JLab Theory Seminar

μ-e scattering at 10ppm

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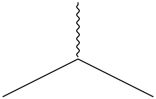
- what is up with $g-2$?
- how is the theory value determined?
- what is MUonE?
- reaching 10^{-5} relative accuracy with McMule

- magnetic moment of a charged lepton: $\vec{\mu} = g\frac{e}{2m}\vec{S}$

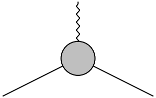
- Dirac: $g_\mu^{\text{Dirac}} = 2$

$(-ie)\bar{u}\gamma^\mu u =$

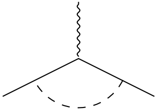


- SM quantum corrections: $g_\mu^{\text{SM}} = 2\times(1+a_\mu) = 2\times(1+F_2(0))$

$$(-ie)\bar{u}\Big[F_1(Q^2)\ \gamma^\mu + F_2(Q^2)\frac{i\sigma^{\mu\nu}Q_\nu}{2m}\Big]u =$$

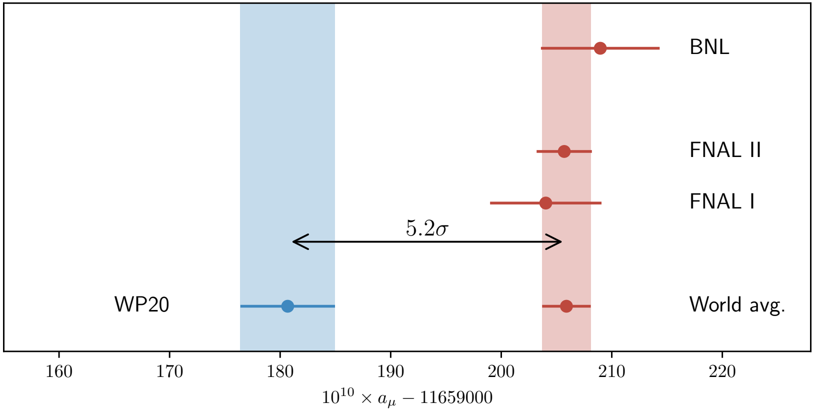


- BSM quantum corrections: $g_\mu^{\text{BSM}} \sim g_\mu^{\text{exp}} - g_\mu^{\text{SM}}$

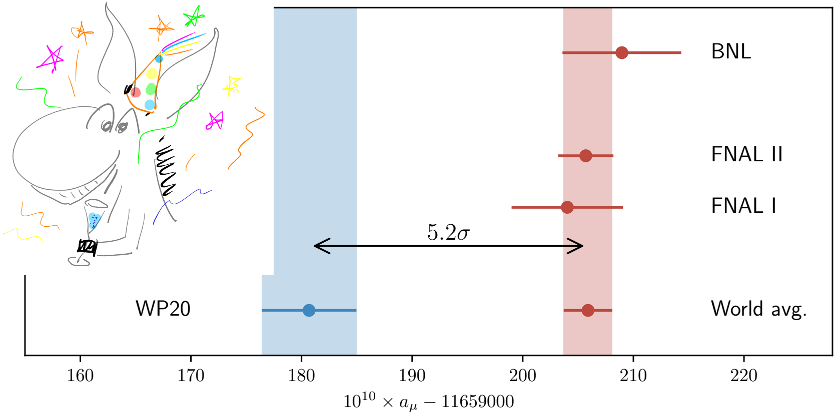


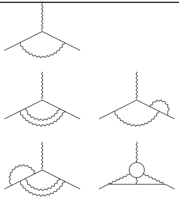
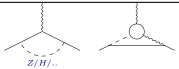
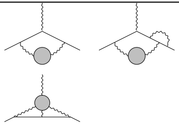
⟨insert favourite BSM⟩

