Quark and Gluon Distributions within Constituent Quarks and the Pion

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Cake Seminar

Based on: Phys. Rev. C 104, 045201 (2021)

December 6, 2021

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Outline

- Introduction
- Quark Target Model
  - Quark distribution of a constituent quark
  - Gluon distribution of a constituent quark
- Quark Target Model with a gluon mass
- Numerical results
- Pion PDF
- Summary and Outlook

Introduction

- Quantum Chromodynamics (QCD) is the underlying theory of the strong force
- The strong force binds quarks and gluons inside hadrons and is responsible for holding neutron and proton inside the atomic nucleus
- QCD is a local, non-Abelian gauge theory whose basic degrees of freedom are quarks and gluons

Introduction

QCD is characterized by a number of distinct phenomena

- **Asymptotic freedom:** At high energy, the coupling strength is small and quarks behave as quasi-free $\Rightarrow$ **Perturbative:** expansion in the coupling

$$\alpha_s(Q^2) = \frac{12\pi}{(11N_c - 2N_f)\log\left(\frac{Q^2}{\Lambda^2_{QCD}}\right)}$$

2004 Nobel Prize in Physics: Gross, Politzer and Wilczek.

However, at low energy:

- **Confinement:** Quarks and gluons are confined inside hadrons
- **DCSB:** Mass generation

Nonperturbative: lattice, DSEs, quark models

Mass Generation

- Chiral symmetry: approximate symmetry of the light quark sector of QCD

$$m_u \quad 2.16\ \mathrm{MeV}, \quad m_d \quad 4.67\ \mathrm{MeV}: \quad m_u \quad m_d$$

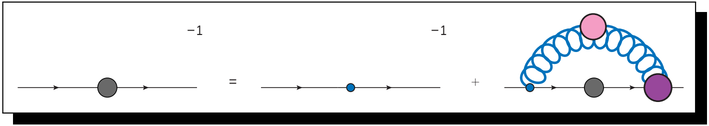
$\mathbf{SU(2)_L \otimes SU(2)_R}$

- Mass splitting between parity partners are big ($m_{a_1} \quad m_\rho \quad 500$ MeV) and cannot be produced by the small current quark masses in the QCD Lagrangian

- Chiral symmetry is broken dynamically and gives rise to:
  - the mass splittings observed in hadron spectrum
  - a nearly massless (pseudo)-Golsdtone boson: **the pion**
  - constituent (dressed) quarks

Mass Generation

- The picture of dressed quarks arises from the dressing of current quarks and gluons due to the strong interactions
  - Constituent quarks are quasi-particles
  - The mass of constituent quarks: $M_q = 300 \quad 400$ MeV
  - Their emergence is described by the gap equation:



- A constituent quark have an indefinitely complicated current quark and gluon substructure

Quark Target Model

- What is the quark and gluon substructure of dressed quarks?
- What are the quark and gluon momentum distributions of a dressed quark?
- Much work has been done into calculating quark distributions within effective theories of QCD
  See Bednar *et. al.* Phys. Rev. Lett. 124, 042002 (2020)
- However, direct calculation of gluon distributions remains largely unexplored
- In general gluons are assumed to be generated entirely perturbatively by DGLAP evolution) **not enough**
  - Gluons are expected to contribute at *all scales*
  - Major focus of future facilities
- Information about gluon distributions have also been inferred
- With this in mind, here, we take the direct approach of calculating the gluonic and quark substructure of a dressed quark

Quark PDF

- PDFs are defined using light-cone correlators
- The distribution of a quark of flavor q in a quark target of flavor Q is:

$$f_{q/Q}(x) = \int \frac{d\lambda}{4\pi}\; e^{ix(P\ \ n)\lambda} h\mathrm{Q}(P)j\bar{\psi}_q(0)\not{n}W(0, n\lambda)\psi_q(n\lambda)j\mathrm{Q}(P)i.$$

The state $j\mathrm{Q}(\mathrm{P})i$ represents the dressed quark state with momentum P and

$$\boxed{x = \frac{k\ \ \ n}{P\ \ \ n}}$$

is the light-cone momentum fraction carried by the bare struck quark.

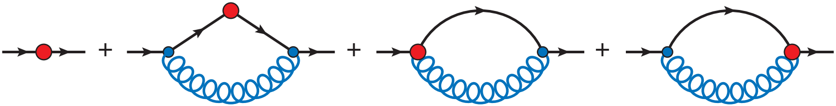
The operator

$$W(0, n\lambda) = \mathrm{P}\, e^{\ \ ig \int_{\lambda}^{0} \mathrm{d}\xi\, n\ A(n\xi)}$$

is the Wilson line which ensures gauge invariance of the PDF.

