Motivation

• QCD allows us to study the **structure of hadrons** in terms of **partons** (quarks, antiquarks, and gluons)

• Use **factorization theorems** to separate hard partonic physics out of soft, non-perturbative objects to quantify structure

• **Define** a structure of hadrons in terms of quantum field theories

• **Identify** theoretical observables that factorize into non-perturbative objects and perturbatively calculable physics

• Perform **global QCD analysis** as structures are universal and are the same in all subprocesses
Pions

• Pion is the **Goldstone boson** associated with spontaneous symmetry breaking of chiral $SU(2)_L \times SU(2)_R$ symmetry

• Lightest hadron as $\frac{m_\pi}{M_N} \ll 1$ and dictates the nature of hadronic interactions at low energies

• Simultaneously a pseudoscalar meson made up of $q$ and $\bar{q}$ constituents
Experiments to Probe Pion Structure

- **Drell-Yan (DY)**
  - Accelerating pion allows for time dilation and longer lifetime

- **Leading Neutron (LN)**
  - Barely striking surface of a target proton knocks out an almost on-shell pion to probe

- **Tagged DIS (TDIS)**
Future Experiments

• TDIS experiment at 12 GeV upgrade from JLab, which will tag a proton in coincidence with a spectator proton

• Gives leading proton observable, complementary to LN, but with a fixed target experiment instead of collider (HERA)

• Proposed COMPASS++/AMBER also give $\pi$-induced DY data

• Both $\pi^+$ and $\pi^-$ beams on carbon and tungsten targets
Kinematic Coverage

- DY data (E615, NA10) exist at large $x_\pi$, while LN (H1, ZEUS) data have small $x_\pi$
- Not much overlap, need more data
Monte Carlo

• Using Bayesian statistics, we describe the probability

\[ P(a|\text{data}) \propto L(\text{data}|a)\pi(a) \]

\[ L(\text{data}|a) = \exp \left( -\frac{1}{2} \chi^2(\text{data}, a) \right) \]

• We quantify the expectation value and variance of our observable \( \mathcal{O} \) as a function of the parameter set \( a_i \)

\[ \mathbb{E}[\mathcal{O}] = \sum_i w_i \mathcal{O}(a_i) \]

\[ \text{Var}[\mathcal{O}] = \sum_i w_i \left[ \mathcal{O}(a_i) - \mathbb{E}[\mathcal{O}] \right]^2 \]
NLO analysis

- PDFs found from fitting to Drell-Yan only (lightly shaded) and for Drell-Yan and Leading Neutron (darkly shaded)
- Yellow shows pion flux model dependence
- Shown at a scale of $\mu^2 = 10 \text{ GeV}^2$

- Top row – Drell-Yan
- Bottom row – Leading neutron
- Good agreement with data
- $\chi^2_{\text{npts}} = 0.979$

- Apparent swap between sea and glue with or without the LN data
Drell-Yan

- $p_T$-dependent DY data also available from E615

\[
\frac{d\sigma}{dQ^2 dY dp_T^2} = \frac{4\pi\alpha^2}{3N_C Q^2 S} \sum_{i,j} e_q^2 \int_{x_\pi}^{1} dx_\pi f^\pi_i (x_\pi, \mu)f^A_j (x_A, \mu) \times \frac{d\sigma_{i,j}}{dQ^2 dt}
\]

$q\bar{q}$ channel

$qg$ channels
Fit results

- Able to describe high-$p_T$ tail
- Best agreement with data comes from scale of $\mu = p_T/2$

- Minimal impact from the $p_T$-dependent DY data on uncertainties and central values of PDFs
Threshold Resummation

- Significant contributions to cross section occur in **soft gluon emissions**
- Terms are predictable to all orders of $\alpha_s$
- Must be careful of the perturbative calculation

Initial quark line from hadron

Annihilates with antiquark to produce virtual photon
Next-to-Leading + Next-to-Leading Logarithm Order Calculation

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<th>NLL</th>
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<th>N^{pLL}</th>
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<td>$\alpha_s^k \log(N)^{2k}$</td>
<td>$\alpha_s^k \left(\log(N)^{2k-1}, \log(N)^{2k-2}\right)$</td>
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<td>$\alpha_s^k \log(N)^{2k-2p} + \cdots$</td>
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Make sure only counted once!
- Subtract the matching
Origin of Landau Pole

Upper Limits imply that $k^2_\perp$ will go to 0

$\alpha_S(\mu^2 = 0)$ is NOT well-defined

Ambiguities on how to deal with this provide needs for prescriptions such as Minimal Prescription (MP) and Borel Prescription (BP)

