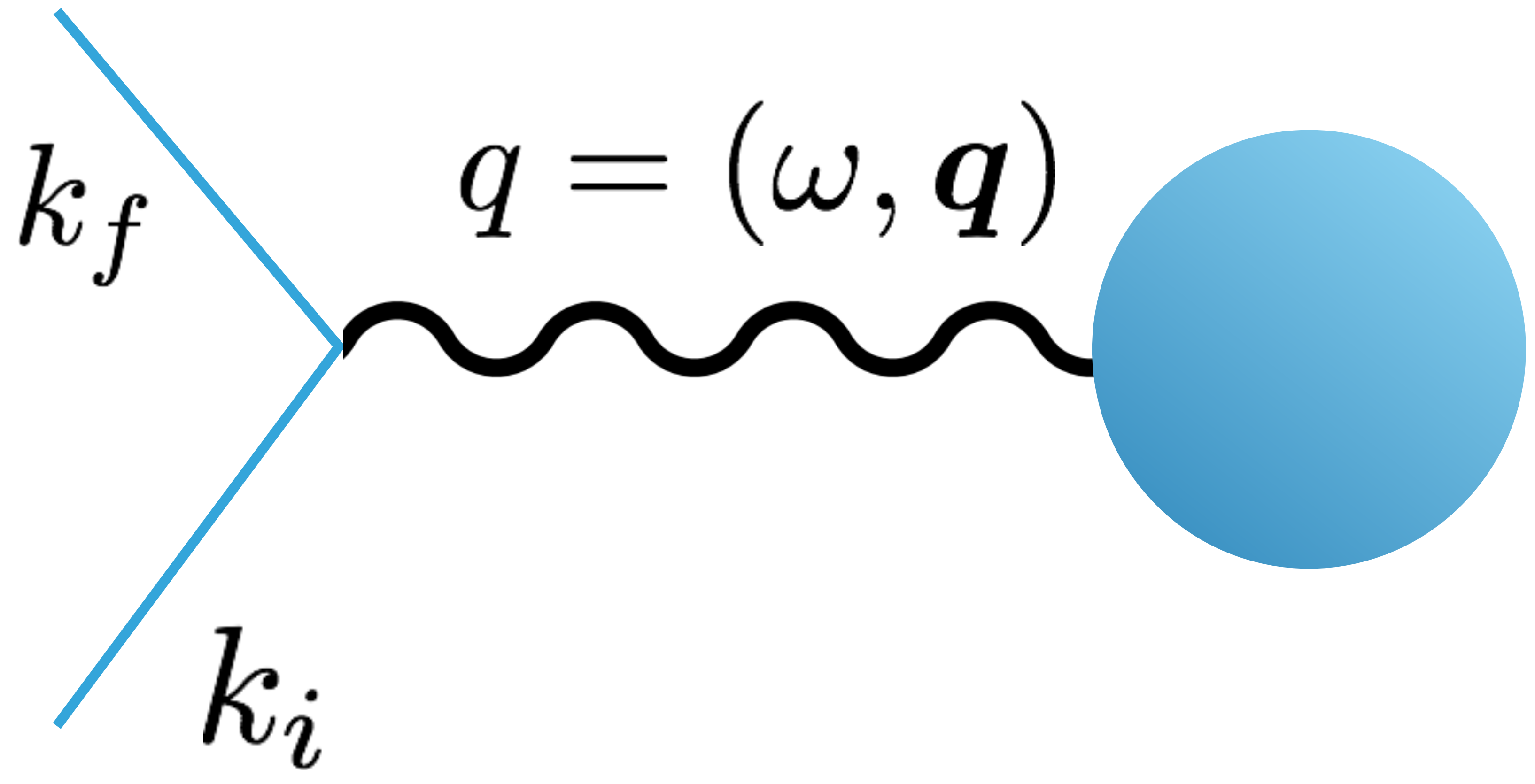
MICHAEL PAOLONE

TEMPLE UNIVERSITY

FOR THE E05-110 COLLABORATION.

COULOMB CORRECTIONS AND THE COULOMB SUM RULE AT JLAB

COULOMB SUM RULE



Inclusive electron scattering cross-section:

$$\frac{d^2\sigma}{d\Omega d\omega} = \sigma_{\text{Mott}} \left[\frac{q^4}{|\boldsymbol{q}|^4} R_L(\omega, |\boldsymbol{q}|) + \left(\frac{q^2}{2|\boldsymbol{q}|^2} + \tan^2\frac{\theta}{2} \right) R_T(\omega, |\boldsymbol{q}|) \right]$$

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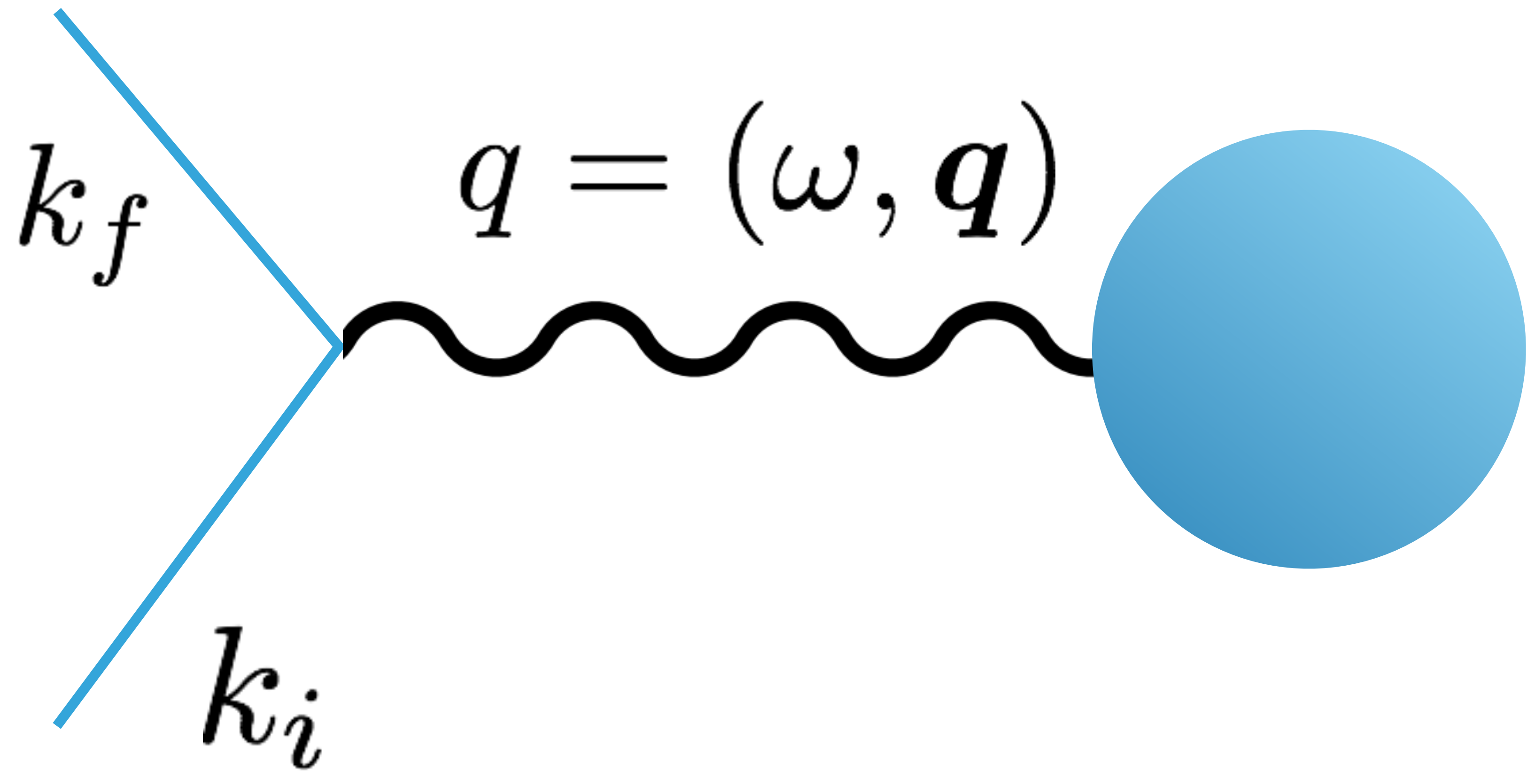

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Scattering response
due to **charge** properties

Scattering response
due to **magnetic** properties

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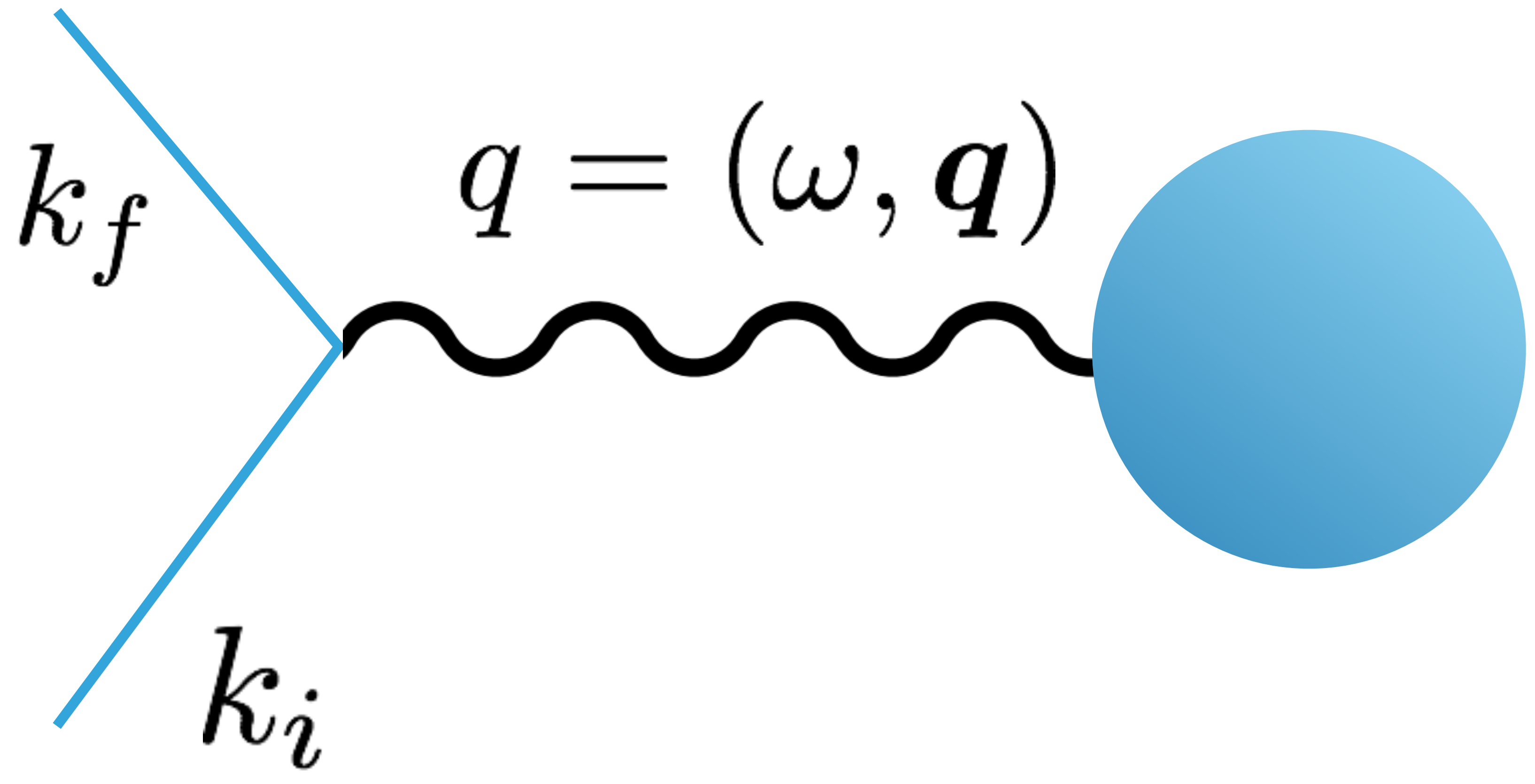

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Scattering response due to **charge** properties

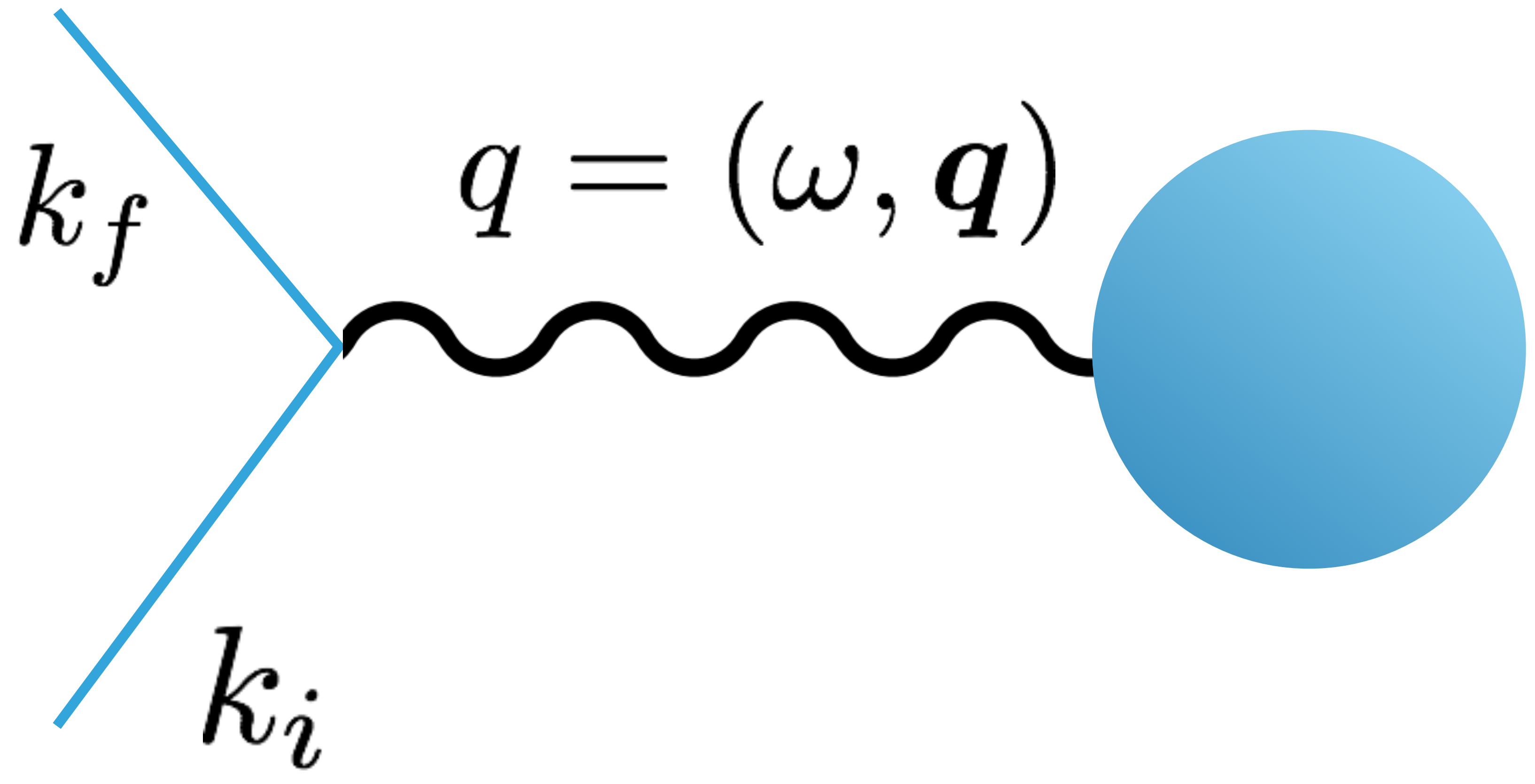
Scattering response due to **magnetic** properties

Coulomb Sum Rule definition:

$$S_L(|\boldsymbol{q}|) = \int_{\omega^+}^{|\boldsymbol{q}|} d\omega \frac{R_L(\omega, |\boldsymbol{q}|)}{Z\tilde{G}^2_{Ep}(Q^2) + N\tilde{G}^2_{En}(Q^2)}$$

If one integrates the charge response divided by the total charge form factor over all available virtual photon energies, naively one might expect the integral to go to unity.

COULOMB SUM RULE



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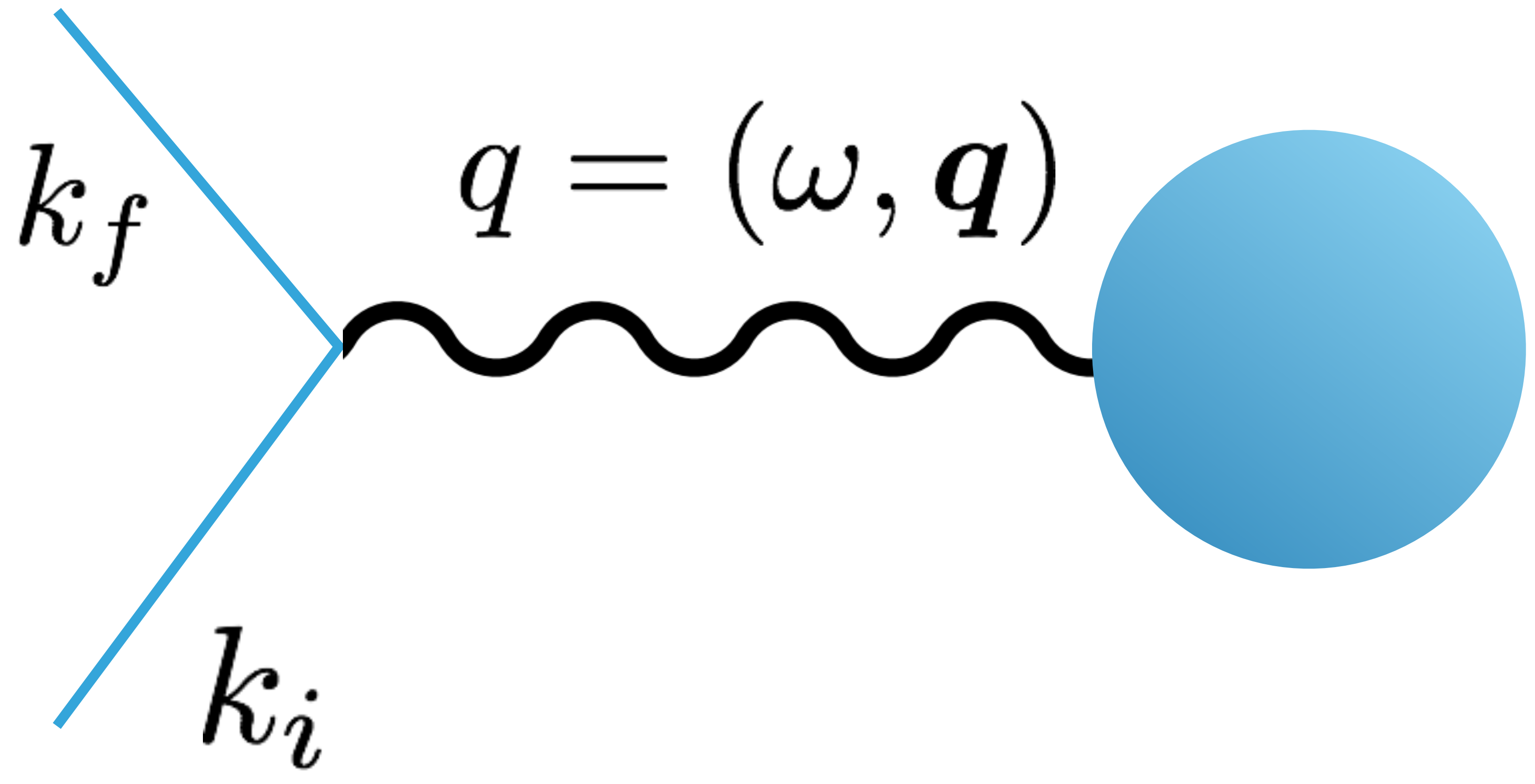

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At large |q| >> $2k_f$, S_L should go to 1. Any significant* deviation from this would be an indication of relativistic or medium effects distorting the nucleon form factor!

*Short range correlations will also quench S_L, but only by < 10%

COULOMB SUM RULE

- Long standing issue with many years of theoretical interest.
- Even most state-of the-art models cannot predict existing data.
- New precise data at larger |q| would provide crucial insight and constraints to modern calculations.

$$S_L(|\boldsymbol{q}|) = \int_{\omega^+}^{|\boldsymbol{q}|} d\omega \frac{R_L(\omega, |\boldsymbol{q}|)}{Z\tilde{G}^2_{Ep}(Q^2) + N\tilde{G}^2_{En}(Q^2)}$$

PRL 116, 032701 (2016) PHYSICAL REVIEW LETTERS week ending 22 JANUARY 2016

Relativistic and Nuclear Medium Effects on the Coulomb Sum Rule

Ian C. Cloët,[1] Wolfgang Bentz,[2] and Anthony W. Thomas[3]
[1]*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*
[2]*Department of Physics, School of Science, Tokai University, Hiratsuka-shi, Kanagawa 259-1292, Japan*
[3]*CSSM and ARC Centre of Excellence for Particle Physics at the Terascale, Department of Physics, University of Adelaide, Adelaide South Australia 5005, Australia*

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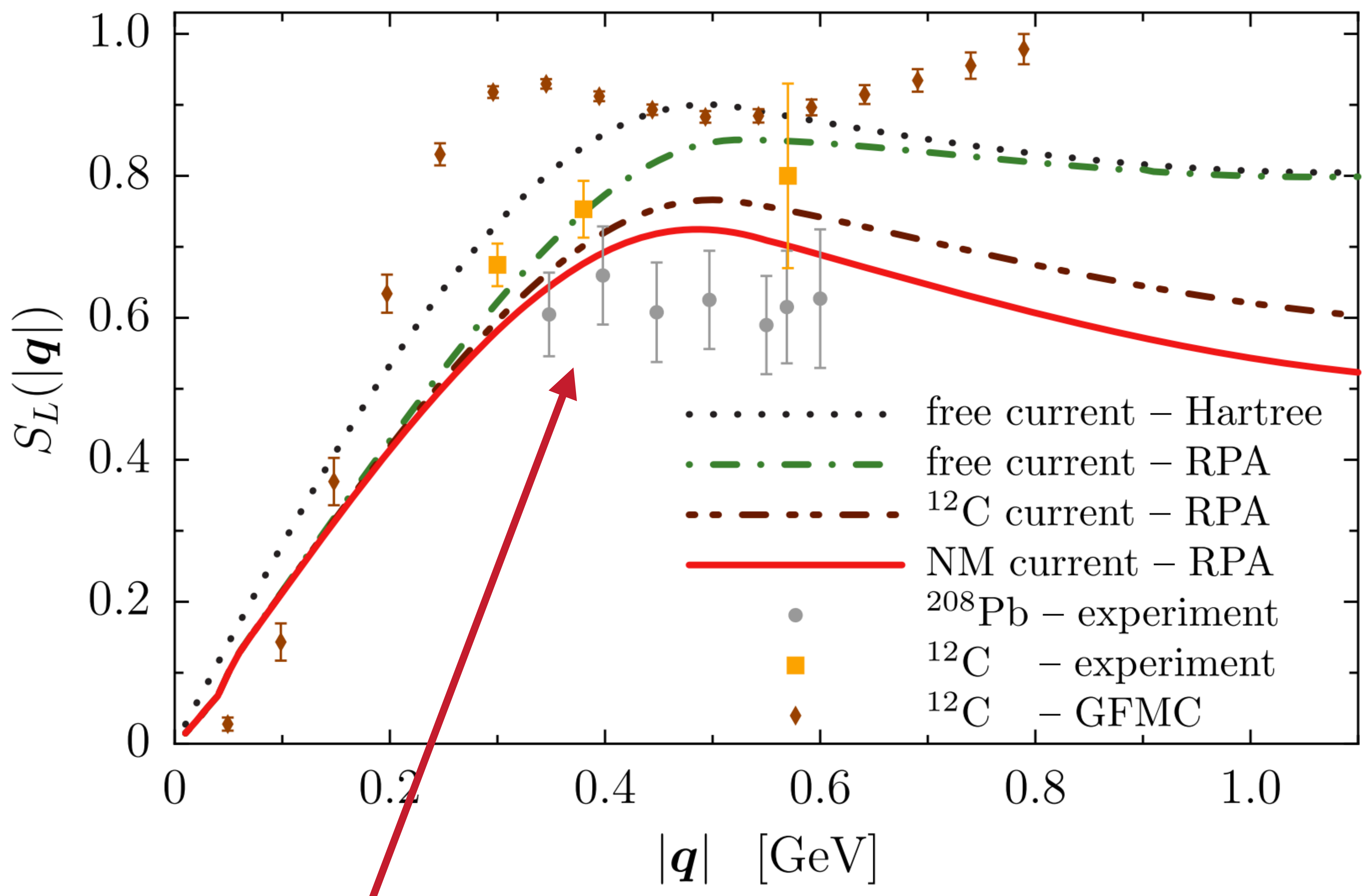

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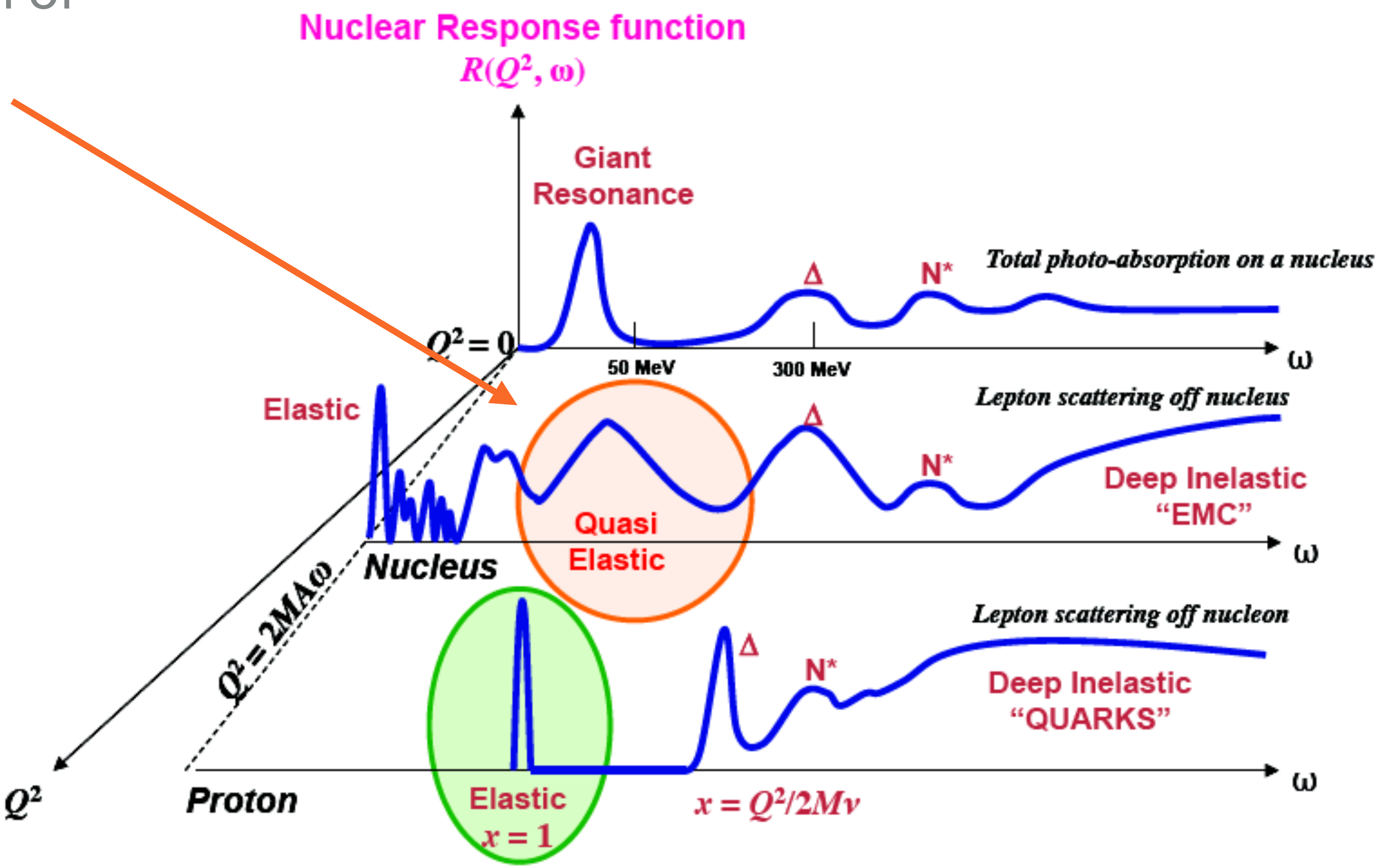
QUASI-ELASTIC SCATTERING

$$S_L(|\boldsymbol{q}|) = \int_{\omega^+}^{|\boldsymbol{q}|} d\omega \frac{R_L(\omega, |\boldsymbol{q}|)}{Z\tilde{G}^2_{Ep}(Q^2) + N\tilde{G}^2_{En}(Q^2)}$$

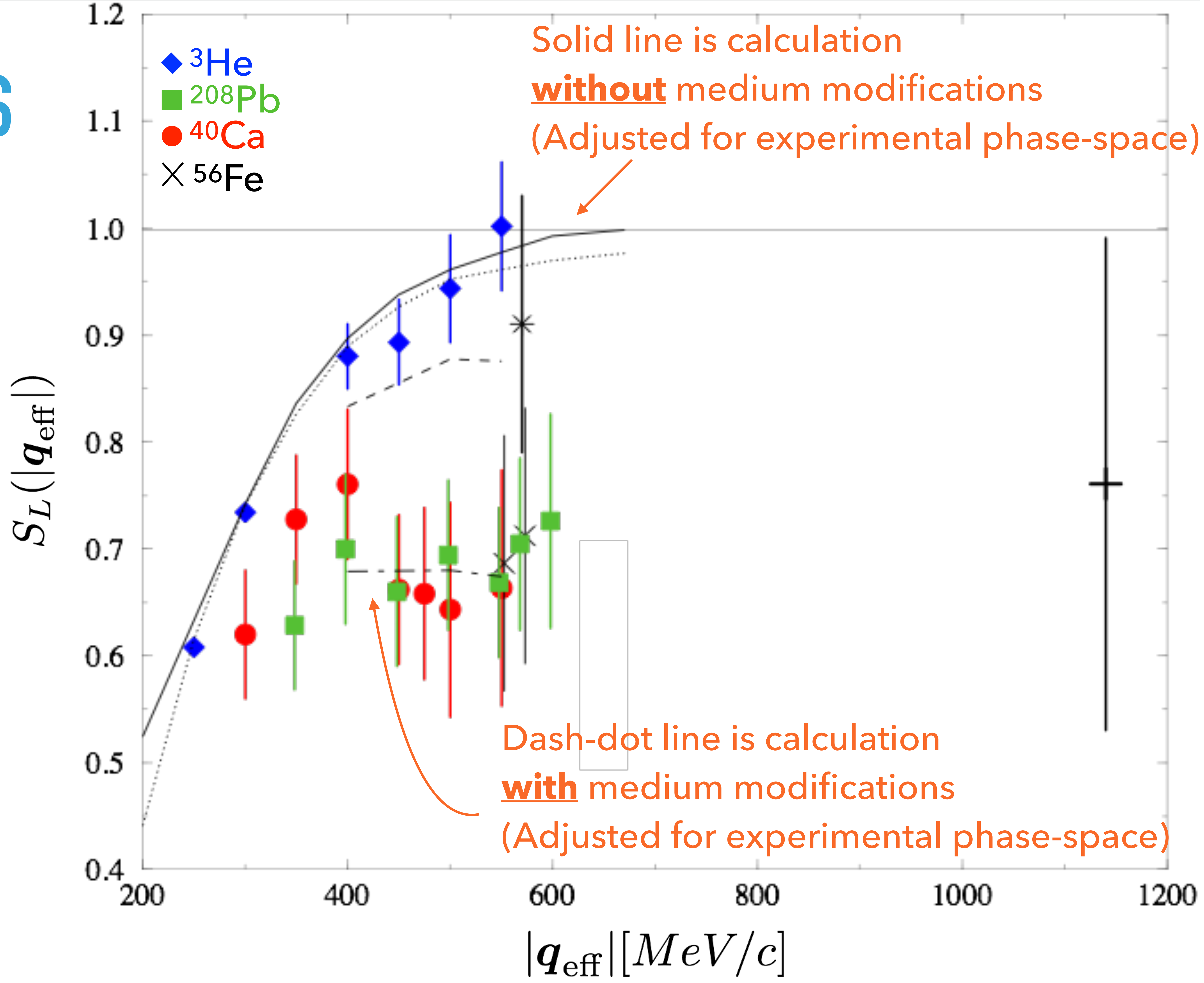
We want to integrate above the coherent elastic peak:
Quasi-elastic is "elastic" scattering on constituent nucleons inside nucleus.

