (2007)

k_T factorization is violated in production of high-transverse-momentum particles in hadron-hadron collisions

John Collins*

Physics Department, Penn State University, 104 Davey Laboratory, University Park Pennsylvania 16802, USA

Jian-Wei Qiu†

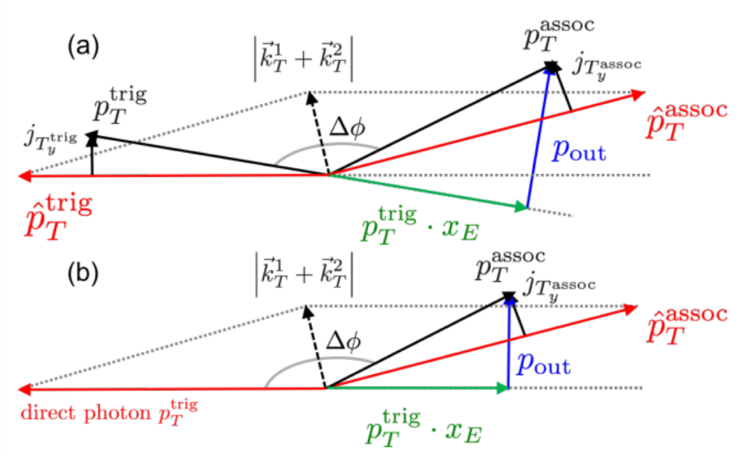
Department of Physics and Astronomy, Iowa State University, Ames Iowa 50011, USA
and High Energy Physics Division, Argonne National Laboratory, Argonne Illinois 60439, USA

(Received 15 May 2007; published 28 June 2007)

Troublesome though it may be for phenomenology, breaking of factorization should be viewed not as some kind of failure, but as an opportunity. Examination of the distribution of high-transverse-momentum hadrons in hadron-hadron collisions will lead to interesting nontrivial phenomena.

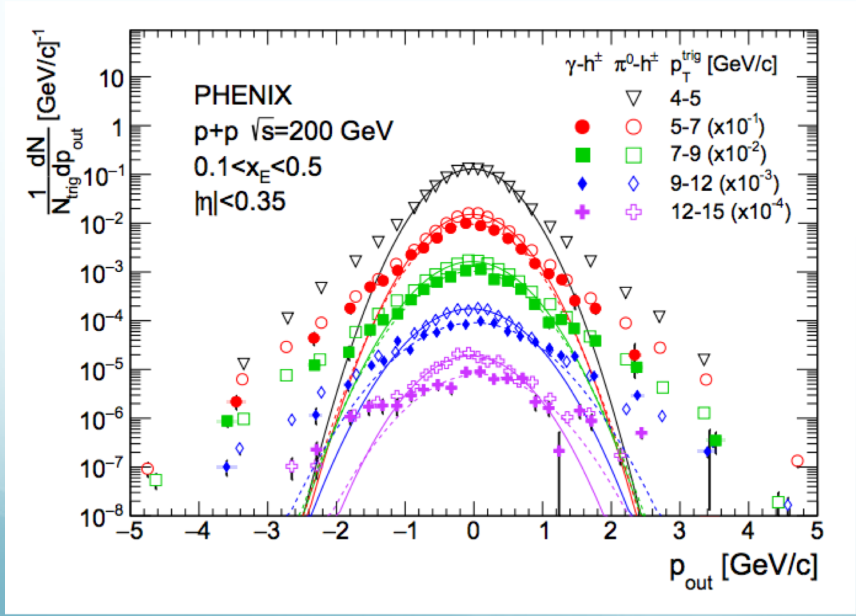
Motivations

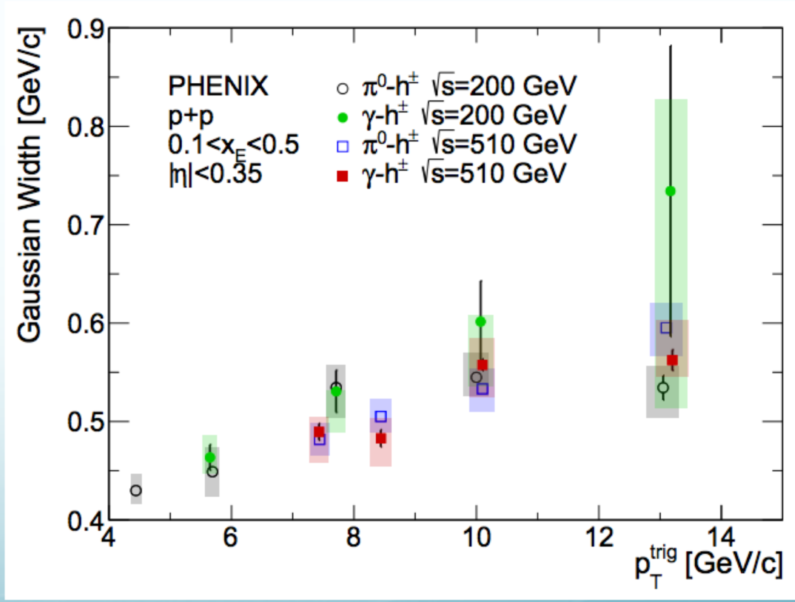
- Experimental measurements: photon+hadron, dihadron



PHENIX 1805.02450, 1609.04769

Even though factorization breaking is present from a theoretical side, we would like to develop a framework for handling such processes





