Cake Seminar

Patrick Barry

2/8/2021

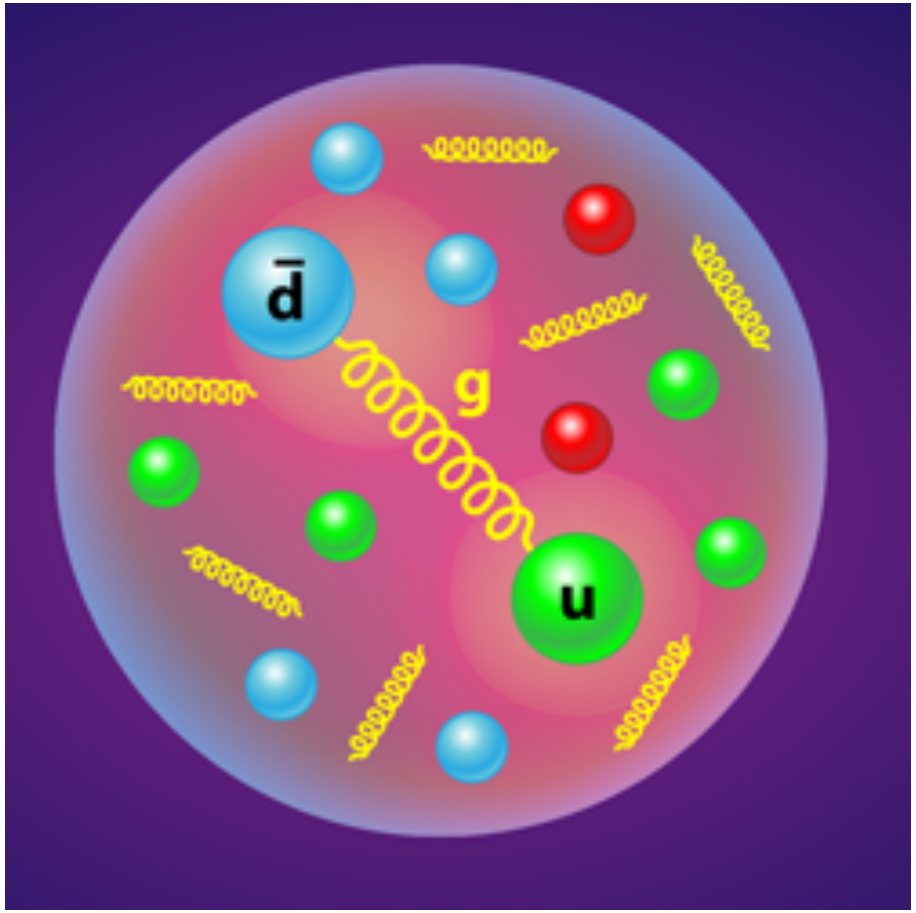
Email: barryp@jlab.org

Motivation

- QCD allows us to study the structure of hadrons in terms of partons (quarks, antiquarks, and gluons)
- Use factorization theorems to separate hard partonic physics out of soft, non-perturbative objects to quantify structure
- Define a structure of hadrons in terms of quantum field theories
- Identify theoretical observables that factorize into non-perturbative objects and perturbatively calculable physics
- Perform global QCD analysis as structures are universal and are the same in all subprocesses

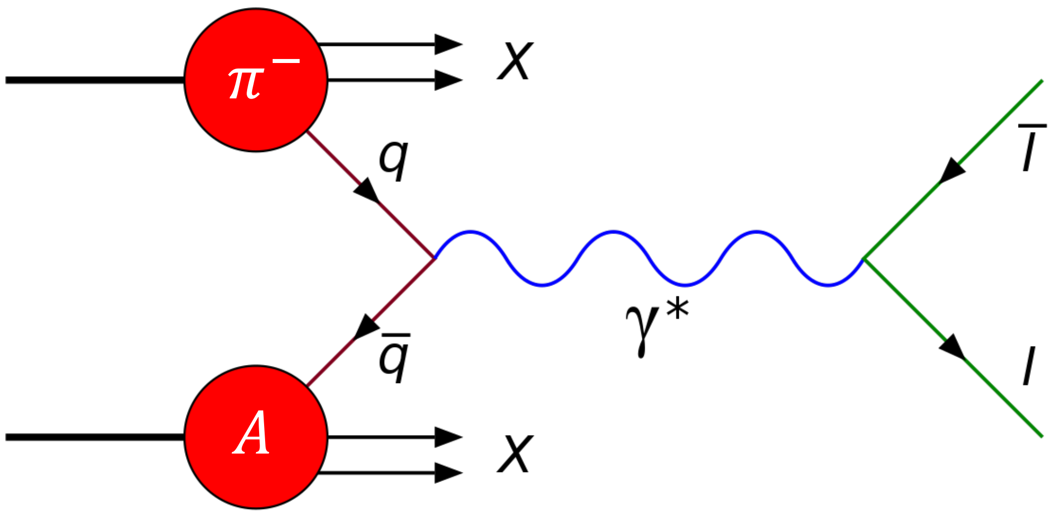
Pions

- Pion is the Goldstone boson associated with spontaneous symmetry breaking of chiral $SU(2)_L \times SU(2)_R$ symmetry
- Lightest hadron as $\frac{m_\pi}{M_N} \ll 1$ and dictates the nature of hadronic interactions at low energies
- Simultaneously a pseudoscalar meson made up of q and $\bar{q}$ constituents



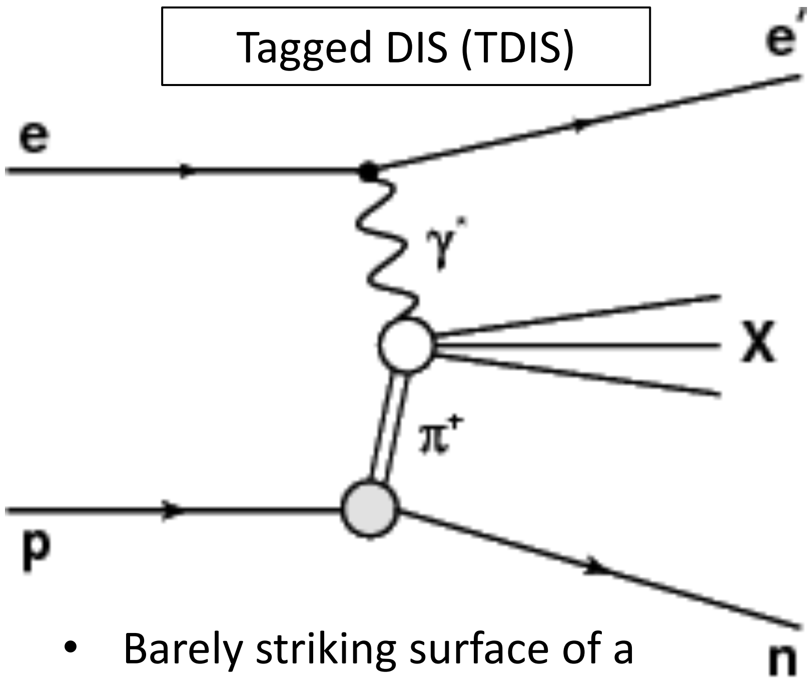
Experiments to Probe Pion Structure

- Drell-Yan (DY)



