

Perturbation Theory of Non-Perturbative QCD

An analytical approach to QCD in the IR from first principles

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OUTLINE

- Screened Massive Expansion:
Yang-Mills: F.S. arXiv:1509.05891; Nucl.Phys.B907(2016) 572-596;
Quarks and analytic prop. F.S. PRD 94 (2016)
- Dynamical mass generation and confinement:
F.S. arXiv:1701.00286; F.S. PRD 96 (2017); G. Comitini + F.S. PRD 97 (2018)
- BRST, generic covariant gauge and Optimization:
F.S. + G. Comitini PRD 98 (2018); F.S. PRD 99 (2019)
- Screened MOM and Strong coupling: F.S. arXiv:1902.04110

The outcome is a very predictive, self-contained, optimized perturbation theory from first principles



Motivations: Dynamical Mass Generation

Higgs mechanism: $SU(2) \times U(1) \longrightarrow U(1)_{em}$

Higgs field vev: $v = 246 \text{ GeV}$

$$m_u \approx m_d \approx 5 - 10 \text{ MeV}$$

$$m_g = 0 \quad \text{SU(3) is unbroken}$$

while

$$M_p \approx 1 \text{ GeV}$$

Two orders of magnitude larger!



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Higgs field vev: $v = 246 \text{ GeV}$

Where do the masses come from?

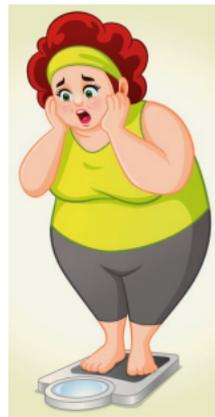
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What glues everything is the GLUON !



Motivations: Confinement

QCD is a confining theory (phenomenological evidence)
but no formal proof yet!

Yang-Mills (no quark):

Center Symmetry \longrightarrow Order Parameter (Polyakov Loop)

Gluons are confining for heavy quarks in a theory without quarks!



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But, what about the confining mechanism of gluons?

A dynamical mechanism arises from their complex mass:
finite damping rate $\rightarrow c\tau \approx 10^{-15}$ m

Dynamical Mass Generation and Confinement are related



Standard Perturbation Theory

Our understanding of QFT relies mainly on PT

Historically based on PT (QED, SM, etc.)

PT has many merits:

- explicit calculations
- analytical results at lowest order and 1-loop
- order by order improved accuracy
- important symmetries embedded in the formalism (gauge inv.)



Standard Perturbation Theory

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Unfortunately, PT breaks down in the IR of QCD

It is a pity since:

- Important phenomenology occurs in the IR (e.g. bound states)
- QCD is believed to be a complete consistent theory at any scale, containing its necessary cut-off



Non-Perturbative Effects

They cannot be addressed by any finite-order truncation

- Typically described by an infinite resummation
- They might be the sign of a wrong expansion point (rather than a failure of PT)
e.g. hydrogen atom is N.P. \rightarrow breaking down of PT in QED?

They are not intrinsic if can be cured by a change of the expansion point. (Well known issue of PT in QM where the accuracy depends on the "good" choice of \hat{H}_0)

What is "perturbative" and what is not?

It might depend on the Expansion Point



Trivial Example of a Wrong Expansion Point

$$\mathcal{L} = \frac{1}{2}\phi(-\partial^2 - m^2)\phi = \frac{1}{2}\phi(-\partial^2)\phi - \frac{m^2}{2}\phi^2, \quad \Delta_0(p) = \frac{1}{p^2}$$

$$\Delta = \frac{1}{p^2} \left[1 - \frac{m^2}{p^2} + \frac{m^4}{p^4} + \dots \right]$$

The pole is at $p = 0$ at any finite order, but

$$\Delta = \frac{1}{p^2} \frac{1}{1 + \frac{m^2}{p^2}} = \frac{1}{m^2 + p^2}$$

The shift of the pole emerges as NP effect by an infinite resumm. of the Dyson expansion.

resummation \iff change of expansion point

$$\text{resummation} = \text{Dyson expansion} + \text{X} + \text{X} \cdot \text{X} + \text{X} \cdot \text{X} \cdot \text{X} + \dots$$



Which Expansion point is the best?

Gauge inv. (BRST) \implies $\left\{ \begin{array}{l} \text{No gluon mass at any} \\ \text{finite order of PT} \end{array} \right.$

Dyn. Mass Generation \implies $\left\{ \begin{array}{l} \text{exact resumm. (NP approach)} \\ \textit{or} \\ \text{change the exp. point of PT} \\ \text{BUT give up exact gauge inv.} \\ \text{at any finite order} \end{array} \right.$



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We can build a viable PT in the IR

but we must give up exact gauge invariance at any finite order.



Exact vs. Approximate Invariance

Suppose we want a SQUARE to be drawn

1) By a computer using a "silly" algorithm which however preserves exact symmetries like

- Rotat. Inv. by $\theta = \frac{\pi}{4}$
- Inversion of axes



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Exact vs. Approximate Invariance

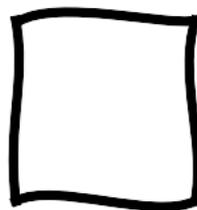
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Does not satisfy any of the symmetries!

If it looks like a square \Rightarrow approximate symmetries
Exact symmetries \nRightarrow correct result

Screened Expansion in a generic covariant gauge

Standard BRST invariant SU(N) YM Lagrangian:

$$\mathcal{L} = \mathcal{L}_{YM} + \mathcal{L}_{fix} + \mathcal{L}_{FP} \leftarrow \text{from Faddeev-Popov Determinant}$$

$$\mathcal{L}_{YM} = -\frac{1}{2} \text{Tr} (\hat{F}_{\mu\nu} \hat{F}^{\mu\nu}), \quad \mathcal{L}_{fix} = -\frac{1}{\xi} \text{Tr} [(\partial_\mu \hat{A}^\mu)(\partial_\nu \hat{A}^\nu)]$$

$$S_0 = \frac{1}{2} \int A_\mu(x) \Delta_0^{-1\mu\nu}(x, y) A_\nu(y) d^4x d^4y + \int \omega^*(x) \mathcal{G}_0^{-1}(x, y) \omega(y) d^4x d^4y$$

$$\Delta_0^{\mu\nu}(p) = \Delta_0(p) [t^{\mu\nu}(p) + \xi \ell^{\mu\nu}(p)]$$

$$\Delta_0(p) = \frac{1}{p^2}, \quad \mathcal{G}_0(p) = -\frac{1}{p^2}$$

$$S_I = \int d^d x [\mathcal{L}_{gh} + \mathcal{L}_3 + \mathcal{L}_4] \quad \text{where:}$$

$$\mathcal{L}_3 = -gf_{abc} (\partial_\mu A_{a\nu}) A_b^\mu A_c^\nu, \quad \mathcal{L}_4 = -\frac{1}{4} g^2 f_{abc} f_{ade} A_{b\mu} A_{c\nu} A_d^\mu A_e^\nu$$

$$\mathcal{L}_{gh} = -gf_{abc} (\partial_\mu \omega_a^*) \omega_b A_c^\mu$$



Screened Expansion in a generic covariant gauge

Same standard, BRST invariant, SU(N) YM Lagrangian:

$$S = S_0 + S_I = \left[S_0 + \frac{1}{2} \int A_\mu \delta\Gamma^{\mu\nu} A_\nu \right] + \left[S_I - \frac{1}{2} \int A_\mu \delta\Gamma^{\mu\nu} A_\nu \right]$$

↙ not BRST inv. ↘

P.T. does not satisfy exact relations imposed by BRST at any finite order

$$\left\{ \begin{array}{l} \Delta_m^{\mu\nu}(p) = \frac{1}{p^2 + m^2} t^{\mu\nu}(p) + \frac{\xi}{p^2} \ell^{\mu\nu}(p) \quad (\text{free propagator}) \\ \delta\Gamma^{\mu\nu} = \left[\Delta_m^{-1\mu\nu} - \Delta_0^{-1\mu\nu} \right] = m^2 t^{\mu\nu}(p) \quad (\text{2-point vertex}) \end{array} \right.$$

↙ Exact since $\Pi^L = 0$

P.T. with the new vertex set

$$\begin{aligned} \mathcal{L}_3 &= -gf_{abc}(\partial_\mu A_{a\nu})A_b^\mu A_c^\nu, & \mathcal{L}_4 &= -\frac{1}{4}g^2 f_{abc}f_{ade}A_{b\mu}A_{c\nu}A_d^\mu A_e^\nu \\ \mathcal{L}_{gh} &= -gf_{abc}(\partial_\mu \omega_a^*)\omega_b A_c^\mu, & \mathcal{L}_m &= -\frac{1}{2}\delta_{ab}\delta\Gamma_{\mu\nu}A_a^\mu A_b^\nu \end{aligned}$$



