Pion Scattering to g-2 and Neutrino Physics in Lattice QCD

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JLab Theory Seminar

BROOKHAVEN
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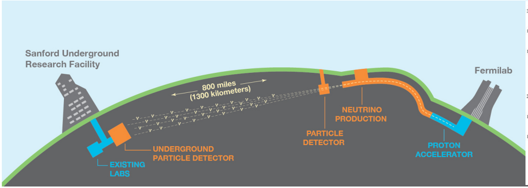
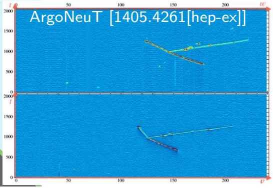
Outline

- Intro & Big Picture: Multiparticle Scattering in Neutrinos
  - Neutrino Oscillation Experiments
  - Neutrino Cross Sections
  - Complications with Neutrino Interactions
  - Resonance interactions in Neutrino Scattering
  - Lattice QCD
- Applications: $\pi\pi$ Scattering in Muon HVP for $g-2$
  - $g-2$ Experiment
  - Dispersive vs. LQCD
  - Error Budget and LQCD Strategy
  - Scattering Spectrum and Matrix Elements
  - Bounding Method
  - Results
- Applications: $\pi\pi$ Scattering Phase Shifts at Physical M_π
  - Scattering States and Resonances: Lattice to Continuum
  - Lüscher Quantization Condition
  - $I=2$ $\pi\pi$ Scattering
  - $I=1$ Scattering and ρ Resonance
- Conclusions

Introduction

Neutrino Oscillation Experiments





Neutrino oscillation experiments are of great interest to the physics community, seeking to do a high-precision measurement of neutrino oscillation parameters

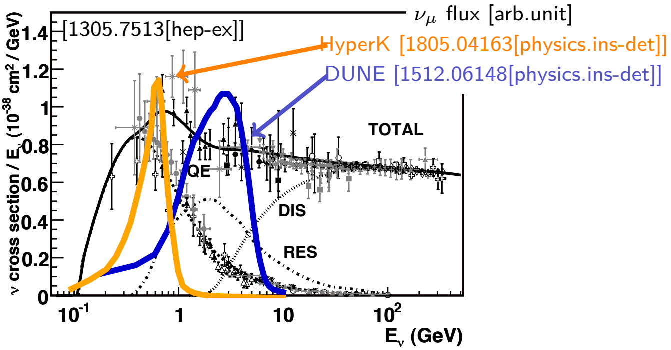
Large program of research goals addressing fundamental physics questions:

- Precision measurements of oscillation parameters, including leptonic CP violation and determination of the mass hierarchy
- Measurements of neutrinos from supernova explosions
- Searches for proton decay

DUNE due to start installation of first detector in 2022, data collection in 2026

This experiment sets a timescale for making theory contributions!

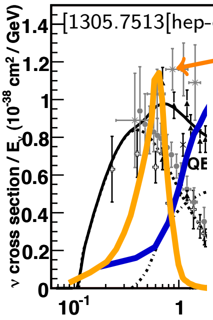
Flux ⊗ Cross Section



Neutrino interactions classified by their interaction products

3 general classes of interaction types:
Quasielastic (QE), Resonance (Res), Deep Inelastic Scattering (DIS)

Flux ⊗ Cross Section



Source of Uncertainty	MINOS ν_e	T2K ν_e	DUNE ν_e	Comments
Beam Flux after N/F extrapolation	0.3%	3.2%	2%	See "Flux Uncertainties" in Section 3.6.2
Interaction Model	2.7%	5.3%	$\sim 2\%$	See "Interaction Model Uncertainties" in Section 3.6.2
Energy scale (ν_μ)	3.5%	included above	(2%)	Included in 5% ν_μ sample normalization uncertainty in DUNE 3-flavor fit.
Energy scale (ν_e)	2.7%	2.5% includes all FD effects	2%	See "ν_e Energy-Scale Uncertainties" in Section3.6.2
Fiducial volume	2.4%	1%	1%	Larger detectors = smaller uncertainty.
Total	5.7%	6.8%	3.6 %	
Used in DUNE Sensitivity Calculations			$5\% \oplus 2\%$	Residual ν_e uncertainty: 2%

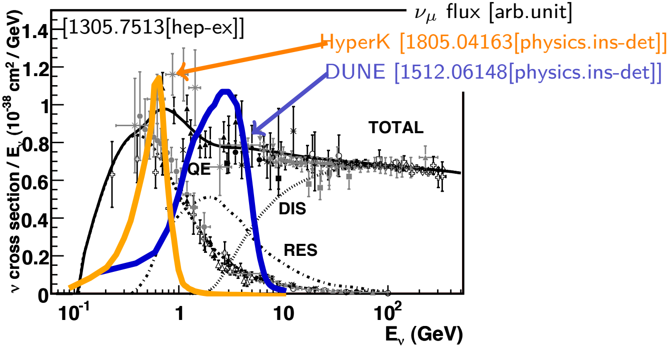
Neutrino interactions classified by their interaction products

3 general classes of interaction types:

Quasielastic (QE), Resonance (Res), Deep Inelastic Scattering (DIS)

DUNE events will be a mixture of all three; $\sim \frac{1}{3}$ events will be resonant

Flux ⊗ Cross Section



Neutrino interactions classified by their interaction products

3 general classes of interaction types:
  Quasielastic (QE), Resonance (Res), Deep Inelastic Scattering (DIS)

DUNE events will be a mixture of all three; $\sim \frac{1}{3}$ events will be resonant

A dominant contribution to systematics in DUNE will be cross section uncertainties

$\Longrightarrow$ Stringent requirements on cross section uncertainties, lots at stake
$\Longrightarrow$ QE coming under control on lattice, next step is resonant