- Quasi-elastic scattering at intermediate Q^2 is the region of interest for our experiment:
  - Nuclei investigated:
    - ^{4}He
    - ^{12}C
    - ^{56}Fe
    - ^{208}Pb



PUBLISHED EXPERIMENTAL RESULTS

- First group of experiments from Saclay, Bates, and SLAC show a quenching of S_L consistent with medium modified form-factors.



$|\mathbf{q}_{\text{eff}}|$ is $|\mathbf{q}|$ corrected for a nuclei dependent mean coulomb potential. Methodology agreed on by Andreas Aste, Steve Wallace and John Tjon.

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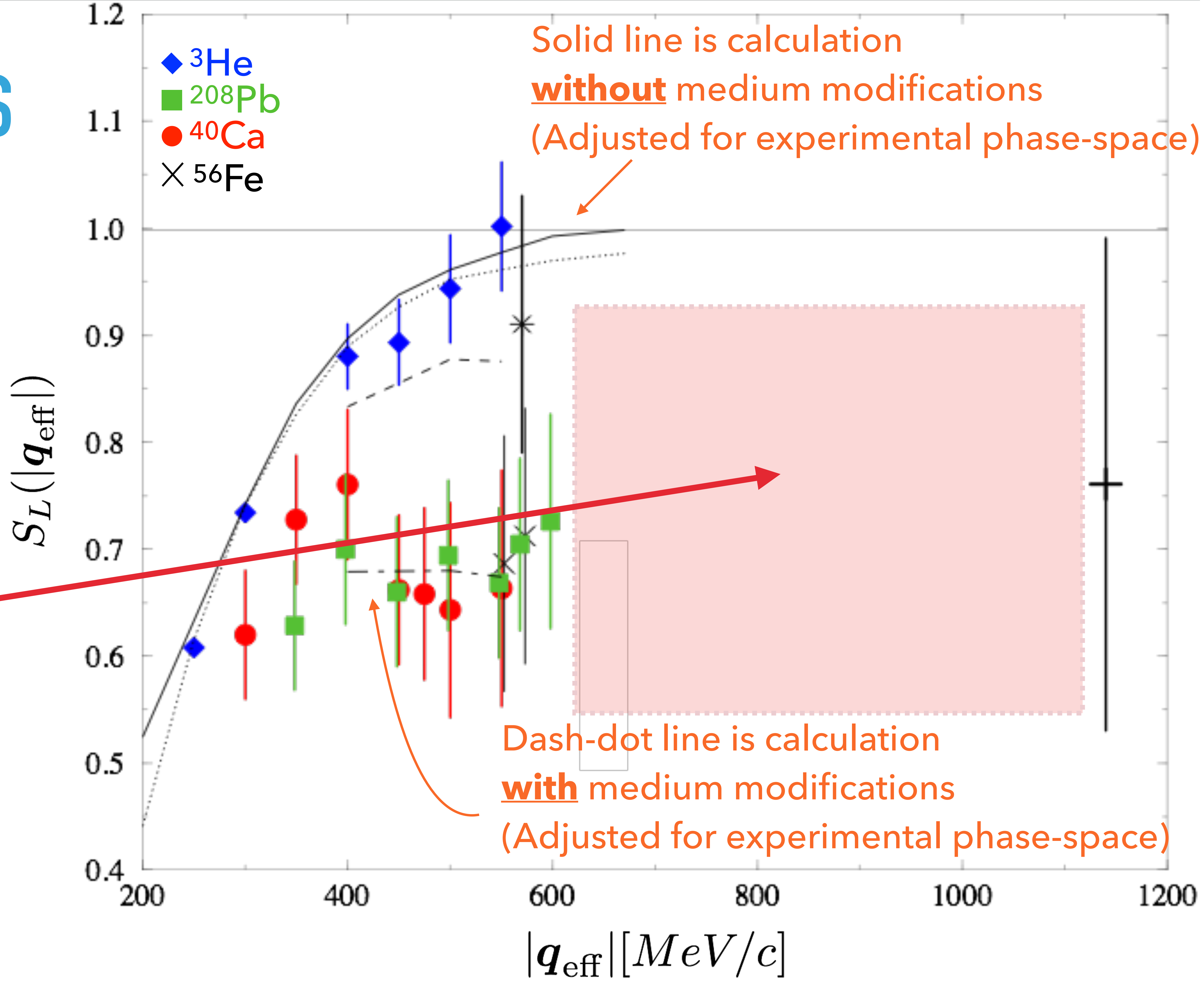
- First group of experiments from Saclay, Bates, and SLAC show a quenching of S_L consistent with medium modified form-factors.
- Very little data above |**q**| of 600 MeV/c, where the cleanest signal of medium effects should exist!
    - Sarclay, Bates limited in beam energy reach up to 800 MeV.
    - SLAC limited in kinematic coverage of scattered electron at |q| below 1150 MeV/c.



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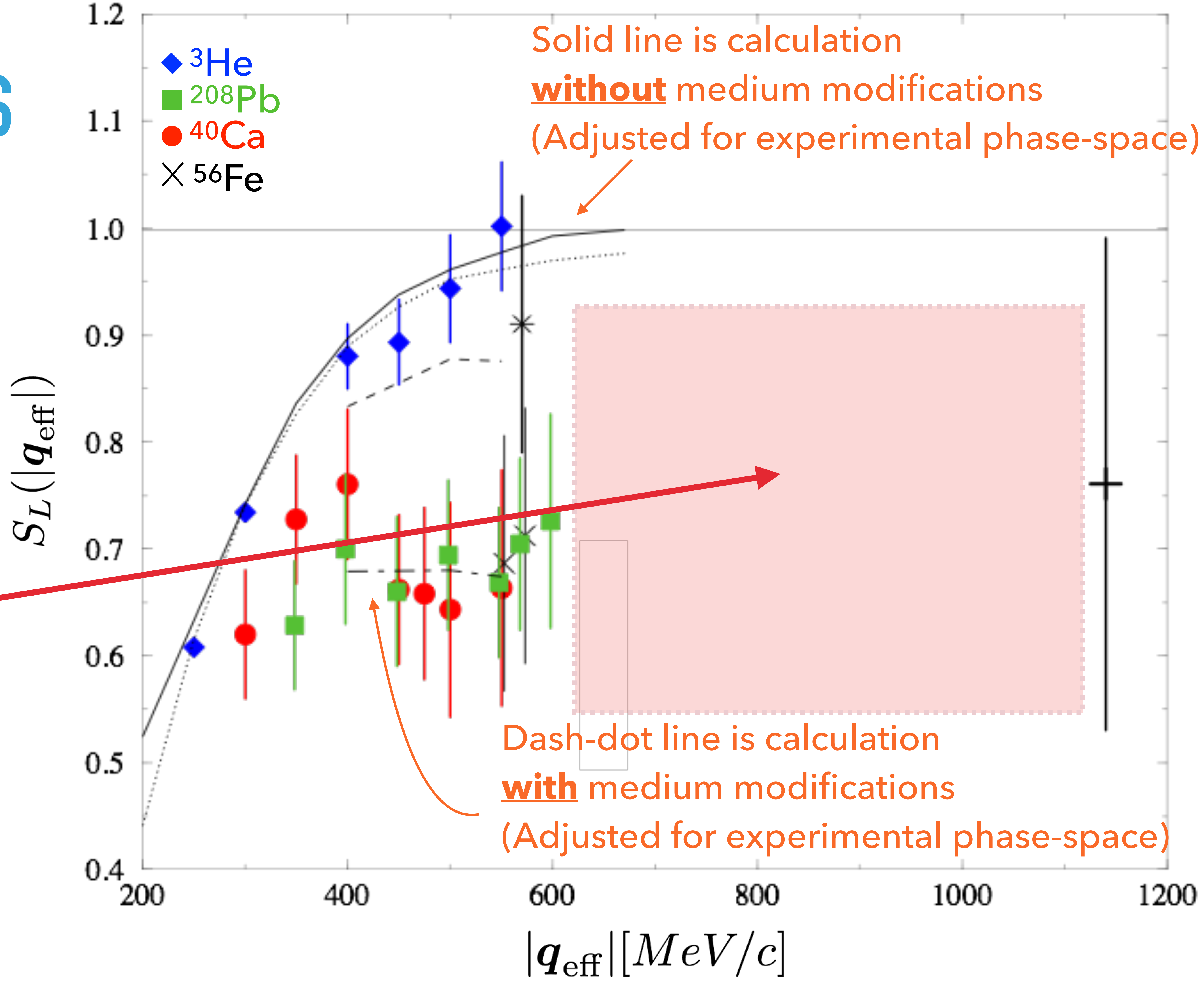

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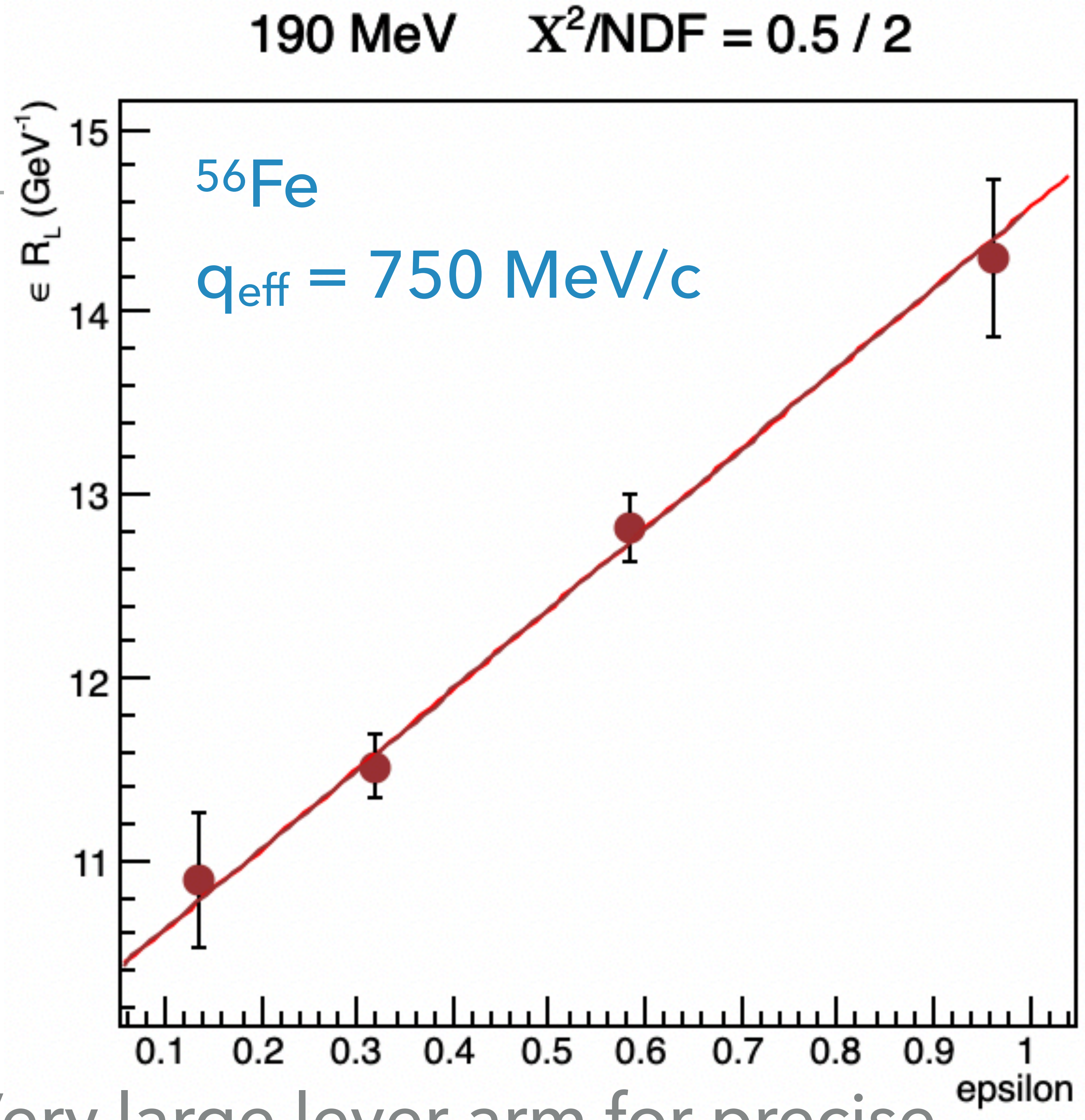
EXPERIMENTAL DESIGN



- Need R_L ⟶ Use Rosenbluth separation!

$$S_L(|\boldsymbol{q}|) = \int_{\omega+}^{|\boldsymbol{q}|} d\omega \frac{\boxed{R_L(\omega, |\boldsymbol{q}|)}}{Z\tilde{G}^2_{Ep}(Q^2) + N\tilde{G}^2_{En}(Q^2)}$$

Slope = $\frac{Q^4}{\vec{q}^4} R_L$

Intercept = $\frac{Q^2}{2\vec{q}^2} R_T$

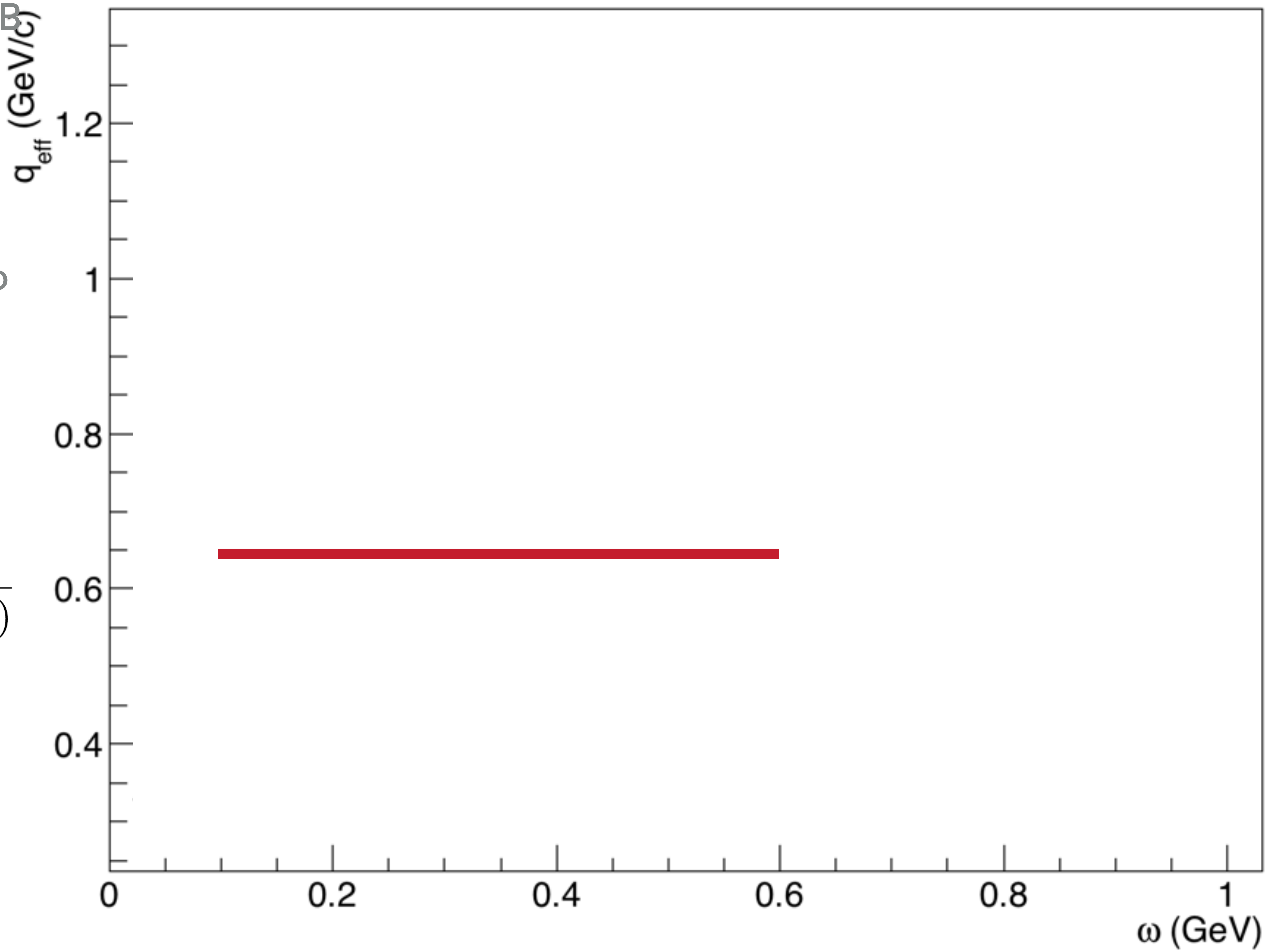
  - Experiment run at 4 angles per target: 15, 60, 90, 120 degs. Very large lever arm for precise calculation of R_L!
- Need data for each angle at a constant |q| over an ω range starting above the elastic peak up to |q|.
  - When running a single arm experiment with fixed beam energy and scattering angle, |q| is NOT constant over your momentum acceptance.
    - Need to take data at varying beam energies, and “map-out” |q| and ω space.

EXPERIMENTAL DESIGN

- If one wants to measure from 100 to 600 MeV ω at constant |q| = 650 MeV/c

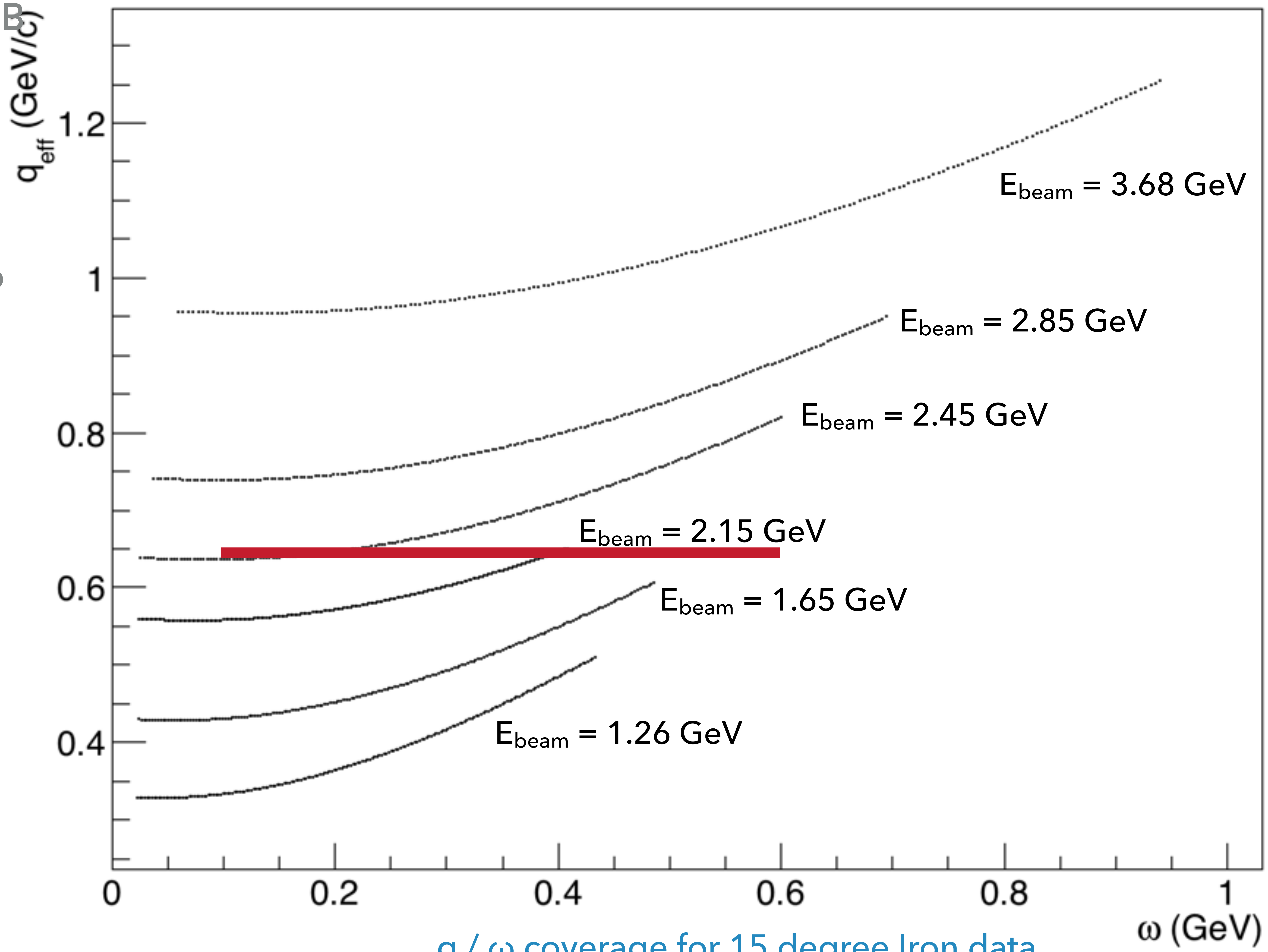
CSR calculated at constant |**q**| !!

$$S_L(|\boldsymbol{q}|) = \int_{\omega^+}^{|\boldsymbol{q}|} d\omega \frac{R_L(\omega, |\boldsymbol{q}|)}{Z\tilde{G}^2_{Ep}(Q^2) + N\tilde{G}^2_{En}(Q^2)}$$



EXPERIMENTAL DESIGN

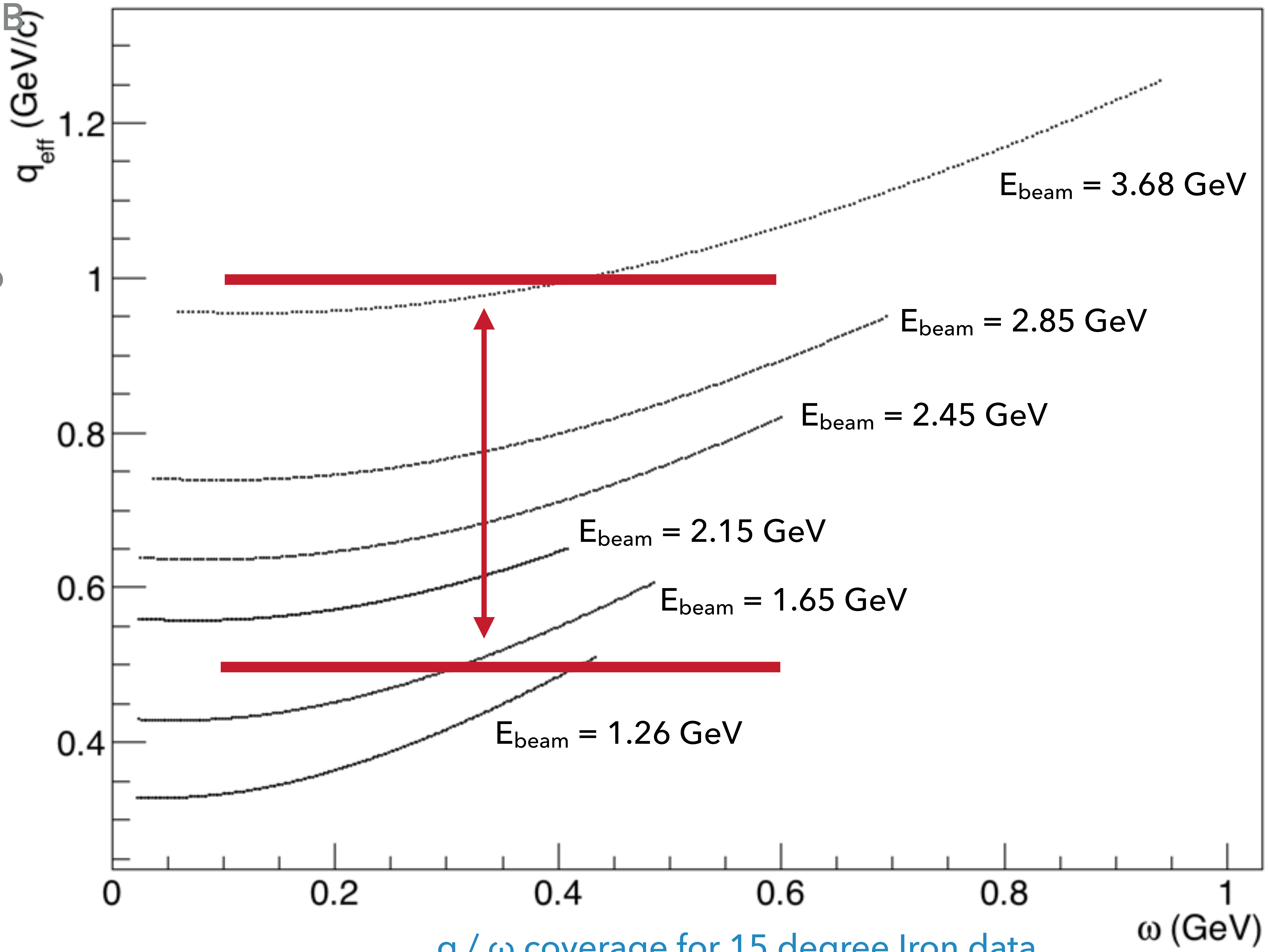
- If one wants to measure from 100 to 600 MeV ω at constant |q| = 650 MeV/c
    - Take data at different beam energies, and interpolate to determine cross-section at constant |q|.



q / ω coverage for 15 degree Iron data

EXPERIMENTAL DESIGN

- If one wants to measure from 100 to 600 MeV ω at constant |q| = 650 MeV/c
  - Take data at different beam energies, and interpolate to determine cross-section at constant |q|.
  - |q| can be selected between 550 and 1000 MeV/c

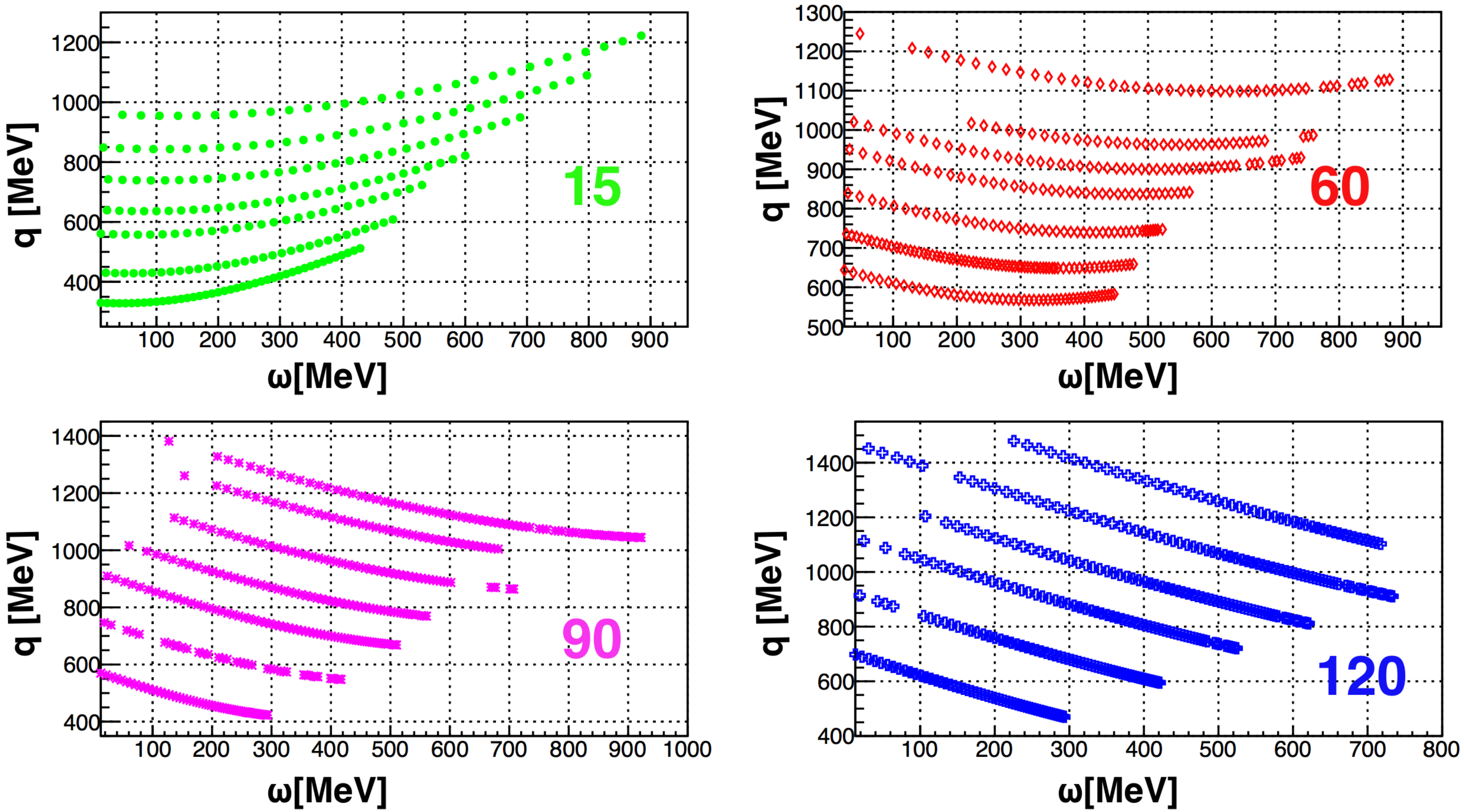


Repeat this "mapping" for 60, 90, and 120 degree spectrometer central angles.

