Particle correlations from initial state

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based on [TA, N. Armesto, arXiv:2004.08185]

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Outline

- Introduction to small x physics and CGC
- Two particle correlations within eikonal approxmiation
- Two particle correlations beyond eikonal approximation
- Discussion

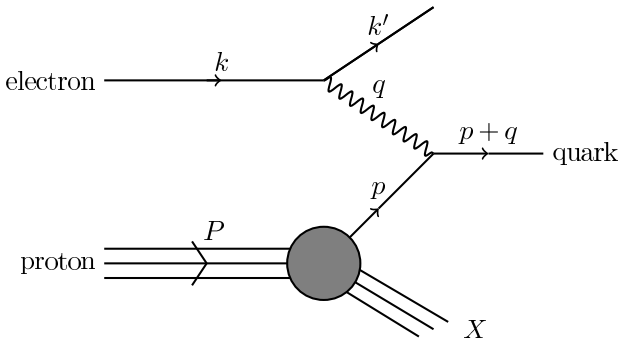
High energy scattering in QCD

High energy scattering in QCD

"hard" scattering
→ large momentum exchange
→ weakly coupled
→ perturbative

"soft" scattering
→ small momentum exchange
→ strongly coupled
→ non-perturbative

DIS in QCD :



Three Lorentz invariant quantities :

1. $q^2 = -Q^2 \equiv$ virtuality of the incoming photon
2. $x = \frac{Q^2}{2P\cdot Q} \equiv$ longitudinal momentum fraction carried by the parton
3. $s \simeq 2P \cdot Q \equiv$ energy of the colliding $\gamma - p$ system

increasing the energy ($s = Q^2/x$) of the system:

Bjorken limit fixed x, $Q^2 \to \infty$

- density of partons decreases.
- system becomes more dilute!
- evolution is given by DGLAP.

Regge-Gribov limit fixed Q^2, $x \to 0$

- density of partons increases.
- system becomes dense!
- causes saturation !

Bjorken limit and DGLAP evolution

In the Bjorken limit: description given by *(QCD-improved) parton model*

proton $\simeq$ set of independent free partons (assumed to be dilute).

γ-p X-section (collinear factorization):

$$\sigma_{T,L}^{\gamma p \to X}(x_{Bj}, Q^2) \propto \sum_{i=q_f,\bar{q}_f,g} \int_{x_{Bj}}^{1} \frac{dz}{z} C_{T,L;i}\left(\frac{x_{Bj}}{z}, \alpha_s(Q^2)\right) f_i\left(z, Q^2\right)$$

$C_{T,L;i}\left(\frac{x_{Bj}}{z}, \alpha_s(Q^2)\right) \equiv$ perturbatively calculable coefficient function

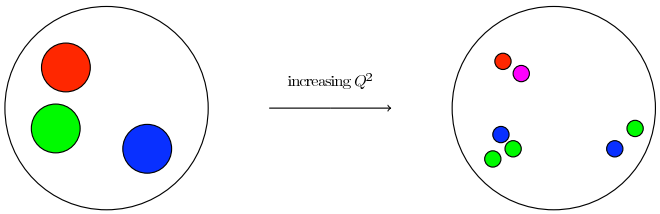
$f_i\left(z, Q^2\right) \equiv$ Parton Distribution Function (PDF): *The number density of partons of type i in the proton seen with transverse resolution $1/Q^2$, carrying a momentum fraction z.*

In the infinite momentum frame:

- transverse size of the photon $\sim 1/Q$ (very small probe).
- can scatter off a quark with the size of $\sim 1/Q$.

$\Rightarrow$ Q is the resolution scale!

$\cdots$ *with increasing Q^2:*



- more substructure resolved by the probe,
- target effectively contains more partons,
- HOWEVER, density of partons decreases!!

DGLAP evolution:

$$\frac{d}{d\log Q^2} f_i(x, Q^2) = \frac{\alpha(Q^2)}{\pi} \int_0^1 \frac{dz}{z} \left\{ P_{q\leftarrow q}(z) f_i\left(\frac{x}{z}, Q\right) + P_{q\leftarrow g}(z) f_g\left(\frac{x}{z}, Q\right) \right\}$$

Regge-Gribov limit: decreasing x at fixed Q^2

[Balitsky, Fadin, Kuraev, Lipatov - 1977, 1978]

First approach: BFKL equation - evolution wrt rapidity $Y = \ln(1/x)$

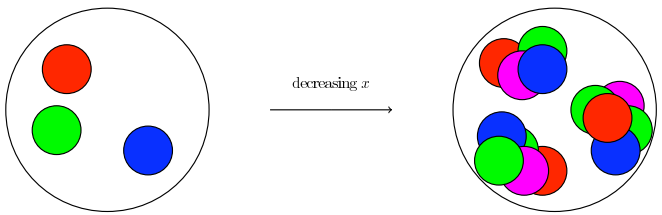
$$\boxed{\frac{\partial \varphi(Y,q)}{\partial Y} = \frac{\alpha_s N_c}{\pi^2} \int d^2k \left[\frac{q^2}{k^2(q-k)^2}\varphi(Y,k) - \frac{1}{2}\frac{q^2}{k^2(q-k)^2}\varphi(Y,q) \right]}$$

$\varphi(Y,q) \equiv$ unintegrated gluon density $\rightarrow$ $\boxed{xf_g(x,Q) = \int_0^{Q^2} \frac{d^2k}{k^2}\varphi(x,k)}$

At very high energies BFKL equation has two major problems:

- Froissart Bound : $\sigma^{total} < \frac{\pi}{m^2} Y^2$
  - X-section calculated by the solution of BFKL equation : $\sigma^{total} \sim e^{cY}$
  - to solve this problem information from the infrared scale of QCD needed.
- violation of unitarity
  - scattering probability grows without a bound, exceeding unity at rapidities of order $Y \simeq \frac{1}{\alpha_s}\ln(1/\alpha_s)$
  - this problem can be addressed by taking into account gluon saturation effects.

$\cdots$ decreasing x at fixed Q^2 (rapidity evolution):

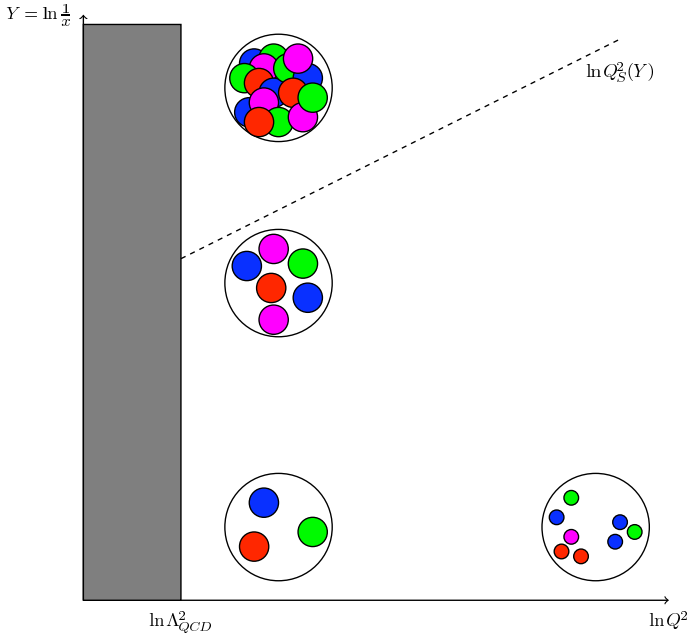


- Nb. of partons increase due to splitting
- Transverse scale doesn't change
- Mother and daughter partons have the same size

$\Rightarrow$ *density of partons increases and causes saturation.*

Color Glass Condensate (CGC) - I

High energy scattering in QCD:



- Regge-Gribov limit : $x \to 0$
- at small $x \to$ **saturation**!
  - $Q_s \equiv$ saturation scale $\equiv \alpha_s \times$ (gluon density per unit area)
  - Q_s is a measure of the strength of the gluon interaction processes that may occur when the gluon density becomes large.
  - $Q_s \gg \Lambda_{QCD} \Rightarrow$ weak coupling methods can still be applied !

[McLerran, Venugopalan - hep-ph/9309289 / hep-ph/9311205]

In the saturation regime the prescription of scattering process: Color Glass Condensate (CGC)

CGC description of a process: "effective degrees of freedom" with respect to a cut off Λ^+

- fast partons : $k^+ > \Lambda^+$ $\to$ described by color sources: $J^\mu(x) = \delta^{\mu+}\rho(x^-, x_\perp)$
- slow partons: $k^+ < \Lambda^+$ $\to$ described by color fields $A^\mu(x)$

interaction between fast and slow partons: $\int d^4x J^\mu(x) A_\mu(x)$

Color Glass Condensate (CGC) - II

Within the CGC framework:

expectation value of an observable $\mathcal{O} \Rightarrow$ $\langle \mathcal{O} \rangle \equiv \int [D\rho] \; W[\rho] \; \mathcal{O}[\rho]$

$W[\rho] \equiv$ distribution function for the color sources ρ^a.

[Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner, 1997-2002]

Rapidity (Y) evolution of the distribution function is governed by the JIMWLK evolution equation:

$$\boxed{\partial_Y W_Y[\rho] = -\mathcal{H}_{JIMWLK}\left[\rho, \frac{\delta}{\delta\rho}\right] W_Y[\rho]}$$

interaction between the projectile and the target:

each parton picks up a Wilson line during the interaction with the target:

$$\boxed{U_{\mathcal{R}}(\mathbf{x}) = \mathcal{P}_+ \exp\left[ig \int dx^+ T^a_{\mathcal{R}} A^-_a(x^+, \mathbf{x})\right]}$$

dipole operator appears in the observable:

$$\boxed{d_{\mathcal{R}}(\mathbf{x}, \mathbf{y}) = \frac{1}{D_{\mathcal{R}}} \mathrm{tr}\left[U_{\mathcal{R}}(\mathbf{x}) U^\dagger_{\mathcal{R}}(\mathbf{y})\right]}$$

$D_{\mathcal{R}} \equiv$ the color dimension of the representation.

Nonlinear evolution of the dipole operator

JIMWLK equation for the dipole operator

$$\frac{\partial\langle d(\mathbf{x},\mathbf{y})\rangle}{\partial Y} = -\frac{\alpha_s N_c}{2\pi^2}\int d^2\mathbf{z}\frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{y}-\mathbf{z})^2}\left\{\langle d(\mathbf{x},\mathbf{y})\rangle - \langle d(\mathbf{x},\mathbf{z})d(\mathbf{z},\mathbf{y})\rangle\right\}$$

Dipole scattering probability: $d(\mathbf{x},\mathbf{y}) = 1 - N(\mathbf{x},\mathbf{y})$

$\langle d(\mathbf{x},\mathbf{z})d(\mathbf{z},\mathbf{y})\rangle \equiv$ simultaneous scattering of both dipoles off the target.