most precise measurement of $g-2$

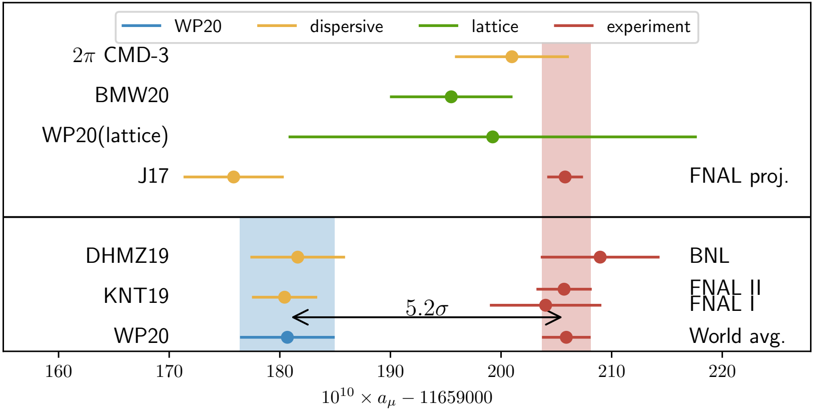


most precise measurement of $g-2$

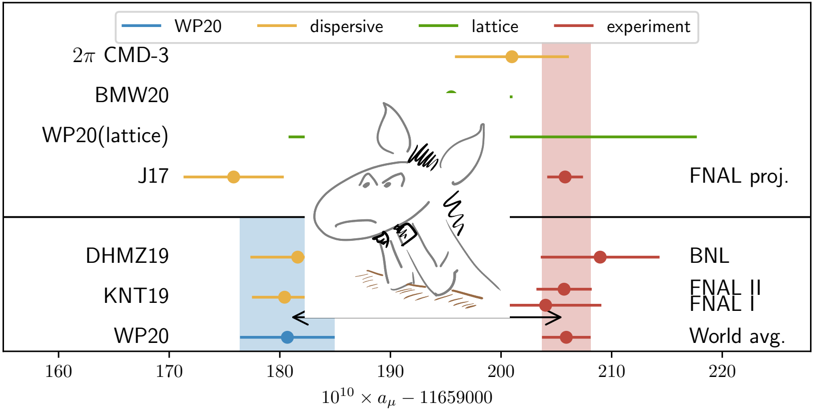


	value	diagrams
QED 1-loop	$\alpha/2\pi = 116\,140\,973$	
QED 2-loop	$-177\,231$	+ 3 others + 1 conspiracy theory
QED 3-loop	$1\,480$	+ 70 others
more QED	-5	
EW	153	
HVP	$6\,845(40)$	 + others
HLbL	$92(17)$	
total	$116\,591\,810(43)$	[$g-2$ white paper 20]
FNAL+BNL	$116\,592\,062(40)$	

largest source of uncertainty & non-perturbative



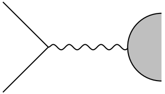
this problem is bigger than $g-2$! [CMD-3 23] [BMW 20] largest source of uncertainty & non-perturbative



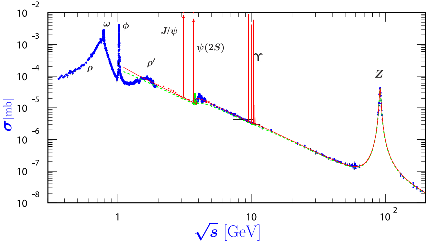
this problem is bigger than $g-2$! [CMD-3 23] [BMW 20] using optical theorem $s > 0$

- measure $ee \to$ hadrons
- remove radiative corrections
- extrapolate to $s \to \infty$ using pQCD
- integrate over s

$$\int \mathrm{d}s \Bigg(K(s)$$



$$\Bigg)$$

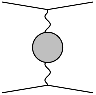


- 72% (78%) of value (uncertainty) from the $ee \to \pi\pi$ channel $s \lesssim 1\,\mathrm{GeV}$

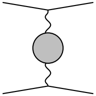
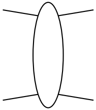
measure low Q^2 regions

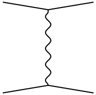
- instead measure in t-channel, i.e. space-like
- no resonances $\to$ much cleaner signal
- HVP is loop-induced $\to$ much smaller signal $(10^{-3} \times \mathrm{LO})$
- competitive extraction @ 10^{-2}

$\Rightarrow$ goal for MUonE: measure $e\mu \to e\mu$ @ 10^{-5}

$$\int \mathrm{d}t \Big(K'(t)$$  $$\Big)$$

[MUonE 19]

 $=$ 

$-$  textbook QED

$-$ QED

5+ years,
4+ workshops,
34+ authors

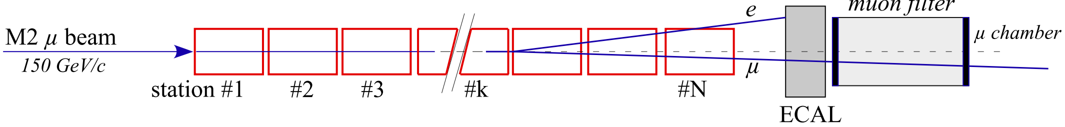
- scattering μ of low-Z material ($_4\mathrm{Be}$)
- pure t-channel $-s \simeq Q^2 \simeq 0$