Strategy on how to procced

- Factorization breaking is due to Glauber region
  - Quote from Rogers and Mulders on original TMD factorization breaking paper 1001.2977: "**We remark that, because the TMD factorization breaking effects are due to the Glauber region** …"
- The proper strategy to move forward would be to ignoring the Glauber modes, and study the factorization properties based on the hard, the collinear, and the soft degrees of freedom
- SCET is perfect for this purpose
  - The original SCET formulation has ignored Glauber modes, contains only hard, collinear, and soft modes
    - Which has been criticized by conventional QCD experts, but nevertheless having great success in applications
  - The Glauber modes are studied by Rothstein and Stewart 1601.04695
- Our starting point: the original SCET formulation

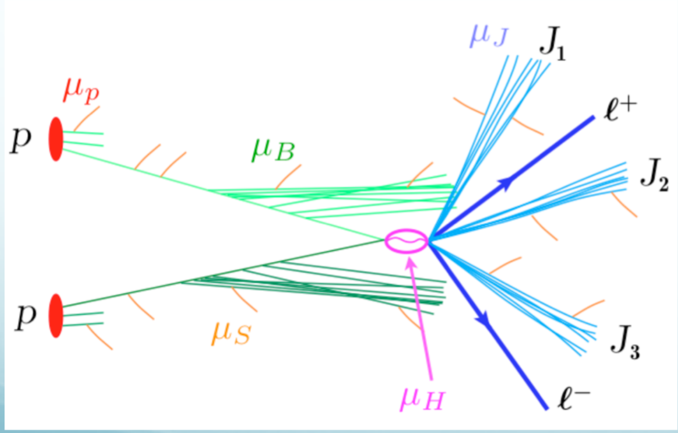
Soft-Collinear Effective Theory (SCET)

- SCET: an effective field theory of QCD — Bauer et al. 01, Pirjol et al. 04
  - Suitable for processes where there are energetic, nearly light-like (collinear) degrees of freedom interacting with one another via soft radiation

- Modes in SCET

modes	$p^\mu = (+, -, \perp)$
hard	$Q(1,1,1)$
collinear	$Q(1,\lambda^2,\lambda)$
soft	$Q(\lambda,\lambda,\lambda)$

- QCD factorization of modes



$$\sigma = H \otimes S \otimes \prod_{i=1}^{n_B} B_i \otimes \prod_{j=1}^{N} J_j$$

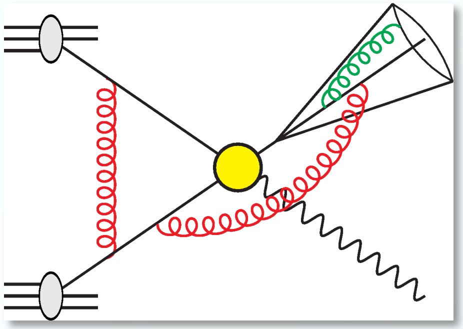
Factorized form in SCET

- Within SCET, one can derive a factorized form in terms of
  - TMD PDFs: collinear d.o.f.
  - Soft functions: soft d.o.f.
  - Jet function: jet production

$$\vec{q}_\perp \equiv \vec{p}_{\gamma\perp} + \vec{p}_{J\perp}$$
$$p_\perp = |\vec{p}_{\gamma\perp} - \vec{p}_{J\perp}|/2$$

$P + P \to \gamma + jet$

$$\frac{d\sigma}{dy_J dy_\gamma dp_\perp d^2\vec{q}_\perp} = \sum_{a,b,c} \int d\phi_J \int \prod_i^4 d^2\vec{k}_{i\perp} \delta^{(2)}(\vec{q}_\perp - \sum_i^4 \vec{k}_{i\perp})$$
$$\times f_a^{\text{unsub}}(x_a, k_{1\perp}^2) f_b^{\text{unsub}}(x_b, k_{2\perp}^2) S_{n\bar{n}n_J}^{\text{global}}(\vec{k}_{3\perp})$$
$$\times S_{n_J}^{cs}(\vec{k}_{4\perp}, R) H_{ab\to c\gamma}(p_\perp) J_c(p_\perp R)$$

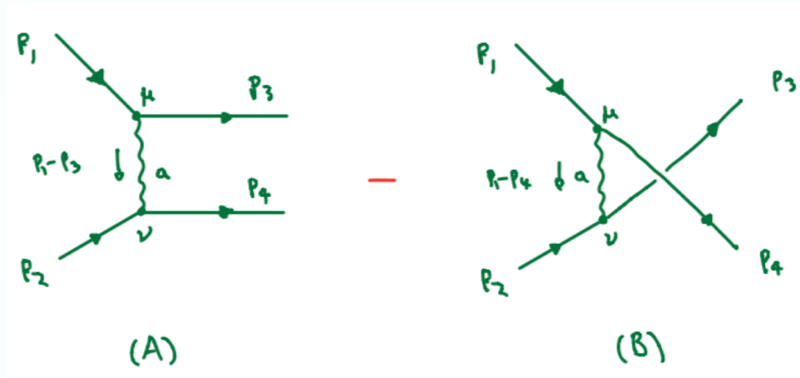


- Two soft functions
  - Global soft function: soft radiation in full phase space
    - Like those in e+e-, SIDIS, DY, but three colored partons
  - Collinear soft function: no rapidity divergence
    - Soft radiation that happens inside the jet does not generate any imbalance
    - Should be along the direction of the jet pT at leading power, for R << 1