Screened Expansion in a generic covariant gauge

At variance with Curci-Ferrari model (Tissier and Wschebor, 2011):

$$\Delta_T(p) = \frac{1}{(p^2 + m^2) - \Pi^T} = \frac{1}{(p^2 + m^2) - (m^2 + \Pi_{Loops}^T)} = \frac{1}{p^2 - \Pi_{Loops}^T}$$

$$\Sigma = \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \quad \delta\Gamma = m^2$$

$$\Pi = \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---}$$

- The pole shift cancels at tree level
- All spurious diverging mass terms cancel without counterterms and/or parameters
- Standard UV behavior

In the \overline{MS} scheme: $\Pi^{diverg.} = \frac{Ng^2}{(4\pi)^2} \left(\frac{2}{\epsilon} + \log \frac{\mu^2}{m^2} \right) p^2 \left(\frac{13}{6} - \frac{\xi}{2} \right)$

Standard UV behavior $\implies \Pi^{finite} \sim -\frac{Ng^2}{(4\pi)^2} p^2 \left(\frac{13}{6} - \frac{\xi}{2} \right) \log \frac{p^2}{\mu^2}$



Screened Expansion at one-loop

Expanding around the best Gaussian vacuum

Setting $s = p^2/m^2 \leftarrow$ **the scale m cannot be fixed by theory!**

$$\Pi_{Loops}^T = -\frac{3Ng^2}{(4\pi)^2} p^2 [F(s) + \xi F_\xi(s)] + \Pi^{diverg.}$$

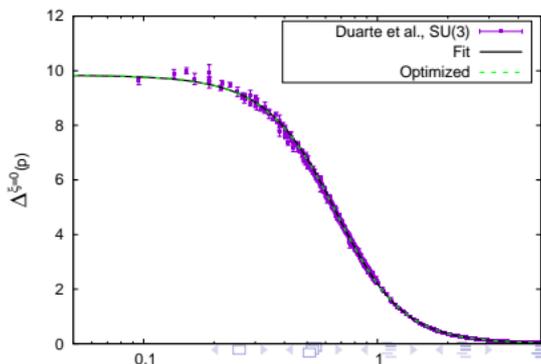
After subtraction (wave function renormalization):

$$\Delta(p) = \frac{Z_\mu}{p^2 + \frac{3Ng^2}{(4\pi)^2} p^2 \left[F(s) + \xi F_\xi(s) - F\left(\frac{\mu^2}{m^2}\right) - \xi F_\xi\left(\frac{\mu^2}{m^2}\right) \right]}$$

$$\Delta(p) = \frac{Z}{p^2 [F(s) + \xi F_\xi(s) + F_0]} \quad \mu \Leftrightarrow F_0$$

- Results depend on $\mu/m \rightarrow F_0$
- Nielsen Identities (BRST) are NOT exactly satisfied

$$\text{Best fit at } \xi = 0 : \begin{cases} m = 0.654 \text{ GeV} \\ F_0 = -0.887 \end{cases}$$



The Dark Side of the Propagators

What do we really know about propagators in the IR of Minkowski space?

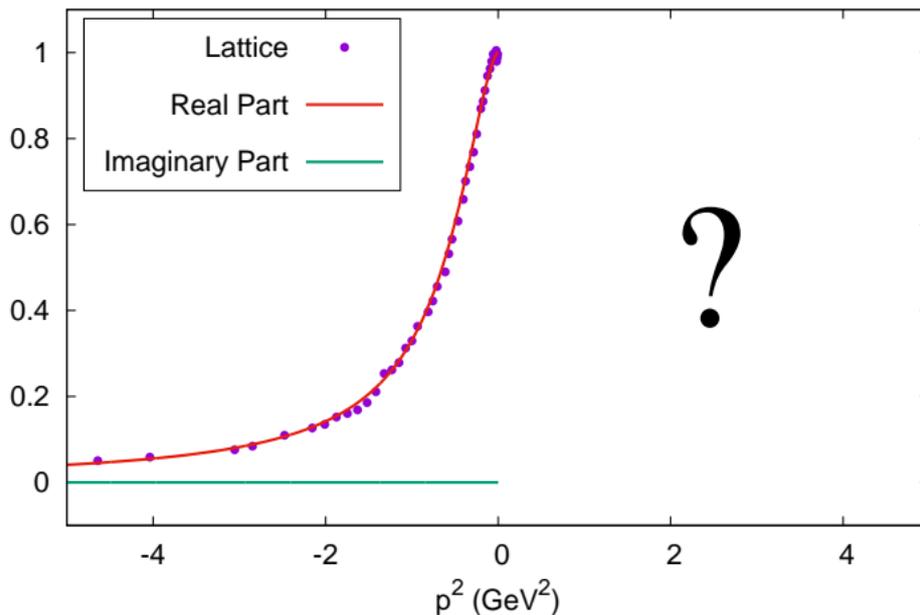


ANALYTIC CONTINUATION

Is there any dynamical pole?

F.S., Nucl. Phys. B 907, (2016); arXiv:1605.07357.

GLUON PROPAGATOR - SU(3)

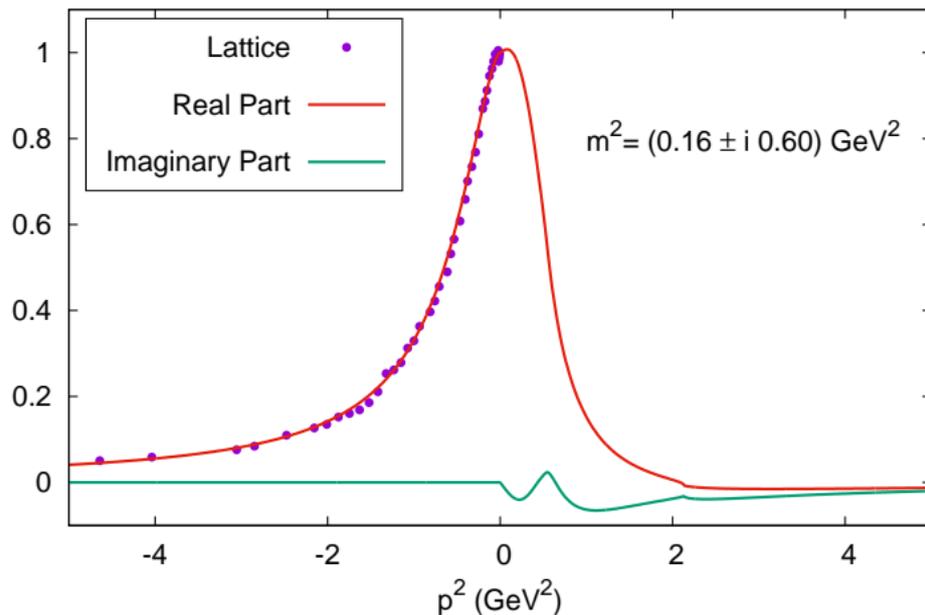


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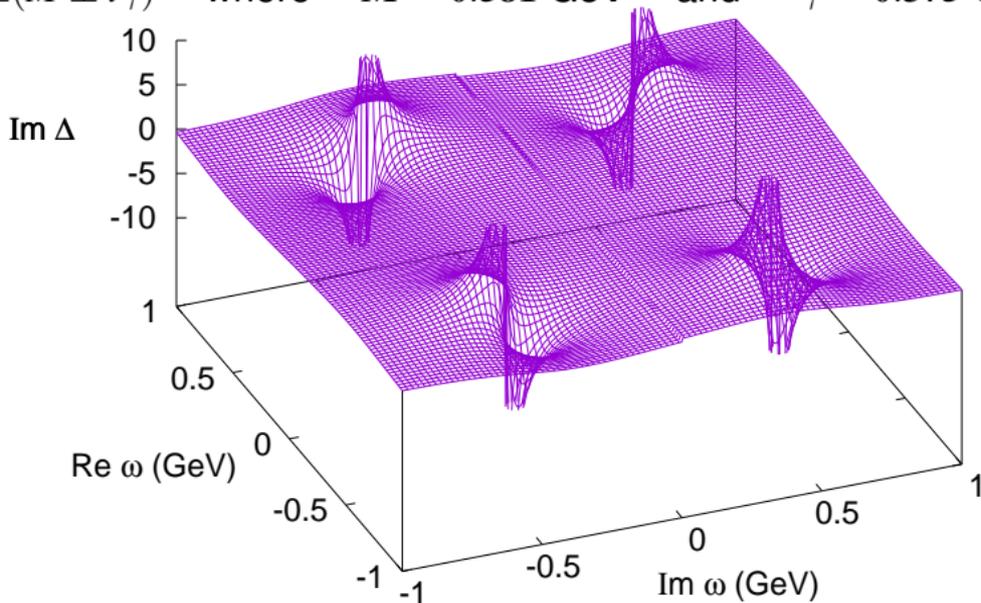
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GLUON PROPAGATOR - SU(3)



ANALYTIC CONTINUATION AND CONFINEMENT

In the long wave-length limit $p^2 = \omega^2 - \mathbf{k}^2 \rightarrow \omega^2$ the poles are at $\omega = \pm(M \pm i\gamma)$ where $\mathbf{M} = 0.581 \text{ GeV}$ and $\gamma = 0.375 \text{ GeV}$.