q / ω coverage for 15 degree Iron data

EXPERIMENTAL SPECIFICS

Each data line represents a constant beam-energy

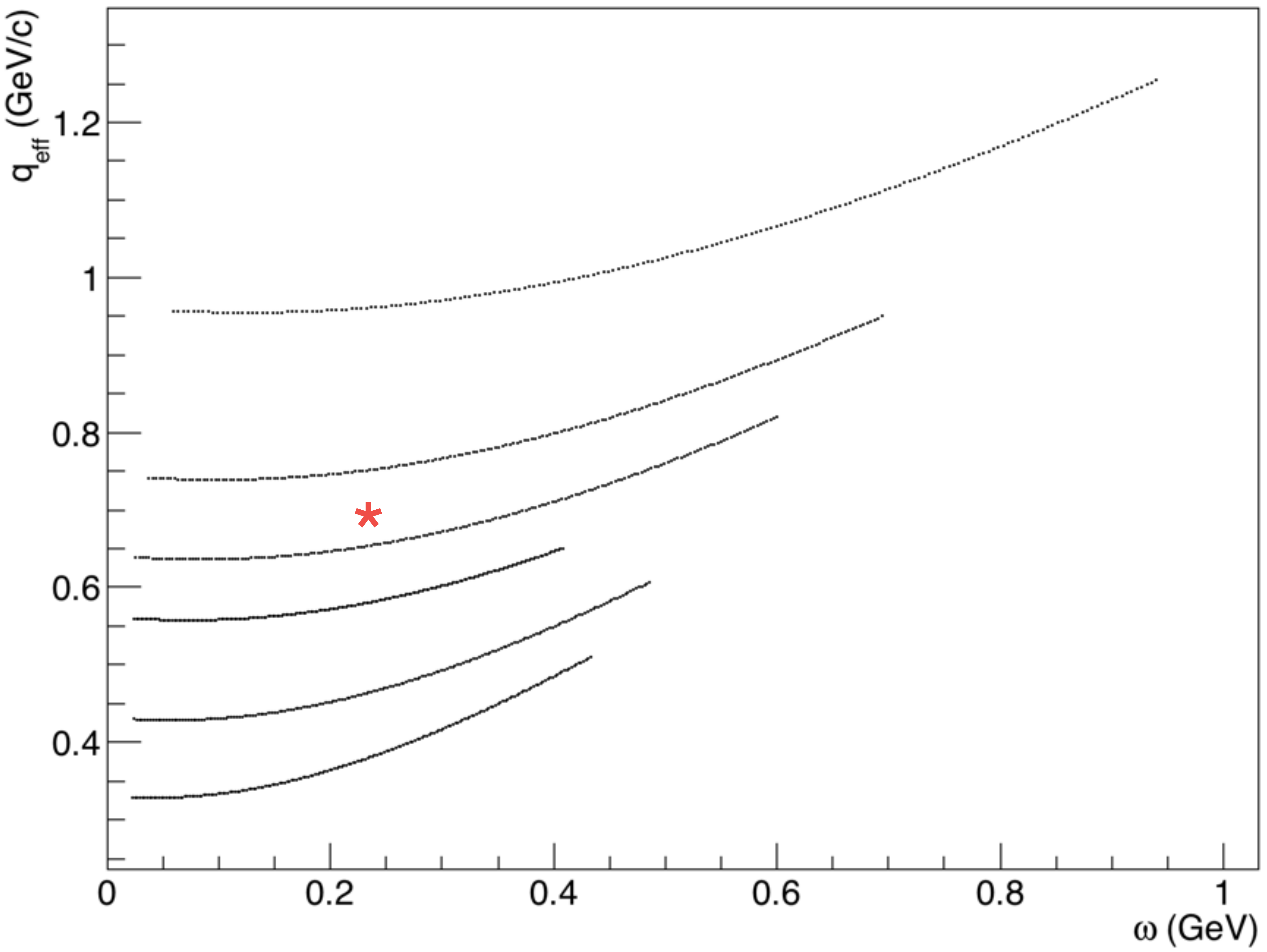


- E05-110:
  - Data taken from October 23rd 2007 to January 16th 2008
  - 4 central angle settings: 15, 60, 90, 120 degs.
  - Many beam energy settings: 0.4 to 4.0 GeV
  - Many central momentum settings: 0.1 to 4.0 GeV
  - LHRS and RHRS independent (redundant) measurements for most settings
  - 4 targets: ^{4}He, ^{12}C, ^{56}Fe, ^{208}Pb.

INTERPOLATION TECHNIQUES

- Interpolation of |q|

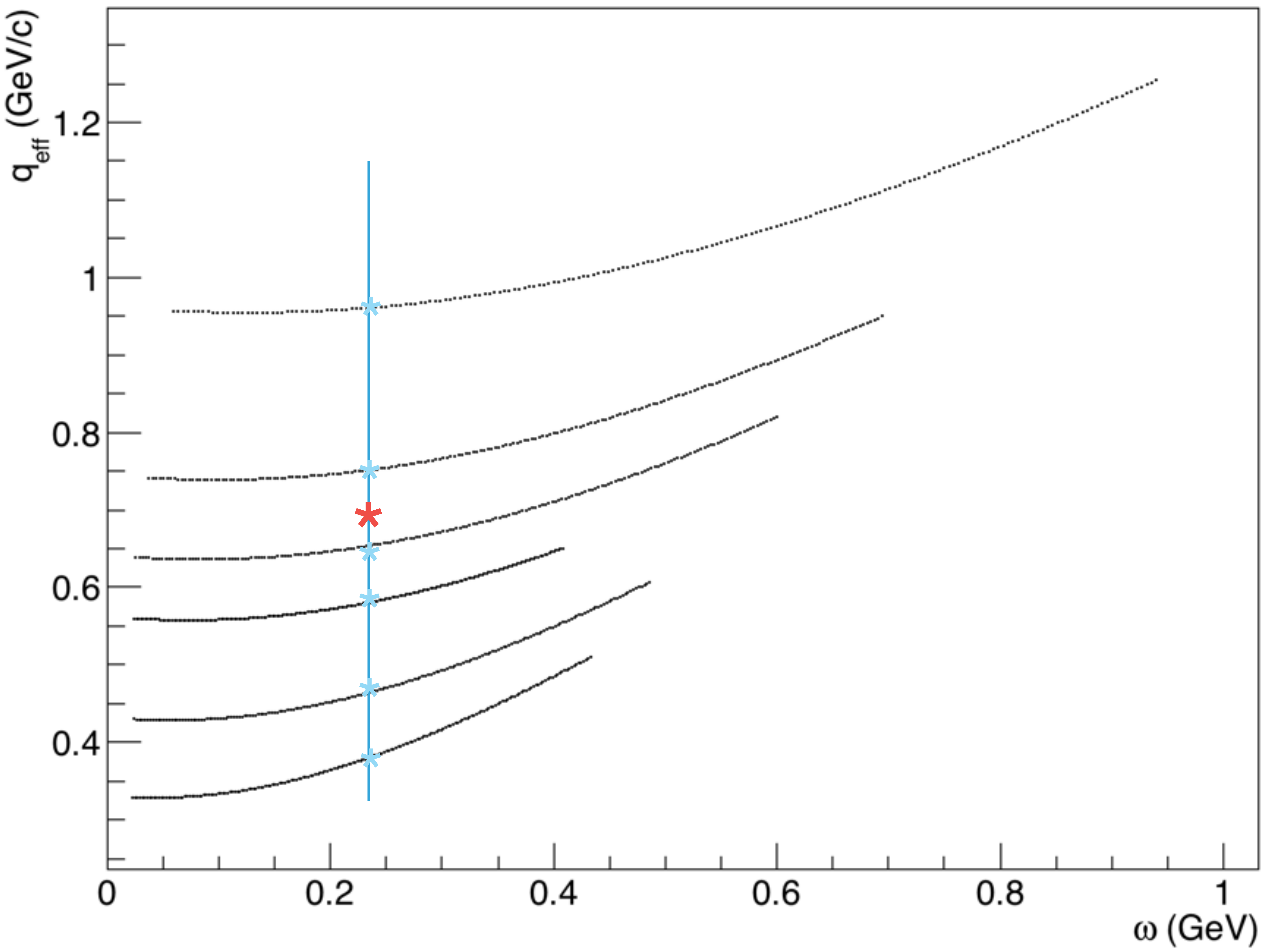
q / ω coverage for 15 degree Iron data



INTERPOLATION TECHNIQUES

- Interpolation of |q|
  - Could go along a constant ω line.
    Not the best option.

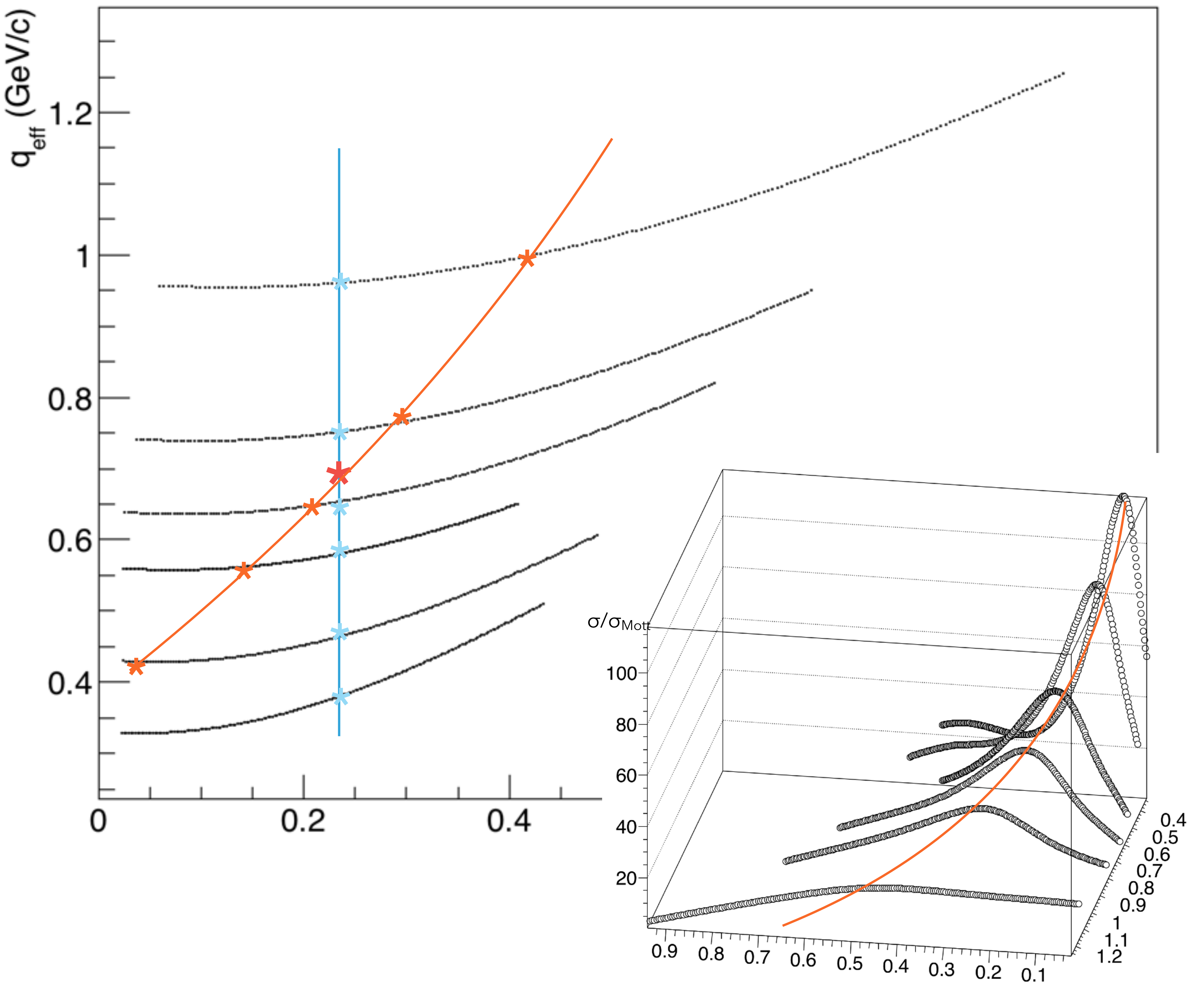
q / ω coverage for 15 degree Iron data



INTERPOLATION TECHNIQUES

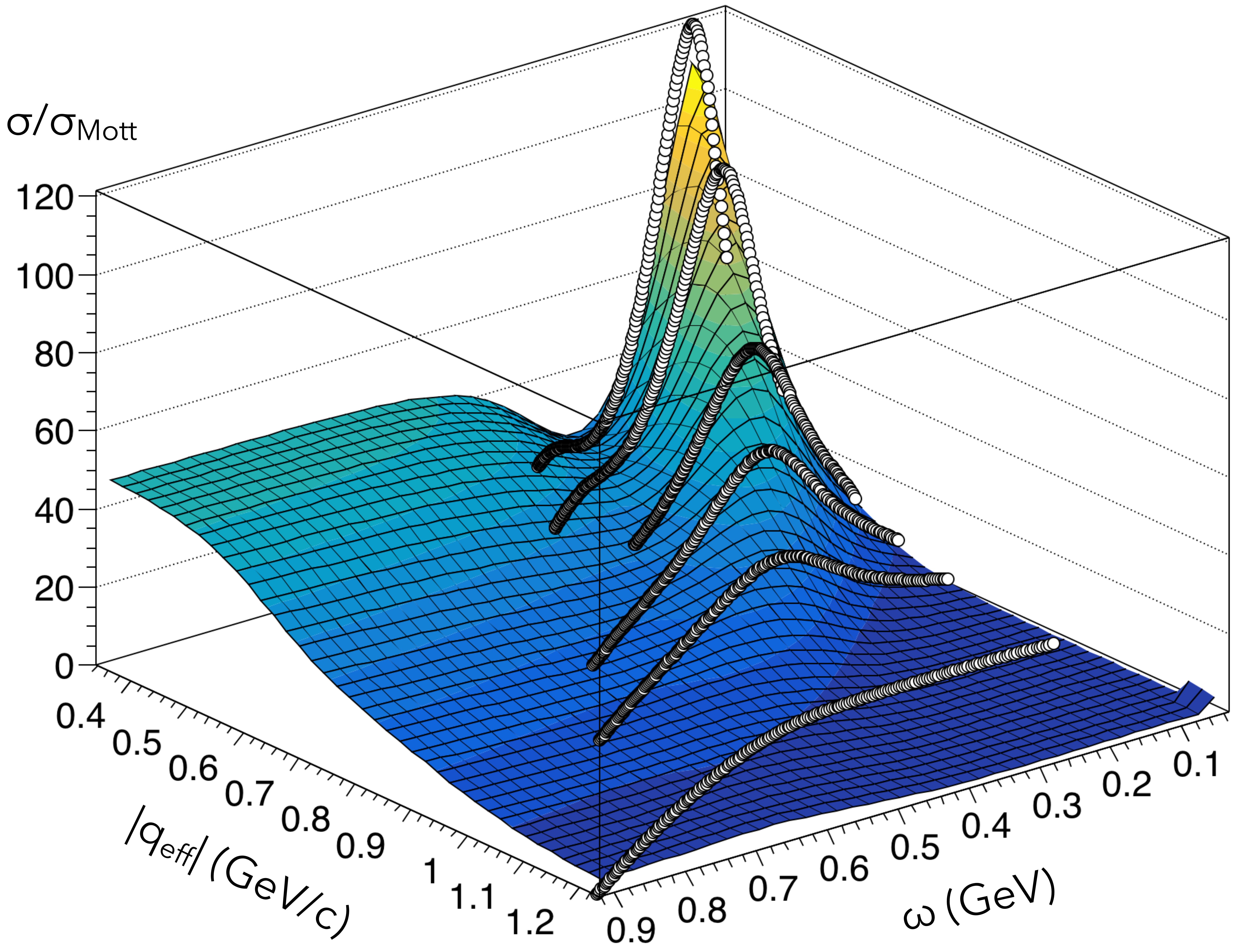
- Interpolation of |q|
  - Could go along a constant ω line. Not the best option.
  - Better: use a constant y line, which will follow the trend of quasi-elastic peak.
  - Alternative: use a constant W line, which should follow the Δ peak.
    - or even a combination of y and W.

q / ω coverage for 15 degree Iron data



INTERPOLATION TECHNIQUES

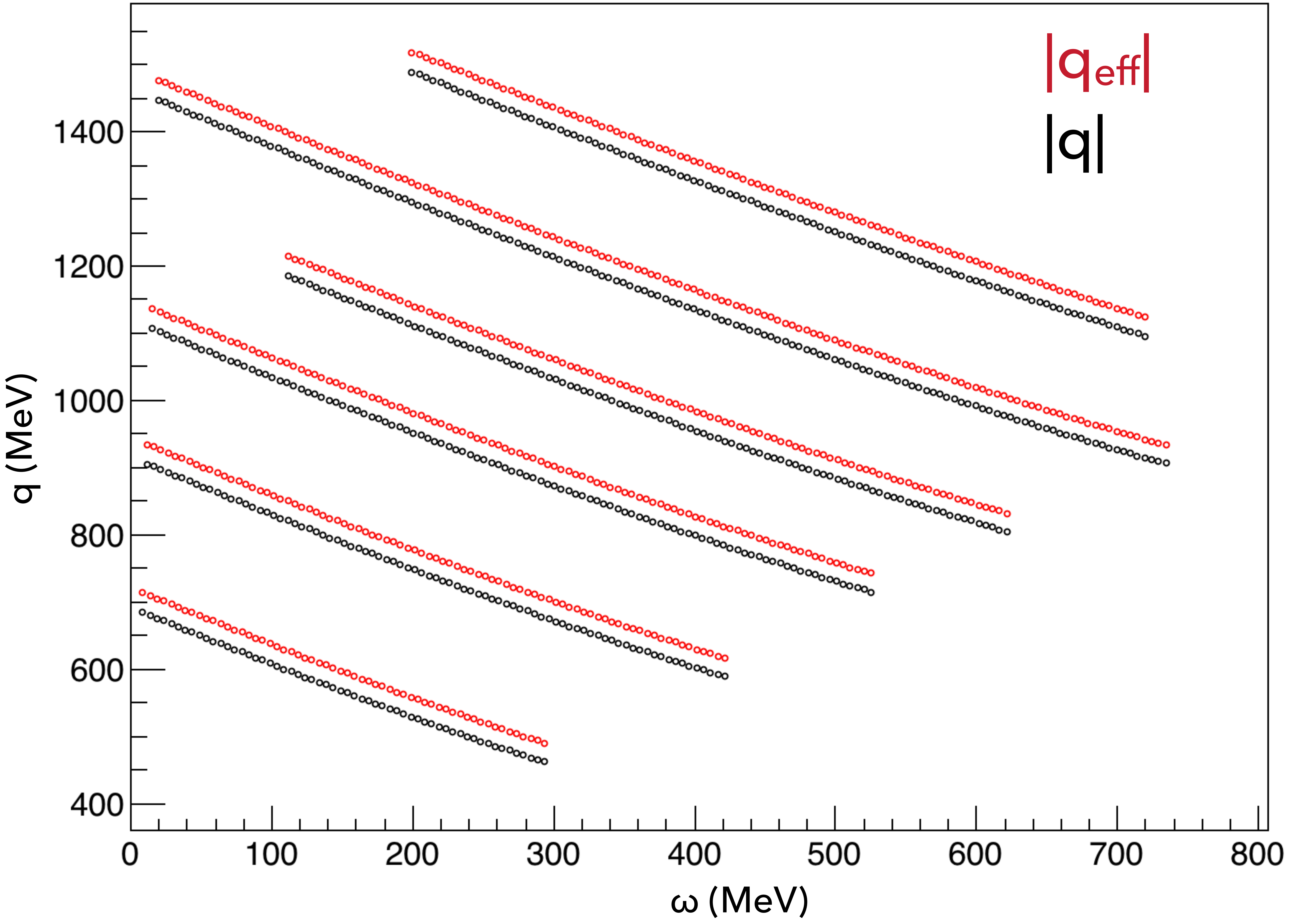
- 3D Machine learning techniques are also available:
  - Unsupervised Neural Network
    - Method uncertainty is hard to pin down.
  - Supervised Gaussian Process Regression.
    - Implemented from scratch.
    - Uncertainties are well constrained.



INTERPOLATION AND q_{eff}

- The offset in the spectra when using the EMA corrected momentum transfer significantly affects the interpolation landscape.
  - Effect is largest at low momenta and in heavier targets

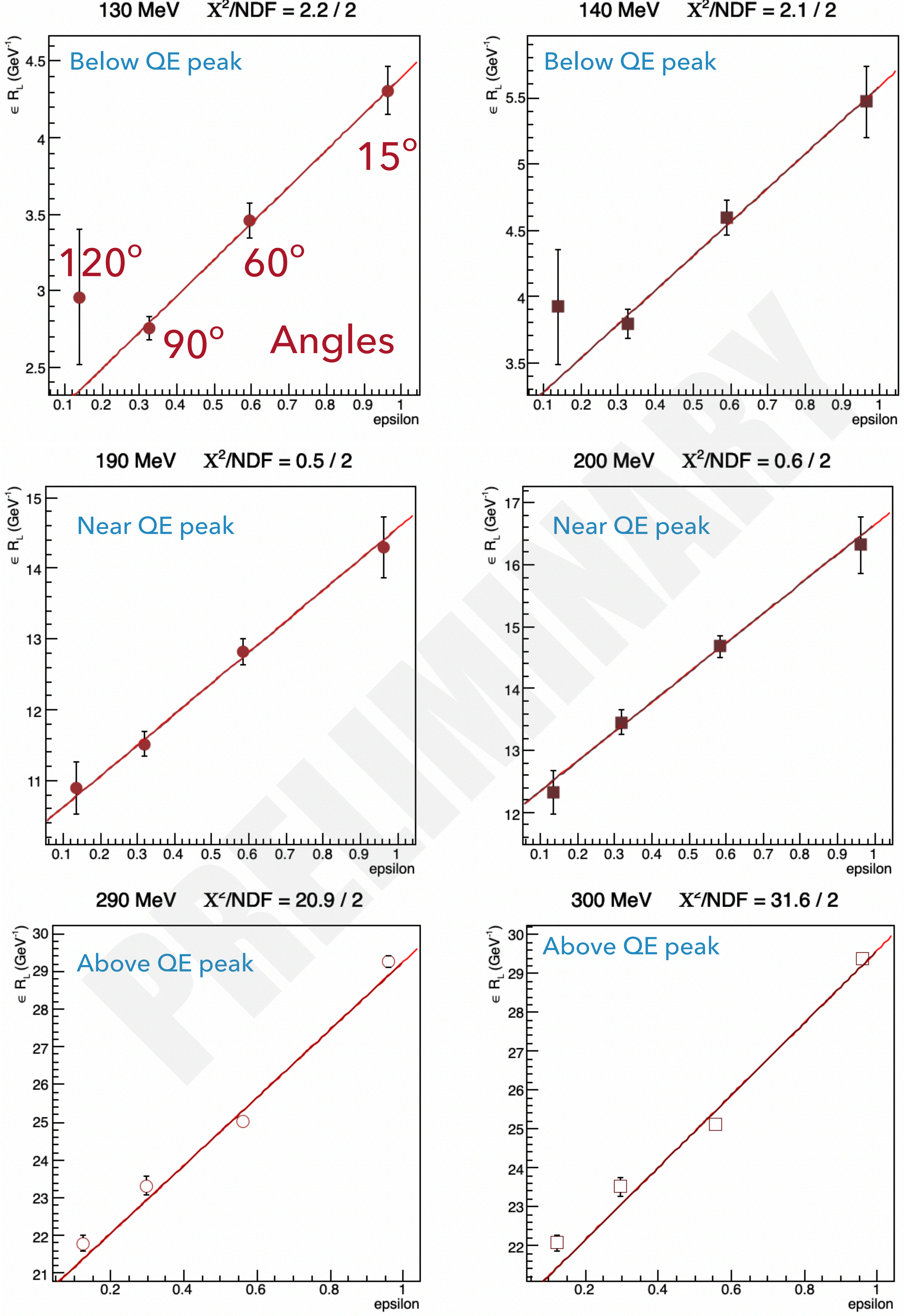
q / ω coverage for 120 degree Pb data



SOME PRELIMINARY RESULTS

- I'd like to discuss ^{56}Fe
  - We use a foil target for Iron (well understood).
  - We are still working on extracting XS from ^{208}Pb, which is housed inside a extended hydrogen cell.
- Preliminary results show some trends in the Rosenbluth extraction:
  - At low energy transfer, the low beam-energy backward angle data pulls the Rosenbluth. At large energy transfer, the high beam-energy forward angle data pulls the fit.
- An EMA correction has been applied.

Rosenbluth fits at q_{eff} = 750 MeV/c



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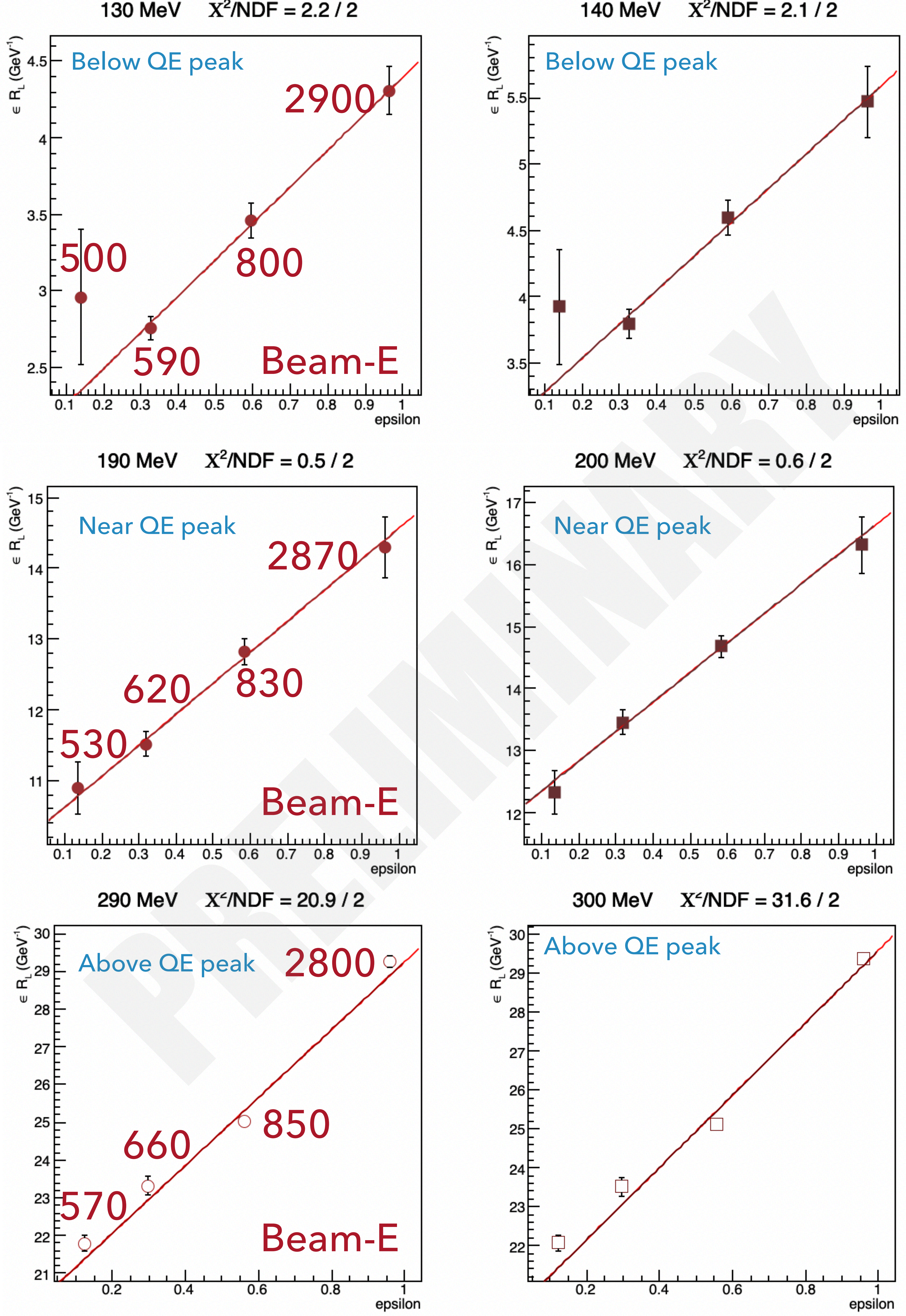

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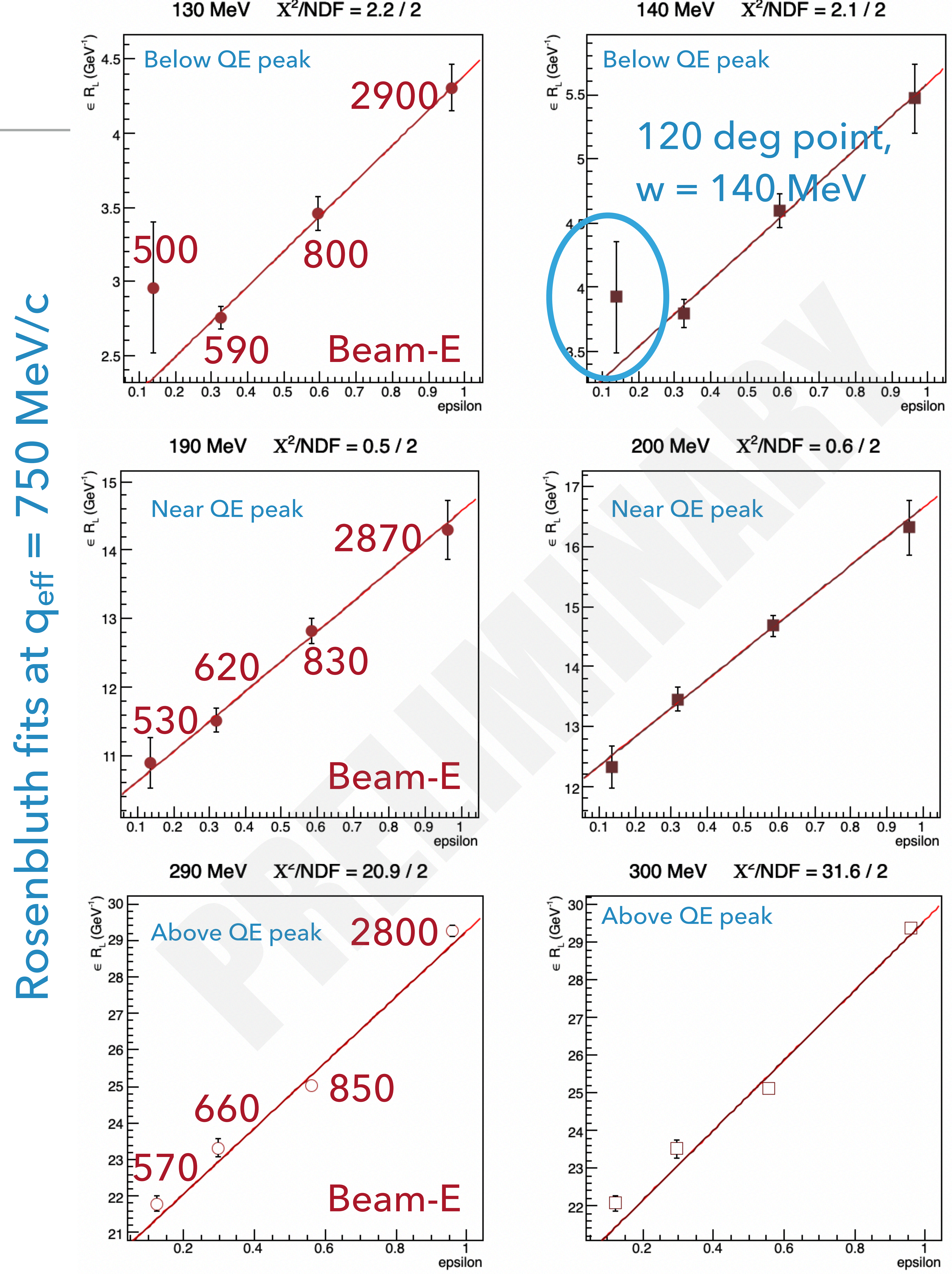

