Noise-aware variational eigensolvers: a dissipative route for lattice gauge theories

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J. Cobos,¹,²,  D. F. Locher,³,⁴, †  A. Bermudez,⁵, ‡  M. Müller,³,⁴, §  and E. Rico¹, ², ⁶, ⁷, ¶

Quantum State Preparation

\[ |ψ(0)\rangle \xrightarrow{U_1 U_3 U_5} |ψ(t)\rangle \]

We show how a variational low-depth circuit can prepare the lowest energy state of a gauge theory
Simulating lattice gauge theories within quantum technologies

Collaborators: M. Dalmonte, S. Montangero, U.-J. Wiese, P. Zoller…

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Simulating lattice gauge theories within quantum technologies

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Quantum Simulations of Lattice Gauge Theories using Ultracold Atoms in Optical Lattices
E. Zohar, J.I. Cirac, B. Reznik

Towards Quantum Simulating QCD
U.-J. Wiese
A fruitful dialogue
(two-way communication)

Quantum Computing for High-Energy Physics
State of the Art and Challenges
Summary of the QC4HEP Working Group

Alberto Di Meglio,¹,* Karl Jansen,²,3, † Ivanov Tavernelli,⁴, † Constantia Alexandrou,⁵,3 Srinivasan Arunachalam,⁶ Christian W. Bauer,⁷ Kerstin Borras,⁸,‡ Stefano Carrazza,¹⁰,¹ Arianna Crippa,²,¹¹ Vincent Croft,¹² Roland de Putter,⁶ Andrea Delgado,¹³ Vedran Dunjko,¹² Daniel J. Egger,⁴ Elias Fernández-Combarro,¹⁴ Elina Fuchs,¹,¹⁵,¹⁶ Lena Funcke,¹⁷ Daniel González-Cuadra,¹⁸,¹⁹ Michele Grossi,¹, Jad C. Halimeh,¹⁰,¹¹ Zoë Holmes,²² Stefan Kühn,² Denis Lacroix,²³ Randy Lewis,²⁴ Donatella Lucchesi,²⁵,²⁶,¹ Miriam Lucio Martínez,²⁷,²⁸ Federico Meloni,⁸ Antonio Mezzacapo,⁶ Simone Montanaro,²⁵,²⁶ Lento Nagano,²⁹ Voica Radescu,³⁰ Enrique Rico Ortega,³¹,³²,³³,⁴ Alessandro Roggero,³⁵,³⁶ Julian Schuhmacher,⁴, Joao Seixas,³⁷,³⁸,³⁹ Pietro Silvi,²⁵,²⁶ Panagiota Spentzouris,⁴⁰ Francesco Tacchino,⁴ Kristian Temme,⁶ Koji Terashi,²⁹ Jordi Tura,¹²,⁴¹ Cenk Tüysüz,²,¹¹ Sofia Vallecorsa,¹ Uwe-Jens Wiese,⁴² Shinjia Yoo,⁴³ and Jinglei Zhang⁴⁴,⁴⁵

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- Quantum Dynamics
- Hybrid Qu-CI
- Optimisation
- Classification
- Quantum Kernels
- VQE/varQITE
- Trotter Dynamics
- TN/QTN
- varQTE
- QNNs
- Quantum Kernels
- Jet/track reconstruction
- Rare signal extraction
- Regression
- For & beyond Standard Model
- Parton Shower
- Experiment Simulation
- Classification
- Quantum Kernels
- QNNs
- QAOA
- Quantum Annealing
- HHL Algorithm
- QBMs
- QCBMs
- QGANs
Simulating lattice gauge theories within quantum technologies

- Implementing the gauge invariant dynamics

\[ \hat{\psi}_{\vec{r}}^+ \quad \hat{U}_{\vec{r}, \vec{r}+\hat{\mu}} \quad \hat{\psi}_{\vec{r}+\hat{\mu}} \]

\[ \vec{r} \quad \vec{r} + \hat{\mu} \]

- Energy penalty
- Color singlet hopping
- Internal symmetry
- Encoding gauge invariant degrees of freedom
Quantum simulation of light-front parton correlators

(semi-inclusive) deep-inelastic lepton scattering
Quantum simulation of light-front parton correlators

(semi-inclusive) deep-inelastic lepton scattering

highly virtual photons resolve inner (partonic) structure
Quantum simulation of light-front parton correlators

modern microscopes

(semi-inclusive) deep-inelastic lepton scattering

highly virtual photons resolve inner (partonic) structure

factorization theorems separate non-calculable from calculable parts
Quantum simulation of light-front parton correlators