Neutrino-Nuclear Cross Sections

Intranuclear rescattering effects can be problematic

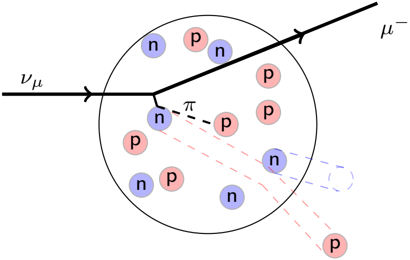
- Nuclear rescattering can change particle energy
- Topologies altered by absorption or emission of other particles

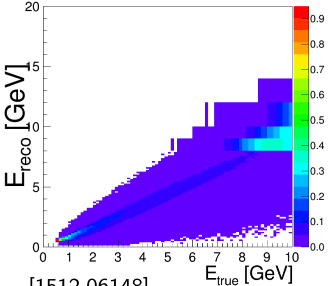
Resulting event-level data is subject to interpretation

$\Longrightarrow$ Neutrino energies cannot be determined on an event-by-event basis

$\Longrightarrow$ Energy spectrum must be reconstructed at the statistical level

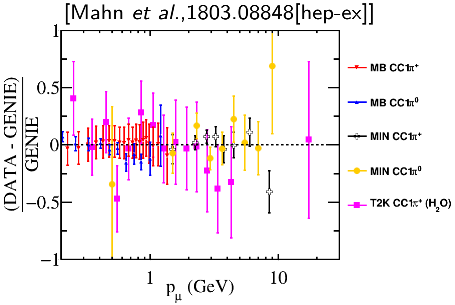
$\Longrightarrow$ Reconstruction depends on the assumed nuclear model





[1512.06148]

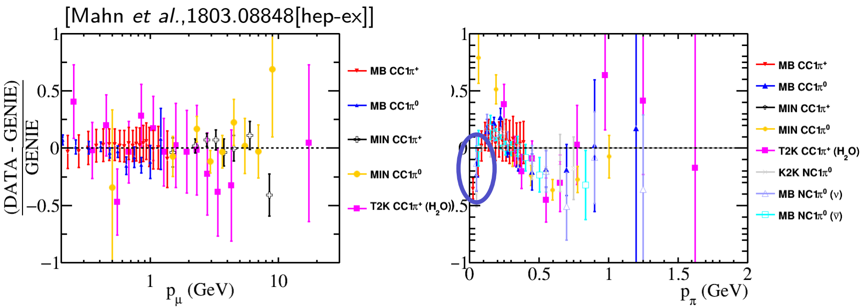
Discrepancies with Monte Carlo



Current state of affairs for CC1π interactions is confusing

Lepton kinematics under control, consistent and agree well with Monte Carlo

Discrepancies with Monte Carlo



Current state of affairs for CC1π interactions is confusing

Lepton kinematics under control, consistent and agree well with Monte Carlo

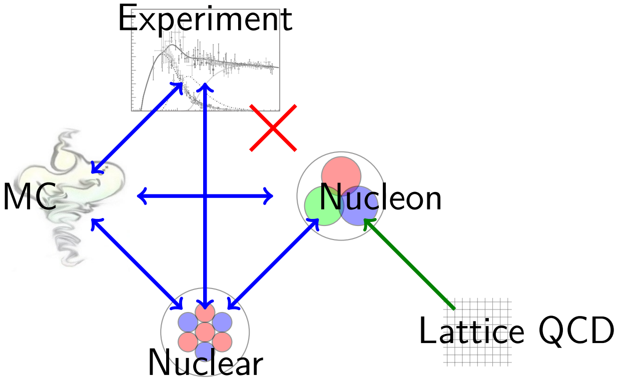
Pion kinematics systematically disagree with shape

Difficult to change pion kinematics without breaking other data

$\Longrightarrow$ Need another handle on pion kinematics!

Ideally a high-statistics *H* or *D* bubble chamber experiment; not likely to happen. . .

Lattice QCD Checklist



Lattice QCD is ideal tool for filling in missing pieces

To have the greatest impact, must satisfy the checklist:

- Process is important for meeting experimental goals ✓
- Current precision not sufficient ✓
- Difficult/impractical to measure experimentally ✓
- Accessible to Lattice QCD ✓

Multiparticle Scattering in LQCD

Multiparticle scattering is a challenging problem in LQCD

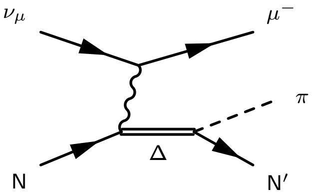
Significant effort and progress has been made over many years

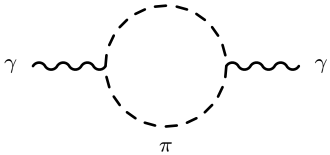
Several timely and interesting physics problems make use of multiparticle scattering:

- Muon $g-2$ HVP contribution from LQCD
- $\pi\pi$ Scattering phase shifts

Less complicated than resonant nucleon interactions, ideal starting place

Long term goals are big risk, big reward:
access to nucleon transition form factors for oscillation experiment!





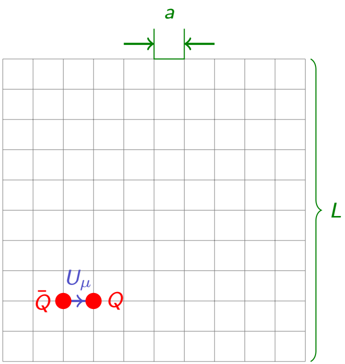
Lattice QCD: Formalism

- Lattice QCD is a technique to numerically evaluate path integral

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}\psi \, \mathcal{D}\overline{\psi} \, \mathcal{D}U \, \exp(-S) \, \mathcal{O}_\psi \, [U]$$

- Discretize spacetime $\implies$ #DOF finite
- Lattice spacing a provides UV cutoff
- Lattice size L provides IR cutoff

- Quark fields on sites $\implies$ $Q(x)$
- Gauge fields between sites $\implies$ $U_\mu(x)$
- Euclidean time $\implies$ correlators $\propto e^{-Et}$



Typical strategy is to construct operators at "source," allow them to propagate through time, then annihilate at "sink"

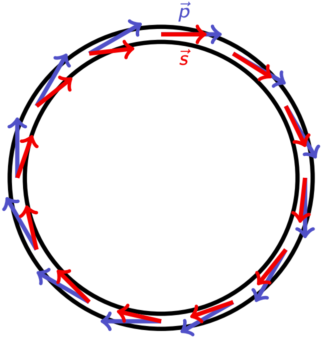
Evaluate correlation functions on fixed background gauge field, compute on many gauge fields for Monte Carlo average

Correlation functions are products of matrix elements times exponentials, e.g.

