Baryon structure in terms of diquarks

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Thanks to Alberto Accardi, César Fernández and the organizers.
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Since 1970s, we know that the nucleon is a bound state of three valence quarks along with a sea of gluons and quark-antiquark pairs.
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• An implication of this understanding is that when energy is dumped into the nucleon ground states, they are excited and can only lose their energy by emitting color singlet states.

• The spectrum of these excited states.

<table>
<thead>
<tr>
<th>$I$</th>
<th>$S$</th>
<th>$J^P$</th>
<th>$3^+$</th>
<th>$5^+$</th>
<th>$1^-$</th>
<th>$3^-$</th>
<th>$5^-$</th>
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<tr>
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<td>$\Xi(1530)$</td>
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<tr>
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<td>$-3$</td>
<td>$\Omega(1672)$</td>
<td>$\Omega(1672)$</td>
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</tr>
</tbody>
</table>
• The Faddeev amplitude $\Psi$ for Baryons in a Bethe-Salpeter approach:

\[
\begin{align*}
  k_d &= -k + P \\
  k_q &= k \\
  l_q &= l \\
  l_d &= -l + P
\end{align*}
\]

[Barabanov:2020jvn]

[Eichmann:2011vu]
The Faddeev amplitude $\Psi$ for Baryons in a Bethe-Salpeter approach:

$$k_d = -k + P$$

$$k = k$$

$$k_q = k$$

$$l_q = l$$

$$l = -l + P$$

$$[\text{Barabanov:2020jvn}]$$

$$S^{-1}(p, \mu) = \frac{i\gamma \cdot p + M(p^2, \mu^2)}{Z(p^2, \mu^2)}$$

$[\text{Eichmann:2011vu}]$
• The Faddeev amplitude $\Psi$ for Baryons in a Bethe-Salpeter approach:

$$\psi^a = \psi^b$$

$[Eichmann:2011vu]$

$[Barabanov:2020jvn]$

$$S^{-1}(p, \mu) = \frac{i\gamma \cdot p + M(p^2, \mu^2)}{Z(p^2, \mu^2)}$$

$$\Delta_{\mu\nu}^{\Omega}(K) = \left[\delta_{\mu\nu} + \frac{K_{\mu}K_{\nu}}{m_{1^+}^2}\right] \frac{1}{K^2 + m_{1^+}^2},$$

$$\Delta^{0^+}(K) = \frac{1}{K^2 + m_{0^+}^2},$$
The Faddeev amplitude $\Psi$ for Baryons in a Bethe-Salpeter approach:

$$
\psi^P(k_i,\alpha_i,\sigma_i) = \left[\Gamma^0(K)\right]^{\alpha_1\alpha_2} \Delta^0(K) \left[ S^P_{\rho}(k;Q)u_\rho(Q) \right]^{\sigma_3}
+ [\tau^i \Gamma^+_\mu] \Delta^1_{\mu\nu} \left[ A^i_{\nu\rho}(k;Q)u_\rho(Q) \right]
+ [\Gamma^0 \Delta^0] \left[ F^P_{\mu}(k;Q)u_\rho(Q) \right]
+ [\Gamma^1_{\mu}] \Delta^1_{\mu\nu} \left[ \nu^P_{\nu\rho}(k;Q)u_\rho(Q) \right],
$$

$$
S^{-1}(p,\mu) = \frac{i\gamma \cdot p + M(p^2,\mu^2)}{Z(p^2,\mu^2)}
$$

$$
\Delta^{1\pm}(K) = \left[ \delta_{\mu\nu} + \frac{K_\mu K_\nu}{m_{1\pm}^2} \right] \frac{1}{K^2 + m_{1\pm}^2},
$$

$$
\Delta^{0\pm}(K) = \frac{1}{K^2 + m_{0\pm}^2},
$$

\[ \text{[Barabanov:2020jvn]} \]
• Ideal for studying non-perturbative phenomena because nothing is assumed about the value of the coupling:

\[
S^{-1}(p, \mu) = Z_{2F} S_{0}^{-1}(p) + Z_{1F} \int \frac{d^4p}{(2\pi)^4} g^2 D_{\rho\nu}(p - q; \mu) \frac{\lambda^a}{2} \gamma_{\rho} S(q; \mu) \Gamma^a_q(q, p; \mu).
\]

[Roberts:1994dr]
Schwinger – Dyson Equations

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Complete Propagator, dressed

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Complete Propagator, dressed
Bare propagator, tree level
Dressed gluon propagator
Dressed quark-gluon vertex

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• Truncation framework: Rainbow-Ladder

\[ Z_1 F g^2 D_{\rho \nu}(p - q; \mu) \frac{\lambda^a}{2} \gamma_\rho S(q; \mu) \Gamma^a_{\nu}(q, p; \mu) \rightarrow k^2 G(k^2) D_{\rho \nu}^0(k; \mu) \frac{\lambda^a}{2} \gamma_\rho S(q; \mu) \frac{\lambda^a}{2} \gamma_\nu. \]

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- Complete Propagator, dressed
- Bare propagator, tree level
- Dressed gluon propagator
- Dressed quark-gluon vertex

Constituent mass vs current mass

\[ m_u + m_u + m_d \approx 10 \text{ MeV} \]

\[ m_{\text{protón}} \approx 1000 \text{ MeV} \]