Buffing, Kang, Lee, Liu, arXiv:1812.07549

Dijet: leading order - 1

- Color structure is more complicated
  - Expand in terms of color basis
- Example: qq→qq



$$\text{Fig-A} = \bar{u}(p_3)\,(-ig\gamma^\mu t^a)\,u(p_1)\;\bar{u}(p_4)\,(-ig\gamma^\nu t^a)\,u(p_2)\;\frac{-ig^{\mu\nu}}{(p_1-p_3)^2}$$

$$= ig^2\,\frac{1}{t}\;\bar{u}(p_3)\,\gamma^\mu t^a\,u(p_1)\;\bar{u}(p_4)\,\gamma_\mu t^a\,u(p_2)$$

$$\text{Fig-B} = -\bar{u}(p_4)\,(-ig\gamma^\mu t^a)\,u(p_1)\;\bar{u}(p_3)\,(-ig\gamma^\nu t^a)\,u(p_2)\;\frac{-ig^{\mu\nu}}{(p_1-p_4)^2}$$

$$= -ig^2\,\frac{1}{u}\;\bar{u}(p_4)\,\gamma^\mu t^a\,u(p_1)\;\bar{u}(p_3)\,\gamma_\mu t^a\,u(p_2)$$

Dijet: leading order - 2

- Color basis

$$\theta_1 = t^a_{i_3, i_1}\, t^a_{i_4, i_2}$$

$$\theta_2 = \mathbb{1}_{i_3, i_1}\, \mathbb{1}_{i_4, i_2} = \delta_{i_3, i_1}\, \delta_{i_4, i_2}$$

- Expand both diagrams

$$\text{figA} = A\, \theta_1$$

$$\text{figB} = B\left(-\frac{1}{C_A}\theta_1 + \frac{C_F}{C_A}\theta_2\right)$$

where

$$A = i g^2 \frac{1}{t}\, \bar{u}(p_3)\gamma^\mu u(p_1)\; \bar{u}(p_4)\gamma_\mu u(p_2)$$

$$B = -i g^2 \frac{1}{u}\, \bar{u}(p_4)\gamma^\mu u(p_1)\; \bar{u}(p_3)\gamma_\mu u(p_2)$$

- Total amplitude

$$\mathcal{M} = \text{fig}A + \text{fig}B$$

$$= \left(A - \frac{1}{C_A}B\right)\theta_1 + \frac{C_F}{C_A}B\,\theta_2 = M_1\theta_1 + M_2\theta_2$$

Dijet: leading order - 3

- Hard and Soft function matrices

$$|M|^2 = (M_1 \theta_1 + M_2 \theta_2)(\bar{M}_1 \bar{\theta}_1 + \bar{M}_2 \bar{\theta}_2)$$

$$H = \begin{pmatrix} M_1 \\ M_2 \end{pmatrix} (\bar{M}_1 \quad \bar{M}_2) = \begin{pmatrix} M_1 \bar{M}_1 & M_1 \bar{M}_2 \\ M_2 \bar{M}_1 & M_2 \bar{M}_2 \end{pmatrix}$$

$$S = \begin{pmatrix} \bar{\theta}_1 \\ \bar{\theta}_2 \end{pmatrix} (\theta_1 \quad \theta_2) = \begin{pmatrix} \bar{\theta}_1 \theta_1 & \bar{\theta}_1 \theta_2 \\ \bar{\theta}_2 \theta_1 & \bar{\theta}_2 \theta_2 \end{pmatrix}$$

- Trace in color space $|M|^2 = \mathrm{Tr}\left[H^{(0)} S^{(0)}\right]$

$$H_{qq\to qq} = \frac{8g^4}{C_A^2} \frac{1}{t^2 u^2} \begin{bmatrix} t^4 + C_A^2 u^4 + s^2 (t - C_A u)^2 & -C_F \left(t^4 + s^2 (t^2 - C_A t u)\right) \\ -C_F \left(t^4 + s^2 (t^2 - C_A t u)\right) & C_F^2 \, t^2 (s^2 + t^2) \end{bmatrix}$$

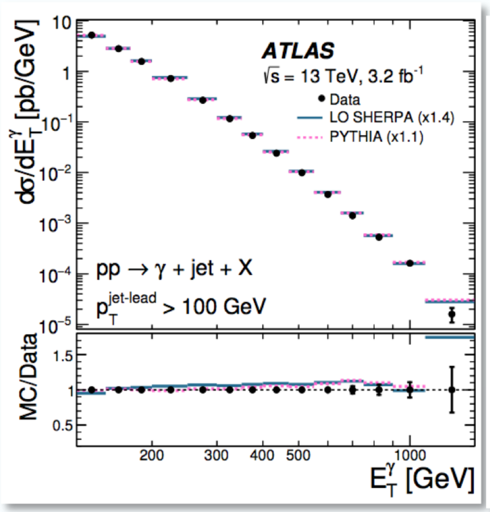
$$S = \begin{pmatrix} \frac{1}{2} C_A C_F & 0 \\ 0 & C_A^2 \end{pmatrix}$$