- Accelerating pion allows for time dilation and longer lifetime

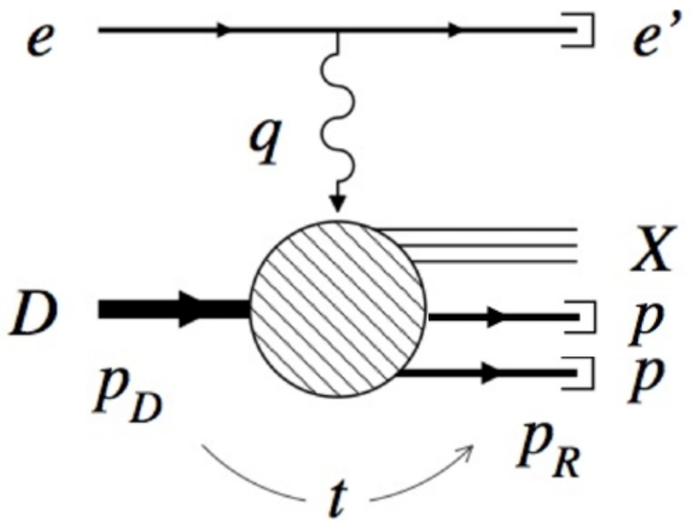
- Leading Neutron (LN)



- Barely striking surface of a target proton knocks out an almost on-shell pion to probe

Future Experiments

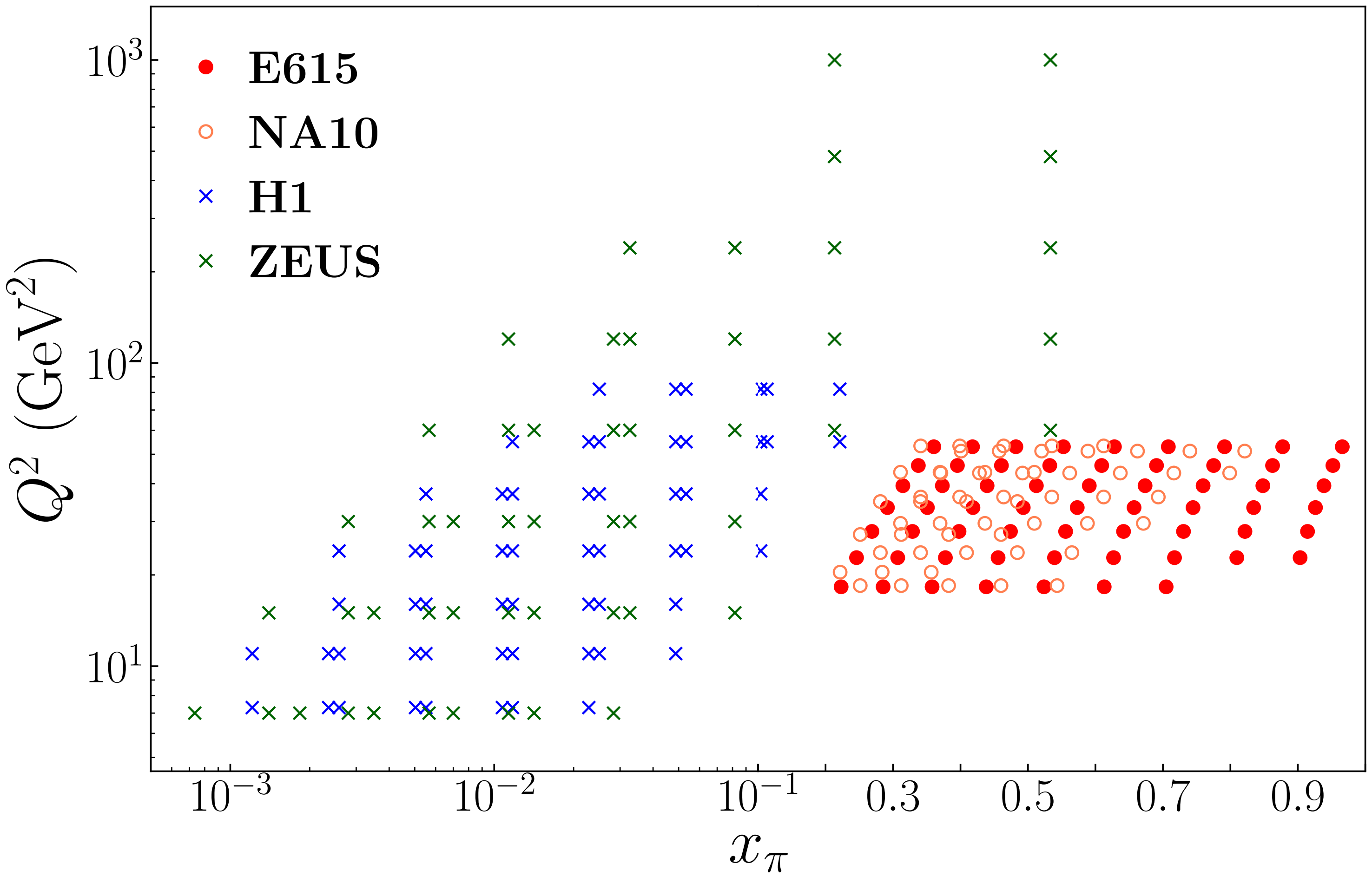
- TDIS experiment at 12 GeV upgrade from JLab, which will tag a proton in coincidence with a spectator proton
- Gives leading proton observable, complementary to LN, but with a fixed target experiment instead of collider (HERA)



- Proposed COMPASS++/AMBER also give π-induced DY data
- Both π^+ and π^- beams on carbon and tungsten targets

Kinematic Coverage

- DY data (E615, NA10) exist at large x_π, while LN (H1, ZEUS) data have small x_π
- Not much overlap, need more data



Monte Carlo

- Using Bayesian statistics, we describe the probability

$$\mathcal{P}(\mathbf{a}|\text{data}) \propto \mathcal{L}(\text{data}|\mathbf{a})\pi(\mathbf{a})$$

$$\mathcal{L}(\text{data}|\mathbf{a}) = \exp\Big(-\frac{1}{2}\chi^2(\text{data},\mathbf{a})\Big)$$