Truncation framework: Rainbow-Ladder

\[ Z_1F g^2 D_{\rho\nu}(p - q; \mu) \frac{\lambda^a}{2} \gamma_\rho S(q; \mu) \Gamma^a_\nu(q, p; \mu) \rightarrow k^2 G(k^2) D_{\rho\nu}^0(k; \mu) \frac{\lambda^a}{2} \gamma_\rho S(q; \mu) \frac{\lambda^a}{2} \gamma_\nu \]

[R Roberts:1994dr]
Simplified QCD model, where UV divergences are regularized to preserve QCD symmetries, compatible with confinement and DCSB.
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- The effective gluons mass in the IR motivates this truncation that replaces the full gluon propagator with a constant in the infrared.

\[ \mathcal{G}(k^2)D^0_{\mu\nu}(k;\mu) \rightarrow \delta_{\mu\nu} \frac{4\pi\alpha_{IR}}{m_g^2} \]
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Interaction strength in infrared \( \alpha_{IR} = 0.93\pi \) compatible with modern computations.
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Gluon mass scale
500 MeV
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Gluon mass scale 500 MeV

[Gutierrez-Guerrero:2010waf, Roberts:2010rn]
The full $q\bar{q}$ scattering matrix or $t$-matrix, contains poles for all $q\bar{q}$ bound states, that is, the physical mesons. [Salpeter:1951sz]

\[
\Gamma^{f\bar{f}_{2}}_H(k; P)_{iu} = \int \frac{d^4q}{(2\pi)^4} \left[ \chi^{f\bar{f}_{2}}_H(q; P) \right]_{sr} K^r_{iu}(q, k; P),
\]

\[
\chi^{f\bar{f}_{2}}_H(q; P) = S_{f_1}(q_+)\Gamma^{f\bar{f}_{2}}_H(q; P)S_{\bar{f}_2}(q_-),
\]

\[
\Gamma^{f}_{H}(k; P) = \gamma^l \gamma_5 \left[ iE_+(k; P) + \gamma \cdot PF_+(k; P) + \gamma \cdot k G_+(k; P) + \sigma_{\mu\nu}k_{\nu}P_{\nu}D_+(k; P) \right],
\]
• The full $q\bar{q}$ scattering matrix or $t$-matrix, contains poles for all $q\bar{q}$ bound states, that is, the physical mesons. [Salpeter:1951sz]

\[
\left[ \Gamma_{H}^{f_{1}f_{2}}(k; P) \right]_{i} = \int \frac{d^{4}q}{(2\pi)^{4}} \chi_{H}^{f_{1}f_{2}}(q; P) K_{i}^{f_{2}}(q, k; P),
\]

\[
\chi_{H}^{f_{1}f_{2}}(q; P) = S_{f_{1}}(q_{+}) \Gamma_{H}^{f_{1}f_{2}}(q; P) S_{f_{2}}(q_{-}).
\]

\[
\gamma_{H}^{i}(k; P) = \tau^{i} \gamma_{5} \left[ iE_{H}(k; P) + \gamma \cdot PF_{H}(k; P) + \gamma \cdot k G_{H}(k; P) + \sigma_{\mu
\nu}k_{\nu}P_{\mu}D_{H}(k; P) \right],
\]

• En el Modelo CI las ABS no depende de $k$, el momento relativo:
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\[
\Gamma_H^{ij}(k; P)_{ij} = \int \frac{d^4q}{(2\pi)^4} \chi_H^{ij}(q; P) K^{rs}_{ij}(q, k; P), \\
\chi_H^{ij}(q; P) = S_{fi}(q_+) \Gamma_H^{ij}(q, P) S_{fj}(q_-),
\]

\[
\Gamma_H^j(k; P) = \tau^j \gamma_5 \left[ iE_H(k; P) + \gamma \cdot PF_H(k; P) + \gamma \cdot k G_H(k; P) + \sigma_{\mu\nu}k_\nu P_\mu D_H(k; P) \right],
\]

• En el Modelo CI las ABS no depende de $k$, el momento relativo:

\[
\Gamma^0(\rho) = \text{Exp}, \\
\Gamma^+ = \gamma_5 \left[ iE^+ + \frac{2M}{2M} \sigma_{\mu\nu}F_{\mu\nu} \right], \\
\Gamma^\mu_\rho = \gamma^\mu E^+ + \frac{1}{2M} \sigma_{\mu\nu}F_{\nu}^\rho, \\
\Gamma^\mu_\sigma = \gamma^\mu E^+ + \frac{1}{2M} \sigma_{\mu\nu}F_{\nu}^\sigma.
\]

\[P^2 = -M_H^2\]

<table>
<thead>
<tr>
<th>Meson</th>
<th>Exp.</th>
<th>CI</th>
<th>Diquarks Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>0.139</td>
<td>0.14</td>
<td>$(qq)_{0^+} = 0.78$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.78</td>
<td>0.93</td>
<td>$(qq)_{1^+} = 1.06$</td>
</tr>
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<td>$\sigma$</td>
<td>1.2</td>
<td>1.22</td>
<td>$(qq)_{0^-} = 1.15$</td>
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<tr>
<td>$a_1$</td>
<td>1.260</td>
<td>1.37</td>
<td>$(qq)_{1^-} = 1.33$</td>
</tr>
</tbody>
</table>

