

Effective field theories for dark matter direct detection



INSTITUTE for
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Seminar

Jefferson Laboratory

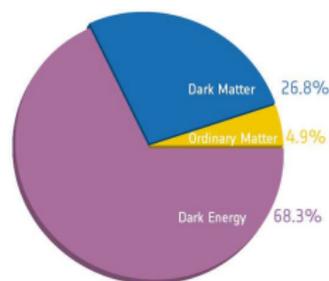
Newport News, April 24, 2019

MH, Klos, Menéndez, Schwenk PLB 746 (2015) 410, PRL 119 (2017) 181803, PRD 94 (2016) 063505, 99 (2019) 055031

Fieguth, MH, Klos, Menéndez, Schwenk, Weinheimer PRD 97 (2018) 103532

XENON collaboration + MH, Klos, Menéndez, Schwenk PRL 122 (2019) 071301

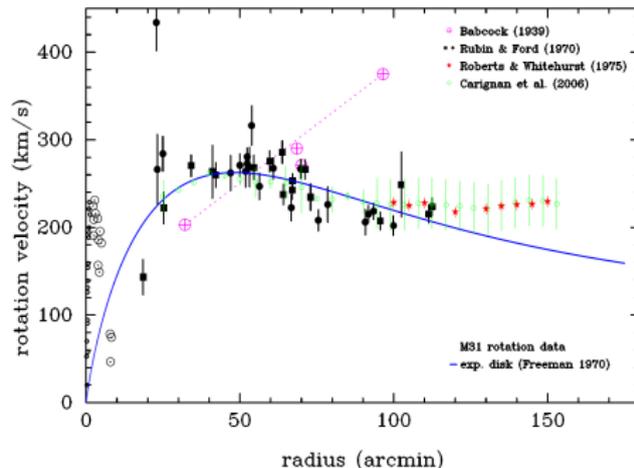
Evidence for dark matter I



Planck 2013

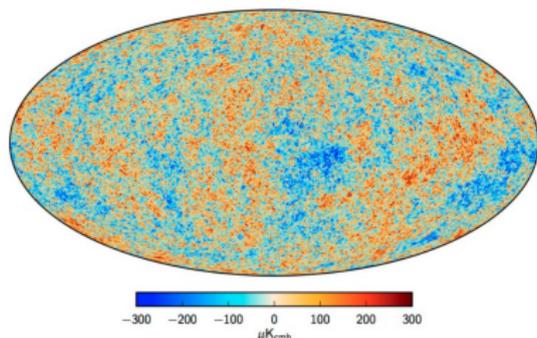
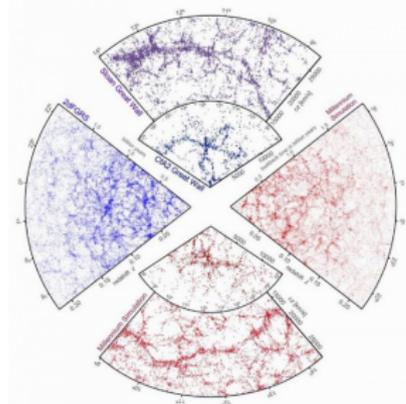
- In hindsight: early evidence from **rotation curves** of galaxies
- Expect $1/\sqrt{R}$ behavior

$$v = \sqrt{\frac{GM(R)}{R}}$$



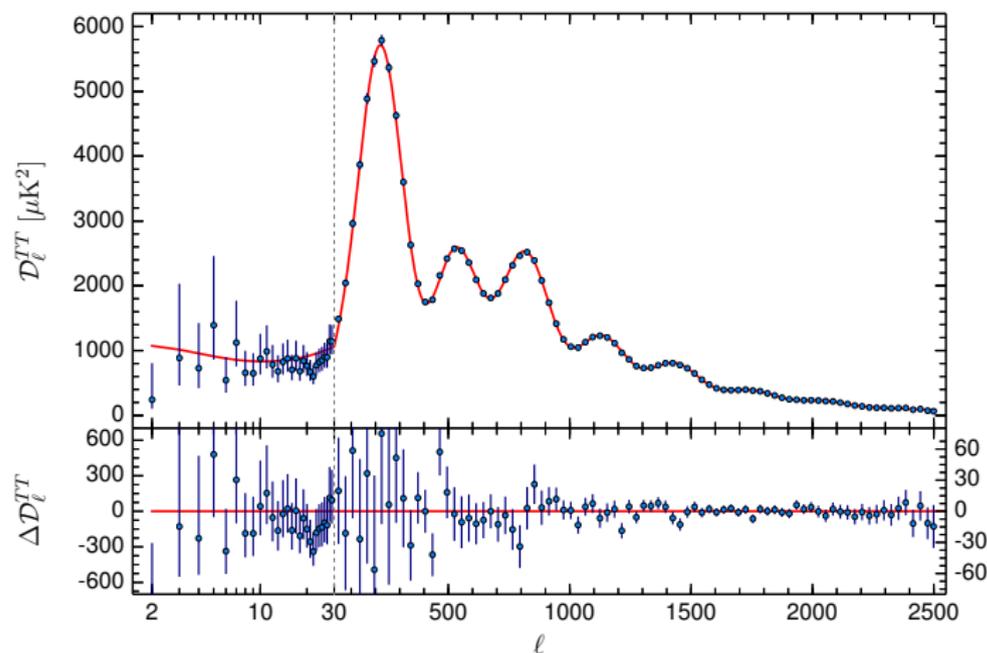
Rotation curve of Andromeda, Bertone, Hooper 2016

Evidence for dark matter II



- **Large scale structure:** N -body simulations only match galaxy surveys with a sufficient amount of cold dark matter 1980s
- **Cosmic microwave background:** temperature variations $\Delta T/T \sim 10^{-5}$ too small for a purely baryonic universe 1990s
- **Microlensing searches:** dark matter cannot be in form of compact objects 1990s
- **Baryon content of the Universe:** light element abundances imply $\Omega_b \lesssim 5\%$ 1990s

Evidence for dark matter III

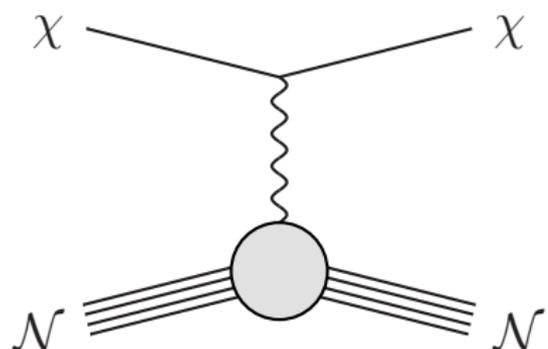
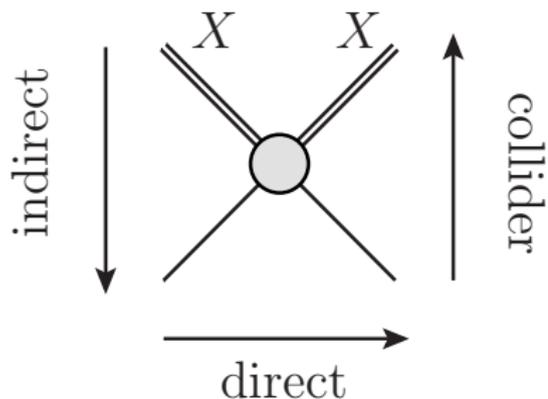


Planck 2015

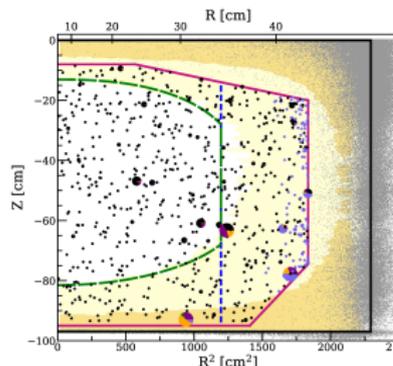
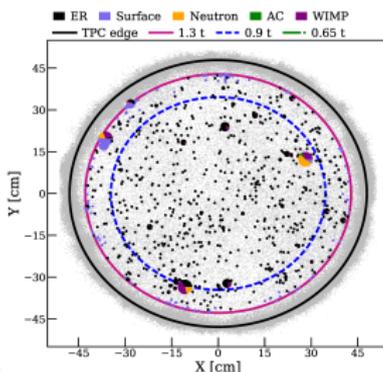
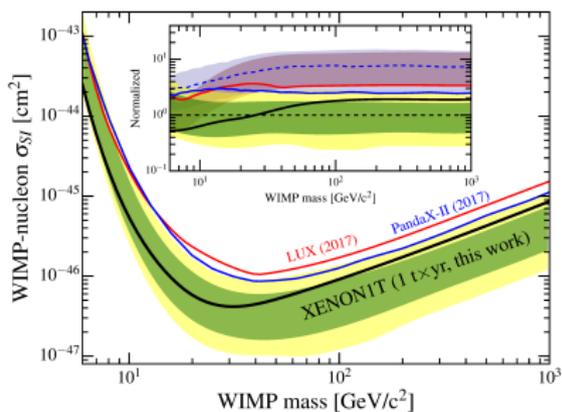
- **Cosmic microwave background**: third peak of the **power spectrum** extremely difficult to get with alternatives to dark matter

How to search for dark matter?

- Assume dark matter exists and is a **weakly interacting massive particle** (WIMP)
- Search strategies: direct, indirect, collider
- **Direct detection**: search for **WIMPs** scattering off nuclei in the large-scale detectors
- Ingredients for interpretation:
 - **Dark matter halo**: velocity distribution
 - **Nucleon matrix elements**: WIMP–nucleon couplings
 - **Nuclear structure factors**: embedding into target nucleus



Direct detection of dark matter: schematics



XENON1T 2018

● Rate for WIMP–nucleus scattering

$$\frac{dR}{dE_r} = \underbrace{\frac{\sigma_{\chi N}^{SI}}{m_{\chi} \mu_N^2}}_{\text{particle + hadronic physics}} \times \underbrace{|\mathcal{F}_+^M(q^2)|^2}_{\text{nuclear physics}} \times \underbrace{\rho_0 \int_{v_{\min}}^{v_{\text{esc}}} \frac{f(\mathbf{v}, t)}{v} d^3v}_{\text{astrophysics}}$$

- Master formula for the rate

$$\frac{dR}{dE_r} = \frac{\rho_0 \sigma_{\chi N}^{\text{SI}}}{m_\chi \mu_N^2} \times |\mathcal{F}_+^M(q^2)|^2 \times \int_{v_{\min}}^{v_{\text{esc}}} \frac{f(\mathbf{v}, t)}{v} d^3v$$

- **Standard halo model**

- Local dark matter density: $\rho_0 = 0.3 \text{ GeV cm}^{-3}$
- Boltzmann distribution: $f(\mathbf{v}) = f_0 e^{-\mathbf{v}^2/v_0^2}$

but large (unquantified) uncertainties

Direct detection of dark matter: astrophysical uncertainties

- Master formula for the rate

$$\frac{dR}{dE_r} = \frac{\rho_0 \sigma_{\chi N}^{\text{SI}}}{m_{\chi} \mu_N^2} \times |\mathcal{F}_+^M(q^2)|^2 \times \int_{v_{\text{min}}}^{v_{\text{esc}}} \frac{f(\mathbf{v}, t)}{v} d^3v$$