$\Rightarrow$ high $s \leftrightarrow$ measure more of the curve

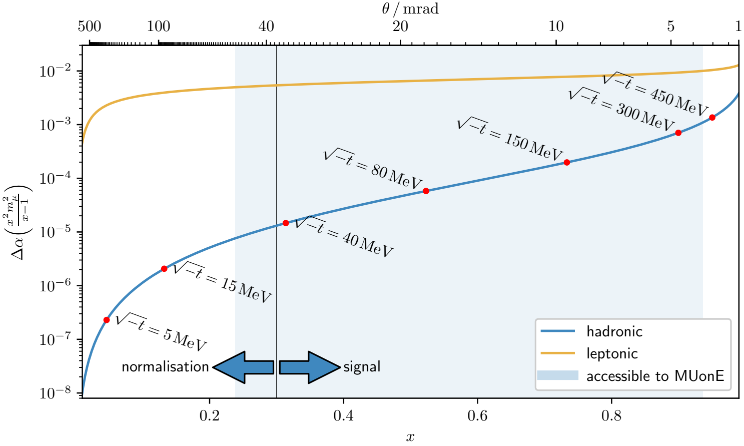
- beam energy needs to be quite high $E_\mu \simeq 160\,\mathrm{GeV}$

$\Rightarrow$ M2 muon beam at CERN North Area

- main measurement: θ_e, θ_μ
  - $+$ E_beam for calibration
  - $+$ E_μ for particle ID



cancel systematic effects $\Big(\mathrm{d}\sigma/\mathrm{d}\theta\Big)_{\mathrm{sig}} \Big/ \Big(\mathrm{d}\sigma/\mathrm{d}\theta\Big)_{\mathrm{norm}}$



6 MUonE (adjacent) theory workshops over 7+ years

Padua 2017

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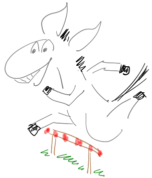
	problem	solution	what?	doable up to?
①	lots of masses	massification	expand in m_e^2/Q^2	LP, three-loop
②	numerical issues in real corrections	NTS stabilisation jettification	expand in $E_\gamma/\sqrt{Q^2}$ expand in $\cos\theta \to 1$	NLP, all-orders LP, one-loop
③	phase space	FKS$^\ell$	YFS-inspired subtraction scheme	all-orders

- NNLO double-boxes: ①
- NNLO real-virtual: ②
- N^{3}LO real-virtual-virtual: ①, ②, jettification



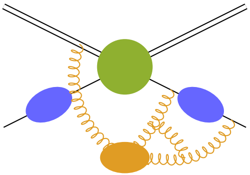
masses are physical in QED $\Rightarrow$ keep masses

- drop polynomially suppressed terms at two-loop $\to$ error $\sim \left(\frac{\alpha}{\pi}\right)^2 \log\frac{m^2}{Q^2} \times \frac{m^2}{Q^2}$
- based on factorisation, SCET, and method of regions

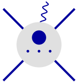
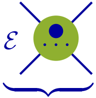
  [Penin 06; Mitov, Moch 06; Becher, Melnikov 07; Engel, Gnendiger, Signer, YU 18]
- process e.g. $e\mu \to e\mu$ at two-loop:

$$\mathcal{A}(m) = \mathcal{S} \times \sqrt{Z} \times \sqrt{Z} \times \mathcal{A}(0) + \mathcal{O}(m) \supset \{1/\epsilon^2, L^2\}$$

- soft: process-dependent $S = 1 +$ fermion loops
  $\to$ compute separately anyway to combine with hadron loops
- collinear: universal Z, converts $1/\epsilon \to \log(m^2/Q^2)$
- hard: massless calculation



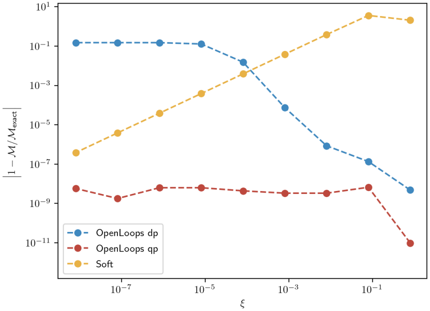
real-virtual corrections trivial in principle, extremely delicate numerically

 $= \frac{1}{E_\gamma^2}\, \mathcal{E}$  $+\mathcal{O}\big(E_\gamma^{-1}\big)$

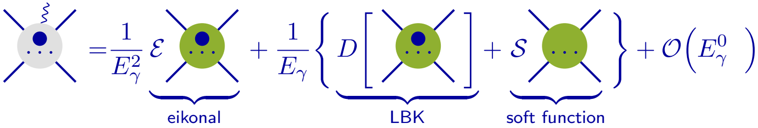
example $ee \to ee\gamma$

[Engel, Signer, YU 21; Kollatzsch, YU 22; Engel 23]

- soft limit $E_\gamma = \xi\,\sqrt{s}/2$
- arbitrary prec. calculation vs dp, qp, eikonal
- stability problem



