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# Quantum Simulation of Light-Front Parton Correlators

Enrique Rico Ortega

Monday, March 8th

Jefferson Lab

U.S. DEPARTMENT OF ENERGY

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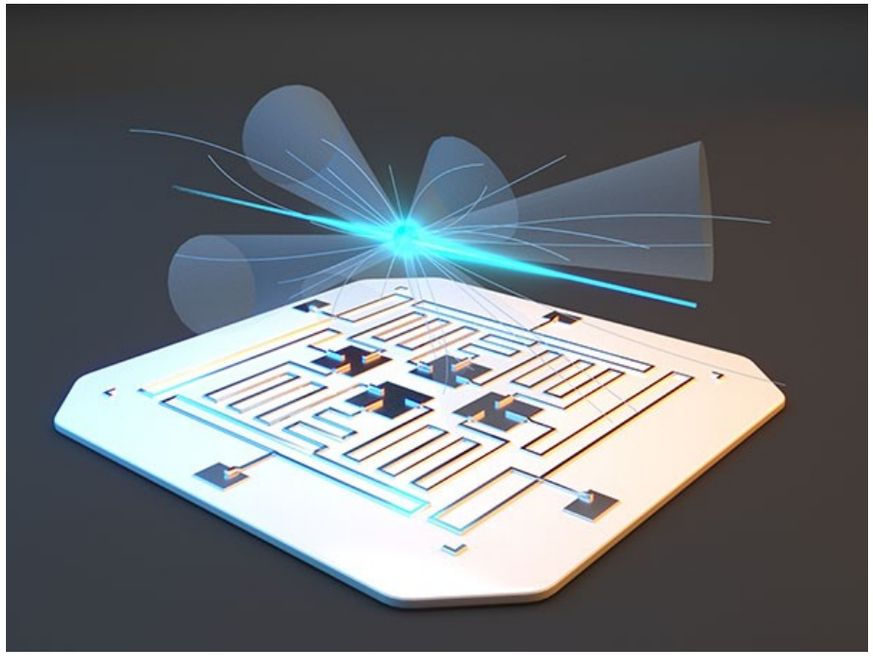
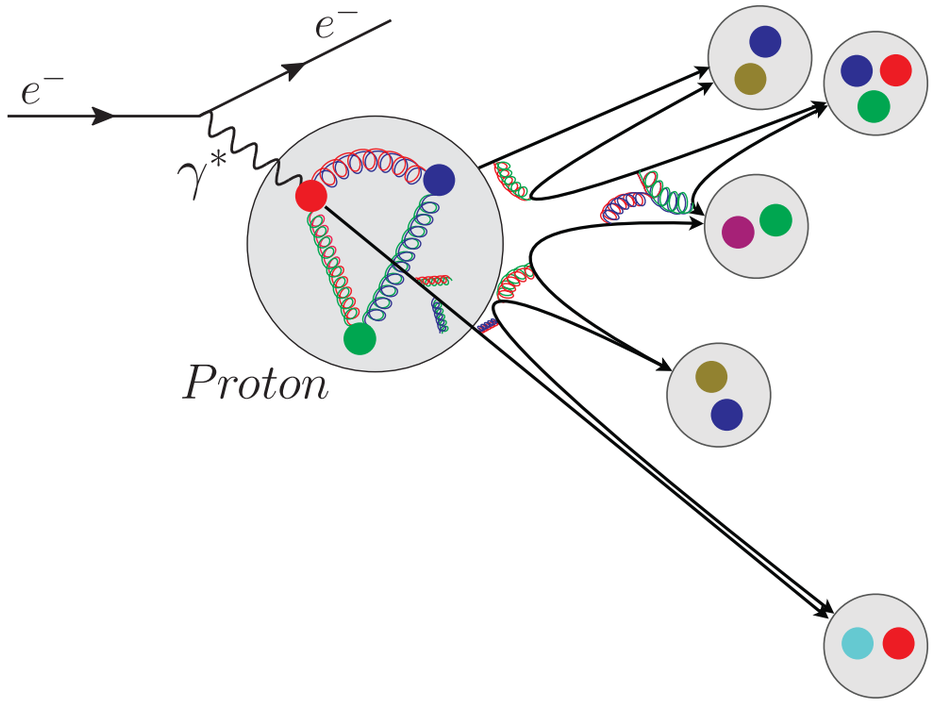
# Simulating lattice gauge theories within quantum technologies



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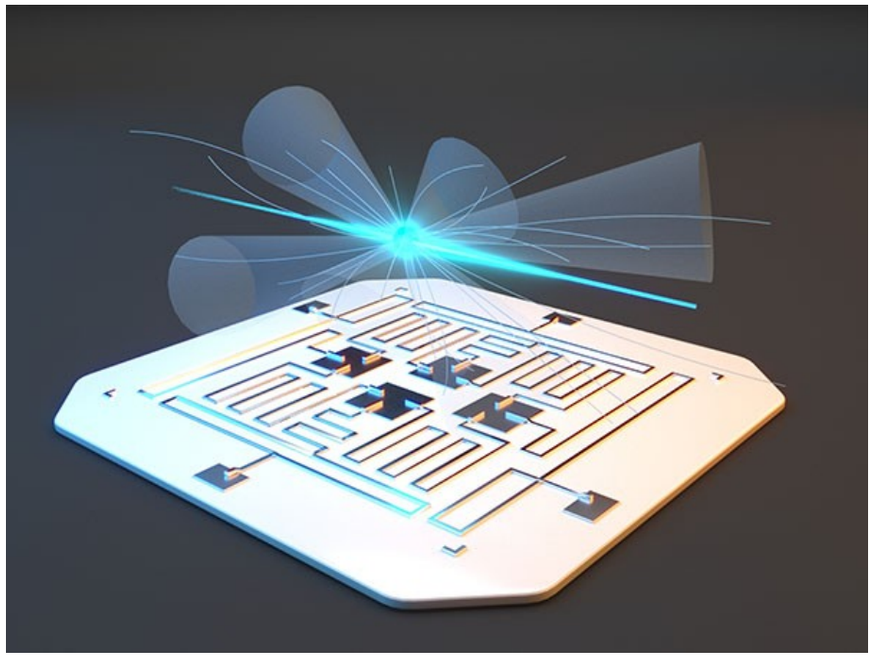

Collaborators: M. Dalmonte, S. Montangero, U.-J. Wiese, P. Zoller…

Eur. Phys. J. D (2020) 74: 165
https://doi.org/10.1140/epjd/e2020-100571-8

THE EUROPEAN PHYSICAL JOURNAL D

Colloquium

## Simulating lattice gauge theories within quantum technologies

Mari Carmen Bañuls[1,2], Rainer Blatt[3,4], Jacopo Catani[5,6,7], Alessio Celi[3,8], Juan Ignacio Cirac[1,2], Marcello Dalmonte[9,10], Leonardo Fallani[5,6,7], Karl Jansen[11], Maciej Lewenstein[8,12,13], Simone Montangero[14,15,a], Christine A. Muschik[3], Benni Reznik[16], Enrique Rico[17,18], Luca Tagliacozzo[19], Karel Van Acoleyen[20], Frank Verstraete[20,21], Uwe-Jens Wiese[22], Matthew Wingate[23], Jakub Zakrzewski[24,25], and Peter Zoller[3]

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# Quantum Simulation of Light-Front Parton Correlators

M.G. Echevarria,[1,*] I.L. Egusquiza,[2,†] E. Rico,[3,4,‡] and G. Schnell[2,4,§]

[1] *Department of Physics and Mathematics, University of Alcalá, 28805 Alcalá de Henares (Madrid), Spain*
[2] *Department of Physics, University of the Basque Country UPV/EHU, Apartado 644, 48080 Bilbao, Spain*
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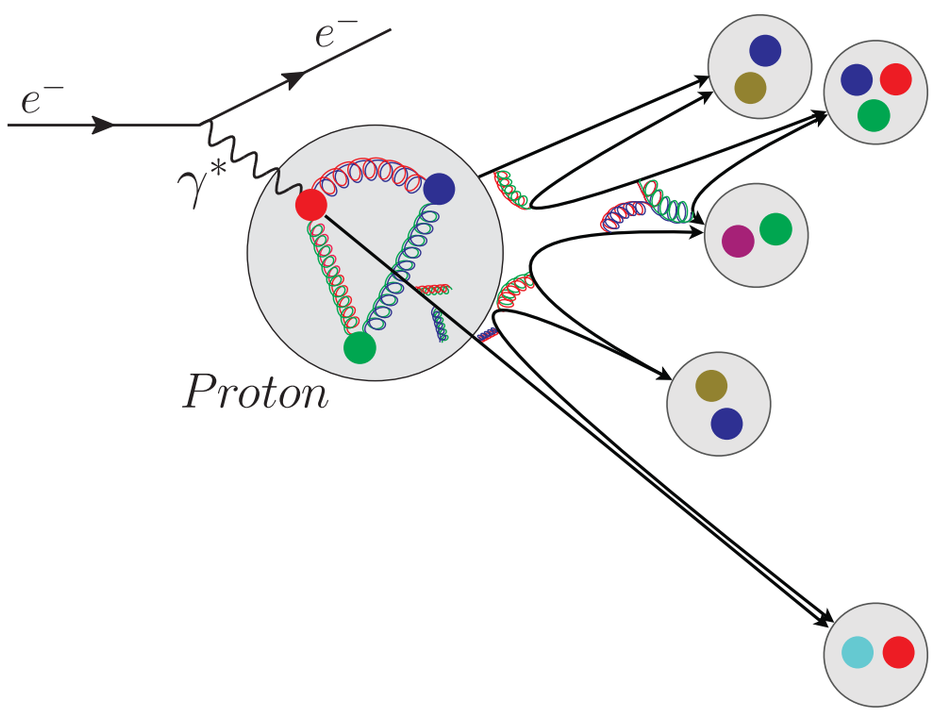
[4] *IKERBASQUE, Basque Foundation for Science, Plaza Euskadi 5, 48009 Bilbao, Spain*

arXiv:2011.01275

Project in progress with: M.G. Echevarria, I.L. Egusquiza, G. Schnell




Quantum simulation of
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# Three ingredients to describe Nature

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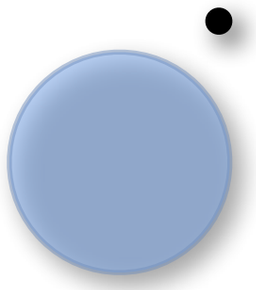
# Three ingredients to describe Nature

- Quantum matter as the basic building block

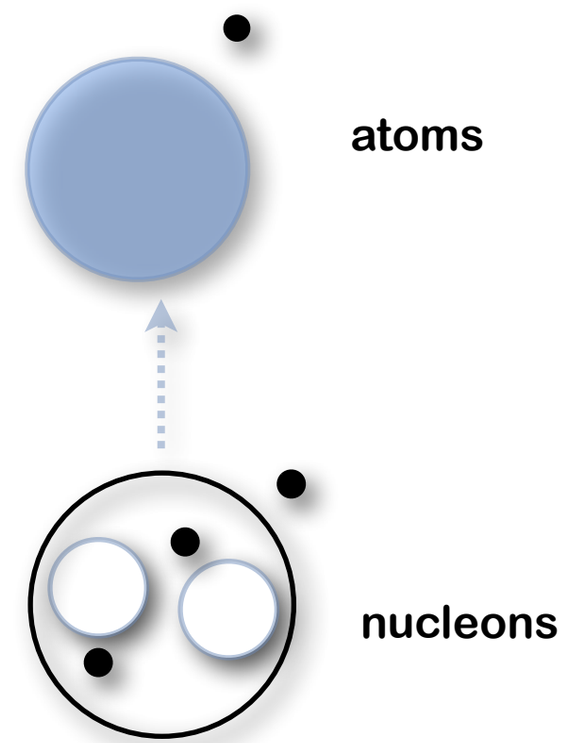


atoms

# Three ingredients to describe Nature

- Quantum matter as the basic building block

- Gauge symmetry as a fundamental principle and at the origin of every force

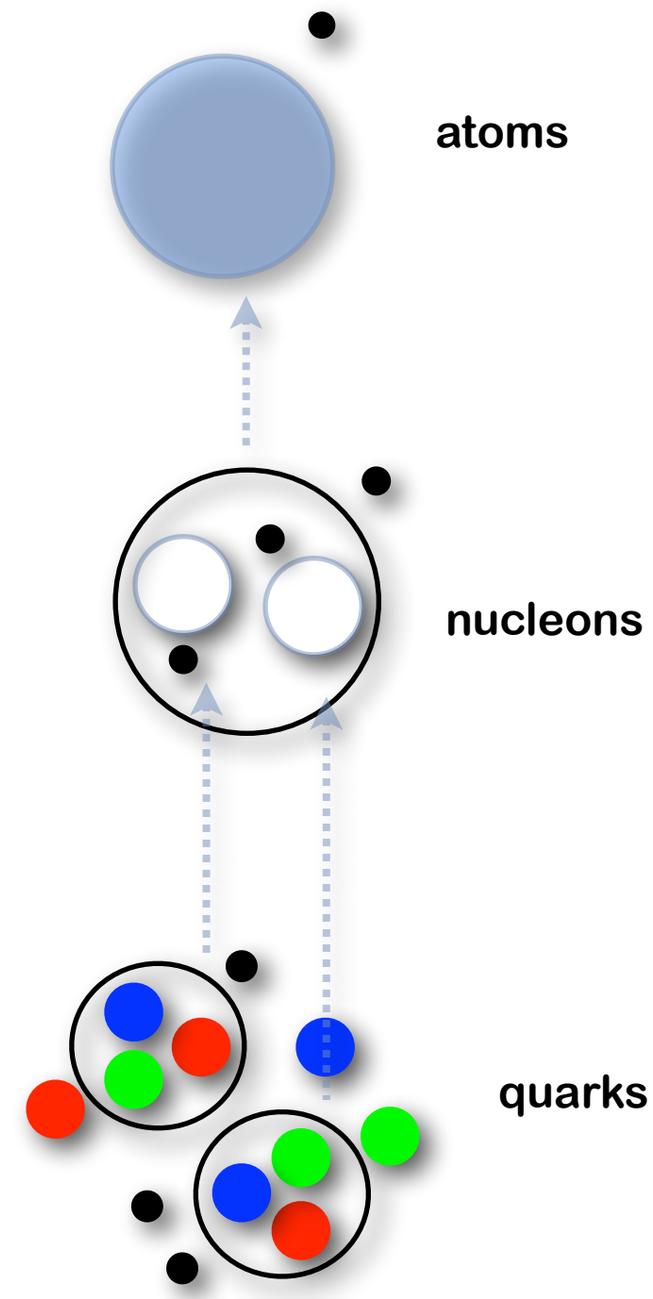


# Three ingredients to describe Nature

- Quantum matter as the basic building block

- Gauge symmetry as a fundamental principle and at the origin of every force

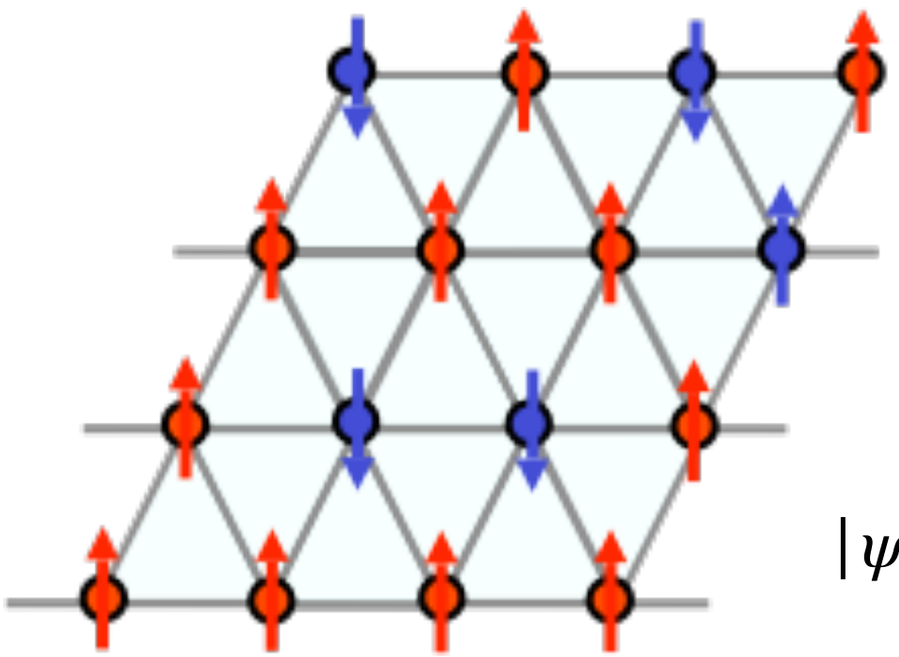
- Renormalisation group as a tool to study Nature at different scales



# Quantum matter as the basic building block

Feynman: “It is difficult to simulate quantum physics on a classical computer”

R.P. Feynman, Int. J. Theor. Phys. (1982)



Huge entanglement

$$|\psi\rangle = c_1|\uparrow\uparrow\cdots\uparrow\rangle + c_2|\uparrow\uparrow\cdots\downarrow\rangle + \cdots + c_{2^N}|\downarrow\downarrow\cdots\downarrow\rangle$$

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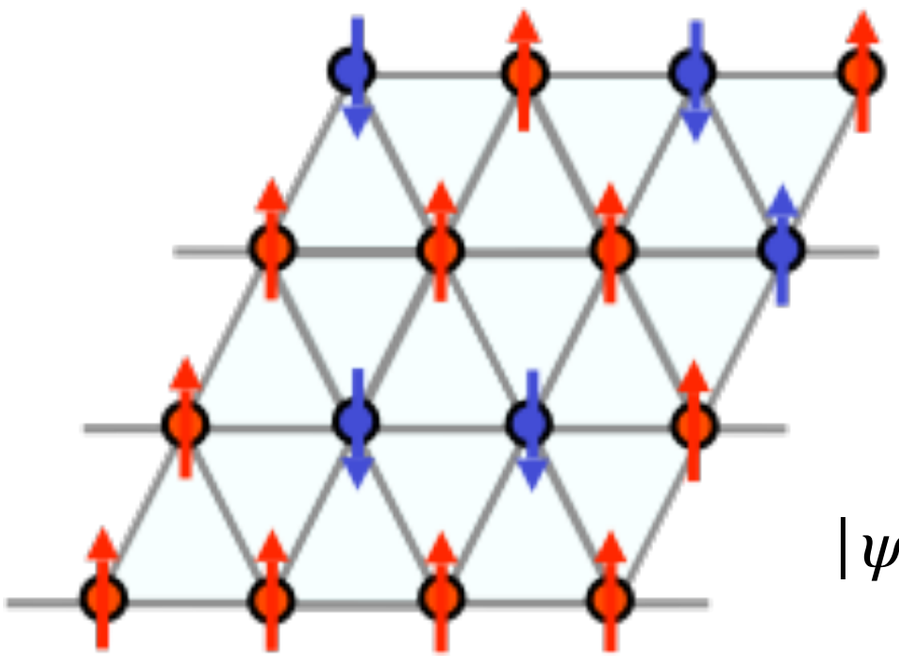

Huge entanglement

$$|\psi\rangle = c_1|\uparrow\uparrow\cdots\uparrow\rangle + c_2|\uparrow\uparrow\cdots\downarrow\rangle + \cdots + c_{2^N}|\downarrow\downarrow\cdots\downarrow\rangle$$

“Nature isn’t classical, dammit, and if you want to make a simulation of Nature, you’d better make it quantum mechanical, and by golly it’s a wonderful problem because it doesn’t look so easy.”

“Let the computer itself be built of quantum mechanical elements which obey quantum mechanical laws.”

# Quantum matter as the basic building block

**Feynman's universal quantum simulator:**
controlled quantum device which efficiently reproduces
the dynamics of any other many-particle quantum system.

More generally, **Quantum Information Technologies** studies
how to transmit and process information via quantum systems

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**Feynman's universal quantum simulator:**
controlled quantum device which efficiently reproduces
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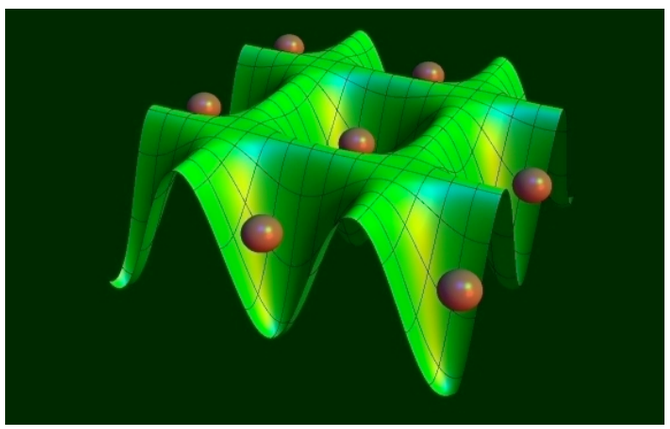
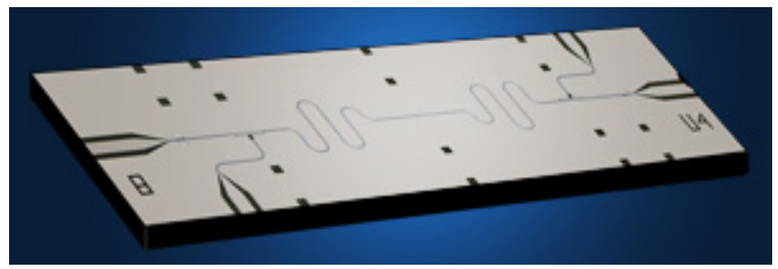
More generally, **Quantum Information Technologies** studies
how to transmit and process information via quantum systems

How?… cold atoms, ions, photons, superconducting circuit, etc.