The Faddeev equation in the CI dynamical quark-diquark picture:

\[
\begin{bmatrix}
S^p(l; P) \\
\mathcal{A}_\mu^p(l; P) \\
\mathcal{P}^p(l; P) \\
\mathcal{V}_\mu^p(l; P)
\end{bmatrix}
= \frac{4}{(2\pi)^4} \int \mathcal{M}_{\mu\nu}^{l}(k, l; P) \begin{bmatrix}
S^p(l; P) \\
\mathcal{A}_\nu^p(l; P) \\
\mathcal{P}^p(l; P) \\
\mathcal{V}_\mu^p(l; P)
\end{bmatrix}
\]

\[S^z = (s^z \mathbb{1}_D) \mathcal{G}^z\]
\[i \mathcal{A}_\mu^z = (a_1^0 \gamma_5 \gamma_\mu - i a_2^0 \gamma_5 \hat{P}_\mu) \mathcal{G}^z\]
\[i \mathcal{P}^z = (p^z \gamma_5) \mathcal{G}^z\]
\[i \mathcal{V}_\mu^z = (v_1^z \gamma_\mu - i v_2^z \gamma_5 \hat{P}_\mu) \mathcal{G}^z\]

\[\Psi_N = \begin{bmatrix}
r_1 u[ud]_{0^+} \\
r_2 d[uu]_{1^+} \\
r_3 u[ud]_{1^-} \\
r_4 u[ud]_{0^-} \\
r_5 u[ud]_{1^-}
\end{bmatrix}\]
The Faddeev equation in the CI dynamical quark-diquark picture:

\[
\begin{bmatrix}
S^p(l; P) \\
\mathcal{A}_\mu^{\alpha \beta}(l; P) \\
\mathcal{P}^p(l; P) \\
\mathcal{V}^p_\mu(l; P)
\end{bmatrix}
\quad u_p = 4 \int \frac{d^4 l}{(2\pi)^4} \mathcal{M}_{\mu \nu}^{\alpha \beta}(k, l, P) 
\begin{bmatrix}
S^\lambda(l; P) \\
\mathcal{A}_\mu^{\alpha \beta}(l; P) \\
\mathcal{P}^\lambda(l; P) \\
\mathcal{V}^\mu_\nu(l; P)
\end{bmatrix}
\quad u_\lambda,
\]

\[
S^z = (s^z 1_D)^\frac{1}{2}
\]

\[
i \mathcal{A}_\mu^{\alpha \beta} = (a_1^{\alpha \beta} \gamma_5 \gamma_\mu - i a_2^{\alpha \beta} \tilde{\gamma}_\mu) G^z
\]

\[
i \mathcal{P}^z = (p^z \gamma_5) G^z
\]

\[
i \mathcal{V}_\mu^z = (v_1^z \gamma_\mu - i v_2^z \gamma_5 \tilde{\gamma}_\mu) G^z
\]

\[
\Psi_N = \begin{bmatrix}
    r_1 u[ud]_0^0 \\
r_2 d[uu]_1^0 \\
r_3 u[ud]_1^1 \\
r_4 u[ud]_0^1 \\
r_5 u[ud]_1^0
\end{bmatrix}
\]

\[
\psi_{\mu \nu}(P) u_\nu = \Gamma_{q_1 q_2}^{1+} \Delta_{\mu \nu, q_1 q_2}^{1+} (\ell_{q_2} q_1) \mathcal{D}_{\nu \rho}(P) u_\rho(P)
\]

\[
\mathcal{D}_{\nu \rho}(P) u_\rho(P) = f^{\rho}(P) \| D u_\nu(P)
\]
The Faddeev equation in the CI dynamical quark-diquark picture:

\[
\begin{bmatrix}
S^f(l; P) \\
\mathcal{A}^f_{\mu}(l; P) \\
\mathcal{P}^f(l; P) \\
\mathcal{V}^f_{\mu}(l; P)
\end{bmatrix}
\left[\begin{array}{c}
S^f(l; P) \\
\mathcal{A}^f_{\mu}(l; P) \\
\mathcal{P}^f(l; P) \\
\mathcal{V}^f_{\mu}(l; P)
\end{array}\right]
= 4 \int \frac{d^4l}{(2\pi)^4} \mathcal{M}^f_{\mu}(k, l; P)
\]

\[
S^z = (s^z \Gamma_{D}) G^z
\]

\[
i \mathcal{A}^f_{\mu} = (a^f \gamma_5 \gamma_\mu - i a^f \gamma_5 \gamma_\mu) G^z
\]

\[
i \mathcal{P}^z = (p^z \gamma_5) G^z
\]

\[
i \mathcal{V}^z = (v^z \gamma_\mu - i v^z \gamma_\mu) G^z
\]

\[
\Psi_N = \begin{bmatrix}
r_1 u[ud]^0 \\
r_2 d[uu]^1 \\
r_3 u[ud]^1 \\
r_4 u[ud]^0 \\
r_5 u[ud]^1
\end{bmatrix}
\]

\[
\psi_{\mu \nu}(P) u_\nu = \Gamma_{qq1+\mu} \Delta_{\mu \nu qq}^1 \mathcal{D}_{\nu \rho}(P) u_\rho(P)
\]

\[
\mathcal{M}^f_{\mu}(q_1, q_2, q_3, q_4) = \Gamma_{[q_1 q_2]}^{T} \Gamma_{[q_3 q_4]}^{T} \left[ g_{PDB}^{P_{DB}} \Gamma_{[q_1 q_2]}^{S_{DB}} \Gamma_{[q_3 q_4]}^{P_{DB}} \right] S_{q_1}(l_{q_2}) \Delta_{[q_1 q_2]}^{f}(l_{q_3 q_4})
\]