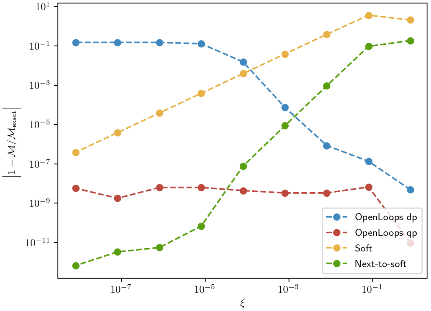
real-virtual corrections trivial in principle, extremely delicate numerically



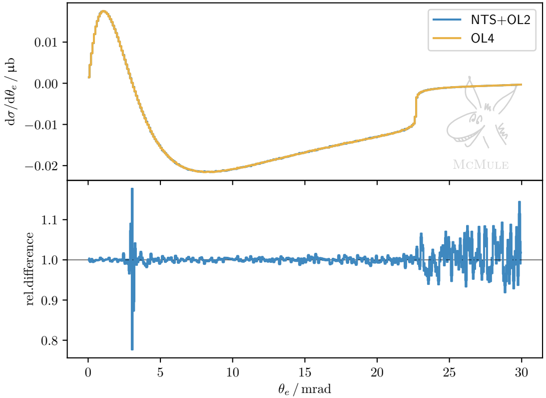
example $ee \to ee\gamma$

[Engel, Signer, YU 21; Kollatzsch, YU 22; Engel 23]

- soft limit $E_\gamma = \xi\,\sqrt{s}/2$
- arbitrary prec. calculation vs dp, qp, eikonal, NTS
- stability problem solved & speed-up



test next-to-soft stabilisation vs OL4 (OpenLoops quad) for $\mu e \to \mu e$ real-virtual



- same statistics, same result
- 70 days vs 4 days
- integrated results for different cuts

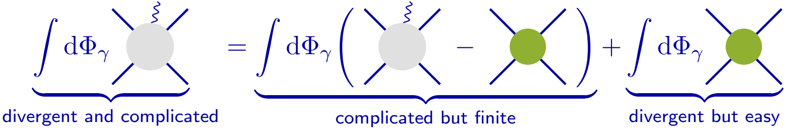
⇒ this is not an approximation but a numerical tool

NTS	OL4
$-0.29268(4)$	$-0.29267(4)$
$-0.44789(6)$	$-0.44778(6)$
$-0.64662(9)$	$-0.64649(9)$

- universal soft limit $\mathcal{M}_{n+1}^{(\ell)} = \mathcal{E}\mathcal{M}_n^{(\ell)} + \mathcal{O}(E_\gamma^{-1})$
- universal pole structure $e^{\hat{\mathcal{E}}} \sum_{\ell=0}^{\infty} \mathcal{M}_n^{(\ell)} = \sum_{\ell=0}^{\infty} \mathcal{M}_n^{(\ell)f}$ = finite

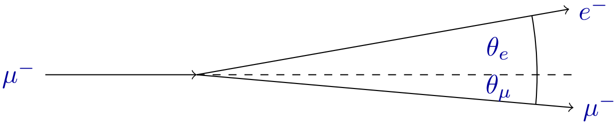
use this to construct an all-order subtraction scheme FKS$^\ell$[Engel, Signer, YU 19]

- nothing complicated needed higher than $\mathcal{O}(\epsilon^0)$
- only one universal CT: $\hat{\mathcal{E}}$