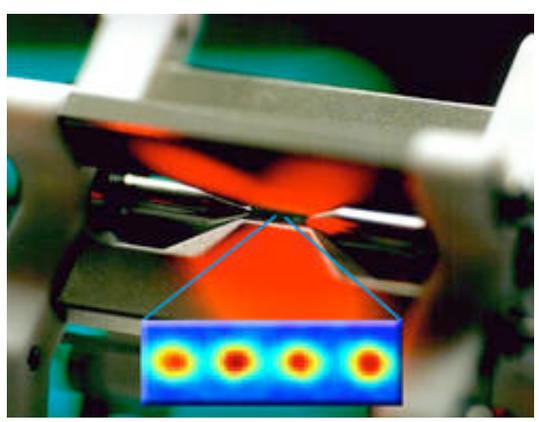


Optical lattices

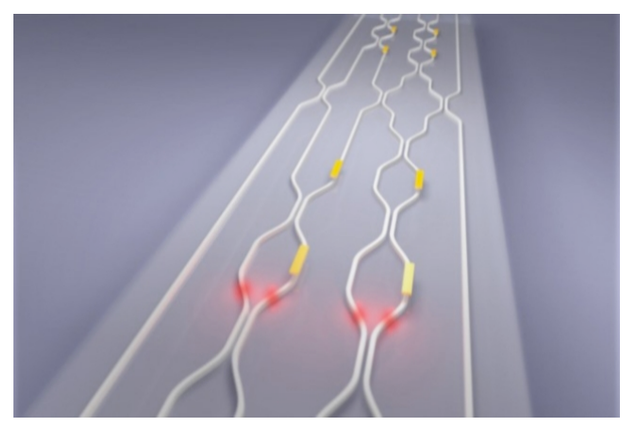


Superconducting circuits

… and several others as
quantum dots, NMR, NV centers



Trapped ions



Quantum photonics

J.I. Cirac, P. Zoller
I. Bloch, J. Dalibard, S. Nascimbène
R. Blatt, C.F. Roos,
A. Aspuru-Guzik, P. Walther
A.A. Hock, H.E. Türeci, J. Koch
Nature Physics Insight -
Quantum Simulation (2012)

# Quantum matter as the basic building block



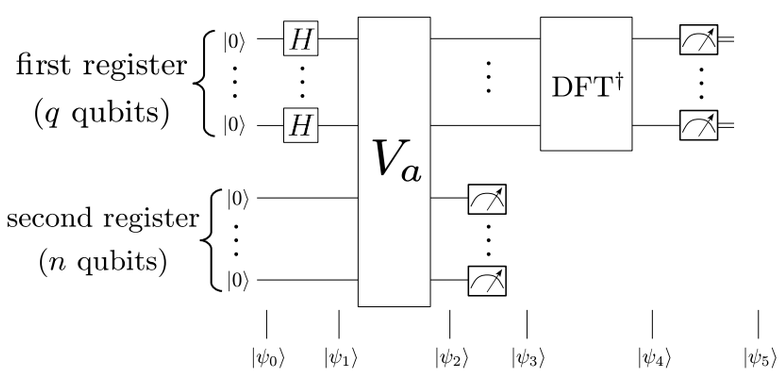
Quantum computing is a computing paradigm that exploits quantum mechanical properties (superposition, entanglement, interference...) in order to do calculations

# Quantum matter as the basic building block

Quantum computing is a computing paradigm that exploits quantum mechanical properties (superposition, entanglement, interference...) in order to do calculations

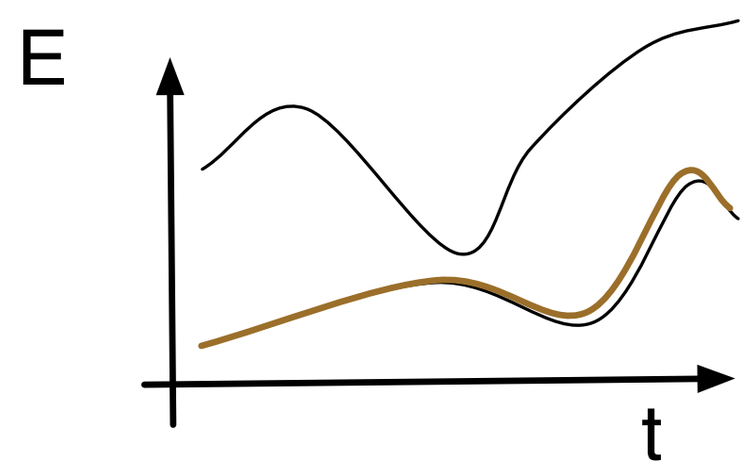
There are several models of quantum computing (they're all equivalent)

Quantum circuits
Adiabatic quantum computing
Measurement based quantum computing (MBQC)

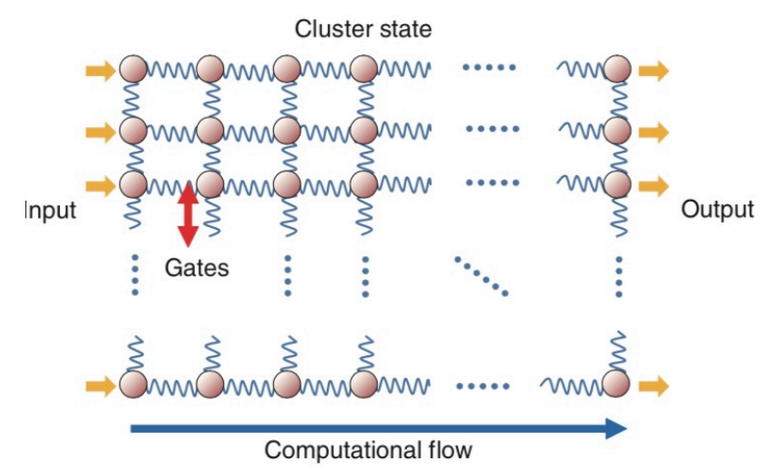



$V_a : |x\rangle|y\rangle \rightarrow |x\rangle|y \oplus a^x \mod N\rangle$

Circuit model



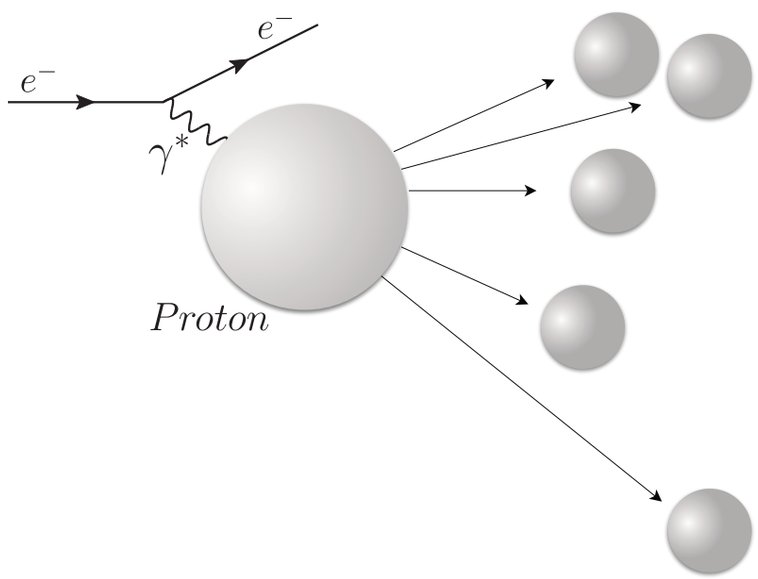

Adiabatic, quantum annealing



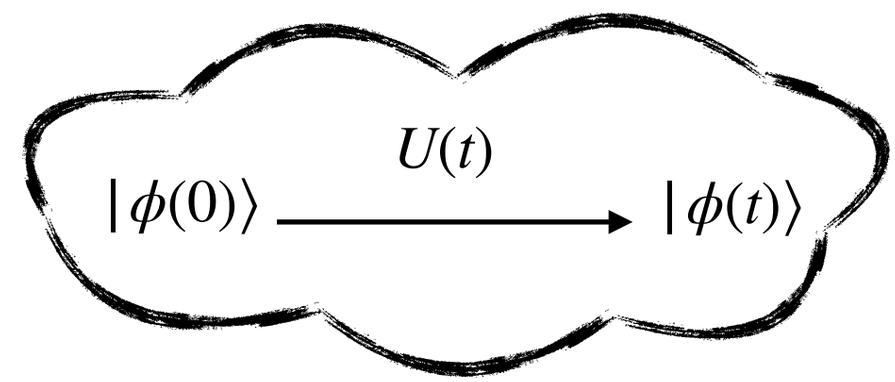

One way quantum computing

# Quantum matter as the basic building block

## Problem to compute




## Quantum system

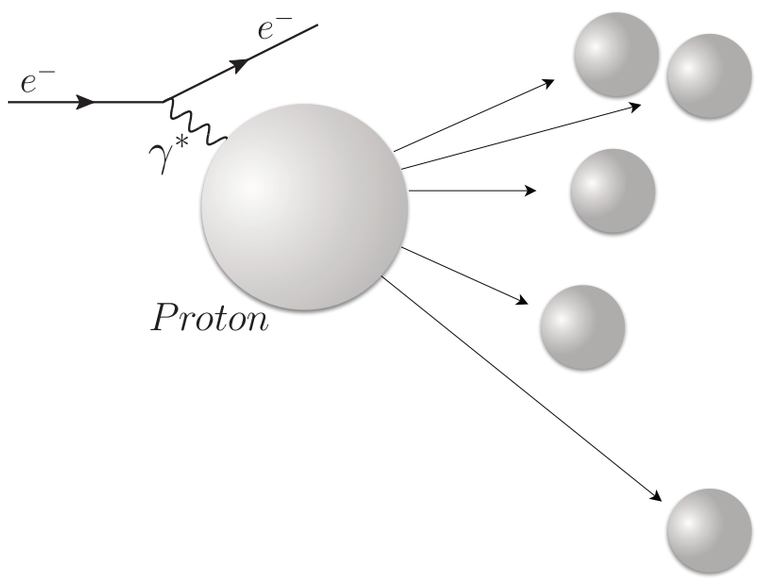



Goal: Simulate the physics of a quantum system of interest
by another system that is easier to control and to measure

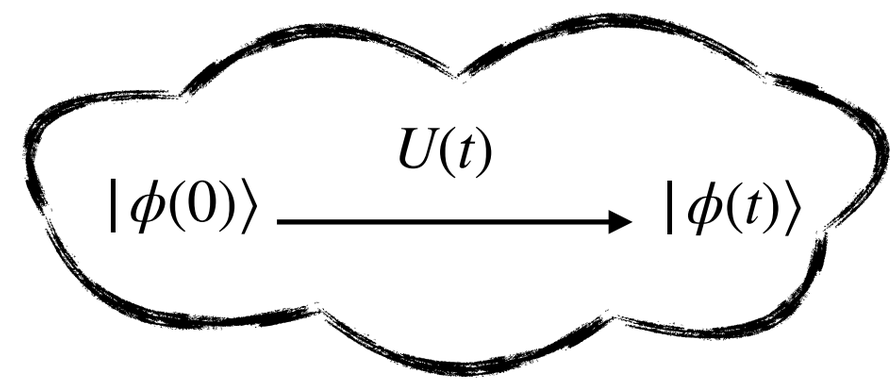
Quantum Simulation,
Rev. Mod. Phys (2014)

# Quantum matter as the basic building block

## Problem to compute




## Quantum system

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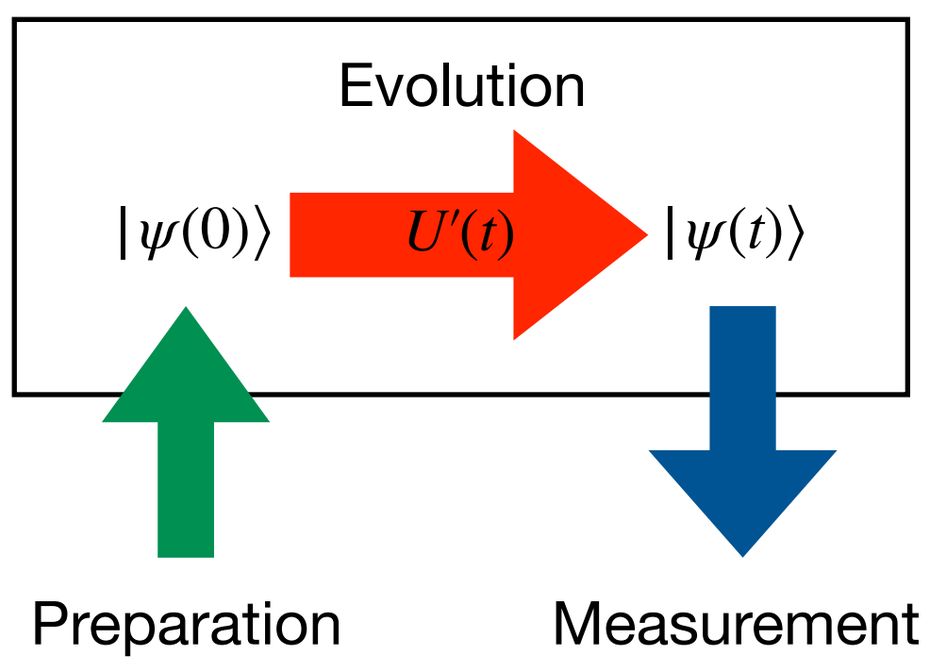
## Quantum simulator

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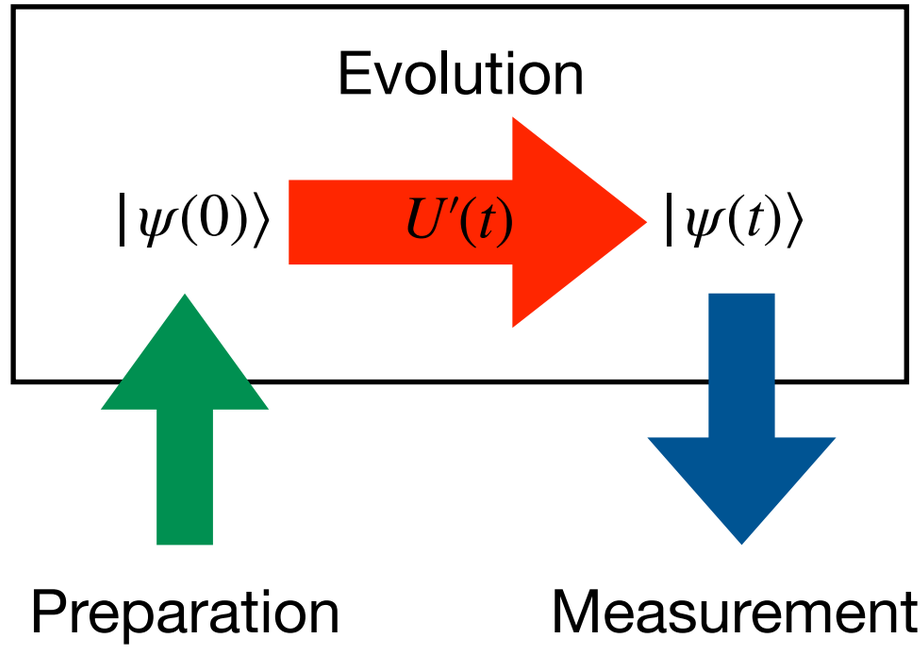
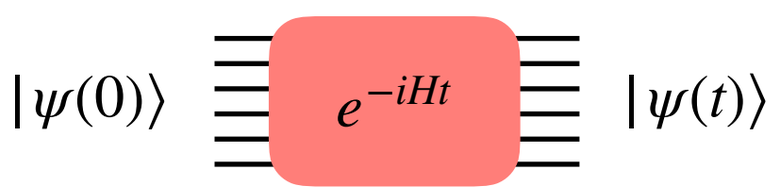
## Quantum simulation approaches



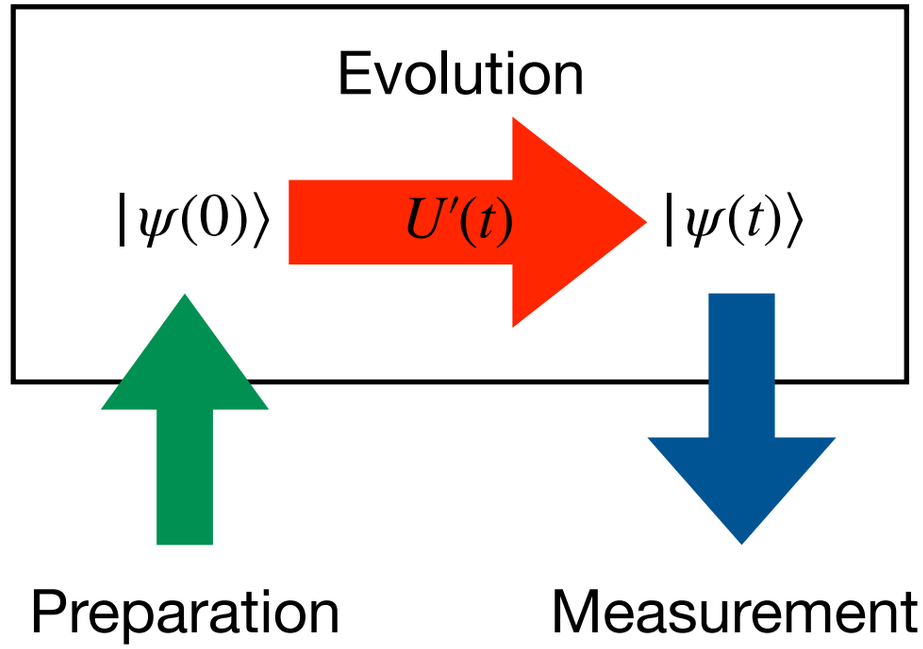

Analog simulation:
Single purposed simulator




Engineer the interactions to
emulate the Hamiltonian of the model

# Quantum matter as the basic building block

## Quantum simulation approaches

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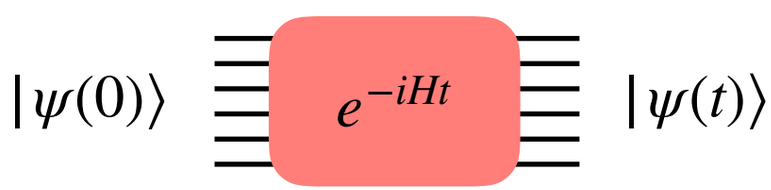
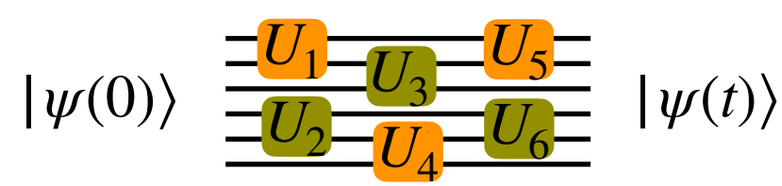



Engineer the interactions to emulate the Hamiltonian of the model

### Digital simulation: Universal simulator




Decompose dynamics into sequence of quantum gates

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# Gauge symmetry as a fundamental principle and at the origin of every force

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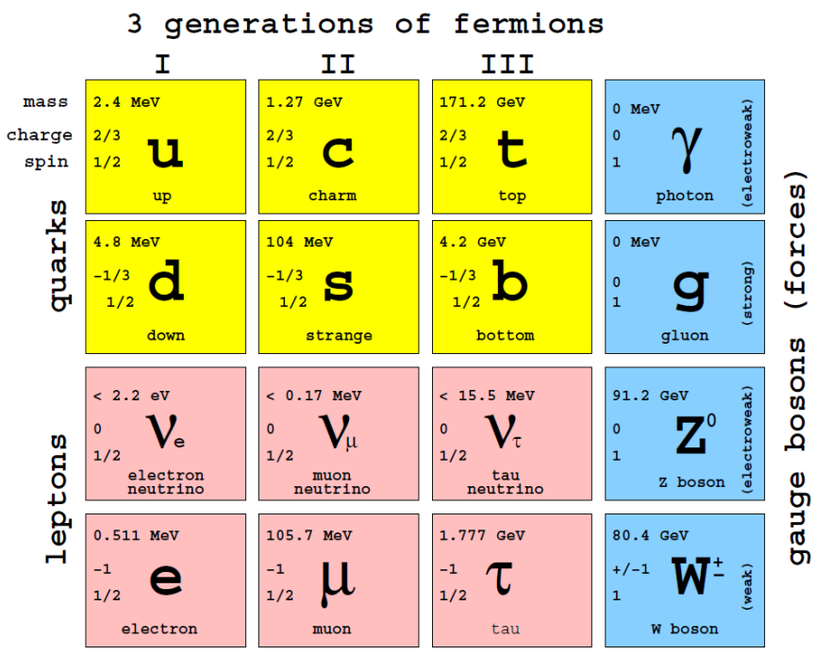


Standard model: for every force there is a gauge boson

The photon is the “carrier” of the electromagnetic force.
The $W_+$, $W_-$ and $Z_0$ are the “carriers” of the weak force.
The gluons are the “carriers” of the strong force.