M. G. Echevarria$^{1,*}$, I. L. Egusquiza$^{2,†}$, E. Rico$^{3,4,‡}$, and G. Schnell$^{2,4,§}$


Project in progress with: M.G. Echevarria, I.L. Egusquiza, G. Schnell
Quantum simulation of light-front parton correlators

cross section:

\[ \sigma(\xi, Q^2) = \sum_f \int_{\xi}^1 d\bar{\xi} \, \hat{\sigma}(\bar{\xi}, Q^2) \, f_f/P(\xi/\bar{\xi}) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{Q}\right) \]

factorization theorems separate non-calculable from calculable parts
Quantum simulation of light-front parton correlators

cross section:

$$\sigma(\xi, Q^2) = \sum_f \int_{\xi}^{1} \d\bar{\xi} \hat{\sigma}(\bar{\xi}, Q^2) f_{f/P}(\xi/\bar{\xi}) + \mathcal{O}\left(\frac{\Lambda_{QCD}}{Q}\right)$$

partonic cross section: calculable

factorization theorems separate non-calculable from calculable parts
Quantum simulation of light-front parton correlators

cross section:

\[ \sigma(\xi, Q^2) = \sum_f \int_1^{\xi/2} d\tilde{\xi} \hat{\sigma}(\tilde{\xi}, Q^2) f_f/P(\xi/\tilde{\xi}) + \mathcal{O}\left(\frac{\Lambda_{QCD}}{Q}\right) \]

partonic cross section: calculable

non-perturbative parametrization of nucleon: PDFs, TMDs etc.

factorization theorems separate non-calculable from calculable parts
Quantum simulation of light-front parton correlators

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Quantum simulation of light-front parton correlators

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corrections

partonic cross section: calculable

non-perturbative parametrization of nucleon: PDFs, TMDs etc.

factorization theorems separate non-calculable from calculable parts

partonic cross section:

\[ f_{f/P}(\xi) = \sum_S \int \frac{dy^-}{2\pi} e^{-i\vec{p}^+ y^-} \langle PS | [\bar{\psi} \mathcal{U}] (y^-) \gamma^+ \mathcal{U}^+ \psi | 0 \rangle | PS \rangle \]
Quantum simulation of light-front parton correlators

cross section:

$$\sigma(\xi, Q^2) = \sum_f \int_{\xi}^{1} d\tilde{\xi} \hat{\sigma}(\tilde{\xi}, Q^2) f_{f/P}(\xi/\tilde{\xi}) + \mathcal{O}\left(\frac{\Lambda_{QCD}}{Q}\right)$$

corrections

partonic cross section:

calculable

non-perturbative parametrization of nucleon:
PDFs, TMDs etc.

factorization theorems separate non-calculable from calculable parts

partonic cross section:

$$f_{f/P}(\xi) = \sum_{S} \int \frac{dy^-}{2\pi} e^{-i\xi p^+ y^-} \langle PS | \bar{\psi} \mathcal{U} (y^-) \gamma^+ \frac{1}{2} \mathcal{U}^\dagger \psi (0) | PS \rangle$$

Non-local (space-time) matrix elements require Wilson lines for gauge invariance

We study the quantum simulation of Wilson loops in space and real-time
Quantum simulation of light-front parton correlators

Non-local (space-time) matrix elements require Wilson lines for gauge invariance. We study the quantum simulation of Wilson loops in space and real-time.

\[
f_{flP}(\xi) = \sum_S \int \frac{dy^-}{2\pi} e^{-i\xi p^+ y^-} \langle PS \mid [\bar{\psi}\mathcal{U}] (y^-) \gamma^+ \mathcal{U}^\dagger \psi (0) \mid PS \rangle
\]

Requirements for the quantum simulation of parton correlators:
Quantum simulation of light-front parton correlators

Non-local (space-time) matrix elements require Wilson lines for gauge invariance
We study the quantum simulation of Wilson loops in space and real-time

\[ f_{f\bar{f}P}(\xi) = \sum_S \int \frac{dy^-}{2\pi} e^{-i\xi p^+ y^-} \langle PS | [\bar{\psi} \mathcal{U}] (y^-) \frac{\gamma^+}{2} [\mathcal{U}^\dagger \psi] (0) | PS \rangle \]

Requirements for the quantum simulation of parton correlators:

- encode in quantum degrees of freedom both matter and gauge fields
- preparation of a reference state, e.g., vacuum, proton, glue-ball
- simulate gauge-invariant quantities, e.g., minimal gauge-matter coupling
- real-time evolution, since the Wilson line is non-local in time
- carry out measurements after the evolution, i.e., quantum interferometer
Quantum simulation of light-front parton correlators

Digital simulation: Universal simulator

\[ |\psi(0)\rangle \rightarrow U_4 \rightarrow \rightarrow U_3 \rightarrow U_2 \rightarrow U_1 \rightarrow |\psi(t)\rangle \]

Decompose dynamics into sequence of quantum gates

Stroboscopic simulation in an analog simulator
Quantum simulation of light-front parton correlators

Digital simulation: Universal simulator

\[ |\psi(0)\rangle \rightarrow U_1 U_3 U_5 |\psi(t)\rangle \]

Decompose dynamics into sequence of quantum gates

Stroboscopic simulation in an analog simulator

Note: in the Hamiltonian formulation the temporal gauge \( A_0 = 0 \) is chosen
Quantum algorithms for quantum state preparation

- Quantum algorithms are recipes that manipulate quantum states

\[ |\psi_0\rangle \xrightarrow{\text{Q-Algorithm}} |\psi_f\rangle \xrightarrow{\text{For our purposes}} |E_0\rangle \]

- We classify algorithms depending on how they manipulate quantum states.
Quantum algorithms for quantum state preparation

- Quantum algorithms are recipes that manipulate quantum states.

\[ |\psi_0\rangle \xrightarrow{\text{Q-Algorithm}} |\psi_f\rangle \xrightarrow{\text{For our purposes}} |E_0\rangle \]

- We classify algorithms depending on how they manipulate quantum states.

Variational quantum algorithms

\[ |\psi(\alpha)\rangle = U_k(\alpha_N)U_{k-1}(\alpha_{N-1}) \ldots U_1(\alpha_1) |\psi_0\rangle \]

\[ |\psi_f\rangle = |\psi(\alpha^*)\rangle \quad \alpha^* = \arg\min_{\alpha} \langle \psi(\alpha) | \hat{H} | \psi(\alpha) \rangle \quad \langle \psi | \hat{H} | \psi \rangle \geq E_0 \ \forall |\psi\rangle \]

Short depth

Hard Optimization
Quantum algorithms for quantum state preparation

- Quantum algorithms are recipes that manipulate quantum states

\[ |\psi_0\rangle \rightarrow \text{Q-Algorithm} \rightarrow |\psi_f\rangle \rightarrow \text{Useful Use} \]

For our purposes \[ |\psi_f\rangle \simeq |E_0\rangle \]

- We classify algorithms depending on how they manipulate quantum states.

**Adiabatic algorithms**

\[ \text{AA} \quad \Rightarrow \quad \mathcal{T}\left\{ \int_0^T \exp \left[ -\frac{it}{\hbar} H(t) \right] \right\} \]

\[ H(t) = [1 - \lambda(t)]H_0 + \lambda(t)H_f \]

\[ \lambda(0) = 0 \quad \lambda(T) = 1 \]

\[ T \sim \mathcal{O}\left(\frac{1}{\Delta}\right) \quad \Delta \rightarrow \text{Min. Gap} \]

Less sensitive to noise

Hamiltonian engineering
Quantum algorithms for quantum state preparation

- Quantum algorithms are recipes that manipulate quantum states

\[ |\psi_0\rangle \xrightarrow{\text{Q-Algorithm}} |\psi_f\rangle \xrightarrow{\text{For our purposes}} |\psi_f\rangle \approx |E_0\rangle \]

- We classify algorithms depending on how they manipulate quantum states.

**Provable algorithms**

\[ \text{QPE} = \begin{array}{c}
\text{H} \\
\vdots \\
\text{H}
\end{array} \quad \begin{array}{c}
U^{2^0} \\
\vdots \\
U^{2^{n-1}}
\end{array} \quad \begin{array}{c}
\text{QFT}^{-1}
\end{array} \]

- Guarantee of success
- Long depths
In the general case, it is known to be a QMA problem (analogue of NP problem)

With unitary circuits, it is known that the depth scales with the system size (topological order)

Bravyi, Hastings, Verstraete (2006)
Quantum State Preparation

\[ |\psi(0)\rangle = U_1 U_3 U_5 |\psi(t)\rangle \]