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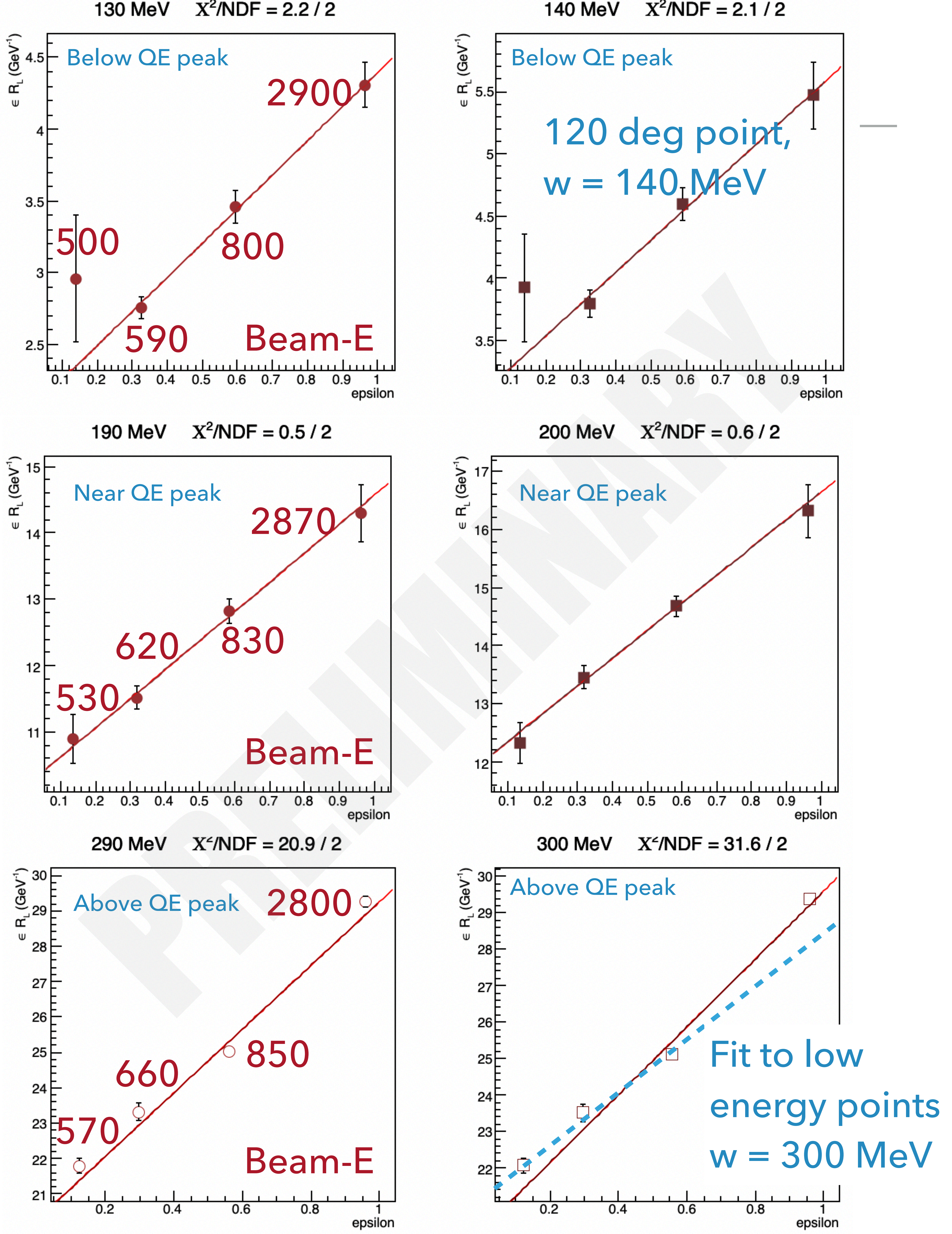

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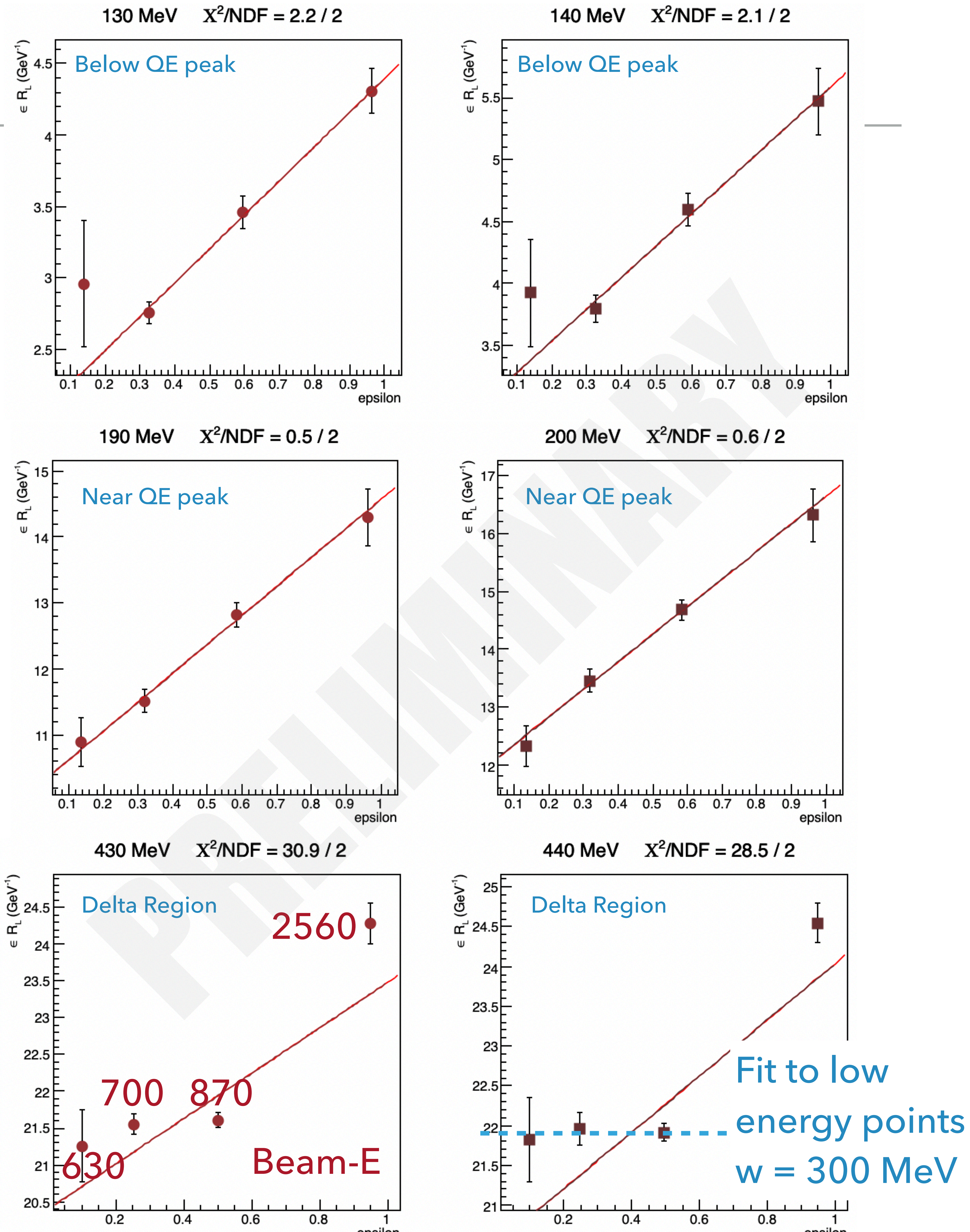
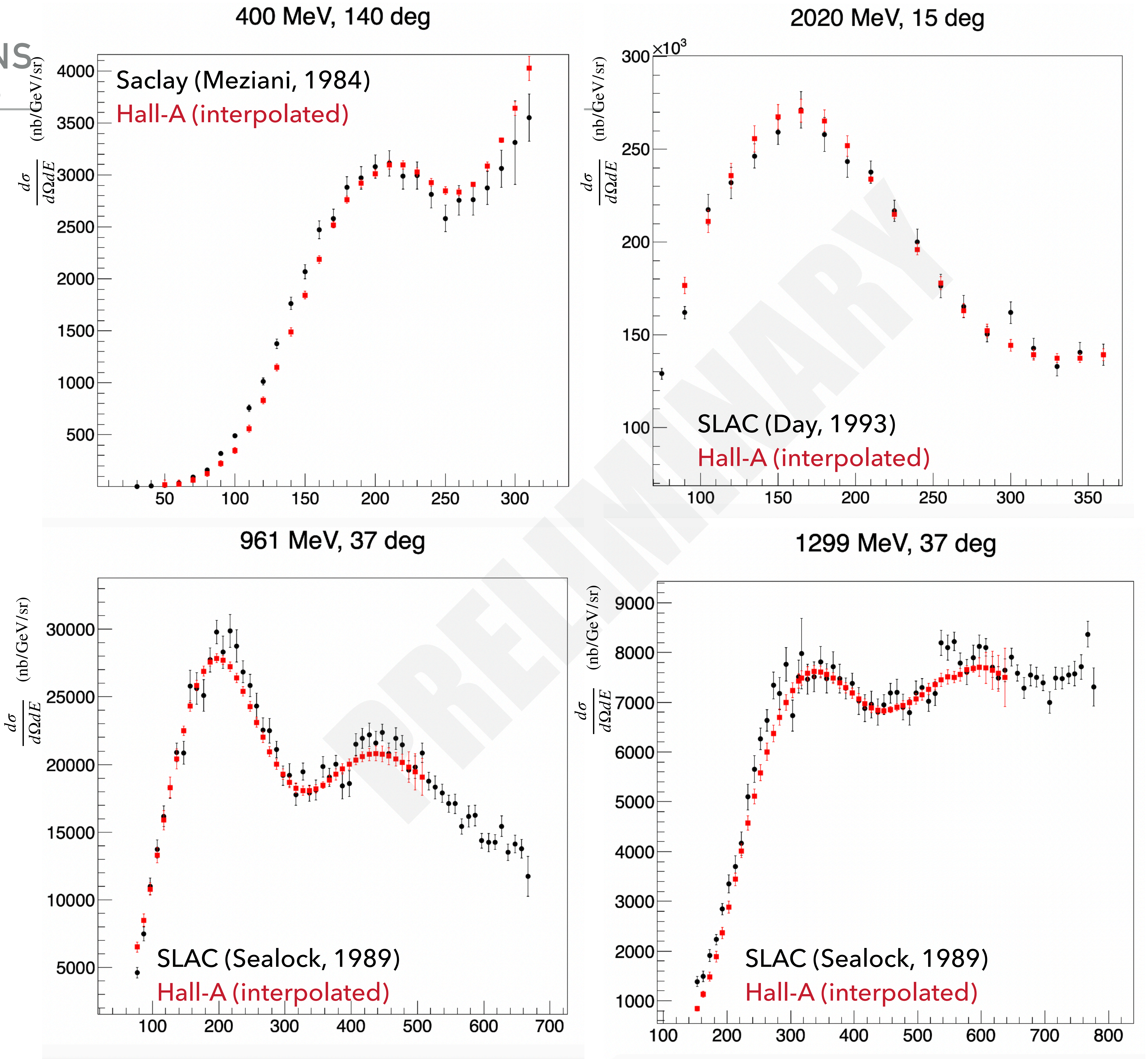


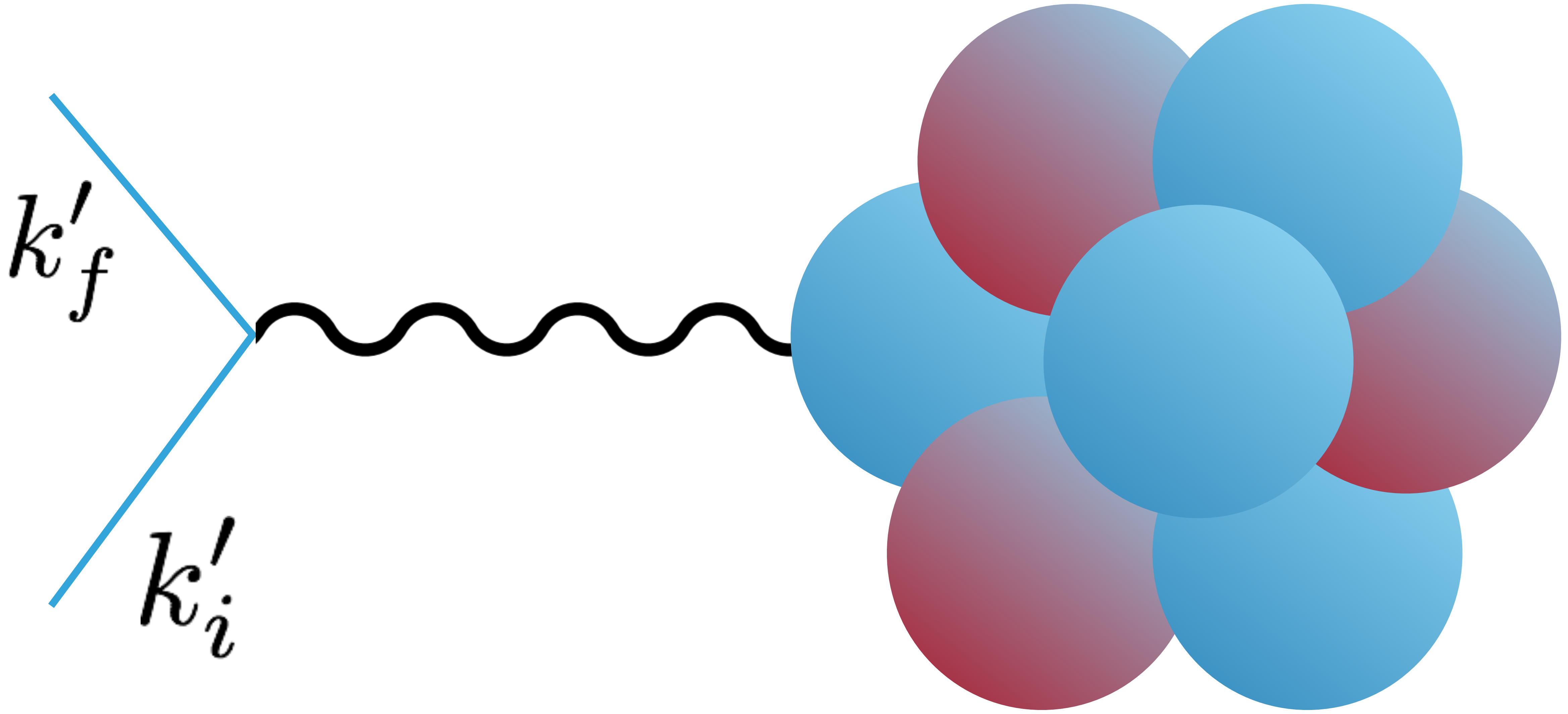
WORLD DATA COMP

- World data comparisons for ^{56}Fe
  - We can use our interpolation space and our Rosenbluth fits to compare to world data.
  - Reasonable agreement suggests that XS extraction is in control:
    - Radiative corrections.
    - Efficiencies and spectrometer response.
    - Interpolation procedures.



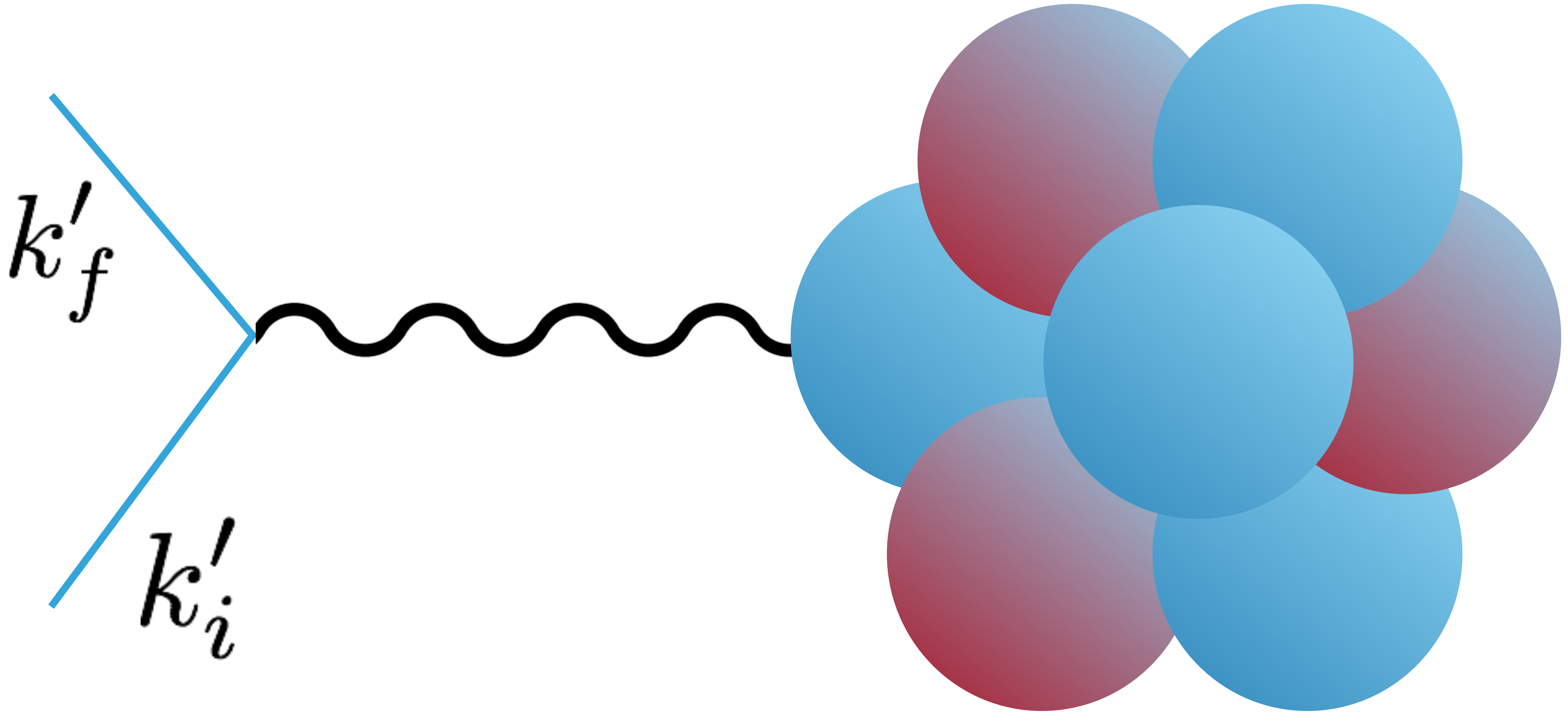
COULOMB CORRECTIONS

- Since we scatter off of a bound nucleon, consideration has to be given to the long-range field effects during the scattering process.
  - One can expect the energy and trajectory of both the incident and scattered electron to be affected:
    - Results in both an energy change and focusing effect.
- First studied by I. Sick and J.S. McCarthy in 1970, and plenty of interest from many theorists through the early 2000's.



COULOMB CORRECTIONS

- Conceptually, the problem is well defined. In practice, calculations can be painful:
  - Solutions to the Dirac equation for electron scattering in the presence of many-body nuclear fields are (laboriously) calculable with partial wave expansion and numerical calculation.



For a systematic analysis of experimental data, the numerical effort is usually too big and unpractical. This is true in particular as often computer codes have been developed for $(e, e'p)$, in which case the (e, e') cross section has to be generated by summing over all possible initial and final states of the knocked-out nucleon.

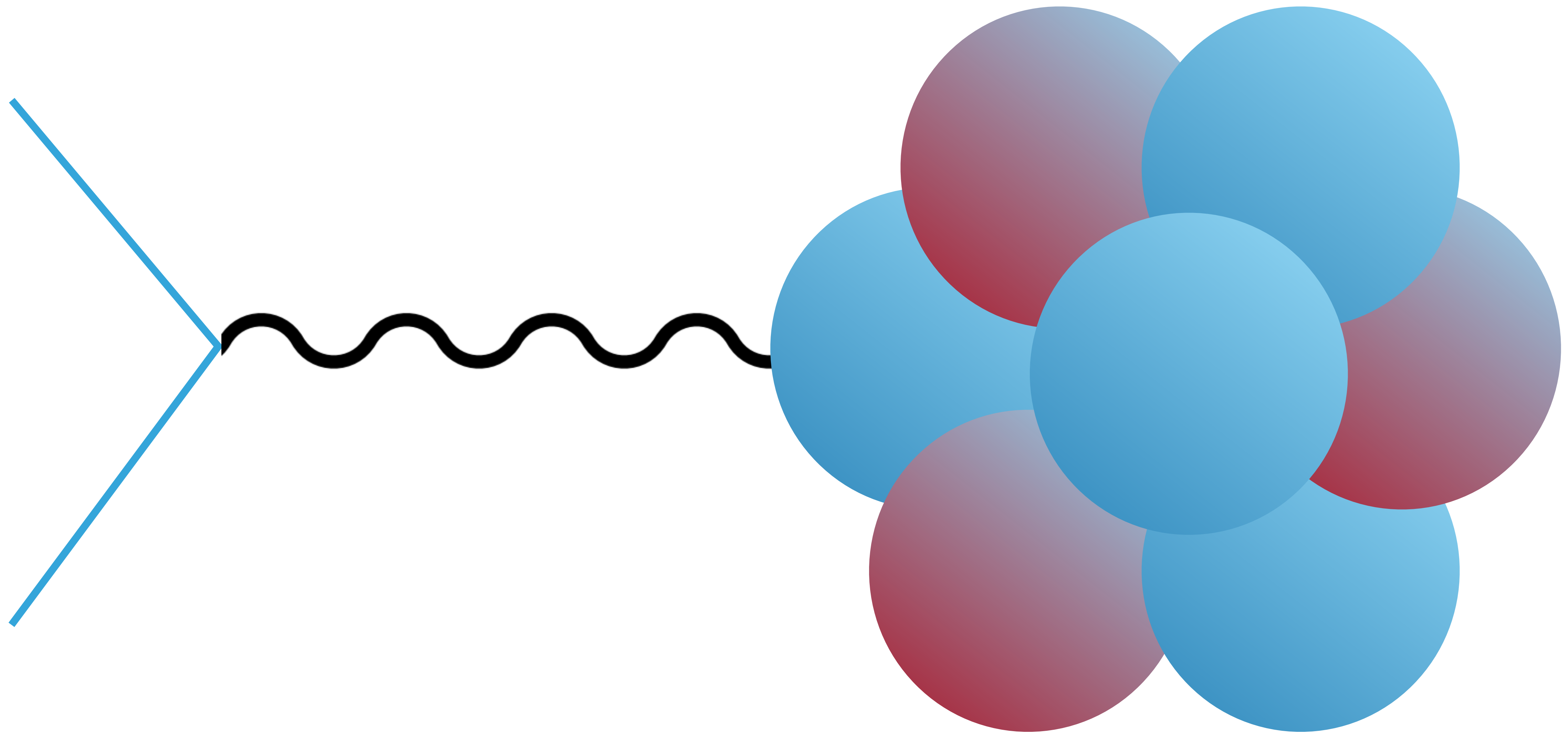
From Benhar, Day, and Sick's review paper "inclusive quasi-elastic electron-nucleus scattering", 2007, arXiv-0603029v2

Maybe not so unpractical anymore?

COULOMB CORRECTIONS AND THE DBWA

The DBWA amplitude goes like:

$$T^{if} = \langle \chi_f^-, \psi_f | H_{\rm int} | \chi_i^+, \psi_i \rangle$$



Traini, Turck-Chieze, Zghiche, Phys. Rev. C38

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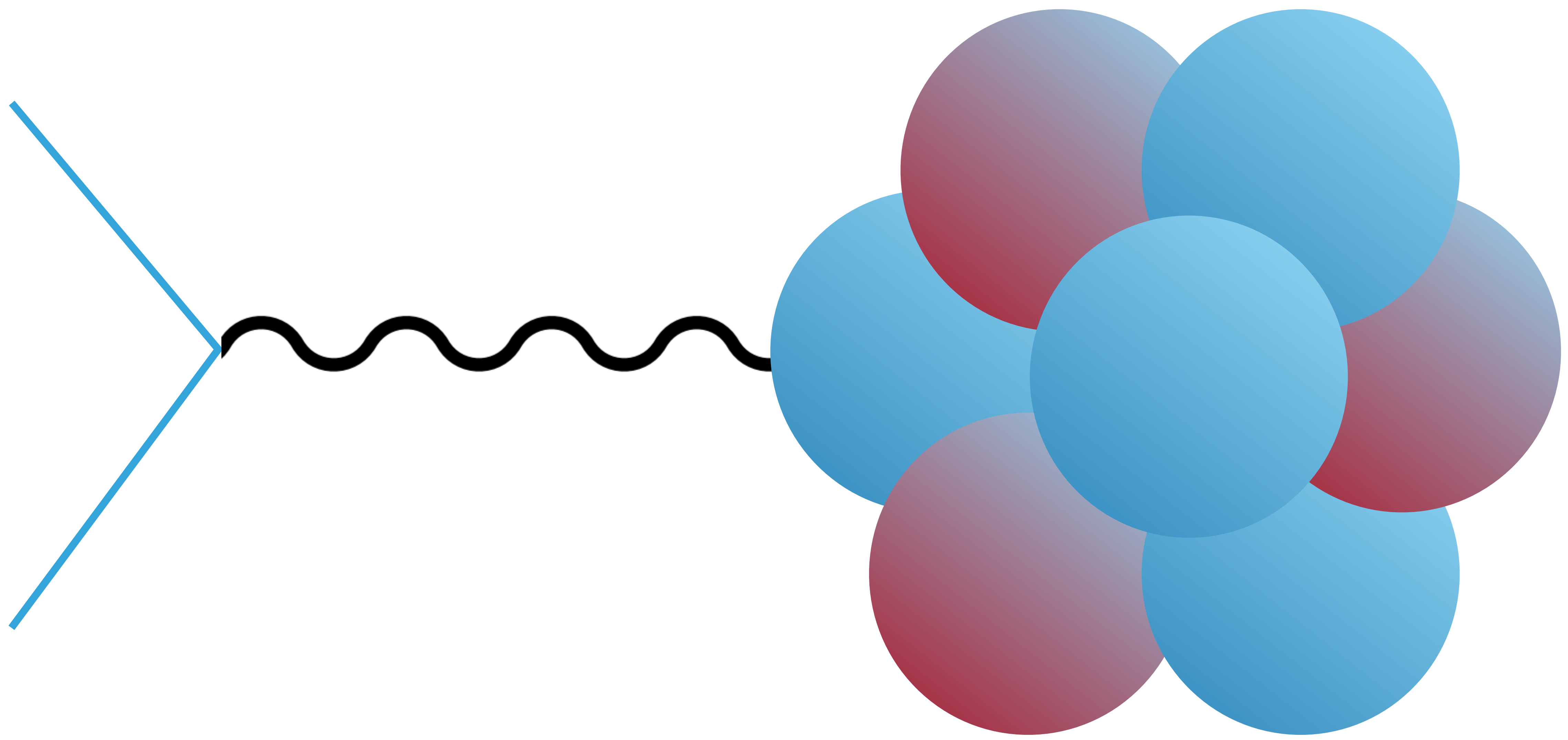
Initial and final nucleus states



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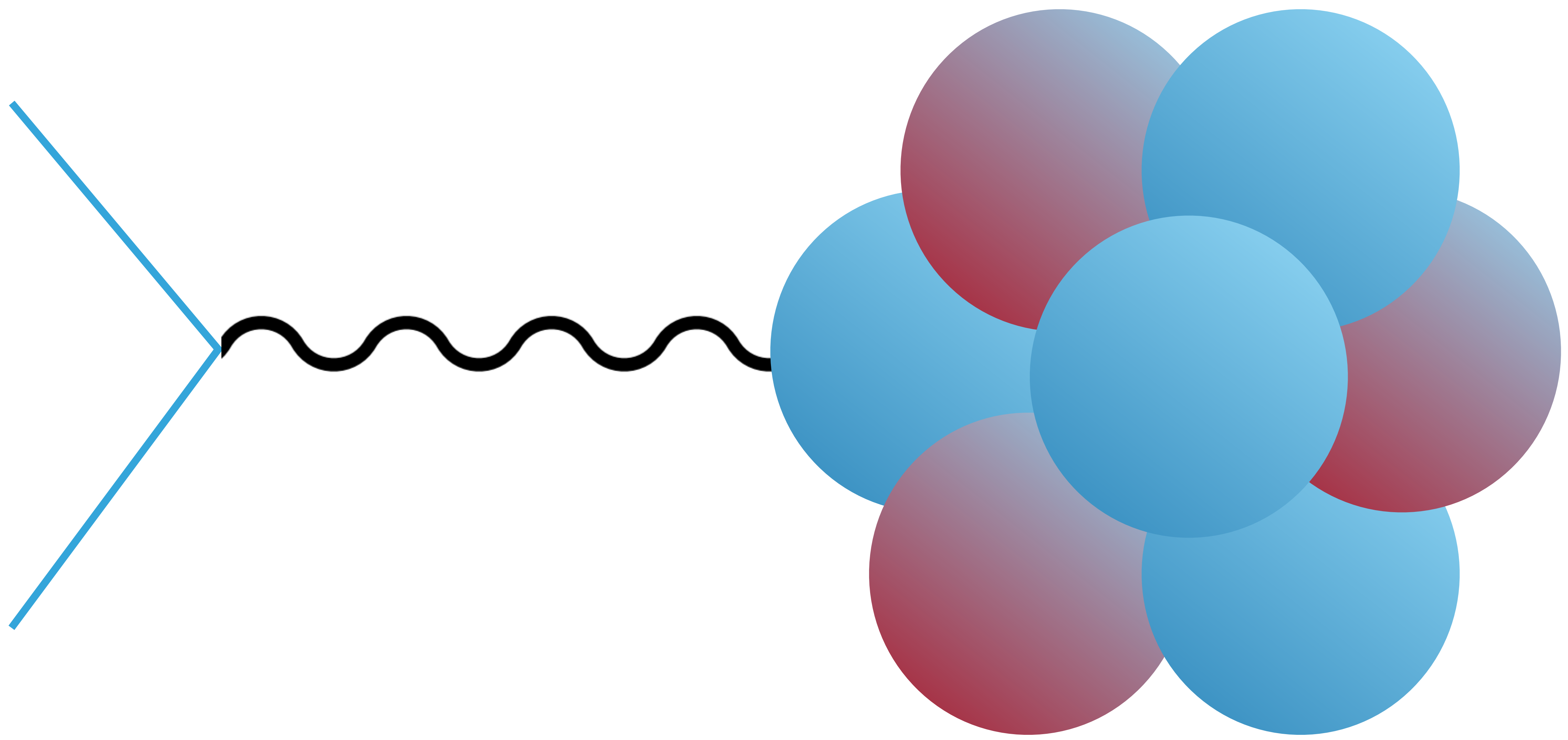
Solution of Dirac equation for incoming and outgoing electron in a static Coulomb field



