



Enrique Rico Ortega Wednesday 03 April 2024











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Quantum State Preparation



We show how a variational low-depth circuit can prepare the lowest energy state of a gauge theory





Simulating lattice gauge theories within quantum technologies



Collaborators: M. Dalmonte, S. Montangero, U.-J. Wiese, P. Zoller...

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Colloquium

THE EUROPEAN PHYSICAL JOURNAL D

Simulating lattice gauge theories within quantum technologies

Mari Carmen Bañuls^{1,2}, Rainer Blatt^{3,4}, Jacopo Catani^{5,6,7}, Alessio Celi^{3,8}, Juan Ignacio Cirac^{1,2}, Marcello Dalmonte^{9,10}, Leonardo Fallani^{5,6,7}, Karl Jansen¹¹, Maciej Lewenstein^{8,12,13}, Simone Montangero^{14,15,a}, Christine A. Muschik³, Benni Reznik¹⁶, Enrique Rico^{17,18}, Luca Tagliacozzo¹⁹, Karel Van Acoleyen²⁰, Frank Verstraete^{20,21}, Uwe-Jens Wiese²², Matthew Wingate²³, Jakub Zakrzewski^{24,25}, and Peter Zoller³



Quantum Technologies for Lattice Gauge Theories



Quantum Simulation for High Energy Physics

C.W. Bauer, Z. Davoudi, A.B. Balantekin, T. Bhattacharya, M. Carena, W.A. de Jong, P. Draper, A. El-Khadra, N. Gemelke, M. Hanada, D. Kharzeev, H. Lamm, Y.-Y. Li, J. Liu, M. Lukin, Y. Meurice, C. Monroe, B. Nachman, G. Pagano, J. Preskill, E. Rinaldi, A. Roggero, D.I. Santiago, M.J. Savage, I. Siddiqi, G. Siopsis, D. Van Zanten, N. Wiebe, Y. Yamauchi, K. Yeter-Aydeniz, S. Zorzetti

arXiv:2204.03381

Lattice gauge theories simulations in the quantum information era

M. Dalmonte, S. Montangero Contemporary Physics 57, 388 (2016)

Quantum Simulations of Lattice Gauge Theories using Ultracold Atoms in Optical Lattices

> E. Zohar, J.I. Cirac, B. Reznik Rep. Prog. Phys. 79, 014401 (2016)

Towards Quantum Simulating QCD

U.-J. Wiese Nucl.Phys. A931, 246-256 (2014)



A fruitful dialogue (two-way communication)



Quantum Computing for High-Energy Physics State of the Art and Challenges Summary of the QC4HEP Working Group

Alberto Di Meglio,^{1,*} Karl Jansen,^{2,3,†} Ivano Tavernelli,^{4,‡} Constantia Alexandrou,^{5,3} Srinivasan Arunachalam,⁶ Christian W. Bauer,⁷ Kerstin Borras,^{8,9} Stefano Carrazza,^{10,1} Arianna Crippa,^{2,11} Vincent Croft,¹² Roland de Putter,⁶ Andrea Delgado,¹³ Vedran Dunjko,¹² Daniel J. Egger,⁴ Elias Fernández-Combarro,¹⁴ Elina Fuchs,^{1,15,16} Lena Funcke,¹⁷ Daniel González-Cuadra,^{18,19} Michele Grossi,¹ Jad C. Halimeh,^{20,21} Zoë Holmes,²² Stefan Kühn,² Denis Lacroix,²³ Randy Lewis,²⁴ Donatella Lucchesi,^{25,26,1} Miriam Lucio Martinez,^{27,28} Federico Meloni,⁸ Antonio Mezzacapo,⁶ Simone Montangero,^{25,26} Lento Nagano,²⁹ Voica Radescu,³⁰ Enrique Rico Ortega,^{31,32,33,34} Alessandro Roggero,^{35,36} Julian Schuhmacher,⁴ Joao Seixas,^{37,38,39} Pietro Silvi,^{25,26} Panagiotis Spentzouris,⁴⁰ Francesco Tacchino,⁴ Kristan Temme,⁶ Koji Terashi,²⁹ Jordi Tura,^{12,41} Cenk Tüysüz,^{2,11} Sofia Vallecorsa,¹ Uwe-Jens Wiese,⁴² Shinjae Yoo,⁴³ and Jinglei Zhang^{44,45}