- **Standard halo model**

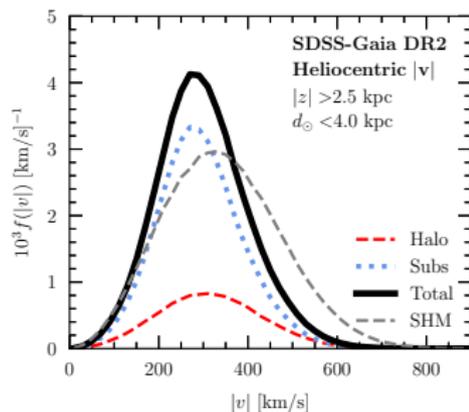
- Local dark matter density: $\rho_0 = 0.3 \text{ GeV cm}^{-3}$
- Boltzmann distribution: $f(\mathbf{v}) = f_0 e^{-v^2/v_0^2}$

but large (unquantified) uncertainties

- New developments thanks to **Gaia**

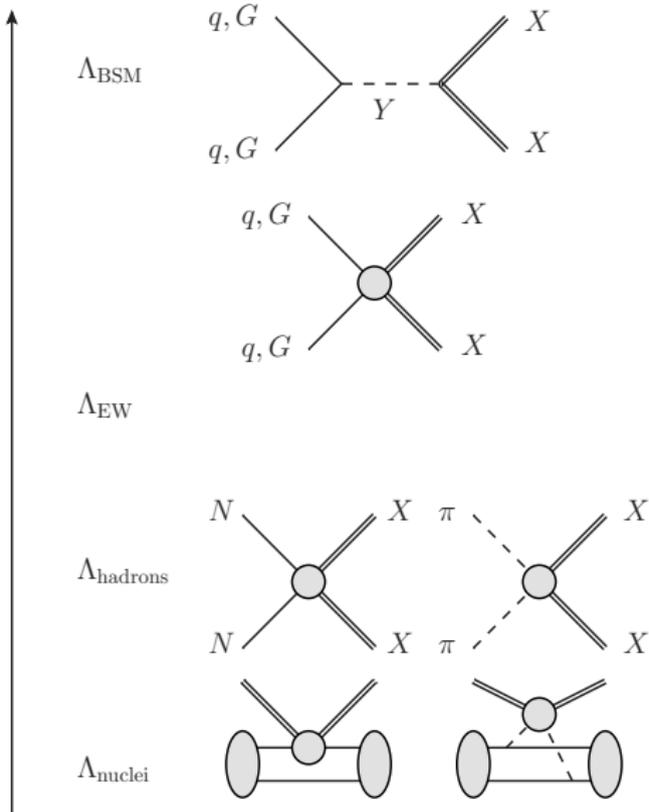
- Better determination of the density, e.g.
 $\rho_0 = 0.30(3) \text{ GeV cm}^{-3}$ Eilers et al. 1810.09466
- Velocity distribution Necib et al. 1807.02519, Evans et al. 1810.11468

↪ astrophysical uncertainties soon to be under much better control than ever before



Necib et al. 2018

Direct detection of dark matter: scales



1 **BSM scale** Λ_{BSM} : \mathcal{L}_{BSM}

2 **Effective Operators**: $\mathcal{L}_{\text{SM}} + \sum_{i,k} \frac{1}{\Lambda_{\text{BSM}}^i} \mathcal{O}_{i,k}$
SMEFT

3 Integrate out **EW physics**

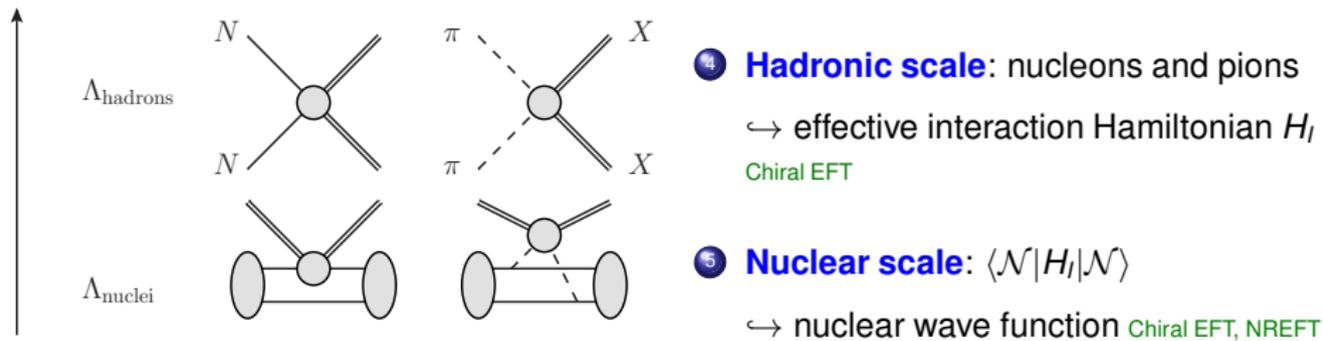
4 **Hadronic scale**: nucleons and pions

↔ effective interaction Hamiltonian H_I
Chiral EFT

5 **Nuclear scale**: $\langle \mathcal{N} | H_I | \mathcal{N} \rangle$

↔ nuclear wave function Chiral EFT, NREFT

Direct detection of dark matter: scales



- Typical WIMP–nucleon **momentum transfer**

$$|\mathbf{q}_{\text{max}}| = 2\mu_{\mathcal{N}\chi}|\mathbf{v}_{\text{rel}}| \sim 200 \text{ MeV} \quad |\mathbf{v}_{\text{rel}}| \sim 10^{-3} \quad \mu_{\mathcal{N}\chi} \sim 100 \text{ GeV}$$

- **Chiral EFT:** pions, nucleons, and WIMPs as degrees of freedom

Prézeau et al. 2003, Cirigliano et al. 2012, 2013, Menéndez et al. 2012, Klos et al. 2013, MH et al. 2015, Bishara et al. 2017

- **NREFT:** all degrees of freedom integrated out but nucleons and WIMPs

Fan et al. 2010, Fitzpatrick et al. 2012, Anand et al. 2013

- 1 Chiral effective field theory
- 2 Cross section and nuclear structure factors
- 3 Applications

- Effective theory of QCD based on **chiral symmetry**

$$\mathcal{L}_{\text{QCD}} = \bar{q}_L i \not{D} q_L + \bar{q}_R i \not{D} q_R - \bar{q}_L M q_R - \bar{q}_R M q_L - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$

- Expansion in momenta p/Λ_χ and quark masses $m_q \sim p^2$

↪ **scale separation**

- Breakdown scale: $\Lambda_\chi = M_\rho \dots 4\pi F_\pi \sim 1 \text{ GeV}$

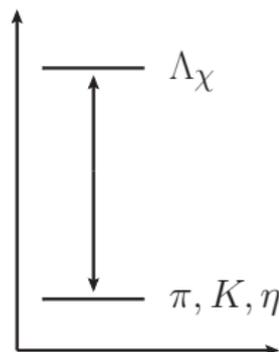
- Two variants

- **$SU(2)$** : u - and d -quark **dynamical**, m_s fixed at **physical value**

↪ expansion in M_π/Λ_χ , usually nice convergence

- **$SU(3)$** : u -, d -, and s -quark dynamical

↪ expansion in M_K/Λ_χ , sometimes tricky



Chiral EFT: a modern approach to nuclear forces

- Traditionally: meson-exchange potentials
- Chiral effective field theory
 - Based on **chiral symmetry** of QCD
 - **Power counting**
 - **Low-energy constants**
 - Hierarchy of multi-nucleon forces
 - Consistency of NN and $3N$

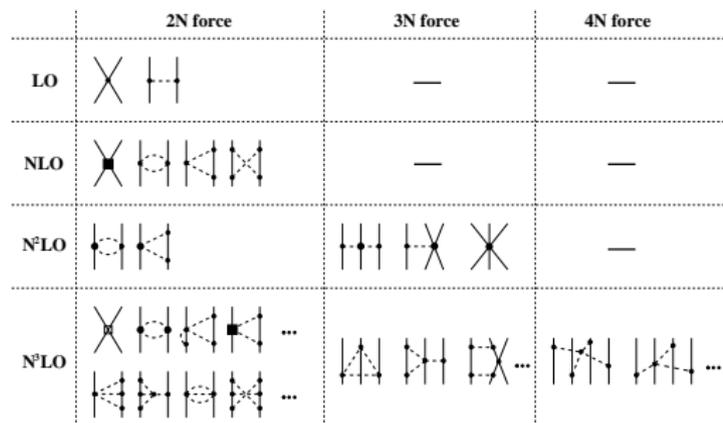
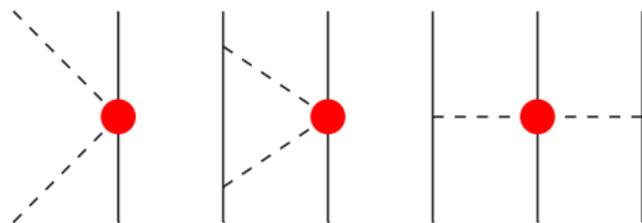


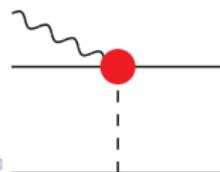
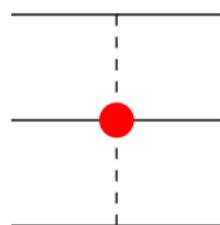
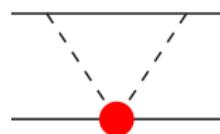
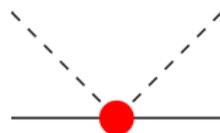
Figure taken from 1011.1343

↔ modern theory of nuclear forces

- Long-range part related to **pion-nucleon scattering**



- Coupling to **external sources** $\mathcal{L}(v_\mu, a_\mu, s, p)$
- Same LECs appear in **axial current**
 $\hookrightarrow \beta$ decay, neutrino interactions, dark matter
- Vast literature for v_μ and a_μ , up to one-loop level
 - With unitary transformations: Kölling et al. 2009, 2011, Krebs et al. 2016, 2019
 - Without unitary transformations: Park et al. 2003, Pastore et al. 2008, Baroni et al. 2015
- For **dark matter** further currents: s , p , tensor, spin-2, θ_μ^μ



- **Effective WIMP Lagrangian** for spin-1/2 SM singlet χ [Goodman et al. 2010](#)

$$\begin{aligned} \mathcal{L}_\chi = & \frac{1}{\Lambda^3} \sum_q \left[C_q^{SS} \bar{\chi} \chi m_q \bar{q} q + C_q^{PS} \bar{\chi} i \gamma_5 \chi m_q \bar{q} q + C_q^{SP} \bar{\chi} \chi m_q \bar{q} i \gamma_5 q + C_q^{PP} \bar{\chi} i \gamma_5 \chi m_q \bar{q} i \gamma_5 q \right] \\ & + \frac{1}{\Lambda^2} \sum_q \left[C_q^{VV} \bar{\chi} \gamma^\mu \chi \bar{q} \gamma_\mu q + C_q^{AV} \bar{\chi} \gamma^\mu \gamma_5 \chi \bar{q} \gamma_\mu q + C_q^{VA} \bar{\chi} \gamma^\mu \chi \bar{q} \gamma_\mu \gamma_5 q + C_q^{AA} \bar{\chi} \gamma^\mu \gamma_5 \chi \bar{q} \gamma_\mu \gamma_5 q \right] \\ & + \frac{1}{\Lambda^3} \left[C_g^S \bar{\chi} \chi \alpha_s G_{\mu\nu}^a G_a^{\mu\nu} \right] \end{aligned}$$

- **Chiral power counting**

$$\partial = \mathcal{O}(p), \quad m_q = \mathcal{O}(p^2) = \mathcal{O}(M_\pi^2), \quad a_\mu, v_\mu = \mathcal{O}(p), \quad \frac{\partial}{m_N} = \mathcal{O}(p^2)$$

↪ construction of effective Lagrangian for nucleon and pion fields

↪ organize in terms of **chiral order** ν , $\mathcal{M} = \mathcal{O}(p^\nu)$

Chiral counting: summary

	Nucleon	V		A	
WIMP		t	\mathbf{x}	t	\mathbf{x}
	1b	0	$1 + 2$	2	$0 + 2$
V	2b	4	$2 + 2$	2	$4 + 2$
	2b NLO	—	—	5	$3 + 2$
	1b	$0 + 2$	1	$2 + 2$	0
A	2b	$4 + 2$	2	$2 + 2$	4
	2b NLO	—	—	$5 + 2$	3

	Nucleon	S	P
WIMP			
	1b	2	1
S	2b	3	5
	2b NLO	—	4
	1b	$2 + 2$	$1 + 2$
P	2b	$3 + 2$	$5 + 2$
	2b NLO	—	$4 + 2$

- $+2$ from NR expansion of WIMP spinors, terms can be dropped if $m_\chi \gg m_N$
- **Red**: all terms up to $\nu = 3$
- 1b: one-body (single-nucleon), 2b: two-body, 2b NLO: two-body at (nominally) next-to-leading order

Example: chiral counting in scalar channel

- Leading pion–nucleon Lagrangian

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi} \left[i\gamma_{\mu} (\partial^{\mu} - i\nu^{\mu}) - m_N + \frac{g_A}{2} \gamma_{\mu} \gamma_5 \left(2\mathbf{a}^{\mu} - \frac{\partial^{\mu} \boldsymbol{\pi}}{F_{\pi}} \right) + \dots \right] \Psi$$

↔ **no scalar source!**

	Nucleon	S
WIMP		
	1b	2
S	2b	3

Example: chiral counting in scalar channel

- Leading pion–nucleon Lagrangian

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi} \left[i\gamma_\mu (\partial^\mu - i\nu^\mu) - m_N + \frac{g_A}{2} \gamma_\mu \gamma_5 \left(2\mathbf{a}^\mu - \frac{\partial^\mu \boldsymbol{\pi}}{F_\pi} \right) + \dots \right] \Psi$$