[Balitsky - 1996, Kovchegov -1999]

Assumption: *The areas of the target on which the dipoles are uncorrelated*:

$$\boxed{\langle d(\mathbf{x},\mathbf{z})d(\mathbf{z},\mathbf{y})\rangle \Rightarrow \langle d(\mathbf{x},\mathbf{z})\rangle\,\langle d(\mathbf{z},\mathbf{y})\rangle} \Rightarrow \boxed{\text{JIMWLK equation} \Rightarrow \text{BK equation}}$$

possible applications in the gluon saturation regime:

- *dilute-dilute scattering* : No saturation effects / BFKL formalism
  - can be applied to: $\gamma^* - \gamma^*$, DIS on p, pp at moderate energies
- *dilute-dense scattering* : saturated target / CGC formalism
  - can be applied to: DIS on A , pA collisions, forward particle production in pp.
- *dense-dense scattering:* saturated projectile and target / non-linear dynamics of Yang-Mills fields
  - can be applied to: pp at very high energies, heavy ion collisions.

saturation sensitive observables in pA collisions:

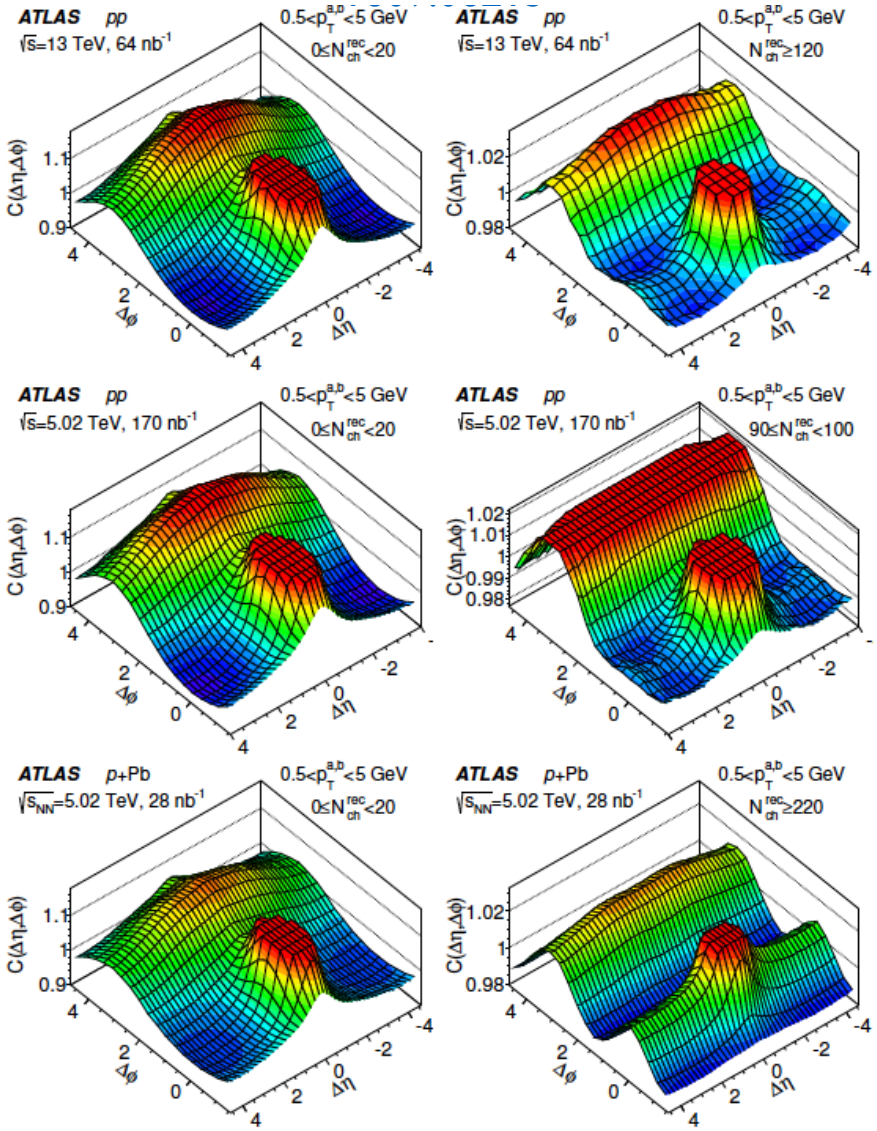
⋆ *forward particle/jet production*

⋆ *two particle correlations*

Two particle correlations

Motivation: Ridge structure

[ATLAS Collaboration - arXiv:1609.06213]



- *correlations between particles over large intervals of rapidity peaking at zero and π relative azimuthal angle.*
- observed first at RHIC in Au-Au collisions.
- observed at LHC for high multiplicity pp and pA collisions.

Correlations within the CGC framework

Ridge in HICs $\leftrightarrow$ collective flow due to strong final state interactions

(good description of the data in the framework of relativistic viscous hydrodynamics)

Ridge in small size systems: similar reasoning looks tenuous but hydro describes the data very well.

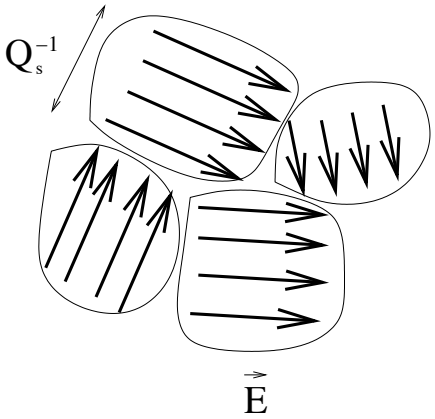
Can it be initial state effect?

idea: *final state particles carry the imprint of the partonic correlations that exist in the initial state.*

Several mechanisms have been suggested to explain the ridge correlations in the CGC framework:

(i) Local anisotropy of the target fields $\rightarrow$ rotational symmetry is broken.

[Kovner, Lublinsky - arXiv:1012.3398 / arXiv:1109.0347 / arXiv:1211.1928]



particles correlated in the incoming w.f.

transverse separation $\ll 1/Q_s$

scatter through the same domain.

initial state correlations $\rightarrow$ final state correlations

Numerical studies based on local anisotropy of the target:

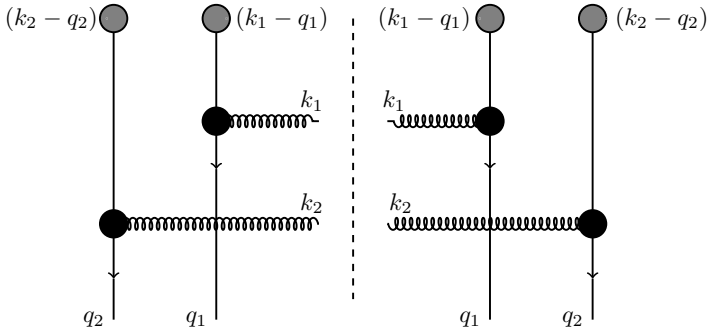
[Dumitru, Skokov - arXiv:1411.6030] / [Dumitru, McLerran, Skokov - arXiv:1410.4844]

Correlations within the CGC framework -II

(ii) Glasma graph approach to two gluon production:

[Dumitru, Gelis, McLerran, Venugopalan - arXiv:0804.3858]

[Dumitru, Dusling, Gelis, Jalilian-Marian, Lappi, Venugopalan - arXiv:1009.5295]



What is the physics behind the glasma graph approximation?

⋆ Glasma graph calculation contains two physical effects:

- Bose enhancement of the gluons in projectile/target wave function

  [TA, Armesto, Beuf, Kovner, Lublinsky - arXiv:1503.07126]

  $$\sigma|_{BE,P} \propto \left\{\delta^{(2)}\left[(k_1 - q_1) - (k_2 - q_2)\right] + \delta^{(2)}\left[(k_1 - q_1) + (k_2 - q_2)\right]\right\}$$

  $$\sigma|_{BE,T} \propto \left\{\delta^{(2)}(q_1 - q_2) + \delta^{(2)}(q_1 + q_2)\right\}$$

- Hanbury-Brown-Twiss (HBT) correlations between gluons far separated in rapidity.

  $$\sigma|_{HBT} \propto \left\{\delta^{(2)}(k_1 - k_2) + \delta^{(2)}(k_1 + k_2)\right\}$$

  [TA, Armesto, Beuf, Kovner, Lublinsky - arXiv:1509.03223]

Correlations within the CGC framework -III

Two particle correlations beyond the glasma graph approach: 2 gluon production in pA collisions

[TA, Armesto, Wertepny - arXiv:1804.02910] $\rightarrow$ $k_\perp$-factorized approach

[TA, Armesto, Kovner, Lublinsky - arXiv:1805.07739] $\rightarrow$ Glasma graph approach .

scattering on a dense target $\rightarrow$ dipole and quadrupole operators. Factorization assumption:

$$\begin{aligned}
\langle Q(x,y,z,v)\rangle_T &\rightarrow d(x,y)d(z,v) + d(x,v)d(z,y) + \frac{1}{N_c^2-1}d(x,z)d(y,v) \\
\langle D(x,y)D(z,v)\rangle_T &\rightarrow d(x,y)d(z,v) + \frac{1}{(N_c^2-1)^2}\left[d(x,v)d(y,z) + d(x,z)d(v,y)\right]
\end{aligned}$$

double inclusive X-section:

$$\frac{d\sigma}{d^3k_1 d^3k_2} \propto \int_{q_1 q_2} \left\{ d(q_1)d(q_2)\left[I_0 + \frac{1}{N_c^2-1}I_1 + \frac{1}{(N_c^2-1)^2}I_2\right] + (k_2 \rightarrow -k_2)\right\} + O\left(\frac{1}{Q_s S_\perp}\right)$$

symmetry under $(k_2 \rightarrow -k_2)$: "**accidental symmetry of the CGC"**

$I_0 \propto \delta^{(2)}(0) \rightarrow$ uncorrelated contribution.