Simulating lattice gauge theories within quantum technologies





Implementing the gauge invariant dynamics





EHU QC EHU Quantum Center



modern microscopes

ikerbasque

Basque Foundation for Science



(semi-inclusive) deep-inelastic lepton scattering



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(semi-inclusive) deep-inelastic lepton scattering highly virtual photons resolve inner (partonic) structure



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(semi-inclusive) deep-inelastic lepton scattering highly virtual photons resolve inner (partonic) structure





M. G. Echevarria^(D),^{1,*} I. L. Egusquiza,^{2,†} E. Rico^(D),^{3,4,‡} and G. Schnell^(D),^{2,4,§}

arXiv:2011.01275 Phys. Rev. D 104, 014512 (2021)

Project in progress with: M.G. Echevarria, I.L. Egusquiza, G. Schnell



Quantum simulation of light-front parton correlators





leptonic part

non-perturbative part

hard

part

cross section:

$$\sigma(\xi, Q^2) = \sum_{f} \int_{\xi}^{1} \mathrm{d}\bar{\xi} \,\hat{\sigma}(\bar{\xi}, Q^2) \,f_{f/P}(\xi/\bar{\xi}) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{Q}\right)$$





leptonic part

non-perturbativo

hard part





partonic cross section: calculable





leptonic part

non-perturbativo

hard part





partonic cross section: calculable

non-perturbative parametrization of nucleon: PDFs, TMDs etc.



corrections

partonic cross section: calculable

non-perturbative parametrization of nucleon: PDFs, TMDs etc.



non-perturbative parametrization of nucleon: PDFs, TMDs etc.

$$f_{f/P}(\xi) = \sum_{S} \int \frac{dy^{-}}{2\pi} e^{-i\xi p^{+}y^{-}} \langle PS | \left[\bar{\psi}\mathcal{U}\right](y^{-})\frac{\gamma^{+}}{2} \left[\mathcal{U}^{\dagger}\psi\right](0) | PS \rangle$$



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Non-local (space-time) matrix elements require Wilson lines for gauge invariance We study the quantum simulation of Wilson loops in space and real-time





Non-local (space-time) matrix elements require Wilson lines for gauge invariance We study the quantum simulation of Wilson loops in space and real-time

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Requirements for the quantum simulation of parton correlators:





Non-local (space-time) matrix elements require Wilson lines for gauge invariance We study the quantum simulation of Wilson loops in space and real-time

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Requirements for the quantum simulation of parton correlators:

- encode in quantum degrees of freedom both matter and gauge fields
- preparation of a reference state, e.g., vacuum, proton, glue-ball
- simulate gauge-invariant quantities, e.g., minimal gauge-matter coupling
- real-time evolution, since the Wilson line is non-local in time
- carry out measurements after the evolution, i.e., quantum interferometer





Digital simulation: Universal simulator



Decompose dynamics into sequence of quantum gates

Stroboscopic simulation in an analog simulator





Discretisation of space-time in a Hamiltonian formulation

Digital simulation: Universal simulator



Decompose dynamics into sequence of quantum gates

Stroboscopic simulation in an analog simulator

Note: in the Hamiltonian formulation the temporal gauge $A_0=0$ is chosen







• Quantum algorithms are recipes that manipulate quantum states





• We classify algorithms depending on how they manipulate quantum states.





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For our purposes $|\psi_f\rangle \simeq |E_0\rangle$

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Variational quantum algorithms







• Quantum algorithms are recipes that manipulate quantum states



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Adiabatic algorithms

$$\qquad \qquad \mathbf{AA} \qquad = \quad \mathcal{T}\left\{\int_0^T \exp\left[-\frac{it}{\hbar}H(t)\right]\right\}$$

$$\begin{split} H(t) &= [1 - \lambda(t)]H_0 + \lambda(t)H_f \\ \lambda(0) &= 0 & \lambda(T) = 1 \\ T &\sim \mathcal{O}\left(\frac{1}{\Delta}\right) & \Delta &\longrightarrow \text{Min. Gap} \end{split}$$







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For our purposes $|\psi_f\rangle \simeq |E_0\rangle$