Diquark breakup and recombination occurs via quark exchange.
• The Faddeev equation in the CI dynamical quark-diquark picture:

\[
\begin{bmatrix}
    S^p(l; P) \\
    \mathcal{A}_\mu^f(l; P) \\
    P^p(l; P) \\
    V_\mu^p(l; P)
\end{bmatrix}
\begin{bmatrix}
    u_p \\
    f^a_{\mu}(k, l, P) \\
    \mathcal{P}^p(l; P) \\
    \mathcal{V}_\mu^p(l; P)
\end{bmatrix}
= 4 \int \frac{d^4l}{(2\pi)^4} \mathcal{M}^{fg}_{\mu\nu}(k, l, P)
\]

\[
\begin{bmatrix}
    S^f(l; P) \\
    \mathcal{A}_\mu^g(l; P) \\
    P^f(l; P) \\
    V_\mu^f(l; P)
\end{bmatrix}
\begin{bmatrix}
    u_p \\
    f^b_{\mu}(k, l, P) \\
    \mathcal{P}^f(l; P) \\
    \mathcal{V}_\mu^f(l; P)
\end{bmatrix}
\]

\[
\psi_{\mu\nu}(P) u_\nu = \Gamma_{q_1 q_2} \Delta_{\mu\nu,q_q}^{1+} (\ell_{q_q}) \mathcal{D}_{\nu\rho}(P) u_\rho(P)
\]

\[
\mathcal{M}^{fg}_{[q_1 q_3][q_1 q_2]} = t^{[q_1 q_2]} [q_1 q_3] t^{q_3} \Gamma^{g}_{DB} \Gamma^{f}_{DB} (l_{q_1 q_2}) S^{T}_{q_1} \mathcal{S}_{q_1}^{P_d P_b} \mathcal{S}_{q_2}^{D_B F_g} \Delta_{q_1 q_3}^{g}(l_{q_1 q_2})
\]

\[
\Psi_N = \begin{bmatrix}
    r_1 u_{[ud]0} \\
    r_2 d_{[uu]}1 \\
    r_3 u_{[ud]}1 \\
    r_4 u_{[ud]}0 \\
    r_5 u_{[ud]}1
\end{bmatrix}
\]

• Diquark breakup and recombination occurs via quark exchange.

• The kernel penalizes the contribution of diquarks whose parity is opposite to that of the baryon using a multiplicative factor \( g_{P_b P_d} \).

• Comparison with other approaches:

<table>
<thead>
<tr>
<th>Method</th>
<th>$\Omega_{ccc}^{++}$</th>
<th>$\Omega_{bbb}^{-*}$</th>
<th>$\Omega_{cbb}^{++}$</th>
<th>$\Omega_{cbb}^{0*}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CI-LP</td>
<td>4.78</td>
<td>14.39</td>
<td>8.03</td>
<td>11.10</td>
</tr>
<tr>
<td>CI-HP</td>
<td>4.93</td>
<td>14.23</td>
<td>8.03</td>
<td>11.12</td>
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• Comparison with other approaches:
• Comparison with other approaches:
• The produced masses and diquark content:

- $N(940)$ and $N(1535)$

- The variation of $g_{DB} \to (1 \pm 0.5)g_{DB}$ produces:

[Lu:2017cln, Raya:2021pyr]
• As expected, the nucleon is mostly composed by scalar diquarks, while also exhibiting a axial- vector diquark component.

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As expected, the nucleon is mostly composed by scalar diquarks, while also exhibiting an axial-vector diquark component.

The nucleon $N(1535)$ shows a similar contribution from $0^+|0^-$ diquarks for $g_{DB} = 0.2$.

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[Lu:2017cln, Raya:2021pyr]
Results for $N^*$

- In collaboration with K. Raya

[Liu:2022nku]
Results for $\Delta^*$

- $1.39$ GeV in Cl
- $1.346$ GeV in Kindred
- $1.99$ GeV in Cl
- $1.786$ GeV in Kindred
- $1.72$ GeV in Cl
- $1.871$ GeV in Kindred
- $1.99$ GeV in Cl
- $2.043$ GeV in Kindred

[Liu:2022ndb]
• We need to normalize the Faddeev amplitudes in the transition diagram and for this we need the properties of the elastic form factors.

• The current for N EFF:

$$J_\mu(K, Q) = ie \Lambda_+(P_f) \Gamma_\mu(K, Q) \Lambda_+(P_i)$$

• The current for Δ EFF:

$$J_{\mu,\lambda}(K, Q) = \Lambda_+(P_f) R_{\lambda\alpha}(P_f) \Gamma_{\mu,\alpha\beta}(K, Q) \Lambda_+(P_i) R_{\beta\omega}(P_f)$$

[Segovia:2014aza, Nicmorus:2010sd]
We need to normalize the Faddeev amplitudes in the transition diagram and for this we need the properties of the elastic form factors.