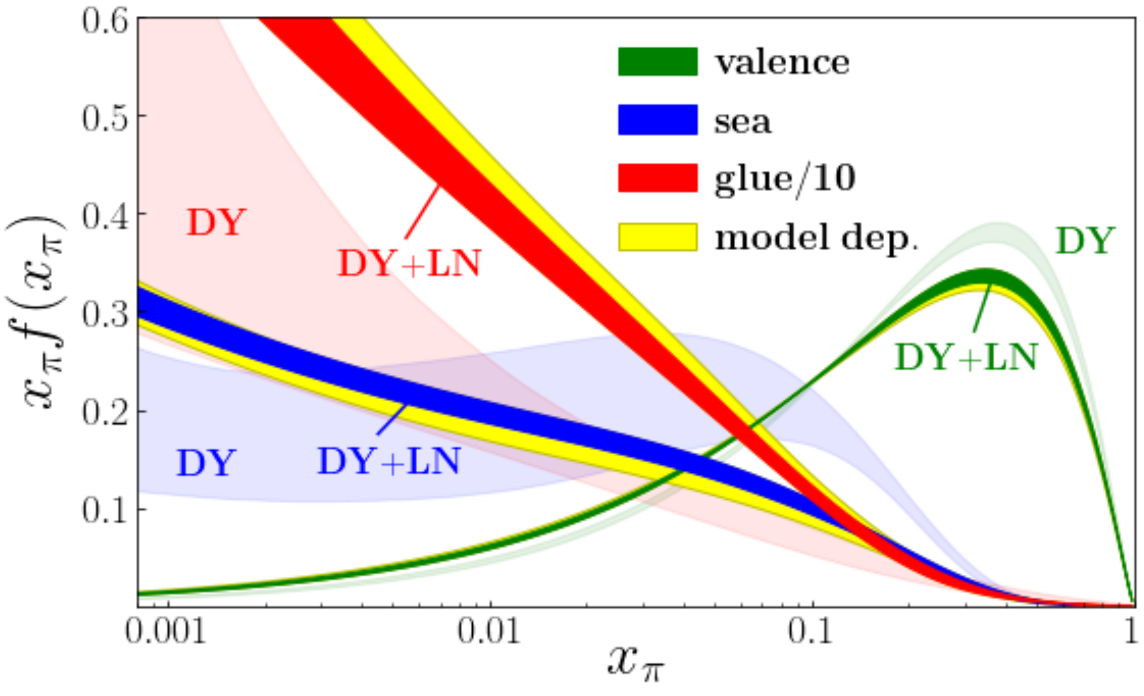
- We quantify the expectation value and variance of our observable $\mathcal{O}$ as a function of the parameter set $\boldsymbol{a}_i$

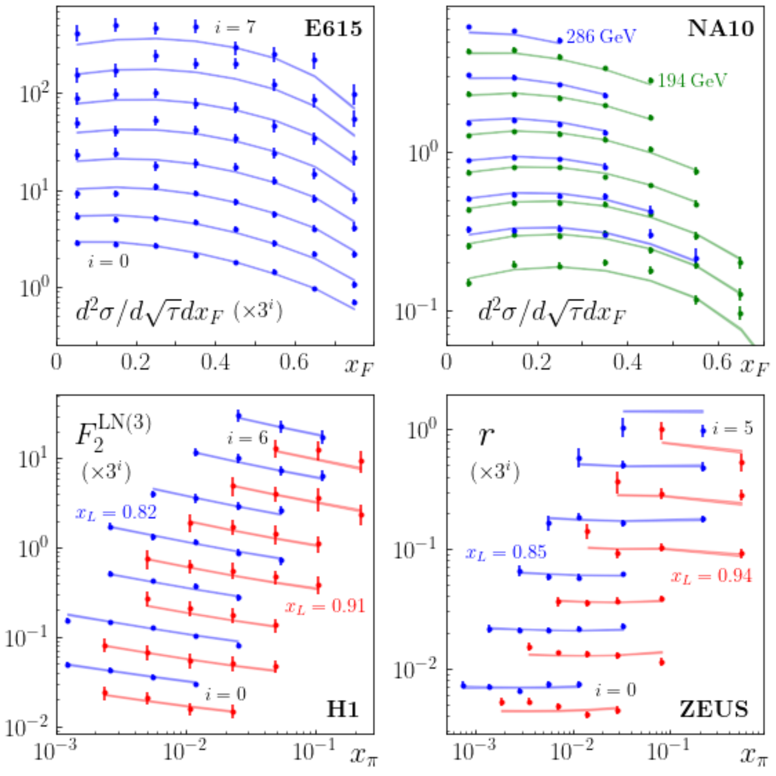
$$E[\mathcal{O}] = \sum_i w_i \mathcal{O}(\mathbf{a}_i)$$

$$V[\mathcal{O}] = \sum_i w_i \big[\mathcal{O}(\mathbf{a}_i) - E[\mathcal{O}]\big]^2$$

NLO analysis

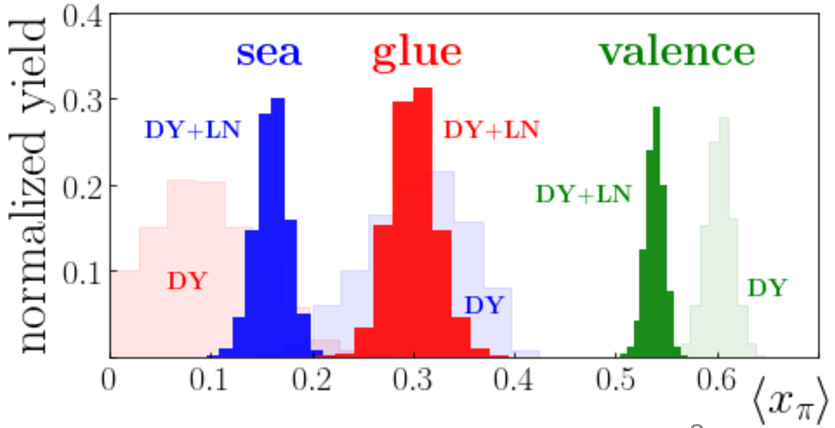
- PDFs found from fitting to Drell-Yan only (lightly shaded) and for Drell-Yan and Leading Neutron (darkly shaded)
- Yellow shows pion flux model dependence
- Shown at a scale of $\mu^2 = 10$ GeV²





- Top row – Drell-Yan
- Bottom row – Leading neutron
- Good agreement with data
- $\chi^2_{\text{npts}} = 0.979$

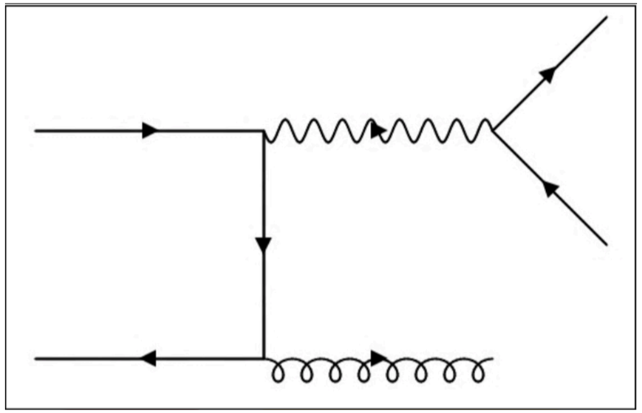
- Apparent swap between sea and glue with or without the LN data