$$I_1 \propto \left\{ \underbrace{f_1\delta^{(2)}\left[(k_1-q_1)-(k_2-q_2)\right]}_{\text{BE. proj.}} + \underbrace{f_2\delta^{(2)}(k_1-k_2)}_{\text{HBT}} \right\}$$

$$I_2 \propto \left\{ \underbrace{g_1\delta^{(2)}(q_1-q_2)}_{\text{BE. target}} + \underbrace{g_2\delta^{(2)}\left[(k_1-q_1)-(k_2-q_2)\right]}_{\text{BE. proj.}} \right\}$$

Tools to study the ridge correlations: azimuthal harmonics

Convenient way to study the two particle correlations: Fourier decomposition into harmonics in $\Delta\phi$

$\Delta\phi \equiv$ azimuthal angle between the produced gluons with transverse momenta $\mathbf{k}_1$ and $\mathbf{k}_2$

$$\frac{dN}{d^2\mathbf{k}_1 d\eta_1 d^2\mathbf{k}_2 d\eta_2} \equiv N(k_1, k_2, \Delta\phi) = a_0(k_1, k_2) + \sum_{n=1}^{\infty} a_n(k_1, k_2)\cos(n\Delta\phi)$$

[T. Lappi, B. Schenke, S. Schlichting, R. Venugopalan - arXiv: 1509.03499]

Spectrum is defined as

$$N(k_1, k_2, \Delta\phi) = a_0(k_1, k_2)\left[1 + \sum_{n=1}^{\infty} 2V_{n\Delta}(k_1, k_2)\cos(n\Delta\phi)\right]$$

where

$$2V_{n\Delta}(k_1, k_2) = \frac{a_n(k_1, k_2)}{a_0(k_1, k_2)} = 2\,\frac{\int_0^\pi N(k_1, k_2, \Delta\phi)\,\cos(n\Delta\phi)\,d\Delta\phi}{\int_0^\pi N(k_1, k_2, \Delta\phi)\,d\Delta\phi}$$

- set $k_1 = p_T^{ref}$ and $k_2 = p_T$. Then, *the azimuthal harmonics are defined as*

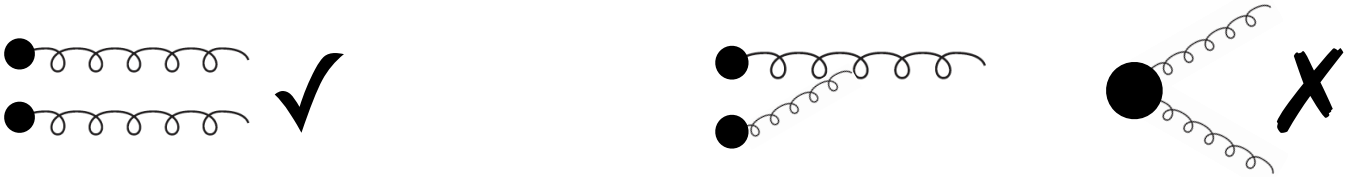
$$v_n(p_T) = \frac{V_{n\Delta}(p_T, p_T^{ref})}{\sqrt{V_{n\Delta}(p_T^{ref}, p_T^{ref})}}$$

challenge: *accidental symmetry in CGC $\Rightarrow$ vanishing odd harmonics!*

Breaking the accidental symmetry in the CGC

(i) density corrections to the projectile:

[Kovner, Lublinsky, Skokov - arXiv:1612.07790] / [Kovchegov, Skokov - arXiv:1802.08166]



$$\frac{dN^{\text{even, odd}}(\mathbf{k}_\perp)}{d^2k dy} = \frac{1}{2}\left(\frac{dN(\mathbf{k}_\perp)}{d^2k dy}\Big[\rho_p, \rho_t\Big] \pm \frac{dN(-\mathbf{k}_\perp)}{d^2k dy}\Big[\rho_p, \rho_t\Big]\right) \quad \Rightarrow \text{non-vanishing odd harmonics.}$$

numerical studies and comparison with data:

[Mace, Skokov, Tribedy, Venugopalan - arXiv:1805.09342 / arXiv:1807.00825 / arXiv:1901.10506]

(ii) subeikonal corrections due to finite width of the target:

[Agostini, TA, Armesto - arXiv:1902.04830 / arXiv:1907.03668]

relax the eikonal approximation by considering a finite width target to compute two particle correlations

Eikonal dilute-dense scattering

For central production (glasma graph approximation) both the target and the projectile are boosted from their rest frame to the center of mass frame:

Boosting the target:

$$A_a^\mu(x) \mapsto \begin{cases} \gamma_t\, A_a^- \left(\gamma_t\, x^+,\ \frac{x^-}{\gamma_t},\ \mathbf{x}\right) \\ \frac{1}{\gamma_t} A_a^+ \left(\gamma_t\, x^+,\ \frac{x^-}{\gamma_t},\ \mathbf{x}\right) \\ A_a^i \left(\gamma_t\, x^+,\ \frac{x^-}{\gamma_t},\ \mathbf{x}\right) \end{cases}$$

- $A_a^- \gg A_a^i \gg A_a^+$ in a generic gauge
- in the light-cone gauge:

  $A_a^\mu(x) = \delta^{\mu-}\delta(x^+)A_a^-(\mathbf{x})$

  target is localized at $x^+ = 0$

  independent of x^-

Boosting the projectile :

$$J_a^\mu(x) \mapsto \begin{cases} \frac{1}{\gamma_p}\, J_a^- \left(\frac{x^+}{\gamma_p},\ \gamma_p x^-,\ \mathbf{x}\right) \\ \gamma_p\, J_a^+ \left(\frac{x^+}{\gamma_p},\ \gamma_p x^-,\ \mathbf{x}\right) \\ J_a^i \left(\frac{x^+}{\gamma_p},\ \gamma_p x^-,\ \mathbf{x}\right) \end{cases}$$

- $J_a^+ \gg J_a^i \gg J_a^-$
- slow x^+ dependence due to Lorentz time dilation

  $J_a^\mu(x) \propto \delta^{\mu+}\delta(x^-)\rho^a(\mathbf{x})$

  projectile is localized at $x^- = 0$

Subeikonal corrections in the CGC

Eikonal approximation amounts dropping the energy suppressed terms!

For realistic values of energy one should go beyond eikonal approximation.

[TA, Armesto, Beuf, Martinez, Salgado - arXiv:1404.2219]
[TA, Armesto, Beuf, Moscoso - arXiv:1505.01400]

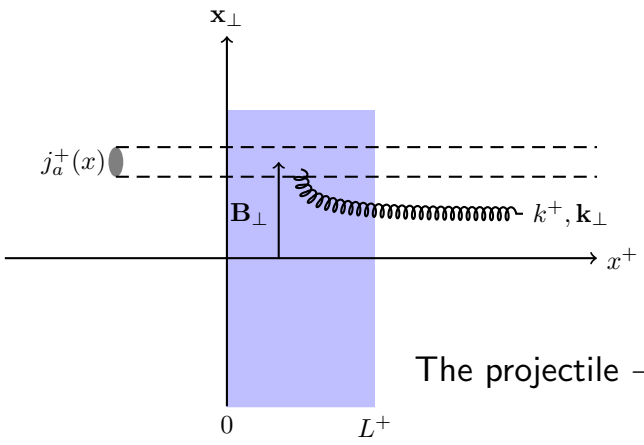
Finite-width-target corrections in single inclusive gluon production in pA collisions:

⋆ dense target is defined by $\mathcal{A}^\mu(x)$ and eikonal approximation amounts to:

1. $\mathcal{A}^\mu_a(x) \simeq \delta^{\mu-}\mathcal{A}^-_a(x)$
2. $\mathcal{A}^\mu_a(x) \simeq \mathcal{A}^\mu_a(x^+, \mathbf{x})$
3. $\mathcal{A}^\mu_a(x) \propto \delta(x^+)$

1. other components of the target background field $\mathcal{A}^\mu_a(x)$
2. dynamics of the target : x^- dependence of $\mathcal{A}^\mu_a(x)$
3. Finite width L^+ of the target along x^+

$$\boxed{\mathcal{A}^\mu = \delta^{\mu-}\delta(x^+)\mathcal{A}^-(\mathbf{x}) \to \mathcal{A}^\mu = \delta^{\mu-}\mathcal{A}^-(x^+, \mathbf{x})}$$



The target $\to \mathcal{A}^\mu(x) \equiv \delta^{\mu-}\mathcal{A}^-_a(x^+, \mathbf{x})$

The projectile $\to$ $j^\mu_a(x) \propto \delta^{\mu+}\delta(x^-)\,\rho^b(\mathbf{x} - \mathbf{B})$

Noneikonal single inclusive production

The single inclusive gluon cross section for pA:

$$(2\pi)^3\,(2k^+)\,\frac{d\sigma}{dk^+\,d^2\mathbf{k}} = \int d^2\mathbf{B}\sum_{\lambda\,\text{phys.}}\left\langle\left\langle|\mathcal{M}^a_\lambda(\underline{k},\mathbf{B})|^2\right\rangle_p\right\rangle_A$$

gluon production amplitude

For a finite-width target: $\mathcal{M}^a_\lambda(\underline{k},\mathbf{B}) \propto \mathcal{G}^{ab}_{k^+}(\underline{x};\underline{y}) \equiv$ scalar background propagator

$\mathcal{G}^{ab}_{k^+}(\underline{x};\underline{y})$ satisfies the scalar Green's eq. whose solution can be written formally as a path integral

$$\mathcal{G}^{ab}_{k^+}(\underline{x};\underline{y}) = \theta(x^+-y^+)\int_{\mathbf{z}(y^+)=\mathbf{y}}^{\mathbf{z}(x^+)=\mathbf{x}}\mathcal{D}\mathbf{z}(z^+)\,\exp\left[\frac{ik^+}{2}\int_{y^+}^{x^+}dz^+\,\dot{\mathbf{z}}^2(z^+)\right]U^{ab}\Big(x^+,y^+,\big[\mathbf{z}(z^+)\big]\Big)$$

with the Wilson line

$$\mathcal{U}^{ab}\Big(x^+,y^+,\big[\mathbf{z}(z^+)\big]\Big) \;=\; \mathcal{P}_+\exp\left\{ig\int_{y^+}^{x^+}dz^+\;T\cdot\mathcal{A}^-\Big(z^+,\mathbf{z}(z^+)\Big)\right\}^{ab}$$

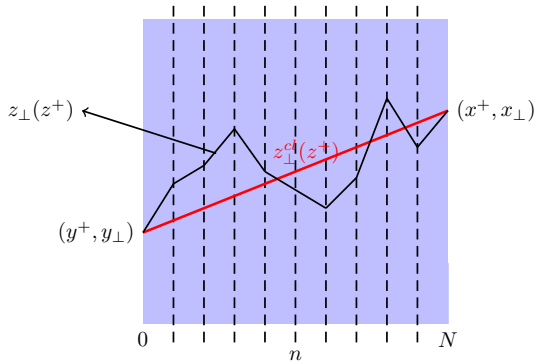
following the Brownian trajectory $\mathbf{z}(z^+)$.