No violation of unitarity and causality (Stingl, 1996):

short-lived quasiguons with lifetime $\tau = 1/\gamma$ are canceled from the asymptotic states



Gauge-Parameter-Independence of Poles and Residues

Proof by Nielsen Identities (BRST)

$$\text{N.I.} \rightarrow \boxed{\frac{\partial}{\partial \xi} \frac{1}{\Delta(p)} = G^T(p) \left[\frac{1}{\Delta(p)} \right]^2} \quad G \sim \langle T [D^\mu \omega_a A_a^\nu \omega_b^* B_b] \rangle$$

The pole $p_0(\xi)$ must be gauge-parameter-independent:

$$\frac{1}{\Delta(p_0(\xi))} = 0; \quad \frac{d}{d\xi} \frac{1}{\Delta(p_0(\xi))} = 0 \quad \Longrightarrow \quad \boxed{\frac{d}{d\xi} p_0(\xi) = 0}$$

The residues are also ξ -independent (first suggested by D.Dudal):

$$\frac{\partial}{\partial \xi} \left[\frac{d}{dp^2} \frac{1}{\Delta} \right] = \left[\frac{d}{dp^2} G^T \right] \left[\frac{1}{\Delta} \right]^2 + 2G^T \frac{1}{\Delta} \left[\frac{d}{dp^2} \frac{1}{\Delta} \right]$$

$$R = \lim_{p \rightarrow p_0} \Delta(p)(p^2 - p_0^2) = \lim_{p \rightarrow p_0} \left[\frac{d}{dp^2} \frac{1}{\Delta(p)} \right]^{-1} \quad \Longrightarrow \quad \boxed{\frac{\partial}{\partial \xi} R = 0}$$

ξ -independent *Principal Part* $\Delta^P(p) = \frac{R}{p^2 - p_0^2} + \frac{R^*}{p^2 - p_0^{*2}} \quad (\text{RGZ})$

Optimized Screened Expansion

Assume that **if** $\begin{cases} F_0 = F_0(\xi) \\ m = m(\xi) \end{cases} \implies$ N.I. are satisfied

and define the complex variable: $z^2 = -p_E^2 = p_M^2, \quad z = x + iy$

$$\Delta = \frac{Z}{p^2 \Psi} \quad \text{where} \quad \boxed{\Psi(z, \xi, F_0, m) = F(-z^2/m^2) + \xi F_\xi(-z^2/m^2) + F_0}$$

Conformal map $\rightarrow \Psi(z_1, \xi_1, F_0(\xi_1), m(\xi_1)) = \Psi(z_2, \xi_2, F_0(\xi_2), m(\xi_2))$

$$\boxed{\text{Fixed Point: } \Psi(z_0, \xi_1, F_0(\xi_1), m(\xi_1)) = \Psi(z_0, \xi_2, F_0(\xi_2), m(\xi_2)) = 0}$$

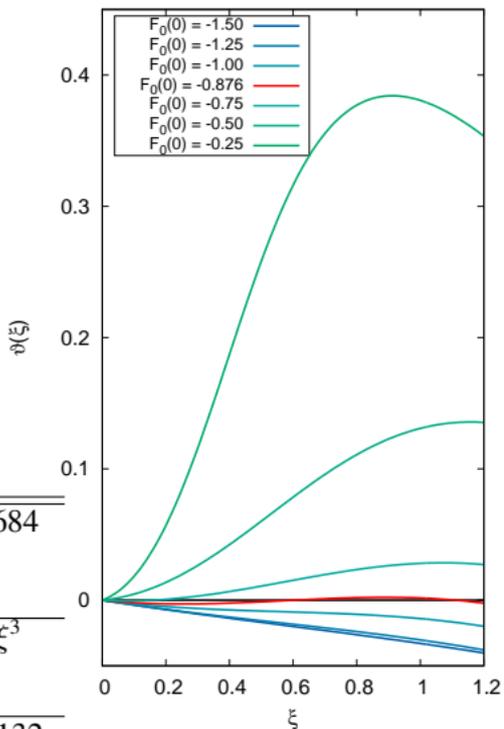
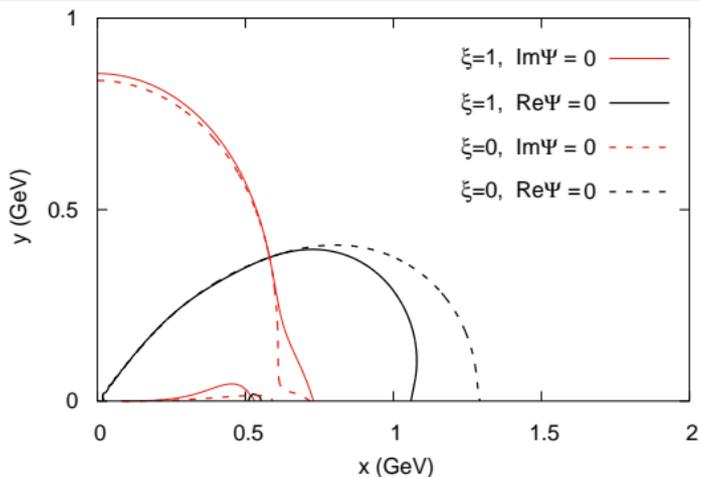
$$\left. \begin{array}{l} \xi_1 = 0 \\ m(\xi_1) = m(0) \\ F_0(\xi_1) = F_0(0) \end{array} \right\} \implies F_0(\xi_2), m(\xi_2) \quad (\text{two real equations})$$

BUT

$$R(\xi) = R(0) e^{i\theta(\xi)}, \quad \theta(\xi) = \text{Arg} \left\{ \frac{\frac{d}{dz} \Psi(z, 0, F_0(0), m(0))}{\frac{d}{dz} \Psi(z, \xi, F_0(\xi), m(\xi))} \right\}_{z=z_0} \neq 0$$

Optimized Screened Expansion

Optimization by ξ -independence of principal part



$$F_0(0) = -0.876, \quad m_0 = m(0) = 0.656 \text{ GeV}, \quad Z(0) = 2.684$$

$$|\theta(\xi)| < 2.76 \cdot 10^{-3}, \quad 0 < \xi < 1.2$$

$$F_0(\xi) \approx -0.8759 - 0.01260\xi + 0.009536\xi^2 + 0.009012\xi^3$$

$$m^2(\xi)/m_0^2 \approx 1 - 0.39997\xi + 0.064141\xi^2$$

$$z_0/m_0 = 0.8857 + 0.5718i, \quad t_R = \text{Im}R(0)/\text{Re}R(0) = 3.132$$

$$M = 0.581 \text{ GeV}, \quad \gamma = 0.375 \text{ GeV} \quad (\text{invariant pole})$$