The current for $N$ EFF:

$$J_\mu(K, Q) = ie \Lambda_+(P_f) \Gamma_\mu(K, Q) \Lambda_+(P_i)$$

The vertex:

$$\Gamma_\mu(K, Q) = \gamma_\mu F_1(Q^2) + \frac{1}{2m_N} \sigma_{\mu\nu} Q_\nu F_2(Q^2)$$

The current for $\Delta$ EFF:

$$J_{\mu,\lambda\omega}(K, Q) = \Lambda_+(P_f) R_{\lambda\alpha}(P_f) \Gamma_{\mu,\alpha\beta}(K, Q) \Lambda_+(P_i) R_{\beta\omega}(P_i)$$

The vertex:

$$\Gamma_{\mu,\alpha\beta}(K, Q) = \left[ (F_1^* + F_2^*) i\gamma_\mu - \frac{F_2^*}{m_\Delta} K_\mu \right] \delta_\alpha\beta - \left[ (F_3^* + F_4^*) i\gamma_\mu - \frac{F_4^*}{m_\Delta} K_\mu \right] \frac{Q_\alpha Q_\beta}{4m^2_\Delta}$$
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- Spatial distribution of charge and magnetic moment:

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m_N^2} F_2(Q^2)$$
$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

- The current for $\Delta\,\text{EFF}$:

$$J_{\mu,\lambda\omega}(K, Q) = \Lambda_+(P_f) R_{\lambda\alpha}(P_f) \Gamma_{\mu,\alpha\beta}(K, Q) \Lambda_+(P_i) R_{\beta\omega}(P_i)$$

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$$\Gamma_{\mu,\alpha\beta}(K, Q) = \left[ (F_1^* + F_2^*) i\gamma_{\mu} - \frac{F_2^*}{m_\Delta} K_{\mu} \right] \delta_{\alpha\beta} - \left[ (F_3^* + F_4^*) i\gamma_{\mu} - \frac{F_4^*}{m_\Delta} K_{\mu} \right] \frac{Q_{\alpha} Q_{\beta}}{4m_\Delta^2}$$

- The $G_{E0}$ form factor in terms of the $F_i$:

$$G_{E0}(Q^2) = \left(1 + \frac{2\tau_\Delta}{3} \right) (F_1^* - \tau_\Delta F_2^*) - \frac{\tau_\Delta}{3} (1 + \tau_\Delta) (F_3^* - \tau_\Delta F_4^*)$$

$$\tau_B = \frac{Q^2}{4m_B^2}$$
The electromagnetic current:

\[ J^{\mu \lambda}(K, Q) = \Lambda_+(P_f) R^{\lambda \alpha}(P_f) i \gamma_5 \Gamma^{\alpha \mu}(K, Q) \Lambda_+(P_i) \]

The vertex with \( G_M^* \) magnetic dipole, \( G_E^* \) electric quadrupole and \( G_C^* \) Coulomb quadrupole:

\[
\Gamma^{\alpha \mu} = b \left[ \frac{i \omega}{2 \lambda_+} (G_M^* - G_E^*) \gamma_5 \varepsilon^{\alpha \mu \gamma \delta} K^\gamma \hat{Q}^\delta - G_E^* T_Q^{\alpha \gamma} T_K^{\gamma \mu} - \frac{i T}{\omega} G_C^* \hat{Q}^{\alpha} K^\mu \right],
\]

\[
P_T^{\mu} = T_Q^{\mu \nu} P^\nu = P^\mu - (P \cdot \hat{Q}) \hat{Q}^\mu,
\]

\[
T_P^{\mu \nu} = \delta^{\mu \nu} - \hat{P}^\mu \hat{P}^{\nu}
\]

\[
\gamma_T^{\mu} = T_P^{\mu \nu} \gamma^{\nu}
\]

\[
\tau := \frac{Q^2}{2(M_N^2 + M_N^2)}, \quad \lambda_+ := \frac{(M_\Delta \pm M_N)^2 + Q^2}{2(M_N^2 + M_N^2)}
\]

\[
\omega := \sqrt{\lambda_+ \lambda_-} \quad \text{and} \quad b := \sqrt{\frac{3}{2}} (1 + M_\Delta/M_N).
\]
In general, the electromagnetic current is:

\[ J_{\mu,x}(P_f, P_i) = \Lambda^P_{+,x}(P_f) \left( i e \Gamma_{\mu,x}(P_f, P_i) \right) \Lambda^P_{+,x}(P_i) \]

\[ \Lambda^\pm_{+,x}(P) = G^\pm \Lambda_{+,x}(P) G^\pm \]
In general, the electromagnetic current is:

\[
\mathcal{J}_{\mu,x}(P_f, P_i) = \Lambda^\mathcal{P}_{+x}(P_f) \left( i e \Gamma_{\mu,x}(P_f, P_i) \right) \Lambda^\mathcal{P}_i(P_i)
\]

\[
\Lambda^\pm_{+x}(P) = G^\pm \Lambda_{+x}(P) G^\pm
\]

In the quark-diquark model, the electromagnetic current is described considering the interaction diagrams of the photon with the diquarks inside baryon.