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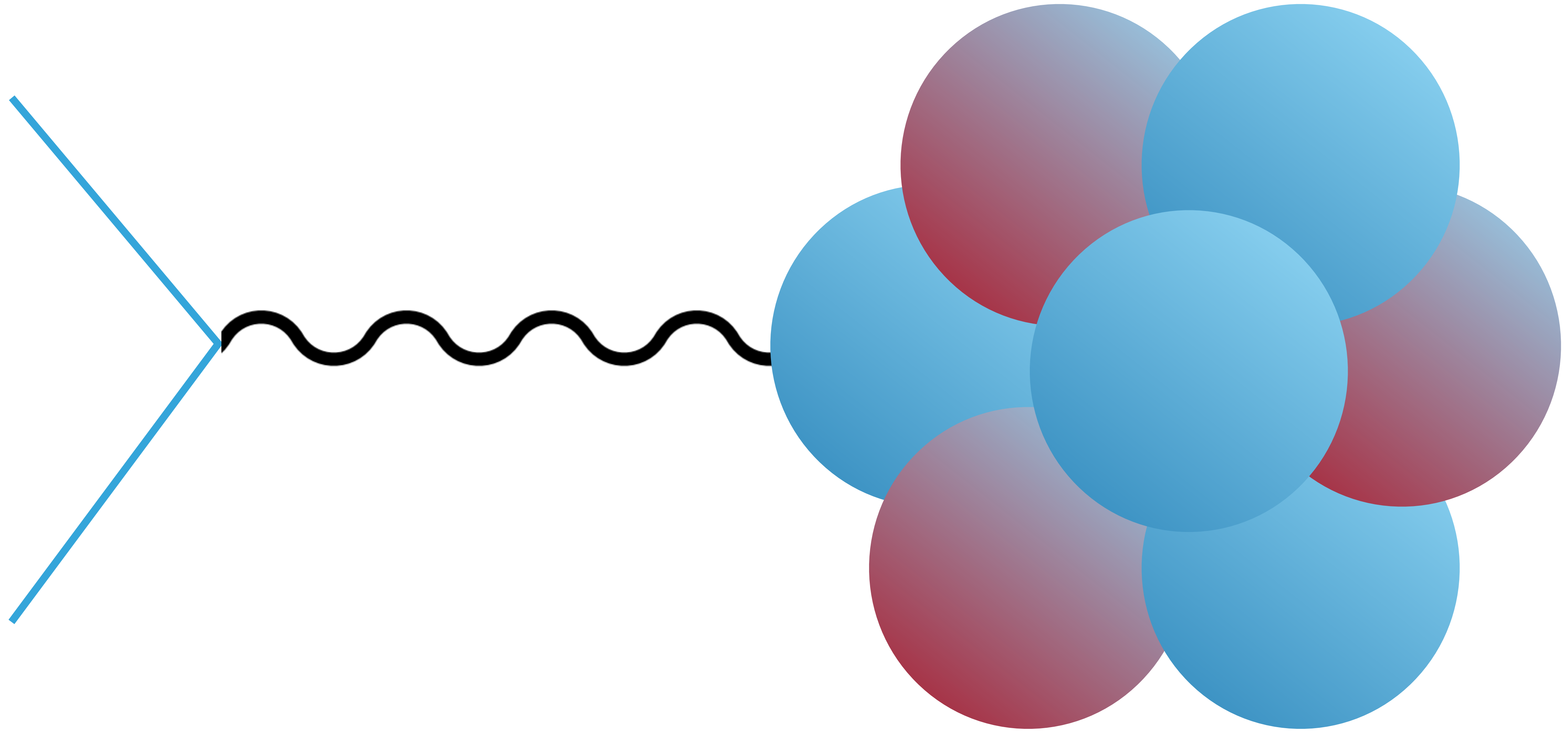
$$= \int d\mathbf{r} \left[\rho_e(\mathbf{r}) \phi_N^{if}(r) - \mathbf{j}_e(\mathbf{r}) \cdot \mathbf{A}_N^{if}(\mathbf{r}) \right]$$



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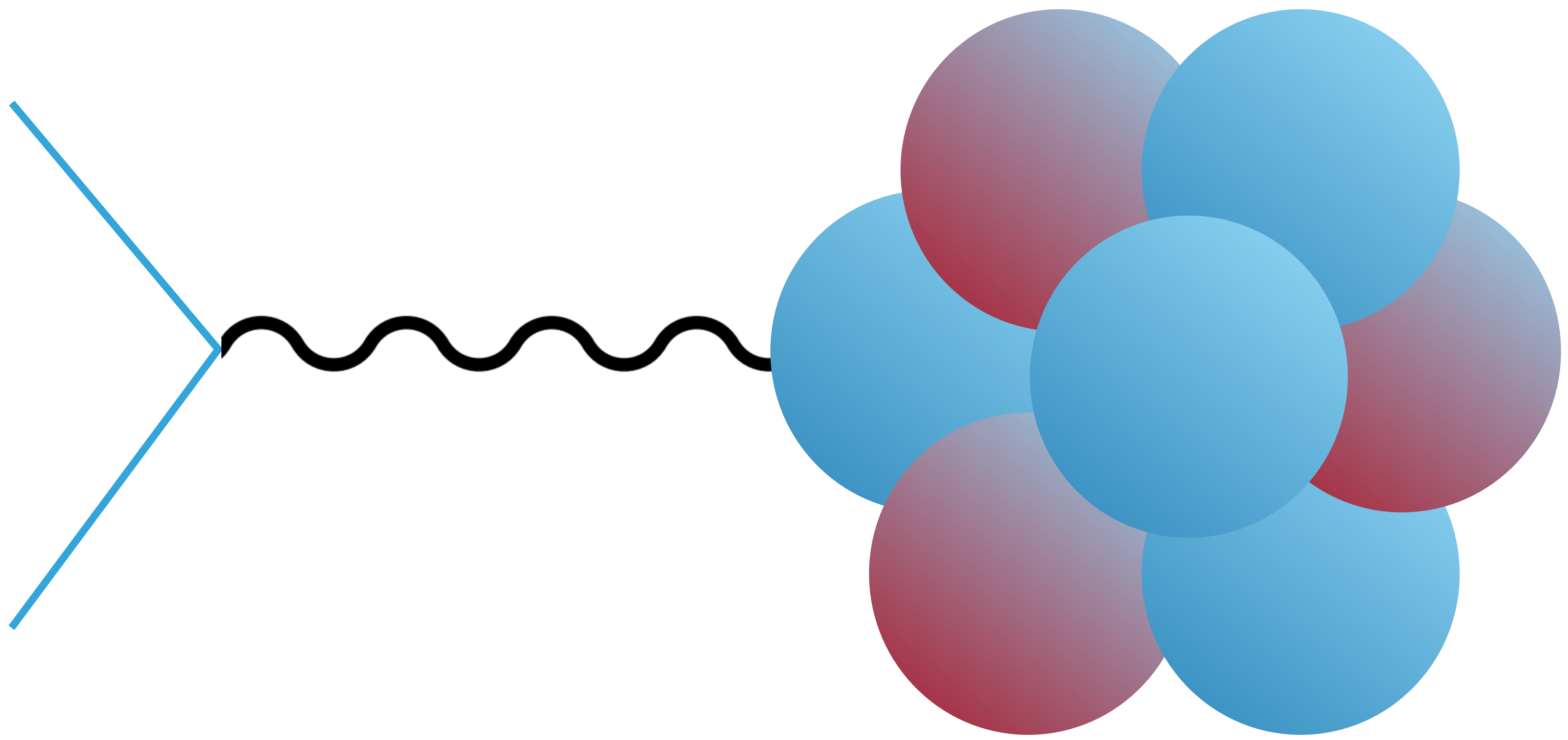
$$= \int d\mathbf{r} \left[\rho_e(\mathbf{r}) \phi_N^{if}(r) - \mathbf{j}_e(\mathbf{r}) \cdot \mathbf{A}_N^{if}(\mathbf{r}) \right]$$

Charge and current transition densities:

$$\rho_e(\mathbf{r}) = \chi_f^- * \chi_i^+, \quad \mathbf{j}_e(\mathbf{r}) = \chi_f^- * \boldsymbol{\sigma} \chi_i^+$$



Traini, Turck-Chieze, Zghiche, Phys. Rev. C38

COULOMB CORRECTIONS AND THE DBWA

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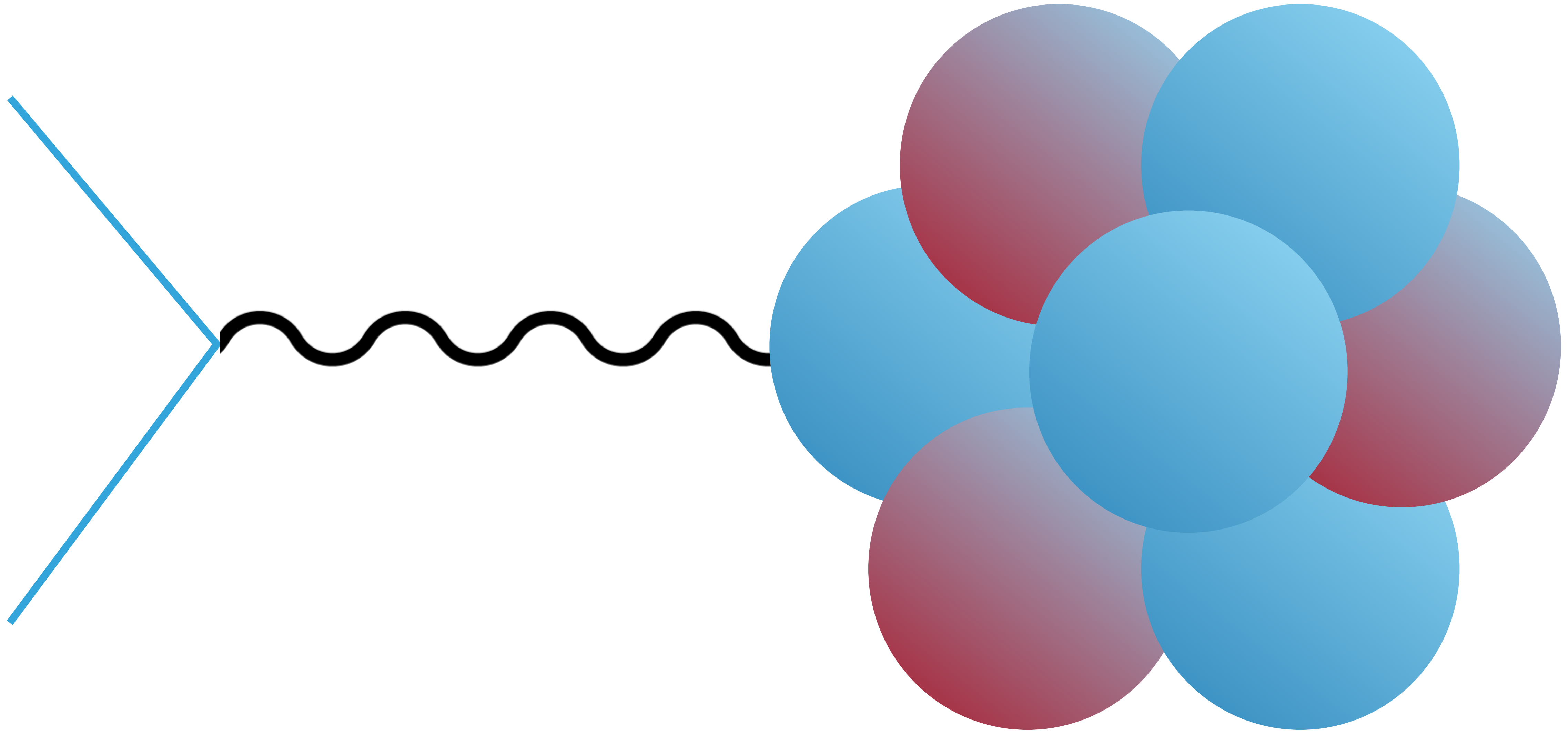
Vector and scalar potentials of the nuclear transition

$$\mathbf{A}_N^{if} = \langle \psi_f | \mathbf{A}_N | \psi_i \rangle, \quad \phi_N^{if} = \langle \psi_f | \phi_N | \psi_i \rangle$$



Traini, Turck-Chieze, Zghiche, Phys. Rev. C38

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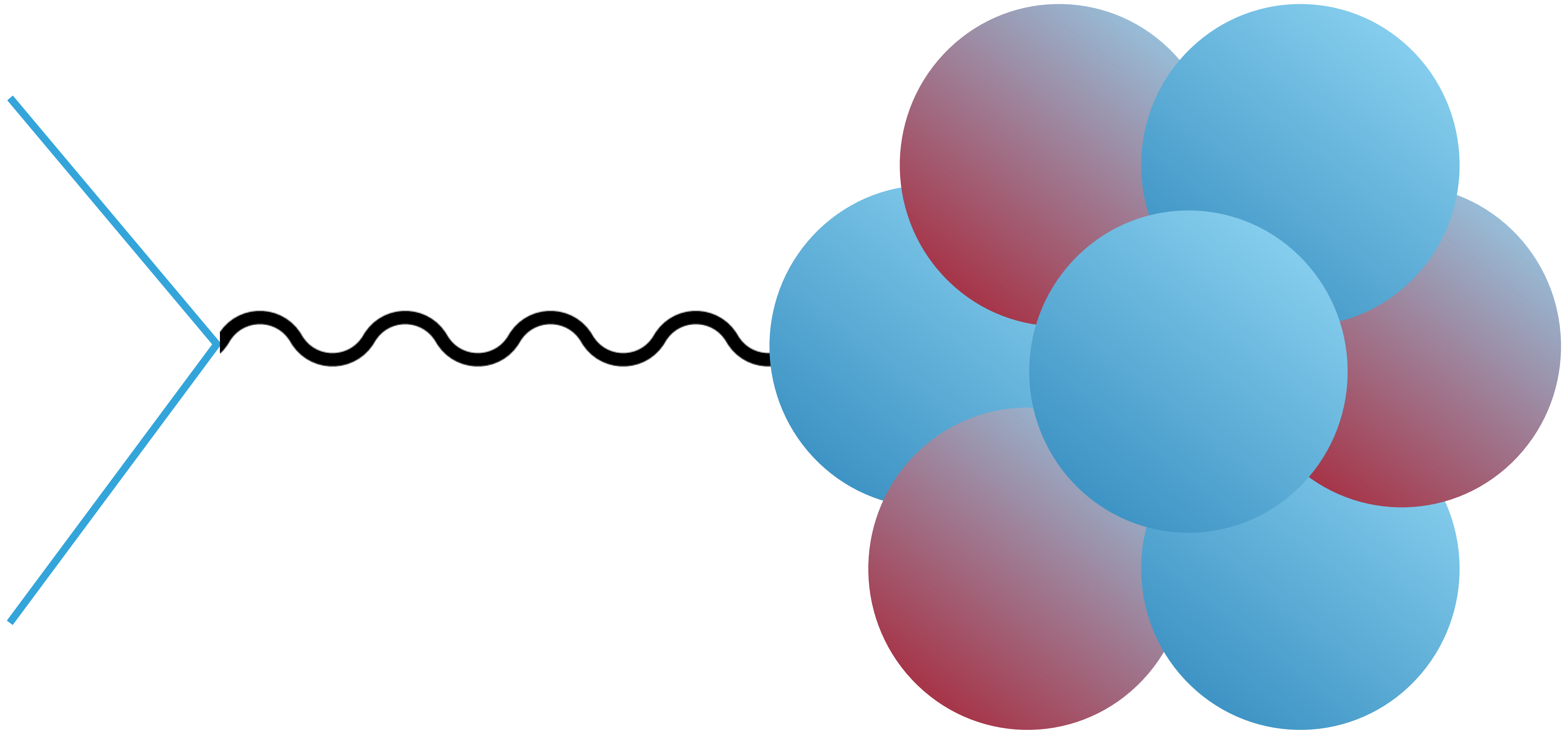


If we replace the exact solutions by plane-waves:

$$\rho_e^{\rm PWBA}(\mathbf{r}) = \rho_{e0}\, e^{i\mathbf{q}\cdot\mathbf{r}}, \quad \mathbf{j}_e^{\rm PWBA} = \mathbf{j}_{e0}\, e^{i\mathbf{q}\cdot\mathbf{r}}$$

With

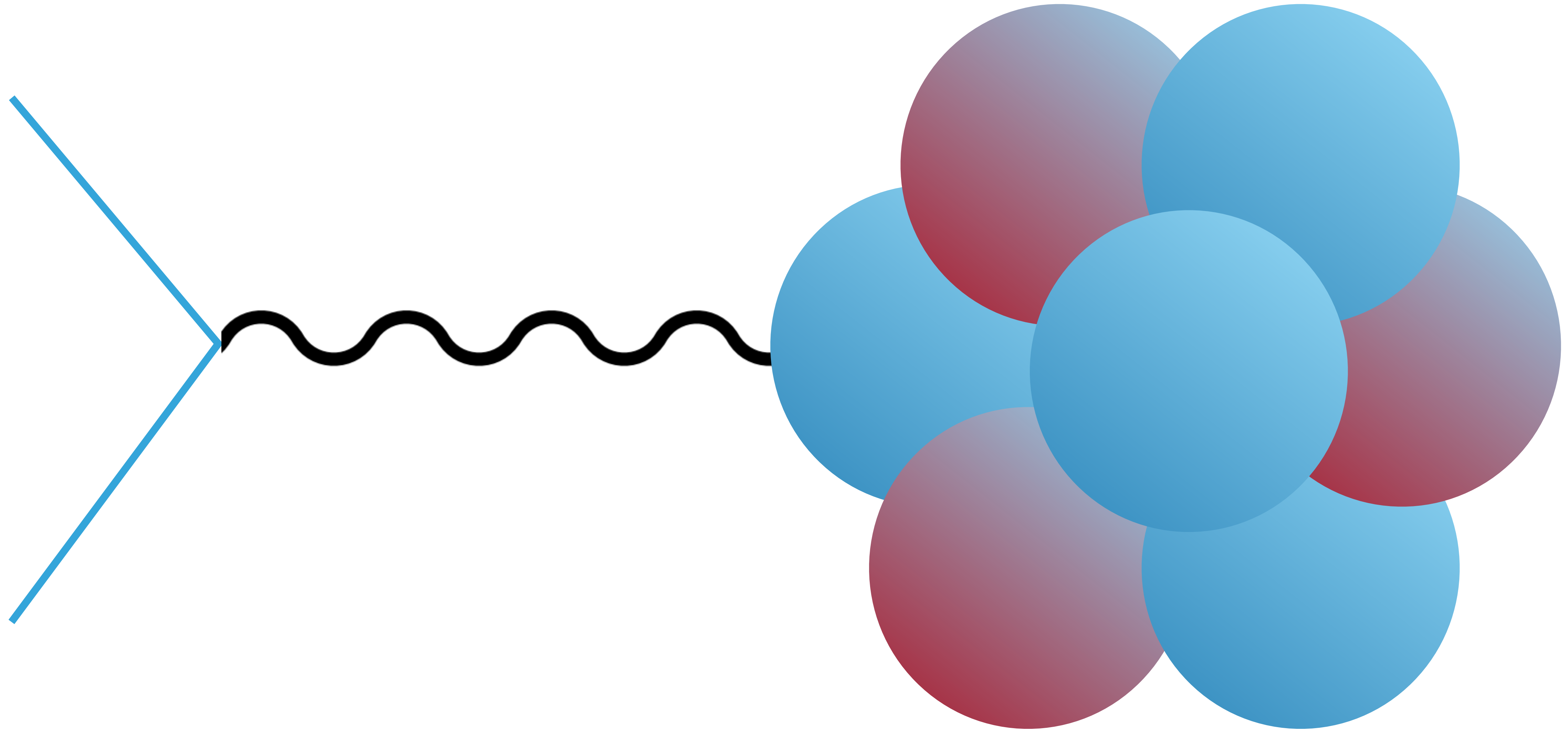
$$\rho_{e0} = u_f^+ u_i, \quad \mathbf{j}_{e0} = u_f^+ \boldsymbol{\sigma} u_i$$

And the amplitude can be written as the Fourier transform: $T^{if\,(\rm PWBA)} = \phi_N^{if}(\mathbf{q})\rho_{e0} - \mathbf{A}_N^{if}(\mathbf{q}) \cdot \mathbf{j}_{e0}$

Traini, Turck-Chieze, Zghiche, Phys. Rev. C38

COULOMB CORRECTIONS, HIGH ENERGY APPROXIMATION

In the high energy limit ($E >> m_e$), Lenz and Rosenfelder derived an analytical expression for the Dirac equation solution, by summing terms in inverse powers of the wave number up to second order:



$$\chi^{\pm}(\mathbf{r}) = e^{\pm i\delta_{1/2}}\eta(\mathbf{r})e^{\pm ib\left(\mathbf{J}^2 - 3/4\right)}e^{i\mathbf{k}\cdot\mathbf{r}\eta(\mathbf{r})}u_{\mathbf{k}}$$

With:

$$\mathbf{J} = \mathbf{L} + \frac{1}{2}\boldsymbol{\sigma} = \mathbf{r}\times\mathbf{p} + \frac{1}{2}\boldsymbol{\sigma} \qquad \eta(\mathbf{r}) = \frac{1}{\mathbf{k}\cdot\mathbf{r}}\int_0^r \left[k - V(\mathbf{r}')\right]d\mathbf{r}' \to \frac{k'}{k}\left(1 + ar^2\right)$$

$$a = -\frac{1}{6k'}\left\langle\frac{d^2V}{dr^2}\right\rangle\Bigg|_{r=0} \cong -\frac{1}{6k'}\frac{Z\alpha}{R^3} \equiv \frac{\tilde{a}}{k'} \qquad b = -\frac{Z\alpha}{4k'^2}\left\langle\frac{1}{r^2}\right\rangle \cong -\frac{3}{4}\frac{Z\alpha}{k'^2R^2} \equiv \frac{\tilde{b}}{k'^2} \qquad k' = k - V(0) \cong k + \frac{3}{2}\frac{Z\alpha}{R}$$

Traini, Turck-Chieze, Zghiche: Phys. Rev. C38 Lenz, Rosenfelder: Nucl. Phys. A176

COULOMB CORRECTIONS AND THE EMA

In the high energy limit (E >> m_e), Lenz and Rosenfelder derived an analytical expression for the Dirac equation solution, by summing terms in inverse powers of the wave number up to second order:

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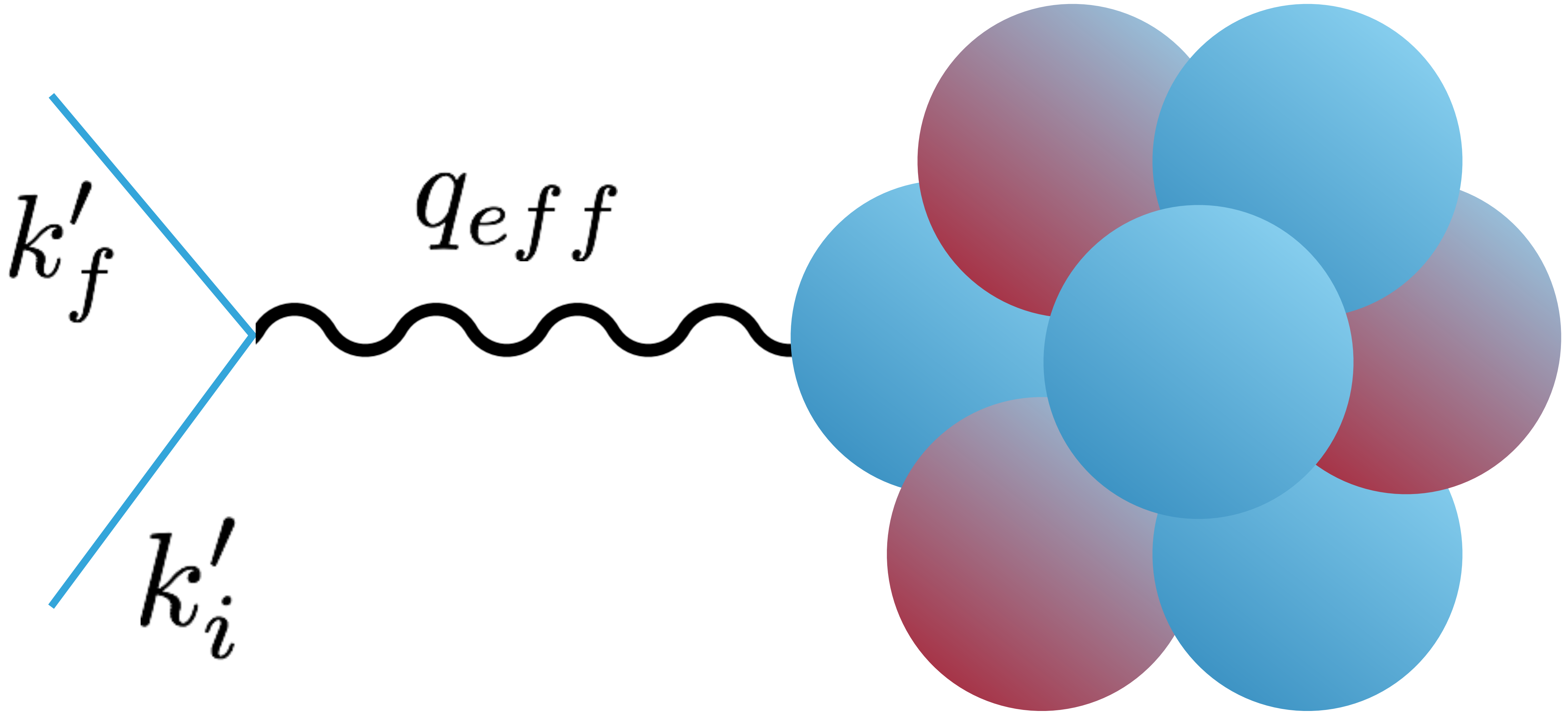


If one expands about r = 0, and keeps terms up to $(\alpha Z)^2$ one gets:

$$\chi^{\pm}(\mathbf{r})\Big|_{\widetilde{a}=\widetilde{b}=0} = e^{\pm i\delta_{1/2}}\,\frac{k'}{k}\,e^{i\mathbf{k}'\cdot\mathbf{r}}u_k$$

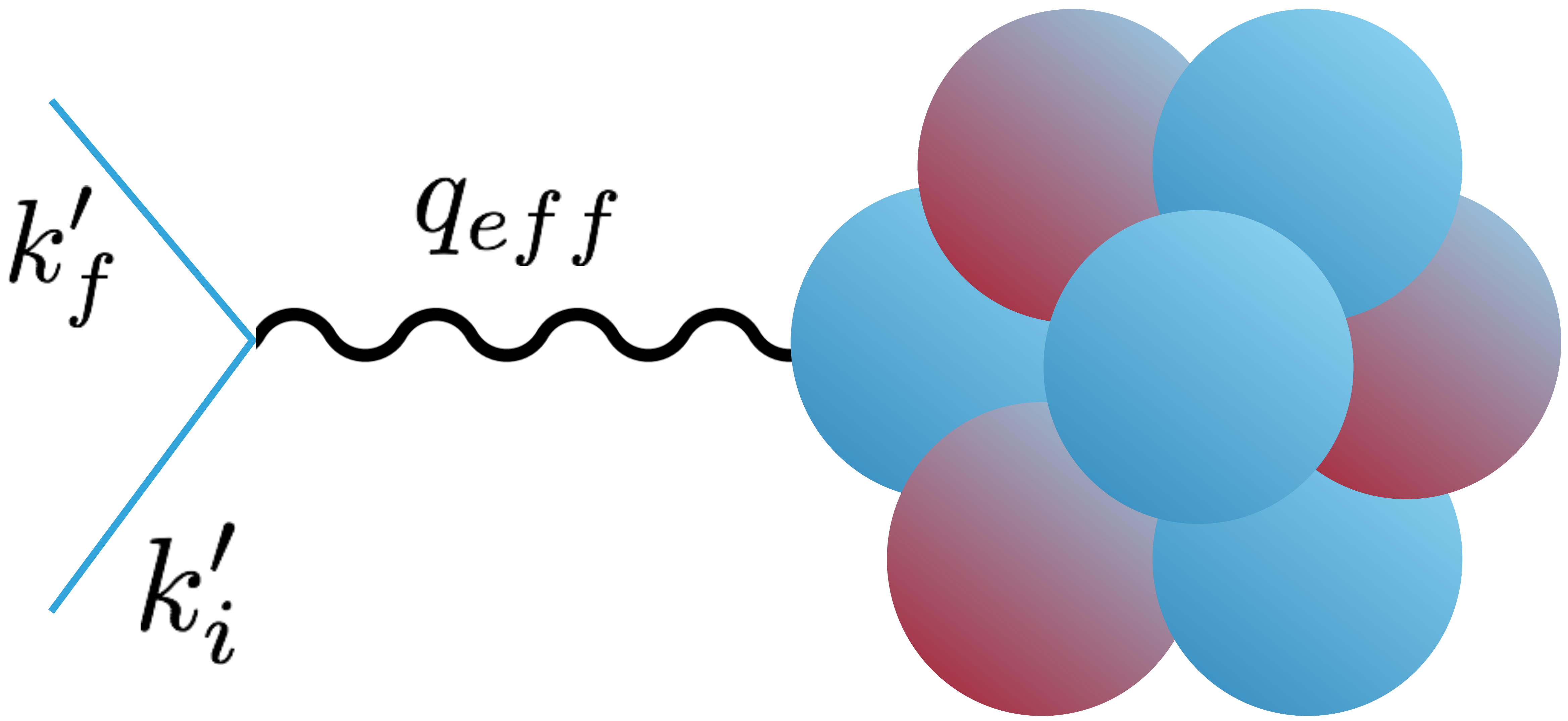
This gives us the basis of the EMA as a low-rider correction:

$$\mathbf{q}_{\text{eff}} = \mathbf{k}'_i - \mathbf{k}'_f = \mathbf{q} - V(0)\left(\hat{\mathbf{k}}_i - \hat{\mathbf{k}}_f\right)$$