Quark PDF

- Leading order diagrams contributing to the quark PDF in the QTM:



 $= Z_2\,\delta(1 \quad x) : \qquad Z_2^{\ \ 1} = 1 \quad \left.\frac{\mathrm{d}\,\Sigma(p)}{\mathrm{d}\not{p}}\right|_{\not{p}=M_q},$

$$\Sigma(p) = ig^2\,C_F \int \frac{\mathrm{d}^4k}{(2\pi)^4}\,\gamma^{\mu}\,S(p \quad k)\,\gamma^{\nu}\,D_{\mu\nu}(k)$$

$$D^{\mathrm{cov}}_{\mu\nu}(k) = \left[\quad g_{\mu\nu} + (1 \quad \xi)\frac{k_\mu k_\nu}{k^2 + i0}\right] D(k^2),$$

$$D^{\mathrm{LC}}_{\mu\nu}(k) = \left[\quad g_{\mu\nu} + \frac{k_\mu n_\nu + n_\mu k_\nu}{(k \quad n)}\right] D(k^2).$$

Quark PDF

- The basic integrals are regularized using the proper-time regularization scheme with both UV and IR regulators ($\Lambda_{IR} \rightarrow$ **confinement**)

$$\frac{1}{A^n} = \frac{1}{(n-1)!}\int_0^{\infty} d\tau\, \tau^{n-1} e^{-\tau A} \rightarrow \frac{1}{(n-1)!}\int_{1/\Lambda_{UV}^2}^{1/\Lambda_{IR}^2} d\tau\, \tau^{n-1} e^{\tau A},$$

such that

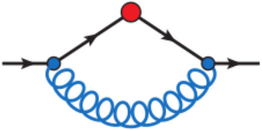
$$I_n(\Delta) \equiv i\int \frac{\mathrm{d}^4 k}{(2\pi)^4}\,\frac{1}{(k^2-\Delta+i\epsilon)^n} \rightarrow I_2(\Delta) = -\frac{1}{16\pi^2}\int_{1/\Lambda_{UV}^2}^{1/\Lambda_{IR}^2} \mathrm{d}\tau\,\frac{1}{\tau}\,e^{-\tau}$$

$$I_3(\Delta) = \frac{1}{16\pi^2}\int_{1/\Lambda_{UV}^2}^{1/\Lambda_{IR}^2} \mathrm{d}\tau\,\frac{1}{2}\,e^{-\tau}$$

$$Z_2^{\text{cov}} = 1 + 2\,g^2\,C_F\int_0^1 \mathrm{d}x\,\left[x\,I_2(\Delta_q(x)) + 4\,M_q^2\,x(1-x)(2-x)\,I_3(\Delta_q(x))\right]$$

$$Z_2^{\text{LC}} = Z_2^{\text{cov}} + 4\,g^2 C_F\int_0^1 \mathrm{d}x\,\frac{x}{1-x}\,I_2(\Delta_q(x))$$

Quark PDF

$f^{\text{tri}}_{q/Q}(x) =$ 

$$= \frac{ig^2}{2}\frac{C_F}{P\cdot n}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\,\delta\left(x-\frac{k\cdot n}{P\cdot n}\right)$$

$$\bar{u}(P)\,\gamma^\mu\,S(k)\,\not{n}\,S(k)\,\gamma^\nu\,D_{\mu\nu}(P-k)\,u(P)$$

- The PDFs are calculated by evaluating the Mellin moments:

$$\boxed{\langle x^{s-1}\rangle_{q,g} = \int_0^1 dx\, x^{s-1} f_{q,g/Q}(x)}$$

$$f^{\text{tri, cov}}_{q/Q}(x) = -2\,g^2 C_F\left[(1-x)\,I_2(\Delta_q(x)) + 4\,M_q^2\,x(1-x)\,I_3(\Delta_q(x))\right],$$

$$f^{\text{tri,LC}}_{q/Q}(x) = f^{\text{tri, cov}}_{q/Q}(x) - 4\,g^2 C_F\,\frac{x}{1-x}\,I_2(\Delta_q(x)),$$