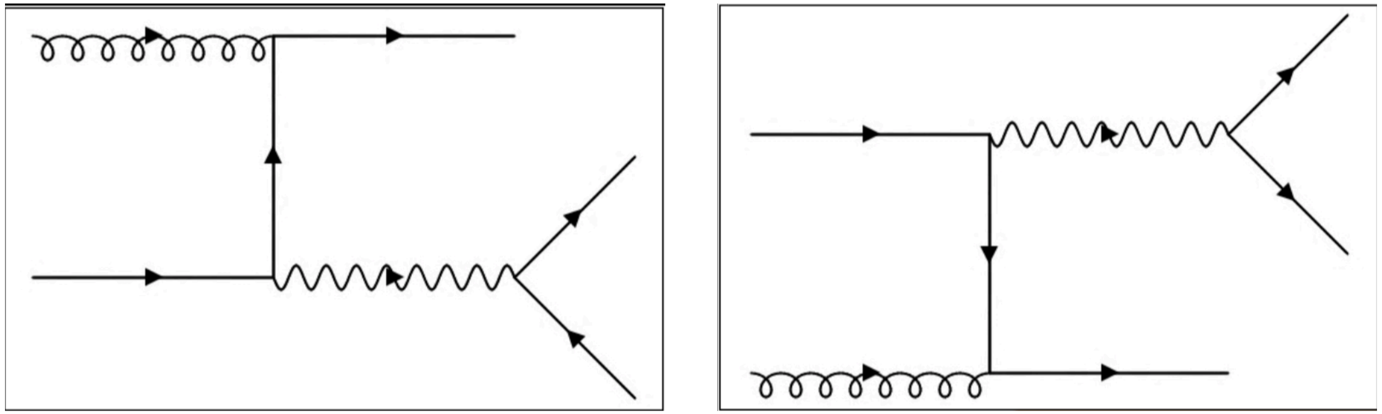
Drell-Yan

- p_T-dependent DY data also available from E615

$$\frac{d\sigma}{dQ^2 dY\, dp_T^2} = \frac{4\pi\alpha^2}{3N_C Q^2 S} \sum_{i,j} e_q^2 \int_{x_\pi^0}^{1} dx_\pi f_i^\pi(x_\pi, \mu) f_j^A(x_A, \mu) \times \frac{d\hat{\sigma}_{i,j}}{dQ^2 d\hat{t}}$$



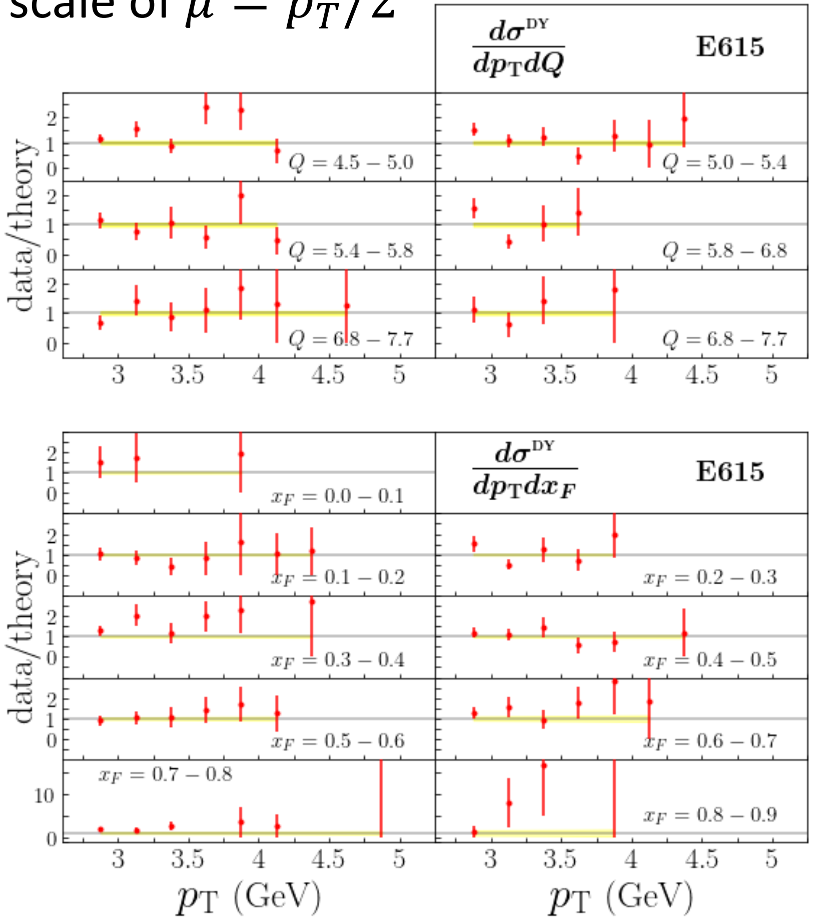
$q\bar{q}$ channel

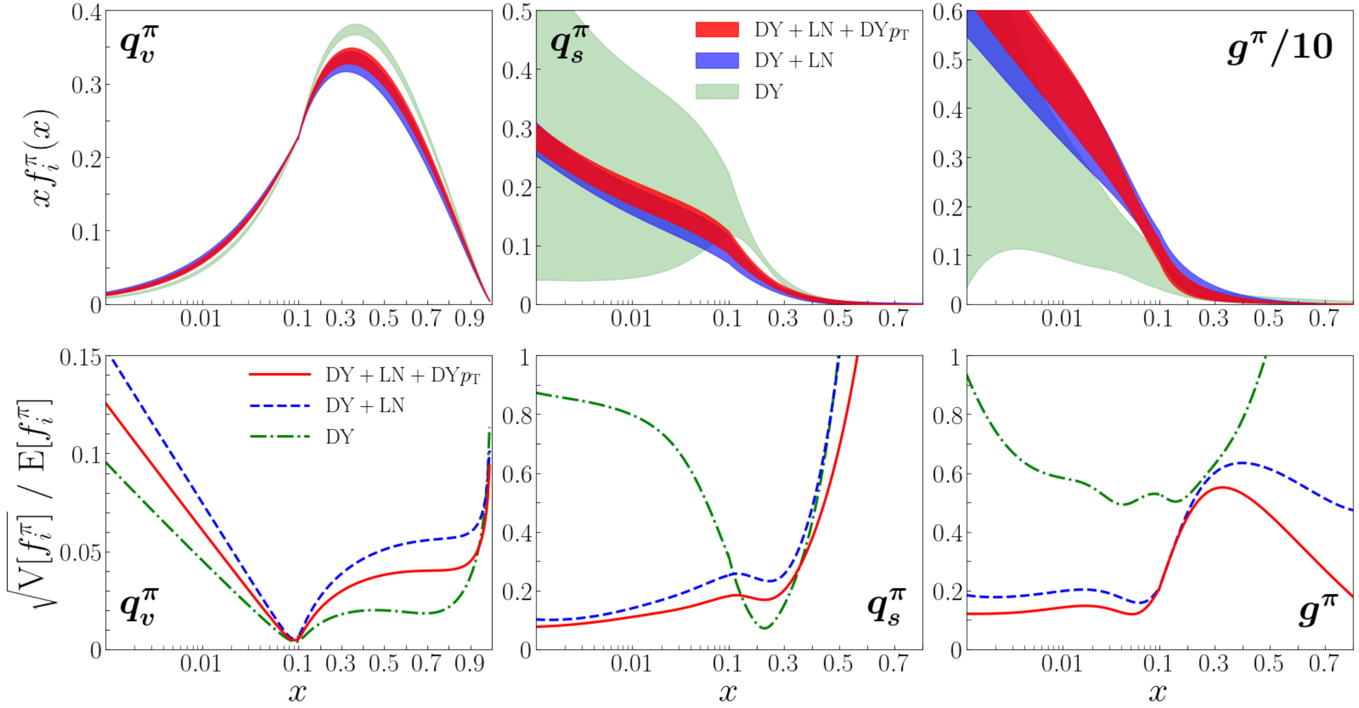


qg channels

Fit results

- Able to describe high-p_T tail
- Best agreement with data comes from scale of $\mu = p_T/2$