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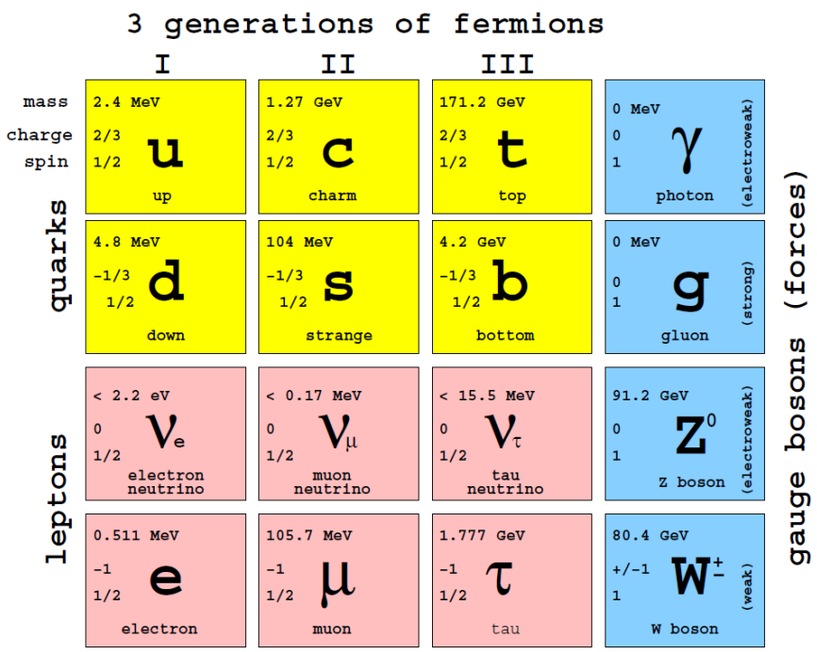


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## Standard model: a successful story

Non-abelian quantum chromodynamics (QCD) responsible for mass in every-day life
50+ years success story of parton model
40 years success story of (p)QCD (1979: discovery of gluon in e+e- at PETRA)

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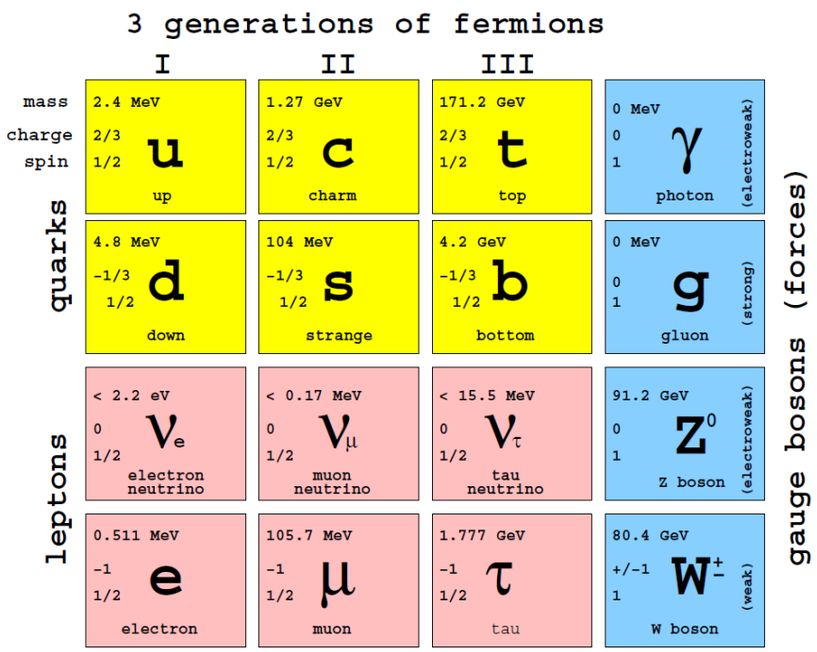


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40 years success story of (p)QCD (1979: discovery of gluon in e+e- at PETRA)

but non-perturbative part (hadron structure and formation) still a vast, partly unexplored field

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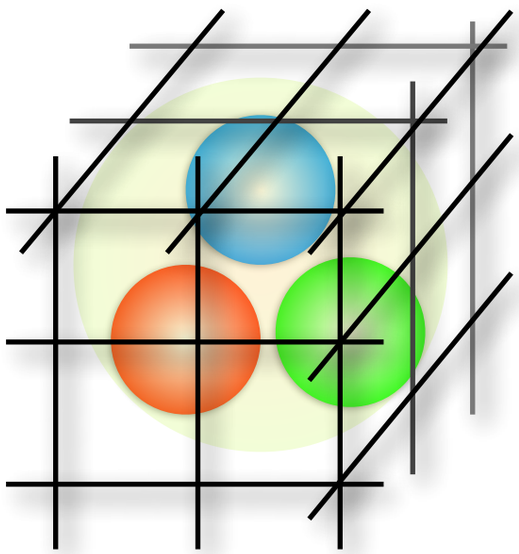
## Gauge theories on a discrete lattice structure.

Non-perturbative approach to fundamental theories of matter, e.g. Q.C.D.



K. Wilson,
Phys. Rev. D (1974)

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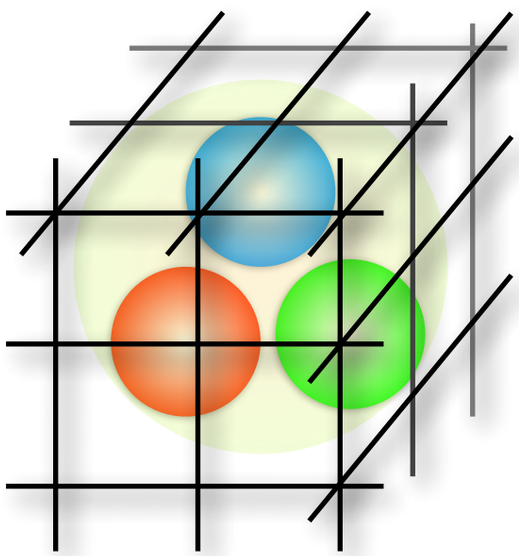

K. Wilson,
Phys. Rev. D (1974)

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}\left[\psi, U\right] e^{-S[\psi,U]} O\left[\psi, U\right]$$

$$\sim \frac{1}{N} \sum_{n=1}^{N} e^{-S[\psi_n,U_n]} O\left[\psi_n, U_n\right]$$

$$\sim \frac{1}{N} \sum_{P[U_n] \propto e^{-S[\psi_n,U_n]}} O\left[\psi_n, U_n\right]$$

Monte Carlo simulation = Classical Statistical Mechanics

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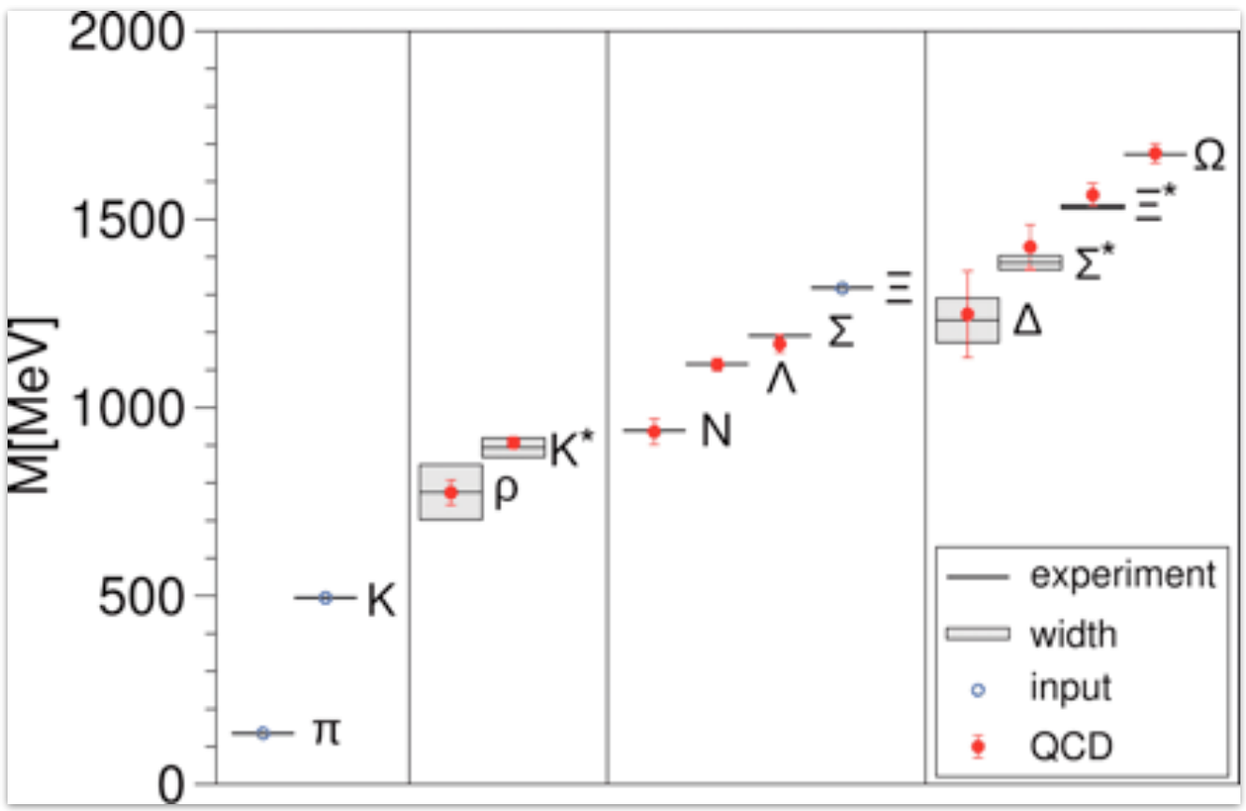
# Gauge symmetry as a fundamental principle and at the origin of every force

## Achievements by classical Monte-Carlo simulations:

first evidence of quark-gluon plasma
ab-initio estimate of the entire hadronic spectrum




S. Dürr, et al.,
Science (2008)

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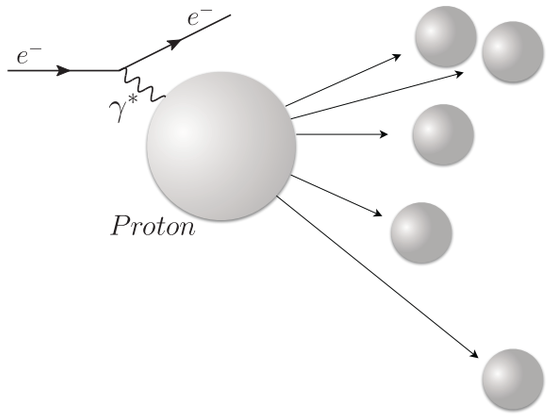
## Various flavours of sign problems in strongly correlated systems

Real time evolution:
Heavy ion experiments
(collisions)



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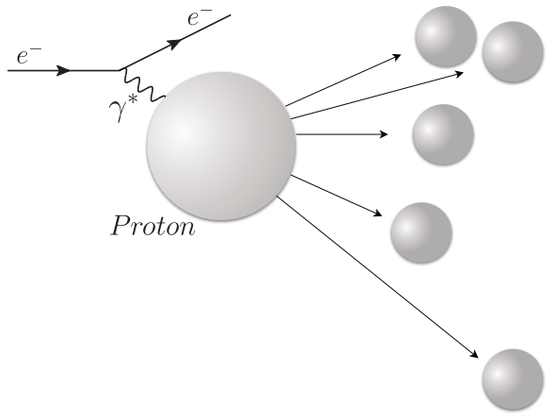
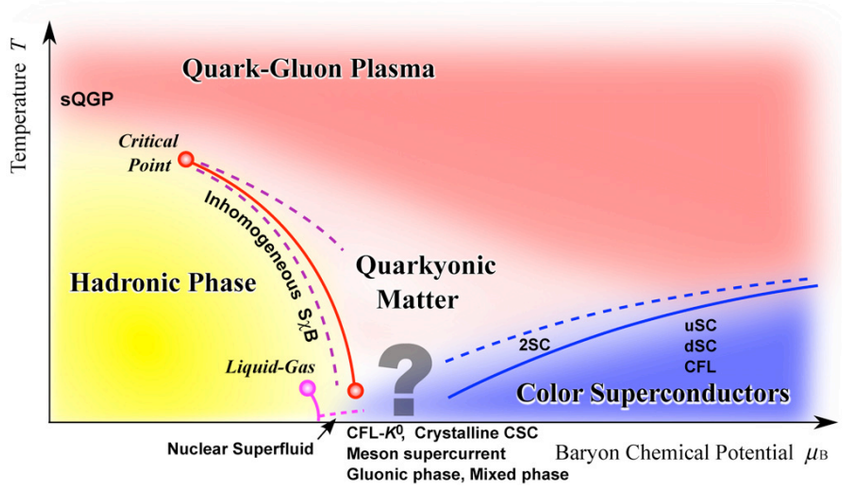



QCD with finite density of fermions:
Dense nuclear matter, color superconductivity
(phase diagram of QCD)




S. Hands, Contemp. Phys. (2001)
M.G. Alford, A. Schmitt, K. Rajagopal, T. Schäfer, Rev. Mod. Phys. (2008)
K. Fukushima, T. Hatsuda, Rep. Prog. Phys. (2011)

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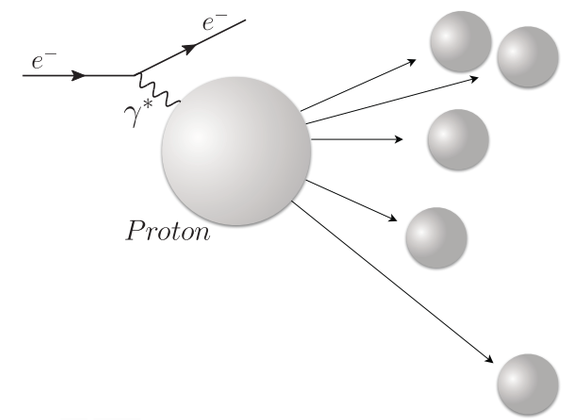


QCD with finite density of fermions:
Dense nuclear matter, color superconductivity
(phase diagram of QCD)

S. Hands, Contemp. Phys. (2001)
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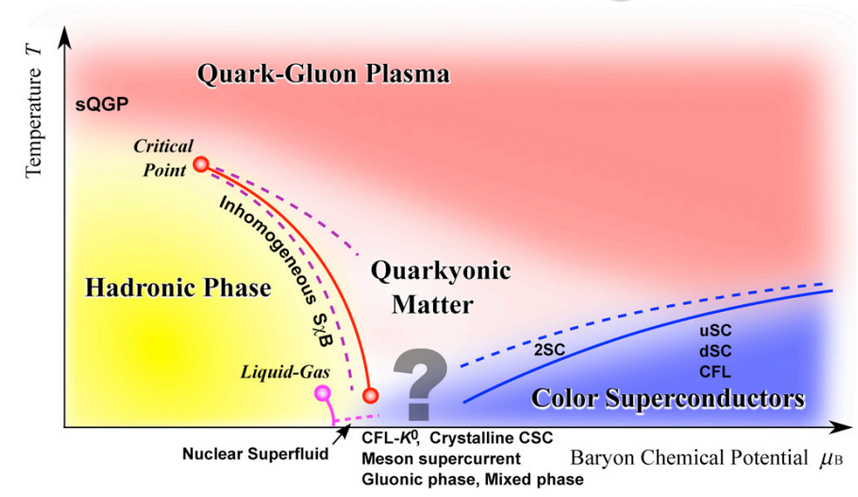
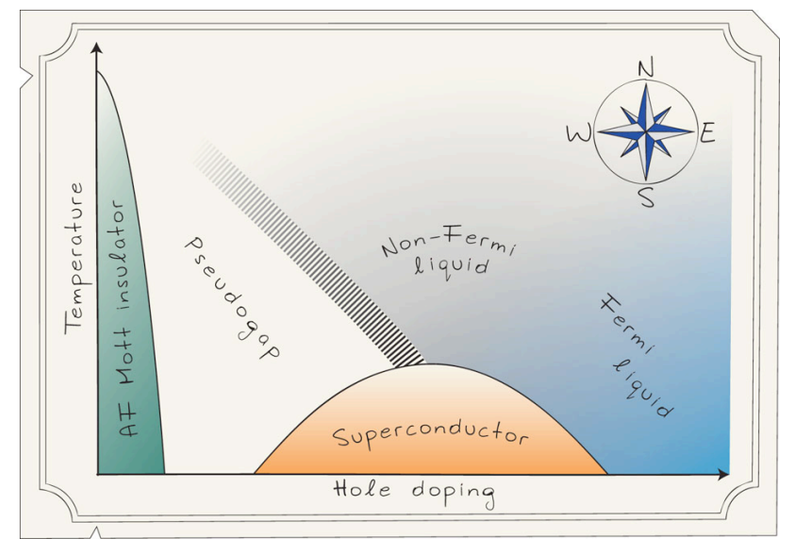



Frustrated spin models:
Spin liquid physics, RVB states
(High Tc superconductivity?)

E. Dagotto, Science (2005)
M.R. Norman, D. Pines, C. Kallinl, Adv. Phys. (2005)
P. Wahl, Nat. Phys. (2012)



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Vision: simulation of “nuclear” physics and dense “quark” matter.

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Need: design a controlled microscopic quantum simulator for lattice gauge theories.

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Goal: development of the AMO physics toolbox implementation of systems with gauge symmetry (abelian and non-abelian)

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# Simulating lattice gauge theories within quantum technologies

Vision: simulation of “nuclear” physics and dense “quark” matter.

Need: design a controlled microscopic quantum simulator for lattice gauge theories.

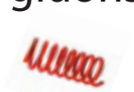
Goal: development of the AMO physics toolbox implementation of systems with gauge symmetry (abelian and non-abelian)

Aim: investigate relevant phenomena, e.g., characterise the phase diagram and dynamics of strongly coupled lattice gauge models.