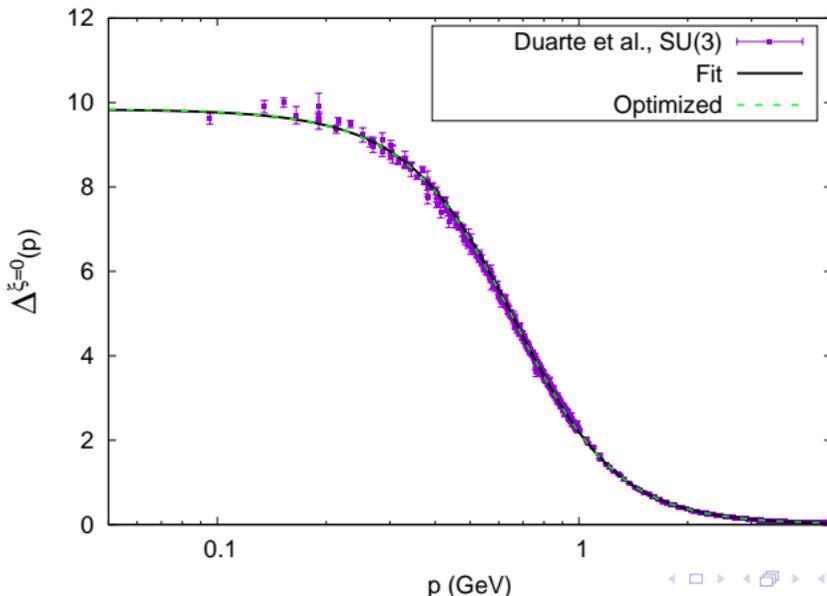
Back to Euclidean Space

Optimized S.E. vs. Lattice data in the Landau gauge

$$\Delta(p) = \frac{Z_\mu}{p^2 + \frac{3Ng^2}{(4\pi)^2} p^2 \left[F(s) + \xi F_\xi(s) - F\left(\frac{\mu^2}{m^2}\right) - \xi F_\xi\left(\frac{\mu^2}{m^2}\right) \right]}$$

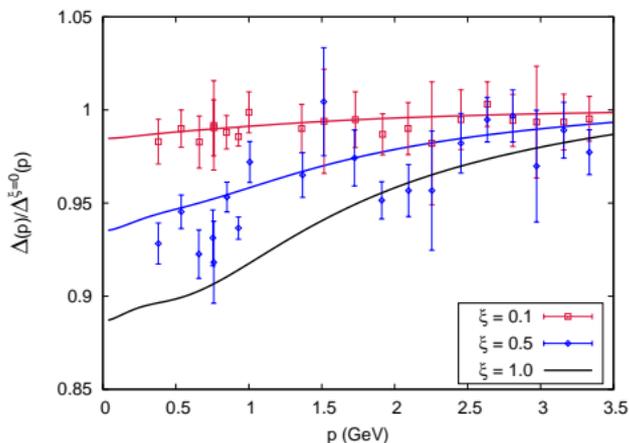
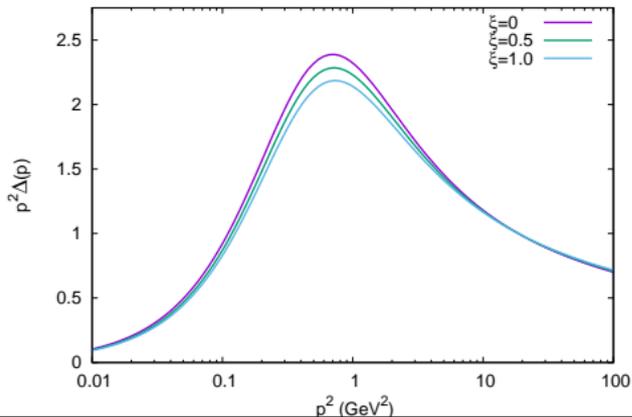
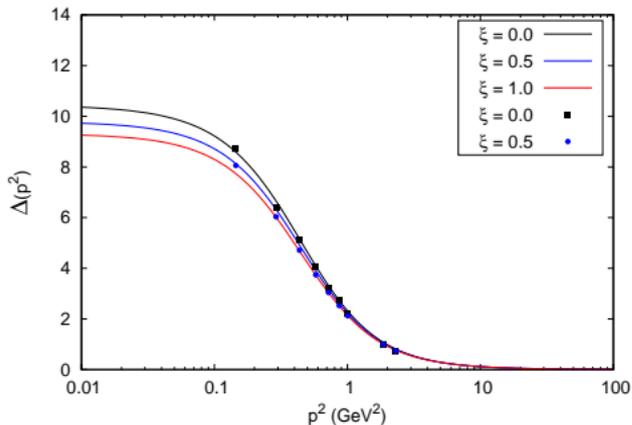
$$\Delta(p) = \frac{Z}{p^2 [F(s) + \xi F_\xi(s) + F_0(\xi)]}$$

$$\mu \Leftrightarrow F_0(\xi), \quad \xi = 0, \quad s = \frac{p^2}{m^2}$$



Back to Euclidean Space: generic covariant gauge $\xi \neq 0$

Optim. S.E. vs. Lattice data of Bicudo, Binosi, Cardoso, Oliveira, Silva PRD 92 (2015)

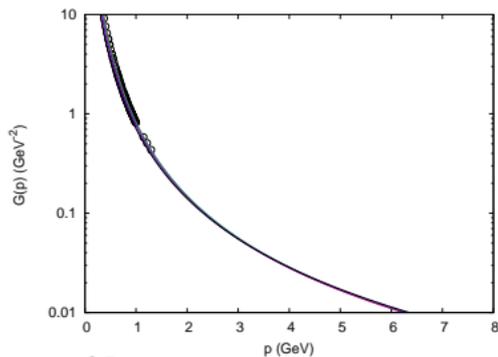


- Optim. in Complex pl. \Rightarrow Euclidean
- Quantitative agreement with lattice
- Qual. agreem. with DS if N.I. are used: [Aguilar, Binosi, Papavassiliou \(2015\)](#)
- Not a fit! No free parameters.
- Quantitative prediction up to and beyond the Feynman gauge ($\xi = 1$) (not accessible by other methods)



Back to Euclidean Space: generic covariant gauge $\xi \neq 0$

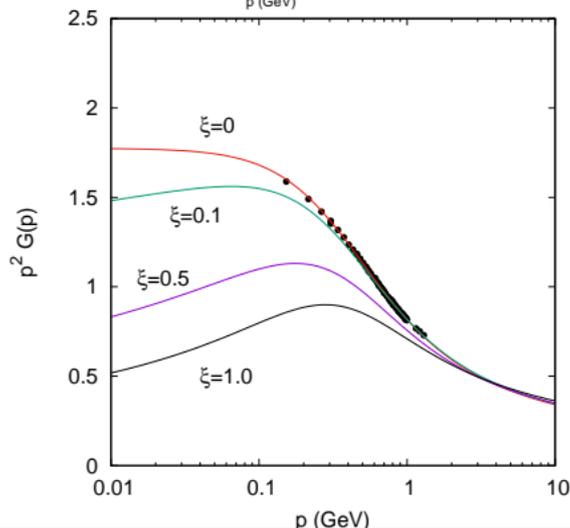
Optim. S.E. vs. Lattice data of Cucchieri, Dudal, Mendes, Oliveira, Roelfs, Silva, PRD 98 (2018).



$$\Sigma(p) = 3N \left(\frac{\alpha_s}{4\pi} \right) p^2 \left[\mathcal{G}(s) - \frac{\xi}{12} \ln s \right]$$

$$p^2 G(p) = \left[\mathcal{G}(s) - \frac{\xi}{12} \ln s + \mathcal{G}_0 \right]^{-1}$$

$$\sim (\xi \ln p)^{-1} \rightarrow 0$$



- Not a fit! No free parameters.
- Ghost Dressing $\rightarrow 0$ in the IR
- Qual. agreement with DS:
[Aguilar, Binosi, Papavassiliou \(2015\)](#)
- Quantitative agreement with lattice ?
- Prediction up to and beyond the Feynman gauge ($\xi = 1$)
(not accessible by other methods)



General Renormalization

Exact Relations

$$\text{BRST} \rightarrow \begin{cases} \text{Slavnov - Taylor} \rightarrow [\text{Multiplicative Renorm.}] \\ \text{Nielsen Id.} \rightarrow [\xi - \text{independence}] \end{cases}$$

$$\frac{Z_1^c}{Z_c} = \frac{Z_1^{3g}}{Z_A} = \left(\frac{Z_1^{4g}}{Z_A} \right)^{\frac{1}{2}}$$

In the Landau gauge:

$$\xi = 0 \implies \boxed{Z_1^c = Z_g Z_c \sqrt{Z_A} = 1} \quad [\text{Taylor (1971), Wschebor(2008)}]$$

$$\frac{\alpha(p)}{\alpha(\mu)} = \left[\frac{Z_g(\mu)}{Z_g(p)} \right]^2 = J(\mu, p) \chi(\mu, p)^2$$

holds exactly if

$$\mathcal{G}(p) = \chi(\mu, p) \mathcal{G}_0(p) \quad (\text{Ghost}), \quad \Delta(p) = J(\mu, p) \Delta_0(p) \quad (\text{Gluon})$$

with $J(\mu, \mu) = \chi(\mu, \mu) = 1$ so that $\Delta(\mu) = \Delta_0(\mu)$, $\mathcal{G}(\mu) = \mathcal{G}_0(\mu)$.