implemented in McMule v0.4.2
`https://mule-tools.gitlab.io`

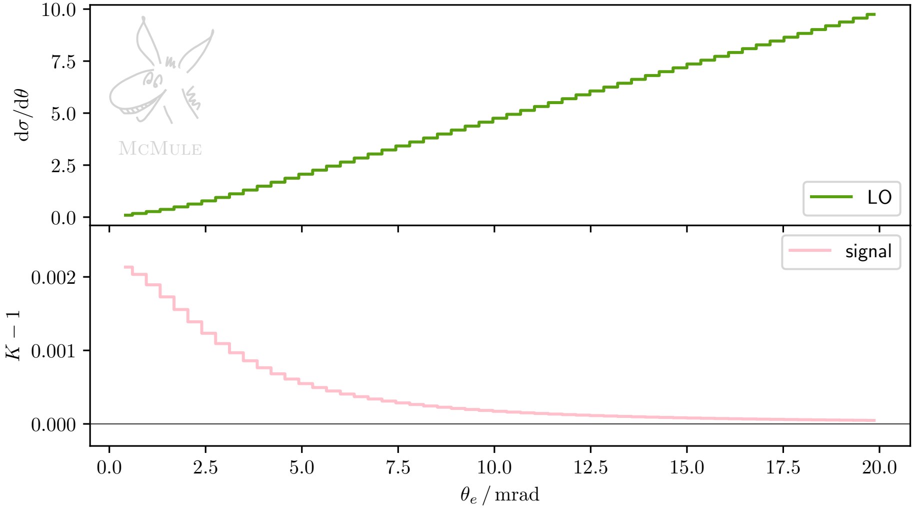
- $\mu^- e^- \to \mu^- e^-$



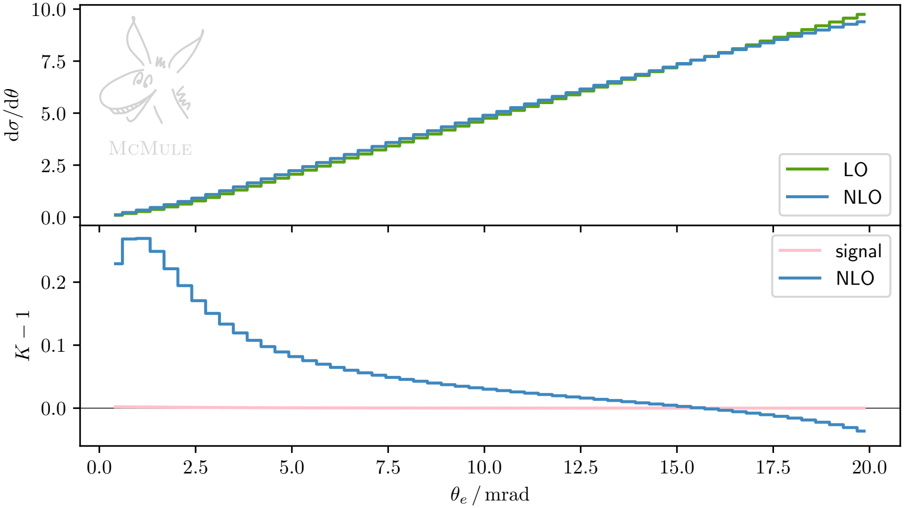
- S1: $E_e > 1\,\mathrm{GeV}$, $\theta_\mu > 0.3\,\mathrm{mrad}$
- run for 2.5 CPU yr
  (290 kWh energy / 3.5 kgCO2e)

[Broggio, Engel, Ferroglia, Mandal, Mastrolia, Rocco, Ronca, Signer, Torres Bobadilla, Zoller, YU 22]

all results and data: `https://mule-tools.gitlab.io/user-library/mu-e-scattering/muone-full-legacy/`