High-energy and nuclear physics in a quantum simulator

quarks

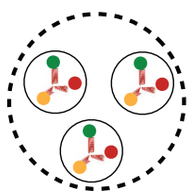
gluons



SU(3) baryon

Fermionic statistics
(confined degrees of freedom)

SU(3) nucleus



Bound states of baryons

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# Simulating lattice gauge theories within quantum technologies

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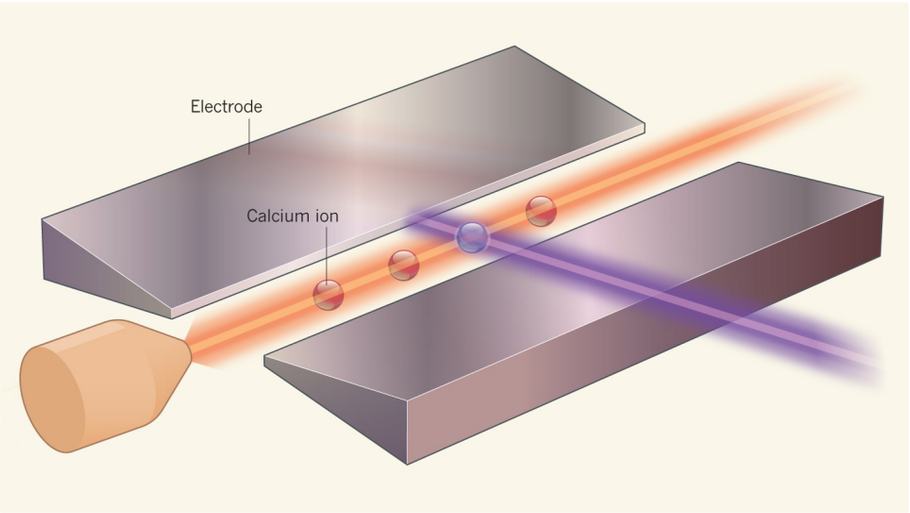



Experimental achievements

## Real-time dynamics of lattice gauge theories with a few-qubit quantum computer

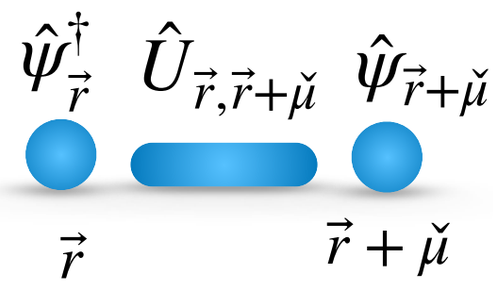
Esteban A. Martinez[1]*, Christine A. Muschik[2,3]*, Philipp Schindler[1], Daniel Nigg[1], Alexander Erhard[1], Markus Heyl[2,4], Philipp Hauke[2,3], Marcello Dalmonte[2,3], Thomas Monz[1], Peter Zoller[2,3] & Rainer Blatt[1,2]

## A scalable realization of local U(1) gauge invariance in cold atomic mixtures

**Alexander Mil[1]*, Torsten V. Zache[2], Apoorva Hegde[1], Andy Xia[1], Rohit P. Bhatt[1], Markus K. Oberthaler[1], Philipp Hauke[1,2,3], Jürgen Berges[2], Fred Jendrzejewski[1]**

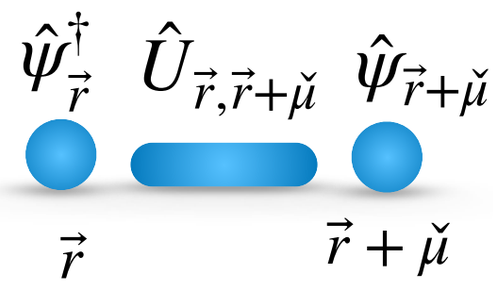
# Simulating lattice gauge theories within quantum technologies

- Quantum link formalism for gauge theories

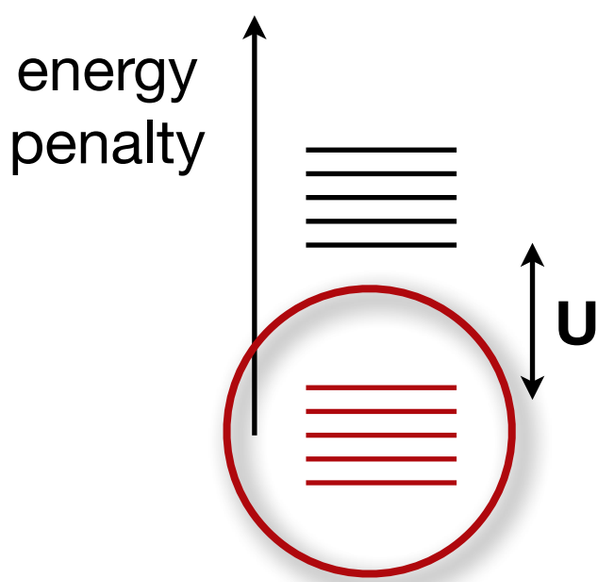


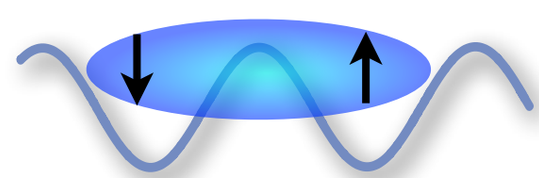
# Simulating lattice gauge theories within quantum technologies

- Quantum link formalism for gauge theories



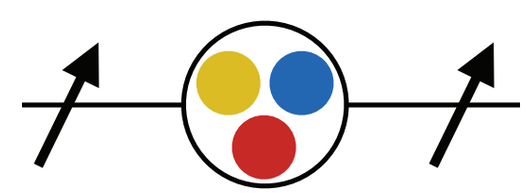

- Implementing the gauge invariance condition



color singlet
hopping

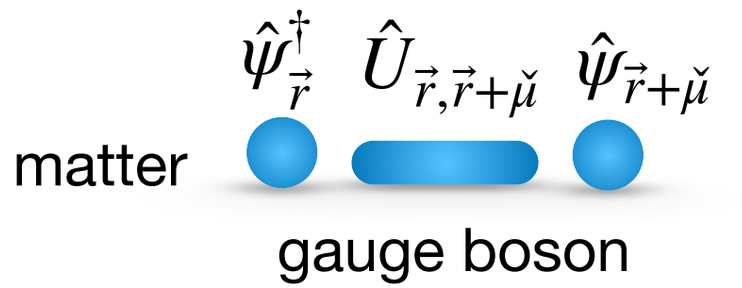
Internal symmetry



encoding
gauge invariant
degrees of freedom

# Simulating lattice gauge theories within quantum technologies

## Gauge invariant quantum Hamiltonian

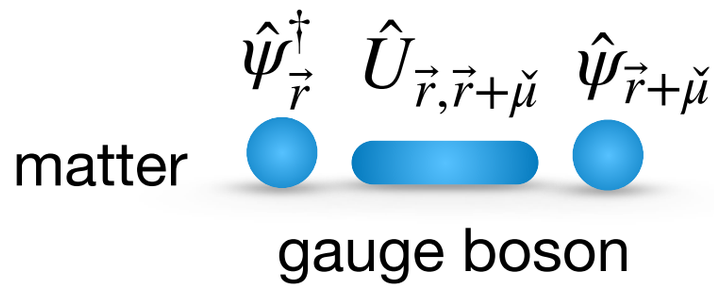



$$\hat{H} = \frac{g^2}{2}\sum_{\vec{r},\breve{\mu}} [E_{\vec{r},\vec{r}+\breve{\mu}}]^2 + \frac{1}{2}\sum_{\vec{r},\breve{\mu}} \hat{\psi}^\dagger_{\vec{r}} \hat{U}_{\vec{r},\vec{r}+\breve{\mu}} \hat{\psi}_{\vec{r}+\breve{\mu}} + m\sum_{\vec{r}} (-1)^{\vec{r}} \hat{\psi}^\dagger_{\vec{r}} \hat{\psi}_{\vec{r}} + \mathrm{h.c.}$$

“Hamiltonian formulation of Wilson's lattice gauge theories” J. Kogut, L. Susskind. PRD (1975)

# Simulating lattice gauge theories within quantum technologies

## Gauge invariant quantum Hamiltonian

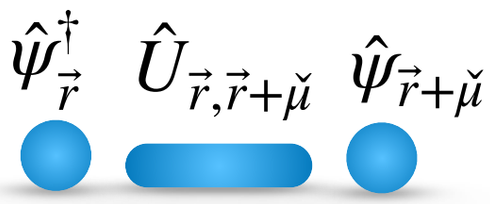



$$\hat{H} = \frac{g^2}{2}\sum_{\vec{r},\breve{\mu}} [E_{\vec{r},\vec{r}+\breve{\mu}}]^2 + \frac{1}{2}\sum_{\vec{r},\breve{\mu}} \hat{\psi}^\dagger_{\vec{r}} \hat{U}_{\vec{r},\vec{r}+\breve{\mu}} \hat{\psi}_{\vec{r}+\breve{\mu}} + m\sum_{\vec{r}} (-1)^{\vec{r}} \hat{\psi}^\dagger_{\vec{r}} \hat{\psi}_{\vec{r}} + \mathrm{h.c.}$$

| electric term (on-site interaction) | matter-gauge interaction (…?…) | staggered mass (lattice potential) |
|---|---|---|

“Hamiltonian formulation of Wilson's lattice gauge theories” J. Kogut, L. Susskind. PRD (1975)

# Simulating lattice gauge theories within quantum technologies




## Local (gauge) symmetry

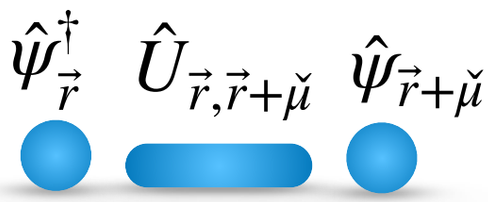
$$\hat{H} = \frac{g^2}{2}\sum_{\vec{r},\breve{\mu}} [E_{\vec{r},\vec{r}+\breve{\mu}}]^2 + \frac{1}{2}\sum_{\vec{r},\breve{\mu}} \hat{\psi}^\dagger_{\vec{r}}\hat{U}_{\vec{r},\vec{r}+\breve{\mu}}\hat{\psi}_{\vec{r}+\breve{\mu}} + m\sum_{\vec{r}} (-1)^{\vec{r}}\hat{\psi}^\dagger_{\vec{r}}\hat{\psi}_{\vec{r}} + \mathrm{h.c.}$$

$$[\hat{H}, \hat{G}_{\vec{r}}] = 0 \ \forall \vec{r}$$

$$\hat{G}_{\vec{r}} = \hat{\psi}^\dagger_{\vec{r}}\hat{\psi}_{\vec{r}} - \sum_{\breve{\mu}} \left(\hat{E}_{\vec{r},\vec{r}+\breve{\mu}} - \hat{E}_{\vec{r}-\breve{\mu},\vec{r}}\right)$$

gauge generator

# Simulating lattice gauge theories within quantum technologies




## Local (gauge) symmetry

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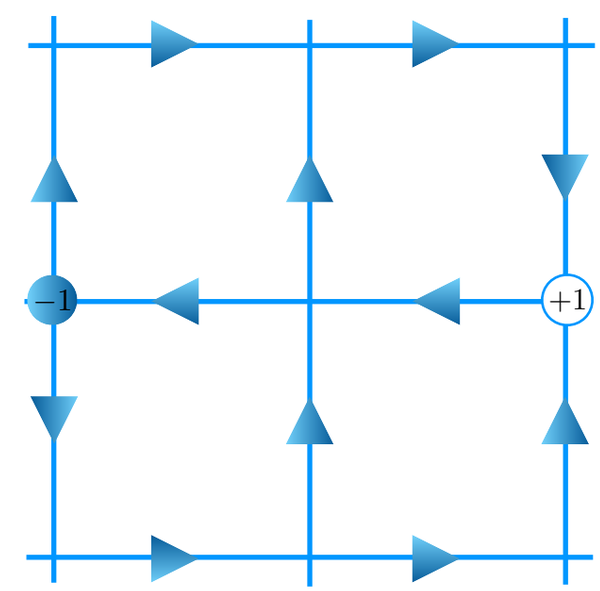
gauge generator

physical Hilbert space

$$\hat{G}_{\vec{r}} |\mathbf{phys}\rangle = 0 \ \forall \vec{r}$$




in the continuum

$$\hat{\rho} = \vec{\nabla} \cdot \hat{\vec{E}}$$

charge is the source of electric field

## Schwinger representation

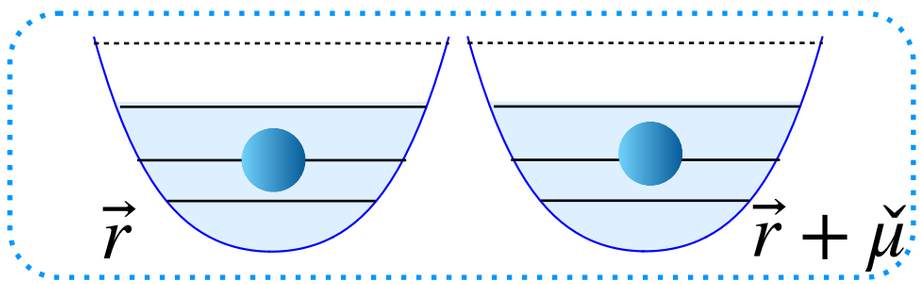
$$\hat{U}_{\vec{r},\vec{r}+\check{\mu}}$$

# Simulating lattice gauge theories within quantum technologies

## Schwinger representation

$$\hat{U}_{\vec{r},\vec{r}+\breve{\mu}}$$

$$\vec{r} \quad \vec{r}+\breve{\mu}$$




$$\hat{U}_{\vec{r},\vec{r}+\breve{\mu}} \equiv \hat{b}_{\vec{r},\breve{\mu}}\hat{b}^{\dagger}_{\vec{r}+\breve{\mu},-\breve{\mu}} \sim e^{i\left(\hat{\phi}_{\vec{r}+\breve{\mu},-\breve{\mu}} - \hat{\phi}_{\vec{r}+\breve{\mu}}\right)}$$

Gauge field = “hopping”

$$\hat{E}_{\vec{r},\vec{r}+\breve{\mu}} \equiv \frac{\hat{n}_{\vec{r}+\breve{\mu},-\breve{\mu}} - \hat{n}_{\vec{r},\breve{\mu}}}{2}$$

Electric field = occupation difference

# Simulating lattice gauge theories within quantum technologies

## Schwinger representation

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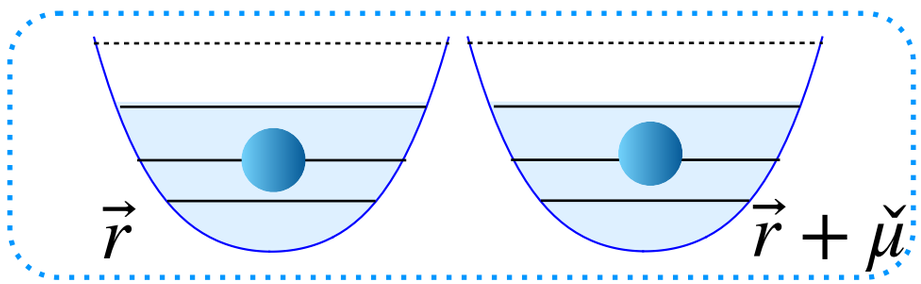


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Electric field = occupation difference

$$[\hat{b}_{\alpha}, \hat{b}^{\dagger}_{\beta}]_{\pm} = \delta_{\alpha,\beta}$$

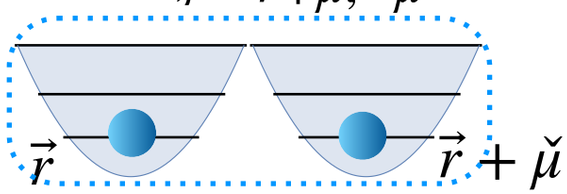
+fermion
-boson

# Simulating lattice gauge theories within quantum technologies

## Schwinger representation with internal indexes

$$\hat{U}^{\alpha\beta}_{\vec{r},\vec{r}+\breve{\mu}}$$


$$\hat{b}^{\alpha}_{\vec{r},\breve{\mu}}\hat{b}^{\beta\dagger}_{\vec{r}+\breve{\mu},-\breve{\mu}}$$




$$\alpha\beta = \begin{cases} 1 & : U(1) \\ \uparrow\downarrow & : U(2) \\ brg & : U(3) \end{cases}$$

# Simulating lattice gauge theories within quantum technologies

## Schwinger representation with internal indexes

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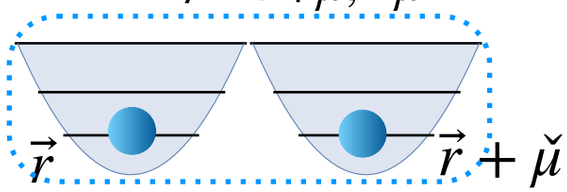
$$\hat{b}^{\alpha}_{\vec{r},\breve{\mu}}\hat{b}^{\beta\dagger}_{\vec{r}+\breve{\mu},-\breve{\mu}}$$

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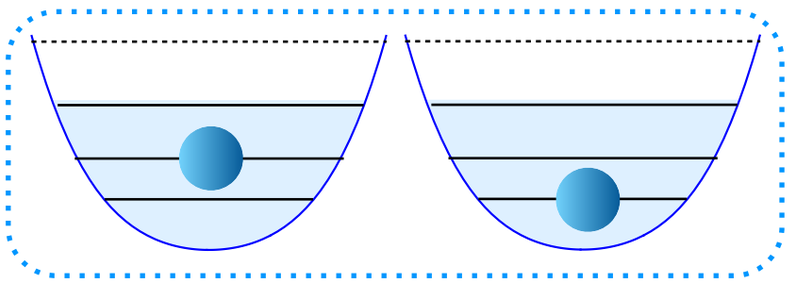
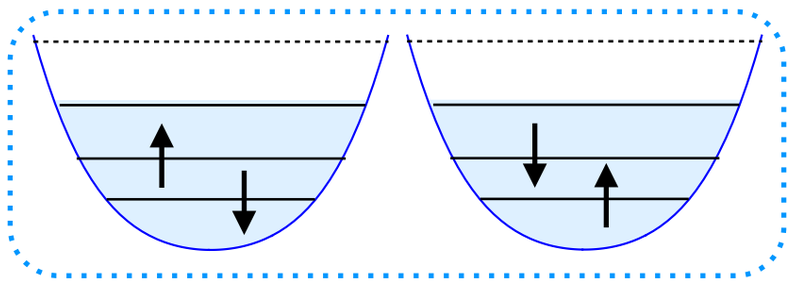
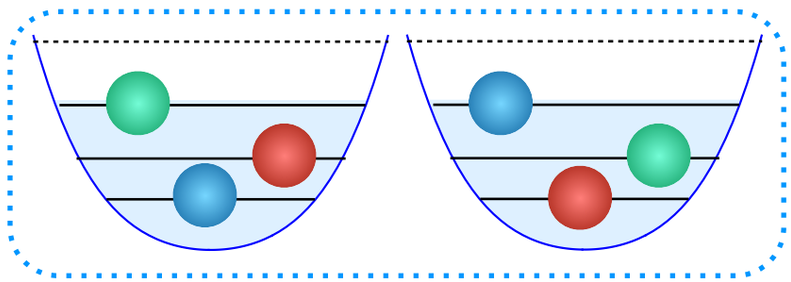


U(1) group



U(2) group



U(3) group

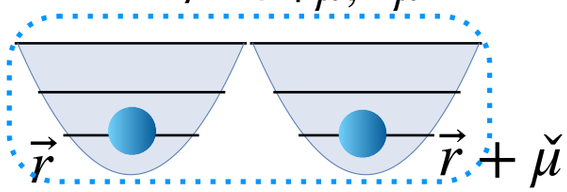
# Simulating lattice gauge theories within quantum technologies

## Schwinger representation with internal indexes

$\hat{U}^{\alpha\beta}_{\vec{r},\vec{r}+\breve{\mu}}$

$\vec{r}$ — $\vec{r}+\breve{\mu}$

$\hat{b}^{\alpha}_{\vec{r},\breve{\mu}}\hat{b}^{\beta\dagger}_{\vec{r}+\breve{\mu},-\breve{\mu}}$

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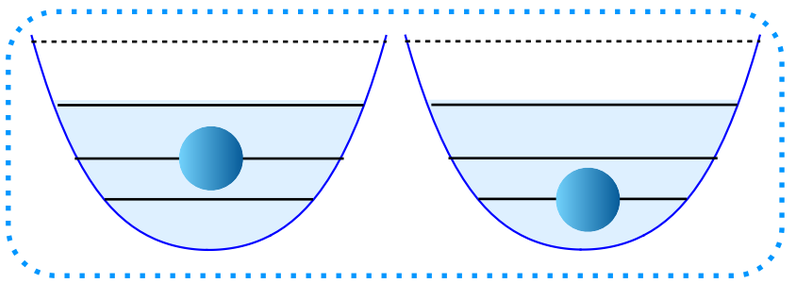
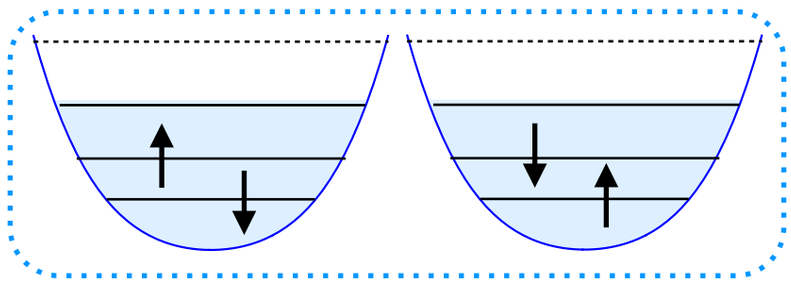
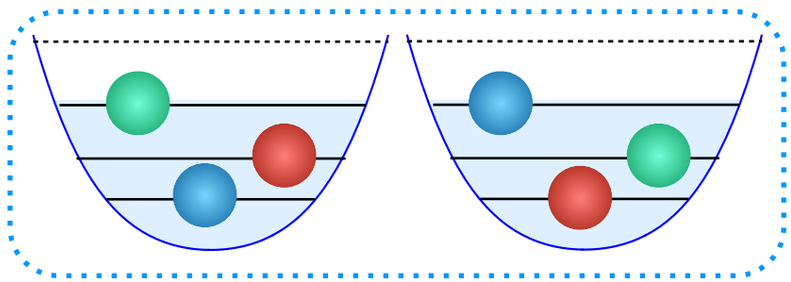


U(1) group



U(2) group



U(3) group

For orthogonal groups:

$$\hat{O}^{\alpha\beta}_{\vec{r},\vec{r}+\breve{\mu}} = \hat{c}^{\alpha}_{\vec{r},\breve{\mu}}\hat{c}^{\beta}_{\vec{r}+\breve{\mu},-\breve{\mu}}$$