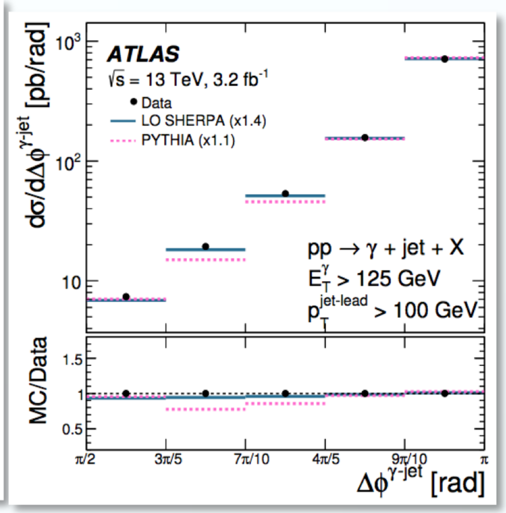
Standard, nothing abnormal

- The above formulation has been well established in the SCET community (also from standard QCD people), for unpolarized cross section
  - ✓ Catani, Grazzini, et.al., transverse momentum resummation for heavy quark/top pair in p+p, arXiv: 1408.4564, 1806.01601, 1901.04005
  - ✓ Li, Li, Shao, Zhu, top quark pair, 1307.2464
  - ✓ Shao, Li, Li, Vector boson+jet, 1309.5015
  - ✓ Chien, Shao, Wu, Z+jet, arXiv: 1905.01335: confirm our factorization for photon+jet

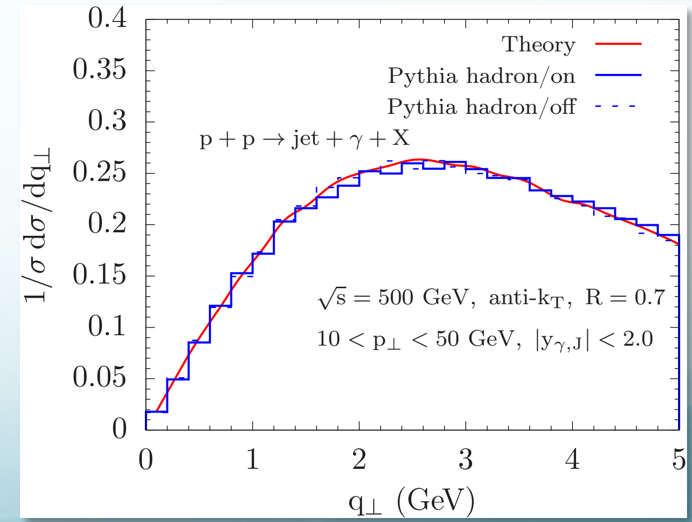
Phenomenology

- No data as a function of imbalance, but lots of LHC data on other variables for photon+jet
  - Pythia describes them well





- Compare with Pythia
  - Extremely well
  - Indicate TMD factorization breaking small??

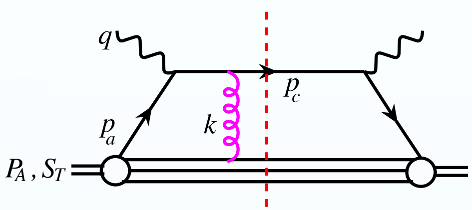


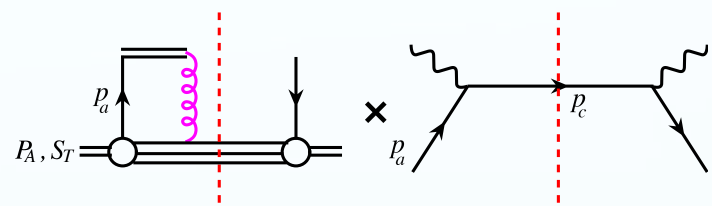
The unsolved issue: gauge link

- While all seem to be great, there are some caveat
  - The TMD PDFs in the above formalism has simple gauge link structure
  - Think of them as either SIDIS or DY: unpolarized TMDs are equal

$$\langle p|\bar{\chi}(y^-, y_\perp)\chi(0)|p\rangle$$

  - This might be enough for unpolarized TMDs, but not for Sivers, in which gauge link leads to process-dependence





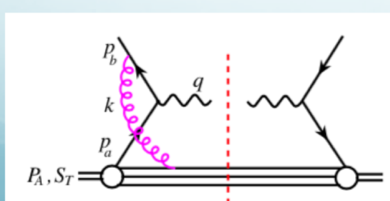
(a) (b)

$$\bar{u}(p_c)(-ig)\gamma^- T^a \frac{i(\not{p}_c - \not{k})}{(p_c - k)^2 + i\epsilon} \approx \bar{u}(p_c)\left[\frac{g}{-k^+ + i\epsilon} T^a\right]$$

$$-i\pi\delta(k^+)$$

$$f_{1T}^{\perp}|_{\rm SIDIS} = -f_{1T}^{\perp}|_{\rm DY}$$

$$+i\pi\delta(k^+)$$



$$\bar{v}(p_b)(-ig)\gamma^- T^a \frac{-i(\not{p}_b + \not{k})}{(p_b + k)^2 + i\epsilon} \approx \bar{v}(p_b)\left[\frac{g}{-k^+ - i\epsilon} T^a\right]$$

How do we handle polarized case?