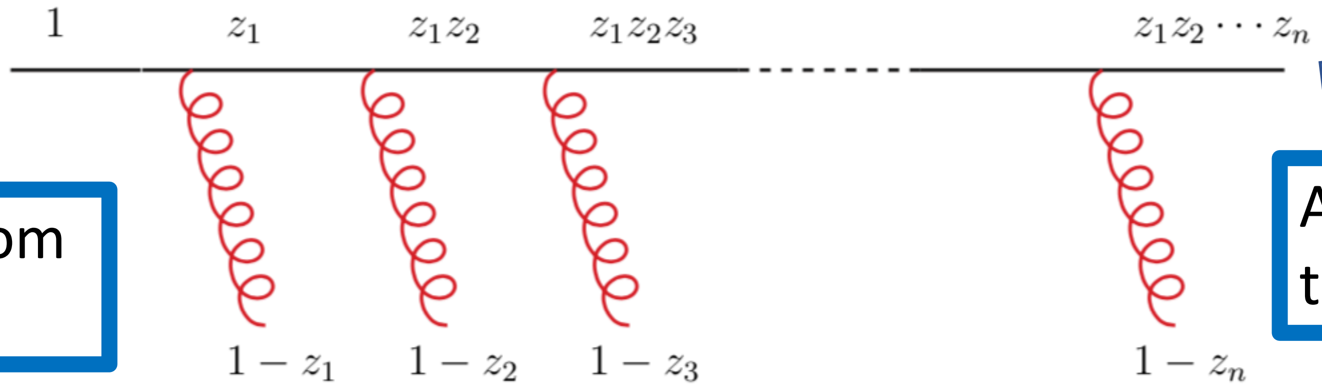




- Minimal impact from the p_T-dependent DY data on uncertainties and central values of PDFs

Threshold Resummation

Initial quark line from hadron



Annihilates with antiquark to produce virtual photon

- Significant contributions to cross section occur in soft gluon emissions
- Terms are predictable to all orders of α_S
- Must be careful of the perturbative calculation

Next-to-Leading + Next-to-Leading Logarithm Order Calculation

Make sure only counted once!
- Subtract the matching

	LL	NLL	...	N^{p}LL
LO	1	--	...	--
NLO	$\alpha_s \log(N)^2$	$\alpha_s \log(N)$	...	--
NNLO	$\alpha_S^2 \log(N)^4$	$\alpha_S^2(\log(N)^2, \log(N)^3)$	...	--
...	...	...	...	...
N^{k}LO	$\alpha_S^k \log(N)^{2k}$	$\alpha_S^k\left(\log(N)^{2k-1}, \log(N)^{2k-2}\right)$	...	$\alpha_S^k \log(N)^{2k-2p} + \cdots$

Origin of Landau Pole

$$\alpha_S C^{(1)}_{\rm soft}(N) = 2\frac{C_F}{\pi}\int_0^1 dz\,\frac{z^{N-1}-1}{1-z}\int_{Q^2}^{(1-z)^2Q^2}\frac{dk_\perp^2}{k_\perp^2}\alpha_S(k_\perp^2)$$

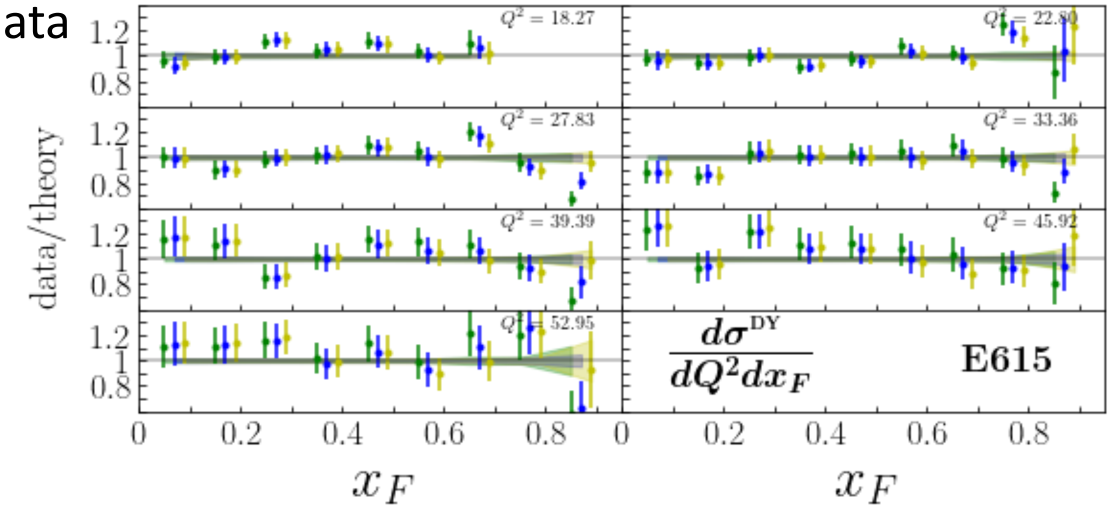
- Upper Limits imply that $k_\perp^2$ will go to 0
- $\alpha_S(\mu^2 = 0)$ is NOT well-defined
- Ambiguities on how to deal with this provide needs for prescriptions such as Minimal Prescription (MP) and Borel Prescription(BP)
- Focus on MP