Quark PDF

- Wilson line operator must also be expanded in powers of α_s
- Wilson line contribution:

$$f_{q/Q}^{W,\mathrm{cov}}(x) =$$

$$= g^2 \frac{C_F}{P \quad n} n^\nu \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \left[\delta\left(x \quad \frac{k \quad n}{P \quad n} \right) \quad \delta(1 \quad x) \right]$$

$$\frac{i}{n \quad (P \quad k) + i0} \bar{u}(P) \gamma^\mu S(k) \not{n} D_{\mu\nu}(P \quad k) u(p).$$

$$f_{q/Q}^{W,\mathrm{cov}}(x) = 4 g^2 C_F \left[\frac{x}{1 \quad x} I_2(\Delta_q(x)) \quad \delta(1 \quad x) \int_0^1 \mathrm{d}y \frac{y}{1 \quad y} I_2(\Delta_q(y)) \right]$$

Quark PDF

- Adding all contributions

$$f_{q/Q}(x) = Z_2\,\delta(1 \quad x) + f^{\text{tri}}_{q/Q}(x) + f^{W}_{q/Q}(x)$$

- Comparison of the light-cone gauge and covariant gauge results:

$$Z_2^{\text{LC}}\,\delta(1 \quad x) + f^{\text{tri,LC}}_{q/Q}(x)$$
$$= Z_2^{\text{cov}}\,\delta(1 \quad x) + f^{\text{tri,cov}}_{q/Q}(x) + f^{W,\text{cov}}_{q/Q}(x)$$

- No Wilson line contribution in the light-cone gauge ($n \quad A = 0$)

- Need careful treatment of the Wilson to draw the right conclusions

- Either one perform the calculation in the light-cone gauge or

- Perform the calculation in a covariant gauge and be consistent by calculating the Wilson link contribution in the **same** gauge

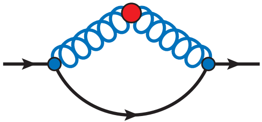
Gluon PDF

- The unpolarized gluon distribution in the quark is defined as

$$x\, f_{g/Q}(x) = \int \frac{d\lambda}{2\pi} \frac{1}{P \cdot n} e^{ix(P\cdot n)\lambda} \langle Q(P)| G^{\,+}_{\mu}(0) W(0, n\lambda) G^{\mu +}(n\lambda) |Q(P)\rangle$$

- Only one diagram contributing at leading order:

$$x\, f_{g/Q}(x) =$$



$$= 2 g^2 C_F \Big[\left[1 + (1-x)^2\right] I_2(\Delta_g(x)) + 4 M_q^2 x^2 (1-x) I_3(\Delta_g(x)) \Big]$$

Sum Rules

Quark number sum rule:

$$\int_0^1 \mathrm{d}x \, f_{q/Q}(x) = 1$$

$$! \quad \int_0^1 \mathrm{d}x \, f_{q/Q}(x) = Z_2^{\text{cov}} + \int_0^1 \mathrm{d}x \, \left[f_{q/Q}^{\,\text{tri, cov}}(x) + f_{q/Q}^{\,\text{W, cov}}(x) \right]$$

$$= 1 + 2 g^2 C_F \int_0^1 \mathrm{d}x \, \frac{\mathrm{d}}{\mathrm{d}x} \left[x(x \quad 1) I_2\left(\Delta_q(x)\right) \right] = 1$$

Momentum sum rule:

$$\int_0^1 \mathrm{d}x \, \left[x \, f_{q/Q}(x) + x \, f_{g/Q}(x) \right] = 1$$

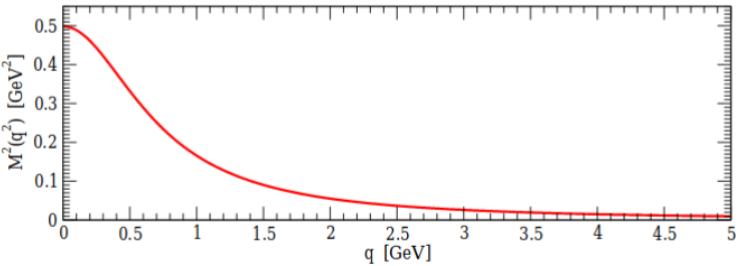
- The symmetry $f_{q/Q}(1 \quad x) = f_{g/Q}(x)$ guarantees momentum sum rule is satisfied once quark number is already satisfied

QTM with a gluon mass

QTM with a gluon mass

- Results from Lattice QCD and DSE suggest that the gluon acquires a dynamical mass that saturates at a finite value in the IR

See O. Oliveira *et. al.* J. Phys. G 38 (2011) 045003 and A. C. Aguilar *et. al.* Phys. Rev. D 84 (2011) 085026



What if we want to incorporate a gluon mass to the model?