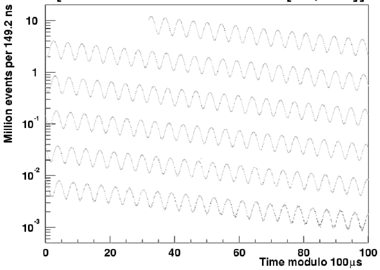
$$C(t) = \sum_n |\langle \Omega | \, \mathcal{O} \, | n \rangle|^2 \, e^{-E_n t}$$

HVP For Muon $g-2$

Muon $g-2$



[Bennett *et al.*, 0602035[hep-ex]]



High-precision experiment of spin precession
relative to momentum direction in storage ring

Anomalous frequency $\omega_a = \frac{g-2}{2}\frac{eB}{m} = a_\mu \frac{eB}{m}$

Experiment to measure the anomalous magnetic moment $g-2$

Sensitive to new physics, and also discrepant with experiment!

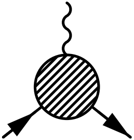
Fermilab Muon $g-2$ Experiment

Experiment has come a long way (and so has theory!)

Aiming for a $4\times$ improvement in uncertainty over the BNL result

Parts of Muon $g-2$ Theory Prediction

Contribution	Value $\times 10^{10}$	Uncertainty $\times 10^{10}$
QED	11 658 471.895	0.008
EW	15.4	0.1
HVP LO	692.5	2.7
HVP NLO	−9.84	0.06
HVP NNLO	1.24	0.01
Hadronic light-by-light	10.5	2.6
Total SM prediction	11 659 181.7	3.8
BNL E821 result	11 659 209.1	6.3
Fermilab E989 target		≈ 1.6



Anomalous magnetic moment a result of quantum corrections to photon interaction

High precision measurement with stringent theoretical requirements

Experiment-Theory difference is 27.4(7.3) $\Longrightarrow$ 3.7σ tension!

Hadronic contributions are least certain
$\Longrightarrow$ Lattice QCD used as a tool to directly access hadronic contributions for $g-2$

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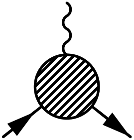
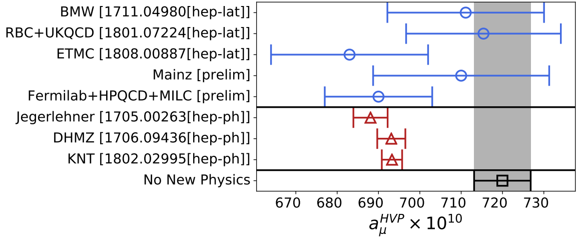
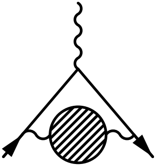




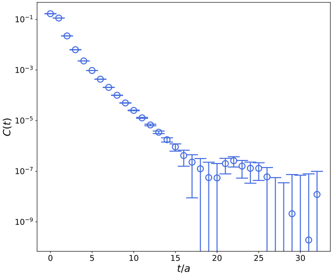


Hadronic Vacuum Polarization is the target measurement

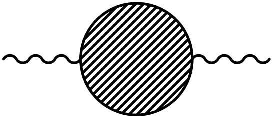
$\Longrightarrow$ Lattice results have larger uncertainty, but are consistently improving

$\Longrightarrow$ Dispersive approach ("R-ratio") results are more precise, but static

Exclusive Channels in the HVP



$$C(t) = \frac{1}{3}\sum_i \left\langle \left[\bar{\psi}\gamma_i\psi\right]_t \left[\bar{\psi}\gamma_i\psi\right]_0 \right\rangle$$
$$\approx \sum_n \left| \langle\Omega|\bar{\psi}\gamma_i\psi|n\rangle \right|^2 e^{-E_n t}$$



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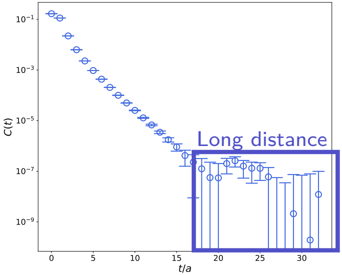

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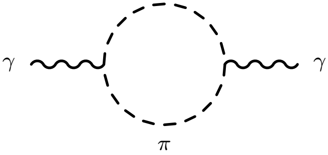


Correlator has large statistical error in long-distance region,
but contributions from high energy states are exponentially suppressed

Long distance correlator dominated by two-pion states,
but overlap of vector current with two-pion states is minimal

Strategy:

- Construct & measure operators that overlap strongly with $\pi\pi$ states
- Correlate these operators with the local vector current
- a_μ^{HVP} computed by integrating with time-momentum representation kernel,
  $a_\mu^{HVP} = \sum_t w_t C(t)$ [Bernecker et al., 1107.4388 [hep-lat]]

Detail: lattice states are admixture of continuum states with definite particle count
This analysis not dependent on particle content of states, only lattice eigenstates

Operator Construction

Operators in $I = 1$ P-wave channel with $\vec{p}_{\text{COM}} = 0$, to impact on a_μ^{HVP}

Designed to have strong overlap with specific target states,
but all operators unavoidably couple to all states in HVP spectrum

Local vector current operator constructed with explicit all-to-all method:

- $\mathcal{O}_0 = \sum_x \bar{\psi}(x)\gamma_\mu\psi(x)$, $\mu \in \{1,2,3\}$

Three 2π operators using distillation ($f \sim$ smearing kernel)
with $\mathcal{O}_{1,2,3}$ given by $\vec{p}_\pi \in \frac{2\pi}{L} \times \{(1,0,0),(1,1,0),(1,1,1)\}$:

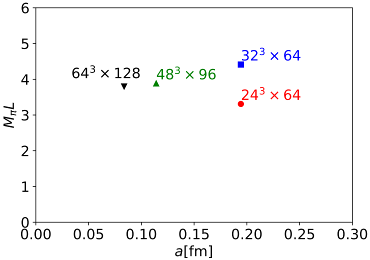
- $\mathcal{O}_n = \left| \sum_{xyz} \bar{\psi}(x) f(x-z) e^{-i\vec{p}_\pi \cdot \vec{z}} \gamma_5 f(z-y)\psi(y) \right|^2$

Correlators arranged in a 4×4 symmetric matrix:

$\otimes$	$\mathcal{O}_0$	$\mathcal{O}_1$	$\mathcal{O}_2$	$\mathcal{O}_3$
$\mathcal{O}_0$	$C_\rho^{(2)}$	$C_{\rho\to\pi\pi}^{(3)}$	$C_{\rho\to\pi\pi}^{(3)}$	$C_{\rho\to\pi\pi}^{(3)}$
$\mathcal{O}_1$		$C_{\pi\pi\to\pi\pi}^{(4)}$	$C_{\pi\pi\to\pi\pi}^{(4)}$	$C_{\pi\pi\to\pi\pi}^{(4)}$
$\mathcal{O}_2$			$C_{\pi\pi\to\pi\pi}^{(4)}$	$C_{\pi\pi\to\pi\pi}^{(4)}$
$\mathcal{O}_3$				$C_{\pi\pi\to\pi\pi}^{(4)}$

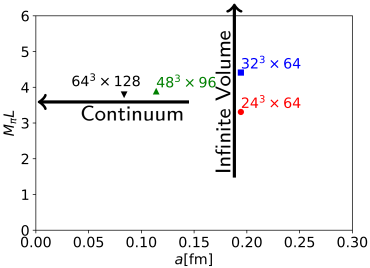
Inclusion of extra operator with $\vec{p}_\pi = \frac{2\pi}{L} \times (2,0,0)$
to estimate systematics from excited state contamination

Computation Details



Computed on $2+1$ flavor Möbius Domain Wall Fermions for valance and sea,
M_π at physical value on all ensembles

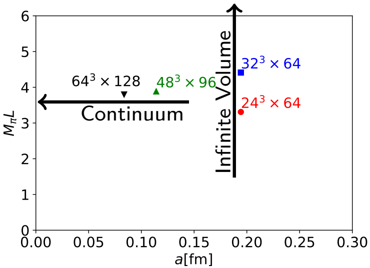
Computation Details



Computed on $2+1$ flavor Möbius Domain Wall Fermions for valance and sea, M_π at physical value on all ensembles

24^3 and 32^3 ensembles used to extrapolate to infinite volume
48^3 and 64^3 ensembles used to extrapolate to continuum limit (lattice spacing $a \to 0$)

Computation Details



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24^3 and 32^3 ensembles used to extrapolate to infinite volume
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Computations using distillation setup with N_{eig} eigenvectors

Results in this talk restricted to $24^3 \times 64$ and $48^3 \times 96$ ensembles:

- 24^3 (24ID): $a \approx 0.20\ \mathrm{fm} \implies 4.8\ \mathrm{fm},\ N_{eig} = 120$
- 48^3 (48I): $a \approx 0.11\ \mathrm{fm} \implies 5.5\ \mathrm{fm},\ N_{eig} = 60$

Future work including other ensembles for finite volume and continuum extrapolations

Generalized EigenValue Problem (GEVP)

Generalized EigenValue Problem to estimate overlap with vector current & energies

$$C(t)\, V = C(t+\delta t)\, V\, \Lambda(\delta t)$$

$$\Lambda_{nn}(\delta t) \sim e^{+E_n \delta t}\,, \quad V_{im} \propto \langle \Omega | \mathcal{O}_i | m \rangle$$

$C(t)$ is the matrix of correlation functions from previous slide

Compute at fixed δt, vary t: plateau for large t

From result, reconstruct exponential dependence of local vector correlation function

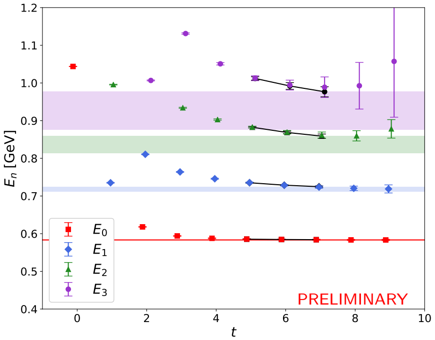
$$C_{ij}^{\text{latt.}}(t) = \sum_{n}^{N} \langle \Omega | \mathcal{O}_i | n \rangle \langle n | \mathcal{O}_j | \Omega \rangle\, e^{-E_n t}$$

In theory, infinite number of states contribute to correlation function

In practice, only finite N necessary to model correlation function

$\implies$ finite GEVP basis is sufficient

GEVP Results



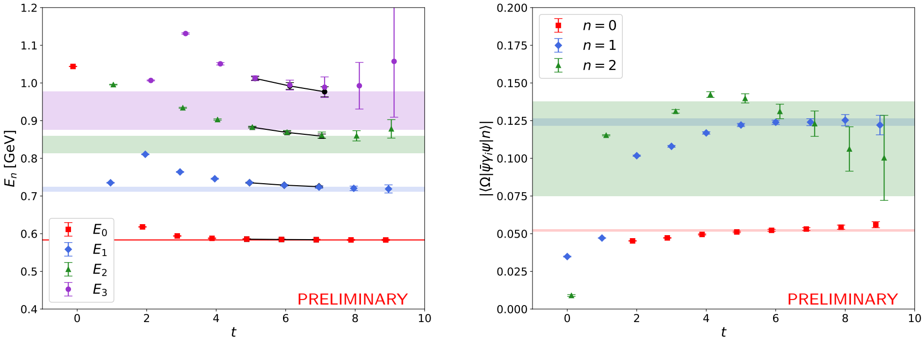
$$C(t)\,V = C(t+\delta t)\,V\,\Lambda(\delta t)\,, \quad \Lambda_{nn}(\delta t) \sim e^{+E_n \delta t}$$