- Start with the generalized TMD formalism from Mulders and collaborators
  - Gauge link is constructed (from collinear gluons)
  - But did not consider the soft gluon radiation yet

Bomhof, Mulders, Vogelsang, Yuan, 07, Qiu, Vogelsang, Yuan, 07

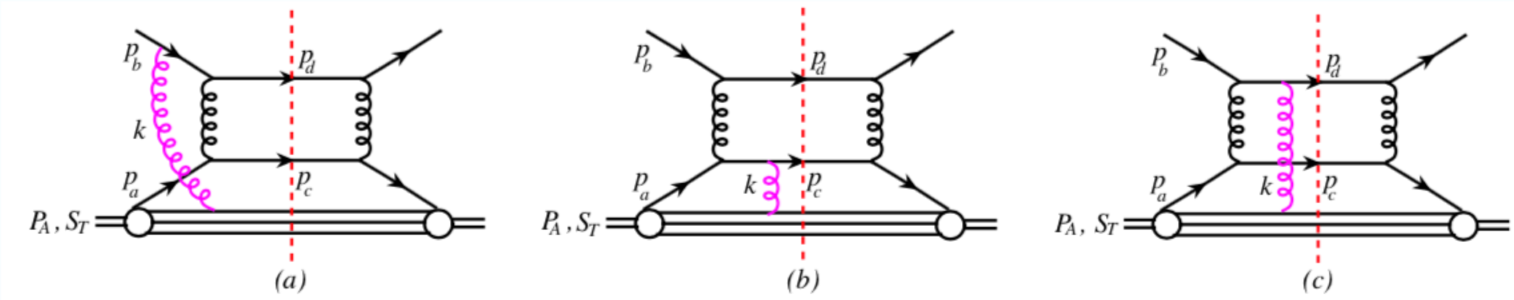
$$\frac{d\Delta\sigma(S_\perp)}{dy_1 dy_2 dP_\perp^2 d^2\vec{q}_\perp} = \frac{\epsilon^{\alpha\beta}S_\perp^\alpha q_\perp^\beta}{\vec{q}_\perp^2}\sum_{ab}\int d^2k_{1\perp}d^2k_{2\perp}d^2\lambda_\perp$$
$$\times\frac{\vec{k}_{1\perp}\cdot\vec{q}_\perp}{M_P}\,x_a\,q_{Ta}^{\rm SIDIS}(x_a,k_{1\perp})\,x_b\,f_b^{\rm SIDIS}(x_b,k_{2\perp})$$
$$\times\left[S_{ab\to cd}(\lambda_\perp)\,H_{ab\to cd}^{\rm Sivers}(P_\perp^2)\right]_c\,\delta^{(2)}(\vec{k}_{1\perp}+\vec{k}_{2\perp}+\vec{\lambda}_\perp-\vec{q}_\perp)\,,$$

$$S_{ab\to cd}(\lambda_\perp) = \delta^2(\lambda_\perp)\ \text{at LO}$$

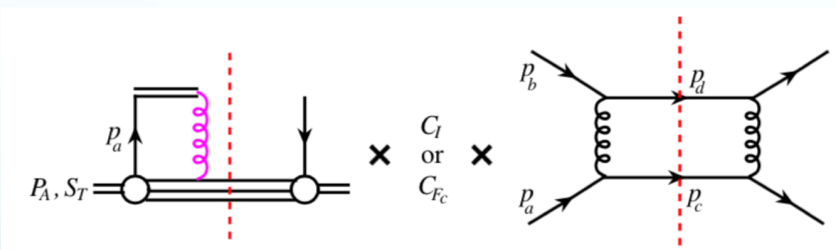
- However, since soft function is spin-independent, we can just use those from unpolarized case

Similar consideration for qq→qq

- Three different gluon attachments



- Perform the same expansion, one might factorize as follows
  - Each diagram has different color factor associated with Sivers functions
  - Perform this step continuously for all the cutting diagrams

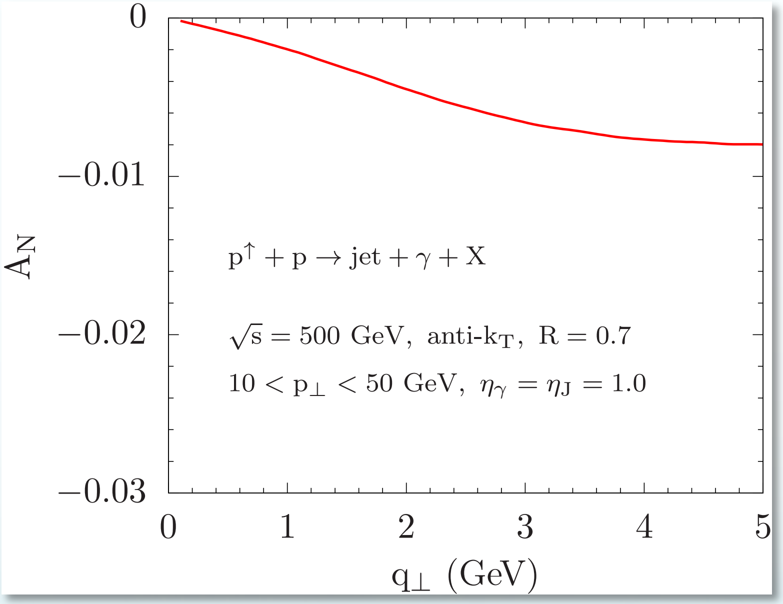


$$-\frac{1}{2N_c^2} \qquad -\frac{1}{4N_c^2} \qquad \frac{N_c^2-2}{4N_c^2}$$

$$|M|^2_{\rm UN} = {\rm Tr}\left[H^{(0)} S^{(0)}\right] \qquad |M|^2_{\rm Sivers} = {\rm Tr}\left[\mathcal{H}^{(0)} S^{(0)}\right]$$

Phenomenology at RHIC

- Prediction for Sivers asymmetry is around 1% level
  - Sivers functions in SIDIS from our earlier extraction 1401.5078
  - TMD evolution has a strong effect (suppress asymmetry), but not so much for unpolarized cross section