\[
\mathcal{J}_{\mu,x}(P_f, P_i) = \sum_{I=\text{Diagrams}} \int \Lambda^\mathcal{P}_{+x}(P_f) \left( \Gamma^I_{\mu,x}(l; P_f, P_i) \right) \Lambda^\mathcal{P}_i(P_i)
\]

\[
= \Lambda^\mathcal{P}_{+x}(P_f) \left[ \sum_d \Pi^d(l; P_f, P_i) + \sum_{d_1, d_2} \Pi^{(d_1, d_2)}(l; P_f, P_i) \right] \Lambda^\mathcal{P}_i(P_i)
\]

Where \( \Pi^d \) represents the diagrams where the photon hits the quark and \( \Pi^{(d_1, d_2)} \) represents the diagrams where the photon hits the diquark.
Each diagram has the following form:

\[ \psi_A \rightarrow \psi_N \]

\[ \int \frac{d^4k}{(2\pi)^4} D_{\alpha}(P_f)S_q(P_f-k)\Gamma^\mu(P_f-k,P_l-k)S_A(P_l-k)a_\mu(-P_l)\delta_{\alpha\beta}(-k) \]

\[ \int \frac{d^4k}{(2\pi)^4} S_q(k)D_{\alpha}(P_f-k)\Gamma^\mu(P_f-k,P_l-k)S_A(P_l-k)\gamma_5(-P_l)\delta_{\alpha\beta}(P_l-k) \]

Diagrams courtesy of Luis Albino
Each diagram has the following form:

\[
\int \frac{d^4k}{(2\pi)^4} D_{\gamma'\gamma}(P_f) S_q'(P_f - k) \Gamma^\mu(P_f - k, P_i - k) S_q(P_i - k) \delta^{\mu
u}(-P_i) \delta_{\alpha\beta}(-k)
\]

In general we need to consider whether the photon hits the quark and the diquarks are spectators (4 contributions), and whether the photon hits the diquarks (16 contributions):

Diagrams courtesy of Luis Albino

[Raya:2021pyr]
• Dirac and Pauli transition form factors.
• It agrees quantitatively in magnitude and qualitatively in trend with data above \( x \geq 2 \).
• The data discrepancy in the \( x \leq 2 \) domain is attributed to the contribution of the meson cloud.
• The dashed-green band is inferred form of the meson cloud contribution from the data fit.

• Similar results for $\gamma^* p \rightarrow N(1535)_{1/2}^-$ and $\gamma^* p \rightarrow N(1520)_{3/2}^-$ are not yet available.
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• An insightful starting point can be provided by the contact interaction.
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• An insightful starting point can be provided by the contact interaction.

• A contact interaction treatment of $\gamma^* p \rightarrow N(1535)\frac{1}{2}^-$ transition amplitudes and form factors provides results providing us insight into its relative diquark content.
• $G_M^*$ magnetic dipole, $G_E^*$ electric quadrupole, $G_C^*$ Coulomb quadrupole.

• In collaboration with L. Albino, K. Raya and J. Segovia.
A description of the nucleon transition form factors to $N(940)_{1/2}^+, \Delta(1232)_{3/2}^+, N(1440)_{1/2}^+, N(1535)_{1/2}^-, \Delta(1600)_{3/2}^+$ in CI and QCD kindred models is already available in the literature.
• A description of the nucleon transition form factors to $N(940)^{1/2+}$, $\Delta(1232)^{3/2+}$, $N(1440)^{1/2+}$, $N(1535)^{1/2-}$, $\Delta(1600)^{3/2+}$ in CI and QCD kindred models is already available in the literature.

• A CI treatment of transition amplitudes and form factors provides first results emphasizing its significant dependence on its structure and relative diquark content.
A description of the nucleon transition form factors to $N(940)_{1/2}^+, \Delta(1232)_{3/2}^+, N(1440)_{1/2}^+, N(1535)_{1/2}^-, \Delta(1600)_{3/2}^+$ in CI and QCD kindred models is already available in the literature.

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A similar analysis for $N(1520)$ and other transitions is required. We have made a start with CI.
Summary and Scope

- A description of the nucleon transition form factors to $N(940)_{1/2}^+$, $\Delta(1232)_{3/2}^+$, $N(1440)_{1/2}^+$, $N(1535)_{1/2}^-$, $\Delta(1600)_{3/2}^+$ in CI and QCD kindred models is already available in the literature.

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- While Contact Interaction analyses have their limitations, they also has the advantage of algebraic simplicity and a demonstrated ability to reveal insights.
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Thank you.
where \( G^{+(-)} = \mathbb{I}_D(i\gamma_5) \) and, with \( T_{\mu\nu} = \delta_{\mu\nu} + \dot{Q}_\mu \dot{Q}_\nu \)
\( \gamma^\perp_\mu = T_{\mu\nu} \gamma_\nu \), \( k^\perp_\mu = T_{\mu\nu} k_\nu \), \( \hat{k}^\perp_\mu \hat{k}^\perp_\mu = 1 \),