QTM with a gluon mass

- Simplest and most straightforward way is to naively add a gluon mass term to the lagrangian:

$$L_{naive} = m_g^2 \mathrm{Tr}\left[A_\mu(x) A^\mu(x)\right]$$

- Obviously, this mass term breaks **gauge invariance**
- Nevertheless, it can serve well as a phenomenological tool
- This is motivated by numerous results See M. Tissier *et. al.* Phys. Rev. D 82, 101701(R)
  - The dressing of the gluon propagator is well described by a propagator in the form

$$D_{\mu\nu}(k) = \left(\quad g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2} \right) D(k^2) \;:\quad D(k^2) = \frac{1}{k^2 \quad m_g^2}$$

QTM with a gluon mass

$$f_{q/Q}(x;m_g) = f_{q/Q}(x) \quad 4\,g^2 C_F\, m_g^2\, x(1 \quad x)\, I_3\left(\Delta_q(x)\right)$$

$$x\, f_{g/Q}^{\rm naive}(x;m_g) = x\, f_{g/Q}(x)$$
$$4\,g^2 C_F\, m_g^2 \left[x^2(1 \quad x) + 2\,(1 \quad x)^2\right] I_3(\Delta_g(x))$$

- Quark number sum rule is satisfied for **any** m_g
- However, this mass term spoils momentum sum rule by an amount

$$f_{q/Q}(x;m_g) \quad f_{g/Q}^{\rm naive}(1 \quad x;m_g) = g^2 C_F \left\{ 8m_g^2 \frac{x^2}{(1 \quad x)}\, I_3\left(\Delta_g(x)\right) \right\}$$

QTM with a gluon mass

An alternative approach:

- A gauge invariant lagrangian containing a gluon mass has been proposed by Cornwall

$$L_{\text{mass}} = m_g^2 \mathrm{Tr}\left[\left(A_\mu(x) \quad \frac{1}{ig}\left(\partial_\mu V(\theta(x))\right) V^{\quad 1}(\theta(x))\right)^2\right],$$

where

$$V(\theta(x)) = e^{i\theta(x)}.$$

transforms under the fundamental representation of the color group, so that its covariant derivative is

$$D_\mu V(\theta(x)) = \partial_\mu V(\theta(x)) \quad igA_\mu(x) V(\theta(x)),$$

$$! \quad L_{\text{mass}} = \frac{m_g^2}{g^2} \mathrm{Tr}\left[\left\{(D_\mu V(\theta(x)))\ V^{\quad 1}(\theta(x))\right\}^2\right]$$

QTM with a gluon mass

- Caveat: The Cornwall Lagrangian is nonrenormalizable
- The effects of the Cornwall mass Lagrangian are two-fold
  1. Massive gluon propagator
  $$D_{\mu\nu}(k) = \left(\quad g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2} \right) \frac{1}{k^2 \quad m_g^2}$$
  2. The introduction of theta fields gives rise to additional contributions to the PDF
- The new fields can also carry momentum

  Additional terms contributing to the energy-momentum tensor (EMT)
- The contribution of the mass Lagrangian to the EMT is found to be:

$$T^{\theta}_{\mu\nu} = \frac{{m_g}^2}{g^2} \Big\{ \quad 2\mathrm{Tr}\Big[(D_\mu V)\, V^{\quad 1} (D_\nu V)\, V^{\quad 1} \Big]$$
$$+ g_{\mu\nu} \mathrm{Tr}\Big[(D^\alpha V)\, V^{\quad 1} (D_\alpha V)\, V^{\quad 1} \Big] \Big\} ; \qquad V \quad V(\theta(x))$$

QTM with a gluon mass

- The bilocal light cone correlator defining the theta-field PDF is defined such as that its second Mellin moment gives the $++$ component of the ETM:

$$x\, f_{\theta/Q}(x) = \quad \frac{2\, m_g^2}{g^2}\ \frac{n^\mu n^\nu}{P \quad n} \int \frac{\mathrm{d}\lambda}{2\pi}\, e^{ix\lambda\, P \quad n}$$
$$\Big\langle Q(P) \Big| \left[D_\mu V\left(\theta(0)\right)\right] V \quad ^1\!\left(\theta(0)\right)$$
$$W_A(0, n\lambda) \left[D_\nu V\left(\theta(n\lambda)\right)\right] V \quad ^1\!\left(\theta(n\lambda)\right) \Big| Q(P) \Big\rangle$$

$$! \qquad x\, f_{\theta/Q}(x) = g^2 C_F\, 8 m_g^2 (1 \quad x)^2\, I_3\left(\Delta_g(x)\right)$$