↔ **no scalar source!**

- Scalar coupling

$$f_N = \frac{m_N}{\Lambda^3} \sum_{q=u,d,s} C_q^{SS} f_q^N + \dots \quad \langle N | m_q \bar{q}q | N \rangle = f_q^N m_N$$

↔ for $q = u, d$ related to **pion–nucleon σ -term** $\sigma_{\pi N}$

- Chiral expansion

$$\sigma_{\pi N} = -4c_1 M_\pi^2 - \frac{9g_A^2 M_\pi^3}{64\pi F_\pi^2} + \mathcal{O}(M_\pi^4) \quad \dot{\sigma} = \frac{5g_A^2 M_\pi}{256\pi F_\pi^2} + \mathcal{O}(M_\pi^2)$$

↔ slow convergence due to strong $\pi\pi$ rescattering

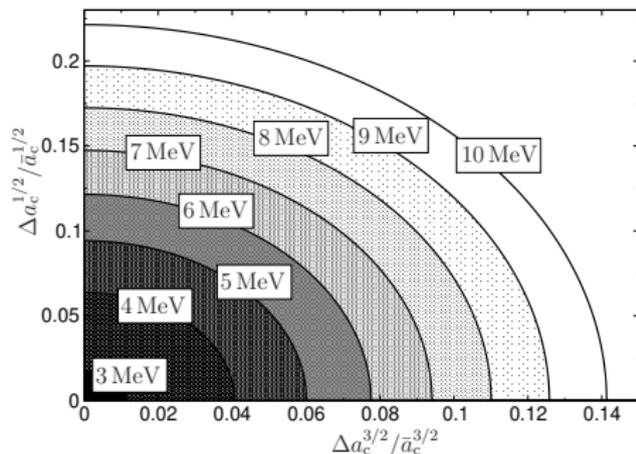
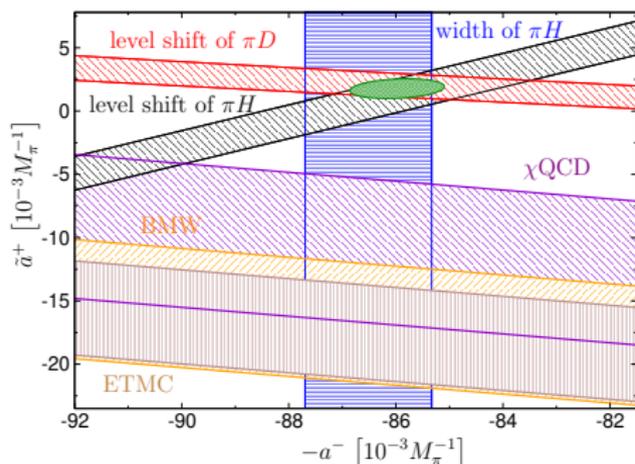
↔ use phenomenology for the full scalar form factor!

	Nucleon	S
WIMP		
	1b	2
S	2b	3

Extracting $\sigma_{\pi N}$ from πN scattering: low-energy theorem

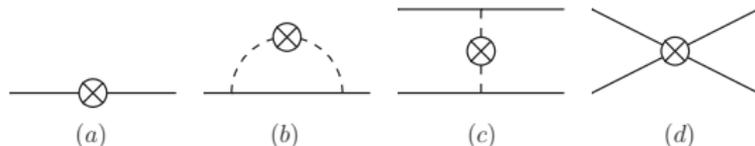
- No scalar probe, but still relation to experiment! How?
↪ **low-energy theorem**
- Topic for another talk [MH, Ruiz de Elvira, Kubis, Meißner PRL 115 \(2015\) 092301, PLB 760 \(2016\) 74, JPG 45 \(2018\) 024001](#)
 - Goes back to [Cheng, Dashen; Brown, Pardee, Peccei 1971](#)
 - Relates $\sigma_{\pi N}$ to πN scattering amplitude, but at **unphysical kinematics**
↪ analytic continuation to the Cheng–Dashen point
 - **No chiral logs** at one-loop order! [Bernard, Kaiser, Meißner 1996](#)
 - **Protected by $SU(2)$**
↪ expected correction: $\sigma_{\pi N} M_\pi^2 / m_N^2 \sim 1 \text{ MeV}$

πN σ -term: a lingering tension with lattice QCD



- Independent experimental constraints from **pionic atoms** and **low-energy cross sections** agree at the level of $\sigma_{\pi N} = 58(5)$ MeV
- Tension can be illustrated in scattering-length plane
 \hookrightarrow independent constraint from lattice calculation of $a_{0+}^{1/2}$ and $a_{0+}^{3/2}$
- This needs to be resolved: **rare opportunity** to benchmark lattice BSM matrix elements from experiment

Cross section and nuclear structure factors



$$\begin{aligned} \frac{d\sigma_{\chi\mathcal{N}}}{dq^2} &= \frac{1}{4\pi v^2} \left| \sum_{l=\pm} \left(c_l^M - \frac{q^2}{m_N^2} \dot{c}_l^M \right) \mathcal{F}_l^M(q^2) + c_\pi \mathcal{F}_\pi(q^2) + a_b \mathcal{F}_b(q^2) + \frac{q^2}{2m_N^2} \sum_{l=\pm} c_l^{\phi''} \mathcal{F}_l^{\phi''}(q^2) \right|^2 \\ &+ \frac{1}{4\pi v^2} \sum_{i=5,8,11} \left| \sum_{l=\pm} \xi_i(q, v_T^\perp) c_l^{M,i} \mathcal{F}_l^M(q^2) \right|^2 \\ &+ \frac{1}{v^2(2J+1)} \left(|a_+|^2 S_{00}(q^2) + \text{Re}(a_+ a_-^*) S_{01}(q^2) + |a_-|^2 S_{11}(q^2) \right) \end{aligned}$$

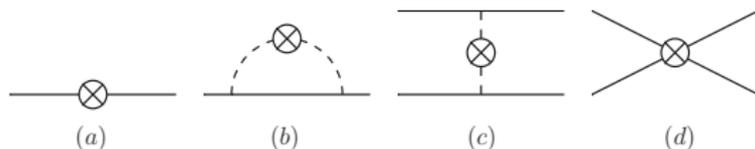
- Decomposition into **nuclear structure factors** \mathcal{F} , S_{ij} and coefficients c , a

- Three classes of contributions:

- **(Sub-) Leading 1b responses** (a): $c_l^M \mathcal{F}_l^M(q^2)$, $c_l^{\phi''} \mathcal{F}_l^{\phi''}(q^2)$, $|a_\pm|^2 S_{ij}(q^2)$
- **Radius corrections** (b): $\dot{c}_l^M \mathcal{F}_l^M(q^2)$
- **Two-body currents** (c), (d): $c_\pi \mathcal{F}_\pi(q^2)$, $a_b \mathcal{F}_b(q^2)$

- (a)+(b) essentially **nucleon form factors**, but (c)+(d) genuinely new effects

Cross section and nuclear structure factors



$$\begin{aligned} \frac{d\sigma_{\chi N}}{dq^2} &= \frac{1}{4\pi v^2} \left| \sum_{l=\pm} \left(c_l^M - \frac{q^2}{m_N^2} \dot{c}_l^M \right) \mathcal{F}_l^M(q^2) + c_\pi \mathcal{F}_\pi(q^2) + c_b \mathcal{F}_b(q^2) + \frac{q^2}{2m_N^2} \sum_{l=\pm} c_l^{\Phi''} \mathcal{F}_l^{\Phi''}(q^2) \right|^2 \\ &+ \frac{1}{4\pi v^2} \sum_{i=5,8,11} \left| \sum_{l=\pm} \xi_i(q, v_T^\perp) c_l^{M,i} \mathcal{F}_l^M(q^2) \right|^2 \\ &+ \frac{1}{v^2(2J+1)} \left(|a_+|^2 S_{00}(q^2) + \text{Re}(a_+ a_-^*) S_{01}(q^2) + |a_-|^2 S_{11}(q^2) \right) \end{aligned}$$

• Nuclear structure interpretation:

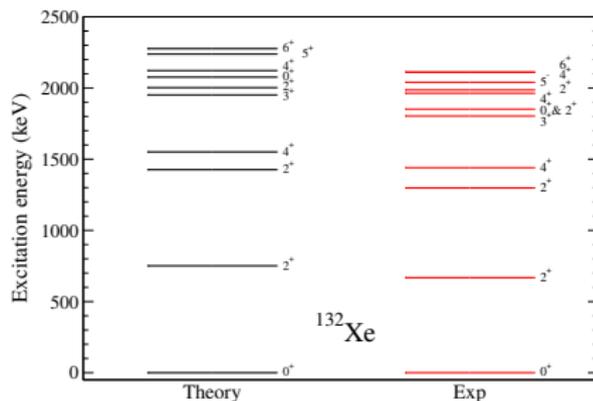
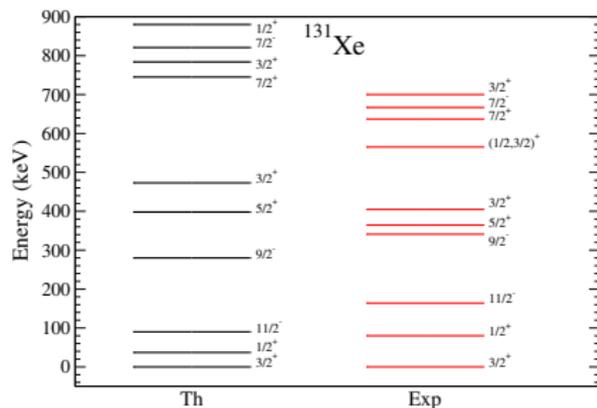
- $\mathcal{F}_l^M(q^2)$: $\mathbb{1} \Rightarrow$ charge distribution (coherent), $\mathcal{F}_\pm^M(0) = Z \pm N$
- $\mathcal{F}_l^{\Phi''}(q^2)$: $i\mathbf{S}_N \cdot (\mathbf{q} \times \mathbf{v}^\perp) \Rightarrow$ spin-orbit interaction (quasi-coherent)
- $S_{ij}(q^2)$: $\mathbf{S}_\chi \cdot \mathbf{S}_N, \mathbf{S}_\chi \cdot \mathbf{q}, \mathbf{S}_N \cdot \mathbf{q} \Rightarrow$ spin average (not coherent),
 $S_{00}(0) \pm S_{01}(0) + S_{11}(0) = \frac{(2J+1)(J+1)}{4\pi J} |\langle \mathbf{S}_{p/n} \rangle|^2$

• Coefficients: convolution of **Wilson coefficients** and **nucleon matrix elements**

$$c_\pm^M = \zeta (f_p \pm f_n) + \dots \quad f_N = \frac{m_N}{\Lambda^3} \sum_{q=u,d,s} C_q^{SS} f_q^N + \dots \quad \langle N | m_q \bar{q}q | N \rangle = m_N f_q^N$$

- ChiralEFT4DM: all results shown in the following available as PYTHON package at <https://theorie.ikp.physik.tu-darmstadt.de/strongint/ChiralEFT4DM.html>
- Includes:
 - (Quasi-) Coherent structure factors for F, Si, Ar, Ge, Xe
 - Nucleon matrix elements and matching relations for spin-1/2 and spin-0 WIMP
 - 1b and 2b responses up to third chiral order
 - S , P , V , A , T , θ_{μ}^{μ} , and spin-2 effective operators that can lead to a coherent response
 - Convolution with Standard Halo Model

Spectra and shell-model calculation



- **Shell-model diagonalization** for Xe isotopes with ^{100}Sn core
- **Uncertainty estimates**: currently phenomenological shell-model interaction
 - ↪ chiral-EFT-based interactions in the future
 - ↪ **ab-initio calculations for light nuclei**

Charge radii and neutron skin

	^{19}F	^{28}Si	^{40}Ar	^{74}Ge	^{132}Xe
$\sqrt{\langle r^2 \rangle_{\text{ch}}}$ [fm] (th)	2.83	3.19	3.43	4.08	4.77
(exp)	2.898(2)	3.122(2)	3.427(3)	4.0742(12)	4.7808(49)
$\sqrt{\langle r_n^2 \rangle} - \sqrt{\langle r_p^2 \rangle}$ [fm]	0.02	0	0.11	0.17	0.28
shell-model interaction	USDB	USDB	SDPF.SM	RG	GCN