$\vec{r}$ — $\vec{r}+\breve{\mu}$

$$\hat{c}^{\alpha}_{\vec{r},\breve{\mu}} = \hat{c}^{\alpha\dagger}_{\vec{r},\breve{\mu}}$$

from complex to real representations

# Simulating lattice gauge theories within quantum technologies

## Schwinger representation with internal indexes

$\hat{U}^{\alpha\beta}_{\vec{r},\vec{r}+\breve{\mu}}$

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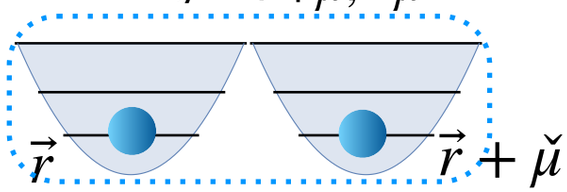
$\hat{b}^{\alpha}_{\vec{r},\breve{\mu}}\hat{b}^{\beta\dagger}_{\vec{r}+\breve{\mu},-\breve{\mu}}$

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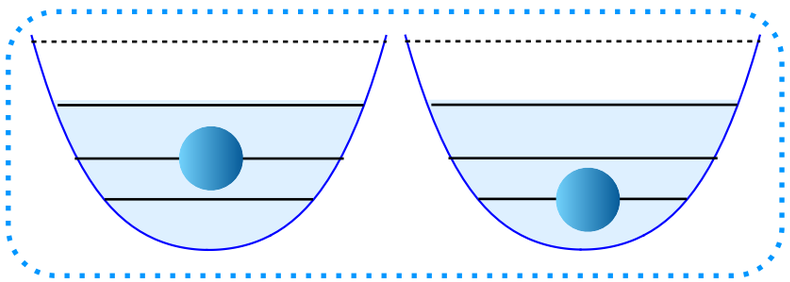
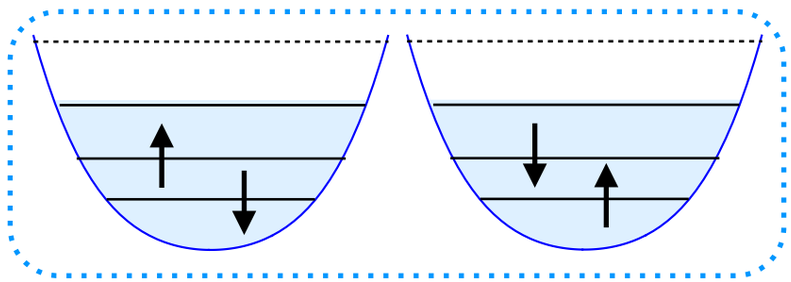
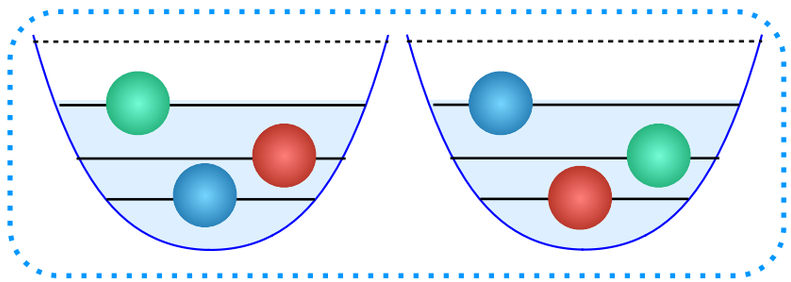


U(1) group



U(2) group



U(3) group

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$$\hat{c}^{\alpha}_{\vec{r},\breve{\mu}} = \hat{c}^{\alpha\dagger}_{\vec{r},\breve{\mu}}$$

$$\hat{c}^{\alpha}_{\vec{r},\breve{\mu}} = \hat{\sigma}^{\alpha}_{\vec{r},\breve{\mu}}$$

O(3) group

from complex to real representations

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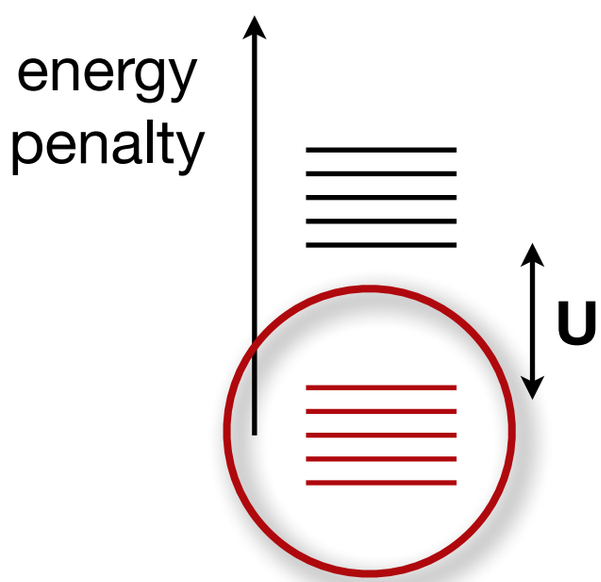
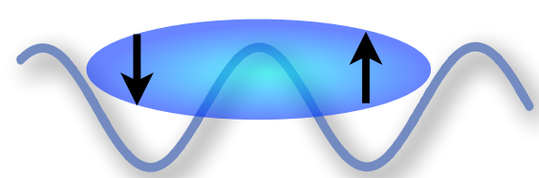
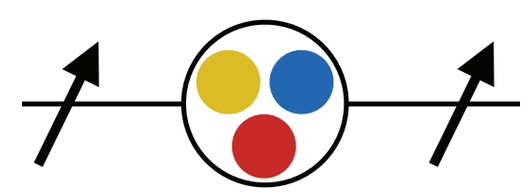
## • Implementing the gauge invariance condition



color singlet
hopping

Internal symmetry



encoding
gauge invariant
degrees of freedom

PRL **109,** 175302 (2012) PHYSICAL REVIEW LETTERS week ending 26 OCTOBER 2012

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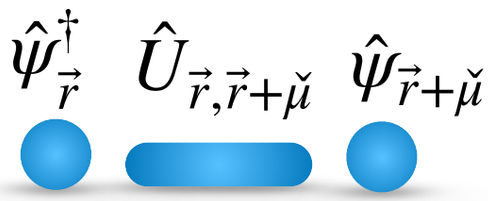
# Atomic Quantum Simulation of Dynamical Gauge Fields Coupled to Fermionic Matter: From String Breaking to Evolution after a Quench

D. Banerjee,[1] M. Dalmonte,[2,3] M. Müller,[4] E. Rico,[2,3] P. Stebler,[1] U.-J. Wiese,[1] and P. Zoller[2,3,5]




$$\hat{H} = \frac{1}{2}\sum_{\vec{r},\check{\mu}} \hat{\psi}^\dagger_{\vec{r}} \hat{U}_{\vec{r},\vec{r}+\check{\mu}} \hat{\psi}_{\vec{r}+\check{\mu}} + \mathrm{h.c.} = \frac{1}{2}\sum_{\vec{\mathrm{r}},\check{\mu}} \hat{\psi}^\dagger_{\vec{\mathrm{r}}} \hat{\mathrm{b}}_{\vec{\mathrm{r}},\check{\mu}} \hat{\mathrm{b}}^\dagger_{\vec{\mathrm{r}}+\check{\mu},-\check{\mu}} \hat{\psi}_{\vec{\mathrm{r}}+\check{\mu}} + \mathrm{h.c.}$$

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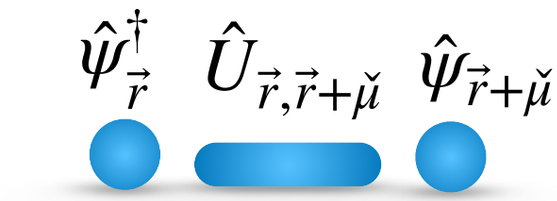


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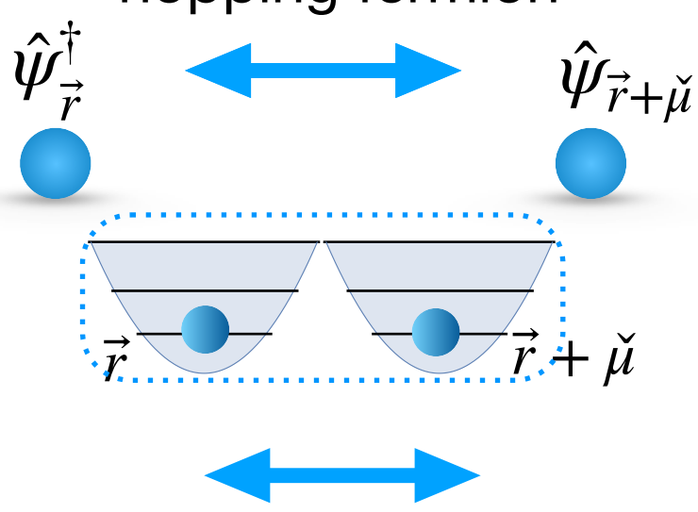
$$\hat{H}_{\mathrm{micro}} = J_F \sum_{\vec{r}} \hat{\psi}^\dagger_{\vec{r}} \hat{\psi}_{\vec{r}+\check{\mu}} + J_B \sum_{\vec{r}} \hat{b}_{\vec{r},\check{\mu}} \hat{b}^\dagger_{\vec{r},\check{\mu}} + \mathrm{h.c.}$$

hopping fermion




$$\hat{b}_{\vec{r},\check{\mu}} \hat{b}^\dagger_{\vec{r}+\check{\mu},-\check{\mu}}$$

hopping boson

PRL 109, 175302 (2012) PHYSICAL REVIEW LETTERS week ending 26 OCTOBER 2012

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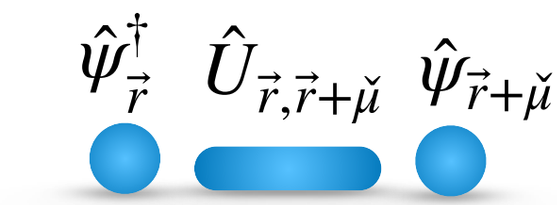
# Atomic Quantum Simulation of Dynamical Gauge Fields Coupled to Fermionic Matter: From String Breaking to Evolution after a Quench

D. Banerjee,[1] M. Dalmonte,[2,3] M. Müller,[4] E. Rico,[2,3] P. Stebler,[1] U.-J. Wiese,[1] and P. Zoller[2,3,5]




$$\hat{H} = \frac{1}{2}\sum_{\vec{r},\check{\mu}} \hat{\psi}^\dagger_{\vec{r}} \hat{U}_{\vec{r},\vec{r}+\check{\mu}} \hat{\psi}_{\vec{r}+\check{\mu}} + \mathrm{h.c.} = \frac{1}{2}\sum_{\vec{\mathrm{r}},\check{\mu}} \hat{\psi}^\dagger_{\vec{\mathrm{r}}} \hat{\mathrm{b}}_{\vec{\mathrm{r}},\check{\mu}} \hat{\mathrm{b}}^\dagger_{\vec{\mathrm{r}}+\check{\mu},-\check{\mu}} \hat{\psi}_{\vec{\mathrm{r}}+\check{\mu}} + \mathrm{h.c.}$$

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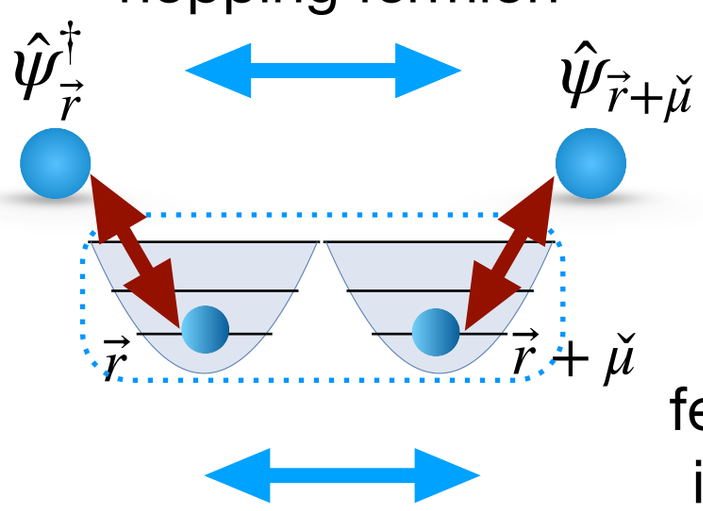
$$+ U \sum_{\vec{r}} \left(\hat{G}_{\vec{r}}\right)^2$$

Fermi-Boson Hubbard model

hopping fermion




fermi-boson interaction

$$\hat{b}_{\vec{r},\check{\mu}} \hat{b}^\dagger_{\vec{r}+\check{\mu},-\check{\mu}}$$

hopping boson

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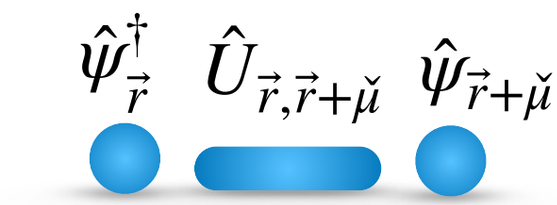


Emergent lattice gauge theory

$$\hat{H} = \frac{1}{2}\sum_{\vec{r},\check{\mu}} \hat{\psi}^\dagger_{\vec{r}} \hat{U}_{\vec{r},\vec{r}} \hat{\psi}_{\vec{r}+\check{\mu}} + \mathrm{h.c.} = \frac{1}{2}\sum_{\vec{r},\check{\mu}} \hat{\psi}^\dagger_{\vec{r}} \hat{b}_{\vec{r},\check{\mu}} \hat{b}^\dagger_{\vec{r}+\check{\mu},-\check{\mu}} \hat{\psi}_{\vec{r}+\check{\mu}} + \mathrm{h.c.}$$

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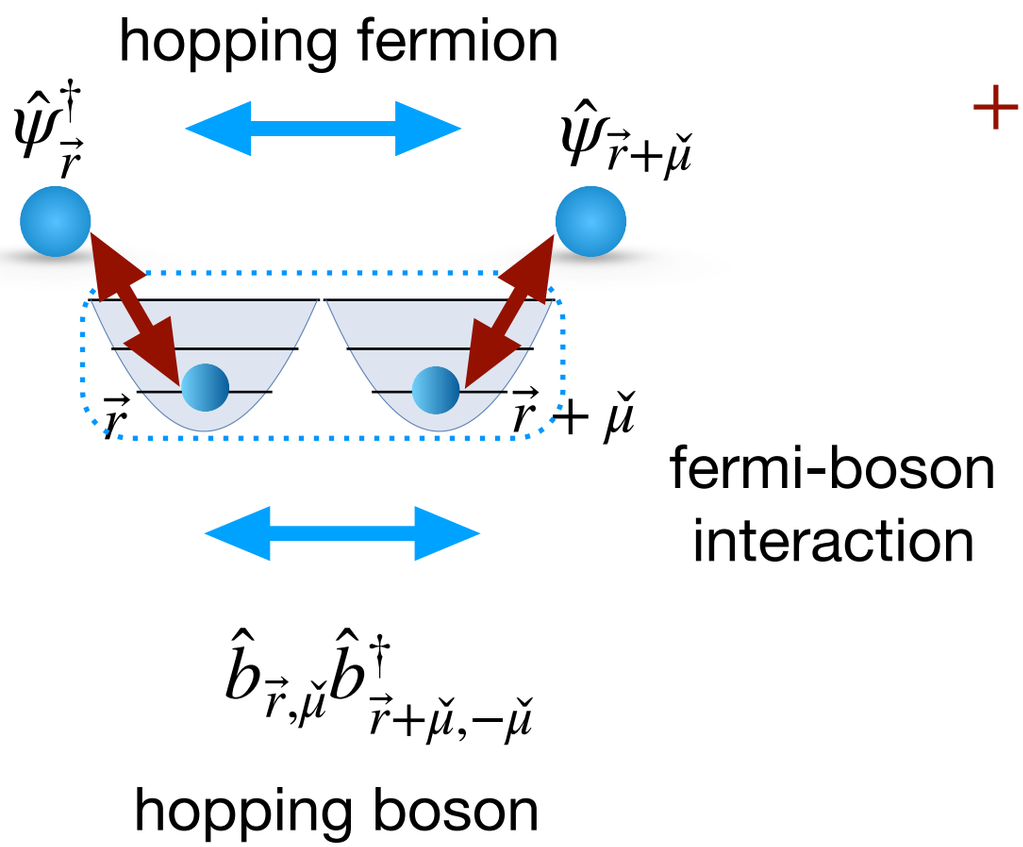


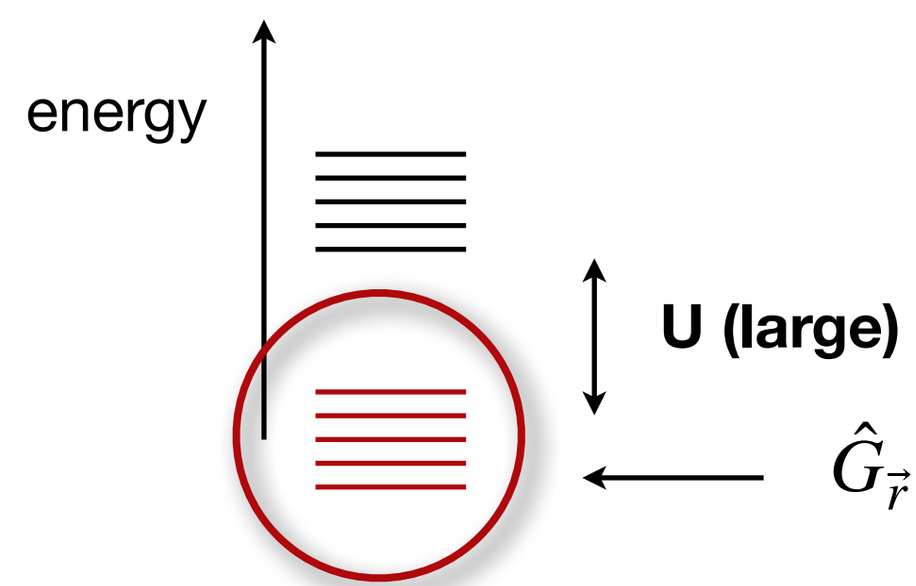


$\hat{G}_{\vec{r}} |\mathbf{phys}\rangle = 0 \;\forall \vec{r}$

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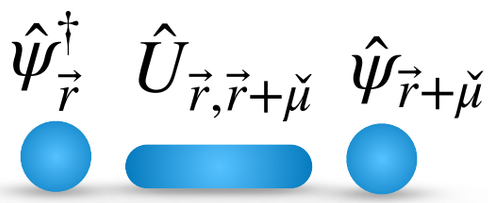


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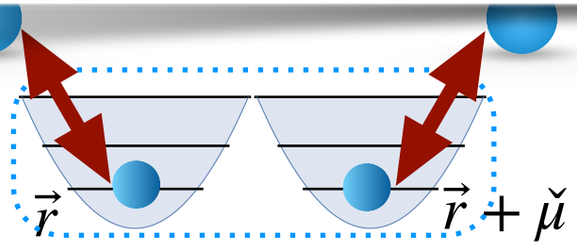
**Features of the model:**

**Real time evolution of string breaking**
**CP-phase transition in QED in (1+1)-dimensions**




fermi-boson interaction

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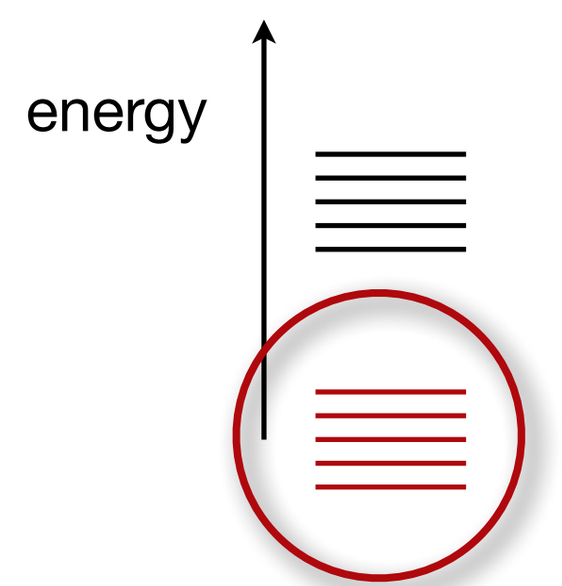
hopping boson