In the general case, it is known to be a QMA problem (analogue of NP problem)

With unitary circuits, it is known that the depth scales with the system size (topological order)

Bravyi, Hastings, Verstraete (2006)

We show how a variational low-depth circuit can prepare the lowest energy state of a gauge theory

\[ \hat{H}_{\mathbb{Z}_2} = - \sum_{\text{link}} \hat{\sigma}_i^x - \lambda \sum_{\text{plaq}} (\hat{\sigma}_i^z \hat{\sigma}_j^z \hat{\sigma}_k^z \hat{\sigma}_l^z) \]

\[ \lambda_c = 3.04438 \]
Noise-aware variational eigensolvers: a dissipative route for lattice gauge theories

J. Cobos, 1, 2, * D. F. Locher, 3, 4, † A. Bermudez, 5, ‡ M. Müller, 3, 4, § and E. Rico 1, 2, 6, 7, ¶


We propose a novel variational ansatz for the ground state preparation of the $\mathbb{Z}_2$ LGT in quantum computers.

**The $\mathbb{Z}_2$ lattice gauge theory**

- **Hamiltonian**
  \[
  \hat{H} = - \sum_{n,i} \hat{\sigma}^{x}_{(n,i)} - \lambda \sum_{n} \hat{P}_{n}
  \]
  - Electric term
  - Magnetic term

- **Gauge invariance**
  \[
  [\hat{G}_k, \hat{H}] = 0 \quad \forall \ k = 0, 1 \ldots N_g
  \]

- **Gauss’ Law**
  \[
  (\nabla \cdot \mathbf{E})(k) \equiv 0 \quad \Rightarrow \quad \hat{G}_k \ket{\psi} = \ket{\psi}
  \]
  \[
  \ket{\psi} \rightarrow \text{Physical states}
  \]

- **Phase diagram**
  \[
  \lambda \quad \text{Topological}
  \]
  \[
  \lambda_{c} = 3.04438
  \]
  - Confined
  - Deconfined

- **Dual magnetization**
  \[
  \hat{M}_n = \prod_{(n,i) \in \partial C_n} \hat{\sigma}^{x}_{(n,i)}
  \]
  \[
  d = 4
  \]
Variational gauge invariant state

$$|\psi\rangle = \frac{e^{\beta \sum_{\text{plaq}} (\hat{\sigma}_x^z \hat{\sigma}_x^z \hat{\sigma}_x^z \hat{\sigma}_x^z)_{\text{plaq}}}}{Z} \otimes_{\text{link}} |+\rangle_l$$

$$\hat{G}_{\text{vertex}} |\psi\rangle = (\hat{\sigma}_x^x \hat{\sigma}_x^x \hat{\sigma}_x^x \hat{\sigma}_x^x)_{\text{vertex}} |\psi\rangle = |\psi\rangle$$

$$\begin{align*}
|\psi\rangle &= \otimes_{\text{link}} |+\rangle_l \\
|\psi\rangle &= \otimes_{\text{plaq}} \frac{1 + (\hat{\sigma}_x^z \hat{\sigma}_x^z \hat{\sigma}_x^z \hat{\sigma}_x^z)_{\text{plaq}}}{2} \otimes_{\text{link}} |+\rangle_l \\
\end{align*}$$

Cardy, Hamber (1980)
Variational gauge invariant state

\[ |\psi\rangle = \frac{e^\beta \sum_{\text{plaq}} (\hat{\sigma}^z \hat{\sigma}^z \hat{\sigma}^z \hat{\sigma}^z)}{Z} \bigotimes_{\text{link}} | + \rangle_l \]

\[ \hat{G}_{\text{vertex}} |\psi\rangle = (\hat{\sigma}_x \hat{\sigma}_x \hat{\sigma}_x \hat{\sigma}_x)_{\text{vertex}} |\psi\rangle = |\psi\rangle \]

\[
\begin{cases}
|\psi\rangle = \bigotimes_{\text{link}} | + \rangle_l & \lambda = 0 \\
\left| \frac{\hat{\sigma}^z \hat{\sigma}^z \hat{\sigma}^z \hat{\sigma}^z}{2} \bigotimes_{\text{link}} | + \rangle_l \right| \lambda \gg 1
\end{cases}
\]

Cardy, Hamber (1980)

\[ \Theta(\beta) = \tan^{-1}(\tanh \beta) \]