Traini, Turck-Chieze, Zghiche: Phys. Rev. C38 Lenz, Rosenfelder: Nucl. Phys. A176

MEAN COULOMB POTENTIAL, EMA, AND POSITRON SCATTERING

- An effective momentum approximation (EMA) takes into account the mean field potential of the target nucleus during quasi-elastic scattering.



$$k'_i = k_i - \kappa_A \frac{V_0}{c} \qquad k'_f = k_f - \kappa_A \frac{V_0}{c}$$

$$V_0 = \frac{3\alpha Z}{2 r_c}$$

$$\omega' = (k'_i - k'_f) = (k_i - k_f) = \omega$$

$$Q'^2 = 4(k'_i)(k'_f)\sin^2\theta/2$$

$$q_{eff} = \sqrt{\omega^2 + Q'^2}$$

Nucleus	V_0 (MeV)
^{12}C	3.46 +/- 0.11
^{56}Fe	9.80 +/- 0.32
^{208}Pb	20.57 +/- 0.66

A. Aste, C. von Arx, and D. Trautmann, EPJ A26 167 (2005)

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$$k_i' = k_i - \boxed{\kappa_A}\frac{V_0}{c} \qquad k_f' = k_f - \boxed{\kappa_A}\frac{V_0}{c}$$

~ 0.66 to 0.8

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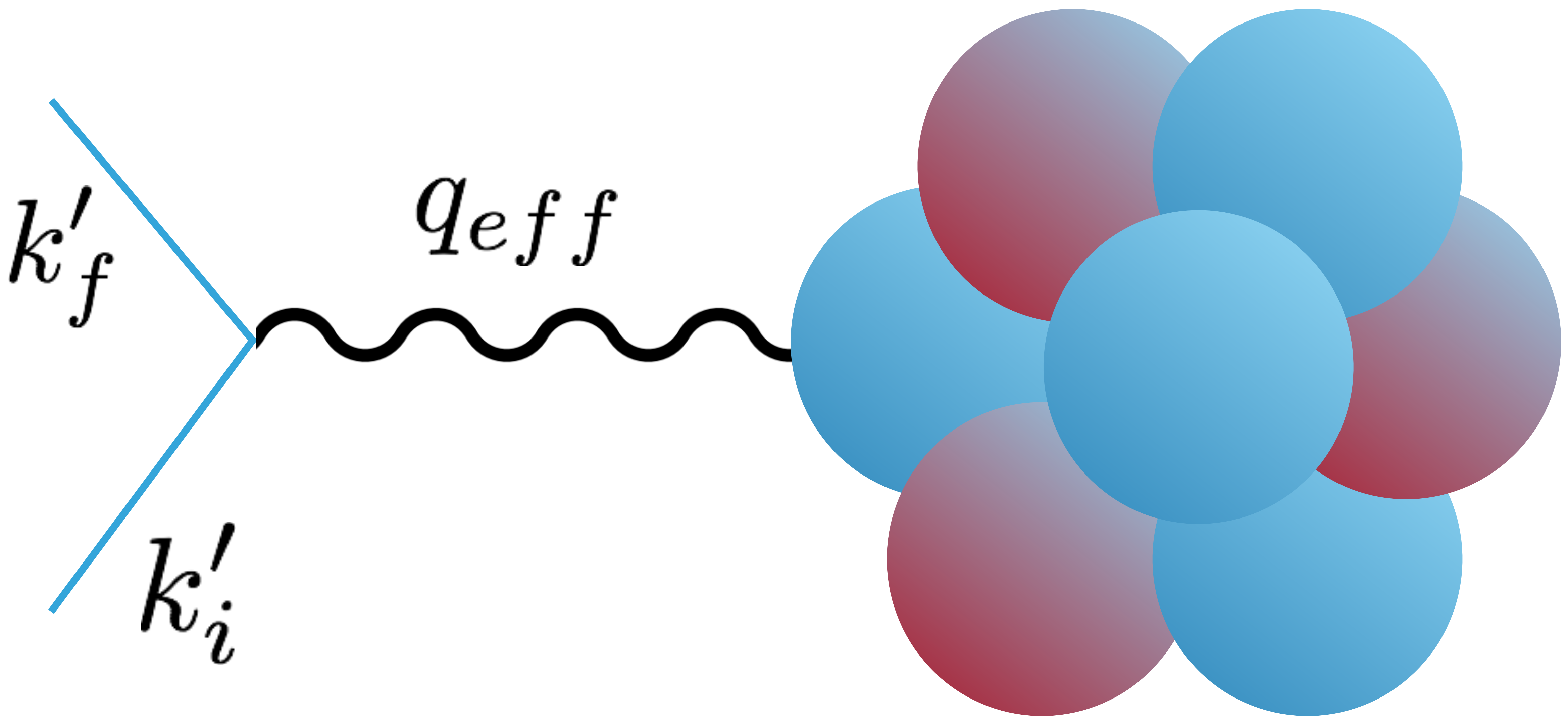

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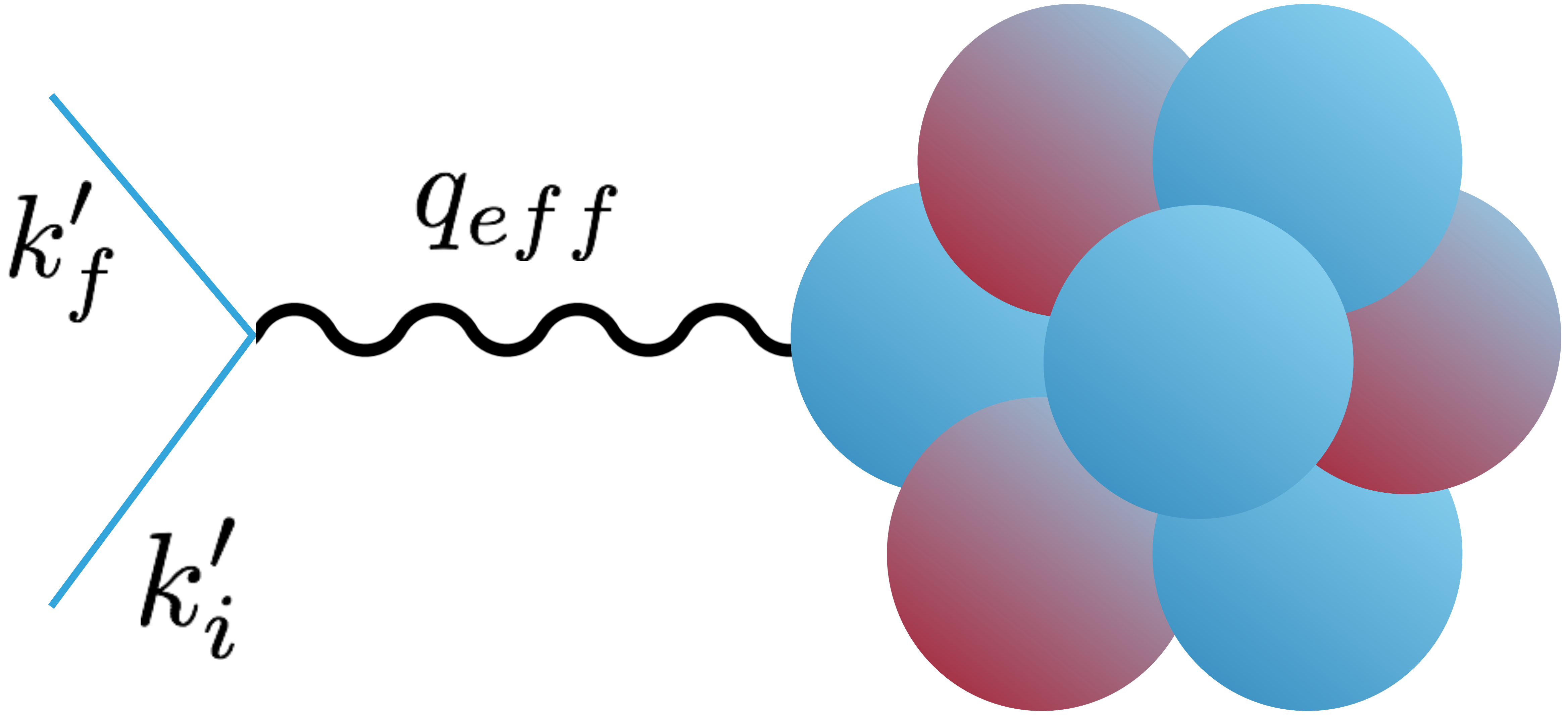

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When scattering with positrons, we effectively change the sign of the mean potential

A. Aste, C. von Arx, and D. Trautmann, EPJ A26 167 (2005)

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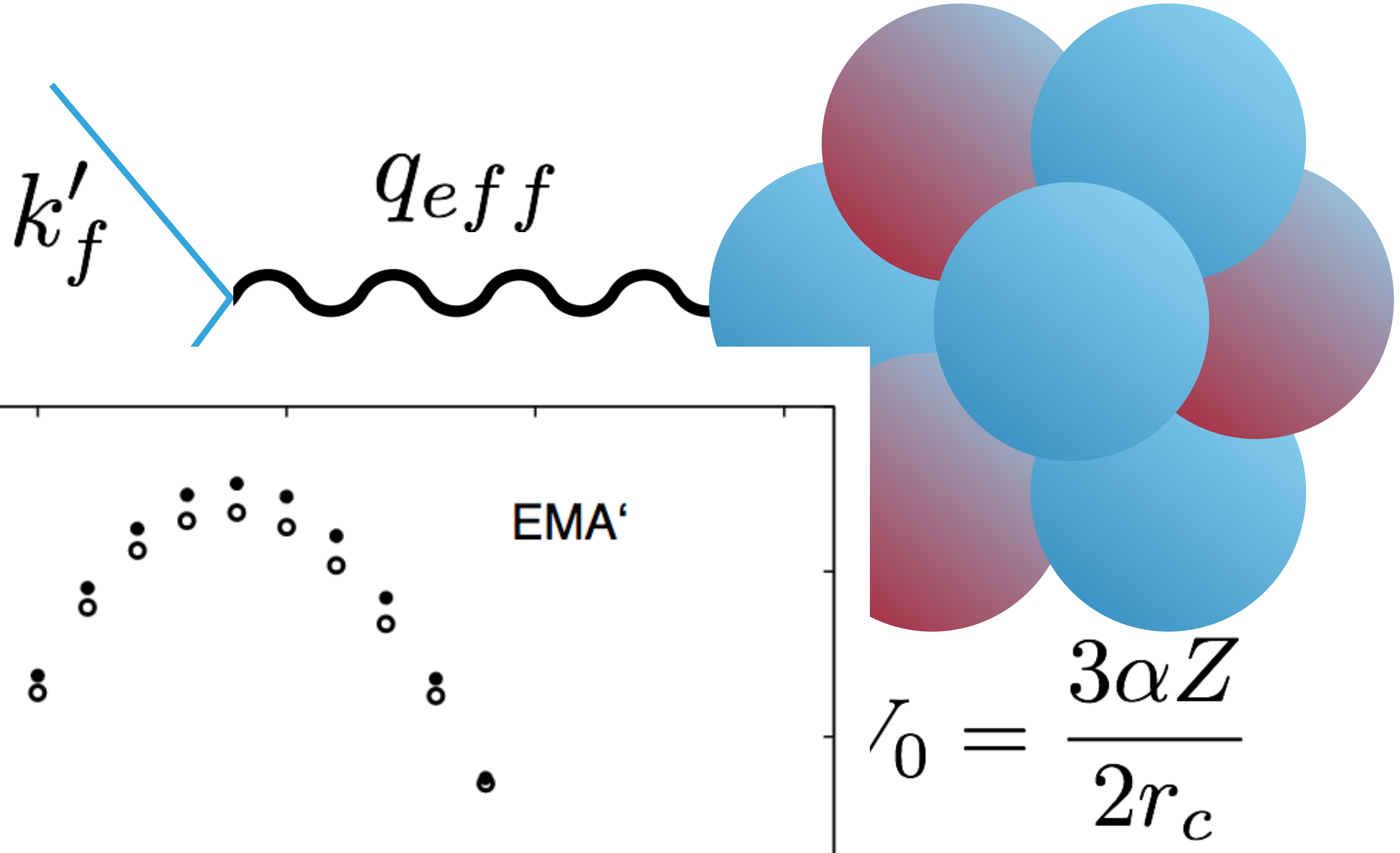
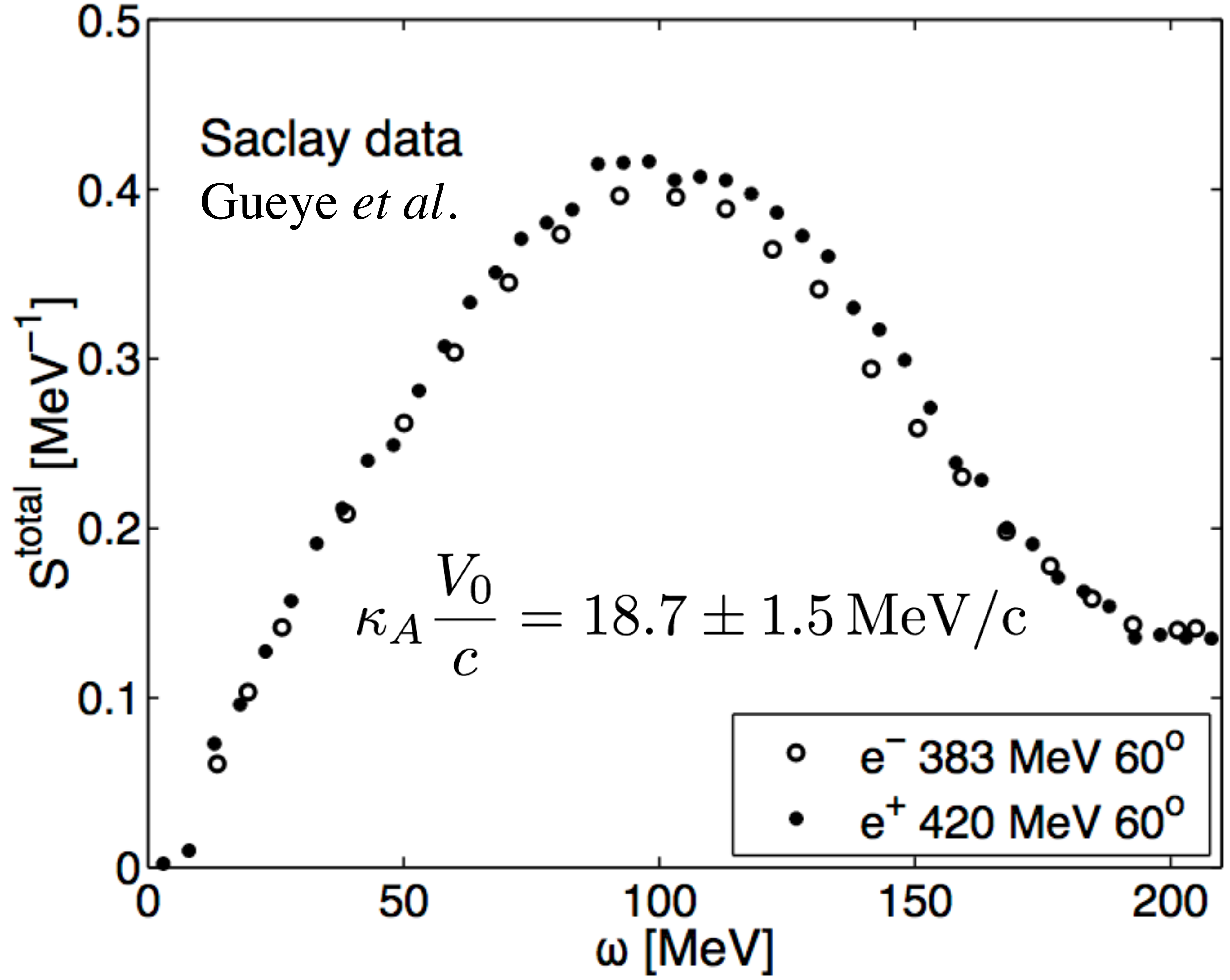
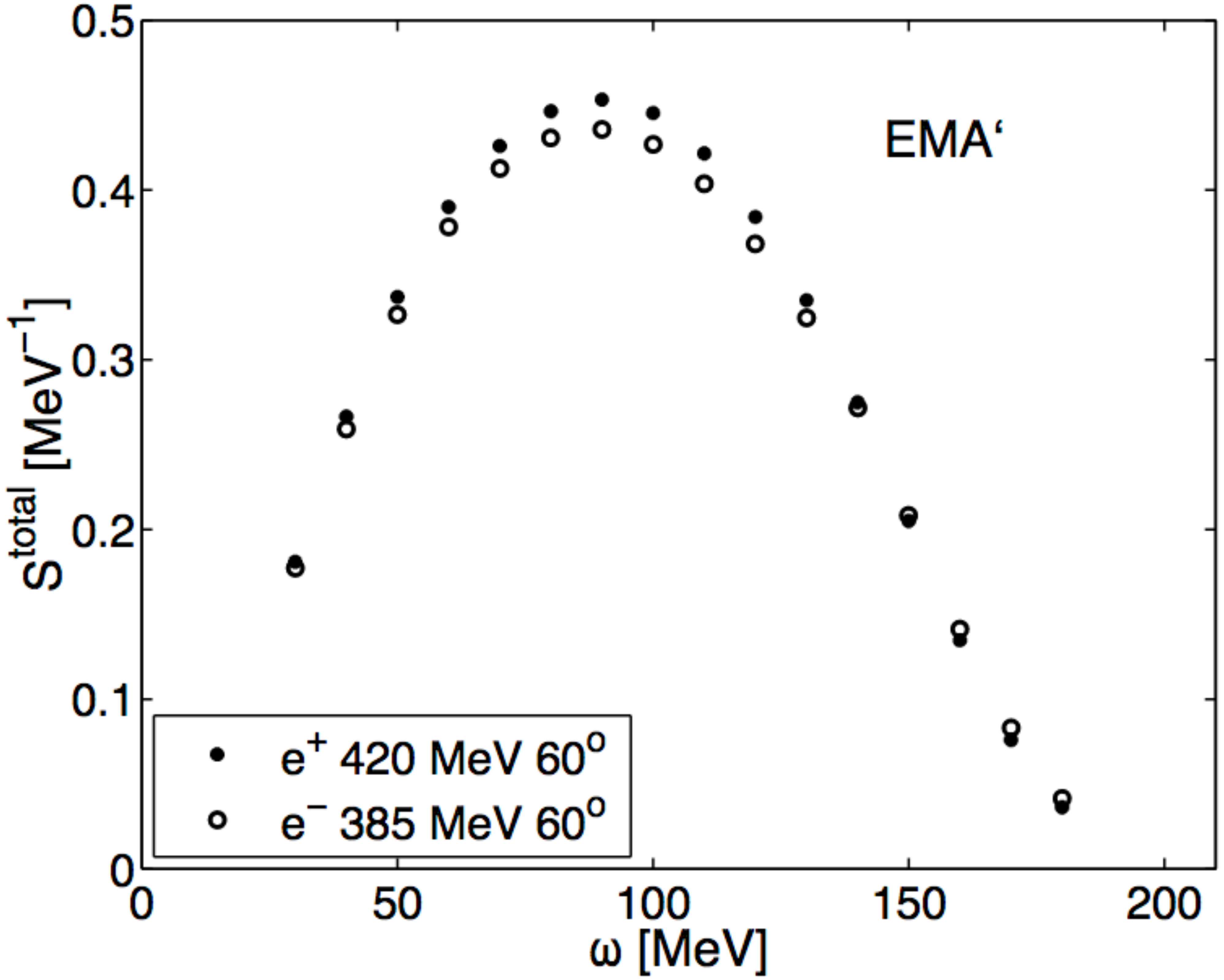
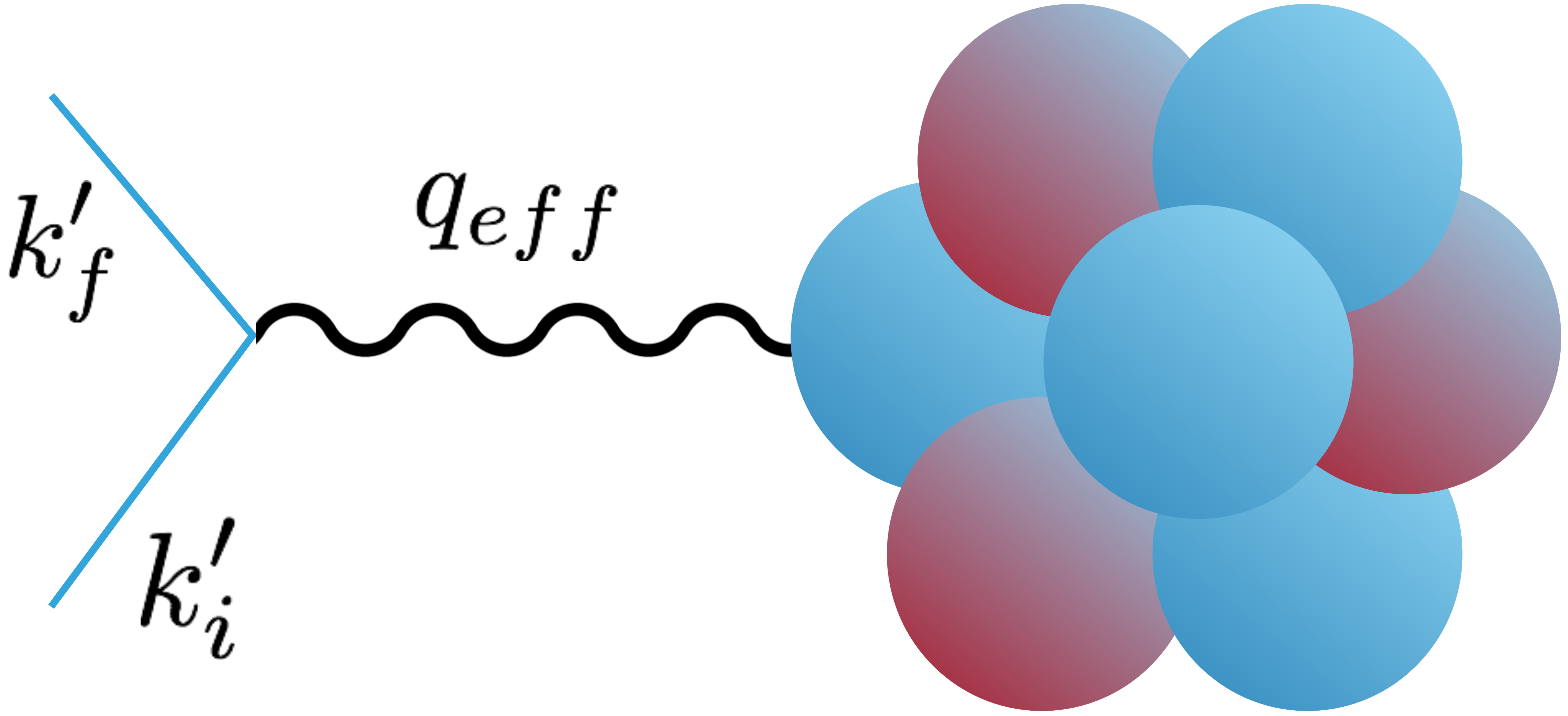






RESPONSE AND THE FOCUSING FACTOR

- Calculating the response Γ :
  - A focusing factor $f(k) = \dfrac{k_i'}{k_i}$ shows up in the amplitude.
  - When calculating the response $\Gamma = \dfrac{\sigma}{\sigma_{\text{Mott}}}$ it turns out the EMA focusing factor cancels out when also applying the corrected beam energy.
    - i.e. $\left(\dfrac{k_i'}{k_i}\right)^2 \sigma_{\text{Mott}}' = \sigma_{\text{Mott}}$
      - And $\Gamma' = \left(\dfrac{k_i'}{k_i}\right)^2 \dfrac{\sigma}{\sigma_{\text{Mott}}'} = \dfrac{\sigma}{\sigma_{\text{Mott}}}$



So, prior studies using the response and no coulomb corrections, according to EMA, only need the energy shift in q and no additional focusing factor.

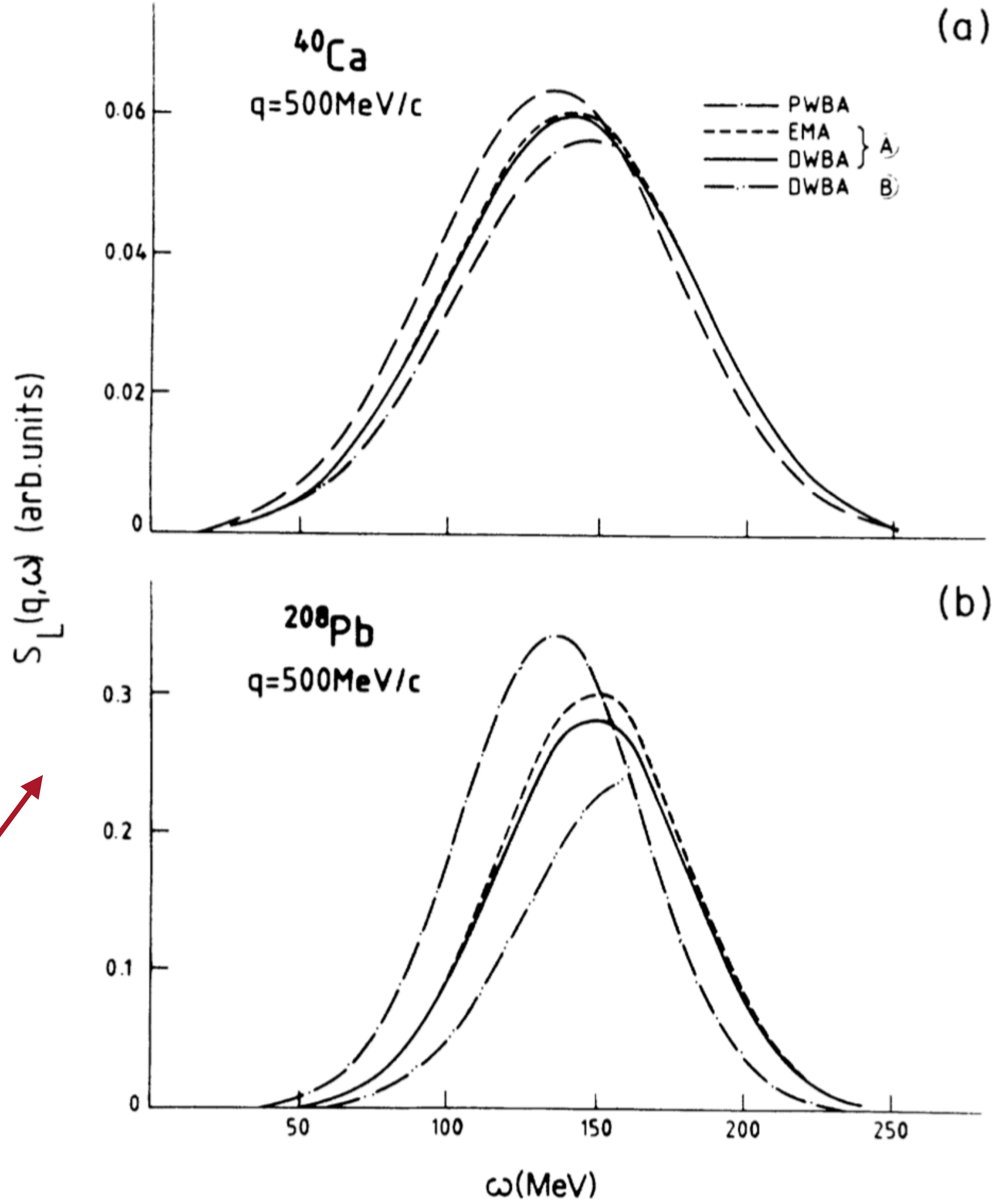
EXISTING CALCULATIONS AND LIMITATIONS

- A number of people have compared EMA to DWBA and PWBA calculations. Many have also created "modified" EMA's that get closer to DWBA calculations in problem regions.
  - The exact region where the EMA is a "good" approximation is only moderately discussed.
  - Most calculations focus on ^{208}Pb (where the EMA is most likely to fail)
- I'll go through some published calculations, but the is not an exhaustive list!!