AIM: Perform an eikonal expansion of $\mathcal{G}^{ab}_{k^+}(\underline{x};\underline{y})$.

Expanding the background propagator

(i) discretize the background propagator.

(ii) Perturbative expansion around free classical path:



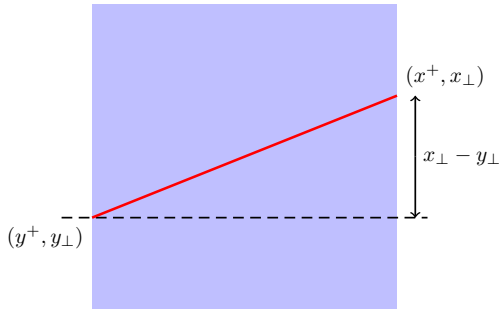
eikonal limit : $\frac{k^+}{(x^+-y^+)} \gg Q_\perp^2$ in the problem

$\mapsto$ large k^+ limit (classical free path!)

$\Rightarrow$ perturbative expansion around the free classical path:

$\mathbf{z}_n = \mathbf{z}_n^{\rm cl} + \mathbf{u}_n$ with $\mathbf{z}_n^{\rm cl} = \mathbf{y} + \frac{n}{N}(\mathbf{x} - \mathbf{y})$

(iii) Expansion around the initial transverse position:



The first expansion is performed for fixed initial and final positions.

In the large k^+ limit , the result has to be re-expanded since $\mathbf{z}^{\rm cl}(z^+) - \mathbf{y}$ is small at each step.

Background scalar propagator at NNE accuracy

After all:

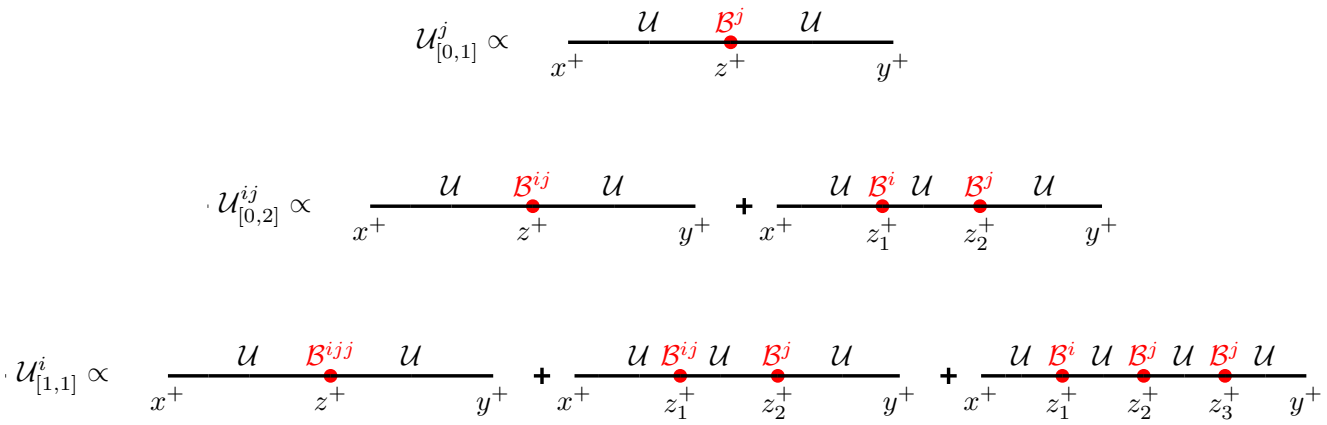
$$\int d^2\mathbf{x}\; e^{-i\mathbf{k}\cdot\mathbf{x}}\; \mathcal{G}^{ab}_{k^+}(\underline{x};\underline{y}) = \theta(x^+-y^+)\; e^{-i\mathbf{k}\cdot\mathbf{y}}\; e^{-ik^-(x^+-y^+)} \Bigg\{ \mathcal{U}(x^+,y^+,\mathbf{y})$$
$$+ \frac{(x^+-y^+)}{k^+}\left[\mathbf{k}^i\, \mathcal{U}^i_{[0,1]}(x^+,y^+,\mathbf{y}) + \frac{i}{2}\,\mathcal{U}_{[1,0]}(x^+,y^+,\mathbf{y})\right]$$
$$+ \frac{(x^+-y^+)^2}{(k^+)^2}\left[\mathbf{k}^i\mathbf{k}^j\mathcal{U}^{ij}_{[0,2]}(x^+,y^+;\mathbf{y}) + \frac{i}{2}\mathbf{k}^i\mathcal{U}^i_{[1,1]}(x^+,y^+;\mathbf{y}) - \frac{1}{4}\mathcal{U}_{[2,0]}(x^+,y^+;\mathbf{y})\right]\Bigg\}^{ab}$$

- $\mathcal{U}(x^+,y^+,\mathbf{y}) \equiv$ standard Wilson lines that appears only at the eikonal level as expected.

$$\mathcal{U}(x^+,y^+,\mathbf{y}) = \mathcal{P}_+\, e^{ig\int_{x^+}^{y^+} dz^+\, T_c\; A_c^-(z^+,\mathbf{y})}$$

- $\mathcal{U}_{[\alpha,\beta]}(x^+,y^+,\mathbf{y}) \equiv$ decorated Wilson lines that only appears beyond eikonal accuracy.
- The subscripts
  - α stands for the order of the expansion around the classical path.
  - β stands for the order of the expansion around the initial transverse position.

Structure of the decorated Wilson lines



with

$$
\begin{aligned}
\mathcal{B}^i(z^+, \mathbf{y}) &\equiv igT \cdot \partial_{\mathbf{y}^i} \mathcal{A}^-(z^+, \mathbf{y}), \\
\mathcal{B}^{ij}(z^+, \mathbf{y}) &\equiv igT \cdot \partial_{\mathbf{y}^i} \partial_{\mathbf{y}^j} \mathcal{A}^-(z^+, \mathbf{y}), \\
\mathcal{B}^{ijl}(z^+, \mathbf{y}) &\equiv igT \cdot \partial_{\mathbf{y}^i} \partial_{\mathbf{y}^j} \partial_{\mathbf{y}^l} \mathcal{A}^-(z^+, \mathbf{y}),
\end{aligned}
$$

Dilute target limit and the modified Lipatov vertex

[T.A., Dumitru - arXiv:1512.00279]

go from pA → pp:

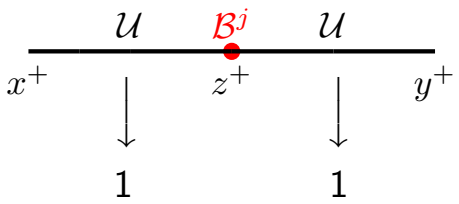
- *dilute limit of the target:*

expand the standard & decorated Wilson lines to first order in the background field.

- *Standard Wilson line*: $U_{ab}(x) \approx 1 + igT^c_{ab} \int_{x^+q} e^{iqx} A^-_c(x^+, q)$

- *the first decorated Wilson line*:

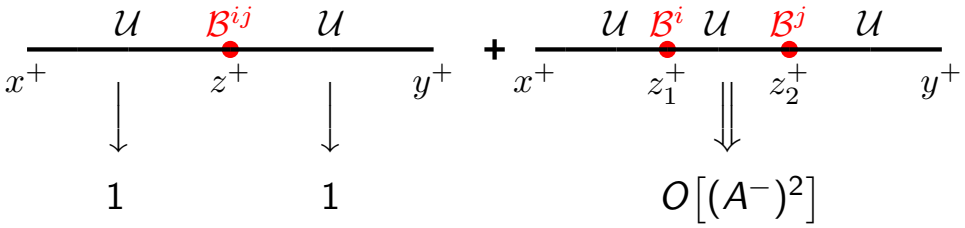
$\mathcal{U}^j_{[0,1]} \propto$



$$U^{i,ab}_{[0,1]}(x^+, y^+, y) = \int_{y^+}^{x^+} dz^+ \frac{z^+ - y^+}{x^+ - y^+} \left[igT^e_{ab} \partial_{y_i} A^{-,e}(z^+, y) \right]$$

- *the second decorated Wilson line*:

$\mathcal{U}^{ij}_{[0,2]} \propto$



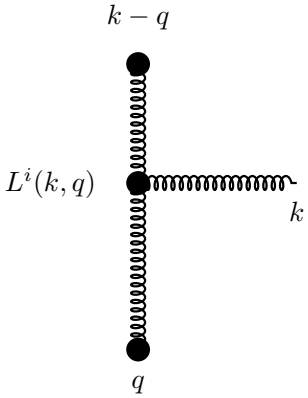
Dilute target limit and the modified Lipatov vertex

[T.A., Dumitru - arXiv:1512.00279]

- summing up all the NEik and NNEik terms in the dilute target limit, one gets

$$\boxed{\mathcal{M} \propto \left[\frac{(k-q)^i}{(k-q)^2} - \frac{k^i}{k^2}\right]\left\{1 + i\frac{k^2}{2k^+}x^+ - \frac{1}{2}\left(\frac{k^2}{2k^+}x^+\right)^2\right\}}$$

- $O(1)$ term $\rightarrow$ eikonal Lipatov vertex.



$$\boxed{L^i(k,q) = \frac{(k-q)^i}{(k-q)^2} - \frac{k^i}{k^2}}$$

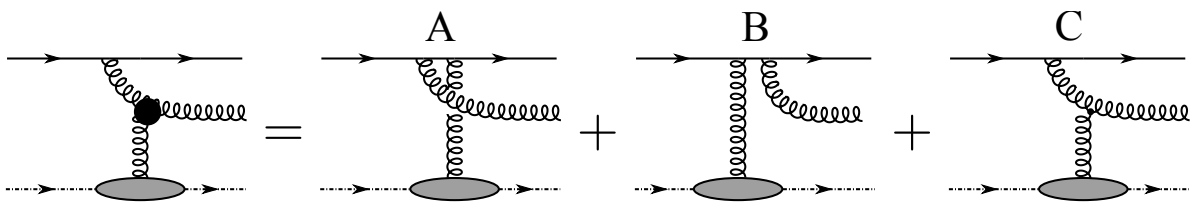
- we get NEik and NNEik corrections to the Lipatov vertex.