Tantivasadakarn, Thorngren, Vishwanath, Verresen (2021)
Variational gauge invariant state

\[ |\psi\rangle = \frac{e^\beta \sum_{\text{plaq}} (\hat{\sigma}^a \hat{\sigma}^b \hat{\sigma}^c \hat{\sigma}^d)_{\text{plaq}}}{Z} \otimes_{\text{link}} | + \rangle_l \]

\[ \hat{G}_{\text{vertex}} |\psi\rangle = (\hat{\sigma}_x \hat{\sigma}_x \hat{\sigma}_x \hat{\sigma}_x)_{\text{vertex}} |\psi\rangle = |\psi\rangle \]

\[
\begin{cases}
|\psi\rangle = \otimes_{\text{link}} | + \rangle_l & \lambda = 0 \\
|\psi\rangle = \otimes_{\text{plaq}} \frac{1 + (\hat{\sigma}^a \hat{\sigma}^b \hat{\sigma}^c \hat{\sigma}^d)_{\text{plaq}}}{2} \otimes_{\text{link}} | + \rangle_l & \lambda \gg 1
\end{cases}
\]

\[ |\psi(0)\rangle_{P_n} \quad |\psi(0)\rangle_{P_n} \xrightarrow{\text{CZ}} \frac{1}{\sqrt{2}} \left\{ \left[ \frac{e^\beta \hat{P}_n}{(\cosh 2\beta)^{Np/2}} \right] |+\rangle_a + \left[ \frac{e^{-\beta \hat{P}_n}}{(\cosh 2\beta)^{Np/2}} \right] |-\rangle_a \right\} |\psi(0)\rangle_{P_n} \]

\[ \hat{\sigma}_m \in P_n \quad e^{-\hat{P}_n} = e^{\hat{P}_n} \hat{\sigma}_m \in P_n \]

\[ \hat{\sigma}_m |\psi(0)\rangle = |\psi(0)\rangle \quad \forall \ m \]
Variational gauge invariant state

We propose a novel variational ansatz for the ground state preparation of the $\mathbb{Z}_2$ LGT in quantum computers.

**Variational ansatz**

$$|\psi(\alpha, \beta)\rangle = \prod_{k=2}^{\ell} e^{i\alpha_k H_E} e^{i\beta_k H_B} e^{i\alpha_1 H_E} \frac{e^{\beta_1 H_B}}{(\cosh 2\beta_1)^{N_f/2}} |\Omega_E\rangle$$

The ansatz captures the ground states of $H_E, H_B$

$$H = -H_E - \lambda H_B$$

$$H_E = \sum_{n,i} \sigma^x_{(n,i)} \quad H_B = \sum_n P_n \quad |\Omega_E\rangle = \bigotimes_{n,i} |+\rangle_{(n,i)}$$

$$|\psi(0, \beta_1 = \infty)\rangle = \prod_n \left[ \frac{1 + P_n}{\sqrt{2}} \right] |\Omega_E\rangle$$

$$\lim_{\tau \to \infty} e^{-\tau \hat{H}} |\psi\rangle \longrightarrow |\text{g.s.}\rangle$$

$$e^{-\tau \hat{H}} |\psi\rangle = e^{-\tau E_0} |\text{g.s.}\rangle + e^{-\tau E_1} |E_1\rangle + e^{-\tau E_2} |E_2\rangle + \ldots$$

$\lambda_c = 3.04438$
We propose a novel variational ansatz for the ground state preparation of the $\mathbb{Z}_2$ LGT in quantum computers.

**Unitary part implementation**

- $e^{i\alpha \sigma^x}$ Single qubit rotations
- Circuit implementation of $e^{i\gamma_k \sigma_z^k}$

**Hamiltonian variational ansatz**

$$
|\phi_{u,c}(\alpha, \beta)\rangle = \left[ \prod_{k=2}^{\ell} e^{i\alpha_k H_E} e^{i\beta_k H_B} \right] |\Omega_E\rangle
$$

$$
|\phi_{u,m}(\alpha, \beta)\rangle = \left[ \prod_{k=2}^{\ell} e^{i\beta_k H_B} e^{i\alpha_k H_E} \right] \left[ \prod_{n=1}^{1} \frac{1 + P_n}{\sqrt{2}} \right] |\Omega_E\rangle
$$