CALCULATIONS IN THE LITERATURE

- Traini, Turck-Chieze, Zghiche:
- Compares a DWBA and PWBA calculations to EMA.
- Conclusions: "For medium weight nuclei (Z <=20) the main effect of Coulomb corrections in (e,e') inclusive scattering, can be embodied in an effective momentum transfer. These effects… are trivial in the sense that they do not need a complete distorted calculation. In heavier nuclei both effective momentum-transfer corrections and focusing effects are important. In that case DWBA are needed… The analytic approach here discussed allows an evaluation of L-T interference… Such extra terms can be neglected in a separation procedure of longitudinal and transverse components only if the kinematical region is restricted up to 140 deg. "

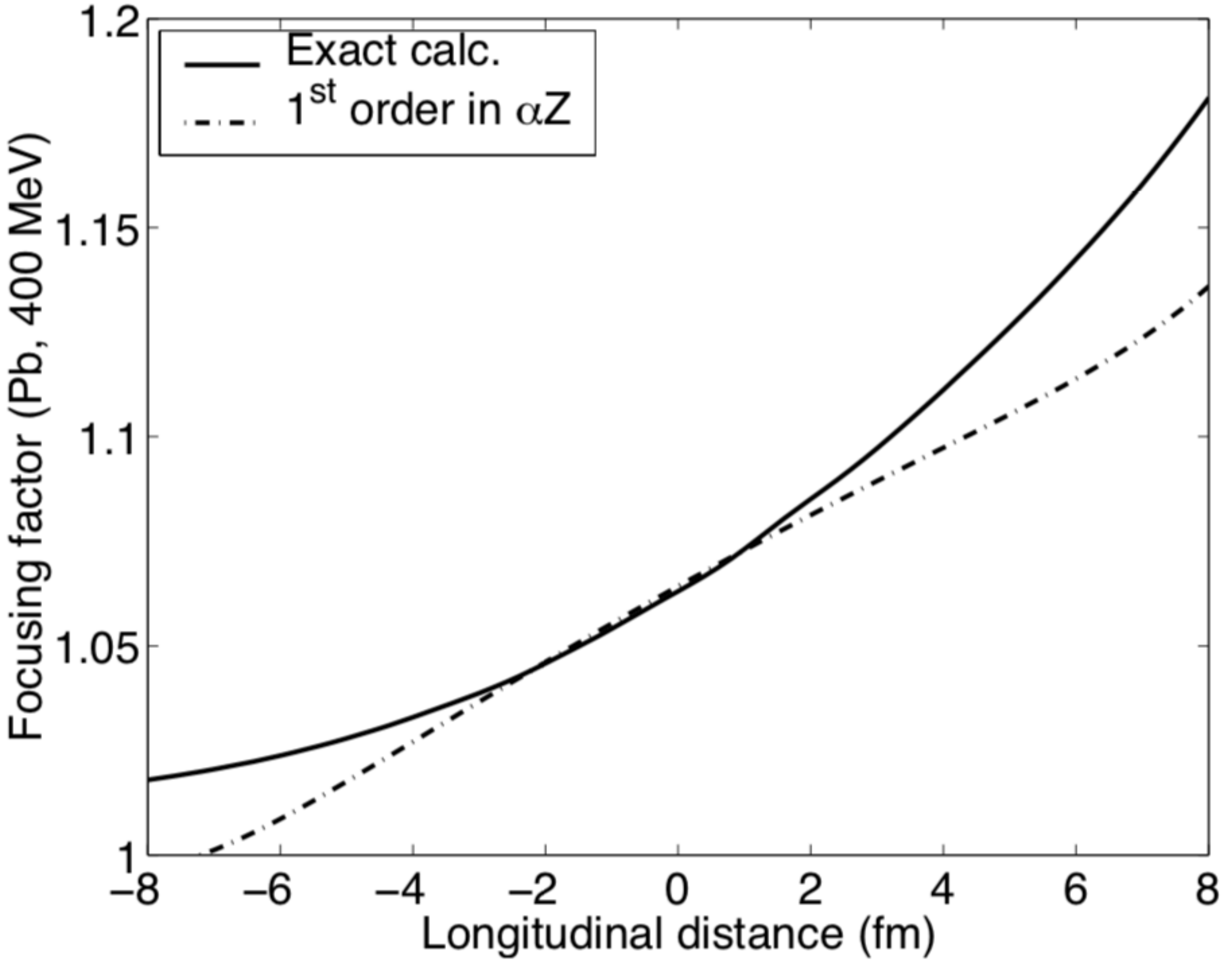
Longitudinal Responsevs. Energy Transfer (ω) in MeV for calcium and lead at q = 500 MeV/c and E = 553 MeV, theta = 60 deg (A) and at E = 331 MeV, theta = 150 deg (B).



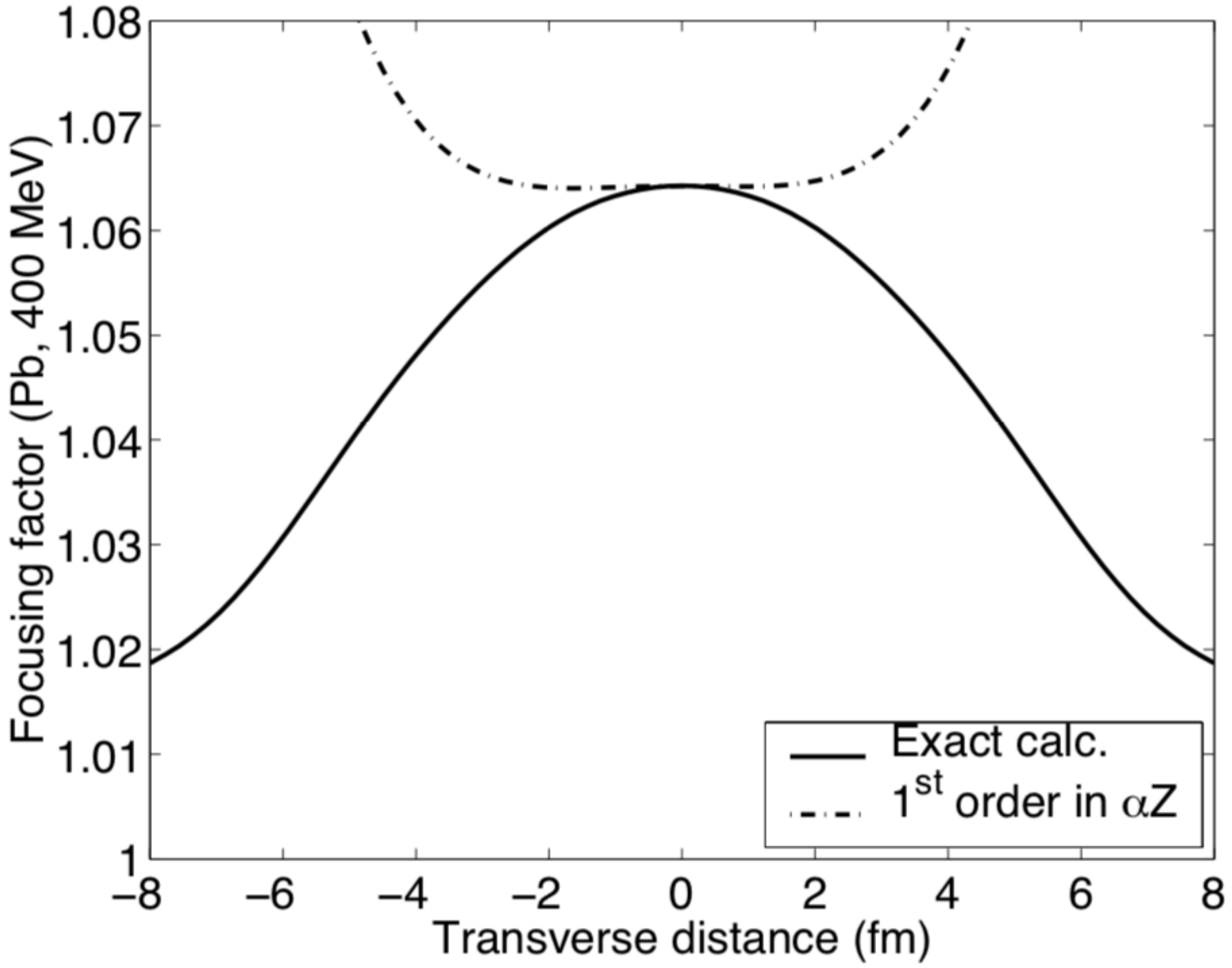
Traini, Turck-Chieze, Zghiche, Phys. Rev. C38 (1988)

CALCULATIONS IN THE LITERATURE

- A. Aste, C. von Arx, and D. Trautmann:
  - Compares focusing effects between exact DWBA calculations and EMA approximations derived from first order expansion.
  - Conclusions: "Naive lowest order approximations in αZ are not suitable for the analysis of Coulomb corrections in scattering experiments, unless they are modified in a well-controlled manner based on exact calculations… It turns out that the effective momentum approximation is not reliable when the central potential value V_0 of the electrostatic field of the nucleus is taken… but a smaller average value V_{avg} ~ (0.75…0.8)V_0 leads to very good results, if the momentum transfer and the energy of the scattered electron large enough… "



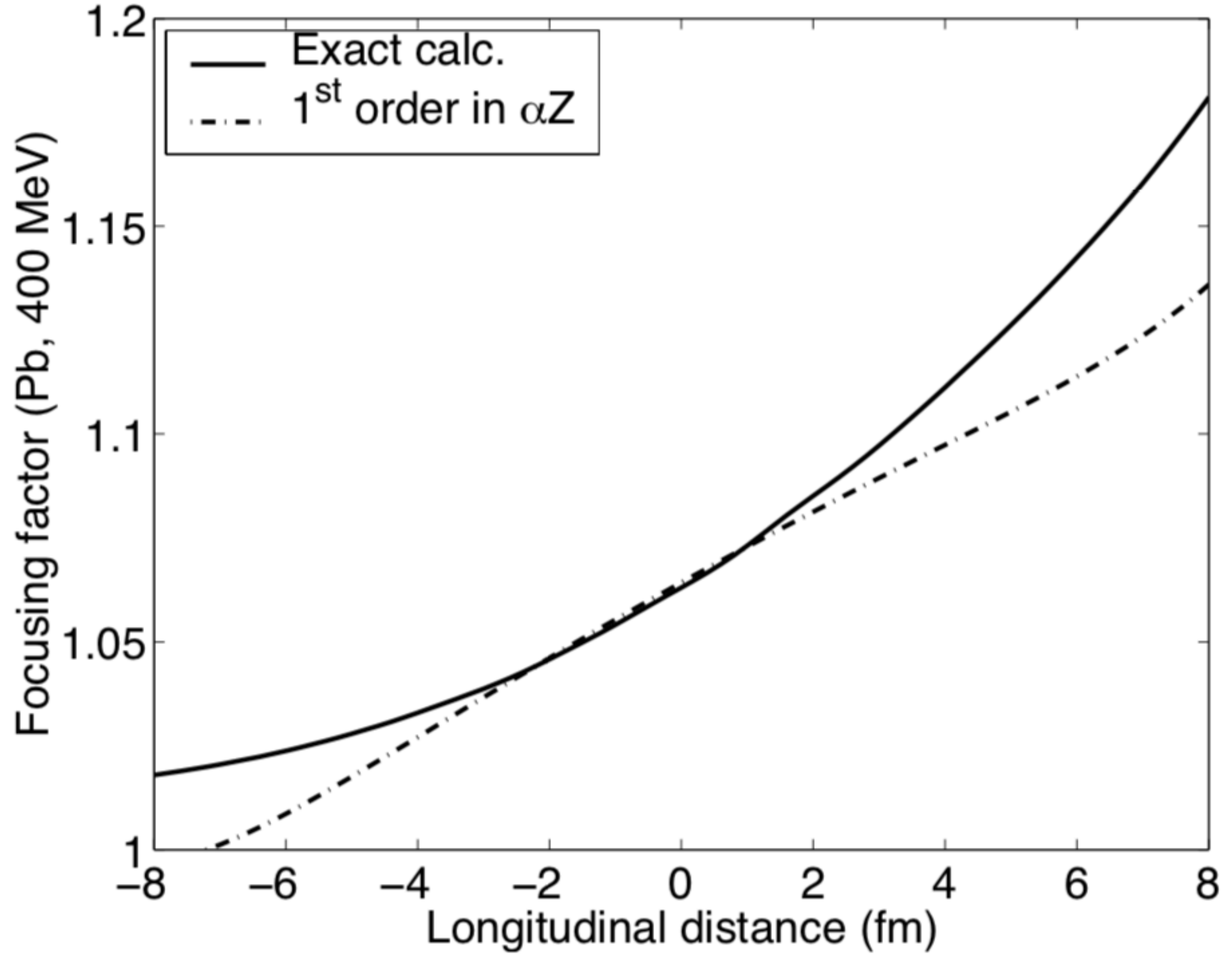
Focusing factor for exact calculation vs 1st order expansion as a function of the longitudinal and transverse scattering distance fo lead at 400 MeV.

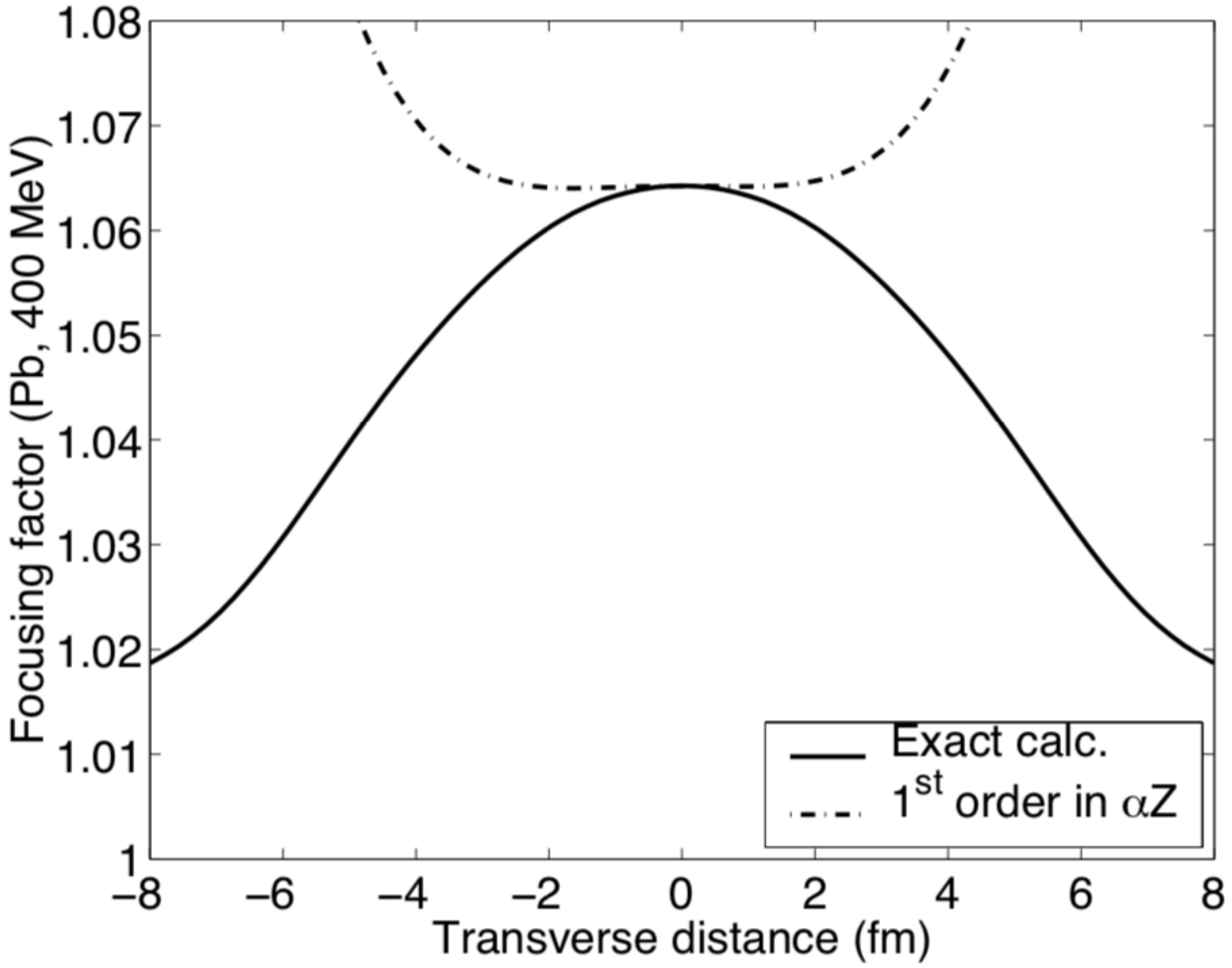


A. Aste, C. von Arx, and D. Trautmann, EPJ A26 167 (2005)

CALCULATIONS IN THE LITERATURE

- A. Aste, C. von Arx, and D. Trautmann:
  - Compares focusing effects between exact DWBA calculations and EMA approximations derived from first order expansion.
  - Conclusions cont: "... An effective potential value of 19 MeV is a very good choice for q ≥ 300 MeV and $Q^2 \geq (400\ \text{MeV})^2$... If the energy of the scattered electron becomes smaller than 300 MeV, the semiclassical description of the final state wave function becomes obsolete, but it is still possible to use an EMA-like approach for the description of the inclusive cross section by using a modified fitted potential value, given the condition that the initial and final energy of the electron and the momentum transfer are not too small."





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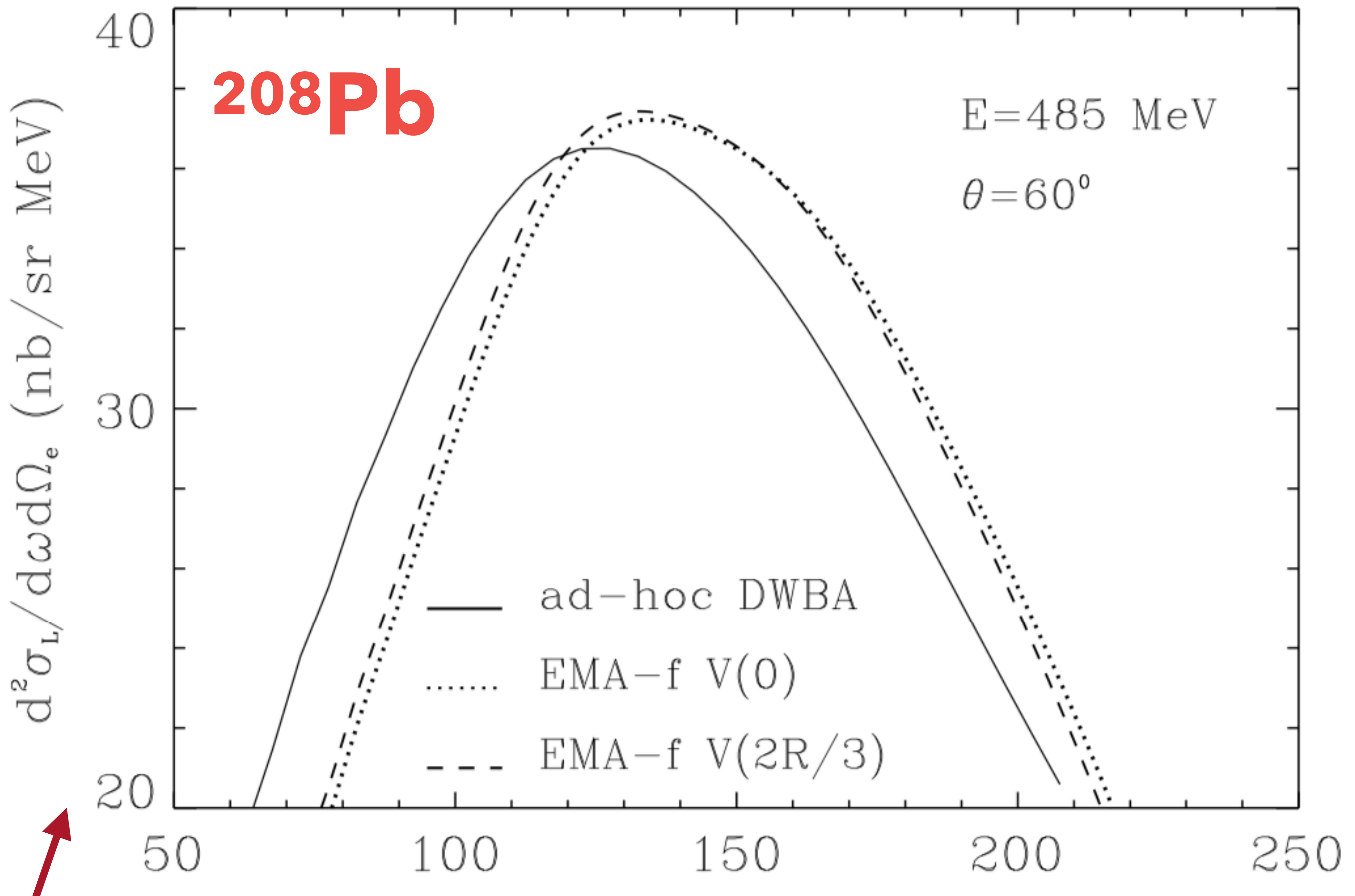
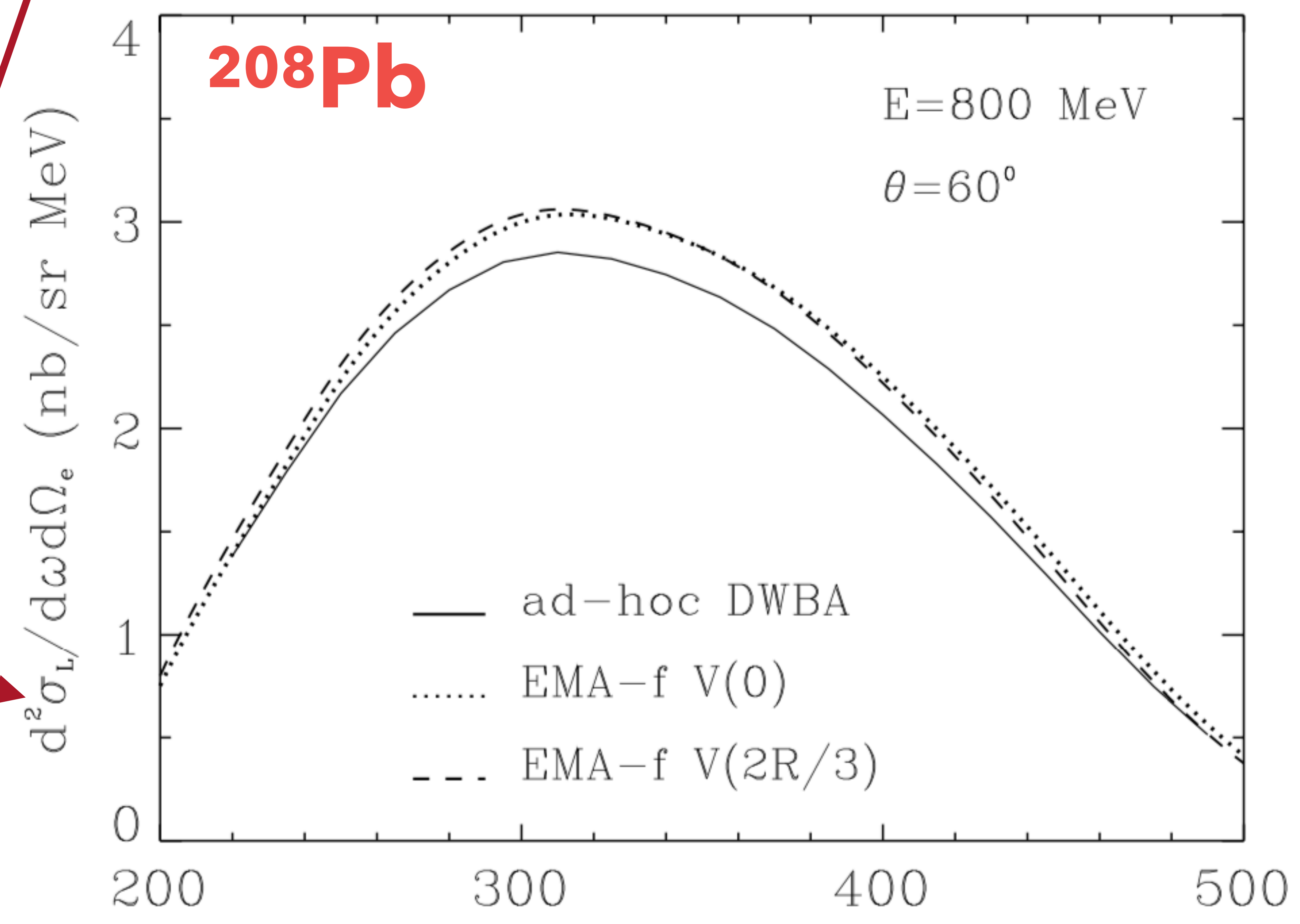
A. Aste, C. von Arx, and D. Trautmann, EPJ A26 167 (2005)

CALCULATIONS IN THE LITERATURE

- Kim and Wright:
  - Compares a focused EMA-f (with potential calculated at r=0, or integrated over the radius) with an ad-hoc DWBA.
  - Conclusions: "In conclusion, we have shown that the effective momentum approximation (using the Coulomb potential at 2R/3) with an overall focusing factor of $(p_i(0)'/p_i)^2$ is a very good approximation of the Coulomb distortion effects for the transverse contributions to the quasi elastic cross section. However, for electron energies less than about 600 MeV it is not a good approximation of the longitudinal contributions."





Longitudinal Cross Section vs. Energy Transfer (ω) in MeV for lead at 60 deg and 485 and 800 MeV beam energy.

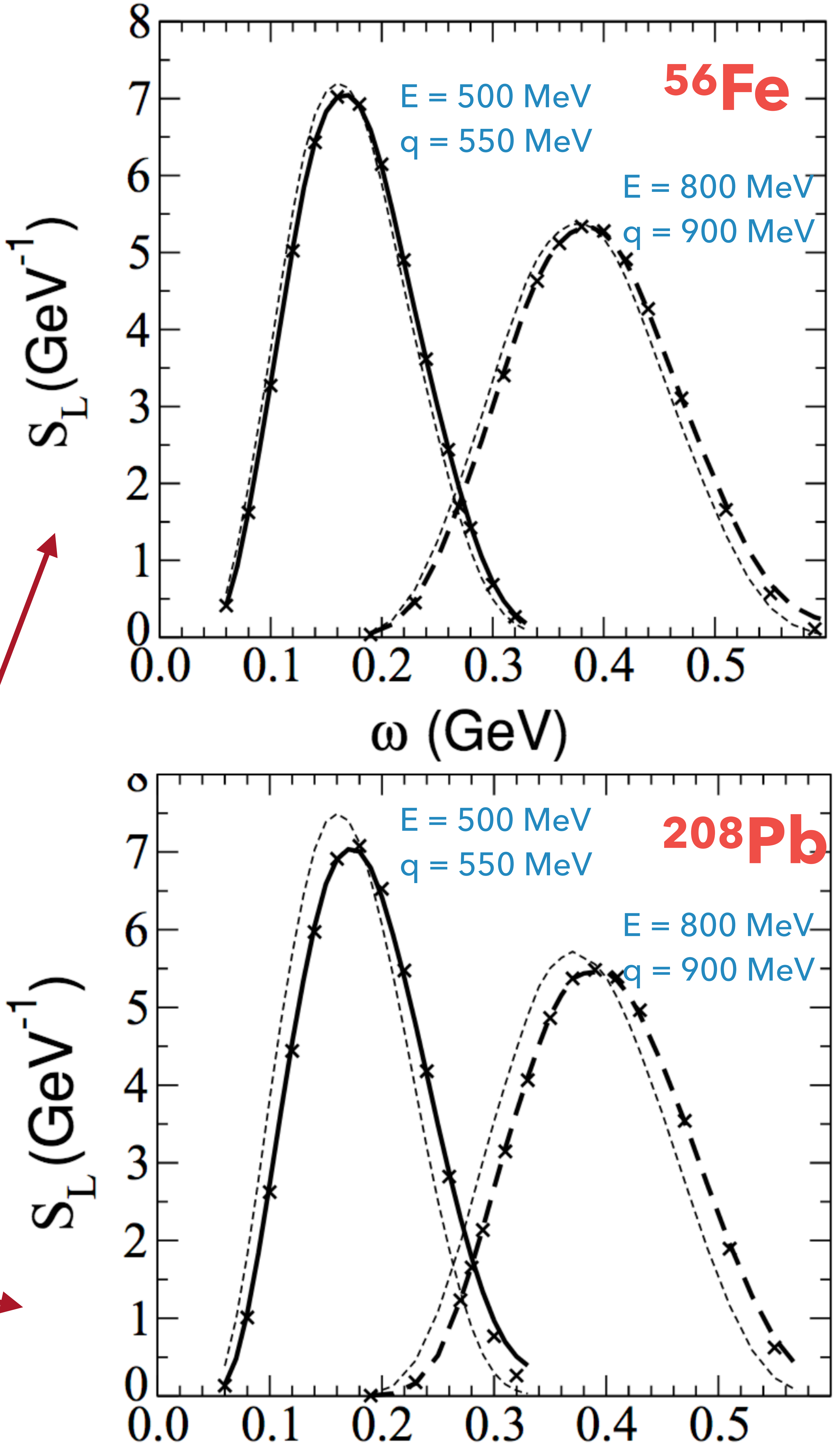
K.S. Kim, L.E. Wright, PRC 72 (2005)

CALCULATIONS IN THE LITERATURE

- S. Wallace and J. Tjon.
  - Compares a modified EMAr, which uses EMA for the hard-photon propagator and form factors, but more carefully treats the r-dependence of Coulomb effects from the electron wave functions.
  - Conclusions: "We find that the spin phases in electron wave functions produce very small effects at energies of 500 MeV or higher... our calculations refute claims... [From Kim and Wright] that the EMA procedure is not accurate at 485 electron energy 60deg scattering angle for a ^{208}Pb target."
  - Other notes: "Because final state interactions, correlations, and pion production have been omitted, the calculated cross sections may differ significantly from experimental cross sections."

Longitudinal Response (S_L) vs. Energy Transfer (ω)

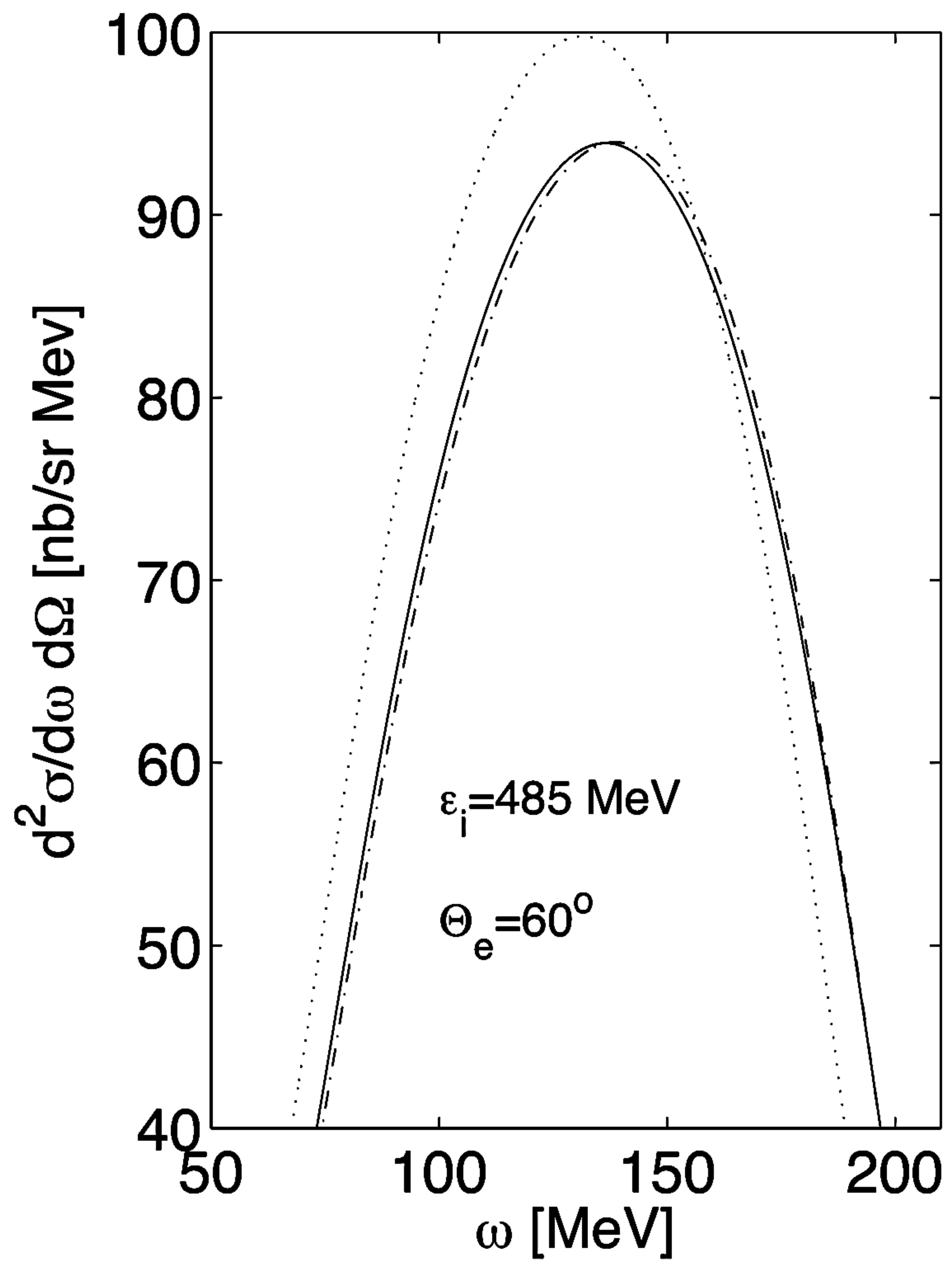
Solid/dashed is EMAr
Fine-dashed is PWIA (no coulomb distortion)



S. Wallace, J. Tjon, PRC 78 (2008)

CALCULATIONS IN THE LITERATURE

- A. Aste, 2008
  - Further studies comparing DWBA, PWBA, and EMA.
  - Conclusions: "As a basic result of this work we conclude that the EMA with an effective potential V = −19 MeV is a valid approximation for the description of Coulomb distortions in the kinematic region where the momentum transfer squared is larger than (300 MeV)^2 (such that the length scale of the exchanged photon is smaller than the typical size of a nucleus), the energy of the scattered electron is larger than 150MeV (such that the semiclassical description of the electron wave functions in the nuclear vicinity is valid) and the energy transfer ω is larger than~140MeV (such that the distortion of the final state nucleon wave functions is moderate)…. For a typical energy range where the initial electron energy is of the order of some hundreds of MeV, the DWBA cross sections are generally larger by 1−2% than the EMA cross sections. If the energy transfer ω is smaller than 140 MeV, the DWBA cross section can still be approximated by an EMA calculation with a phenomenological effective potential V and a minor amplitude correction. However, in such a case one has to rely on the model used to describe the nuclear current. "



Cross section for 208Pb at 485 MeV and 60 deg.