- the form suggests exponentiation. However, we do not know the corrections beyond NNEik accuracy!

Dilute target limit and the modified Lipatov vertex

[Agostini, TA, Armesto - arXiv:1902.04830 / arXiv:1907.03668]

- calculate the diagrams by keeping the phase $e^{ik^- x^+}$ which is taken to be 1 in the eikonal limit.



The total amplitude reads

$$i(\mathcal{M}_A + \mathcal{M}_B + \mathcal{M}_C) \propto \int \frac{d^2q}{(2\pi)^2} L^i(k,q) e^{ik^- x_1^+} A_a^-(k^-,q) e^{-iq\cdot x_1}$$

with $L^i(k,q)$ is the standard Lipatov vertex

$$\boxed{L^i(k,q) = \frac{(k-q)^i}{(k-q)^2} - \frac{k^i}{k^2}}$$

and the non-eikonal Lipatov vertex being

$$\boxed{L^i_{\mathrm{NE}}(\underline{k},q;x^+) = \left[\frac{(k-q)^i}{(k-q)^2} - \frac{k^i}{k^2}\right] e^{ik^- x^+}}$$

$k^- = \frac{k^2}{2k^+}$

Dilute target limit and the modified Lipatov vertex

Double inclusive cross section with Non-Eik Lipatov vertex $\left[\underline{k} \equiv (k^+, k_\perp)\right]$

$$\left.\frac{d\sigma}{d^2k_1 d\eta_1 d^2k_2\eta_2}\right|_{\text{dilute}}^{\text{NE}} \propto \int_{q_1 q_2} \left\{ \left[f(k_1, q_1, k_2, q_2) + \mathcal{G}_2^{\text{NE}}(k_1^-, k_2^-; L^+)\, g(k_1, q_1, k_2, q_2) \right] + (\underline{k}_2 \to -\underline{k}_2) \right\}$$

all non-eikonal effects are encoded in

$$\boxed{\mathcal{G}_2^{\text{NE}}(k_1^-, k_2^-; L^+) = \left\{ \frac{2}{(k_1^- - k_2^-)L^+} \sin\left[\frac{(k_1^- - k_2^-)}{2} L^+ \right] \right\}^2}$$

In the double inclusive production X-section:

- certain terms are accompanied by $\mathcal{G}_2^{\text{NE}}(k_1^-, k_2^-; L^+)$
- and their mirror images given by $(\underline{k}_2 \to -\underline{k}_2)$ are accompanied by $\mathcal{G}_2^{\text{NE}}(k_1^-, -k_2^-; L^+)$.
- $\mathcal{G}_2^{\text{NE}}(k_1^-, k_2^-; L^+)$ is not symmetric under $(\underline{k}_2 \to -\underline{k}_2)$!!

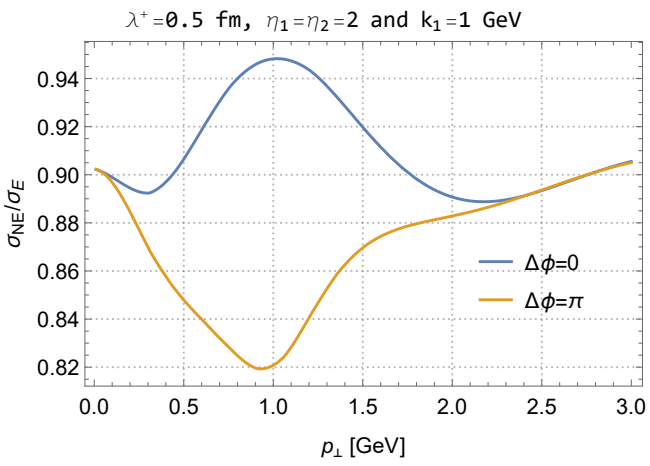
In certain kinematics the behavior of $\mathcal{G}_2^{\text{NE}}(k_1^-, k_2^-; L^+)$ differs completely from $\mathcal{G}_2^{\text{NE}}(k_1^-, -k_2^-; L^+)$:

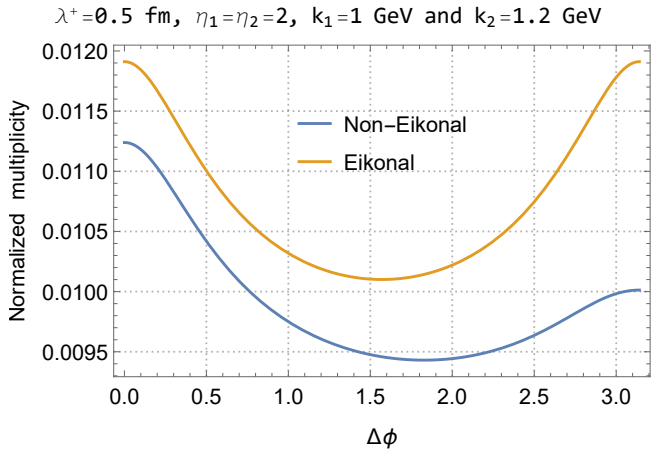
- in the region where $k_1^- \sim k_2^-$ we get

$$\boxed{\mathcal{G}_2^{\text{NE}}(k_1^-, k_2^-; L^+) \gg \mathcal{G}_2^{\text{NE}}(k_1^-, -k_2^-; L^+)}$$

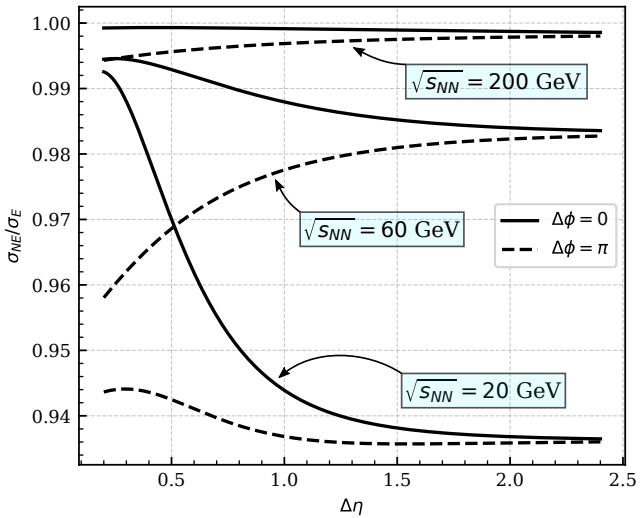
Dilute limit of non-eikonal double inclusive X-section

- *This asymmetry created by the non-eikonal effects immediately reminds the asymmetry between the forward and backward peaks of the ridge structure observed in two particle production.*



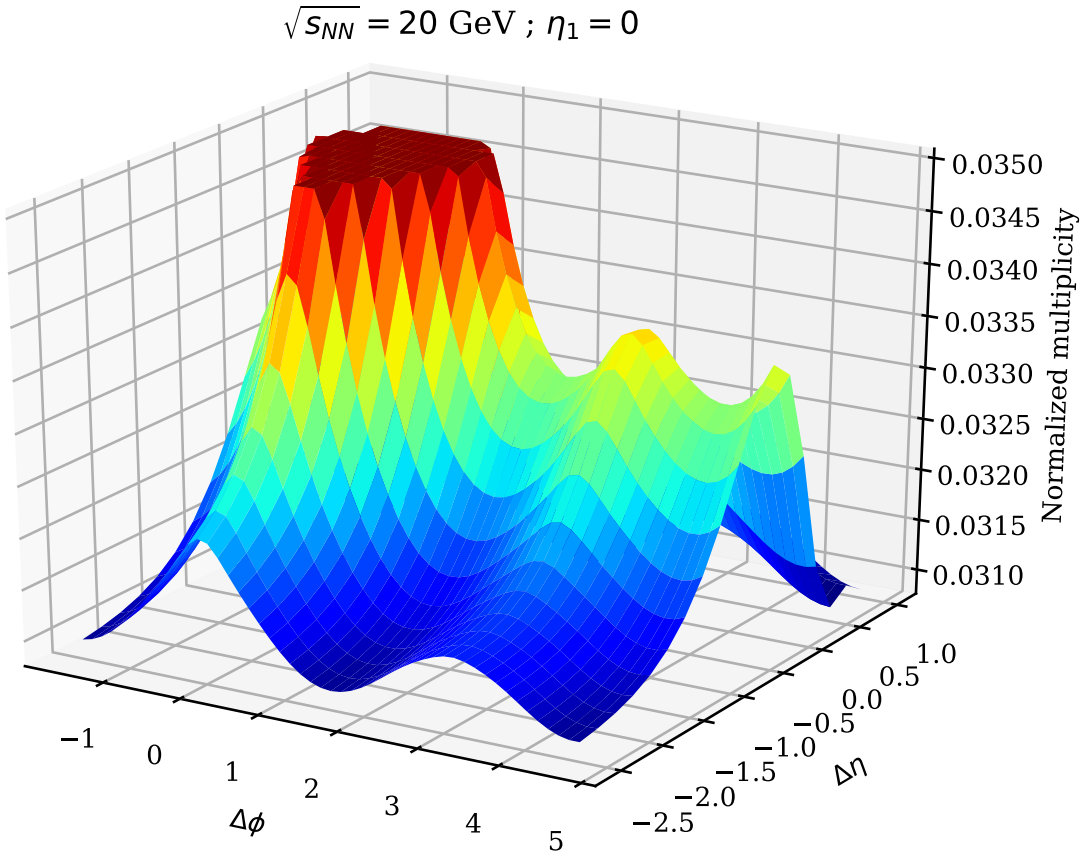


X-section is completely symmetric with respect to $\Delta\phi = \pi/2$ in the eikonal case, while an asymetric behavior is seen for the non-eikonal case.