- Full gluon PDF $=)$ $f_{g/Q}^{\text{naive}}(x; m_g) + f_{\theta/Q}(x; m_g)$

$$f_{g/Q}(x; m_g) = f_{g/Q}(x) \quad 4\, g^2 C_F\, m_g^2\, x\, (1 \quad x)\, I_3\left(\Delta_g(x)\right).$$

QTM with a gluon mass

- And now we observe the symmetry

$$f_{q/Q}(x; m_g) = f_{g/Q}^{\text{naive}}(1 \quad x; m_g) + f_{\theta/Q}(1 \quad x; m_g) \quad f_{g/Q}(1 \quad x; m_g)$$

- The introduction of theta fields gives rise to additional contributions to the PDF
- This contribution is such that the PDF is gauge-invariant even with an explicit gluon mass
- The theta-field PDF fixes momentum sum rule

Numerical Results

- Parameters used

Λ_{IR}	Λ_{UV}	M_q	α_s	m_g
240 MeV	645 MeV	400 MeV	0.5	0

Table: Results for the second Mellin moment $\langle x \rangle = \int_0^1 dx\, x\, f(x)$ of the QTM PDFs.

	$Z_2 + \int_0^1 dx\, x\, f_{q/Q}^{\mathrm{tri}}(x)$	$\int_0^1 dx\, x\, f_{q/Q}^{W}(x)$	$\int_0^1 dx\, x\, f_{q/Q}(x)$	$\int_0^1 dx\, x\, f_{g/Q}(x)$
α_s	$Z_2 + \langle x \rangle_q^{\mathrm{tri}}$	$\langle x \rangle_q^{W}$	$\langle x \rangle_q$	$\langle x \rangle_g$
0.25	0.9934	-0.0887	0.9046	0.0954
0.5	0.9867	-0.1775	0.8093	0.1907
0.75	0.9800	-0.2662	0.7139	0.2861
1	0.9735	-0.3549	0.6185	0.3815

Numerical Results

- Wilson lines are necessary to obtain momentum conservation

- Wilson lines carry negative momenta

- The momentum carried by the gluons increases with the coupling strength

- Momentum sum rule would be oversaturated without Wilson lines

  1.1176 **instead of** 1

  If the gluon momentum fraction would be infered from imposing momentum sum rule and neglecting the Wilson lines, one would end up with gluons carrying a much smaller momentum fraction than they actually do (0.013 **instead of** 0.19)

- Gluon carries 28% of the quark target momentum at $\alpha_s = 0.75$. This is approximately equal to the fraction of the pion's momentum carried by gluons (experiment)

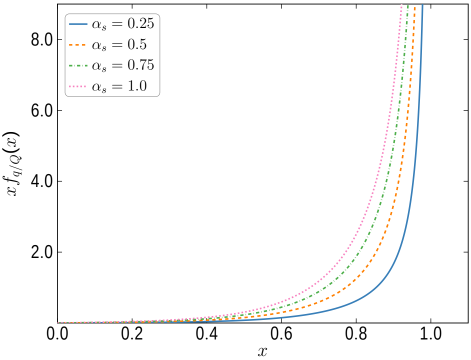
Numerical Results

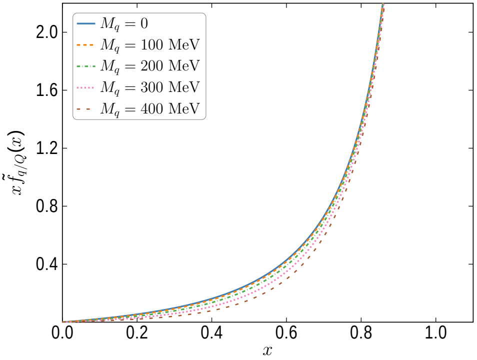
s	$Z_2 + \langle x^{s-1} \rangle_q^{\mathrm{tri}}$	$\langle x^{s-1} \rangle_q^{W}$	$\langle x^{s-1} \rangle_q$	$\langle x^{s-1} \rangle_g$
1	1	0	1	–
2	0.9867	0.1775	0.8093	0.1907
3	0.9866	0.3034	0.6832	0.0647
4	0.9877	0.4011	0.5867	0.0351
5	0.9888	0.4807	0.5081	0.0235
6	0.9896	0.5479	0.4418	0.0176
7	0.9903	0.6059	0.3843	0.0140
8	0.9908	0.6571	0.3337	0.0118
9	0.9912	0.7027	0.2884	0.0101
10	0.9915	0.7440	0.2475	0.0088
⋮	⋮	⋮	⋮	⋮
18	0.9926	0.9791	0.0135	0.0045
19	0.9927	1.0010	**0.0084**	0.0043
20	0.9927	1.0219	**0.0292**	0.0040