- Excellent agreement for charge radii

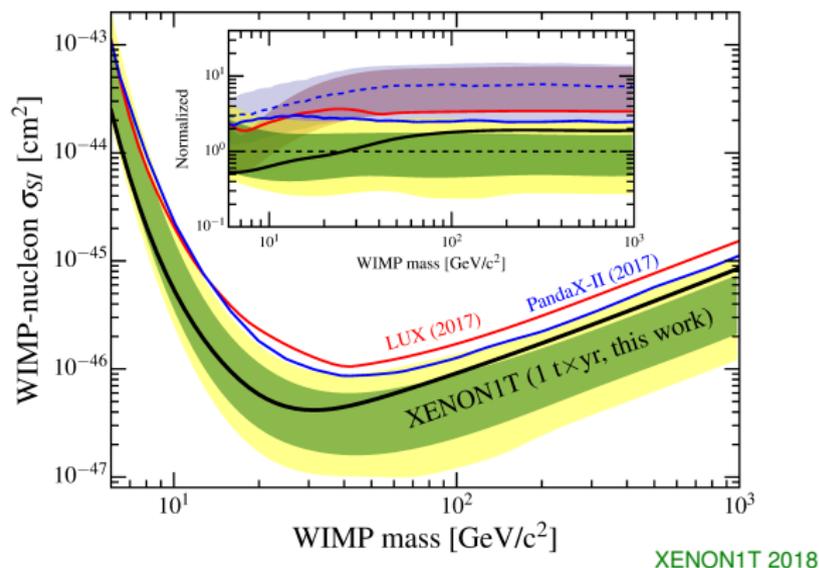
$$\langle r_{\text{ch}}^2 \rangle = \langle r_p^2 \rangle + \langle r_{E,p}^2 \rangle + \frac{N}{Z} \langle r_{E,n}^2 \rangle + \langle r_{\text{rel}}^2 \rangle + \langle r_{\text{spin-orbit}}^2 \rangle$$

- Point-neutron radii more uncertain
- Related to structure factors by

$$\langle r_p^2 \rangle = -\frac{3}{Z} \frac{d}{dq^2} (\mathcal{F}_+^M(q^2) + \mathcal{F}_-^M(q^2)) \Big|_{q^2=0} \quad \langle r_n^2 \rangle = -\frac{3}{N} \frac{d}{dq^2} (\mathcal{F}_+^M(q^2) - \mathcal{F}_-^M(q^2)) \Big|_{q^2=0}$$

- Further cross checks: electric quadrupole and magnetic dipole moments, transition matrix elements

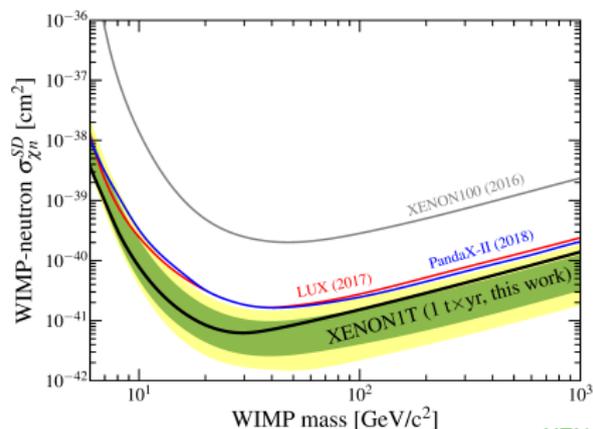
Case 1: spin-independent scattering



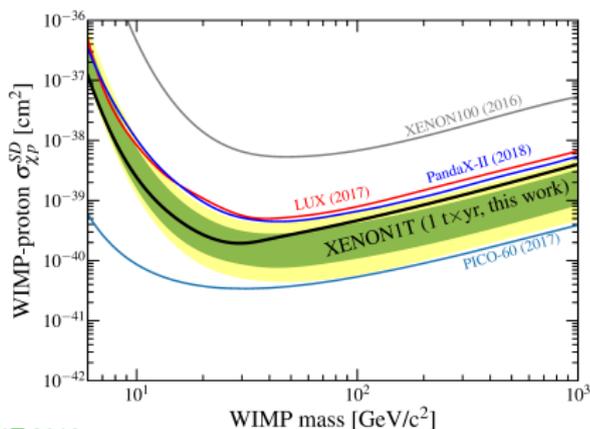
- All $c = 0$, $a = 0$ except for c_+^M : **spin-independent scattering**

$$\frac{d\sigma_{\chi\mathcal{N}}}{dq^2} = \frac{\sigma_{\chi\mathcal{N}}^{\text{SI}}}{4\mu_N^2 v^2} |\mathcal{F}_+^M(q^2)|^2 \quad \sigma_{\chi\mathcal{N}}^{\text{SI}} = \frac{\mu_N^2}{\pi} |c_+^M|^2 \quad \mu_N = \frac{m_N m_\chi}{m_N + m_\chi}$$

Case 2: spin-dependent scattering

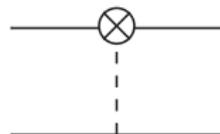


XENON1T 2019



- All $c = 0$, $a_+ = \pm a_-$: **spin-dependent scattering**

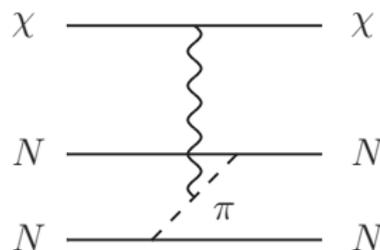
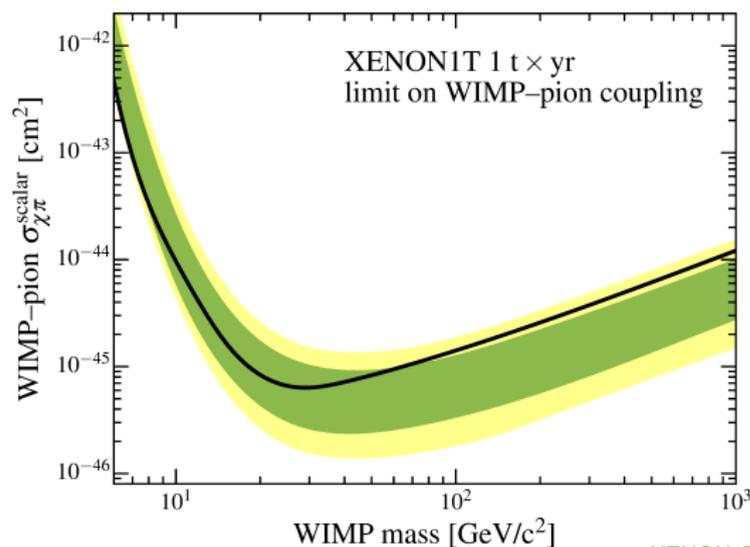
$$\frac{d\sigma_{XN}}{dq^2} = \frac{\sigma_{XN}^{SD}}{3\mu_N^2 v^2} \frac{\pi}{2J+1} S_N(q^2) \quad \sigma_{XN}^{SD} = \frac{3\mu_N^2}{\pi} |a_+|^2$$



- Xe sensitive to proton spin due to **two-body currents**

Klos, Menéndez, Gazit, Schwenk 2013

Case 3: WIMP–pion scattering



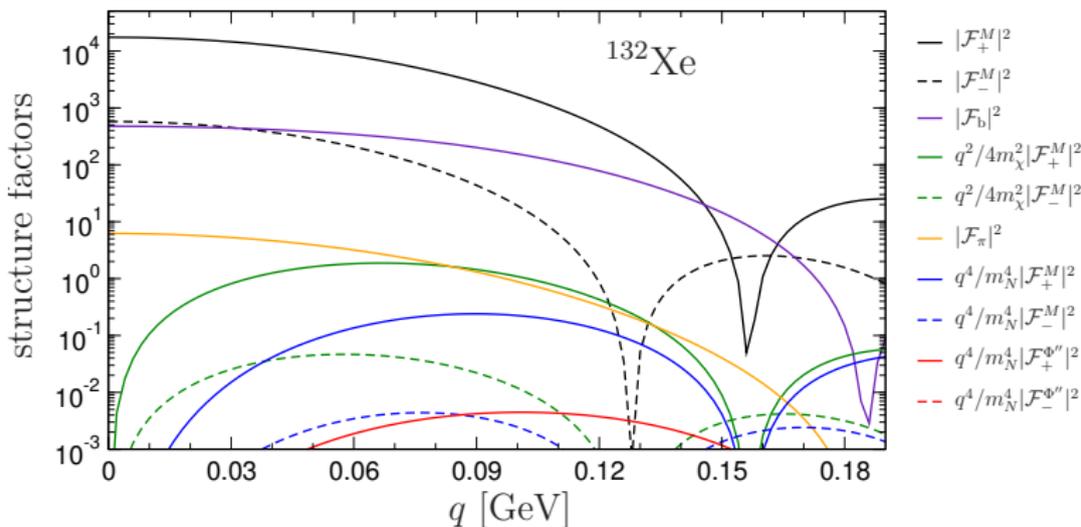
XENON1T + MH, Klos, Menéndez, Schwenk 2019

- Only c_π nonzero: **WIMP–pion scattering**

$$\frac{d\sigma_{\chi\mathcal{N}}}{dq^2} = \frac{\sigma_{\chi\pi}^{\text{scalar}}}{\mu_\pi^2 v^2} |\mathcal{F}_\pi(q^2)|^2 \quad \sigma_{\chi\pi}^{\text{scalar}} = \frac{\mu_\pi^2}{4\pi} |c_\pi|^2 \quad \mu_\pi = \frac{m_\chi M_\pi}{m_\chi + M_\pi}$$

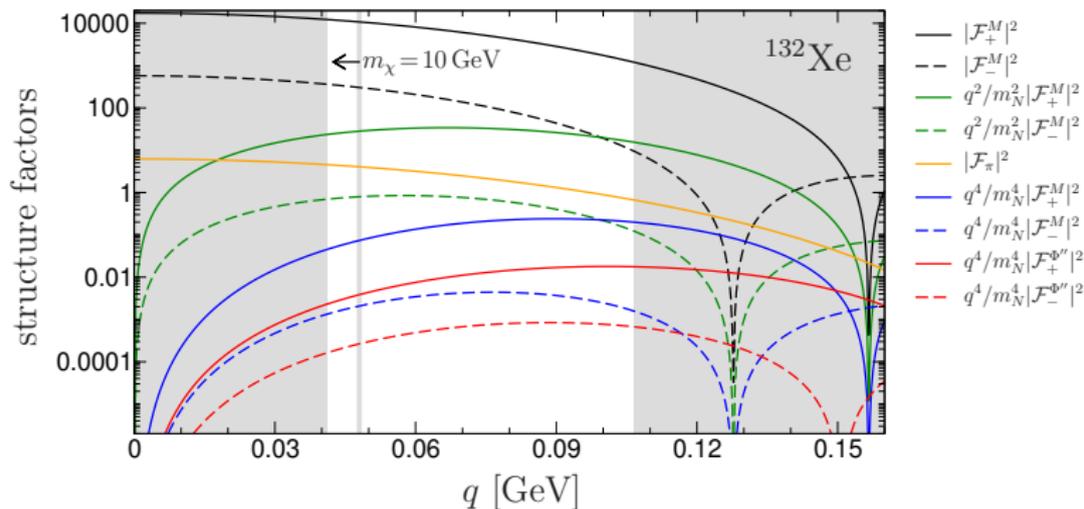
- Expression in terms of cross section depends on underlying operator, here for a scalar $\bar{\chi}\chi\bar{q}q$

Full set of coherent contributions



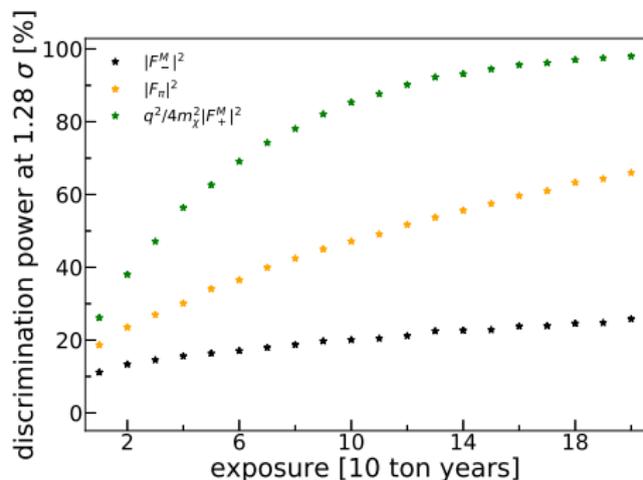
$$\begin{aligned}
 \frac{d\sigma_{\chi\mathcal{N}}}{dq^2} &= \frac{1}{4\pi v^2} \left| \sum_{l=\pm} \left(c_l^M - \frac{q^2}{m_N^2} \tilde{c}_l^M \right) \mathcal{F}_l^M(q^2) + c_\pi \mathcal{F}_\pi(q^2) + c_b \mathcal{F}_b(q^2) + \frac{q^2}{2m_N^2} \sum_{l=\pm} c_l^{\Phi''} \mathcal{F}_l^{\Phi''}(q^2) \right|^2 \\
 &+ \frac{1}{4\pi v^2} \sum_{i=5,8,11} \left| \sum_{l=\pm} \xi_i(q, v_T^\perp) c_l^{M,i} \mathcal{F}_l^M(q^2) \right|^2 \\
 &+ \frac{1}{v^2(2J+1)} \left(|a_+|^2 S_{00}(q^2) + \text{Re}(a_+ a_-^*) S_{01}(q^2) + |a_-|^2 S_{11}(q^2) \right)
 \end{aligned}$$