**U (large)**

$\hat{G}_{\vec{r}}|\mathbf{phys}\rangle = 0 \;\forall \vec{r}$

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$$\hat{\psi}^{\alpha\dagger}_{\vec{r}}\ \hat{U}^{\alpha\beta}_{\vec{r},\vec{r}+\breve{\mu}}\ \hat{\psi}^{\beta}_{\vec{r}+\breve{\mu}}$$

$$\hat{H} = \frac{1}{2}\sum_{\vec{r},\breve{\mu}} \hat{\psi}^{\alpha\dagger}_{\vec{r}} \hat{U}^{\alpha\beta}_{\vec{r},\vec{r}+\breve{\mu}} \hat{\psi}^{\beta}_{\vec{r}+\breve{\mu}} + \mathrm{h.c.} = \frac{1}{2}\sum_{\vec{\mathrm{r}},\breve{\mu}} (\hat{\psi}^{\alpha\dagger}_{\vec{\mathrm{r}}} \hat{\mathrm{b}}^{\alpha}_{\vec{\mathrm{r}},\breve{\mu}})(\hat{\mathrm{b}}^{\beta\dagger}_{\vec{\mathrm{r}}+\breve{\mu},-\breve{\mu}} \hat{\psi}^{\beta}_{\vec{\mathrm{r}}+\breve{\mu}}) + \mathrm{h.c.}$$

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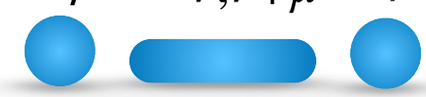

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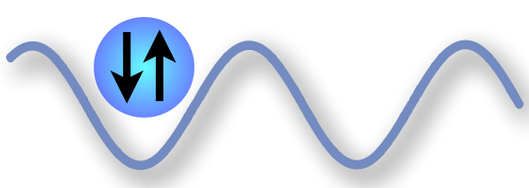

color singlet hopping

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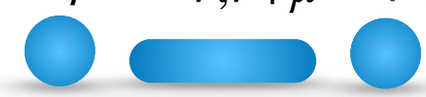

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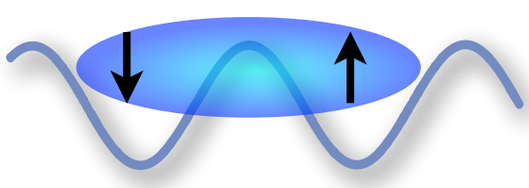

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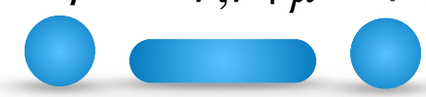

$$\hat{H} = \frac{1}{2}\sum_{\vec{r},\breve{\mu}} \hat{\psi}^{\alpha\dagger}_{\vec{r}} \hat{U}^{\alpha\beta}_{\vec{r},\vec{r}+\breve{\mu}} \hat{\psi}^{\beta}_{\vec{r}+\breve{\mu}} + \mathrm{h.c.} = \frac{1}{2}\sum_{\vec{\mathrm{r}},\breve{\mu}} (\hat{\psi}^{\alpha\dagger}_{\vec{\mathrm{r}}} \hat{\mathrm{b}}^{\alpha}_{\vec{\mathrm{r}},\breve{\mu}})(\hat{\mathrm{b}}^{\beta\dagger}_{\vec{\mathrm{r}}+\breve{\mu},-\breve{\mu}} \hat{\psi}^{\beta}_{\vec{\mathrm{r}}+\breve{\mu}}) + \mathrm{h.c.}$$

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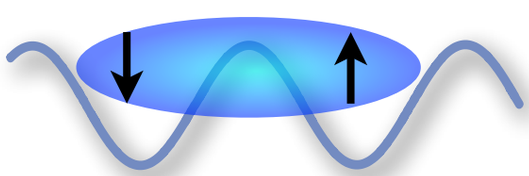
$$+U\sum_{\vec{r},\breve{\mu}} \hat{N}^{2}_{\vec{r},\breve{\mu}}$$



color singlet
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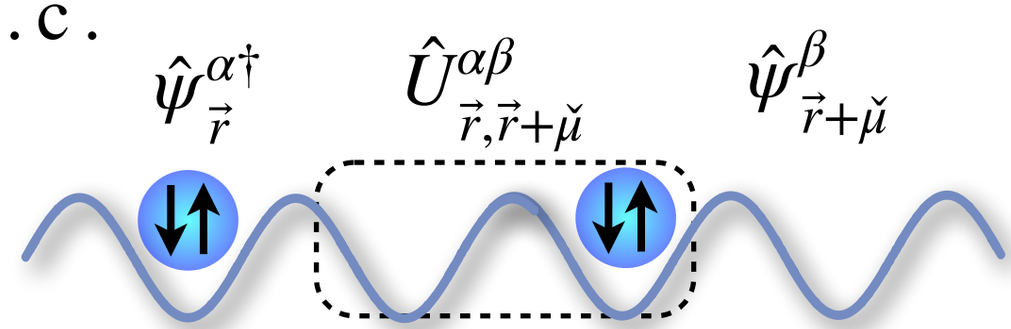


color singlet (density-density) interaction
conservation number of excitation

$$\hat{N}_{\vec{r},\breve{\mu}} = \hat{b}^{\alpha\dagger}_{\vec{r},\breve{\mu}} \hat{b}^{\alpha}_{\vec{r},\breve{\mu}} + \hat{b}^{\alpha\dagger}_{\vec{r}+\breve{\mu},-\breve{\mu}} \hat{b}^{\alpha}_{\vec{r}+\breve{\mu},-\breve{\mu}}$$

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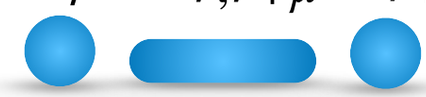

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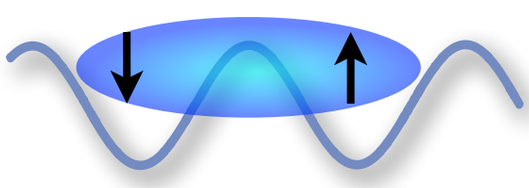
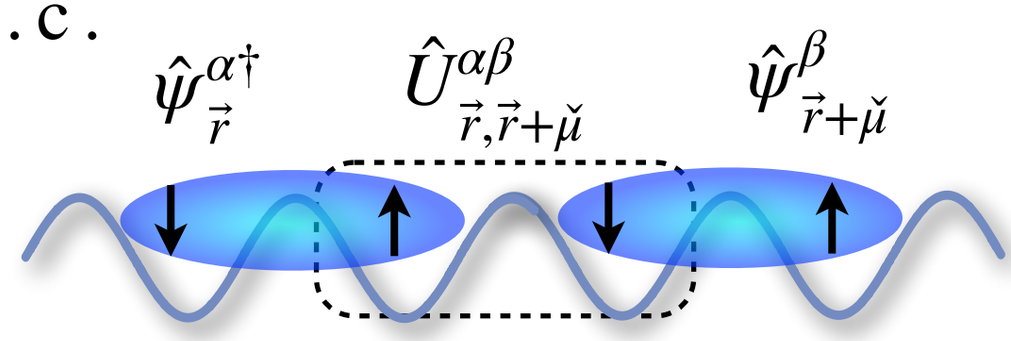


color singlet
hopping

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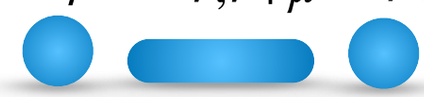
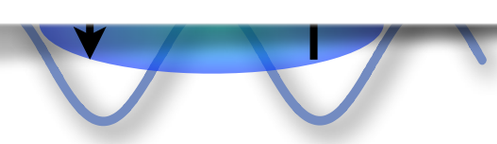


$$\hat{H} = -\frac{1}{\ }\sum \hat{\psi}^{\alpha\dagger}_{\vec{r}}\hat{U}^{\alpha\beta}_{\vec{r},\vec{r}+\check{\mu}}\hat{\psi}^{\beta}_{\vec{r}+\check{\mu}} + \mathrm{h.c.} - \frac{1}{\ }\sum (\hat{\psi}^{\alpha\dagger}_{\vec{r}}\hat{b}^{\alpha}_{\ })(\hat{b}^{\beta\dagger}_{\ }\hat{\psi}^{\beta}_{\ }) + \mathrm{h.c.}$$

**Features of the model:**

**Chiral dynamics in real time**

**Chiral SB and restoration at non-zero fermion density**



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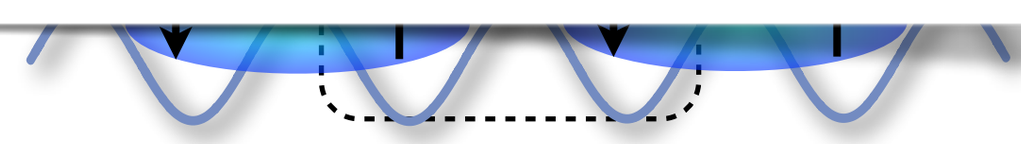

color singlet hopping

color singlet (density-density) interaction conservation number of excitation

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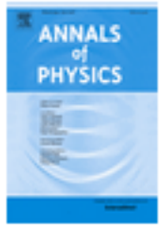
Annals of Physics
Volume 393, June 2018, Pages 466-483

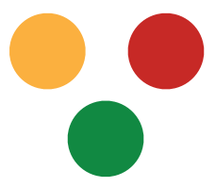



# $SO(3)$ "Nuclear Physics" with ultracold Gases ☆

E. Rico [a, b], M. Dalmonte [c], P. Zoller [d], D. Banerjee [e, f], M. Bögli [e], P. Stebler [e], U.-J. Wiese [e]

SU(3) vs. SO(3)

quarks



gluons

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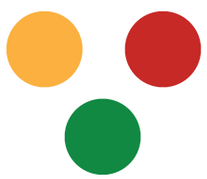
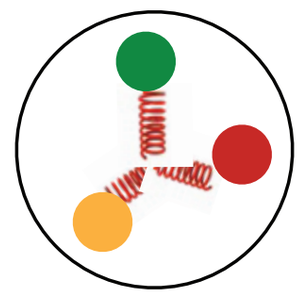
## SU(3) vs. SO(3)

quarks

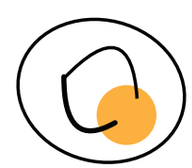


gluons

SU(3) baryon



SO(3) 'baryon'



**Fermionic statistics**

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# $SO(3)$ "Nuclear Physics" with ultracold Gases ☆

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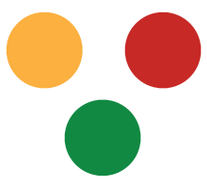
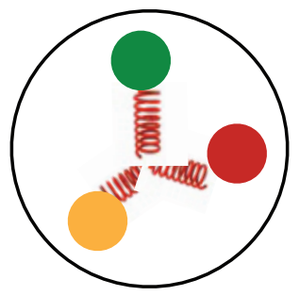
## SU(3) vs. SO(3)

quarks



gluons

SU(3) baryon



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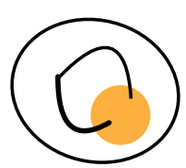

**Fermionic statistics**

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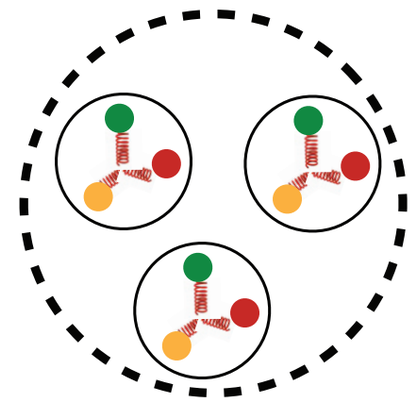
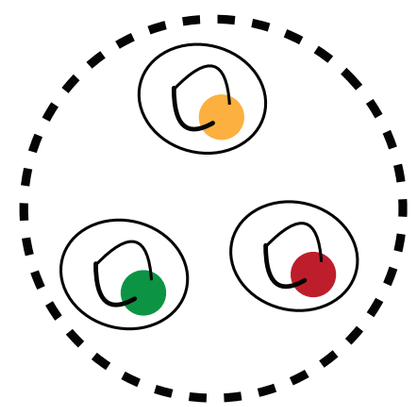


SO(3) nucleus



**Bound states of baryons**

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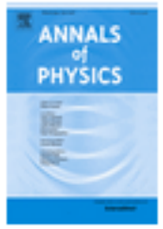


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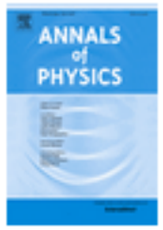


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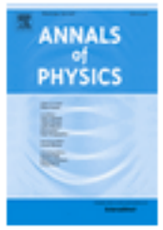


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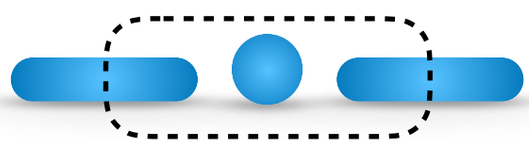

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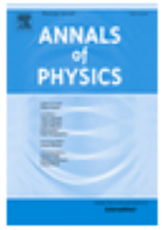


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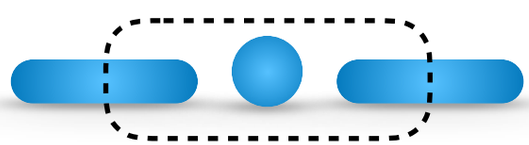

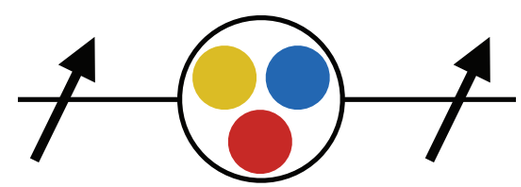

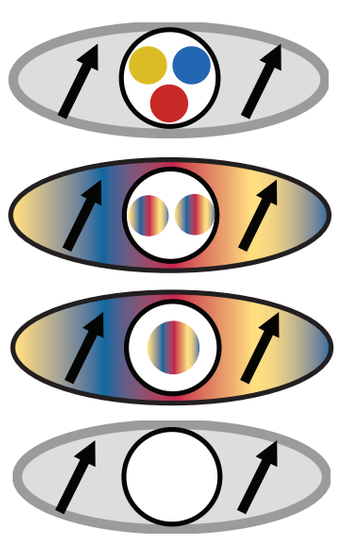

Gauge invariant Hilbert space
Singlet among the matter and gauge fields

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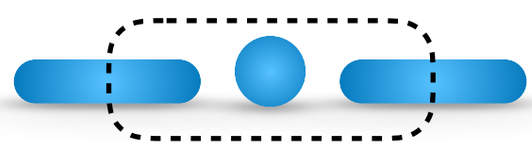

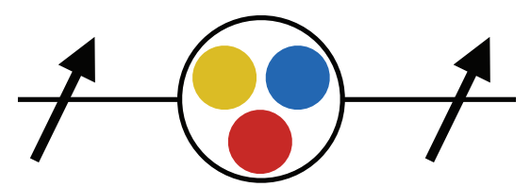

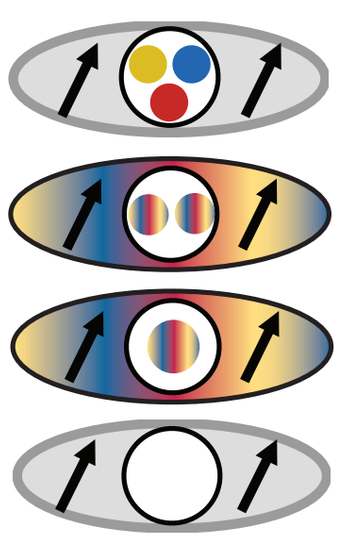

Gauge invariant Hilbert space
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Exact encoding to a spin-3/2 model
Matter content maps to z spin component

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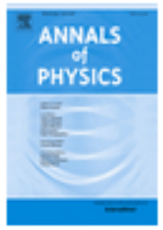


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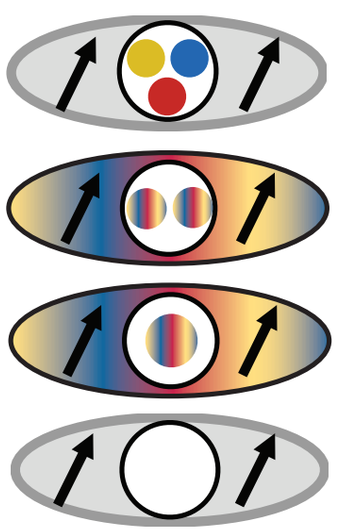
## Features of the model:

**SB of chiral symmetry and its restoration at finite baryon density**
**Existence of stable bound states (binding energy)**



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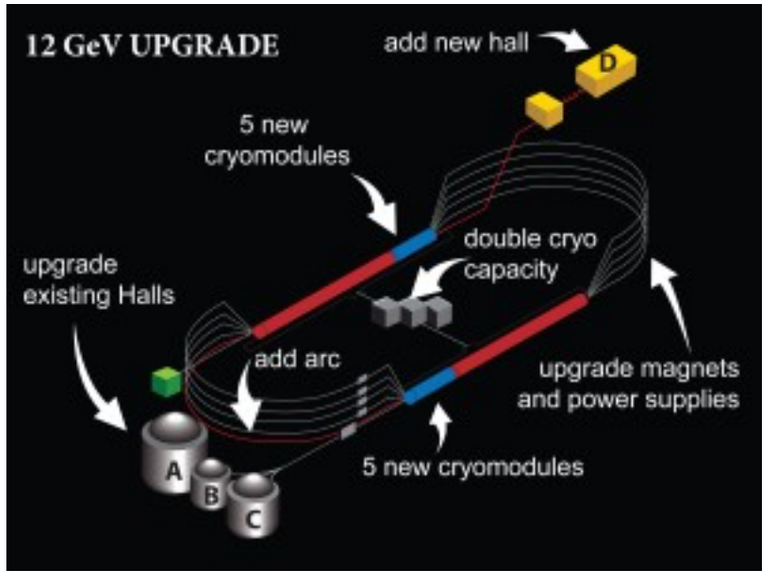
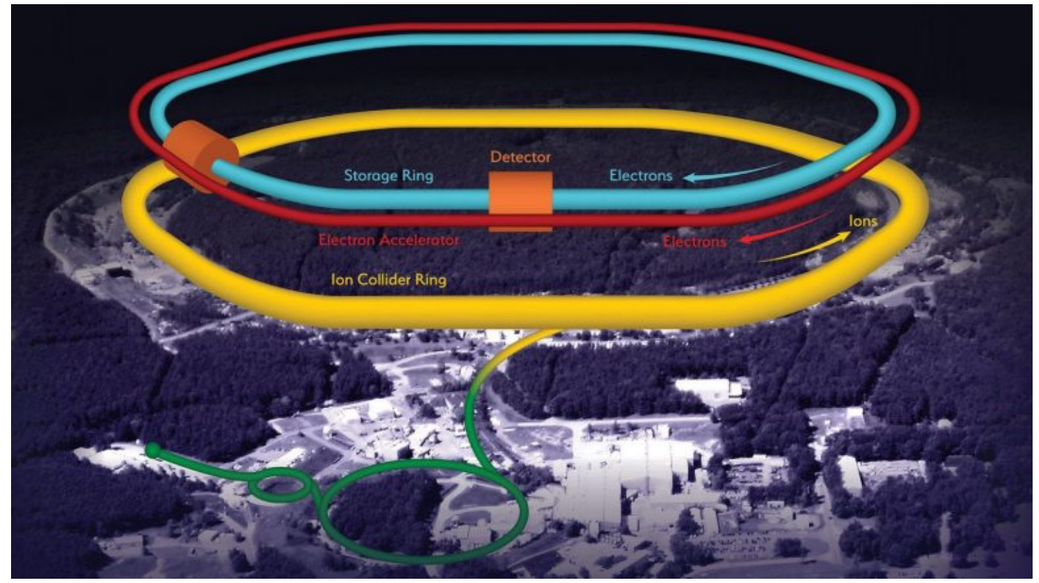
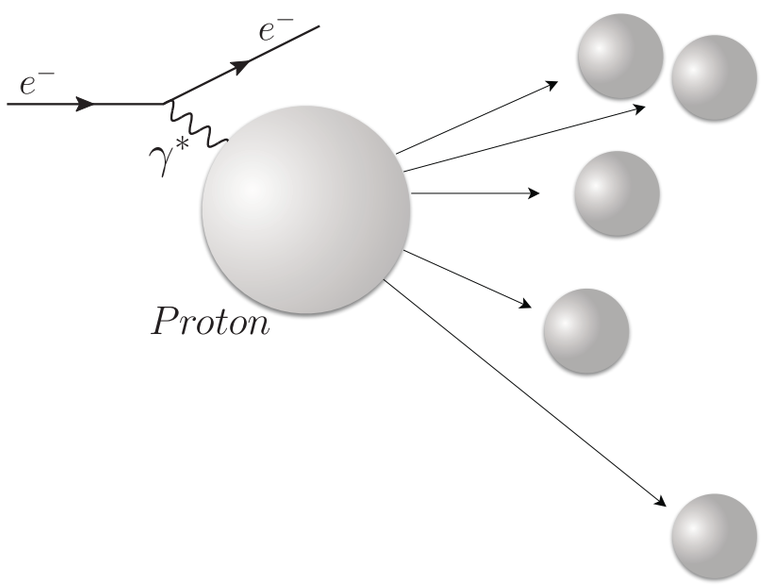
# Quantum simulation of light-front parton correlators




modern microscopes




(semi-inclusive)
deep-inelastic lepton
scattering

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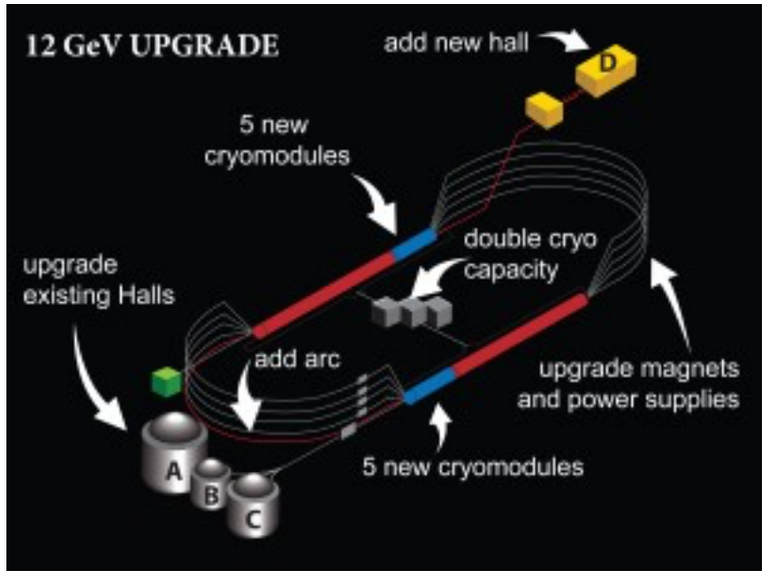