Screened MOM scheme (SMOM)

$$\Delta(p) = J(\mu, p)\Delta_0(p) \iff \Delta(p) = J_m(\mu, p)\Delta_m(p) = \frac{J_m(\mu, p)}{m^2 + p^2}$$

$$\text{MOM} \begin{cases} \mathcal{G}(\mu) = -\frac{1}{\mu^2} \\ \Delta(\mu) = \frac{1}{\mu^2} \\ J(\mu, p) = p^2 \Delta(p) \end{cases} \quad \text{SMOM} \begin{cases} \mathcal{G}(\mu) = -\frac{1}{\mu^2} \\ \Delta(\mu) = \frac{1}{m^2 + \mu^2} \\ J_m(\mu, p) = (p^2 + m^2) \Delta(p) \end{cases}$$

$$\text{MOM} \iff \text{SMOM}: \quad J_m(\mu, p) = \frac{p^2 + m^2}{p^2} J(\mu, p)$$

$$[\alpha(p)]_{\text{SMOM}} = \left(\frac{p^2 + m^2}{p^2} \right) [\alpha(p)]_{\text{MOM}}$$

$$\alpha_{\text{MOM}} \sim p^2 \implies \alpha_{\text{SMOM}} \sim (p^2 + m^2) \quad \text{is finite in the IR}$$



Screened MOM scheme (SMOM)

Natural units of m :

$$s = \frac{p^2}{m^2}, \quad t = \frac{\mu^2}{m^2}, \quad \alpha = 3N \left(\frac{\alpha_s}{4\pi} \right)$$

$$\begin{cases} \Pi_{1-loop}^{finite}(p) = -p^2 [\alpha F(s)], & \Rightarrow J_m(t, s) = \frac{(1 + 1/s)}{(1 + 1/t) + \alpha(t) [F(s) - F(t)]} \\ \Sigma_{1-loop}^{finite}(p) = p^2 [\alpha G(s)], & \Rightarrow \chi(t, s) = \frac{1}{1 + \alpha(t) [G(s) - G(t)]} \end{cases}$$

$$\begin{cases} J_m(t, s) \alpha(t) = \frac{(1 + 1/s)}{F(s) + F_0(t)}, & \Rightarrow F_0(t) = [\alpha(t)^{-1}] (1 + 1/t) - F(t) \\ \chi(t, s) \alpha(t) = \frac{1}{G(s) + G_0(t)}, & \Rightarrow G_0(t) = [\alpha(t)^{-1}] - G(t) \end{cases}$$

$$G_0(t) = [F_0(t) + F(t)] \left(\frac{t}{1+t} \right) - G(t)$$



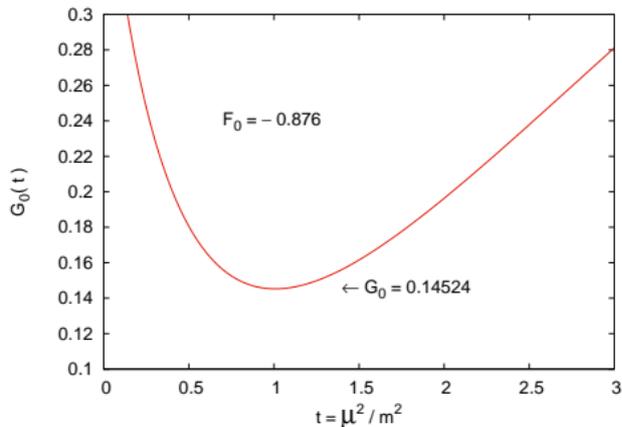
SMOM scheme

Optimization by Variation of RS: Stevenson 1981, 2013, 2016

$$\frac{\partial G_0}{\partial t} = 0 \implies \mu^* \approx m \quad (t^* = 1.0037)$$

Only one scale $\mu \approx m \implies$ no scale at all!

and no large logs $\ln(\mu/m)$!

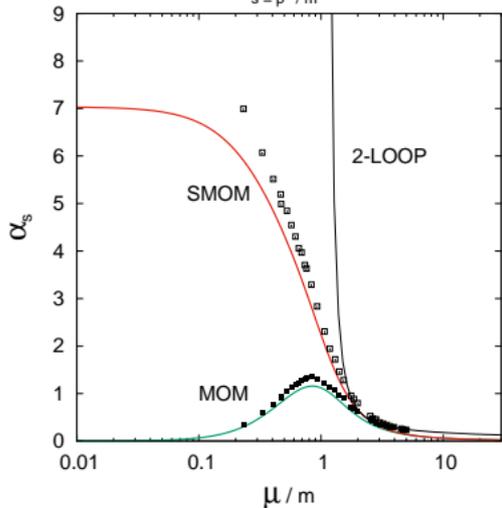
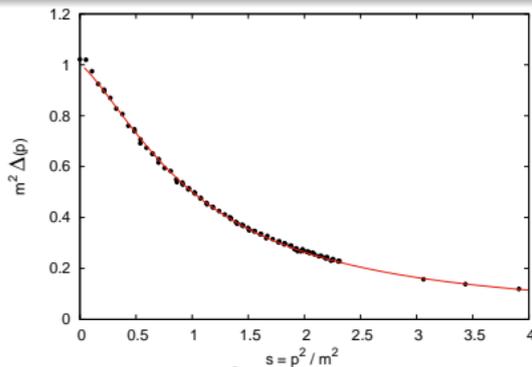
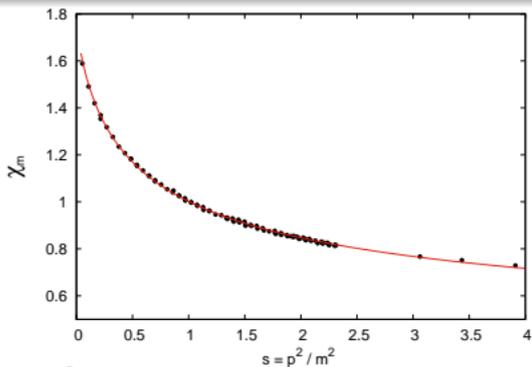


- $F_0(t^*) = -0.876, G_0(t^*) = 0.14524$
- @ $\mu^* \approx m$:
 $[\alpha_s]_{SMOM} = 2.2255, [\alpha_s]_{MOM} = 1.117$
- If α_s is known \rightarrow units are fixed!
- If the units are fixed $\rightarrow \alpha_s$ is predicted!
- $\alpha_s(t) = \alpha_s(t^*)J_m(t^*, t)\chi(t^*, t)^2$
Analytical!

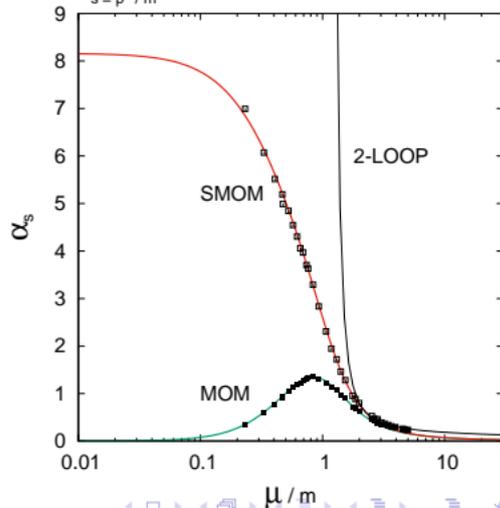


Lattice Data in the SMOM scheme: 16% higher!