Discriminating different response functions



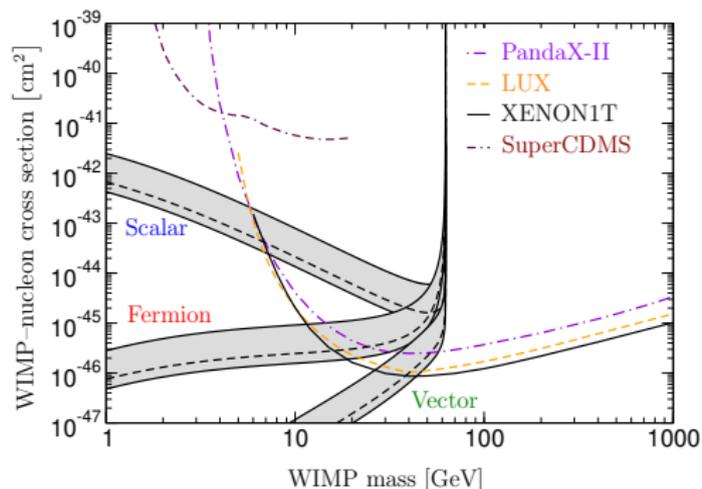
- White region accessible to XENON-type experiment
- Can one tell these curves apart in a realistic experimental setting?
- Consider XENON1T-like, XENONnT-like, DARWIN-like settings

Discriminating different response functions



- DARWIN-like setting, $m_\chi = 100 \text{ GeV}$
- q -dependent responses more easily distinguishable
- If interaction not much weaker than current limits, DARWIN could discriminate most responses from standard SI structure factor

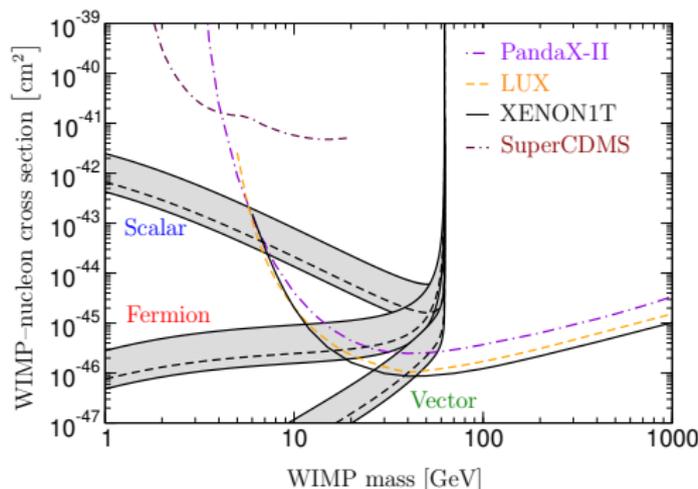
- **Higgs Portal:** WIMP interacts with SM via the Higgs
 - **Scalar:** $H^\dagger H S^2$
 - **Vector:** $H^\dagger H V_\mu V^\mu$
 - **Fermion:** $H^\dagger H \bar{f} f$
- If $m_h > 2m_\chi$, should happen at the LHC
 - ↔ limits on **invisible Higgs decays**



- **Higgs Portal:** WIMP interacts with SM via the Higgs

- **Scalar:** $H^\dagger H S^2$
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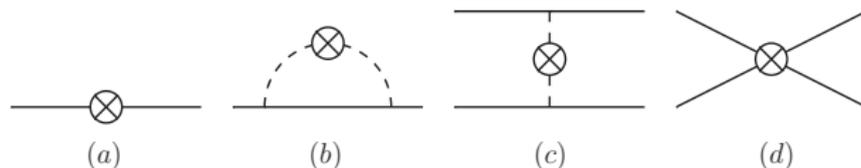
- If $m_h > 2m_\chi$, should happen at the LHC
 \hookrightarrow limits on **invisible Higgs decays**



- Translation requires input for **Higgs–nucleon coupling**

$$f_N = \sum_{q=u,d,s,c,b,t} f_q^N = \frac{2}{9} + \frac{7}{9} \sum_{q=u,d,s} f_q^N + \mathcal{O}(\alpha_s) \quad m_N f_q^N = \langle N | m_q \bar{q} q | N \rangle$$

- Issues: input for $f_N = 0.260 \dots 0.629$ outdated, 2b currents missing



- **One-body contribution**

$$f_N^{1b} = 0.307(9)_{ud}(15)_s(5)_{\text{pert}} = 0.307(18)$$

- Limits on WIMP–nucleon cross section subsume **2b effects**

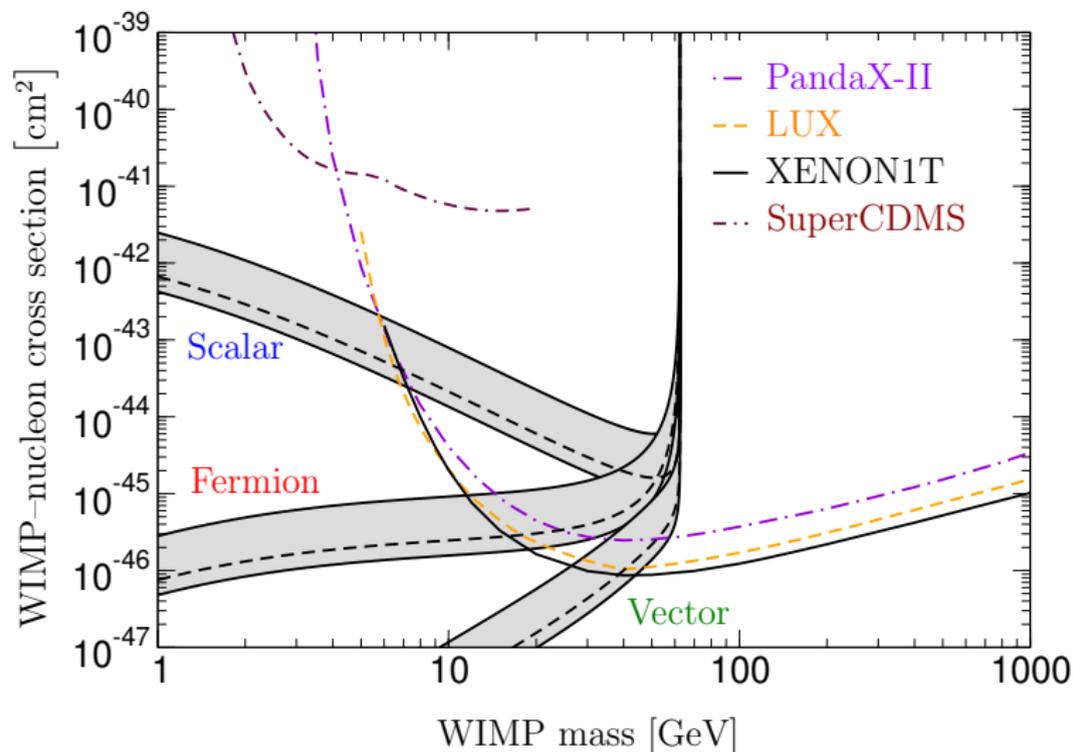
↪ have to be included for meaningful comparison

- **Two-body contribution**

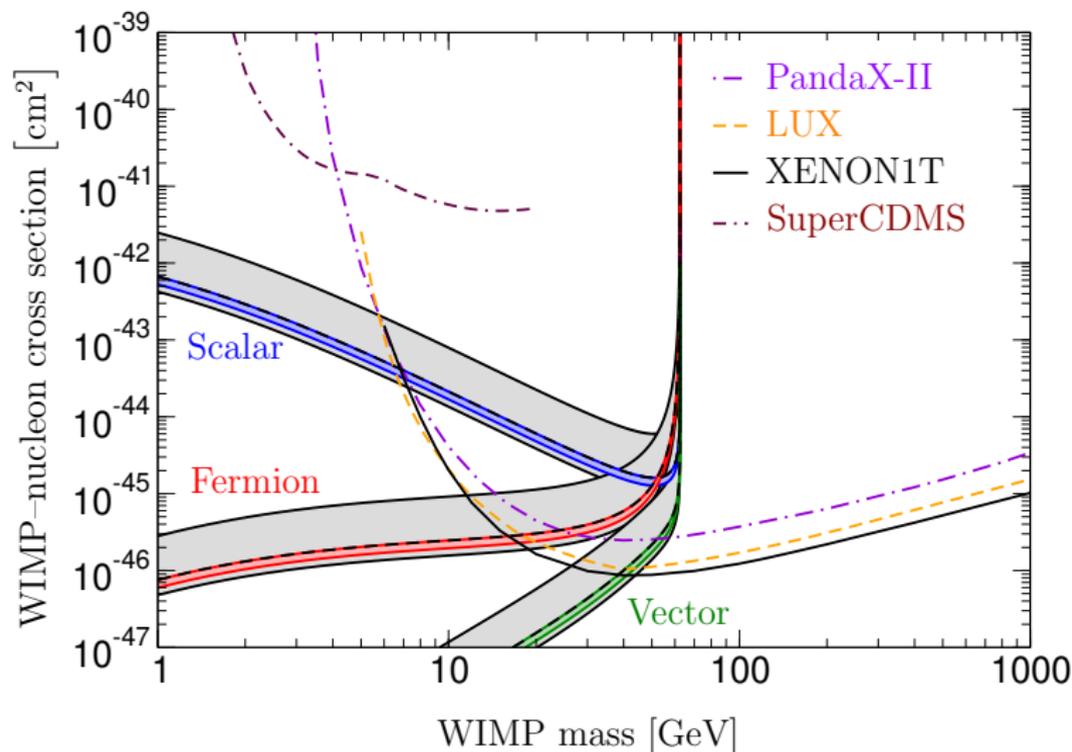
- Need s and θ_μ^μ currents
- Treatment of θ_μ^μ tricky: several ill-defined terms combine to $\langle \Psi | T + V_{NN} | \Psi \rangle = E_b$
- A cancellation makes the final result anomalously small

$$f_N^{2b} = [- 3.2(0.2)_A(2.1)_{\text{ChEFT}} + 5.0(0.4)_A] \times 10^{-3} = 1.8(2.1) \times 10^{-3}$$

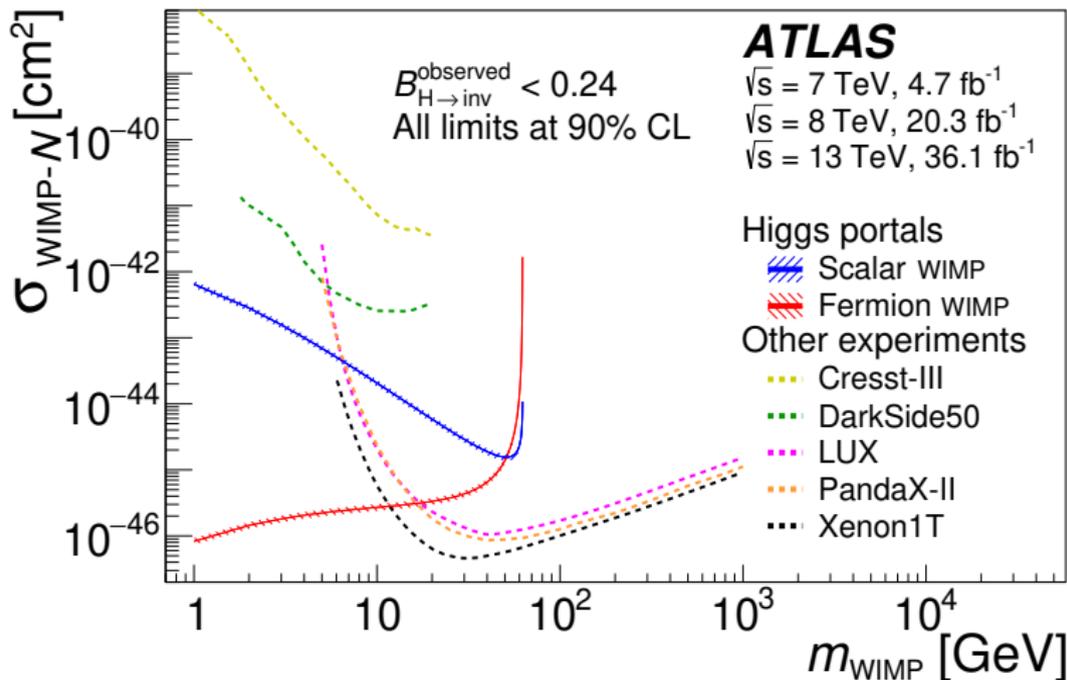
Improved limits for Higgs Portal dark matter