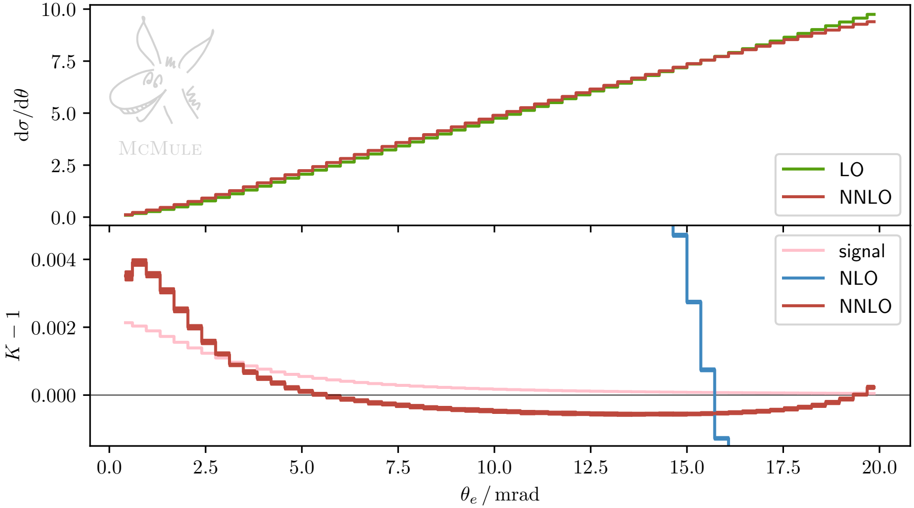


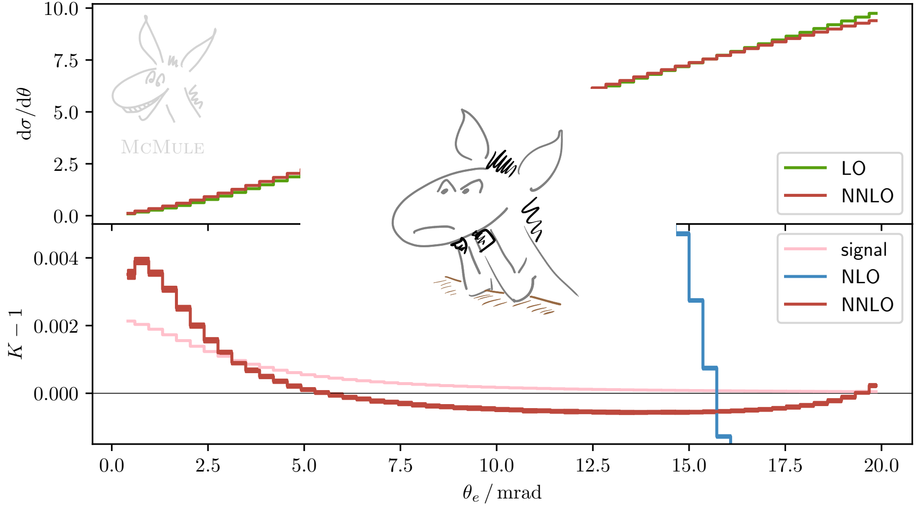
McMule
LO
signal
10.0
7.5
5.0
2.5
0.0
$d\sigma/d\theta$
0.002
0.001
0.000
$K-1$
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
θ_e / mrad



10.0
7.5
5.0
2.5
0.0
dσ/dθ
McMule
LO
NLO
signal
NLO
0.2
0.1
0.0
K − 1
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
θe / mrad

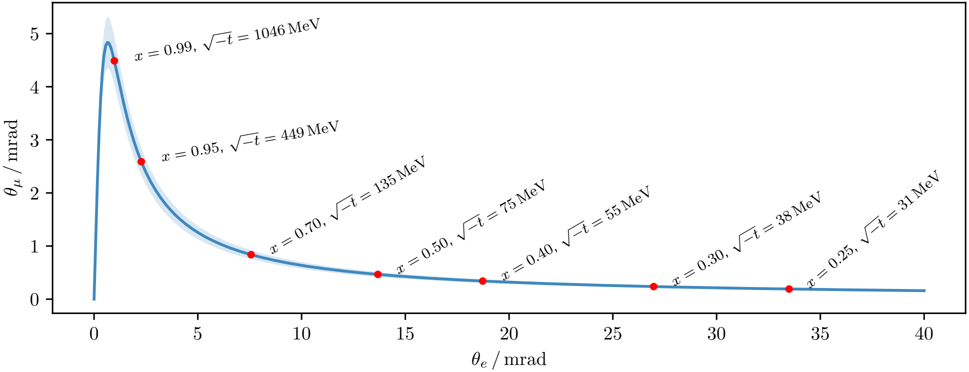
S1 θ_e spectrum

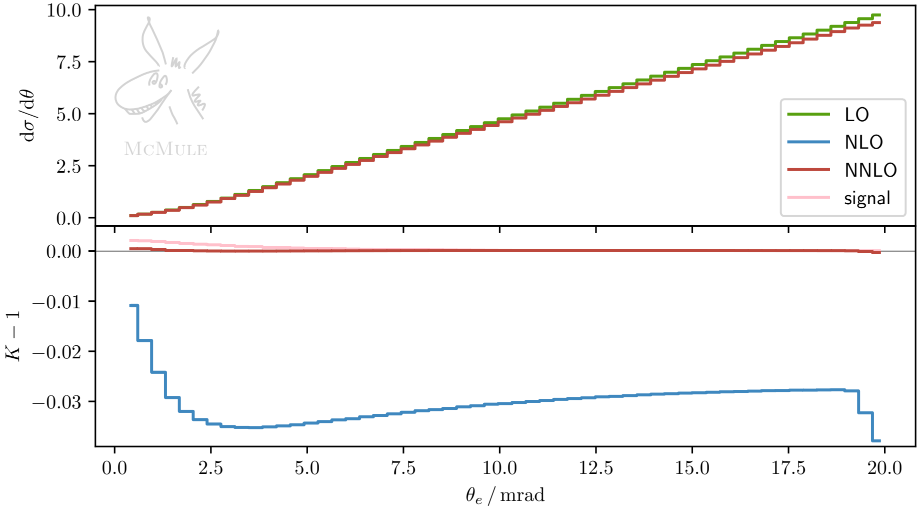



10.0
7.5
5.0
2.5
0.0
dσ/dθ
McMule
LO
NNLO
0.004
0.002
0.000
K − 1
signal
NLO
NNLO
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
θe / mrad