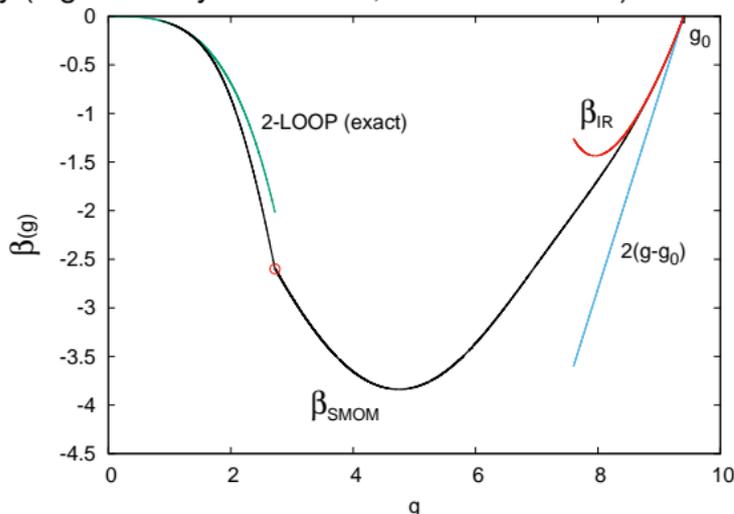
Comparison with $\beta = 6.0$ and $L = 80$ (Duarte, Oliveira, Silva, 2016) for $m = 0.656$ GeV



$\times 1.16 \Rightarrow$



IR stable fixed point: qualitative agreement with FR (e.g. Gies 2002) and AdS duality (e.g. Brodsky et al. 2010, Deur et al. 2016)



$$\beta_{IR}(g) \approx 2(g - g_0) \left\{ 1 - \left[\frac{a}{b} + \ln \frac{b g_0}{g - g_0} + \ln \left(\ln \frac{b g_0}{g - g_0} \right) \right]^{-1} \right\}$$

$$g_0 = \frac{4\pi}{\sqrt{3} c N} \frac{1}{k_0 \alpha(t^*)} \approx 9.4017 \quad \left(\frac{\alpha_s}{\pi} = 2.239 \right)$$



- **PT works well in IR**
Approx. BRST \implies self-contained optimization at $\mu = m$
Analytical and Predictive tool from First Principles
(Minkowski, ξ -gauge, α_s).
- **Optimization in Complex Plane \rightarrow Agreem. in Euclidean**
Complex Poles are genuine!
Direct evidence for Gluon Confinement
- **QCD has its own cut-off m (complete theory)**
 $m \approx 0.6$ GeV \rightarrow Gribov copies irrelevant!
The mass m is as effective as the Gribov parameter for screening the theory (Gao,Qin,Roberts,Rodriguez-Quintero,2018);
Faddeev-Popov \rightarrow very good approx. if PT works well.
- **Analytical propagators: basic blocks for phenomenology?**
Include Quarks!
Intrinsic NP problems: PDFs, Hadron Masses...



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THANK YOU!



BACKUP SLIDES

Dynamical Mass Generation

Variational argument by the Gaussian Effective Potential (GEP)

P.M. Stevenson PRD 32 (1985); P.M. Stevenson Z.Phys.C35 (1987)

$$m_B = 0 \implies \mathcal{L} = \left[\frac{1}{2} \phi (-\partial^2 - m^2) \phi \right] - \left[\frac{\lambda}{4!} \phi^4 - m^2 \phi^2 \right]$$

$$V_{GEP}(\langle \phi \rangle, m^2) = \left\{ \begin{array}{l} \text{SCALAR} \\ \text{SU}(N) \end{array} \right.$$

Diagrams for SCALAR:

Diagrams for SU(N):

“Precarious” renormalization in $d = 4 + \epsilon$, P.M. Stevenson, (1987):

$$\frac{\partial V_{GEP}(\langle \phi \rangle = 0, m^2)}{\partial m^2} = 0 \implies \begin{cases} m = m_0 \neq 0 \\ V_{GEP}(\langle \phi \rangle = 0, m_0^2) = -\frac{m_0^4}{128\pi^2} < 0 \end{cases}$$

Gluon Mass

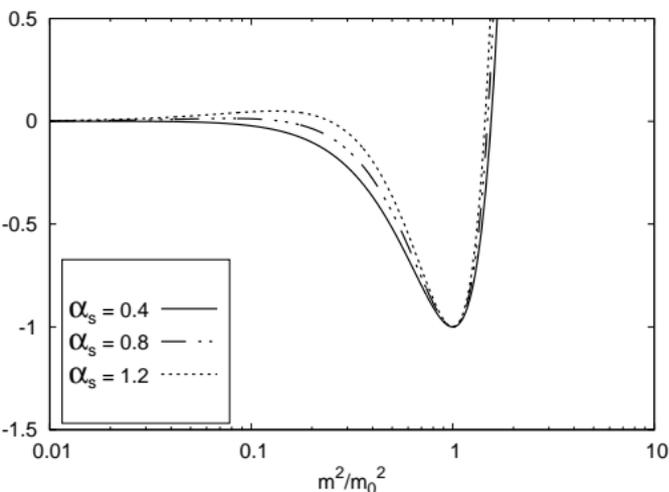
Same identical result for SU(N) YM in any covariant ξ -gauge
(gauge parameter independent! G. Comitini + F.S. PRD 97 (2018))



Gaussian Effective Potential (GEP)

Renormalized Effective Potential in units of the best mass m_0

$$V(m) = \frac{m^4}{128\pi^2} \left[\alpha \left(\log \frac{m^2}{m_0^2} \right)^2 + 2 \log \frac{m^2}{m_0^2} - 1 \right]$$



From the gap eq.:

$$\delta_\epsilon = m_0 \exp(-1/\alpha)$$

The vacuum energy does not depend on δ_ϵ and α :

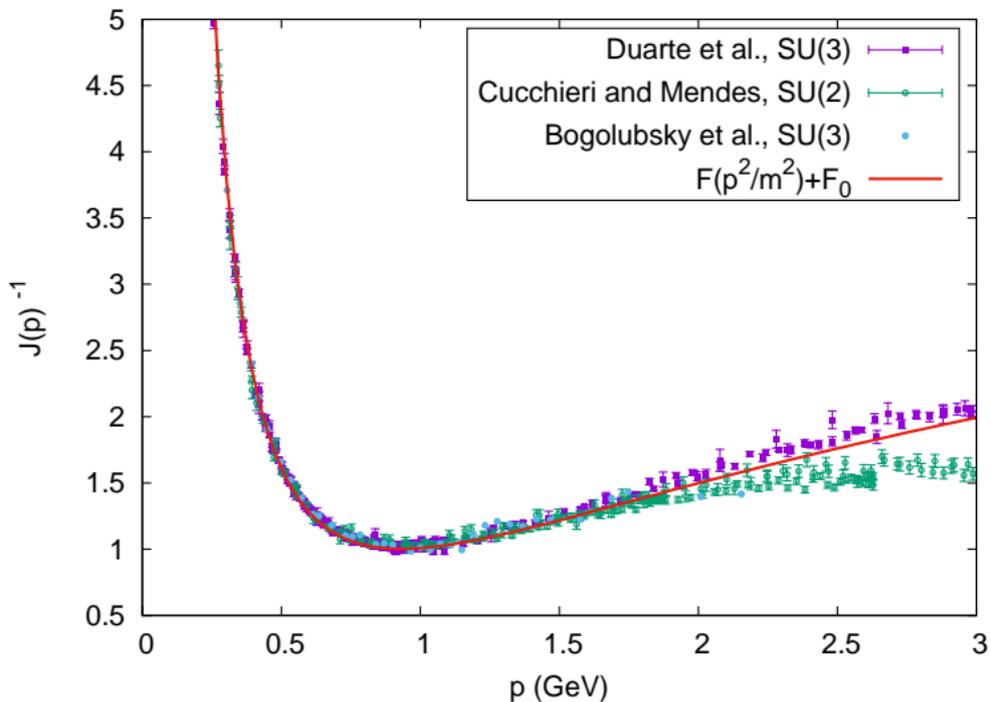
$$V(m_0) = -\frac{m_0^4}{128\pi^2} < 0$$

Gluon mass generation: the same identical result for SU(N) Yang-Mills Theory in any covariant ξ -gauge if $\alpha = 9N\alpha_s/(8\pi)$



UNIVERSAL SCALING

GLUON INVERSE DRESSING FUNCTION (Landau gauge $\xi = 0$)

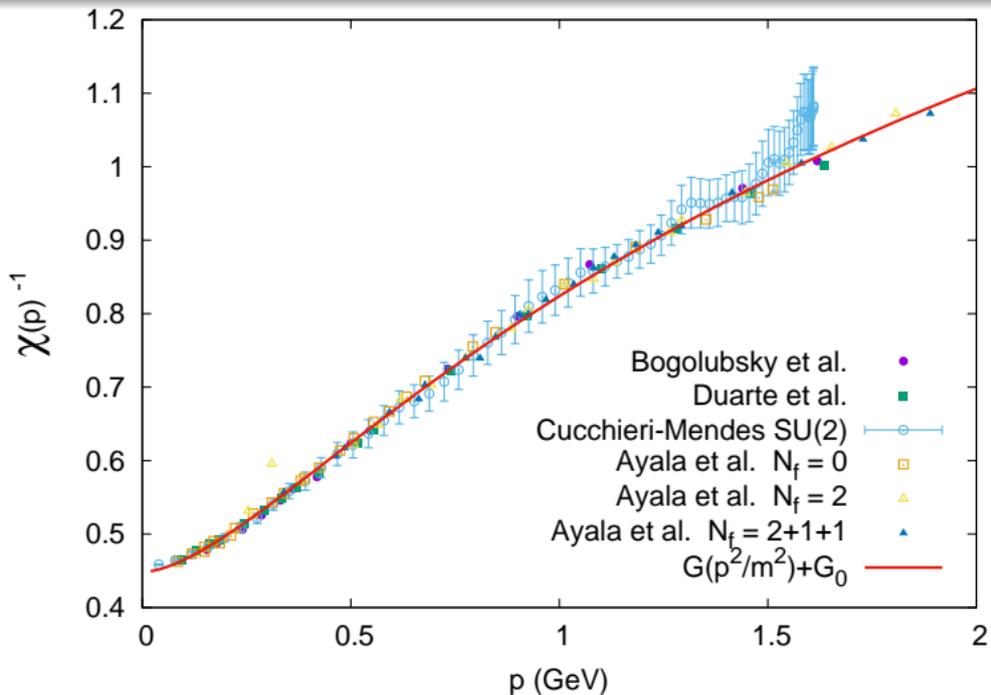


UNIVERSAL SCALING

GHOST INVERSE DRESSING FUNCTION (Landau gauge $\xi = 0$)