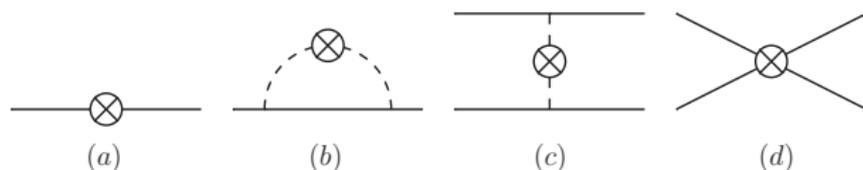
Improved limits for Higgs Portal dark matter



Improved limits for Higgs Portal dark matter



ATLAS 2019



- **Chiral EFT** for WIMP–nucleon scattering
 - Connects nuclear and hadronic scales
 - For cross section need **nucleon matrix elements** and **nuclear structure factors**
- Applications
 - Scalar WIMP–pion cross section
 - Discriminating nuclear responses
 - Higgs-portal models
- Systematic approach to the nuclear responses thanks to EFT

Extracting $\sigma_{\pi N}$ from πN scattering: strategy

- **Scalar form factor** of the nucleon

$$\sigma(t) = \langle N(p') | \hat{m}(\bar{u}u + \bar{d}d) | N(p) \rangle \quad t = (p' - p)^2 \quad \sigma_{\pi N} = \sigma(0)$$

- **Low-energy theorem** Cheng, Dashen 1971

$$F_{\pi}^2 \bar{D}^+(\nu = 0, t = 2M_{\pi}^2) = \sigma(2M_{\pi}^2) + \Delta_R$$

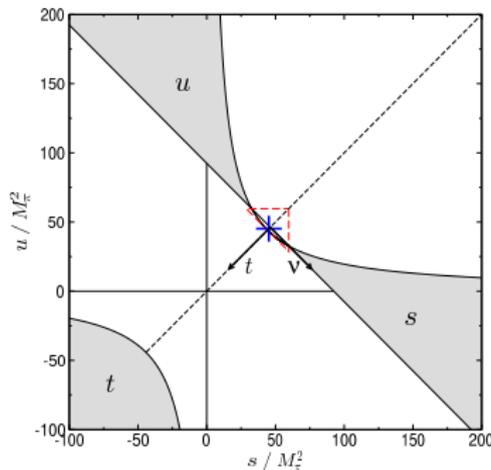
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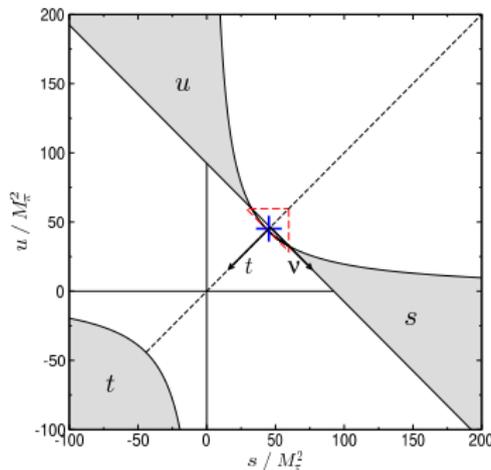
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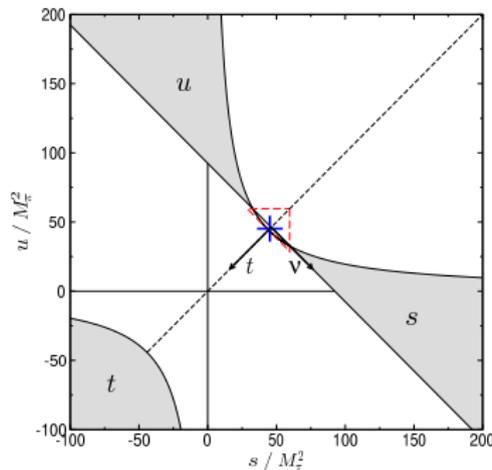
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$$F_{\pi}^2 (\alpha_{00}^+ + 2M_{\pi}^2 \alpha_{01}^+) + \Delta_D$$

- Remainder $|\Delta_R| \lesssim 2 \text{ MeV}$ small Bernard, Kaiser, Meißner 1996



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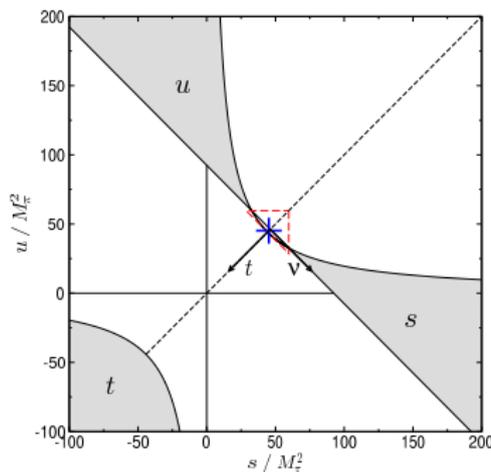
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- **Dispersive approach** Gasser, Leutwyler, Sainio 1991,
update MH, Ditsche, Kubis, Meißner 2012

$$\Delta_D - \Delta_{\sigma} = (-1.8 \pm 0.2) \text{ MeV}$$



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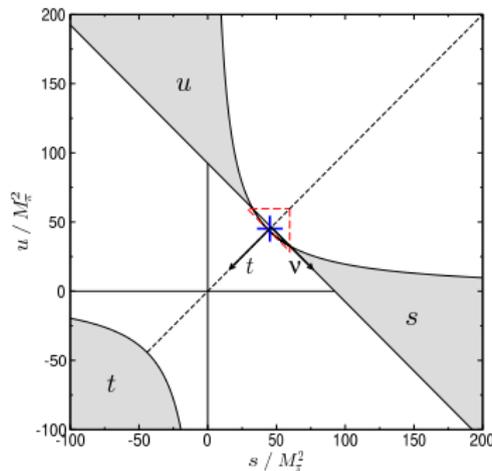
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$$\Delta_D - \Delta_{\sigma} = (-1.8 \pm 0.2) \text{ MeV}$$

- Need to determine subthreshold parameters d_{00}^+ , d_{01}^+

↔ **Roy–Steiner equations for πN scattering**



Solution of Roy–Steiner equations: constraints from pionic atoms

- $\pi H/\pi D$: bound state of π^- and p/d
 \hookrightarrow spectrum sensitive to threshold

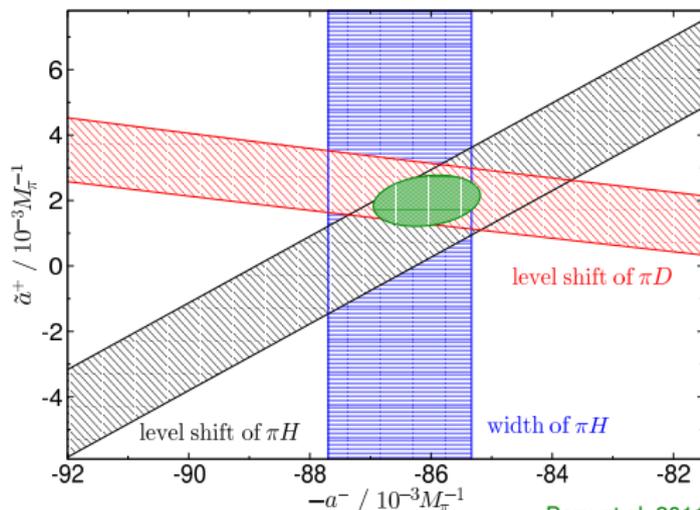
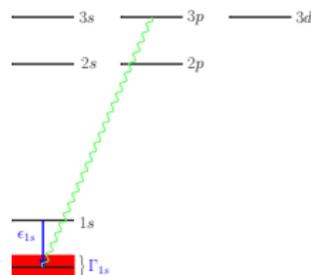
πN scattering PSI 1995–2010

- **Combined analysis** of πH and πD

$$\bar{a}_{0+}^{1/2} = 169.8(2.0) \times 10^{-3} M_\pi^{-1}$$

$$\bar{a}_{0+}^{3/2} = -86.3(1.8) \times 10^{-3} M_\pi^{-1}$$

- Impose $\bar{a}_{0+}^{1/2}$ and $\bar{a}_{0+}^{3/2}$ in the solution of the Roy–Steiner equations



Baru et al. 2011

Central result

$$\sigma_{\pi N} = 59.1(3.1) \text{ MeV} + \sum_{I_S} c_{I_S} (a_{0+}^{I_S} - \bar{a}_{0+}^{I_S}) = 59.1(3.5) \text{ MeV}$$

$$c_{1/2} = 0.242 \text{ MeV} \times 10^3 M_\pi \quad c_{3/2} = 0.874 \text{ MeV} \times 10^3 M_\pi$$

• Error budget

Low-energy theorem				
Cheng–Dashen remainder	Isospin breaking	Scattering lengths	Roy–Steiner systematics	Total
2 MeV	2.2 MeV	1.6 MeV	0.9 MeV	3.5 MeV

Solution of Roy–Steiner equations: πN σ -term

Central result

$$\sigma_{\pi N} = 59.1(3.1) \text{ MeV} + \sum_{I_S} c_{I_S} (a_{0+}^{I_S} - \bar{a}_{0+}^{I_S}) = 59.1(3.5) \text{ MeV}$$

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• Error budget

Low-energy theorem				
Cheng–Dashen remainder	Isospin breaking	Scattering lengths	Roy–Steiner systematics	Total
2 MeV	2.2 MeV	1.6 MeV	0.9 MeV	3.5 MeV

• Lattice QCD

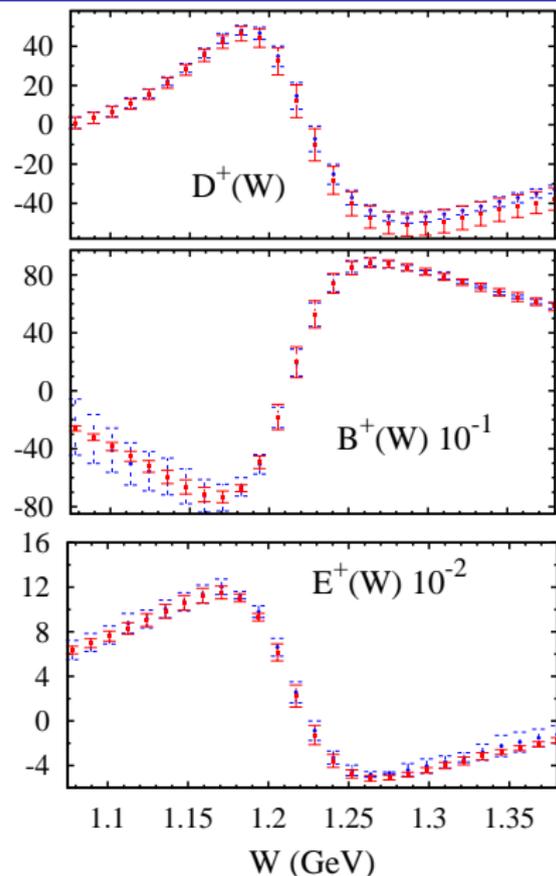
Collaboration	BMW	χ QCD	ETMC	RQCD
$\sigma_{\pi N}$ [MeV]	38(3)(3)	45.9(7.4)(2.8)	37.2(2.6)($\frac{4.7}{2.9}$)	35(6)
Tension	3.8 σ	1.5 σ	3.4 σ	3.5 σ

FAQ 1: Why does your number differ from Gasser, Leutwyler, Sainio?