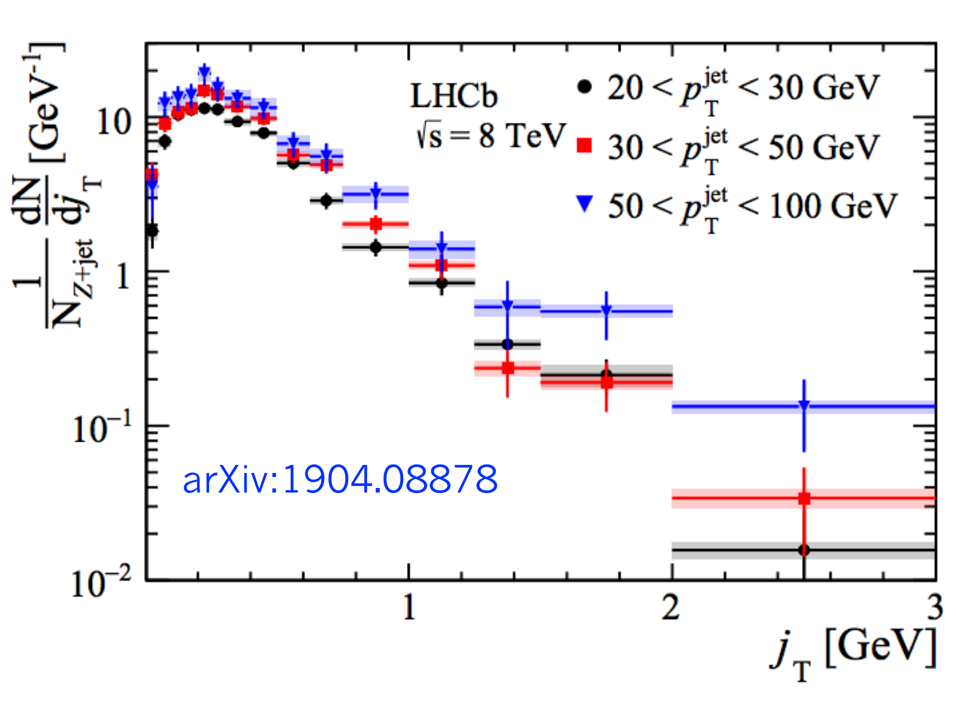
Buffing, Kang, Lee, Liu, arXiv:1812.07549

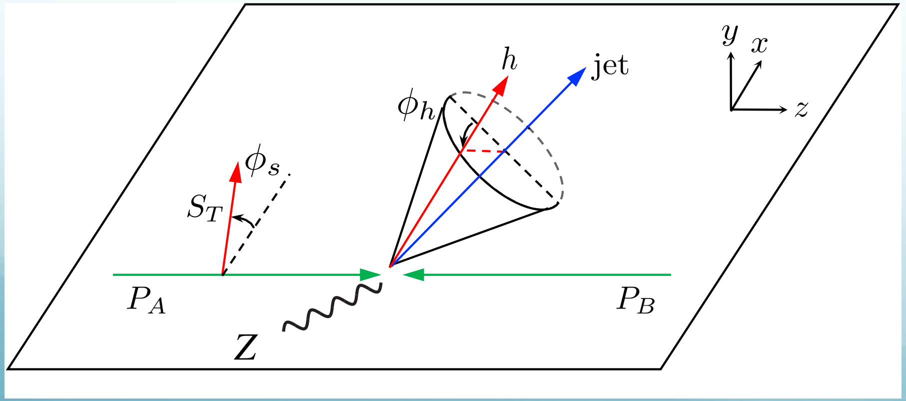
Open the door is good for us

- Once you open this door (processes beyond standard ones), a new world is open for you

TMD fragmentation function in Z+jet

- Recent measurements at the LHCb (also many others)