$\lambda$ ~ Confined ~ Deconfined

$\lambda_c = 3.04438$
Modified variational gauge invariant state

\[ |\psi(\alpha, \beta, \gamma, \theta)\rangle = \left[ \prod_{k=2}^{L} e^{i\theta_k \hat{H}_E} e^{i\gamma_k \hat{H}_B} \right] e^{i\alpha \hat{H}_E} \frac{e^{\beta \hat{H}_B}}{(\cosh 2\beta)^{N_p/2}} \left( \bigotimes_{n=0}^{N} |+\rangle \right) \]
Modified variational gauge invariant state

\[ \left| \psi(\alpha, \beta, \gamma, \theta) \right\rangle = \left[ \prod_{k=2}^{L} e^{i \theta_k \hat{H}_E} e^{i \gamma_k \hat{H}_B} \right] e^{i \alpha \hat{H}_E} \frac{e^{\beta \hat{H}_B}}{(\cosh 2\beta)^{N_p/2}} \left( \bigotimes_{n=0}^{N} | + \rangle \right) \]

Energy deviation

- Energy difference with the exact ground state

![Energy deviation graphs]

\( \ell = 2 \)

\( d = 3 \)
\( d = 4 \)
\( d = 5 \)
Modified variational gauge invariant state

$$\left| \psi(\alpha, \beta, \gamma, \theta) \right\rangle = \left[ \prod_{k=2}^{L} e^{i\theta_k \hat{H}_E} e^{i\gamma_k \hat{H}_B} \right] e^{i\alpha \hat{H}_E} \frac{e^{\beta \hat{H}_B}}{(\cosh 2\beta)^{N_p/2}} \left( \bigotimes_{n=0}^{N} |+\rangle \right)$$
Modified variational gauge invariant state

\[ |\psi(\alpha, \beta, \gamma, \theta)\rangle = \left[ \prod_{k=2}^{L} e^{i\theta_k \hat{H}_E} e^{i\gamma_k \hat{H}_B} \right] e^{i\alpha \hat{H}_E} \frac{e^{\beta \hat{H}_B}}{(\cosh 2\beta)^{N_p/2}} \left( \bigotimes_{n=0}^{N} |+\rangle \right) \]

**Dual magnetization**

|         | Dissipative | Monte Carlo | Exact diag. | Unitary $|\phi_{u,e}\rangle$ | Unitary $|\phi_{u,m}\rangle$ |
|---------|-------------|-------------|-------------|----------------------------|----------------------------|
| $\lambda_c$ | 3.24        | 3.04        | 3.06        | 2.56                       | 2.09                       |
| $\beta$  | 0.35        | 0.33        | 0.36        | 0.04                       | -1.27                      |
| $\nu$    | 0.59        | 0.63        | 0.64        | -0.20                      | -0.40                      |
Modified variational gauge invariant state

\[ |\psi(\alpha, \beta, \gamma, \theta)\rangle = \left[ \prod_{k=2}^{L} e^{i\theta_k \hat{H}_E'} e^{i\gamma_k \hat{H}_B'} \right] e^{i\alpha \hat{H}_E'} \frac{e^{\beta \hat{H}_B'}}{(\cosh 2\beta)^{N_p/2}} \left( \bigotimes_{n=0}^{N} \right) \]

Topological entanglement entropy

\[ S_{\text{topo}} = S_A + S_B + S_C - S_{AB} - S_{AC} - S_{BC} + S_{ABC} \]
Modified variational gauge invariant state

\[ |\psi(\alpha, \beta, \gamma, \theta)\rangle = \left[ \prod_{k=2}^{L} e^{i\theta_k \hat{H}_E'} e^{i\gamma_k \hat{H}_B'} \right] e^{i\alpha \hat{H}_E'} \frac{e^{\beta \hat{H}_B'}}{(\cosh 2\beta)^{N_p/2}} \left( \bigotimes_{n=0}^{N} |+\rangle \right) \]

State preparation with noisy gates

\[ \lambda = 3.00 \]
Quantum State Preparation

\[ |\psi(0)\rangle \xrightarrow{U_1 U_3 U_5} |\psi(t)\rangle \]

We show how a variational low-depth circuit can prepare the lowest energy state of a gauge theory
A fruitful dialogue (two-way communication)

- The first successful implementations of gauge-field theory dynamics on quantum simulators have emerged for small systems.
- Efficient Hamiltonian formulations for (non-Abelian) gauge theories along with best approaches to state preparation and measurement will continue to develop.
- Abelian and non-Abelian lattice gauge theories in higher than 1+1 dimensions present significant challenge but progress is being made.
- Theory-experiment collaborations will be highly beneficial.
- New results in the frontier between HEP and Quant-Ph