this clearly isn't working

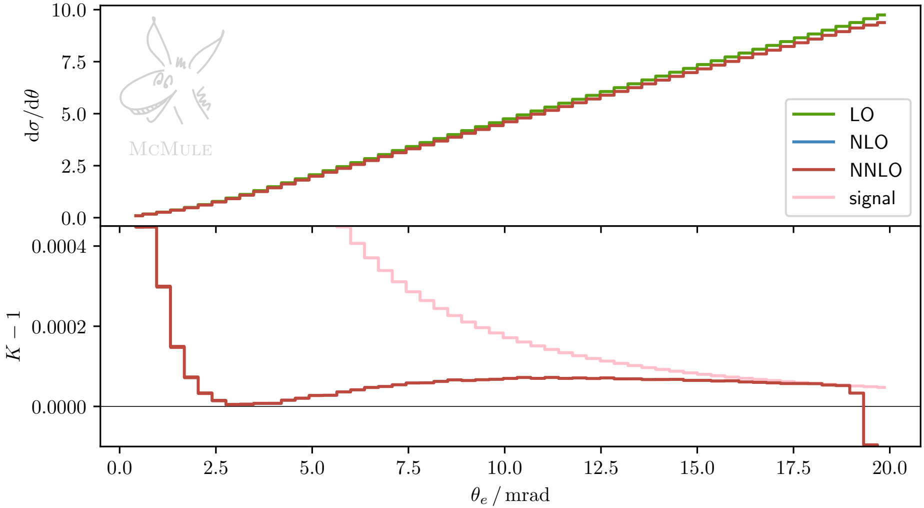
- at this rate ($\sim 10\%$ NLO, $\sim 0.1\%$ NNLO), we would need N^{4}LO to reach 10^{-5}
- most of this is due to hard radition
- S2: same as S1 + needs to be in the band





MCMULE
10.0
7.5
5.0
2.5
0.0
$d\sigma/d\theta$
LO
NLO
NNLO
signal
0.00
−0.01
−0.02
−0.03
$K-1$
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
θ_e / mrad

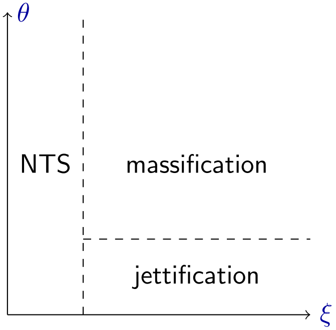
S2 θ_e spectrum



$ee \to \gamma^*$ can be taken to N^3 LO

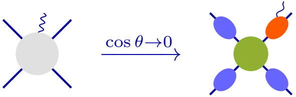
- VVV: known
  [Fael, Lange, Schönwald, Steinhauser 22]
- RRR: “trivial”
- RRV: OpenLoops + NTS stabilisation
- RVV: massless known (three-jet @ NNLO), massive (DiffExp?)

$\Rightarrow$ LBK + jettification at two-loop



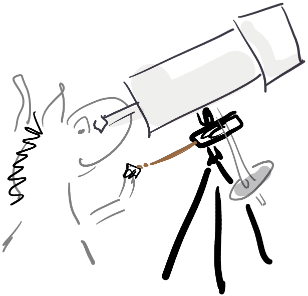
jettification

- expand for small emission angles



$= J \times Z \times Z \times Z \times H$

- ✓ first NNLO with multiple external masses
  [Broggio, Engel, Ferroglia, Mandal, Mastrolia, Rocco, Ronca, Signer, Torres Bobadilla, Zoller, YU 22]
- ✓ event generation (not in McMule)
- ✓ iterative HVP extraction procedure
  [Fael 18]
- ✓ precision now: $\mathcal{O}(10^{\{-3,-4\}})$, goal: $\mathcal{O}(10^{-5})$
- lots of optimisation still possible
  (observable, μ^+ vs. μ^- beam, polarisation etc)
- resummation (analytic & parton shower)
- partial N^{3}LO ($Q_e^8 Q_\mu^2$)