- High quark moments become become negative due to Wilson line
- This will have consequences when using the QTM to calculate hadron PDFs

Numerical Results

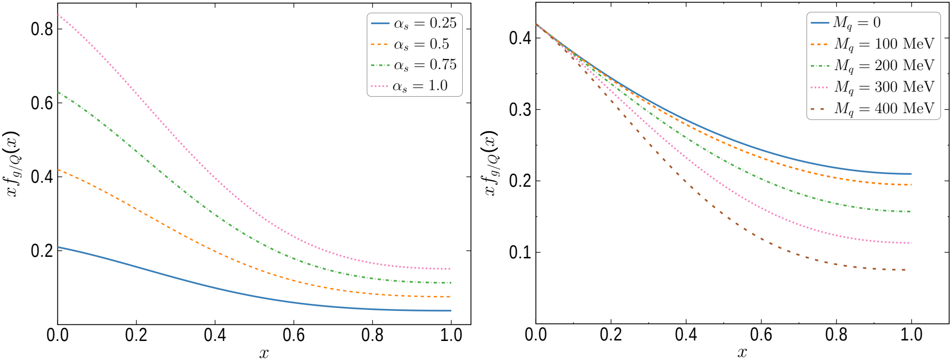
Quark PDF for various values of the coupling and quark mass.





Numerical Results

Gluon PDF for various values of the coupling and quark mass.



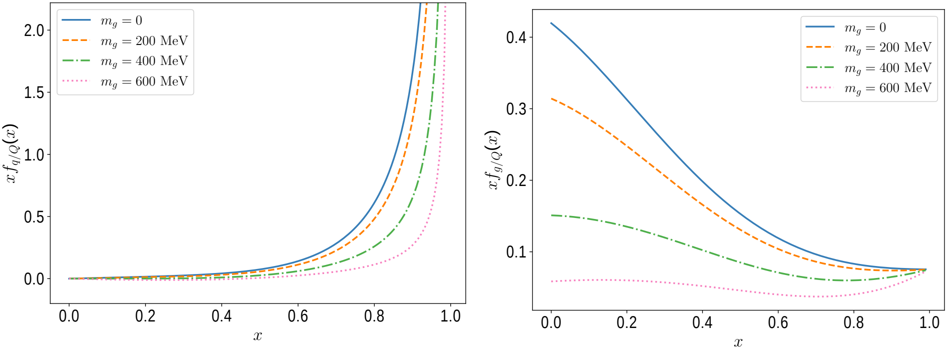
Numerical Results

- Numerical results with gluon mass

Table: Contributions to the quark and gluon momentum fractions for various values of the Cornwall gluon mass (in GeV).

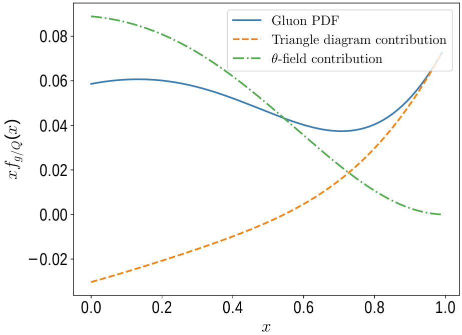
m_g	$Z_2 + \langle x \rangle_q^{\text{tri}}$	$\langle x \rangle_q^W$	$\langle x \rangle_q$	$\langle x \rangle_g^{\text{naive}}$	$\langle x \rangle_\theta$	$\langle x \rangle_g$
0	0.9867	0.1775	0.8093	0.1907	0	0.1907
0.2	0.9885	0.1454	0.8431	0.1293	0.0276	0.1569
0.4	0.9920	0.0878	0.9042	0.0434	0.0524	0.0958
0.6	0.9951	0.0462	0.9489	0.0045	0.0466	0.0511

Numerical Results



- The gluon mass supresses both the quark and gluon PDFs
- A larger gluon mass results in gluons carrying a smaller proportion of the quark target's momentum – The radiation of a gluon becomes more costly and difficult when the gluon has mass.