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Provable algorithms







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Quantum State Preparation

$$|\psi(0)\rangle = \frac{U_1 - U_3 - U_5}{U_2 - U_4} |\psi(t)\rangle$$

In the <u>general case</u>, it is know to be a QMA problem (analogue of NP problem)

With <u>unitary circuits</u>, it is know that the depth scales with the system size (topological order)

Bravyi, Hastings, Verstraete (2006)





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Quantum State Preparation

$$|\psi(0)\rangle = \frac{U_1 - U_3 - U_5}{U_2 - U_4 - U_6} |\psi(t)\rangle$$

In the <u>general case</u>, it is know to be a QMA problem (analogue of NP problem)

With <u>unitary circuits</u>, it is know that the depth scales with the system size (topological order)

Bravyi, Hastings, Verstraete (2006)

We show how a variational low-depth circuit can prepare the lowest energy state of a gauge theory

$$\hat{H}_{\mathbb{Z}_{2}} = -\sum_{\text{link}} \hat{\sigma}_{l}^{x} - \lambda \sum_{\text{plaq}} (\hat{\sigma}^{z} \hat{\sigma}^{z} \hat{\sigma}^{z})_{\text{plaq}}$$

$$\lambda_{c} = 3.04438$$





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We propose a novel variational ansatz for the ground state preparation of the \mathbb{Z}_2 LGT in quantum computers.

The \mathbb{Z}_2 lattice gauge theory







$$|\psi\rangle = \frac{e^{\beta \sum_{\text{plaq}} (\hat{\sigma}^{z} \hat{\sigma}^{z} \hat{\sigma}^{z} \hat{\sigma}^{z})_{\text{plaq}}}}{Z} \otimes_{\text{link}} |+\rangle_{l} \qquad \begin{cases} |\psi\rangle = \otimes_{\text{link}} |+\rangle_{l} & \lambda = 0 \\ |\psi\rangle = \otimes_{\text{plaq}} \frac{\mathbb{I} + (\hat{\sigma}^{z} \hat{\sigma}^{z} \hat{\sigma}^{z} \hat{\sigma}^{z})_{\text{plaq}}}{2} \otimes_{\text{link}} |+\rangle_{l} & \lambda \gg 1 \end{cases}$$

.

$$\hat{G}_{\text{vertex}} |\psi\rangle = \left(\hat{\sigma}_x \hat{\sigma}_x \hat{\sigma}_x \hat{\sigma}_x\right)_{\text{vertex}} |\psi\rangle = |\psi\rangle$$

Cardy, Hamber (1980)





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Cardy, Hamber (1980)



Tantivasadakarn, Thorngren, Vishwanath, Verresen (2021)





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$$\hat{G}_{\text{vertex}} |\psi\rangle = \left(\hat{\sigma}_x \hat{\sigma}_x \hat{\sigma}_x \hat{\sigma}_x\right)_{\text{vertex}} |\psi\rangle = |\psi\rangle$$



$$\frac{|0\rangle_a + \tanh\beta|1\rangle_a}{\sqrt{1 + \tanh^2\beta}} \left| |\psi(0)\rangle_{P_n} \xrightarrow{CZ} \frac{1}{\sqrt{2}} \left\{ \left[\frac{e^{\beta\hat{P}_n}}{(\cosh 2\beta)^{N_p/2}} \right] |+\rangle_a + \left[\frac{e^{-\beta\hat{P}_n}}{(\cosh 2\beta)^{N_p/2}} \right] |-\rangle_a \right\} |\psi(0)\rangle_{P_n}$$





We propose a novel variational ansatz for the ground state preparation of the \mathbb{Z}_2 LGT in quantum computers.

Variational ansatz











We propose a novel variational ansatz for the ground state preparation of the \mathbb{Z}_2 LGT in quantum computers.

Unitary part implementation

- $e^{i\alpha \hat{\sigma}^x}$ Single qubit rotations
- Circuit implementation of $e^{i\gamma_k \hat{P}_n}$