- The difference between the peaks at $\Delta\phi = 0$ and at $\Delta\phi = \pi$ is a sign of generating non-zero odd harmonics.
- $\eta_1 = 0 \rightarrow \Delta\eta = \eta_2$ & $k_1 = 1$ GeV and $k_2 = 1.2$ GeV.
- *With increasing energy the difference between the peaks gets smaller $\rightarrow$ non-eikonal corrections gets smaller.*
- *the asymmetry exists in an interval of roughly two units of rapidity.*

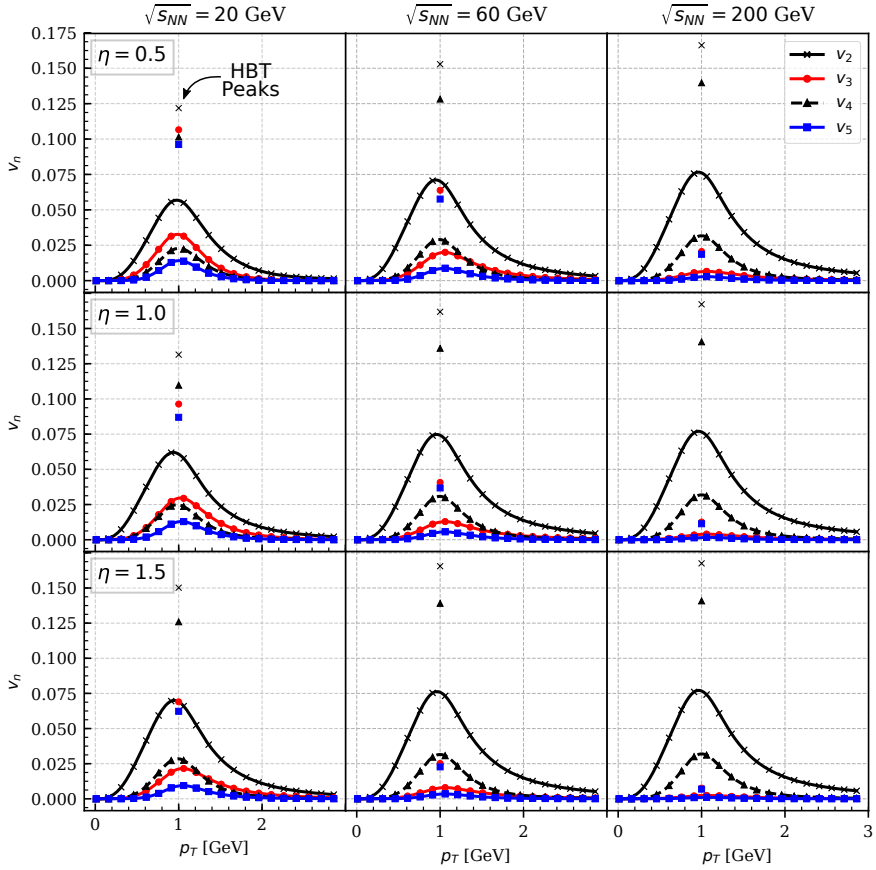
odd-harmonics from the non-eikonal corrections?



- The difference between the peaks is max for $\Delta\eta = 0$ (max. v_3 as well).
- The difference between the peaks vanishes after two units of rapidity.

odd-harmonics from the non-eikonal corrections?

non-eikonal corrections do generate non-zero odd harmonics!



$$V_{n\Delta}(k_1, k_2) = \frac{\int_0^\pi N(k_1, k_2, \Delta\phi)\, \cos(n\Delta\phi)\, d\Delta\phi}{\int_0^\pi N(k_1, k_2, \Delta\phi)\, d\Delta\phi}$$

$$v_n(p_T) = \frac{V_{n\Delta}(p_T, p_T^{ref})}{\sqrt{V_{n\Delta}(p_T^{ref}, p_T^{ref})}}$$

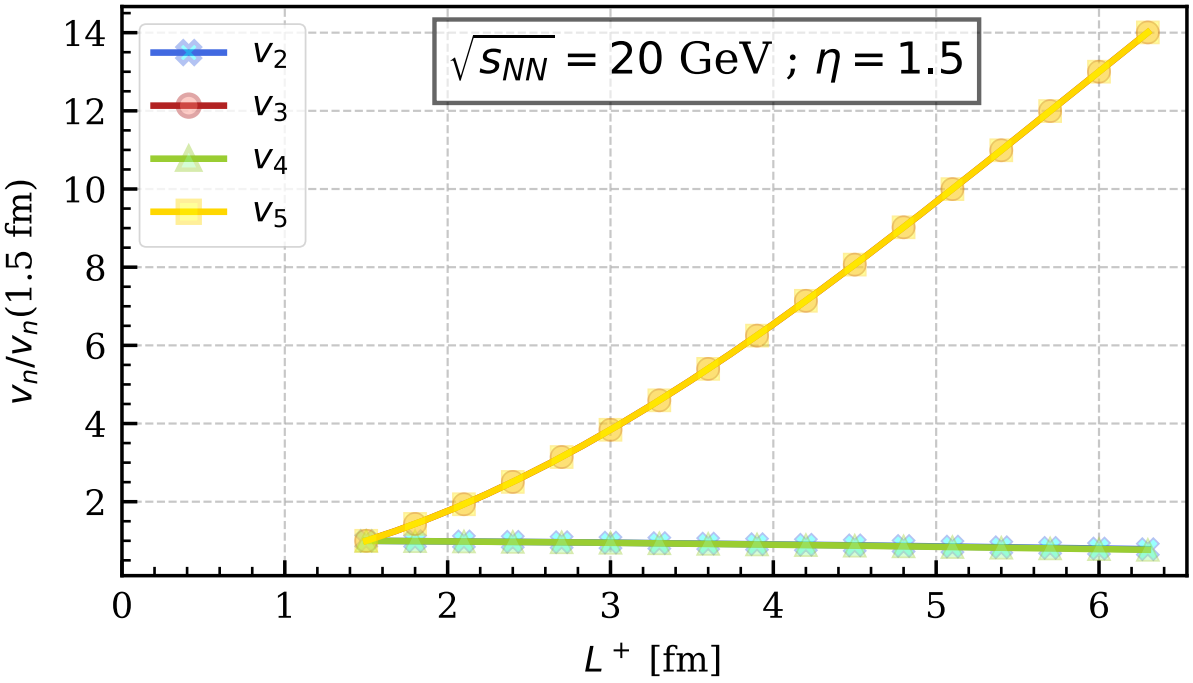
- $L^+ = 6$ fm in the rest frame and we scale it with the γ factor for different energies.
- $\mu_T = 0.4$ GeV and $\mu_P = 0.2$ GeV (these are the values that maximize v_3).
- $\eta_1 = \eta_2$ & $p_t^{ref} = 1$ GeV.

Non-eikonal effects alone can not explain the odd-harmonics HOWEVER there is a contribution originating from these effects for certain kinematic region.

Azimuthal harmonics from non-eikonal production

A side remark:

even harmonics do not depend on L^+ but odd harmonics do.



Summary / Remarks / Discussions

$$\gamma \sim \sqrt{S_{NN}}/(2m_{NN}) \qquad \& \qquad L^+ = L/(\gamma\sqrt{2}) \qquad \& \qquad \textit{eikonal parameter} : (p_T\, L^+\, e^{-\eta})$$

- At LHC energies $\sqrt{S_{NN}} > 2$ TeV $\Rightarrow \gamma \simeq 1000 \Rightarrow L^+ \simeq 10^{-2}\ \mathrm{GeV}^{-1}$
  - LHC small p_T (0-3 GeV): (i) vanishing odd harmonics
    (ii) $\mathcal{G}_2^{\mathrm{NE}} \to 1 \Rightarrow$ Non-eikonal expressions $\to$ eikonal ones.
  - LHC high p_T (3-10 GeV): Does $\mathcal{G}_2^{\mathrm{NE}} \to 1$? $\Rightarrow$ Non-eikonal terms might be still important.
- At RHIC energies $\sqrt{S_{NN}} < 200$ GeV $\Rightarrow \gamma < 100 \Rightarrow L^+ > 0.3\ \mathrm{GeV}^{-1}$
  - RHIC small p_T (0-3 GeV): (i) difference between the peaks
    (ii) non-vanishing odd harmonics
  - RHIC high p_T (3-10 GeV): (i) no difference between the peaks.
    (ii) $\mathcal{G}_2^{\mathrm{NE}}(k_1^-, k_2^-, L^+) \to \mathcal{G}_2^{\mathrm{NE}}(k_1^-, -k_2^-, L^+)$
    (iii) vanishing odd harmonics

Summary / Remarks / Discussions

- *With the change of the azimuthal angle from $\Delta\phi = 0$ to $\Delta\phi = \pi$ the magnitude of the non-eikonal parameter is changing $\rightarrow$ breaks the accidental symmetry of the CGC and generates non-zero odd harmonics.*

- Other corrections to the eikonal limit may carry a similar effect:
  - including the transverse component of the background field will bring k^+ dependence. [in preparation - TA, Beuf, Tymowska]
  - the dynamics of the target: x^- dependence of the target field ??

- *Non-eikonal effects alone can not explain the odd-harmonics HOWEVER there is a contribution originating from these effects for certain kinematic region.*

Back up slides

Bose enhancement in a nutshell (1)

Consider a state with fixed occupation numbers of N species of bosons at different momenta:

$$\boxed{\left|\{n^i(p)\}\right\rangle \equiv \prod_{i,p} \frac{1}{\sqrt{n^i(p)!}} \left(\frac{a_i^\dagger(p)}{\sqrt{V}}\right)^{n^i(p)} |0\rangle}$$

with a finite volume V and periodic boundary conditions so that momenta are discrete. $(i = 1, 2, \cdots, N)$

The mean particle density :

$$\boxed{n \equiv \left\langle \{n^i(p)\}|a^{\dagger i}(x)a^i(x)|\{n^i(p)\}\right\rangle = \sum_{i,p} n^i(p)}$$

The 2-particle correlator:

in x-space $\rightarrow$ $\boxed{D(x,y) \equiv \langle\{n(p)\}|a^{\dagger i}(x)a^{\dagger j}(y)a^i(x)a^j(y)|\{n(p)\}\rangle}$

in p-space $\rightarrow$ $\boxed{D(p,k) \equiv \langle\{n(p)\}|a^{\dagger i}(p)a^{\dagger j}(q)a^i(l)a^j(m)|\{n(p)\}\rangle}$

$$\Rightarrow D(p,k) = \delta(p-l)\delta(q-m)\sum_i n^i(p)\sum_j n^j(q) + \delta(p-m)\delta(q-l)\sum_i n^i(p)n^i(q)$$

Bose enhancement in a nutshell (2)

Using these results, the 2-particle correlator in coordinate space:

$$D(x,y) = n^2 + \underbrace{\sum_i \left| \int \frac{d^3p}{(2\pi)^3}\, e^{ip(x-y)} n^i(p) \right|^2}_{\text{the Bose enhancement term}}$$

in momentum space:

$$D(p,k) = \left[\sum_i n_i(p)\right]\left[\sum_j n_j(k)\right] + \overbrace{\delta(p-k)\sum_i [n_i(p)]^2}$$

- It vanishes when the points are far away!
- It gives $O(1/N)$ enhancement when the points coincide!