Numerical Results



- When the gluon is massive, a large portion of the gluon's light cone momentum is carried by the theta-field
- In fact, for $m_g = 600$ MeV, the theta-field carries about 90%
- For large values of the gluon mass, the triangle diagram goes negative at small x resulting gluon PDF, however, is positive definite
- The resulting gluon PDF, however, is positive definite

Hadrons PDFs

- QTM PDFs are not observables but shed light on the inner structure of dressed quarks
- Ideally, QCD models should allow to make predictions
- It is important that these models mantain symmetries and properties of the theory
- One such model is the NJL model
  - A four-point interacting theory
  - Exhibits **dynamical chiral symmetry breaking**
  - Confinement can be simulated in the model by a proper choice of regularization scheme
- In the NJL model hadrons are made of dressed quarks

How do we combine these results to obtain a description of hadrons PDFs in terms of its current quark and gluon substructure ?

Pion PDF

- The pion has an important role in QCD as it is the Goldstone boson associated with DCSB

- The QTM PDFs are folded into a model pion PDF through a convolution equation to give a QTM-modified pion PDF

$$f_{q,g/\pi}(x) = \sum_{Q} \iint_0^1 \mathrm{d}z\, \mathrm{d}y\, \delta(x \quad yz)\, f_{q,g/Q}(y)\, f_{Q/\pi}(z)$$

- $f_{Q/\pi}(z)$ is the dressed quark distribution in the pion for which there are existing results

- $f_{Q/\pi}(z)$ is a "body pion" PDF that would be the pion PDF if the dressed quark were structureless

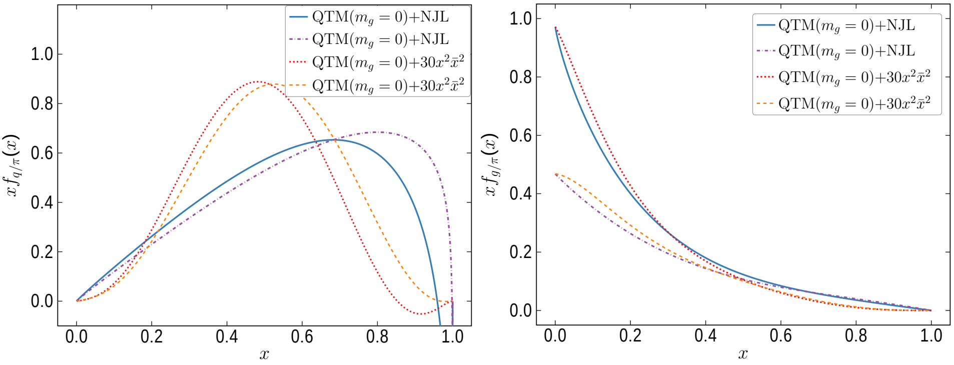
- In particular, the dressed quark target of the QTM is identified with the dressed quark of the effective theory's quark degree of freedom

Pion PDF

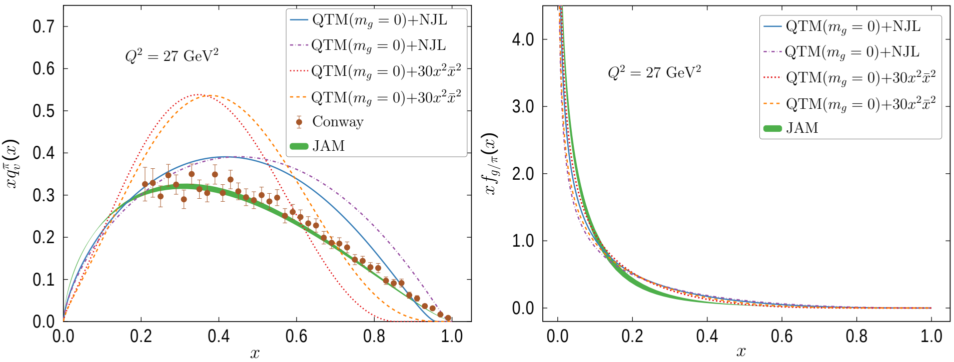
- To compare the predictions of the model with experimental data as well as the empirical parameterizations, it is necessary to determine the model scale
- The model scale is determined by requiring that the pion's gluon momentum fraction content match that found by the JAM analysis after NLO DGLAP evolution.

m_g	α_{s0}	Q_0^2
0	0.579	0.82 GeV^2
0.4 GeV	0.775	0.58 GeV^2

Model scale pion PDFs



Evolved pion PDFs



Summary and Outlook

- We have constructed a QTM at leading order in the quark-gluon coupling strength and quark and gluon PDFs of a dressed quark target were calculated directly
- The calculation included for the first time the Wilson line contributions in covariant gauges
- An important finding was that the Wilson lines can make sizeable contributions to the quark PDF
- Therefore, calculations that do not take into account the Wilson lines contribution to quark PDFs in covariant gauges **might** be missing a portion of the momentum carried by the gluons