Hamiltonian variational ansatz

$$\begin{split} |\phi_{\rm u,e}(\boldsymbol{\alpha},\boldsymbol{\beta})\rangle &= \left[\prod_{k=2}^{\ell} e^{i\alpha_k H_E} e^{i\beta_k H_B}\right] |\Omega_E\rangle \\ |\phi_{\rm u,m}(\boldsymbol{\alpha},\boldsymbol{\beta})\rangle &= \left[\prod_{k=2}^{\ell} e^{i\beta_k H_B} e^{i\alpha_k H_E}\right] \left[\prod_n \frac{1+P_n}{\sqrt{2}}\right] |\Omega_E\rangle \\ & \frac{\lambda}{\text{Confined}} \xrightarrow{\text{Topological}} \\ & \lambda_c = 3.04438 \end{split}$$





$$\left|\psi(\alpha,\beta,\boldsymbol{\gamma},\boldsymbol{\theta})\right\rangle = \left[\prod_{k=2}^{L} e^{i\theta_{k}\hat{H}'_{E}} e^{i\gamma_{k}\hat{H}'_{B}}\right] e^{i\alpha\hat{H}'_{E}} \frac{e^{\beta\hat{H}'_{B}}}{(\cosh 2\beta)^{N_{p}/2}} \left(\bigotimes_{n=0}^{N}\left|+\right\rangle\right)$$







$$\left|\psi(\alpha,\beta,\boldsymbol{\gamma},\boldsymbol{\theta})\right\rangle = \left[\prod_{k=2}^{L} e^{i\theta_{k}\hat{H}'_{E}} e^{i\gamma_{k}\hat{H}'_{B}}\right] e^{i\alpha\hat{H}'_{E}} \frac{e^{\beta\hat{H}'_{B}}}{(\cosh 2\beta)^{N_{p}/2}} \left(\bigotimes_{n=0}^{N} \left|+\right\rangle\right)$$



Energy deviation

• Energy difference with the exact ground state







$$\left|\psi(\alpha,\beta,\boldsymbol{\gamma},\boldsymbol{\theta})\right\rangle = \left[\prod_{k=2}^{L} e^{i\theta_{k}\hat{H}'_{E}} e^{i\gamma_{k}\hat{H}'_{B}}\right] e^{i\alpha\hat{H}'_{E}} \frac{e^{\beta\hat{H}'_{B}}}{(\cosh 2\beta)^{N_{p}/2}} \left(\bigotimes_{n=0}^{N} \left|+\right\rangle\right)$$





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Dual magnetization

	Dissipative	Monte	Exact	Unitary	Unitary
		Carlo	diag.	$ \phi_{ m u,e} angle$	$ \phi_{ m u,m} angle$
λ_c	3.24	3.04	3.06	2.56	2.09
β	0.35	0.33	0.36	0.04	-1.27
ν	0.59	0.63	0.64	-0.20	-0.40







$$\left|\psi(\alpha,\beta,\boldsymbol{\gamma},\boldsymbol{\theta})\right\rangle = \left[\prod_{k=2}^{L} e^{i\theta_{k}\hat{H}'_{E}} e^{i\gamma_{k}\hat{H}'_{B}}\right] e^{i\alpha\hat{H}'_{E}} \frac{e^{\beta\hat{H}'_{B}}}{(\cosh 2\beta)^{N_{p}/2}} \left(\bigotimes_{n=0}^{N} \left|+\right\rangle\right)$$

Topological entanglement entropy



 $S_{\text{topo}} = S_A + S_B + S_C - S_{AB} - S_{AC} - S_{BC} + S_{ABC}$





$$\left|\psi(\alpha,\beta,\boldsymbol{\gamma},\boldsymbol{\theta})\right\rangle = \left[\prod_{k=2}^{L} e^{i\theta_{k}\hat{H}'_{E}} e^{i\gamma_{k}\hat{H}'_{B}}\right] e^{i\alpha\hat{H}'_{E}} \frac{e^{\beta\hat{H}'_{B}}}{(\cosh 2\beta)^{N_{p}/2}} \left(\bigotimes_{n=0}^{N}\left|+\right\rangle\right)$$

State preparation with noisy gates

 $\lambda = 3.00$







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Quantum State Preparation



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A fruitful dialogue (two-way communication)







Quantum Information Science and Technology

- The first successful implementations of gauge-field theory dynamics on quantum simulators have emerged for small systems.
- Efficient Hamiltonian formulations for (non-Abelian) gauge theories along with best approaches to state preparation and measurement will continue to develop.
- Abelian and non-Abelian lattice gauge theories in higher than 1+1 dimensions present significant challenge but progress is being made.
- Theory-experiment collaborations will be highly beneficial.
- New results in the frontier between HEP and Quant-Ph