Solid line: DWBA
Dashed line: PWBA
Dot-Dashed line: EMA with V = -19 MeV

A. Aste, Nucl. Phys. A806 (2008)

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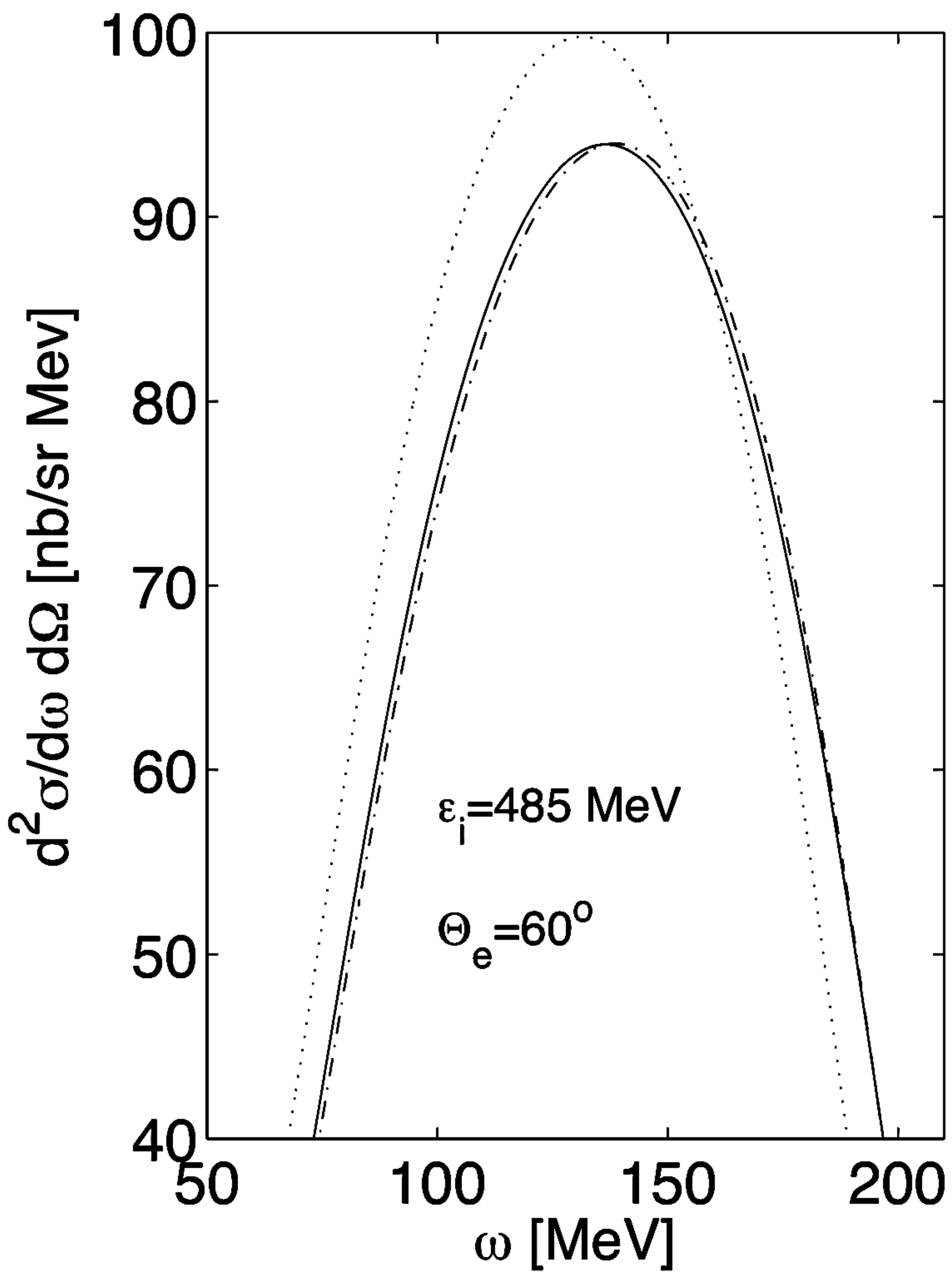
Our data spans from (250 MeV)^2 to (560 MeV)^2



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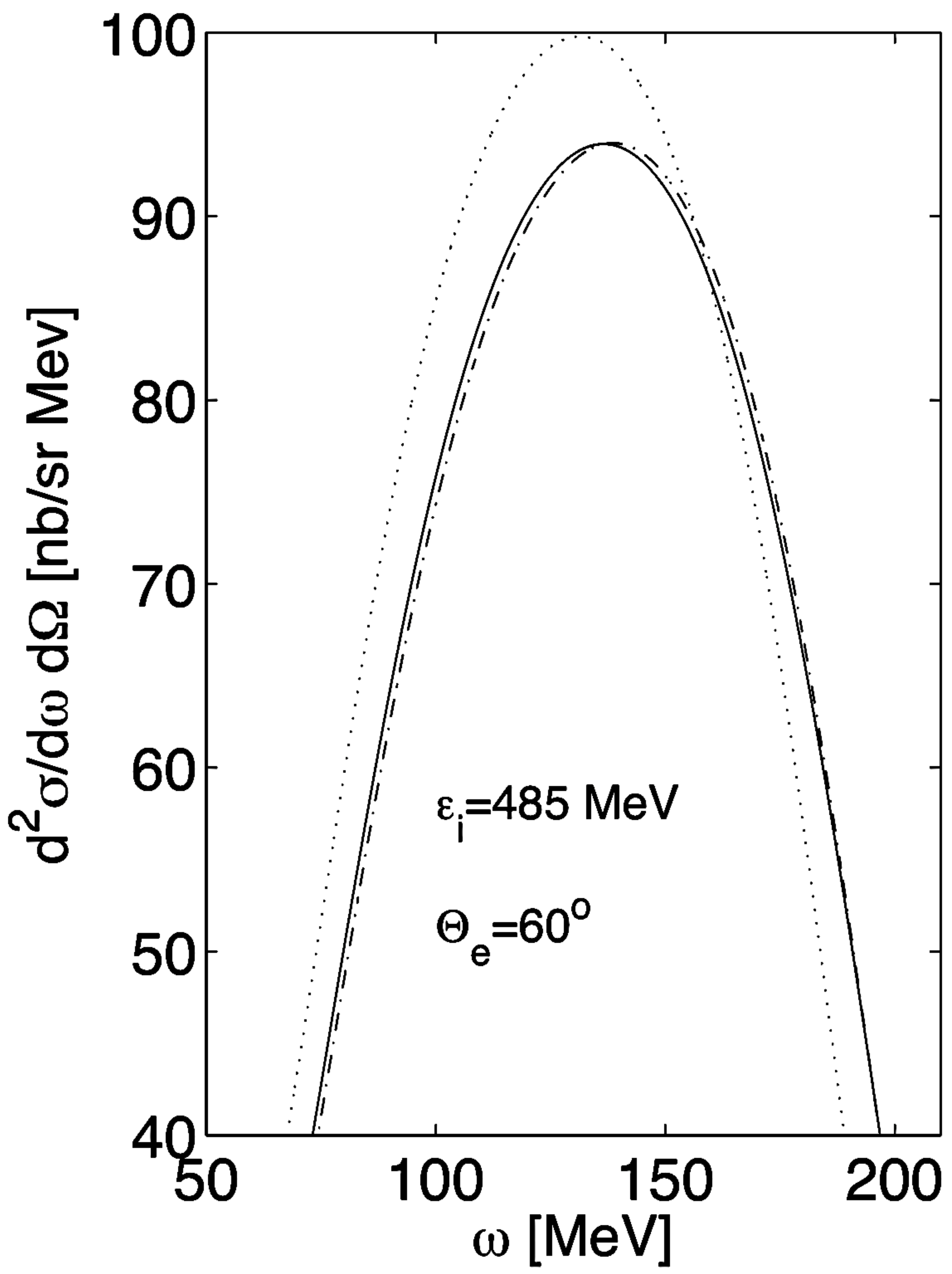
Our data goes as low as 116 MeV (although uncertainties are large below 150 MeV).



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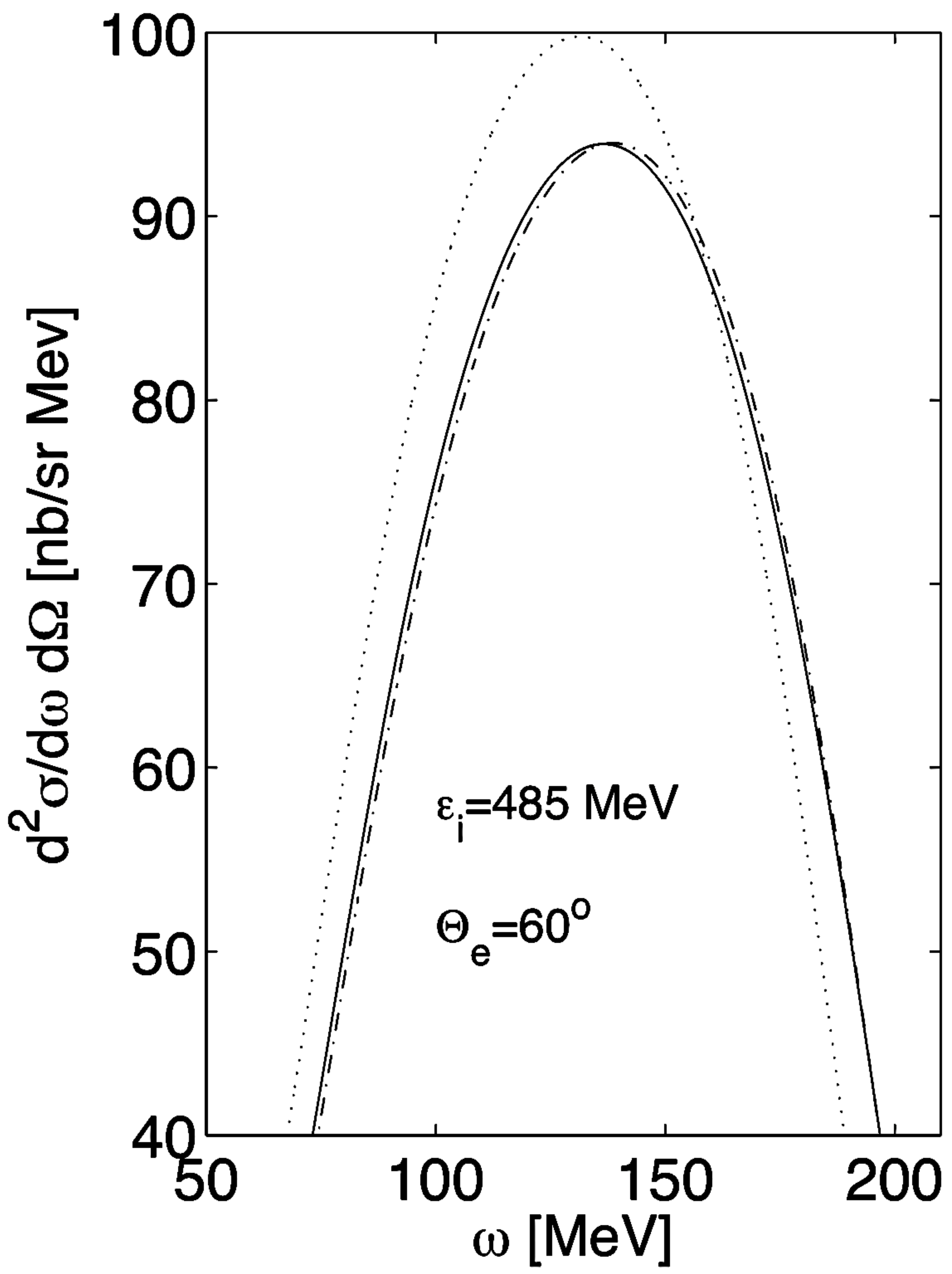
For lower |q|, our kinematic range goes as low as ω = 80 MeV.



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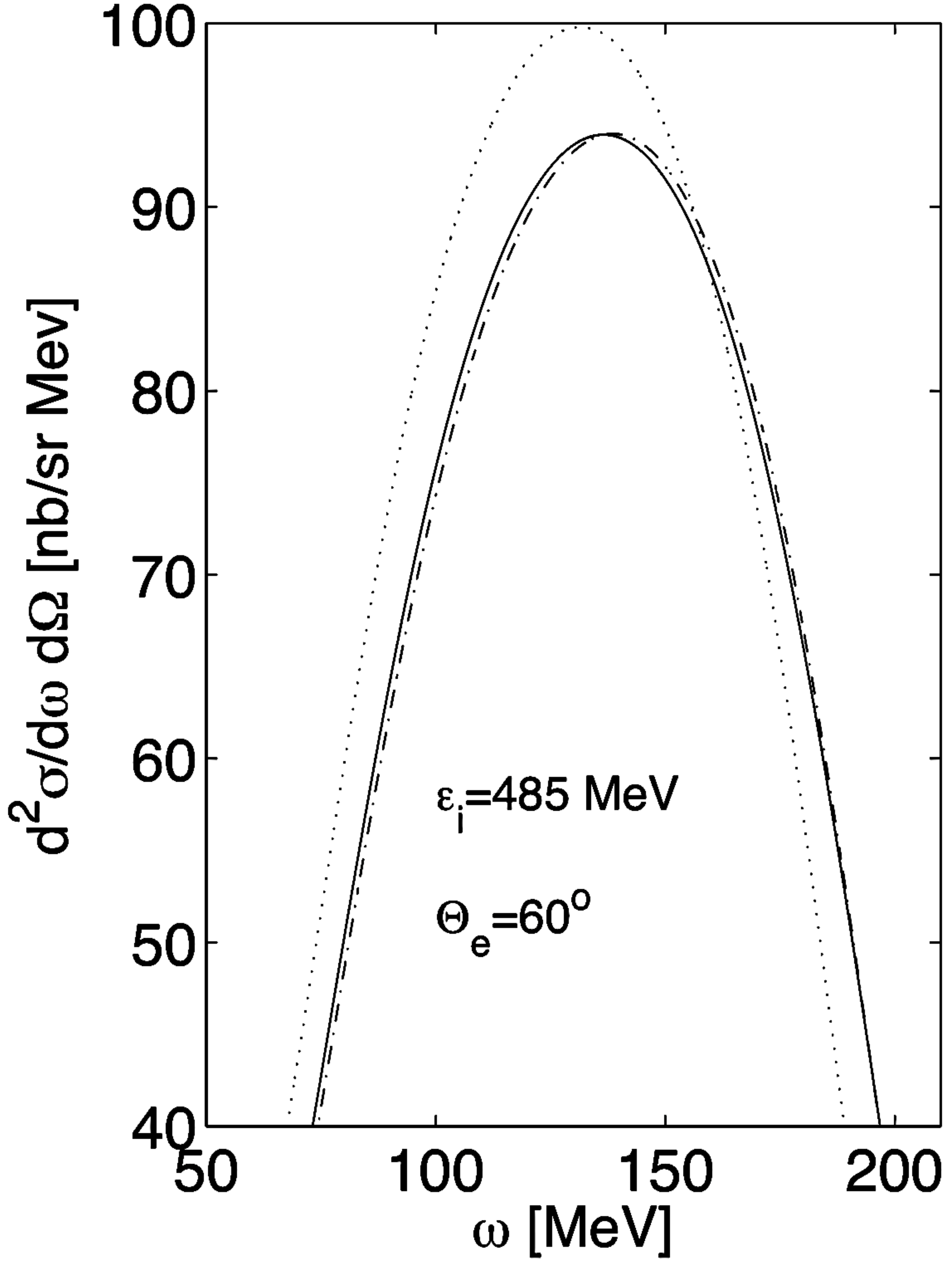
We are aiming for cross-section precision on the order of 1% to 2% in that region.



Cross section for 208Pb at 485 MeV and 60 deg.

Solid line: DWBA
Dashed line: PWBA
Dot-Dashed line: EMA with V = -19 MeV

A. Aste, Nucl. Phys. A806 (2008)

DISPERSIVE CORRECTIONS AND LEDEX

- First introduced by I. Sick (1970) and later extended by Friar and Rosen (1978), dynamic corrections due to the nucleus coulomb field were used to describe the shift of the diffractive minima in elastic cross-sections.
- The LEDEX group introduced the correction as:

$$\sigma_{disp} = \sigma_{stat}\left[1 + \delta\left(E_{inc}\right)\right]$$

- Where a zeroth order dispersive correction corresponds to a Coulomb correction:

$$\sigma_{stat} = \sigma_{Born}\left[1 + \delta(0)\right]$$

- The LEDEX group found a much stronger effect than predicted by Friar and Rosen:
  - For 600 MeV beam energy on 12C, a 15% correction is on the minimum of diffraction is expected from data extrapolation.
    - The predicted value from Friar and Rosen is only 2% (7.5 times lower).
    - If a 3% effect is expected/calculated on the longitudinal response, but too small by a factor of 7.5 times.... The total correction would be on the order of 22%. Implying a much larger correction on even carbon than predicted.

Gueye et al, (LEDEX and HAll-A collaboration) arXiv:1805.12441

CONCLUSIONS

- Data from Hall-A is being analyzed to extract the Coulomb Sum Rule at momentum transfer between 500 and 1000 MeV/c.
- Corrections due to the electric field of the nucleus are critical in the analysis.
  - The EMA has been used exclusively for targets of ^{56}Fe and lighter, but the exact limits and their contribution to uncertainties is not well defined in our kinematic regime.
  - For ^{208}Pb, we will not be able to use EMA and will need calculations corresponding to our kinematic coverage.

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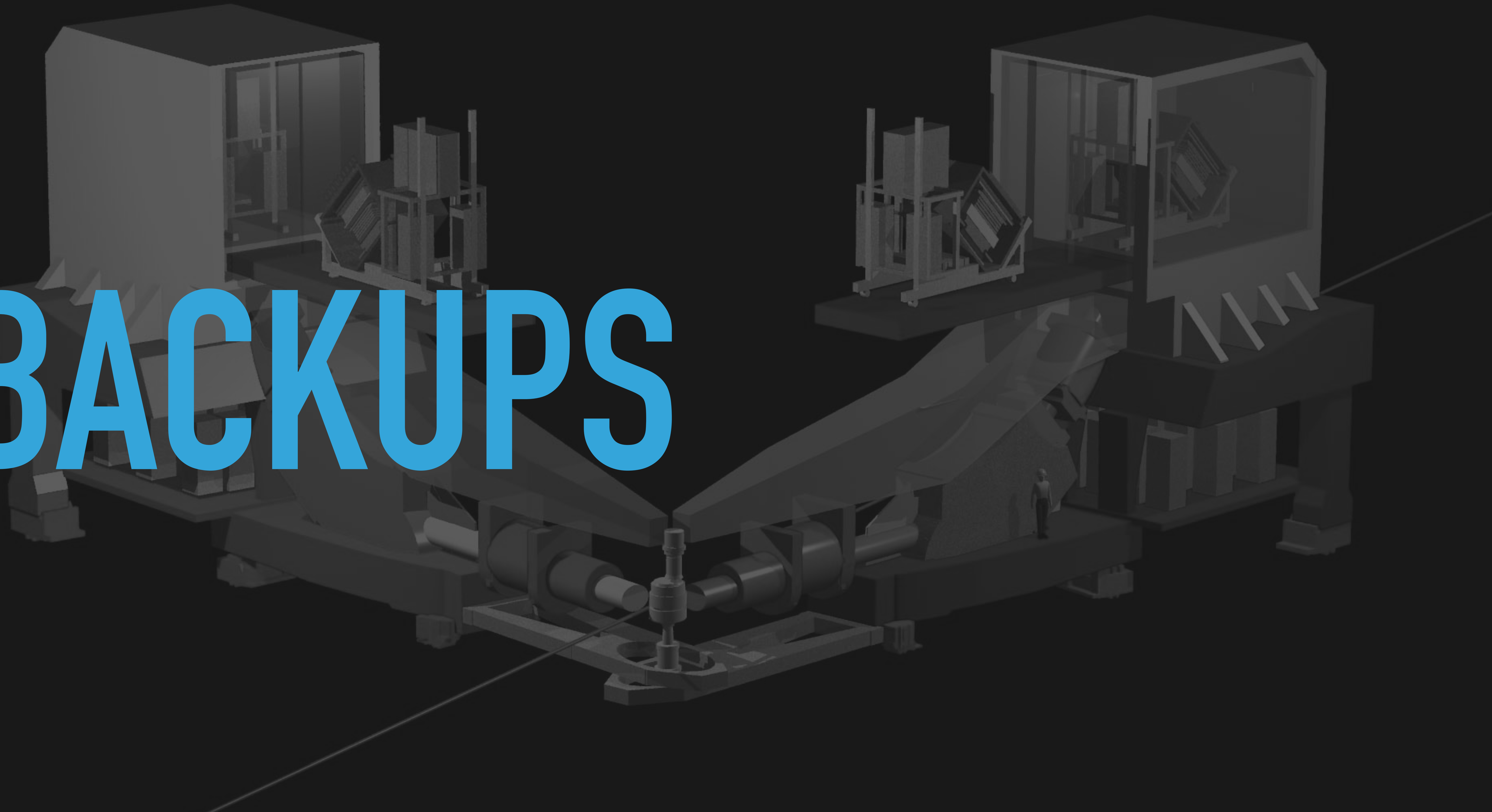
PEOPLE

Kalyan Allada, Korand Aniol, Jon Arrington, **Hamza Atac**, Todd Averett, Herat Bandara, Werner Boeglin, Alexandre Camsonne, Mustafa Canan, **Jian-Ping Chen**, Wei Chen, Khem Chirapatpimol, **Seonho Choi**, Eugene Chudakov, Evaristo Cisbani, Francesco Cusanno, Rafelle De Leo, Chiranjib Dutta, Cesar Fernandez-Ramirez, David Flay, Salvatore Frullani, Haiyan Gao, Franco Garibaldi, Ronald Gilman, Oleksandr Glamazdin, Brian Hahn, Ole Hansen, Douglas Higinbotham, Tim Holmstrom, Bitao Hu, Jin Huang, **Yan Huang**, Florian Itard, Liyang Jiang, Xiaodong Jiang, **Kai Jin**, Hoyoung Kang, Joe Katich, Mina Katramatou, Aidan Kelleher, Elena Khrosinkova, Gerfried Kumbartzki, John LeRose, Xiaomei Li, Richard Lindgren, Nilanga Liyanage, Joaquin Lopez Herraiz, Lagamba Luigi, Alexandre Lukhanin, Michael Paolone, Maria Martinez Perez, Dustin McNulty, **Zein-Eddine Meziani**, Robert Michaels, Miha Mihovilovic, Joseph Morgenstern, Blaine Norum, **Yoomin Oh**, Michael Olson, Makis Petratos, Milan Potokar, Xin Qian, **Yi Qiang, Arun Saha, Brad Sawatzky, Elaine Schulte**, Mitra Shabestari, Simon Sirca, Patricia Solvignon, Jeongseog Song, **Nikolaos Sparveris, Ramesh Subedi, Vincent Sulkosky**, Jose Udias, Javier Vignote, Eric Voutier, Youcai Wang, John Watson, Yunxiu Ye , **Xinhu Yan**, **Huan Yao**, Zhihong Ye, Xiaohui Zhan, Yi Zhang, Xiaochao Zheng, Lingyan Zhu

and
Hall-A collaboration

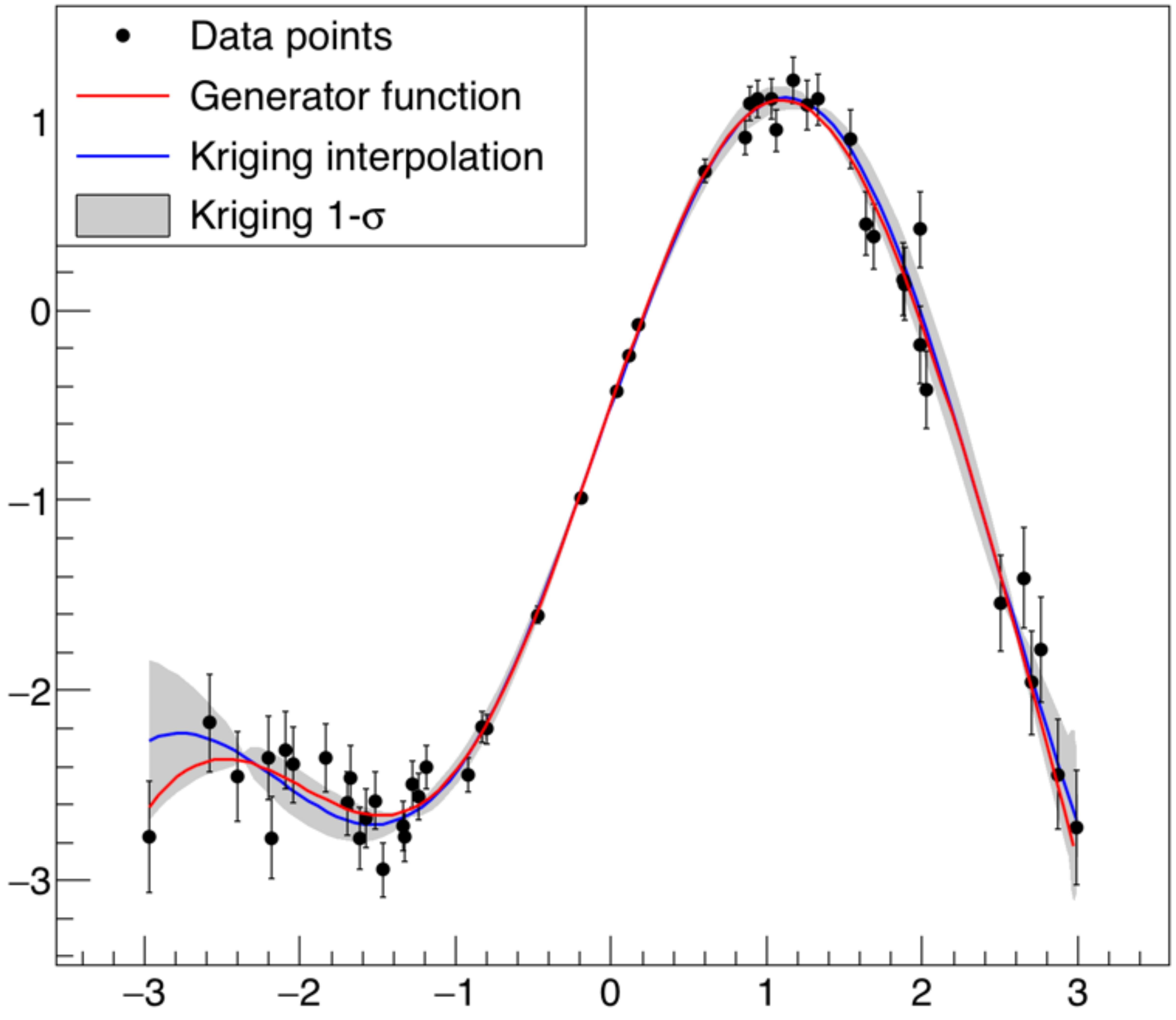
Graduate students **Spokespersons** **Run Coordinators**

BACKUPS



SUPERVISED GAUSSIAN PROCESS REGRESSION (KRIGING)

- Advantages:
  - Can provide "smoothing" of distribution.
  - Does not need an input function (like least squared fitting).
  - Well constrained uncertainties.
- Disadvantages:
  - Interpolation options are still needed:
    - The exact covariant function (gaussian, matern)
    - The "scale" and "width" parameter of the covariant function must be set:
      - A small width parameter will pick out more "bumps".
        - As sigma goes to zero, the interpolation will directly go through every point.
      - A larger width will smooth the distribution.



VARIOUS DEFINITIONS AND CORRECTIONS

Basic kinematic definitions:

$$\epsilon(|\boldsymbol{q}|, \omega, \theta) = \left[1 + \frac{2\boldsymbol{q}^2}{\boldsymbol{q}^2 - \omega^2}\tan^2\frac{\theta}{2}\right]^{-1}$$

$$Q^2 = \boldsymbol{q}^2 - \omega^2$$

$$\frac{d\sigma}{d\Omega}_{\mathrm{Mott}} = \frac{\alpha^2 \cos^2(\theta/2)}{4E^2 \sin^4(\theta/2)}$$

$$W^2 = M_N^2 + 2M_N\omega - Q^2$$

Relativistic correction to nucleon form-factor:

$$\tilde{G}_E^2 = G_E^2 \frac{1 + Q^2/4M^2}{1 + Q^2/2M^2}$$

VERIFICATION OF RADIATIVE CORRECTIONS