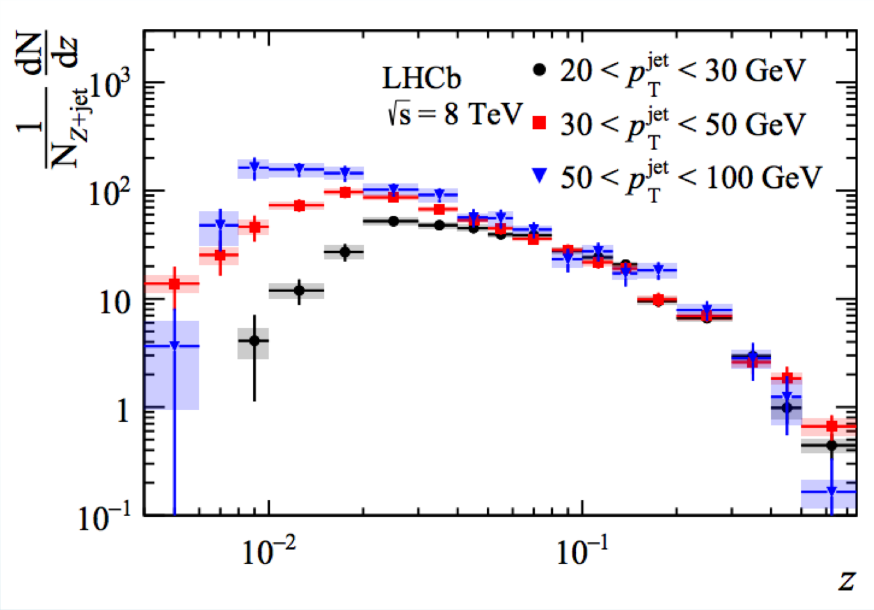


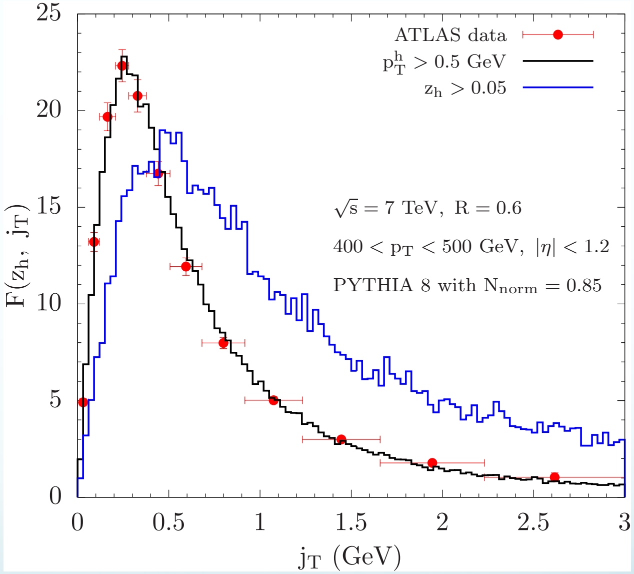


✓ Sensitive to TMD PDFs
✓ Sensitive to TMD FFs in jet

Comment: differential in z_h

- The current j_T distribution is integrated over the entire $0<z_h<1$ region
  - Low z_h part tricky: not available in most FFs fits, driven by soft physics ($\ln(z_h)$ resummation)
  - Only the further binning in z_h gives us direct connection to TMD FFs





From 1705.08443

TMD factorization on top of TMD factorization

- Jet production:

$$\frac{d\sigma}{d\mathcal{PS}} = \sum_{a,b,c} \int d\phi_J \int \prod_{i=1}^{4} d^2\boldsymbol{k}_{iT}\, \delta^2(\boldsymbol{q}_T - \sum_i^4 \boldsymbol{k}_{iT}) f_a(x_a, k_{1T}^2, \mu, \nu) f_b(x_b, k_{2T}^2, \mu, \nu)$$

$$\times\, S_{n\bar{n}n_J}^{\text{global}}(\boldsymbol{k}_{3T}, \mu, \nu) S_{n_J}^{cs}(\boldsymbol{k}_{4T}, R, \mu) H_{ab\to cZ}(p_T, m_Z, \mu)\, J_c(p_{JT}R, \mu)$$

- Hadron inside the jet: zh-dependence
  - Related to standard collinear FFs

$$\frac{d\sigma^h}{d\mathcal{PS}\, dz_h} = \text{same} \times \mathcal{G}_c^h(z_h, p_{JT}R, \mu)$$

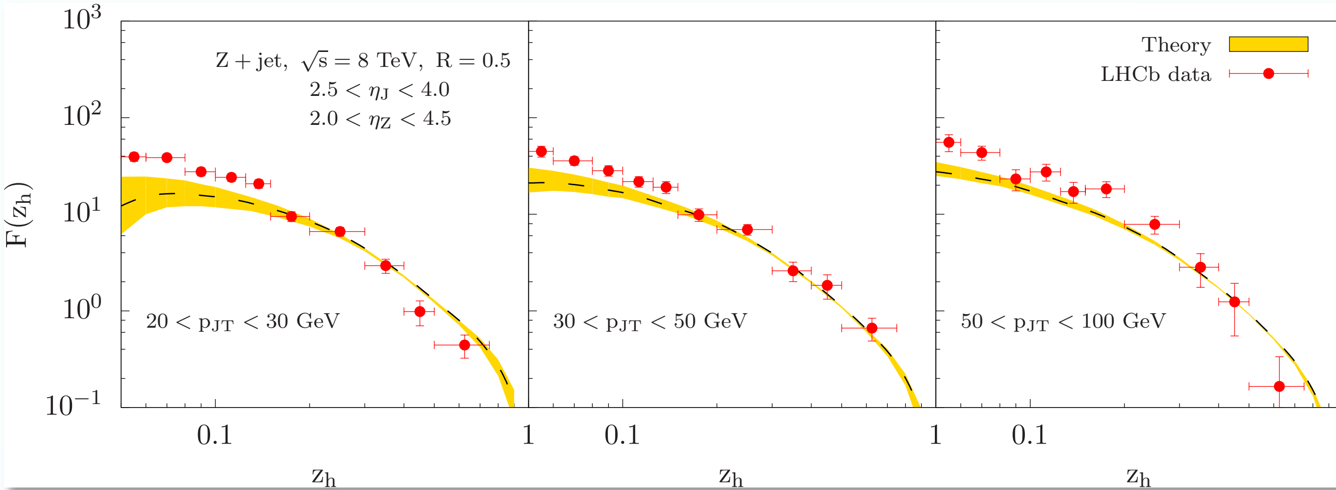
- Hadron TMD distribution inside the jet: j_T-dependence
  - Related to TMD FFs

$$\frac{d\sigma^h}{d\mathcal{PS}\, dz_h\, d^2\boldsymbol{j}_\perp} = \text{same} \times \mathcal{G}_c^h(z_h, p_{JT}R, \boldsymbol{j}_\perp, \mu)$$