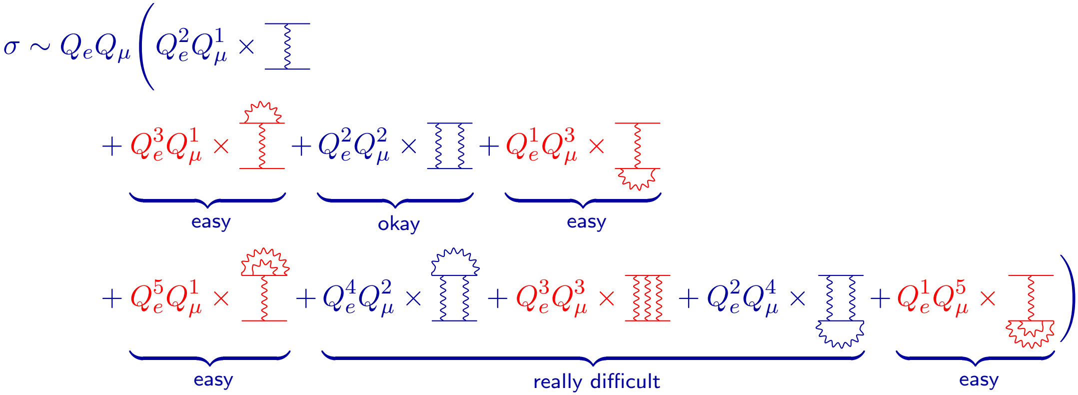


f.l.t.r.: F.Hagelstein (Mainz), A.Coutinho (IFIC), N.Schalch (Bern), L.Naterop (Zurich & PSI), S.Kollatzsch (Zurich & PSI), A.Signer (Zurich & PSI), M.Rocco (PSI), T.Engel (Freiburg), V.Sharkovska (Zurich & PSI), Y.Ulrich (Bern), A.Gurgone (Pavia)
not pictured: P.Banerjee (IIT Guwahati), D.Moreno (PSI), D.Radic (PSI)

McMule

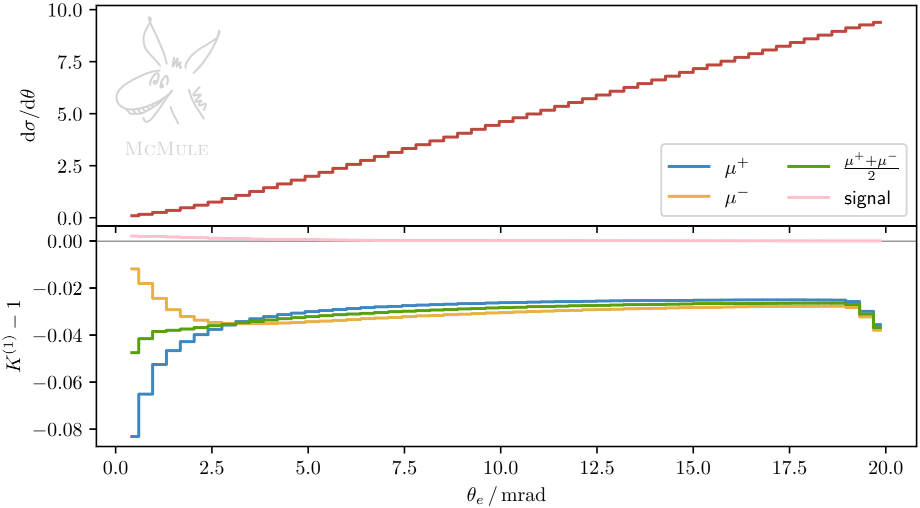
mule-tools.gitlab.io

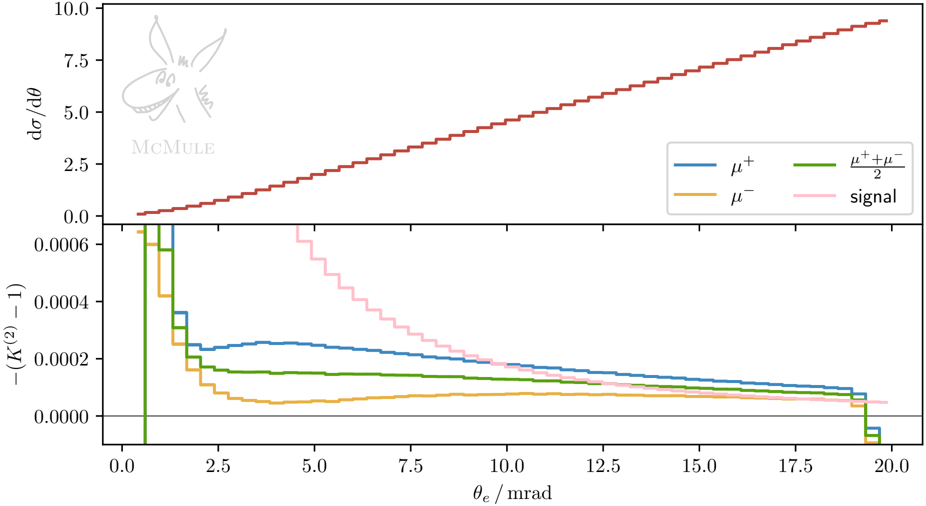
the beam can do both μ^+ and μ^-



- proposal $\sigma(\mu^+) + \sigma(\mu^-)$

$\Rightarrow$ some of the difficult stuff cancels


10.0
7.5
5.0
2.5
0.0
dσ/dθ
McMule
μ^+
μ^-
$\frac{\mu^+ + \mu^-}{2}$
signal
0.00
−0.02
−0.04
−0.06
−0.08
$K^{(1)} - 1$
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
θ_e / mrad



10.0
7.5
5.0
2.5
0.0
dσ/dθ
McMule
μ+
μ−
(μ+ + μ−)/2
signal
0.0006
0.0004
0.0002
0.0000
−(K(2) − 1)
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
θe / mrad