\[
\begin{align*}
X^1_\rho(k; Q) &= i\sqrt{3} \hat{k}^\perp_\rho \gamma_5 , \\
X^2_\rho(k; Q) &= i\gamma \cdot \hat{k}^\perp X^1_\rho(k; Q), \\
\mathcal{Y}^1_{\nu\rho}(k; Q) &= \delta_{\nu\rho} \mathbb{I}_D, \\
\mathcal{Y}^2_{\nu\rho}(k; Q) &= \frac{i}{\sqrt{5}} [2\gamma^\perp_\nu \hat{k}^\perp_\rho - 3\delta_{\nu\rho} \gamma \cdot \hat{k}^\perp], \\
\mathcal{Y}^3_{\nu\rho}(k; Q) &= -i\gamma^\perp_\nu \hat{k}^\perp_\rho , \\
\mathcal{Y}^4_{\nu\rho}(k; Q) &= \sqrt{3} \dot{Q}_\nu \hat{k}^\perp_\rho , \\
\mathcal{Y}^5_{\nu\rho}(k; Q) &= 3\hat{k}^\perp_\nu \hat{k}^\perp_\rho - \delta_{\nu\rho} - \gamma^\perp_\nu \hat{k}^\perp_\rho \gamma \cdot \hat{k}^\perp , \\
\mathcal{Y}^6_{\nu\rho}(k; Q) &= \gamma^\perp_\nu \hat{k}^\perp_\rho \gamma \cdot \hat{k}^\perp , \\
\mathcal{Y}^7_{\nu\rho}(k; Q) &= -i\gamma \cdot \hat{k}^\perp \mathcal{Y}^4_{\nu\rho}(k; Q), \\
\mathcal{Y}^8_{\nu\rho}(k; Q) &= \frac{i}{\sqrt{5}} [\delta_{\nu\rho} \gamma \cdot \hat{k}^\perp \\
&\quad + \gamma^\perp_\nu \hat{k}^\perp_\rho - 5\hat{k}^\perp_\nu \hat{k}^\perp_\rho \gamma \cdot \hat{k}^\perp].
\end{align*}
\]
• General Faddeev amplitudes for Baryons

where \( G^{+(-)} = \mathbb{I}_D(i\gamma_5) \) and, with \( T_{\mu\nu} = \delta_{\mu\nu} + \bar{Q}_\mu \bar{Q}_\nu \)

\[
\gamma_{\mu}^+ = T_{\mu\nu} \gamma_\nu, \quad k_{\mu}^+ = T_{\mu\nu} k_\nu, \quad \gamma_{\mu}^- k_{\mu}^- = 1,
\]

\[
\begin{align*}
\chi_{\rho}^1(k; Q) &= i\sqrt{3} \hat{k}_{\rho}^+ \gamma_5, \\
\chi_{\rho}^2(k; Q) &= i\gamma \cdot \hat{k}^\perp \chi_{\rho}^1(k; Q), \\
\gamma_{\nu\rho}^1(k; Q) &= \delta_{\nu\rho} \mathbb{I}_D, \\
\gamma_{\nu\rho}^2(k; Q) &= \frac{i}{\sqrt{5}} \left[ 2 \gamma_{\nu}^\perp \hat{k}_{\rho}^\perp - 3 \delta_{\nu\rho} \gamma \cdot \hat{k}^\perp \right], \\
\gamma_{\nu\rho}^3(k; Q) &= -i \gamma_{\nu}^\perp \hat{k}_{\rho}^\perp, \\
\gamma_{\nu\rho}^4(k; Q) &= \sqrt{3} \bar{Q}_\nu \hat{k}_{\rho}^+ , \\
\gamma_{\nu\rho}^5(k; Q) &= 3 \hat{k}_{\nu}^\perp \hat{k}_{\rho}^+ - \delta_{\nu\rho} - \gamma_{\nu}^\perp \hat{k}_{\rho}^+ \gamma \cdot \hat{k}^\perp, \\
\gamma_{\nu\rho}^6(k; Q) &= \gamma_{\nu}^\perp \hat{k}_{\rho}^+ \gamma \cdot \hat{k}^\perp, \\
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&\quad + \gamma_{\nu}^\perp \hat{k}_{\rho}^+ - 5 \hat{k}_{\nu}^\perp \hat{k}_{\rho}^+ \gamma \cdot \hat{k}^\perp \right].
\end{align*}
\]

\[\Gamma = \quad + \quad [ \quad + \quad + \quad + \quad + \quad + \ldots ]\]
With the general structure:

\[ \Pi'(l; P_f, P_i) = q_r \int_1^\infty \bar{\psi}^{(r)}(l_f^+ \Gamma_{\mu}(Q) S(l_i^+) \psi^{(r)} \Delta(-l) \]

\[ l_{f,i} = \pm l + P_{f,i} \]
With the general structure:

\[ \Pi'(l, P_f, P_i) = q_r \int \bar{\psi}^{(r)} S(l_f^+) \Gamma_{\mu}^{q}(Q) S(l_i^+) \psi^{(r)} \Delta'(-l) \]

Where the quark photon vertex is:

\[ \Gamma_{\mu}^{qq}(Q) = \xi \frac{r_0}{Q^2} Q_{\mu} + P_T(Q^2) \left( \gamma_{\mu} - \frac{r_0}{Q^2} Q_{\mu} \right) + \eta \sigma_{\mu\nu} Q_{\nu} \]

\[ l^+_{f,i} = \pm l + P_{f,i} \]
With the general structure:

\[ \Pi'(l; P_f, P_i) = q_r \int l \bar{\psi}^{(r)} S(l_f^+) \Gamma_{\mu}^{\nu}(Q) S(l_i^-) \psi^{(r)} \Delta'(-l) \]