Focus on MP
Methods of Resummation

• Rapidity distribution $\frac{d\sigma}{dQ^2 dY}$ adds more complications

• We can perform a Mellin-Fourier transform to account for the rapidity
  • A cosine appears while doing Fourier transform; options:
    1) Take first order expansion, cosine $\approx 1$
    2) Keep cosine intact

• Can additionally perform a Double Mellin transform

• Explore the different methods and analyze effects
Resummation Fit Results

• Different prescriptions give different agreements with data

PDFs shown at input scale

Noticeably different behavior of $q_v(x_\pi \to 1) \sim (1-x_\pi)^{b_v}$

• Not only $(1 - x_\pi)^2$
Other Applications of Pion Distributions

• Provide potential impact from EIC leading baryon experiment
• High integrated luminosity shrinks statistical uncertainties

• Aim to describe both high- and low-\(p_T\) data
• Use CSS TMD factorization to describe low-\(p_T\) data
Backup Slides
Drell-Yan (DY)

\[ \sigma \propto \sum_{i,j} f_{i}^{\pi}(x_{\pi}, \mu) \otimes f_{j}^{A}(x_{A}, \mu) \otimes C_{i,j}(x_{\pi}, x_{A}, Q/\mu) \]
Leading Neutron (LN)

\[
\frac{d\sigma}{dxdQ^2d\bar{x}_L} \propto f_{\pi N}(\bar{x}_L) \times \sum_i \int_{x/\bar{x}_L}^1 \frac{d\xi}{\xi} C'\left(\xi\right) f_i\left(\frac{x/\bar{x}_L}{\xi}, \mu^2\right)
\]
Parametrization of the PDF

• We use a general template for the PDF by parameterizing the valence, sea, and gluon PDFs

\[ f_i(x_\pi, \mu_0^2) = \frac{N_i x_\pi^{a_i} (1 - x_\pi)^{b_i}}{B[a_i + 2, b_i + 1]} \]

• \( B \) is the Euler beta function, and we normalize to the second Mellin moment
Issues with Perturbative Calculations

\[ \hat{\sigma} \sim \delta(1 - z) + \alpha_s \log(1 - z) \]

\[ \hat{\sigma} \sim \delta(1 - z)[1 + \alpha_s \log (1 - \tau)] \]

• If \( \tau \) is large, can potentially spoil the perturbative calculation
• Improvements can be made by resumming \( \log(1 - z) \) terms
Threshold Resummation

• Phase space needs to be broken up and factorized
• A convenient way to do this is by **Mellin transforms**

\[ \log (1 - z) \rightarrow \log N \]

• Kernels will exponentiate in Mellin space
Resummed Kernel

\[ \ln C_{\text{NLL}}^{\text{res}}(N, \alpha_S) = C_q + 2h^{(1)}(\lambda) \ln \bar{N} + 2h^{(2)}(\lambda, \frac{Q^2}{\mu^2}) \]

\[ \bar{N} = Ne^{\gamma_E} \]

\[ \lambda = b_0 \alpha_S(\mu^2) \ln \bar{N} \]

\[ C_q = \frac{\alpha_S}{\pi} C_F \left( -4 + \frac{2\pi^2}{3} + \frac{3}{2} \ln \frac{Q^2}{\mu^2} \right) \]
Resummed Kernel

\[ h^{(1)}(\lambda) = \frac{A_q^{(1)}}{2\pi b_0 \lambda} \left[ 2\lambda + (1 - 2\lambda) \ln (1 - 2\lambda) \right] \]

\[ h^{(2)}(\lambda) = (\pi A_q^{(1)} b_1 - b_0 A_q^{(2)}) \frac{2\lambda + \ln (1 - 2\lambda)}{2\pi^2 b_0^3} + \frac{A_q^{(1)} b_1}{4\pi b_0^3} \ln^2 (1 - 2\lambda) + \frac{A_q^{(1)}}{2\pi b_0} \ln (1 - 2\lambda) \ln \frac{Q^2}{\mu^2} \]
Landau Pole

• Leading logarithm (LL) resummation term

\[ h^{(1)}(\lambda) = \frac{A^{(1)}_q}{2\pi b_0 \lambda} [2\lambda + (1 - 2\lambda) \ln (1 - 2\lambda)] \]

• The argument of a logarithm cannot be \( \leq 0 \)
• The value of \( N \) which this occurs is the Landau pole

\[ N_{\text{Landau}} = \exp \left( \frac{1}{2b_0 \alpha_s} - \gamma_E \right) \]
Minimal Prescription (MP)

• Need to Mellin invert to $z$ space to compare with data
• The MP makes use of a contour that does not enclose Landau Pole
Resummation Momentum Fractions

- Momentum fraction from the high-$x$ valence quarks tend to move to the low-$x$ gluon
- Even more gluons in the pion (40% of total momentum)