	$\frac{g^2}{4\pi}$	$a_{0+}^{1/2} [10^{-3} M_\pi^{-1}]$	$a_{0+}^{3/2} [10^{-3} M_\pi^{-1}]$	$\sigma_{\pi N} [\text{MeV}]$
KH80 input	14.28	173(3)	-101(4)	47(5)
pionic-atom input	13.7(2)	169.8(2.0)	-86.3(1.8)	59.1(3.5)

- It's the input.
- $\sigma_{\pi N} \sim 45 \text{ MeV}$ from [Gasser, Leutwyler, Sainio 1991](#) relies on Karlsruhe–Helsinki partial-wave analysis (PWA) “KH80”
- We checked that:
 - KH80 internally consistent
 - Reproduce $\sigma_{\pi N} \sim 45 \text{ MeV}$ with KH80 input for scattering lengths and coupling constant
- **KH80 at odds with modern pionic-atom experiments**

FAQ 1: Why does your number differ from Gasser, Leutwyler, Sainio?



- Gasser, Leutwyler, Sainio 1991 do not use Roy–Steiner equations, but **Forward Dispersion Relations**
- Checked FDR for our amplitudes (blue/red \Leftrightarrow LHS/RHS)

FAQ 2: Could it be isospin-breaking corrections?

- Highly unlikely.
- Isospin-breaking corrections large in **isoscalar scattering lengths** Gasser et al. 2002
↪ effect goes with $\Delta_\pi = M_{\pi^\pm}^2 - M_{\pi^0}^2$ times large coefficient
- This is included in our version of the low-energy theorem

$$\begin{aligned}\sigma_{\pi N} &= F_\pi^2 (d_{00}^+ + 2M_\pi^2 d_{01}^+) + \underbrace{\Delta_D - \Delta_\sigma - \Delta_R}_{-1.8(2.0) \text{ MeV}} + \underbrace{\frac{81g_A^2 M_\pi \Delta_\pi}{256\pi F_\pi^2}}_{+3.4 \text{ MeV}} + \underbrace{\frac{e^2}{2} F_\pi^2 (4f_1 + f_2)}_{-0.4(2.2) \text{ MeV}} \\ &= F_\pi^2 (d_{00}^+ + 2M_\pi^2 d_{01}^+) + 1.2(3.0) \text{ MeV}\end{aligned}$$

↪ main effect from Δ_π goes into **“wrong” direction**

- Isospin amplitudes defined by $\pi^\pm p$ channels
↪ this is what (mainly) enters the PWAs
- $\bar{a}_{0+}^{1/2}$ and $\bar{a}_{0+}^{3/2}$ from pionic atoms defined consistent with these conventions

FAQ 3: Why not use ChPT to extract the σ -term?

	KH	GW	EM
heavy-baryon, ϵ counting Fettes, Meißner 2001	45.5(2.7)		58.5(5.4)
EOMS, δ counting Alarcón et al. 2012	43(5)	59(4)	59(2)

+ many more ChPT-based studies with similar conclusions

- You get out what you put in.
- Need to determine low-energy constants from somewhere
 - ↪ result for $\sigma_{\pi N}$ **depends on the PWA**
- Better control over analytic continuation to Cheng–Dashen point
- Systematics of the low-energy theorem
 - Relations only fulfilled perturbatively, known to fail for Δ_D, Δ_σ
 - Effect from t -channel D -waves missing
 - Relating the subthreshold and the physical region problematic
 - Isospin breaking

↪ **1-loop ChPT not sufficient for rigorous extraction**

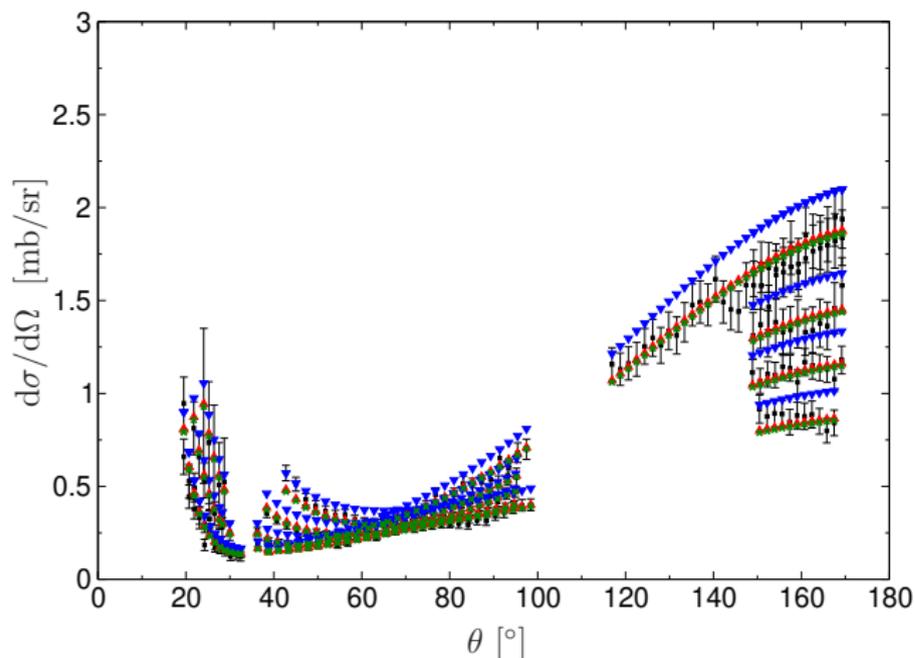
FAQ 4: What if the pionic-atom measurements are wrong?

- Fair enough. That's why we looked at cross sections.
- Challenges in extracting $\sigma_{\pi N}$ from **low-energy cross sections**
 - Normalizations
 - Isospin breaking
- Cannot use existing compilations due to bias from respective fit model

FAQ 4: What if the pionic-atom measurements are wrong?

- Fair enough. That's why we looked at cross sections.
- Challenges in extracting $\sigma_{\pi N}$ from **low-energy cross sections**
 - Normalizations
 - Isospin breaking
- Cannot use existing compilations due to bias from respective fit model
- Strategy
 - Roy–Steiner representation with scattering length as free parameter
 \hookrightarrow separately for $\pi^+ p \rightarrow \pi^+ p$, $\pi^- p \rightarrow \pi^- p$, and $\pi^- p \rightarrow \pi^0 n$
 - Normalizations as additional fit parameters (with GW as starting point)
 - Keep Coulomb piece of Tromborg corrections [Tromborg et al. 1977](#), consider the rest as error estimate
 - Iterative fit to avoid d'Agostini bias

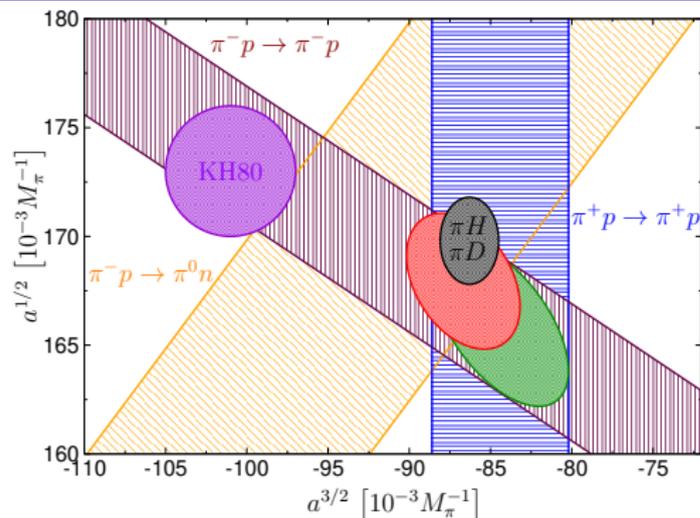
FAQ 4: What if the pionic-atom measurements are wrong?



- Example from $\pi^+ p \rightarrow \pi^+ p$ fit: **KH80**, **pionic atoms**, **fit to data**

↪ can see by eye that **KH80 is disfavored**

FAQ 4: What if the pionic-atom measurements are wrong?



	$a_{0+}^{1/2} [10^{-3} M_{\pi}^{-1}]$	$a_{0+}^{3/2} [10^{-3} M_{\pi}^{-1}]$	$\sigma_{\pi N} [\text{MeV}]$
all channels	167.9(3.2)	-86.7(3.5)	58.3(4.2)
only $\pi^{\pm} p \rightarrow \pi^{\pm} p$	166.0(3.8)	-84.4(4.2)	59.8(4.5)
pionic atoms	169.8(2.0)	-86.3(1.8)	59.1(3.5)
KH80	173(3)	-101(4)	47(5)

FAQ 4: What if the pionic-atom measurements are wrong?

Extracting the σ -term from experimental data

$$\sigma_{\pi N}|_{\text{pionic atoms}} = 59.1(3.5) \text{ MeV} \quad \sigma_{\pi N}|_{\text{cross sections}} = 58(5) \text{ MeV}$$

- Determinations from two **completely independent experiments**
- By-product: scattering lengths in three charge channels
- **Isospin-breaking in the scattering lengths**

$$R = 2 \frac{a_{\pi^+ p \rightarrow \pi^+ p} - a_{\pi^- p \rightarrow \pi^- p} - \sqrt{2} a_{\pi^- p \rightarrow \pi^0 n}}{a_{\pi^+ p \rightarrow \pi^+ p} - a_{\pi^- p \rightarrow \pi^- p} + \sqrt{2} a_{\pi^- p \rightarrow \pi^0 n}} = -3.6(4.4)\%$$

\leftrightarrow in agreement with ChPT expectation $R = 0.6(4)\%$

- **Rate**

$$\frac{dR}{dq^2} = \frac{\rho M}{m_A m_\chi} \int_{v_{\min}}^{v_{\text{esc}}} d^3v |\mathbf{v}| f(|\mathbf{v}|) \frac{d\sigma_{\chi\mathcal{N}}}{dq^2}$$

- **Halo-independent methods** Drees, Shan 2008, Fox, Liu, Weiner 2010, ...

- **Nuclear structure factors** Engel, Pittel, Vogel 1992

$$\frac{d\sigma_{\chi\mathcal{N}}}{dq^2} = \frac{8G_F^2}{(2J+1)v^2} [S_A(q) + S_S(q)]$$

- Normalization at $|\mathbf{q}| = 0$:

$$S_S(0) = \frac{2J+1}{4\pi} |c_0 A + c_1(Z-N)|^2$$

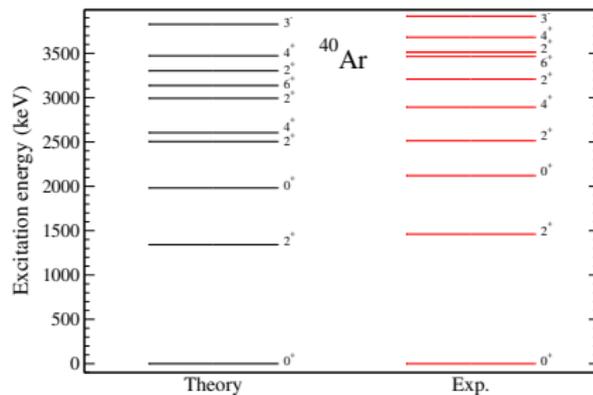
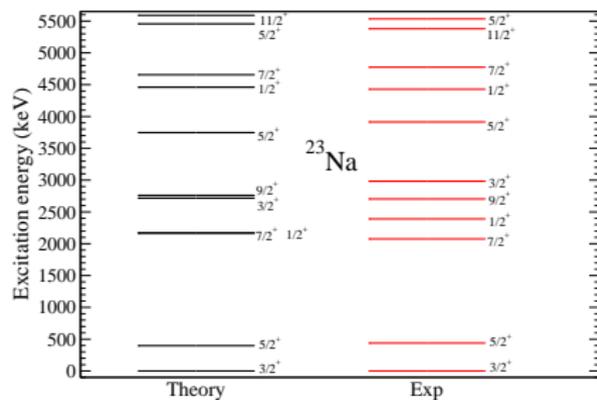
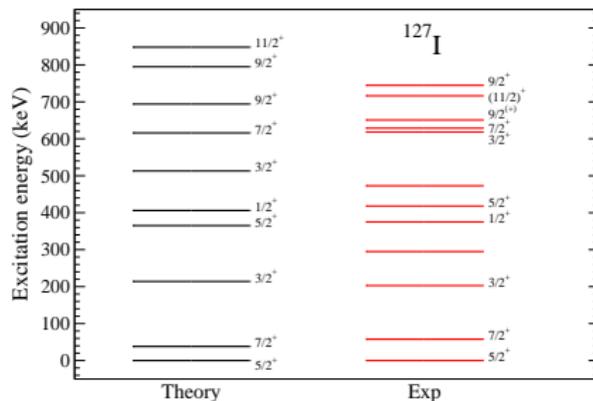
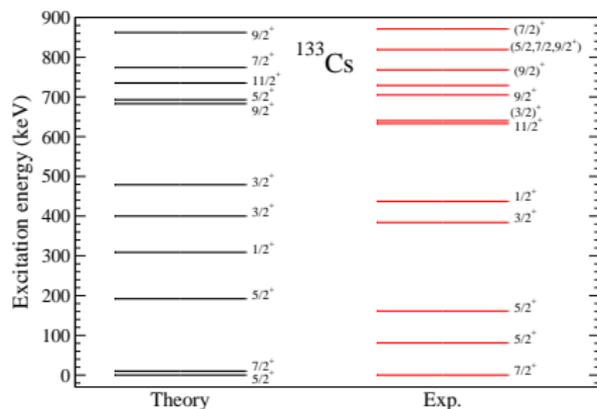
$$S_A(0) = \frac{(2J+1)(J+1)}{4\pi J} |(a_0 + a_1)\langle \mathbf{S}_p \rangle + (a_0 - a_1)\langle \mathbf{S}_n \rangle|^2$$