Colored scatter points from solving GEVP at fixed δt

Black lines are $f_i(t)$ result from fit to ansatz: $f_i(t) = E_i + a_i e^{-(E_N - E_i)t}$

Colored bands are $E_i \pm \delta E_i$ retsult from fit to ansatz

GEVP Results



$$C(t)\,V = C(t+\delta t)\,V\,\Lambda(\delta t)\,, \quad V_{im} \propto \langle\Omega|\,\mathcal{O}_i\,|m\rangle$$

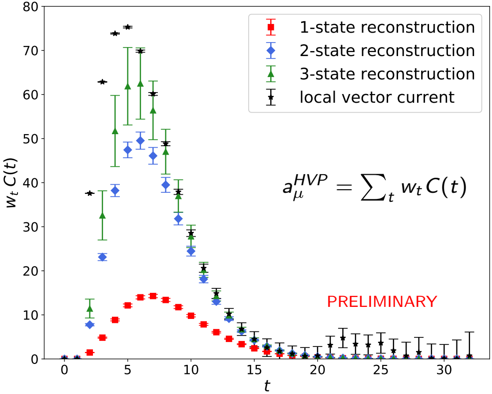
Overlaps determined from picking single-t GEVP result, different t for $n = 0, 1, 2$

ts picked to get approximately same excited state contamination for each

Bands include systematic for difference between 4- and 5-operator GEVP basis

First two overlaps well-determined, third state has larger systematics

Correlation Function Reconstruction



Plotted: (weight kernel) $\times$ (correlation function), integrated to get a_μ^{HVP}

Results from GEVP fits used to reconstruct long-distance correlator

More states reconstructed $\implies$ switchover at smaller t $\implies$ better statistics

(Improved) Bounding Method

Traditional bounding method uses correlation function at medium distance to constrain a_μ^{HVP} with strict upper & lower bound on functional form:

$$\widetilde{C}(t; t_{\rm max}, E) = \begin{cases} C(t) & t < t_{\rm max} \\ C(t_{\rm max}) e^{-E(t-t_{\rm max})} & t \geq t_{\rm max} \end{cases}$$

Upper bound: $E = E_0$, lowest state in spectrum

Lower bound: $E = \log[\frac{C(t_{\rm max})}{C(t_{\rm max}+1)}]$, "local effective mass"

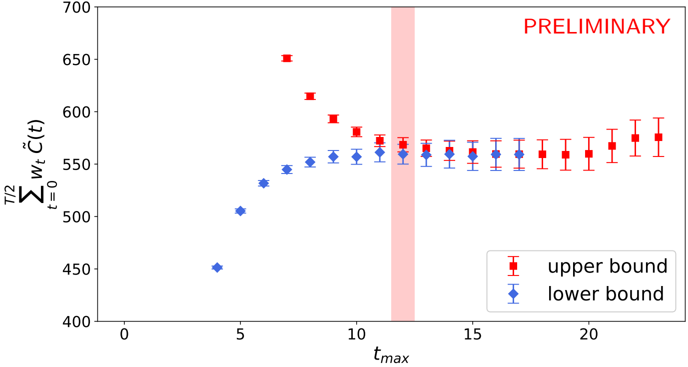
Bounding method "improved" by subtracting out reconstruction of lowest states

Replace $C(t) \to C(t) - \sum_n^N |c_n|^2 e^{-E_n t}$ and apply bounding procedure for $a_\mu - \delta a_\mu$

$\implies$ Upper bound now $\propto e^{-E_{N+1} t}$, lower bound falls faster
$\implies$ Smaller overall contribution from neglected states

After bounding, add back $\delta a_\mu = \sum_{t=t_{\rm max}}^{\infty} w_t \sum_n^N |c_n|^2 e^{-E_n t}$

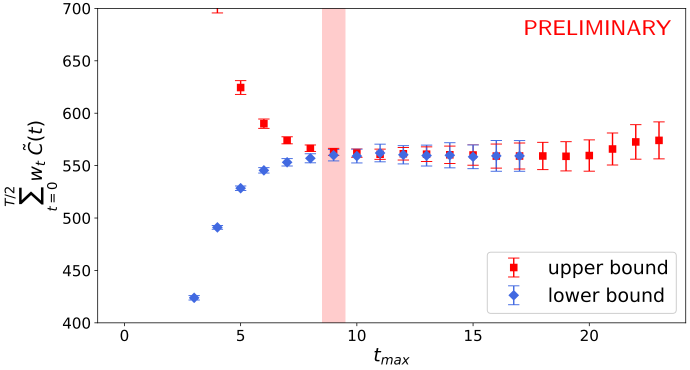
Improved Bounding Method



No bounding method: $a_\mu^{HVP} = 577(31) \times 10^{-10}$

Bounding method $t_{\mathrm{max}} = 2.3\ \mathrm{fm}$, no improvement: $a_\mu^{HVP} = 564.0(9.1) \times 10^{-10}$

Improved Bounding Method



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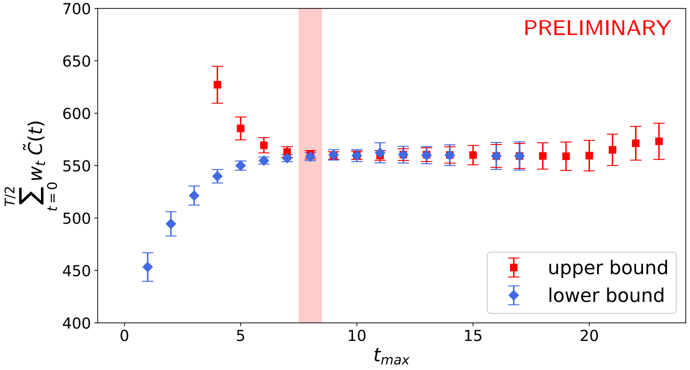