The $O(1/N)$ suppression is due to the fact the second term contains a single sum over the species index!

The physics: **Only bosons of the same species are correlated with each other.**

Noneikonal single inclusive gluon production

Single inclusive gluon production in pA collisions (eikonal accuracy):

$$\frac{d\sigma}{d^2k d\eta} \propto \int_{zx\bar{z}y} e^{ik(z-\bar{z})}\, A^i(x-z)\, A^j(\bar{z}-y)\, \langle \rho^a(x)\rho^b(y)\rangle_P \left\langle [U_z - U_x]^{ac} [U^\dagger_{\bar{z}} - U^\dagger_y]^{cb} \right\rangle_T$$

- *projectile averaging:* in x-space $\rightarrow \langle \rho^a(x)\rho^b(y)\rangle_P = \delta^{ab}\mu^2(x,y)$
  in p-space $\rightarrow \langle \rho^a(k)\rho^b(p)\rangle_P = \delta^{ab}\mu^2(k,p) = \delta^{ab}\, T\left(\frac{k-p}{2}\right) F\left[(k+p)R\right]$

T $\rightarrow$ transverse momentum dependent distribution of the color charge densities
F $\rightarrow$ soft form factor which is peaked when its argument vanihes

Single inclusive gluon production in pp collisions (eikonal accuracy):

- *dilute target limit* $\rightarrow U_{ab}(x) \approx 1 + igT^c_{ab} \int_{x^+q} e^{iqx}\, A^-_c(x^+, q)$

$$\left.\frac{d\sigma}{d^2k d\eta}\right|_{\text{dilute}} \propto \int_{x_1^+ x_2^+ q_1 q_2} L^i(k,q_1)\, L^i(k,q_2)\, \mu^2\left[k-q_1, k-q_2\right] \left\langle A^-_c(x_1^+, q_1) A^-_{\bar{c}}(x_2^+, q_2)\right\rangle_T$$

- *go from eikonal to non-eikonal:* $L^i(k,q) \rightarrow L^i_{\text{NE}}(\underline{k}, q; x^+)$

$\underline{k} \equiv (k^+, k)$

Noneikonal single inclusive gluon production

target averaging:

- Adopt a modified expression for the correlator of two target fields:

Since the target has finite longitudinal length, the target fields can be located at two different longitudinal positions. We consider a generalization of the MV model in which the two color fields are located at different longitudinal positions.

$$\boxed{\left\langle A_c^-(x_1^+, q_1) A_{\bar{c}}^-(x_2^+, q_2)\right\rangle_T = \delta^{c\bar{c}} n(x_1^+) \frac{1}{2\lambda^+} \Theta\left(\lambda^+ - |x_1^+ - x_2^+|\right)(2\pi)^2 \delta^{(2)}(q_1 - q_2)|a(q_1)|^2}$$

- $\lambda^+ \equiv$ color correlation length in the target ($\lambda^+ \ll L^+$)

- $n(x^+) \equiv$ 1-d target density along longitudinal direction

($n(x^+) = n_0$ for $0 \leq x^+ \leq L^+$ and 0 elsewhere)

- $a(q) \equiv$ functional form of the potential in p-space

It is Yukawa type $\rightarrow |a(q)|^2 = \frac{\mu_T^2}{(q^2+\mu_T^2)^2}$ with μ_T is Debye screening mass.

In the limit $\lambda^+ \rightarrow 0$ together with a constant potential $|a(q)|^2$ and constant 1-d target density, the correlator goes to standard MV model one.

Noneikonal single inclusive gluon production

When we plug this back in the X-section we get

$$\left.\frac{d\sigma}{d^2k d\eta}\right|_{\text{dilute}}^{\text{NE}} \propto \int_q |a(q)|^2 \mu^2 [k-q, q-k] L^i(k,q) L^i(k,q)\, n_0 \frac{1}{2\lambda^+} \int_0^{L^+} dx_1^+ \int_{x_1^+-\lambda^+}^{x_1^+-\lambda^+} dx_2^+ \, e^{i\frac{k^2}{2k^+}(x_1^+-x_2^+)}$$

- The NE Lipatov vertex is incorporated in the phase.
- The θ-function in the correlator provides the integration limits.
- The 1-d target density is taken to be constant for $0 \leq x_1^+ \leq L^+$.
- integration over x_1^+ gives a factor of $(n_0 L^+)$ which corresponds to number of scattering centers in inside the finite length L^+. Since in the dilute target limit we only take into account a single scattering in the amplitude and c.c. amplitude, this factor can be set to 1.

Noneikonal single inclusive gluon production

After all said and done:

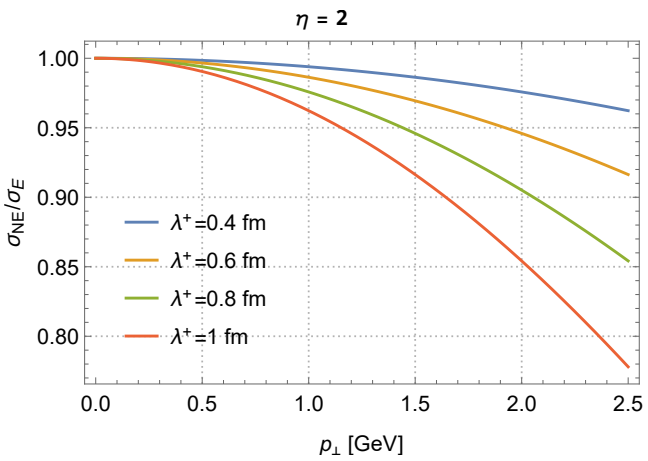
$$\left.\frac{d\sigma}{d^2k d\eta}\right|_{\text{dilute}}^{\text{NE}} \propto \mathcal{G}_1^{\text{NE}}(k^-;\lambda^+)\int_q \mu^2\big[k-q,q-k\big]L^i(k,q)L^i(k,q)\big|a(q)\big|^2$$

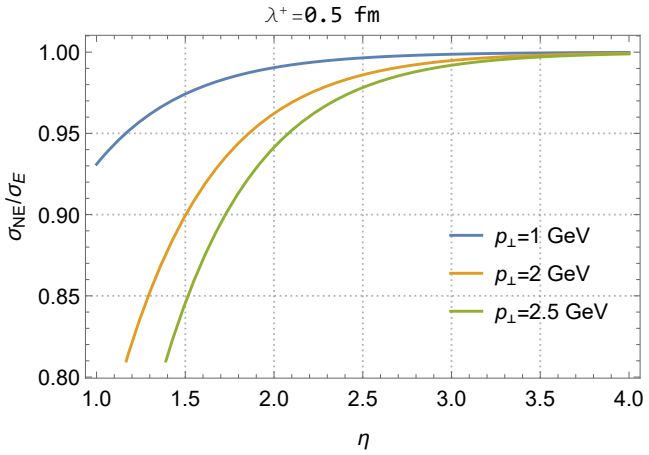
the function that encodes the non-eikonal effects

$$\boxed{\mathcal{G}_1^{\text{NE}}(k^-;\lambda^+) = \frac{1}{k^-\lambda^+}\sin(k^-\lambda^+)}$$

in the eikonal limit:

$$\lim_{(k^-\lambda^+)\to 0}\mathcal{G}_1^{\text{NE}}(k^-;\lambda^+) = 1$$





($N_c = 3$, $\mu_T = 0.2$ GeV, $\mu^2(k,q) = \delta^{(2)}(k+q)$ with a projectile size $S_\perp = 4\,GeV^{-2}$.)

Double inclusive gluon production and glasma graphs

Same procedure can be adopted to calculate the double inclusive gluon production.

The double inclusive gluon production X-section for dilute-dense scattering:

$$\frac{d\sigma}{d^2k_1 d\eta_1 d^2k_2\eta_2} \propto \int_{z_1 z_2 x_1 x_2 \bar{z}_1 \bar{z}_2 y_1 y_2} e^{ik_1(z_1-\bar{z}_1)+ik_2(z_2-\bar{z}_2)} A^i(x_1-z_1)A^i(\bar{z}_1-y_1)A^j(x_2-z_2)A^j(\bar{z}_2-y_2)$$
$$\times \left\langle \rho^{a_1}_{x_1}\rho^{a_2}_{x_2}\rho^{b_1}_{y_1}\rho^{b_2}_{y_2}\right\rangle_P \left\langle [U_{z_1}-U_{x_1}]^{a_1 c}[U^\dagger_{\bar{z}_1}-U^\dagger_{y_1}]^{cb_1}[U_{z_2}-U_{x_2}]^{a_2 d}[U^\dagger_{\bar{z}_2}-U^\dagger_{y_2}]^{db_2}\right\rangle_T$$

- *projectile averaging:* pair wise Wick contraction:

$$\left\langle \rho^{a_1}_{x_1}\rho^{a_2}_{x_2}\rho^{b_1}_{y_1}\rho^{b_2}_{y_2}\right\rangle_P = \left\langle \rho^{a_1}_{x_1}\rho^{a_2}_{x_2}\right\rangle_P \left\langle \rho^{b_1}_{y_1}\rho^{b_2}_{y_2}\right\rangle_P + \left\langle \rho^{a_1}_{x_1}\rho^{b_1}_{y_1}\right\rangle_P \left\langle \rho^{a_2}_{x_2}\rho^{b_2}_{y_2}\right\rangle_P + \left\langle \rho^{a_1}_{x_1}\rho^{b_2}_{y_2}\right\rangle_P \left\langle \rho^{a_2}_{x_2}\rho^{b_1}_{y_1}\right\rangle_P$$

- *projectile averaging:* use the same two color charge correlator:

$$\text{in x-space} \rightarrow \langle \rho^a(x)\rho^b(y)\rangle_P = \delta^{ab}\mu^2(x,y)$$

$$\text{in p-space} \rightarrow \langle \rho^a(k)\rho^b(p)\rangle_P = \delta^{ab}\mu^2(k,p) = \delta^{ab}T\left(\frac{k-p}{2}\right)F\left[(k+p)R\right]$$