The ghost universal function is just

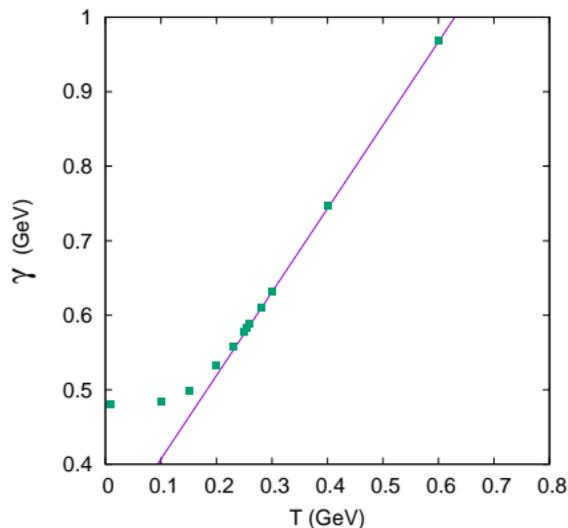
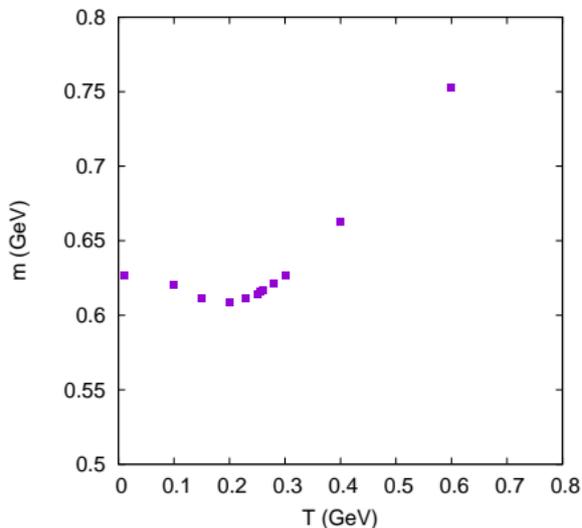
$$G(s) = \frac{1}{12} \left[2 + \frac{1}{s} - 2s \log s + \frac{1}{s^2} (1+s)^2 (2s-1) \log(1+s) \right]$$



Finite T

Trajectory of poles in the complex plane

In the limit $\mathbf{k} \rightarrow 0$ the pole $\omega = \pm(m \pm i\gamma)$ is the same for Δ_L, Δ_T .
Using $m_0 = 0.73$ GeV and $F_0 = -1.05$ (fixed at $T = 0$):



The line is the fit $\gamma = \gamma_0 + bT$ with $\gamma_0 = 0.295$ GeV and $b = 1.12$.
(Hard thermal loops: $\gamma/T = 3.3\alpha_s$)



Complex Poles and Confinement

Failure of Wick Rotation:

Observables in Minkowski require extra contribution from poles

Tiburzi, Detmold, Miller, PRD **68**, (2003).

$$\begin{array}{ccc} \Delta_M(t) & \Leftarrow \text{Fourier Transform} \Rightarrow & \Delta_M(p_M) \\ \boxed{t = -it_E ?} & & p^0 = ip^4 \text{ (analytic)} \\ \Delta_E(t_E) & \Leftarrow \text{Fourier Transform} \Rightarrow & \Delta_E(p_E) \end{array}$$

The Schwinger function $\Delta_E(t_E)$ is NOT the analytic contin. of $\Delta_M(t)$

NO Källen-Lehmann Representation

But the numerical reconstruction improves if the poles are added

(Binosi and Tripolt arXiv:1904.08172)

Complex Poles and Confinement

Schwinger function vs. Minkowski

$$\Delta_E(t_E) = \int_{-\infty}^{+\infty} \frac{dp_4}{2\pi} e^{ip_4 t_E} \Delta(\vec{p} = 0, p_4) \quad (t_E = \text{Euclidean time})$$

$$\left\{ \begin{array}{l} \Delta_E(p) = \frac{R}{p^2 + z_0^2} + \frac{R^*}{p^2 + z_0^{*2}} \quad \text{where } \boxed{z_0 = M + i\gamma} \\ \Delta_E(t_E) = \left[\frac{|R|}{\sqrt{M^2 + \gamma^2}} \right] e^{-M|t_E|} \cos(\gamma|t_E| - \phi), \quad \phi = \text{Arg}[R] - \tan^{-1} \frac{\gamma}{M} \end{array} \right.$$

$$\Delta_M(t) = \int_{-\infty}^{+\infty} \frac{dp_0}{2\pi} e^{ip_0 t} \Delta_M(p_0, \vec{p} = 0) \quad (t = \text{real time})$$

$$\left\{ \begin{array}{l} \Delta_M(p) = \frac{R}{-p^2 + z_0^2} + \frac{R^*}{-p^2 + z_0^{*2}} \\ \Delta_M(t) = \left[\frac{|R|}{\sqrt{M^2 + \gamma^2}} \right] e^{-\gamma|t|} \sin(M|t| + \phi) \implies \Delta_M(t) \neq i\Delta_E(it) \end{array} \right.$$

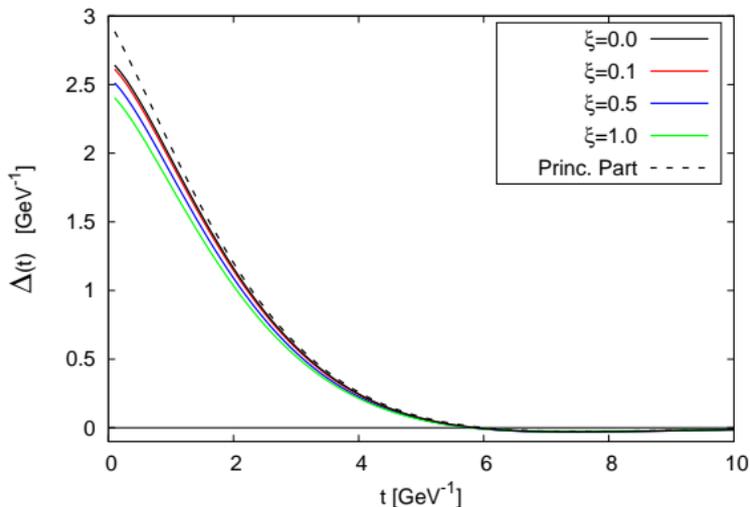


Complex Poles and Confinement

Schwinger function

$$\Delta(t) = \int_{-\infty}^{+\infty} \frac{dp_4}{2\pi} e^{ip_4 t} \Delta(\vec{p} = 0, p_4) \quad (t = \text{Euclidean time})$$

$$\begin{cases} \Delta^P(p) = \frac{R}{p^2+z_0^2} + \frac{R^*}{p^2+z_0^{*2}} = Z_{GZ} \frac{p^2 + M_1^2}{p^4 + M_2^2 p^2 + M_3^4} & \text{(RGZ language)} \\ \Delta^P(t) = \left[\frac{|R|}{\sqrt{M^2 + \gamma^2}} \right] e^{-Mt} \cos(\gamma t - \phi) & \text{where } \phi = \text{Arg}[R] - \tan^{-1} \frac{\gamma}{M} \end{cases}$$