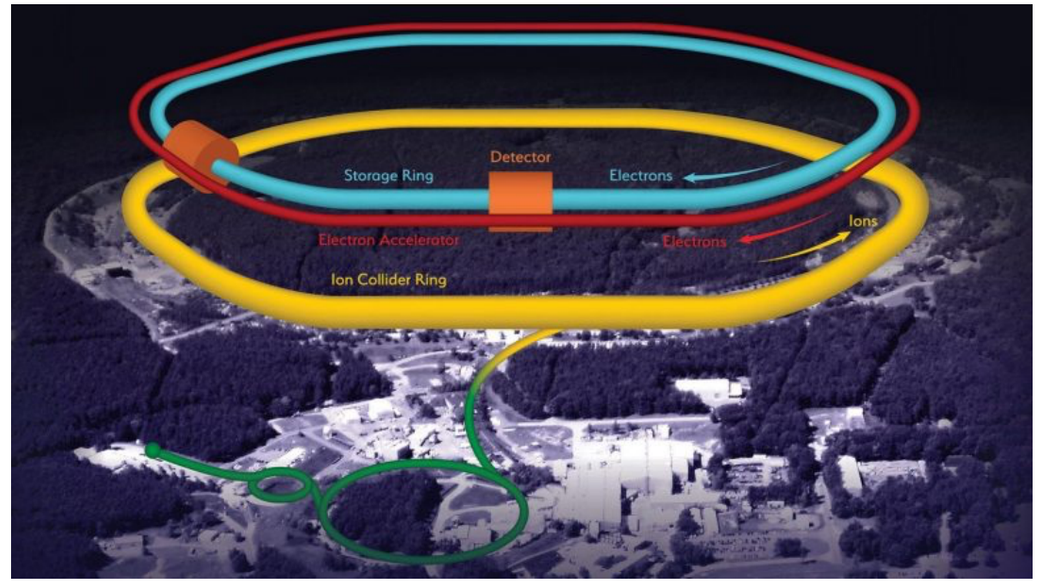


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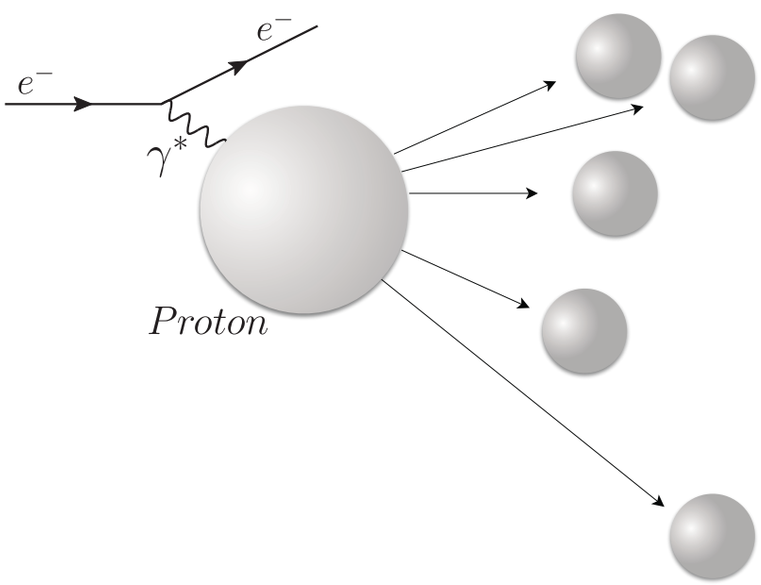


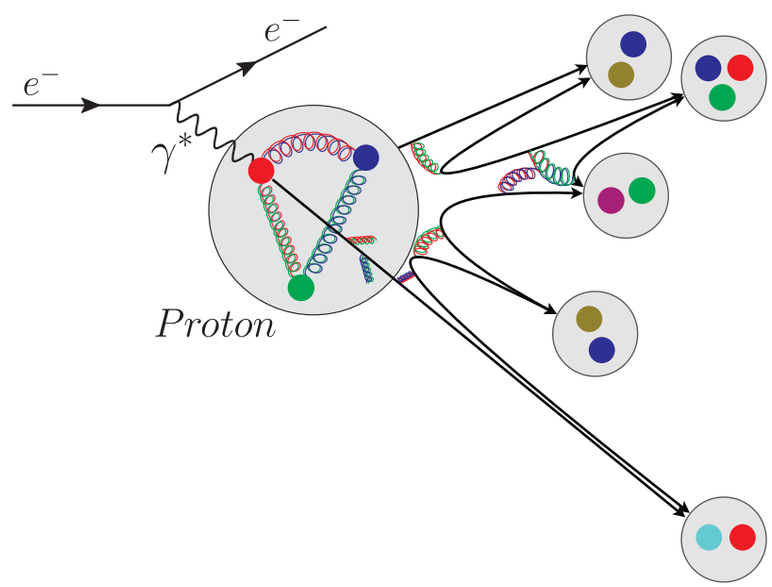


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deep-inelastic lepton
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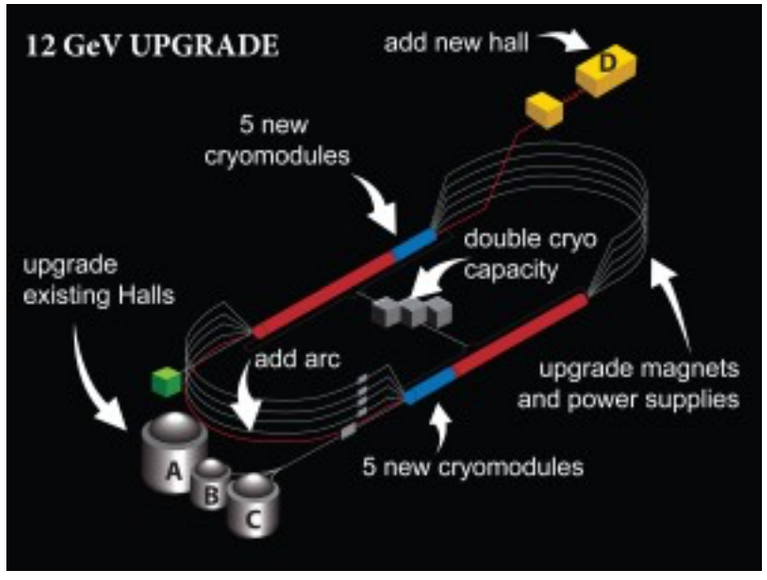
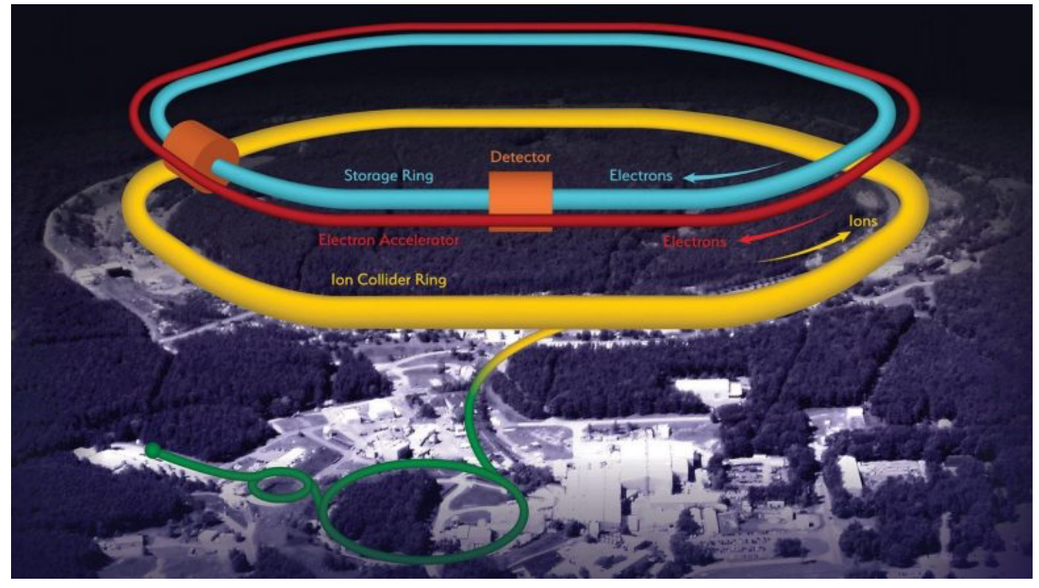
highly virtual photons
resolve inner (partonic)
structure

# Quantum simulation of light-front parton correlators

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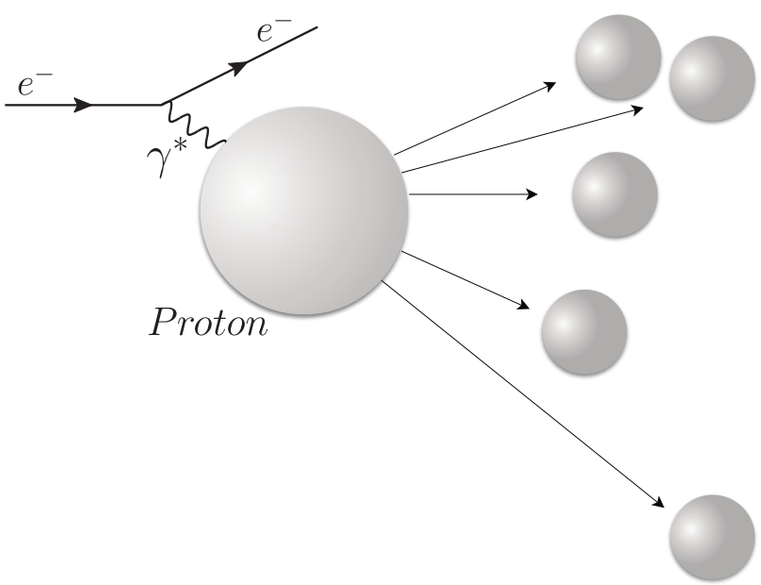


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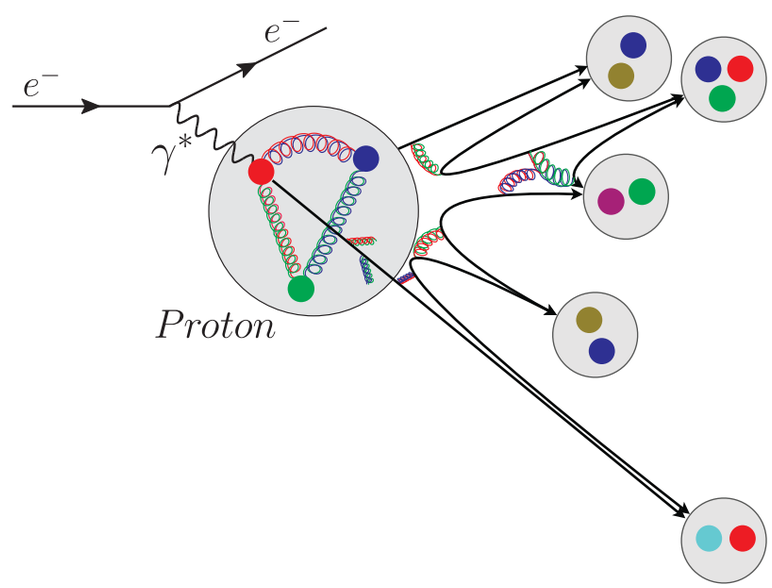


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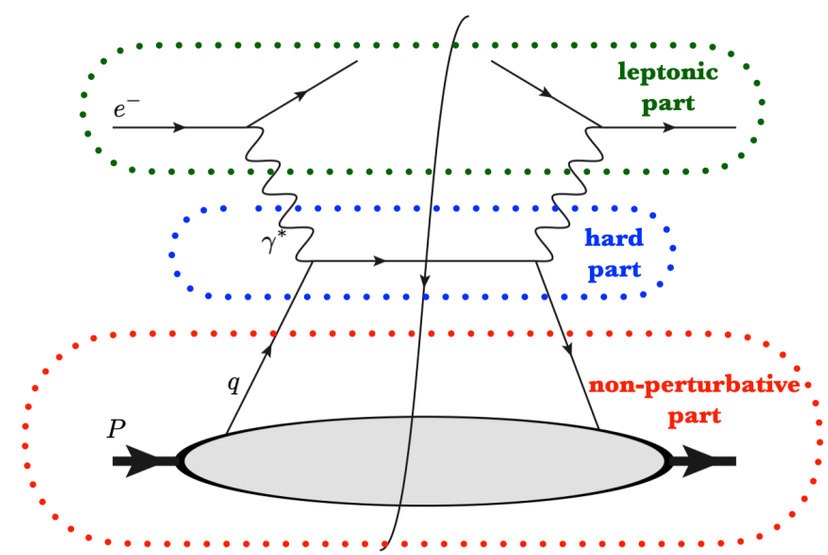
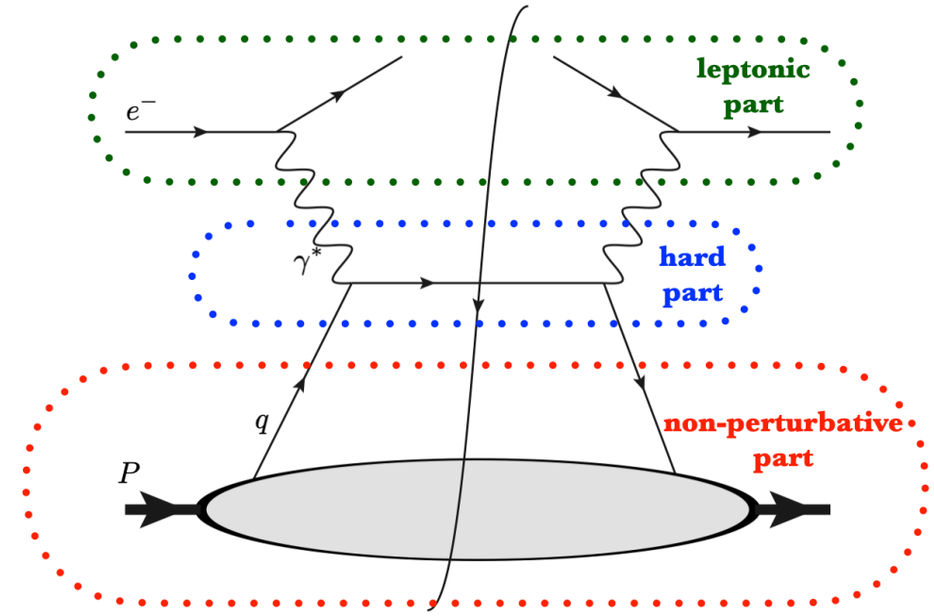



factorization theorems separate non-calculable from calculable parts

# Quantum simulation of light-front parton correlators

cross section:

$$\sigma(\xi, Q^2) = \sum_f \int_\xi^1 \mathrm{d}\bar{\xi}\, \hat{\sigma}(\bar{\xi}, Q^2)\, f_{f/P}(\xi/\bar{\xi}) \quad + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)$$

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partonic cross section:
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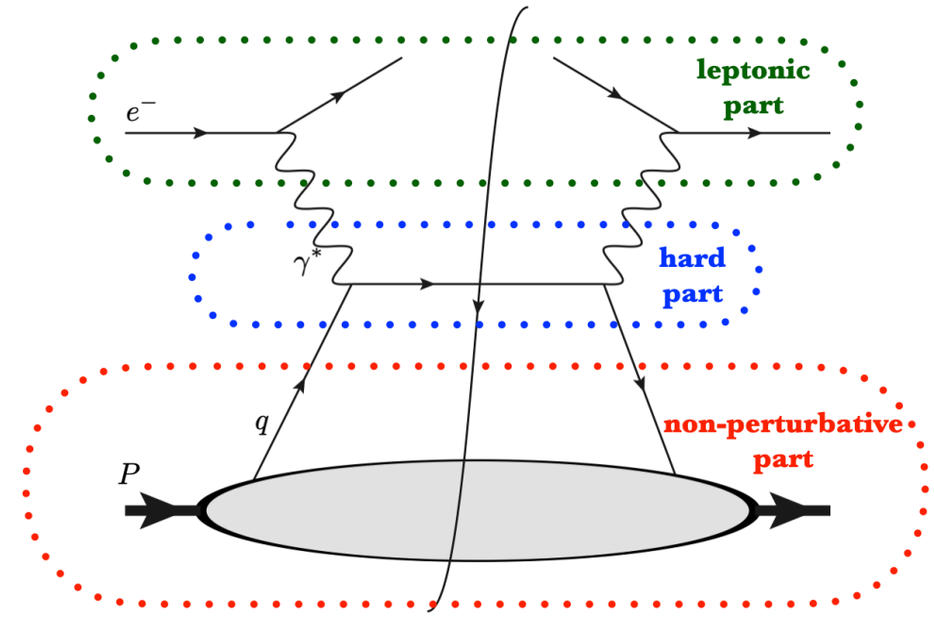


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non-perturbative
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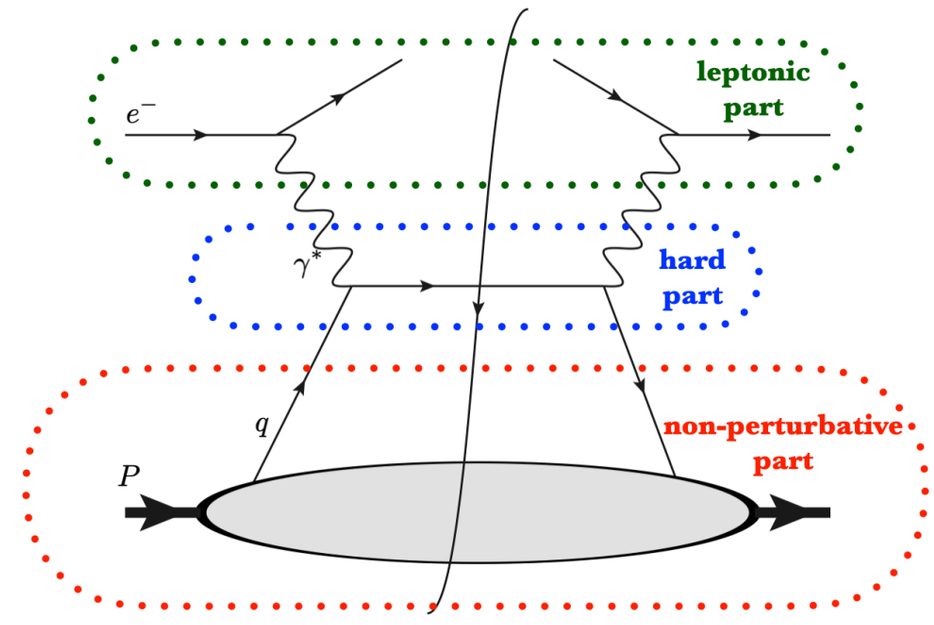