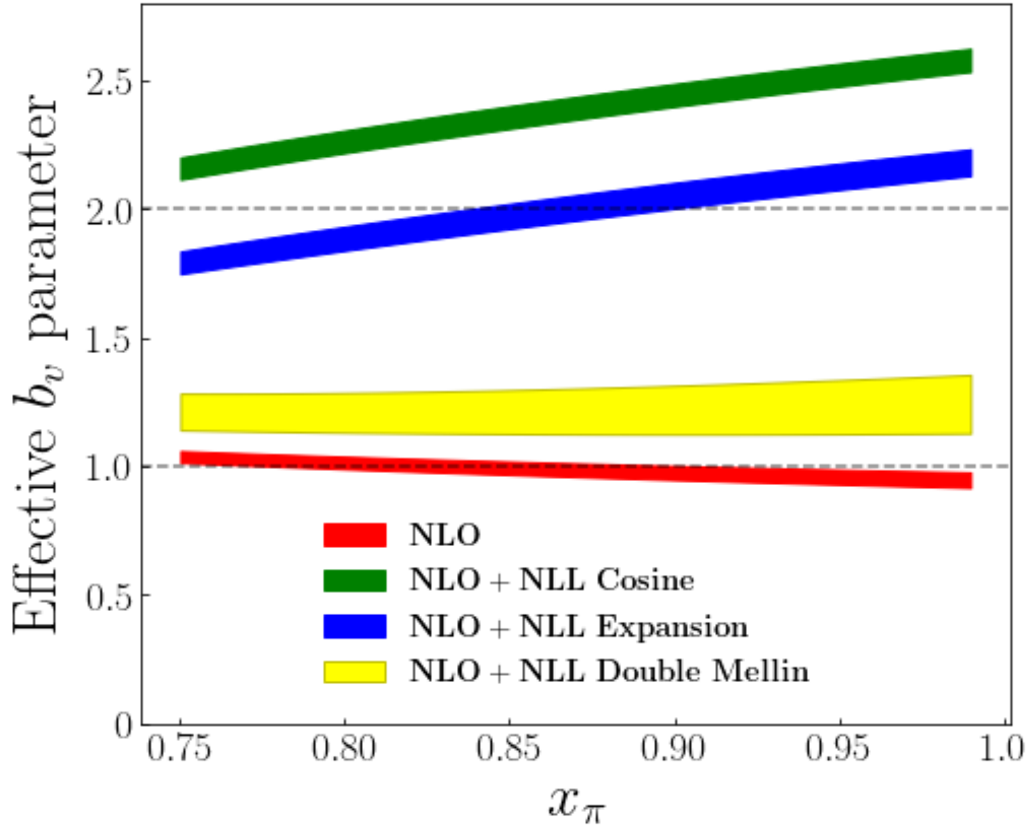
Methods of Resummation

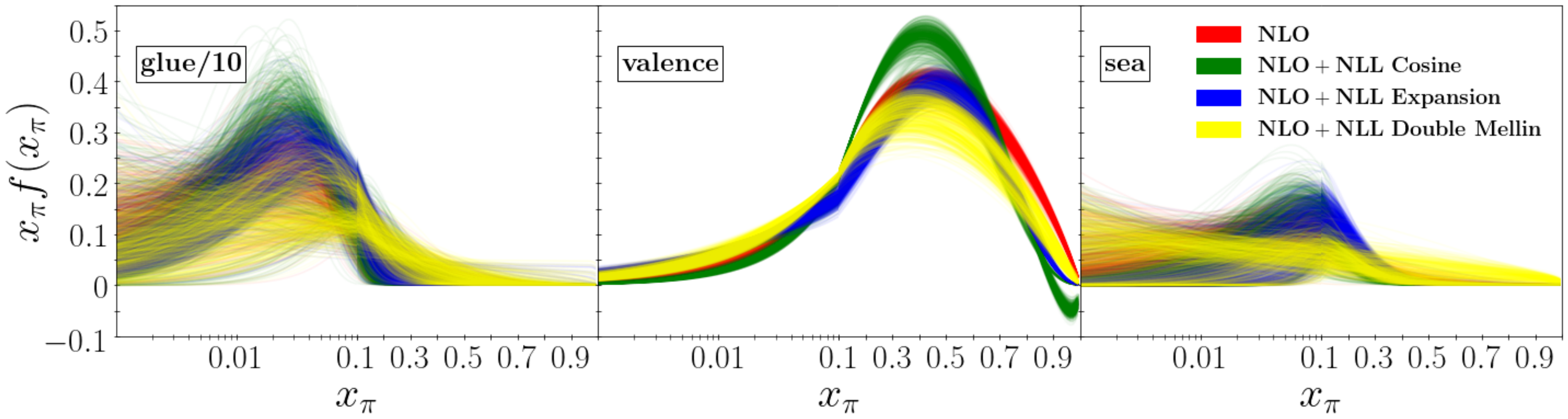
- Rapidity distribution $\frac{d\sigma}{dQ^2dY}$ adds more complications
- We can perform a Mellin-Fourier transform to account for the rapidity
  - A cosine appears while doing Fourier transform; options:
    1) Take first order expansion, cosine ≈ 1
    2) Keep cosine intact
- Can additionally perform a Double Mellin transform
- Explore the different methods and analyze effects

Resummation Fit Results

- Different prescriptions give different agreements with data



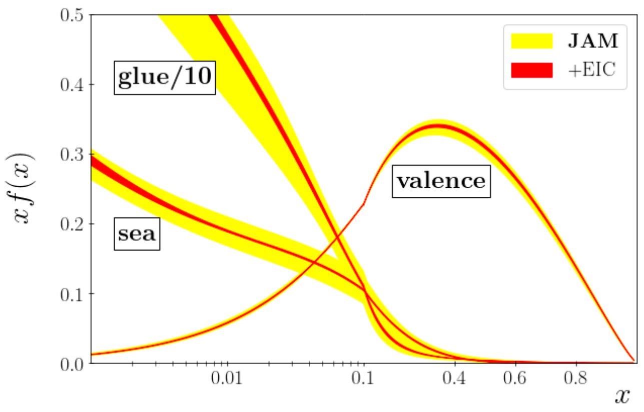


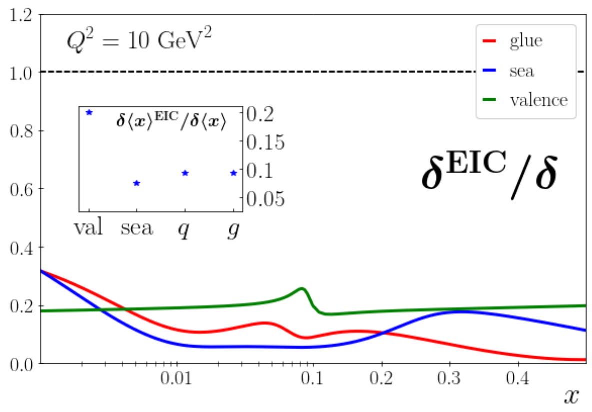


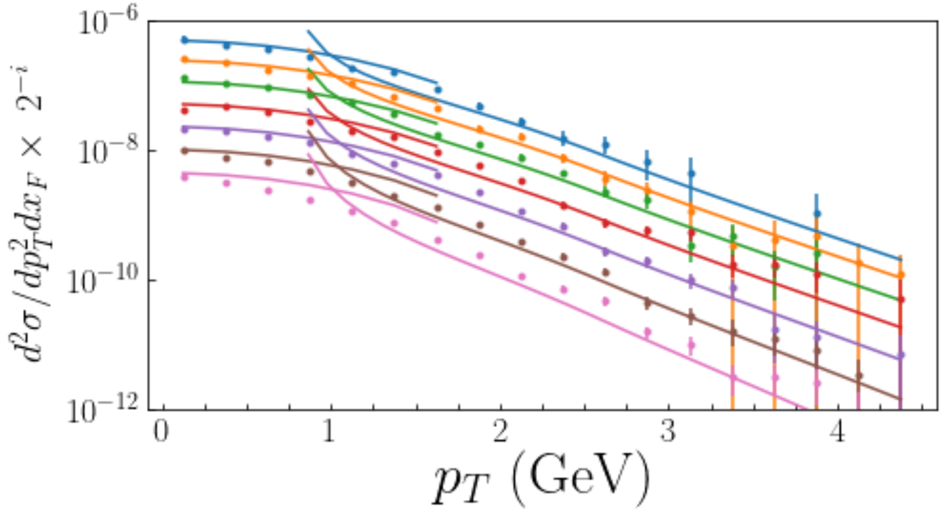
- PDFs shown at input scale
- Noticeably different behavior of $q_v(x_\pi \to 1) \sim (1 - x_\pi)^{b_v}$
- Not *only* $(1 - x_\pi)^2$