Double inclusive gluon production and glasma graphs

- *dilute target limit* $\rightarrow U_{ab}(x) \approx 1 + igT^c_{ab}\int_{x^+q} e^{iqx}\, A^-_c(x^+, q)$
- *go from eikonal to non-eikonal:* $L(k,q) \rightarrow L_{\rm NE}(\underline{k}, q; x^+)$
- *target averaging:* pair wise Wick contraction

$$\left\langle A^-_a(x_1^+, q_1)A^-_b(x_2^+, q_2)A^-_c(x_3^+, q_3)A^-_d(x_4^+, q_4)\right\rangle_T = \left\langle A^-_a(x_1^+, q_1)A^-_b(x_2^+, q_2)\right\rangle_T \left\langle A^-_c(x_3^+, q_3)A^-_d(x_4^+, q_4)\right\rangle_T$$
$$+\left\langle A^-_a(x_1^+, q_1)A^-_d(x_4^+, q_4)\right\rangle_T \left\langle A^-_c(x_3^+, q_3)A^-_b(x_2^+, q_2)\right\rangle_T + \left\langle A^-_a(x_1^+, q_1)A^-_c(x_3^+, q_3)\right\rangle_T \left\langle A^-_b(x_2^+, q_2)A^-_d(x_4^+, q_4)\right\rangle_T$$

- *target averaging:* use the same two field correlator:

$$\boxed{\left\langle A^-_c(x_1^+, q_1)A^-_{\bar c}(x_2^+, q_2)\right\rangle_T = \delta^{c\bar c} n(x_1^+)\frac{1}{2\lambda^+}\Theta\big(\lambda^+ - |x_1^+ - x_2^+|\big)(2\pi)^2\delta^{(2)}(q_1 - q_2)|a(q_1)|^2}$$

The dilute limit with non-eikonal corrections:

$$\left.\frac{d\sigma}{d^2k_1 d\eta_1 d^2k_2\eta_2}\right|^{\rm NE}_{\rm dilute} \propto \int_{q_1 q_2} |a(q_1)|^2|a(q_2)|^2 \mathcal{G}^{\rm NE}_1(k_1^-;\lambda^+)\,\mathcal{G}^{\rm NE}_1(k_2^-;\lambda^+)\left\{I^{(0)}_{2\rm tr} + \frac{1}{N_c^2-1}\left[I^{(1)}_{2\rm tr} + I^{(1)}_{1\rm tr}\right]\right\}$$

Glasma graphs / two particle correlations

In our set up:

- $k_1 - q_1$ and $k_2 - q_2$: momenta of the two gluons in the projectile.
- k_1 and k_2: momenta of the two gluons in the final state.
- q_1 and q_2: momenta transferred from the target to the projectile during the interaction.

In such a set up:

- (forward/backward) Bose enhancement of the gluons in the projectile $\Rightarrow F\left[|(k_1 - q_1) \mp (k_2 - q_2)|R\right]$
- (forward/backward) HBT correlations of the final state gluons $\Rightarrow F\left[|k_1 \mp k_2|R\right]$
- (forward/backward) Bose enhancement of the gluons in the target $\Rightarrow F\left[|q_1 \mp q_2|R\right]$

Glasma graphs / two particle correlations

identification of the terms:

$$I^{(0)}_{2\,\mathrm{tr}} = \left(\mu^2\left[k_1-q_1,q_1-k_1\right]\,L^i(k_1,q_1)L^i(k_1,q_1)\right)\,\left(\mu^2\left[k_2-q_2,q_2-k_2\right]\,L^j(k_2,q_2)L^j(k_2,q_2)\right)$$

- Square of the single inclusive production / uncorrelated production.

$$I^{(1)}_{2\,\mathrm{tr}} = \Big\{\mathcal{G}^{\mathrm{NE}}_2(k_1^-,k_2^-;L^+)\mu^2\left[k_1-q_1,q_2-k_1\right]\,\mu^2\left[k_2-q_2,q_1-k_2\right] \times L^i(k_1,q_1)L^i(k_1,q_2)\,L^j(k_2,q_2)L^j(k_2,q_1)\Big\} + (\underline{k}_2 \to -\underline{k}_2)$$

- $\underline{k} \equiv (k^+,k)$
- $\mu^2[k_1-q_1,q_2-k_1] \propto F\left[|q_1-q_2|R\right] \Rightarrow$ Bose enhancement of the target gluons.
- A new function appears that accounts for non-eikonal effects:

$$\boxed{\mathcal{G}^{\mathrm{NE}}_2(k_1^-,k_2^-;L^+) = \left\{\frac{2}{(k_1^- - k_2^-)L^+}\sin\left[\frac{(k_1^- - k_2^-)}{2}L^+\right]\right\}^2}$$

- in the eikonal limit:

$$\lim_{L^+\to 0}\mathcal{G}^{\mathrm{NE}}_2(k_1^-,k_2^-;L^+) = 1$$

Glasma graphs / two particle correlations

identification of the terms:

$$I^{(1)}_{1\,\mathrm{tr}} = \Big\{ \mu^2\big[k_1 - q_1, q_2 - k_2\big]\, \mu^2\big[k_2 - q_2, q_1 - k_1\big]\, L^i(k_1, q_1) L^i(k_1, q_1)\, L^j(k_2, q_2) L^j(k_2, q_2)$$
$$+ \mathcal{G}_2^{\mathrm{NE}}(k_1^-, k_2^-; L^+) \Big(\mu^2\big[k_1 - q_1, q_1 - k_2\big]\, \mu^2\big[k_2 - q_2, q_2 - k_1\big] + \frac{1}{2}\mu^2\big[k_1 - q_1, k_2 - q_2\big]\, \mu^2\big[q_2 - k_1, q_1 - k_2\big] \Big)$$
$$\times L^i(k_1, q_1) L^i(k_1, q_2)\, L^j(k_2, q_1) L^j(k_2, q_2) \Big\} + (\underline{k}_2 \to -\underline{k}_2)$$

- $\mu^2\big[k_1 - q_1, q_2 - k_2\big] \propto F\big[|(k_1 - q_1) - (k_2 - q_2)|R\big] \Rightarrow$ Bose enhancement of the projectile gluons (forward peak).
- $\mu^2\big[k_1 - q_1, q_1 - k_2\big] \propto F\big[|k_1 - k_2|R\big] \Rightarrow$ HBT correlations of the produced gluons.
- $\mu^2\big[k_1 - q_1, k_2 - q_2\big] \propto F\big[|(k_1 - q_1) + (k_2 - q_2)|R\big] \Rightarrow$ Bose enhancement of the projectile gluons (backward peak).

The nature of $\mathcal{G}_2^{\rm NE}(k_1^-, k_2^-; L^+)$

In the double inclusive production X-section:

- certain terms are accompanied by $\mathcal{G}_2^{\rm NE}(k_1^-, k_2^-; L^+)$
- and their mirror images given by $(\underline{k}_2 \to -\underline{k}_2)$ are accompanied by $\mathcal{G}_2^{\rm NE}(k_1^-, -k_2^-; L^+)$.

$$\boxed{\mathcal{G}_2^{\rm NE}(k_1^-, k_2^-; L^+) = \left\{ \frac{2}{(k_1^- - k_2^-)L^+} \sin\left[\frac{(k_1^- - k_2^-)}{2} L^+\right] \right\}^2}$$

- $k^- = k^2/2k^+$
- $\mathcal{G}_2^{\rm NE}(k_1^-, k_2^-; L^+)$ is not symmetric under $(\underline{k}_2 \to -\underline{k}_2)$!!

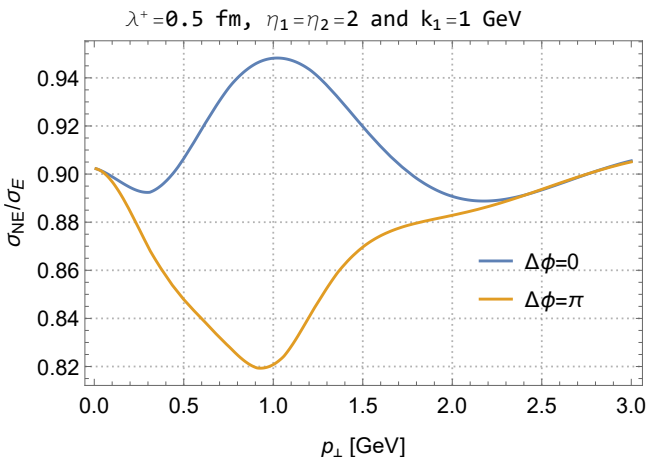
In certain kinematics the behavior of $\mathcal{G}_2^{\rm NE}(k_1^-, k_2^-; L^+)$ differs completely from $\mathcal{G}_2^{\rm NE}(k_1^-, -k_2^-; L^+)$:

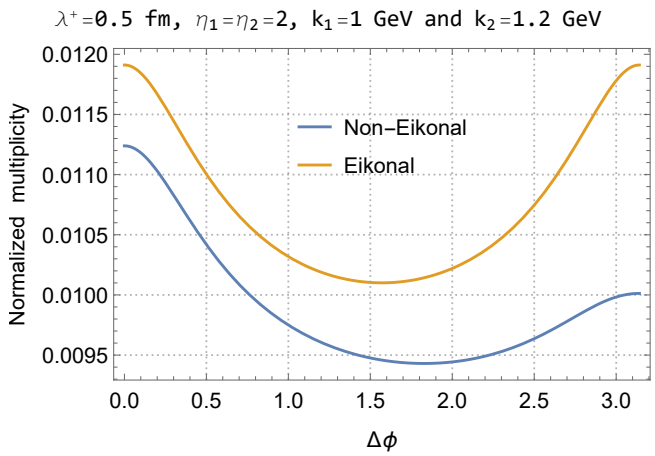
- in the region where $k_1^- \sim k_2^-$ we get

$$\boxed{\mathcal{G}_2^{\rm NE}(k_1^-, k_2^-; L^+) \gg \mathcal{G}_2^{\rm NE}(k_1^-, -k_2^-; L^+)}$$

Dilute limit of non-eikonal double inclusive X-section

• *This asymmetry created by the non-eikonal effects immediately reminds the asymmetry between the forward and backward peaks of the ridge structure observed in two particle production.*





• $L^+ = 6\,fm$ and $N_c = 3$

• $\mu_T = 0.2$ GeV

• translational invariance: $\mu^2(k,q) = \delta^{(2)}(k+q)$ with a projectile size $S_\perp = 4 GeV^{-2}$.

• regulate the denominators that give rise to infrared divergencies by substituting the corresponding squared transverse momenta $l^2 \to l^2 + \mu_P$ where we have used the numerical value $\mu_P = 0.2$ GeV.

X-section is completely symmetric with respect to $\Delta\phi = \pi/2$ in the eikonal case, while an asymetric behavior is seen for the non-eikonal case.