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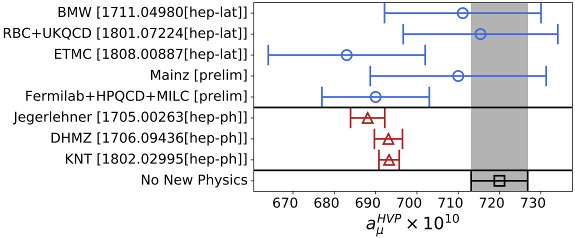
Bounding method $t_{\max} = 1.7$ fm, 1 state improvement: $a_\mu^{HVP} = 561.5(4.5) \times 10^{-10}$

Bounding method $t_{\max} = 1.6$ fm, 2 state improvement: $a_\mu^{HVP} = 559.5(3.8) \times 10^{-10}$

Exclusive study + improved bounding method give $\times 10$ statistical improvement!

Error Budget and Timeline



Update to RBC-UKQCD calculation including exclusive study within two months

$\Longrightarrow$ precision improvement $\times 2$, error on a_μ^{HVP} at 7×10^{-10}
$\Longrightarrow$ to be included in $g-2$ Theory WP before release of Fermilab first results

Further reduction will require full RBC-UKQCD program of computations

Work on the exclusive channel study using bounding method has led to world-first estimation of finite volume corrections to a_μ^{HVP} at physical M_π

Complete analysis with full suite of systematic improvements by 2020
$\Longrightarrow$ precision improvement $\times 10$ over original, error on a_μ^{HVP} at 2×10^{-10}

$\pi\pi$ Scattering Phase Shifts

LQCD Two-Particle States and Resonances

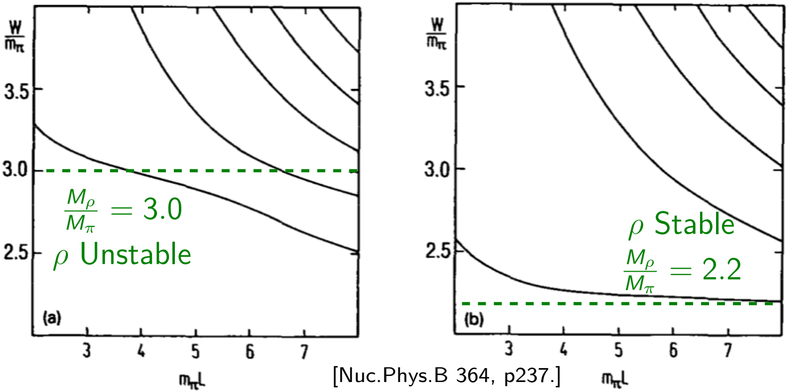
For solo stable particles, interpretation of spectrum is clean $\implies E_{\text{measured}} = E_{\text{state}}$

Multiparticle states and unstable states are not so straightforward:

- Two particles confined to box cannot be isolated $\implies$ no asymptotic states
- Need to enforce energy/momentum conservation with discretized momenta
- Avoided level crossings $\implies$ E eigenstates are superposition of N-particle states

Argument can be turned on its head:

Corrections from finite volume give access to scattering phase shifts on the lattice



[Nuc.Phys.B 364, p237.]

Scattering phase shifts

Phase shifts from studying deviation of $\pi\pi$ spectrum from noninteracting values

Pion states with $\vec{p}_{COM} = 0$ are assumed to have the form

$$E_{\pi\pi} = 2\sqrt{k^2 + m_\pi^2}$$

In noninteracting case, this is dispersion relation with k quantized
With interactions in lattice QCD, $E_{\pi\pi}$ is modified and must be measured
k is determined as a function of $E_{\pi\pi}$

Phase shift determined from formula [Nucl.Phys.B354,531(1991)],

$$\det[e^{2i\delta(k)}\mathbb{1} - U(k)] = 0$$

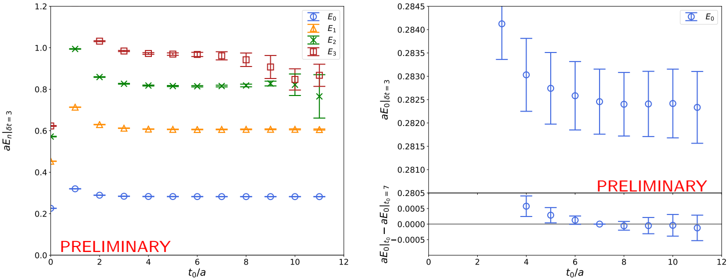
If lowest partial wave assumed to dominate, $\ell = 0$ partial wave determined from

$$\tan\delta(q) = \frac{q\pi^{3/2}}{\mathcal{Z}(1, q^2)}, \quad q = \frac{kL}{2\pi}$$

with $\mathcal{Z}$ the analytic continuation of the Riemann zeta function,

$$\mathcal{Z}(s, q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\vec{n} \in \mathbb{Z}^3} (\vec{n}^2 - q^2)^{-s}$$

$I = 2\ \ell = 0$ Spectrum

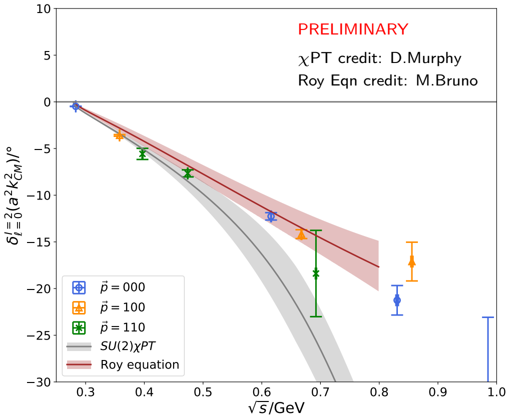


Isospin-2 S-wave channel:

4×4 basis of 2π terms with $\frac{L^2}{4\pi^2}\vec{p}_\pi^{\,2} = 0, 1, 2, 3$

- Sizeable around-the-world terms ($\mathcal{O}(1\%)$)
  due to single-π states propagating through BCs
  $\Longrightarrow$ removed with dedicated matrix element calculation
- No resonances to modify spectrum, shifts due to pion finite volume rescattering

$I = 2$ $\ell = 0$ Phase Shift



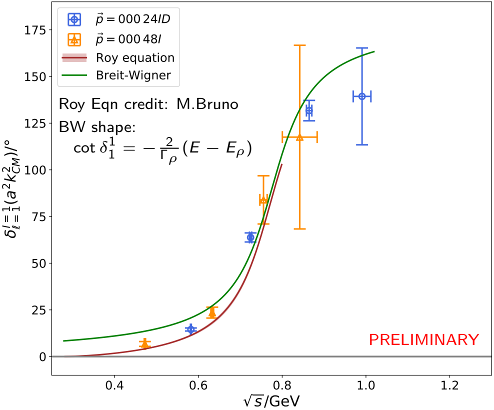
Using data from rest frame and moving frames, can fill out phase shift curve

Roy equations [Phys.Lett.B36(1971)] based on optical theorem, crossing symmetry

Breakdown of SU(2)χPT expected at around 500 MeV

Good agreement with phenomenology and SU(2)χPT

$I = 1$ $\ell = 1$ Phase Shift



Isospin-1 P-wave channel, same data as HVP:

4×4 basis with local vector current $\bar{\psi}\gamma_i\psi$ and 2π terms with $\frac{L^2}{4\pi^2}\vec{p}_\pi^{\,2} = 1, 2, 3$

ρ resonance channel $\implies$ phase shift expected to go through 180° increase

Looking Ahead

Additional studies for $\pi\pi$ scattering phase shifts:

- Other lattice irreps $\implies$ more partial waves
- Finish analyses on both 24ID and 48I ensembles
- Isospin 0? Correlation functions might be too noisy
- One of a series of upcoming RBC+UKQCD $\pi\pi$ scattering phase shift papers
  $\implies$ timescale $\sim$1 month?

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Calculations for neutrino physics will be more challenging

What technical issues can we expect?

- Technical challenges for fermionic spin states, unequal particle masses
- More computationally costly (more Wick contractions)
- Exponential degradation in signal to noise (Lepage scaling)
- More than two particle scattering states, e.g. $N\pi\pi$, open up quickly

Conclusions

Conclusions

Neutrino oscillation experiments in upcoming decade target precision measurements

- Deducing neutrino energy spectrum requires precise control of cross sections for many neutrino interaction channels
- Some interaction channels involve weak matrix elements, which are impractical to measure experimentally or depend on models
- One especially prominent example is nucleon resonant interactions, which will account for about 1/3 of DUNE's total events
- Lattice QCD offers an avenue to study these interaction channels, but LQCD has its own difficulties

Pion scattering physics makes for a simple playground to learn about multiparticle scattering in lattice QCD while accomplishing physics goals:

- Exclusive channel studies using 2π correlation functions reduced uncertainty on LQCD calculation of muon HVP contribution
- Measurements of $\pi\pi$ scattering phase shifts at physical M_π

With experience gained from studying $\pi\pi$ scattering, will move on to tackle more challenging problems with $N\pi$ scattering states and transition form factors

Thank you!

BACKUP

Distillation [0905.2160 [hep-lat]]

Correlation functions with more quark lines are more costly to compute
$\Longrightarrow$ More efficient computationally to use distillation

Projection matrices constructed from eigenvectors of Laplacian operator

$$\mathcal{P}^{ab}_{t;xy} = \sum_{i=0}^{M-1} \langle x | i^a_t \rangle \langle i^b_t | y \rangle$$

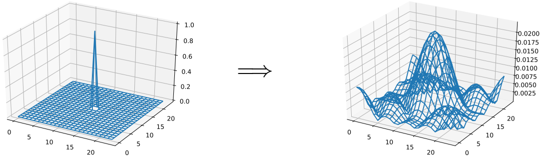
Inserting distillation projection matrices smears quarks in bilinear

$$\bar{Q}\Gamma Q \to \bar{Q}\mathcal{P}\Gamma\mathcal{P}Q = \sum_{x,y} \bar{Q}(x) f(x-z) \Gamma f(z-y) Q(y)$$

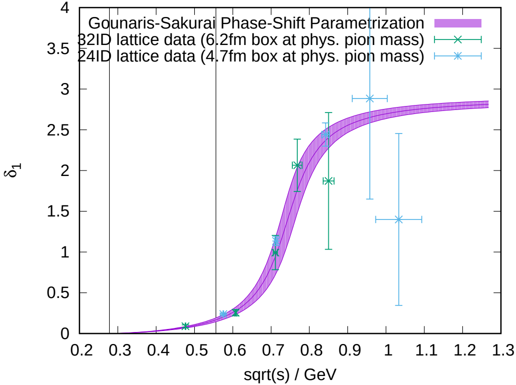
Computations with "perambulators," propagators contracted with eigenvectors

$$M^{ji} = \langle j | D^{-1} | i \rangle$$

Orthogonality of eigenvectors used in place of lattice site index
$\Longrightarrow$ significantly reduced computational burden
$\Longrightarrow$ ideal for creating multiparticle correlation functions



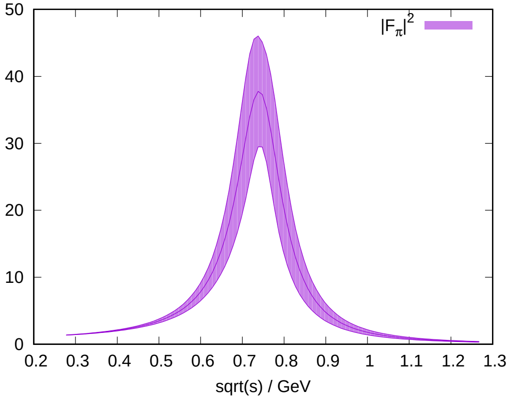
First constrain the p-wave phase shift from our $L = 6.22$ fm physical pion mass lattice:



$E_\rho = 0.766(21)$ GeV (PDG 0.77549(34) GeV)
$\Gamma_\rho = 0.139(18)$ GeV (PDG 0.1462(7) GeV)

[Lehner, Mainz 2018]

Predicts $|F_\pi(s)|^2$:



We can then also predict matrix elements and energies for our other lattices; successfully checked!