Other Applications of Pion Distributions

- Provide potential impact from EIC leading baryon experiment
- High integrated luminosity shrinks statistical uncertainties



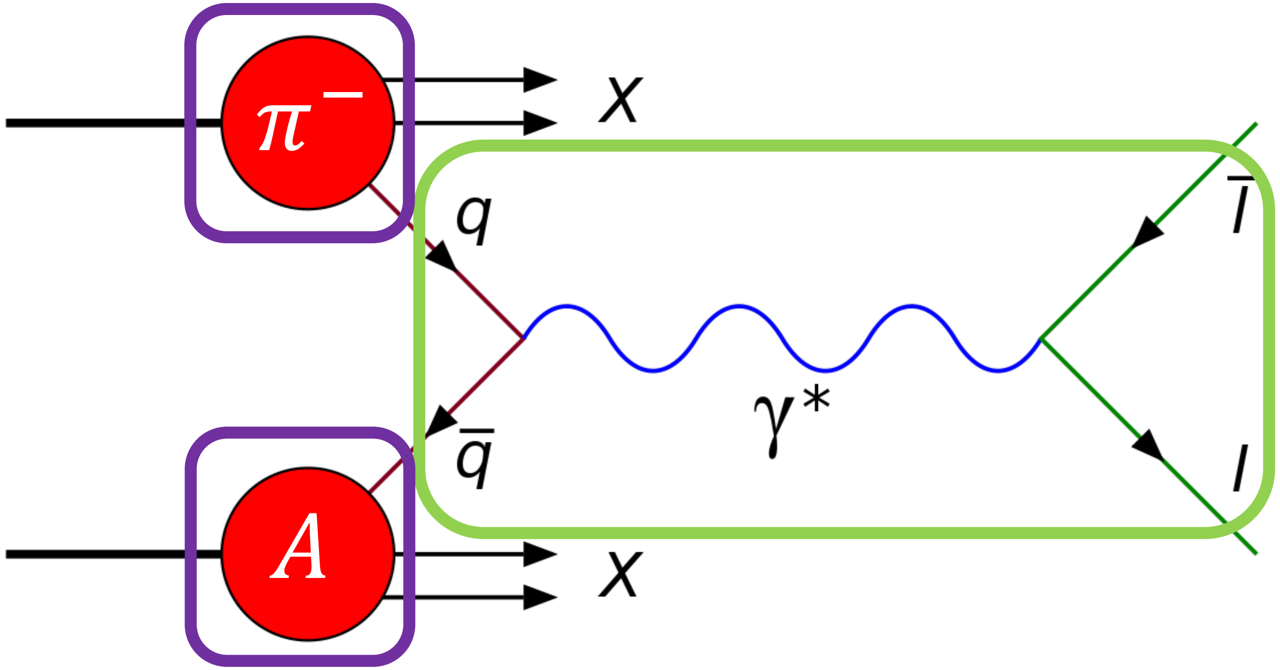




- Aim to describe both high- and low-p_T data
- Use CSS TMD factorization to describe low-p_T data