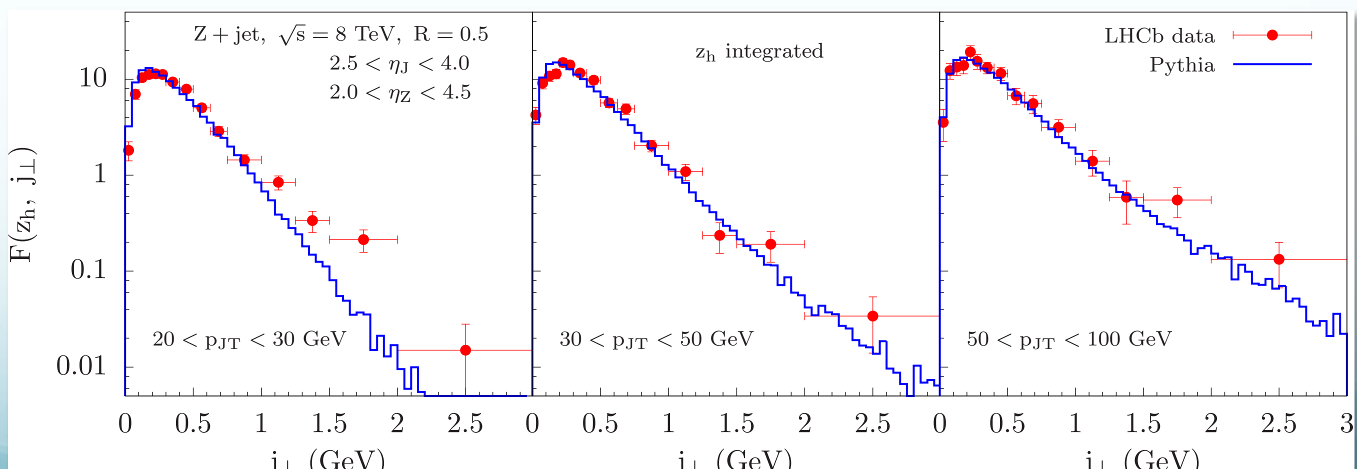
Jet fragmentation functions in Z+jet

- z_h distribution

Kang, Lee, Terry, Xing, arXiv:1906.07187

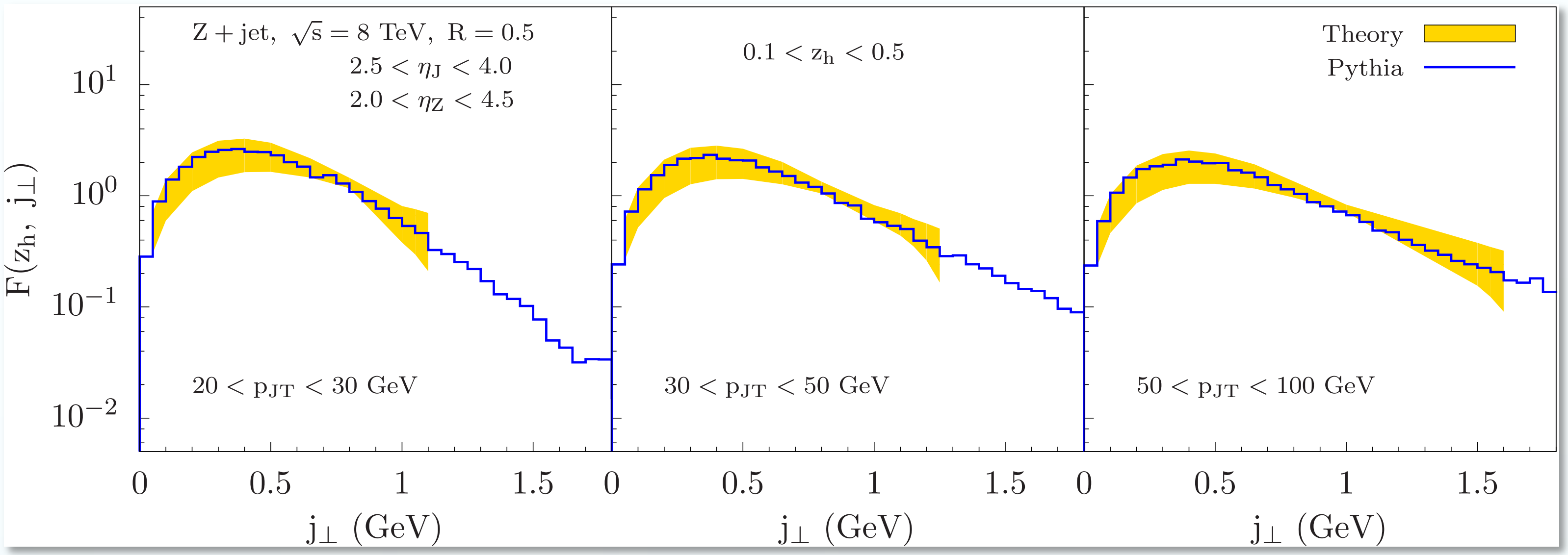


- Again, since zh integrated over [0,1], first check Pythia



For physical region zh: [0.1,0.5]

- For zh integrated [0.1,0.5]



- Works very well

TMD study

- Study on TMDs are extremely active in the past few years, lots of progress have been made
- With great excitement, we look forward to the future experimental results from COMPASS/RHIC, as well as Jefferson Lab, of course also LHC, most importantly, the EIC
- Hadron TMD distribution inside single inclusive jets are good opportunities for TMD FFs
- Back-to-back dijet/photon+jet in both p+p and e+p are new opportunities for TMDs
  - Note: dijet production in EIC has no factorization breaking issue
  - It is now the opportune time to develop the QCD formalism for them

Thank you!