[Lehner, Mainz 2018]

Finite Volume Corrections on the Lattice

Complete error budget needs extrapolation to infinite volume

FV shift can be measured directly from results of exclusive study

$\Longrightarrow$ First time this shift resolved from zero at physical M_π!

$\Longrightarrow$ Previous bound at $10(26) \times 10^{-10}$, $M_\pi = 146$ MeV [1805.04250[hep-lat]]

Can compare FV shift predictions from phenomenological estimations:

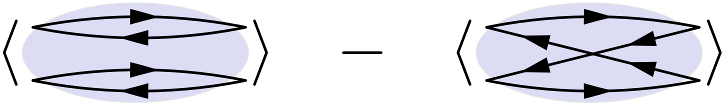
Gounaris-Sakurai-Lüscher [Phys.Rev.Lett. 21, 244, Nucl.Phys.B 354]

and scalar QED

$$a_\mu^{HVP}(L = 6.2\ \mathrm{fm}) - a_\mu^{HVP}(L = 4.7\ \mathrm{fm}) = \begin{cases} 21.6(6.3) \times 10^{-10} & \text{LQCD} \\ 20(3) \times 10^{-10} & \text{GSL} \\ 12.2 \times 10^{-10} & \text{sQED} \end{cases}$$

Good agreement with GSL in range of energies probed by LQCD

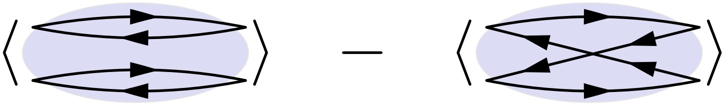
Correlation with Free Pions



Isospin-2 two-pion correlation functions contain two Wick contractions

Signal dominated by contraction on left

Correlation with Free Pions

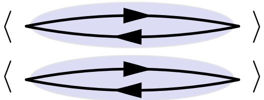


Isospin-2 two-pion correlation functions contain two Wick contractions

Signal dominated by contraction on left

Left contraction also looks like two noninteracting pions

$\implies$ signal improved by taking correlated difference of
[above] “interacting” 2-pion spectrum with [below] “free” 2-pion spectrum

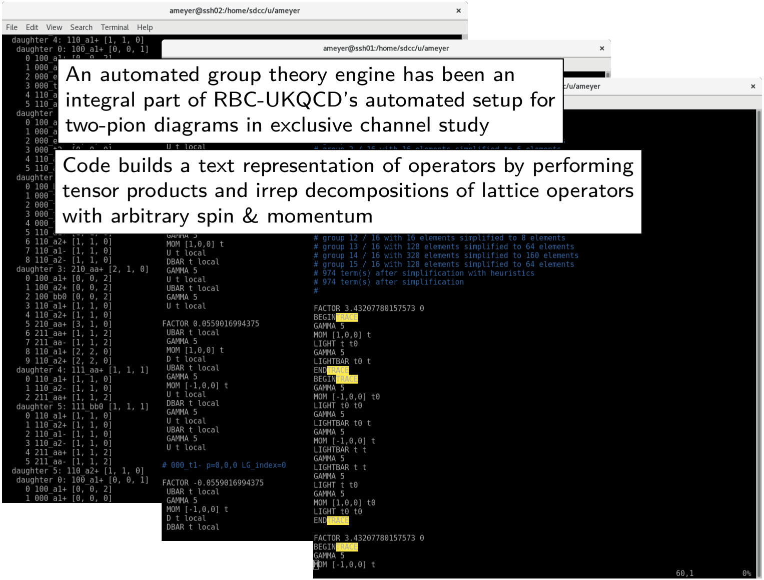


Correlations largely cancel statistical errors, discretization errors

Group Theory & Contraction Engine

An automated group theory engine has been an integral part of RBC-UKQCD's automated setup for two-pion diagrams in exclusive channel study

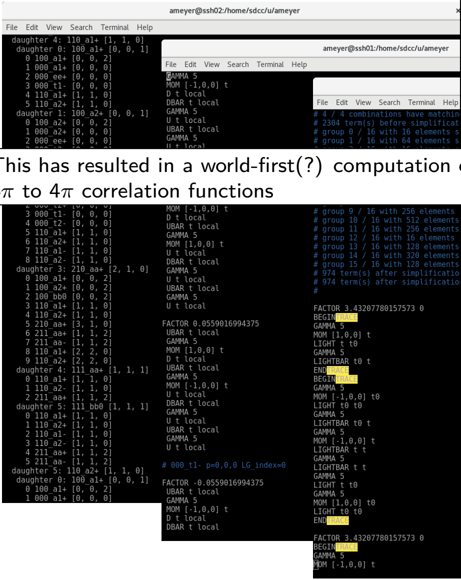
Group Theory & Contraction Engine



An automated group theory engine has been an integral part of RBC-UKQCD's automated setup for two-pion diagrams in exclusive channel study

Code builds a text representation of operators by performing tensor products and irrep decompositions of lattice operators with arbitrary spin & momentum

Group Theory & Contraction Engine



This has resulted in a world-first(?) computation of 4π to 4π correlation functions