- $t_0 \approx 5.8 \text{ GeV}^{-1} \approx \text{hadron size}$:
physical gauge-invariant scale ?
Conject. by Alkofer, Detmold,
Fischer, Maris PRD 70 (2004)
- Large t behavior dominated by
singularities (i.e. ξ -independent
principal part)
- $t_0 \approx \frac{1}{\gamma} \left(\text{Arg}[R] - \tan^{-1} \frac{\gamma}{M} + \frac{\pi}{2} \right)$
 $t_0 \rightarrow \infty$ if $\gamma \rightarrow 0$



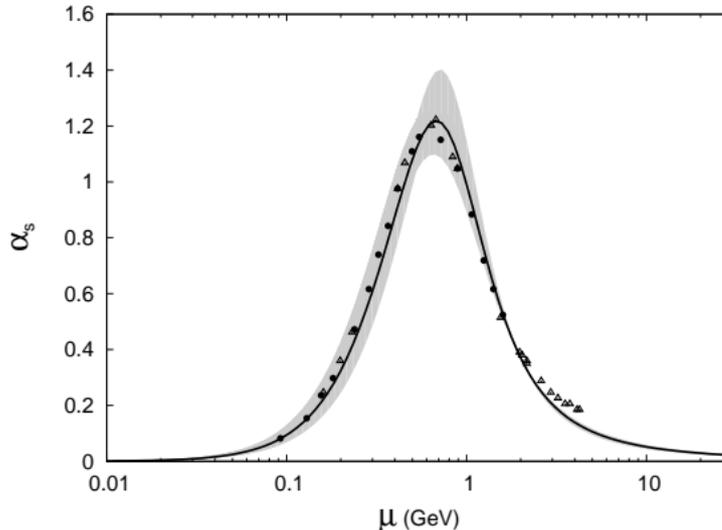
Running Coupling

Pure Yang-Mills SU(3)

RG invariant product (Landau Gauge – MOM-Taylor scheme):

$$\alpha_s(\mu) = \alpha_s(\mu_0) \frac{J(\mu)\chi(\mu)^2}{J(\mu_0)\chi(\mu_0)^2}$$

What if $\delta F_0 = \delta G_0 = \pm 25\%$?



$\mu_0 = 2$ GeV, $\alpha_s = 0.37$, data of Bogolubsky et al.(2009).



Quark propagator:

$$S(p) = S_p(p^2)\not{p} + S_M(p^2)$$

NO COMPLEX POLES \implies Standard Dispersion Relations

$$\rho_M(p^2) = -\frac{1}{\pi} \text{Im} S_M(p^2)$$

$$\rho_p(p^2) = -\frac{1}{\pi} \text{Im} S_p(p^2)$$

$$S(p) = \int_0^\infty dq^2 \frac{\rho_p(q^2)\not{p} + \rho_M(q^2)}{p^2 - q^2 + i\epsilon}.$$

Positivity Conditions:

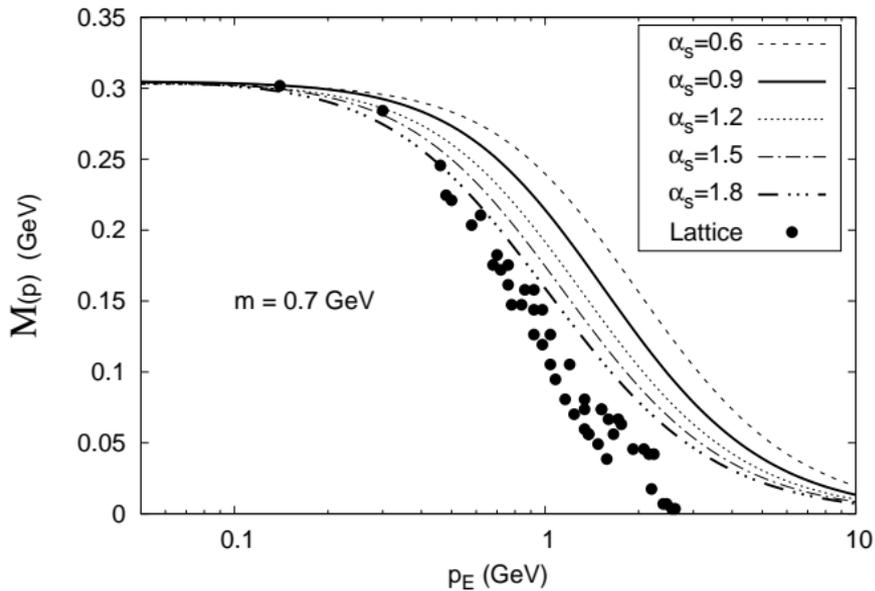
$$\rho_p(p^2) \geq 0,$$

$$p \rho_p(p^2) - \rho_M(p^2) \geq 0$$



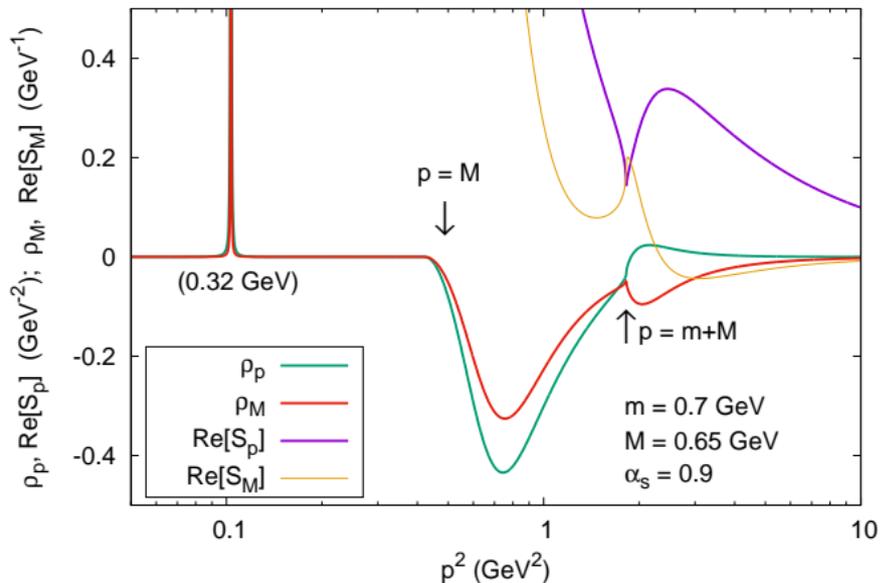
CHIRAL QCD

Quark sector: $m = 0.7$ GeV, M is fixed by requiring that $M(0) \approx 0.32$ GeV



CHIRAL QCD

Quark sector: $N_f = 2$, $M = 0.65$ GeV, $m = 0.7$ GeV



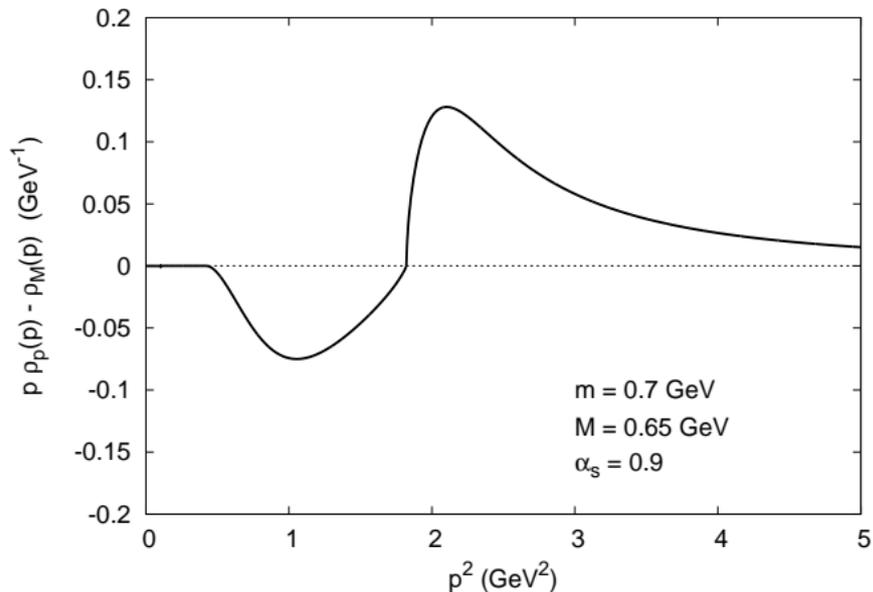
Positivity Conditions:

$$\rho_p(p^2) \geq 0, \quad p \rho_p(p^2) - \rho_M(p^2) \geq 0$$



CHIRAL QCD

Quark sector: $N_f = 2$, $M = 0.65$ GeV, $m = 0.7$ GeV



Positivity Conditions:

$$\rho_p(p^2) \geq 0,$$

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