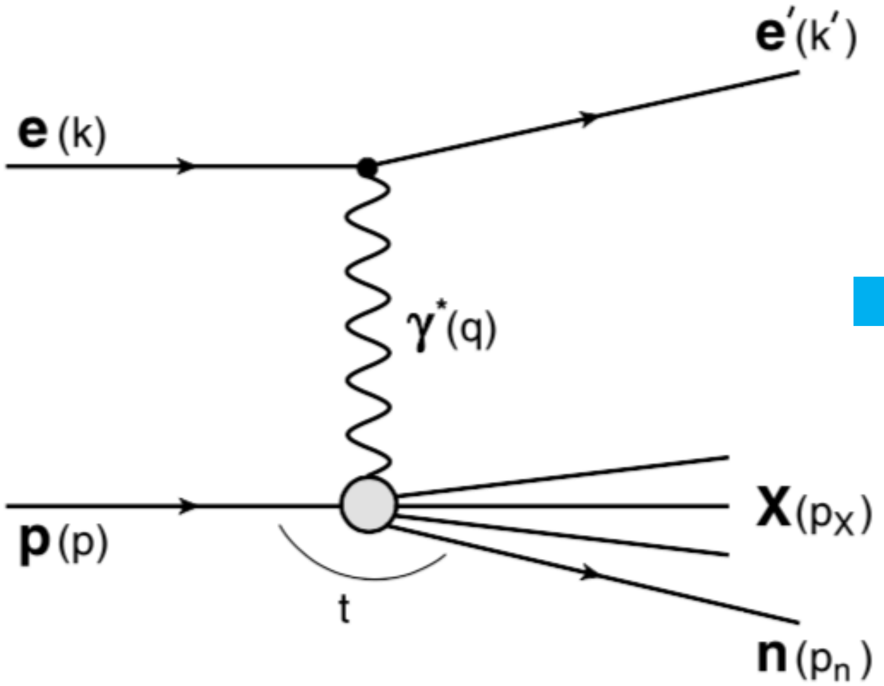
Backup Slides

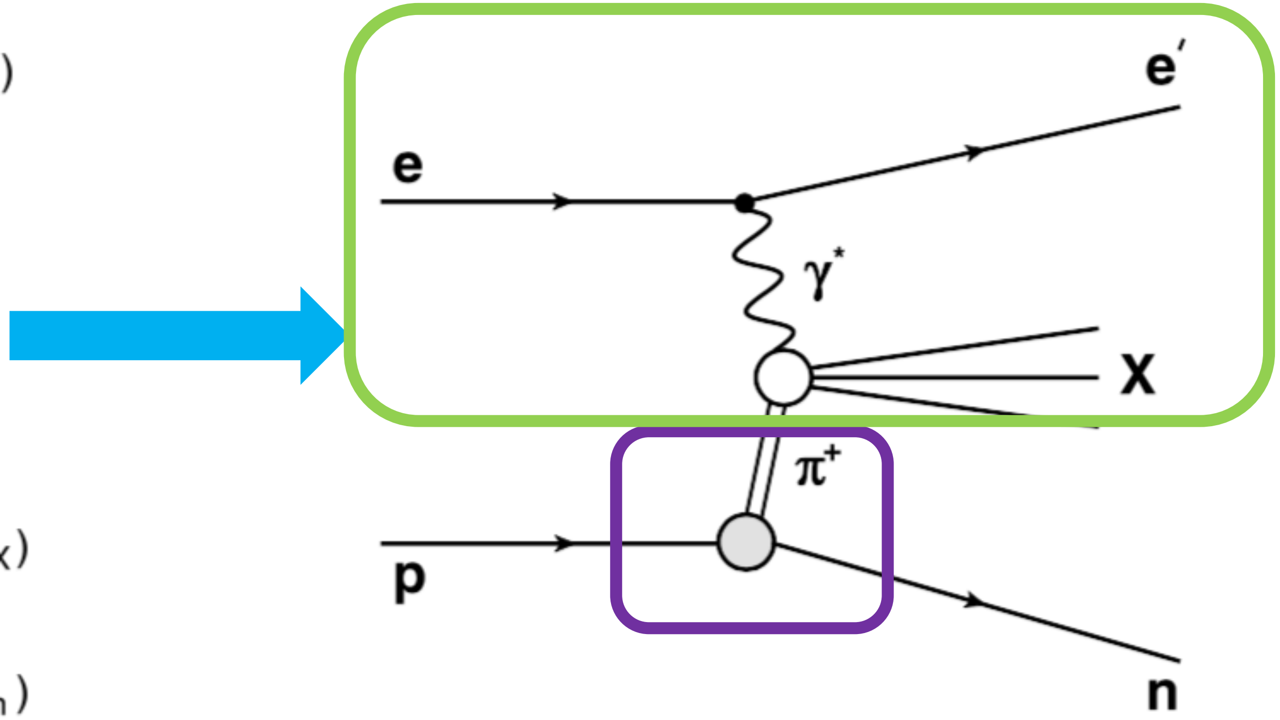
Drell-Yan (DY)



$$\sigma \propto \sum_{i,j} f_i^{\pi}(x_\pi, \mu) \otimes f_j^{A}(x_A, \mu) \otimes C_{i,j}(x_\pi, x_A, Q/\mu)$$

Leading Neutron (LN)





$$\frac{d\sigma}{dx\,dQ^2\,d\bar{x}_L} \propto f_{\pi N}(\bar{x}_L) \times \sum_i \int_{x/\bar{x}_L}^{1} \frac{d\xi}{\xi} C(\xi) f_i\left(\frac{x/\bar{x}_L}{\xi}, \mu^2\right)$$

Parametrization of the PDF

- We use a general template for the PDF by parameterizing the valence, sea, and gluon PDFs

$$f_i(x_\pi, \mu_0^2) = \frac{N_i x_\pi^{a_i} (1 - x_\pi)^{b_i}}{B[a_i + 2, b_i + 1]}$$

- B is the Euler beta function, and we normalize to the second Mellin moment

Issues with Perturbative Calculations

$$\hat{\sigma} \sim \delta(1-z) + \alpha_S(\log(1-z))_+ \longrightarrow \hat{\sigma} \sim \delta(1-z)[1 + \alpha_S \log(1-\tau)]$$

- If τ is large, can potentially spoil the perturbative calculation
- Improvements can be made by resumming $\log(1-z)_+$ terms

Threshold Resummation

- Phase space needs to be broken up and factorized
- A convenient way to do this is by Mellin transforms

$$\log(1-z) \to \log N$$

- Kernels will exponentiate in Mellin space

Resummed Kernel

$$\ln C^{\text{res}}_{\text{NLL}}(N, \alpha_S) = C_q + 2h^{(1)}(\lambda)\ln \bar{N} + 2h^{(2)}(\lambda, \frac{Q^2}{\mu^2})$$

$$\bar{N} = N e^{\gamma_E}$$

$$\lambda = b_0 \alpha_S(\mu^2) \ln \bar{N}$$

$$C_q = \frac{\alpha_S}{\pi} C_F \Big(-4 + \frac{2\pi^2}{3} + \frac{3}{2} \ln \frac{Q^2}{\mu^2} \Big)$$

Resummed Kernel

$$h^{(1)}(\lambda) = \frac{A_q^{(1)}}{2\pi b_0 \lambda}[2\lambda + (1-2\lambda)\ln(1-2\lambda)]$$

$$h^{(2)}(\lambda) = (\pi A_q^{(1)} b_1 - b_0 A_q^{(2)}) \frac{2\lambda + \ln(1-2\lambda)}{2\pi^2 b_0^3}$$

$$+ \frac{A_q^{(1)} b_1}{4\pi b_0^3} \ln^2(1-2\lambda) + \frac{A_q^{(1)}}{2\pi b_0} \ln(1-2\lambda) \ln\frac{Q^2}{\mu^2}$$

Landau Pole

- Leading logarithm (LL) resummation term

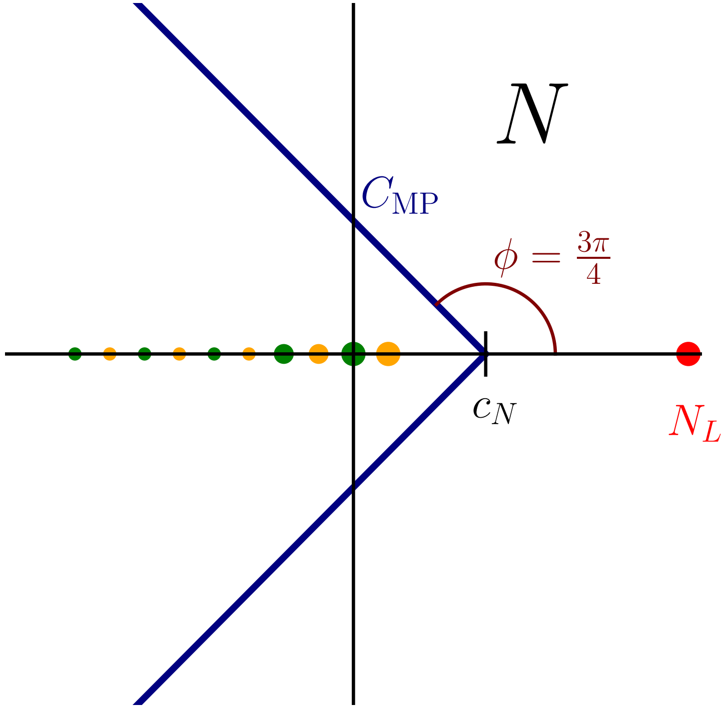
$$h^{(1)}(\lambda) = \frac{A_q^{(1)}}{2\pi b_0 \lambda}[2\lambda + (1-2\lambda)\ln(1-2\lambda)]$$

- The argument of a logarithm cannot be ≤ 0
- The value of N which this occurs is the Landau pole

$$N_{\text{Landau}} = \exp\left(\frac{1}{2b_0\alpha_S} - \gamma_E\right)$$

Minimal Prescription (MP)

- Need to Mellin invert to z space to compare with data
- The MP makes use of a contour that does not enclose Landau Pole



Resummation Momentum Fractions

- Momentum fraction from the high-x valence quarks tend to move to the low-x gluon
- Even more gluons in the pion (40% of total momentum)