Summary and Outlook

- The inclusion of gluons into an effective field of only quarks (as the NJL) will also modify the hadron's Bethe-Salpeter vertices

- In this work

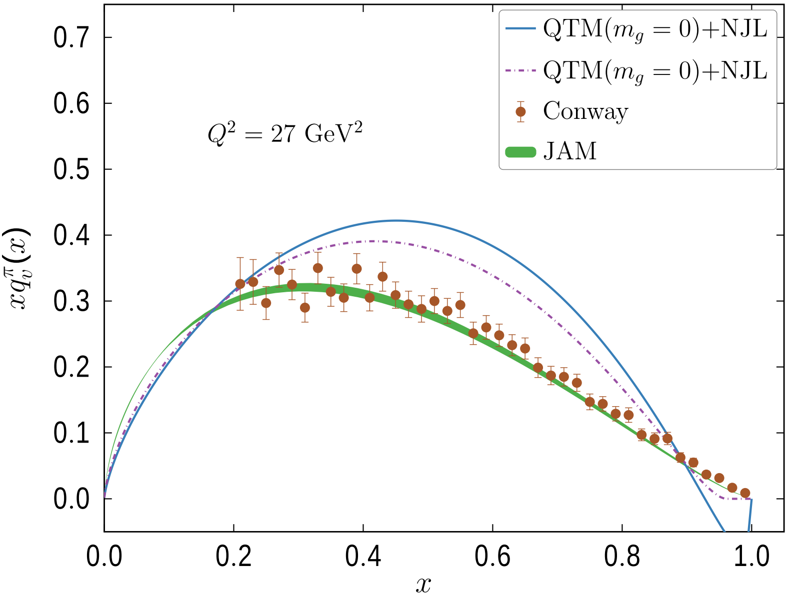
  Gluons were incorporated in a limited way

  No gluon exchange between the dressed quarks

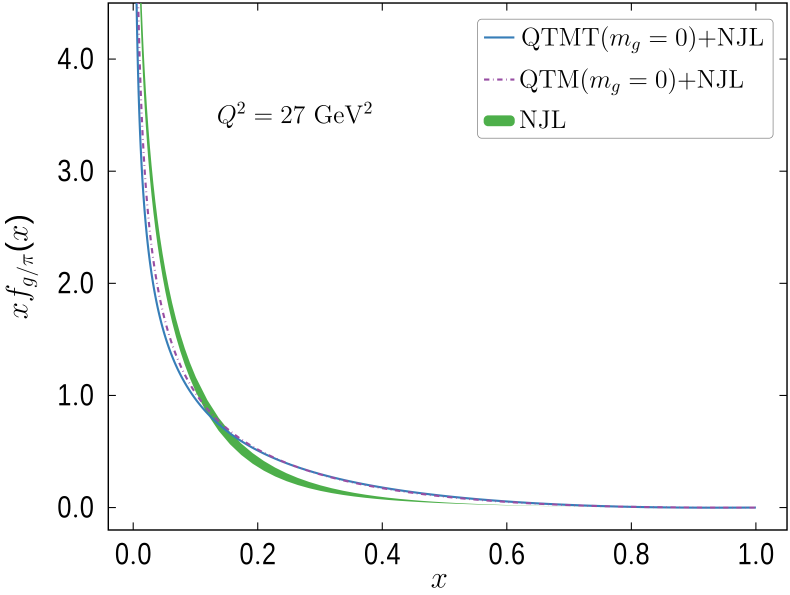
- In the QTM+convolution approach taken in this work the quark PDF becomes negative 1 owing to the Wilson line

- We expect that by considering higher order diagrams will remove the domain of negative support of the quark PDF

Thank you!



QTM($m_g = 0$)+NJL
QTM($m_g = 0$)+NJL
Conway
JAM
$Q^2 = 27$ GeV^2
$xq_v^\pi(x)$
x
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.0
0.2
0.4
0.6
0.8
1.0



QTMT($m_g = 0$)+NJL
QTM($m_g = 0$)+NJL
NJL
$Q^2 = 27$ GeV^2
$x\,f_{g/\pi}(x)$
x
4.0
3.0
2.0
1.0
0.0
0.0
0.2
0.4
0.6
0.8
1.0

- The coupling α_s is different for each value of m_g and are chosen to give $\langle x \rangle_g = 0.1907$

- The area under the curves are all the same