- Assume $c_1 = 0$ and SI scattering

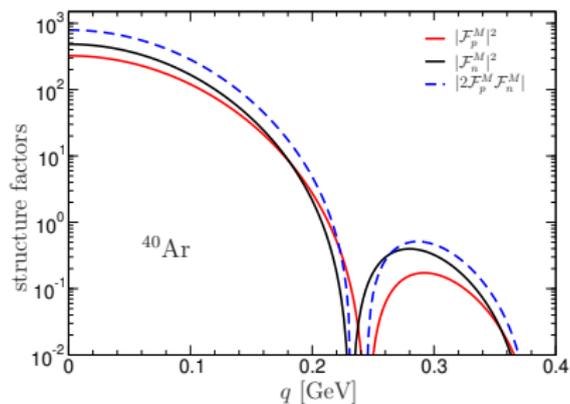
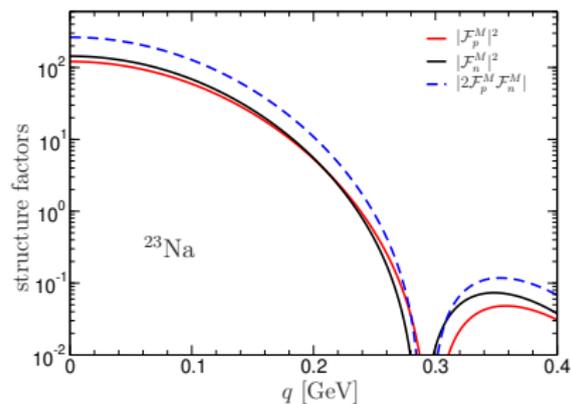
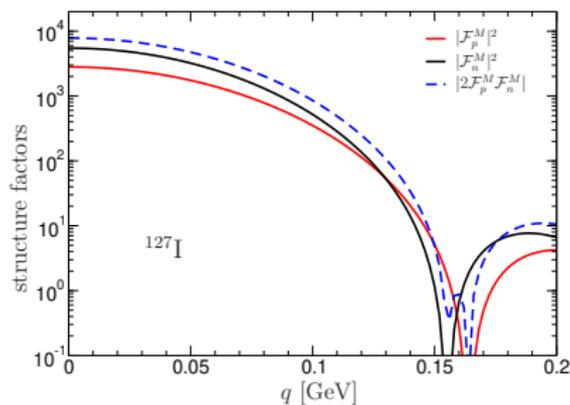
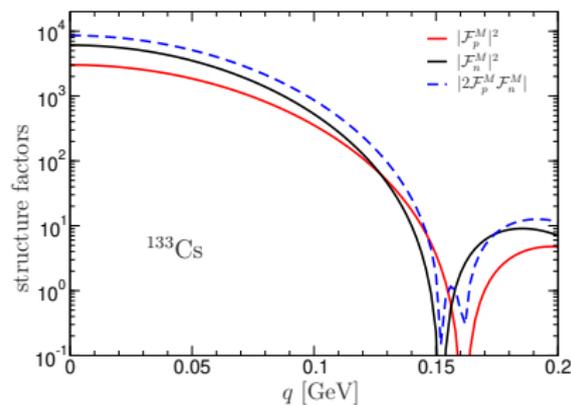
$$\frac{d\sigma_{\chi\mathcal{N}}^{\text{SI}}}{dq^2} = \frac{\sigma_{\chi\mathcal{N}}^{\text{SI}}}{4v^2 \mu_N^2} \mathcal{F}_{\text{SI}}^2(\mathbf{q}^2)$$

↪ phenomenological **Helm form factor** $\mathcal{F}_{\text{SI}}^2(\mathbf{q}^2)$

Spectra



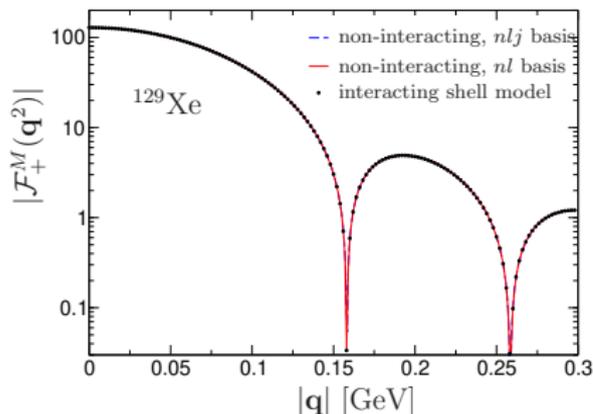
Structure factors



Scalar two-body currents: oscillator model

$$\begin{aligned}
 \mathcal{F}_\pi(\mathbf{q}^2) &= \frac{M_\pi}{2} \left(\frac{g_A}{2F_\pi} \right)^2 \sum_{n_1 l_1 n_2 l_2} \sum_{\tau_1 \tau_2} \int \frac{d^3 p_1 d^3 p_2 d^3 p'_1 d^3 p'_2}{(2\pi)^6} R_{n_1 l_1}(|\mathbf{p}'_1|) R_{n_2 l_2}(|\mathbf{p}'_2|) R_{n_1 l_1}(|\mathbf{p}_1|) R_{n_2 l_2}(|\mathbf{p}_2|) \\
 &\times \frac{(2l_1 + 1)(2l_2 + 1)}{16\pi^2} P_{l_1}(\hat{\mathbf{p}}'_1 \cdot \hat{\mathbf{p}}_1) P_{l_2}(\hat{\mathbf{p}}'_2 \cdot \hat{\mathbf{p}}_2) (2\pi)^3 \delta^{(3)}(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}'_1 - \mathbf{p}'_2 - \mathbf{q}) \\
 &\times (3 - \tau_1 \cdot \tau_2) \frac{\mathbf{q}_1^{\text{ex}} \cdot \mathbf{q}_2^{\text{ex}}}{((\mathbf{q}_1^{\text{ex}})^2 + M_\pi^2)((\mathbf{q}_2^{\text{ex}})^2 + M_\pi^2)}
 \end{aligned}$$

- Two-body current defines genuinely **new structure factor**
- Checked the oscillator model for 1b case
 \hookrightarrow reproduces perfectly the $L = 0$ multipole



Scalar two-body currents: numerical estimates

- Early claims: could be as large as 60% [Prézeau, Kurylov, Kamionkowski, Vogel 2003](#)

$$\frac{\mathcal{A}_{\pi\pi}}{\mathcal{A}_{NN}} \simeq (0.21 \pm 0.08) r \frac{\mathcal{N}_{\pi\pi}}{A} \quad r = \frac{S_u m_u + S_d m_d}{(S_u m_u + S_d m_d) \frac{\epsilon_s^0}{2} + \sum_{q=s,c,b,t} S_q m_q \epsilon_s^q}$$

- They find $1 < \frac{\mathcal{N}_{\pi\pi}}{A} < 2$ and consider r as large as 1.5. This brings you to 60%.

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- They find $1 < \frac{\mathcal{N}_{\pi\pi}}{A} < 2$ and consider r as large as 1.5. This brings you to 60%.
- Decomposition is actually scale dependent, they quote

$$\epsilon_s^0 = \langle N | \bar{u}u + \bar{d}d | N \rangle \simeq 16 \pm 8$$

- Without such cancellations ($C_s^{SS} = C_g^{IS} = 0$) MH et al. 2016

$$2 \frac{2f_\pi}{f_p + f_n} \frac{\mathcal{F}_\pi(0)}{A} = -9\%$$

- Claimed enhancement not at all related to power counting, but to cancellations in BSM parameter space
- To actually check power counting: **scalar current in light nuclei + lattice**

σ -terms work in progress, NPLQCD 2013, 2018, Körber et al. 2017



- Scalar source suppressed for $(N^\dagger N)^2$
 - ↔ **long-range contribution dominant** (in Weinberg counting)
- Typical size **(5–10)%**
 - ↔ reflected by results for structure factors
 - ↔ more important in case of cancellations
- Contact terms do appear for other sources, e.g. θ_μ^μ
 - ↔ related to **nuclear binding energy** E_b
- Same structure factor in spin-2 two-body currents

Coherence effects

- Six distinct nuclear responses

Fitzpatrick et al. 2012, Anand et al. 2013

- $M \leftrightarrow \mathcal{O}_1 \leftrightarrow SI$
- $\Sigma', \Sigma'' \leftrightarrow \mathcal{O}_4, \mathcal{O}_6 \leftrightarrow SD$
- $\Phi'' \leftrightarrow \mathcal{O}_3 \leftrightarrow$ quasi-coherent, spin-orbit operator
- $\Delta, \tilde{\Phi}'$: not coherent

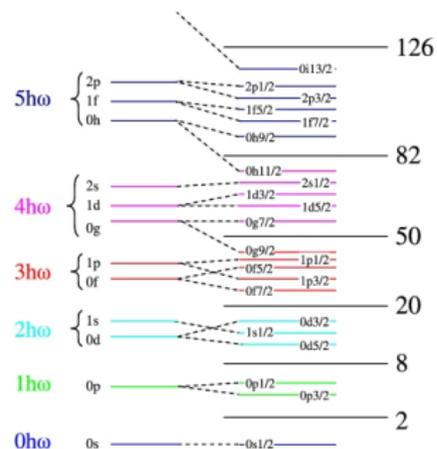
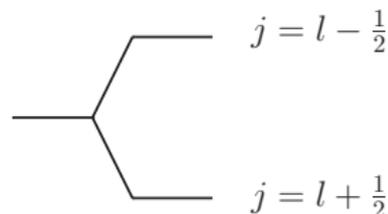
- **Quasi-coherence** of Φ''

- Spin-orbit splitting
- Coherence until mid-shell
- About 20 coherent nucleons in Xe
- Interference $M-\Phi'' \leftrightarrow \mathcal{O}_1-\mathcal{O}_3$

- Coherent 2b currents:

- Scalar $\propto N + Z$
- Vector $\propto N - Z$

\hookrightarrow concentrate on scalar case



Spin-2 and coupling to the energy-momentum tensor

- Effective Lagrangian truncated at dim-7, but if WIMP heavy $m_\chi/\Lambda = \mathcal{O}(1)$

↪ heavy-WIMP EFT [Hill, Solon 2012, 2014](#)

$$\mathcal{L} = \frac{1}{\Lambda^4} \left\{ \sum_q C_q^{(2)} \bar{\chi} \gamma_\mu i \partial_\nu \chi \frac{1}{2} \bar{q} \left(\gamma^{\{\mu} i D_{-}^{\nu\}} - \frac{m_q}{2} g^{\mu\nu} \right) q + C_g^{(2)} \bar{\chi} \gamma_\mu i \partial_\nu \chi \left(\frac{g_{\mu\nu}}{4} G_{\lambda\sigma}^a G_a^{\lambda\sigma} - G_a^{\mu\lambda} G_{a\lambda}^\nu \right) \right\}$$

↪ leading order: **nucleon pdfs**

↪ similar two-body current as in scalar case, pion pdfs, EMC effect

- Coupling of trace anomaly θ_μ^μ to $\pi\pi$

$$\theta_\mu^\mu = \sum_q m_q \bar{q} q + \frac{\beta_{\text{QCD}}}{2g_s} G_{\mu\nu}^a G_a^{\mu\nu} \quad \Leftrightarrow \quad \langle \pi(p') | \theta_{\mu\nu} | \pi(p) \rangle = p_\mu p'_\nu + p'_\mu p_\nu + g_{\mu\nu} (M_\pi^2 - p \cdot p')$$

↪ probes gluon Wilson coefficient C_g^S