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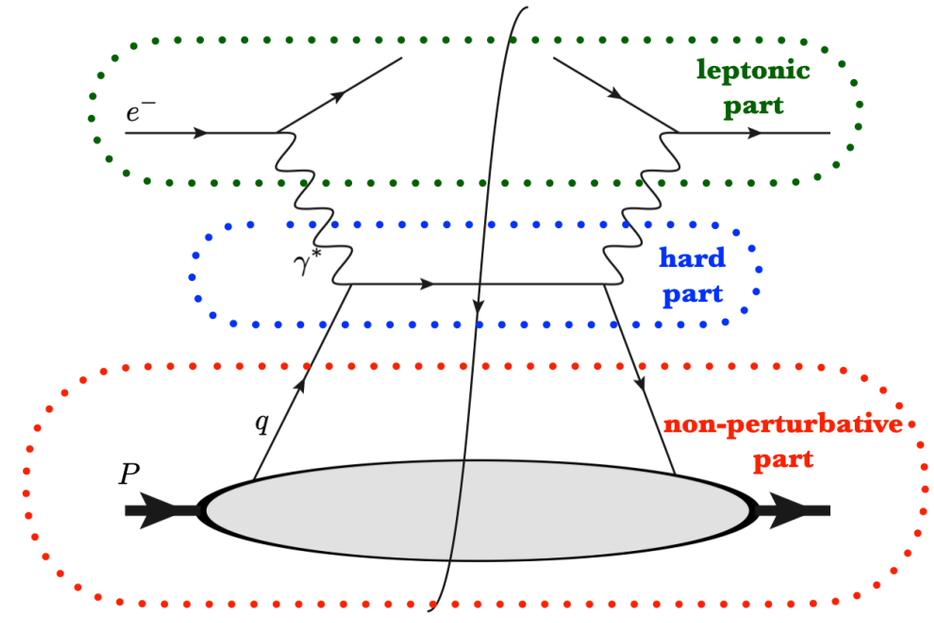
corrections

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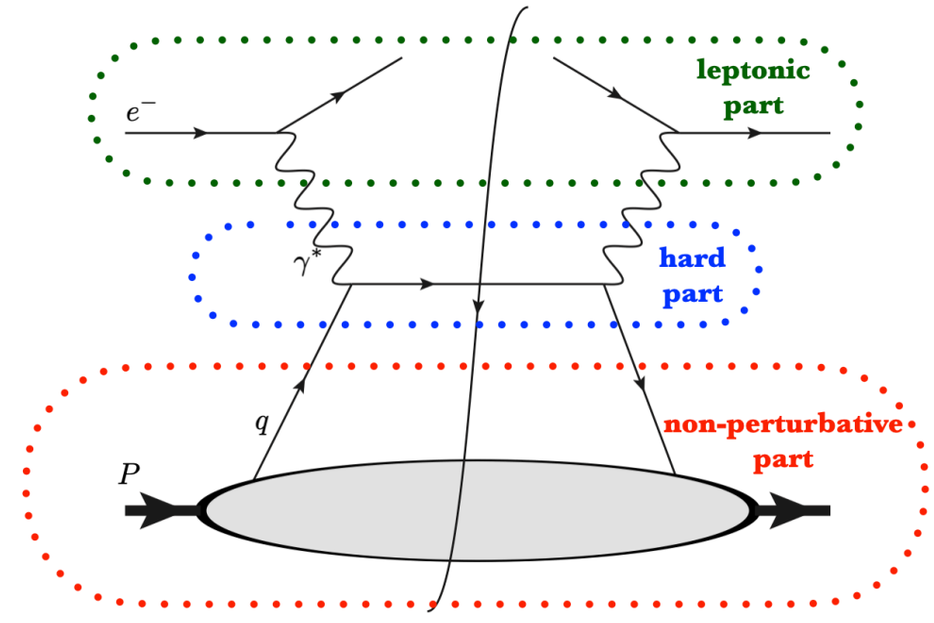


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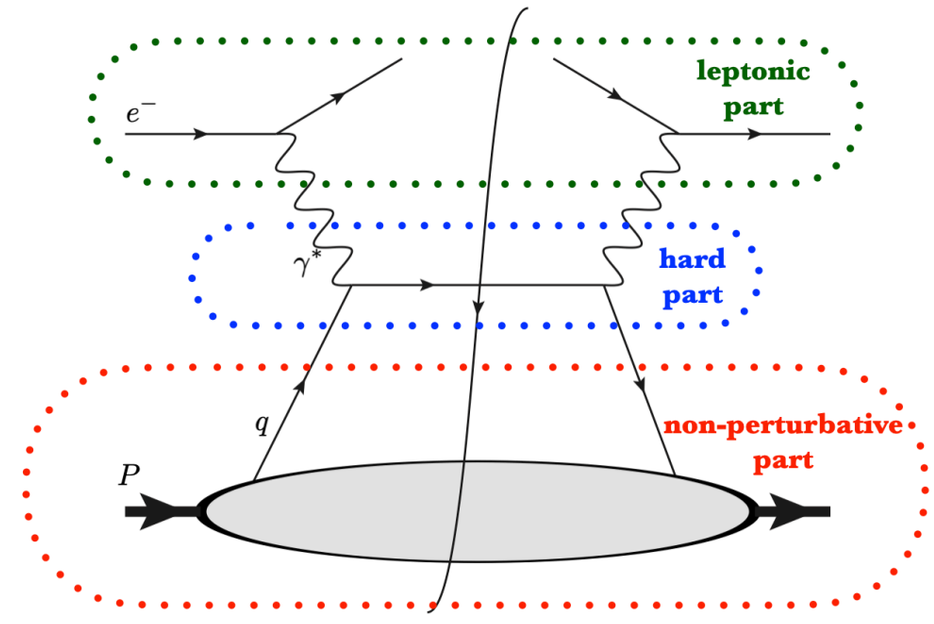


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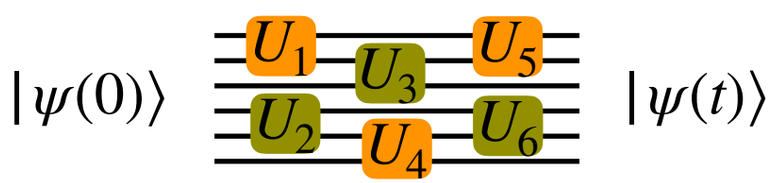
non-local matrix elements (in space-time)
require Wilson lines for gauge invariance

# Quantum simulation of light-front parton correlators

Digital simulation:
Universal simulator

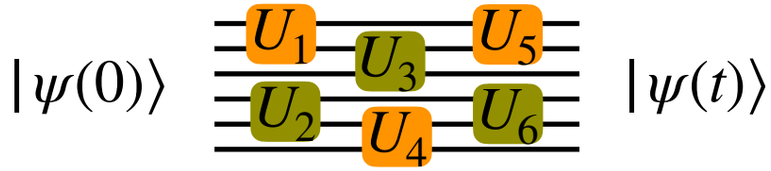



Decompose dynamics into
sequence of quantum gates

# Quantum simulation of light-front parton correlators

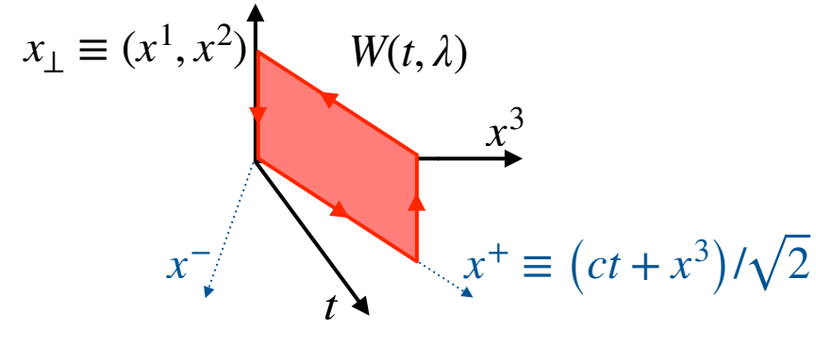
Discretisation of space-time
in a Hamiltonian formulation

Digital simulation:
Universal simulator

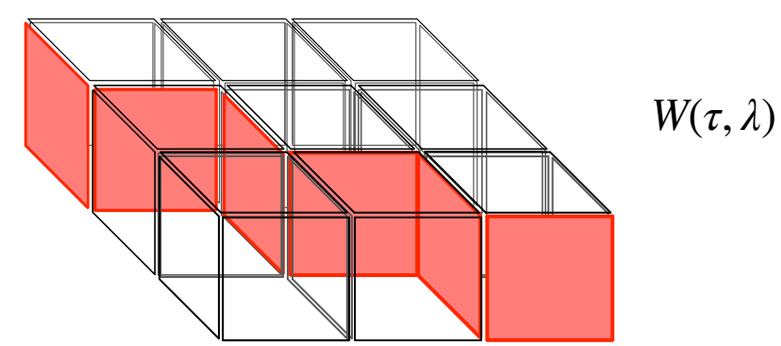



Decompose dynamics into
sequence of quantum gates

(a)

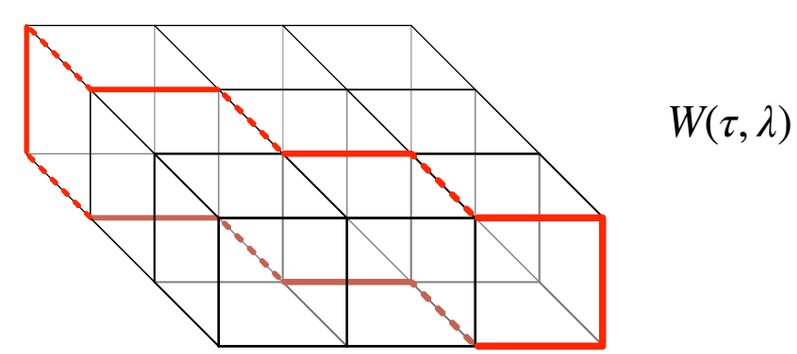



(b)




$$W(\tau,\lambda) = W_{C_1} W_{\tau_1} W_{C_2} W_{\tau_2} \cdots W_{C_k} W_{\tau_k} \cdots$$

(c)




$$W(\tau,\lambda) = \mathcal{U}_1 e^{-i\tau_1 H} \mathcal{U}_2 e^{-i\tau_2 H} \ldots \mathcal{U}_k e^{-i\tau_k H} \ldots \mathcal{U}_N$$

Note: in the Hamiltonian formulation
the temporal gauge $A_0=0$ is chosen

# Quantum simulation of light-front parton correlators

## Moving a single quark:

$$u_{12} = \exp\left\{ \frac{-i\pi}{2} \sum_{\alpha\beta} \left[ \psi^\dagger_{\alpha,1} U_{\alpha\beta}(e)\, \psi_{\beta,2} + \mathrm{h.c.} \right] \right\}$$

$$\to (-i) \left[ \psi^\dagger_{\alpha,1} U_{\alpha\beta}(e)\, \psi_{\beta,2} + \mathrm{h.c.} \right],$$

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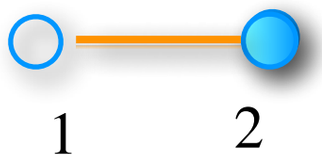

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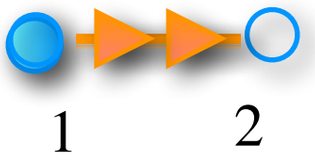

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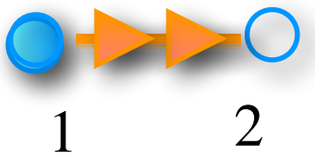


Starting from a “meson” state:

$$|\,\mathrm{m}\rangle \equiv \frac{1}{\mathrm{N}^{1/2}} \sum_{\alpha=1}^{\mathrm{N}} |\,\alpha(1), \bar{\alpha}(2)\rangle$$

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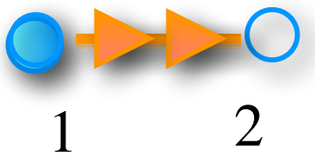


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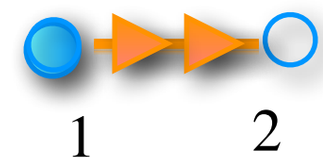


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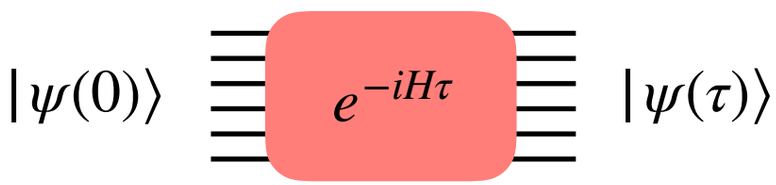
$$\mathcal{U}(A_1, B_L) = \frac{1}{N^{1/2}} \sum_{\alpha\beta\cdots\mu\nu\omega\cdots\theta\phi} |\alpha(A_1)\rangle U_{\alpha\beta}(e_1)\cdots U_{\mu\nu}(e_{L/2-1}) U^*_{\omega\nu}(e_{L/2})\cdots U^*_{\phi\theta}(e_{L-1}) |\bar{\phi}(B_L)\rangle$$

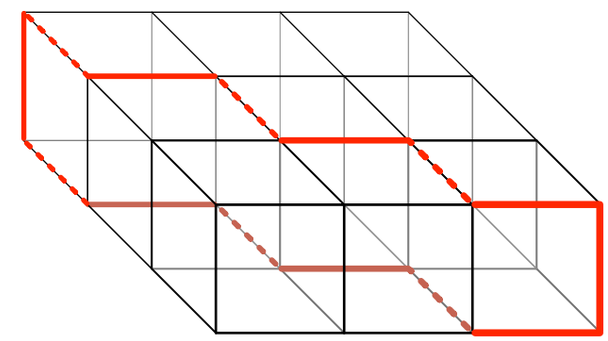
$$= \frac{1}{N^{1/2}} \sum_{\alpha\phi} |\alpha(A_1)\rangle \mathcal{U}_{\alpha\phi}(e_1, \cdots, e_{L-1}) |\bar{\phi}(B_L)\rangle$$

we built a spatial Wilson line

# Quantum simulation of light-front parton correlators

Time-evolution by
a single time step



$W(\tau, \lambda)$

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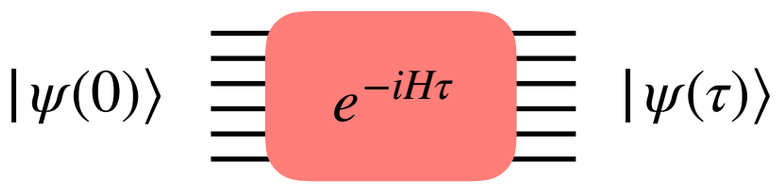


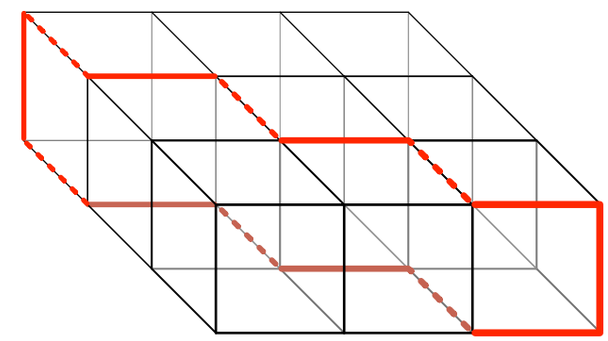

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Digital simulation can simulate any model but requires many gate operations

Decompose dynamics induced by systems Hamiltonian into sequence of quantum gates

$$H = H_{\rm el} + H_{\rm mag}$$

Efficient for local interactions

$$e^{-iH} \simeq \left[ e^{-iH_{\rm el}/2n_T} e^{-i\lambda H_{\rm mag}/n_T} e^{-iH_{\rm el}/2n_T} \right]^{n_T}$$

Trotter-Suzuki approximation

S. Lloyd, Science (1996)

# Quantum simulation of light-front parton correlators

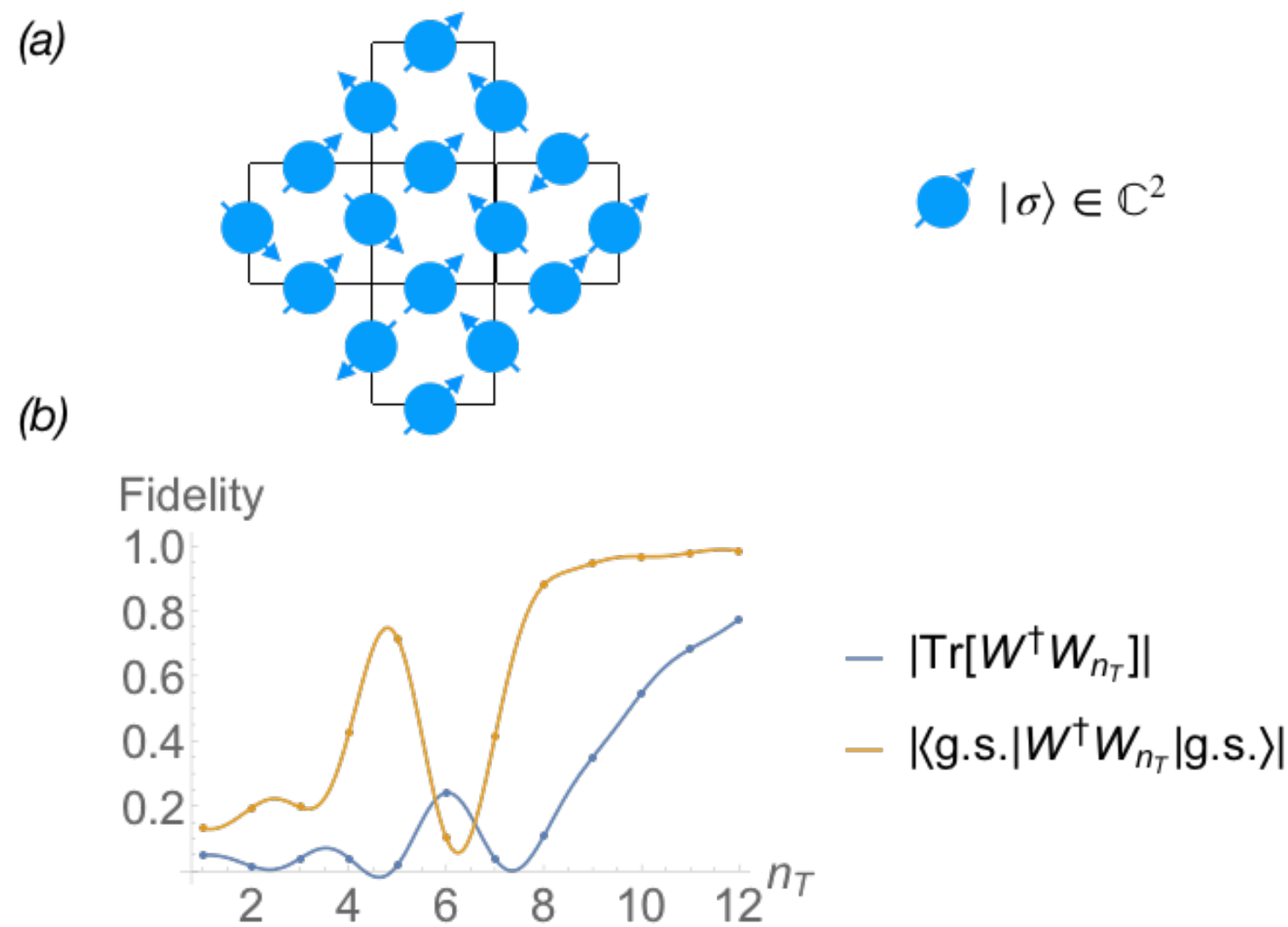
## Proof of principle: $Z_2$ pure gauge model

operator norm:

$$\left| \mathrm{Tr}\left[ \mathscr{W}^\dagger \mathscr{W}_{n_T} \right] \right|$$

ground state fidelity:

$$\left| \langle \mathrm{g.s.} | \mathscr{W}^\dagger \mathscr{W}_{n_T} | \mathrm{g.s.} \rangle \right|$$




within a few Trotter steps a fidelity closed to one is achieved

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# Quantum Simulation of Light-Front Parton Correlators

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[4] *IKERBASQUE, Basque Foundation for Science, Plaza Euskadi 5, 48009 Bilbao, Spain*

arXiv:2011.01275

Project in progress with: M.G. Echevarria, I.L. Egusquiza, G. Schnell

Collaborators: M. Dalmonte, S. Montangero, U.-J. Wiese, P. Zoller…

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THE EUROPEAN PHYSICAL JOURNAL D

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# Simulating lattice gauge theories within quantum technologies

Mari Carmen Bañuls[1,2], Rainer Blatt[3,4], Jacopo Catani[5,6,7], Alessio Celi[3,8], Juan Ignacio Cirac[1,2], Marcello Dalmonte[9,10], Leonardo Fallani[5,6,7], Karl Jansen[11], Maciej Lewenstein[8,12,13], Simone Montangero[14,15,a], Christine A. Muschik[3], Benni Reznik[16], Enrique Rico[17,18], Luca Tagliacozzo[19], Karel Van Acoleyen[20], Frank Verstraete[20,21], Uwe-Jens Wiese[22], Matthew Wingate[23], Jakub Zakrzewski[24,25], and Peter Zoller[3]

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Enrique Rico (University of the Basque Country, Spain

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- **Philipp Hauke** (Università di Trento, Italy) Open Quantum Systems and Optical Lattices
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- **Mari Carmen Bañuls** (Max-Planck-Institute of Quantum Optics, Germany) Tensor network methods: from quantum information to solving lattice gauge theories
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