Where the quark photon vertex is:

\[ \Gamma_{\mu}^{\nu}(Q) = \xi \frac{\nu \cdot Q}{Q^2} Q_{\mu} + P_T(Q^2) \left( \gamma_{\mu} - \frac{\nu \cdot Q}{Q^2} Q_{\mu} \right) + \eta \sigma_{\nu\nu} Q_{\nu} \]

Where \( \eta \) is the contribution of the anomalous magnetic moment of the quark (AMM), the origin of this contribution is related to DCSB.

\[ \eta = \eta_0 \frac{-Q^2}{4 M_q^2} \frac{e}{2 M_q} \]
With the general structure:

\[ \Pi'(l; P_f, P_i) = q_r \int_l^\infty \bar{\psi}(r) S(l_f^+) \Gamma_{\mu}^{\gamma\nu}(Q) S(l_i) \psi(r) \Delta'(-l) \]

Where the quark photon vertex is:

\[ \Gamma_{\mu}^{\gamma\nu}(Q) = 3 \frac{r Q_\mu}{Q^2} Q_\nu + P_T(Q^2) \left( \gamma_\mu - \frac{r Q_\mu}{Q^2} Q_\nu \right) + \eta \sigma_{\mu\nu} Q_\nu \]

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\[ \eta = \eta_0 \frac{-Q^2}{4 M_q^2} \frac{e}{2 M_q} \]

Where \( P_T \) is a dressing function of the inhomogeneous BSE and its behavior recovers the tree-level vertex.
• With the general structure:

\[
\Pi^{(d_1,d_2)}(l; P_f, P_i) = q_{d_2\bar{d}_1} \int_l \psi^{f(d_2)} S(l) \psi^{\bar{i}(d_1)} \Delta^r(-l) \Gamma^{(d_1,d_2)}_{\mu,x}(l_f^-, l_i^-) \Delta^{\bar{d}_1}(l_i^-)
\]

\[l_{f,i}^\pm = \pm l + P_{f,i}\]

Backups  Photon hits the diquark

With the general structure:

$$\Pi^{(d_1,d_2)}(I; P_f, P_i) = q_{d_2}d_1 \int l \psi^{f^{(d_2)}} S(l) \bar{\psi}^{j(d_1)} \Delta^{r(-l)}(l) \Gamma^{(d_1,d_2)\gamma\mu}(l^-) \Delta^{d_1}(l^-)$$

$$l_{f,i}^\pm = \pm l + P_{f,i}$$

The diquark photon vertex depends on the type of the interaction. Elastic:

$$\Gamma^{(qq,1+)}(k_f = K + Q/2, k_i = K - Q/2) = \sum_{j=1}^{3} T_{\mu,\rho\sigma}^j(K, Q) F_j^{(q_1,1+)}(Q^2),$$

$$T_{\mu,\rho\sigma}^1(K, Q) = 2K_\mu \mathcal{P}_{\rho\sigma}^T(k^\nu) \mathcal{P}_{\alpha\sigma}^T(k^\nu),$$

$$T_{\mu,\rho\sigma}^2(K, Q) = \left[ Q_\rho - k_\rho^2 \frac{Q^2}{2m_{(qq,1+)}^2} \right] \mathcal{P}_{\mu\alpha}^T(k^\nu) - \left[ Q_\sigma + k_\sigma^2 \frac{Q^2}{2m_{(qq,1+)}^2} \right] \mathcal{P}_{\mu\rho}^T(k^\nu),$$

$$T_{\mu,\rho\sigma}^3(K, Q) = \frac{K_\mu}{m_{(qq,1+)}^2} \left[ Q_\rho - k_\rho^2 \frac{Q^2}{2m_{(qq,1+)}^2} \right] \left[ Q_\sigma + k_\sigma^2 \frac{Q^2}{2m_{(qq,1+)}^2} \right],$$

$$\mathcal{P}_{\rho\sigma}^T(p) = \delta_{\rho\sigma} - p_\rho p_\sigma/p^2.$$
• With the general structure:

\[ \Pi^{(d_1,d_2)}(l; P_f, P_i) = q_{d_2 \bar d_1} \int_S \psi_f^{(d_2)} S(l) \bar \psi_i^{(d_1)} \Delta^r(-l) \Gamma^{(d_1,d_2)}_{\mu,x} \gamma(l^-_f, l^-_i) \Delta^{d_1}(l^-_i) \]

\[ l^\pm_{f,i} = \pm l + P_{f,i} \]

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\[ \Gamma^{(qq,1+)}_{\rho\sigma}(k_f = K + Q/2, k_i = K - Q/2) = \sum_{j=1}^{3} T^j_{\mu,\rho\sigma}(K, Q) F_j^{(qq,1+)}(Q^2), \]

\[ T^1_{\mu,\rho\sigma}(K, Q) = 2K_\mu \mathcal{P}^T_{\rho\sigma}(k^i) \mathcal{P}^T_{\alpha\sigma}(k^f), \]

\[ T^2_{\mu,\rho\sigma}(K, Q) = \left[ Q_\rho - k^i_\rho \frac{Q^2}{2m^2_{(qq,1+)}} \right] \mathcal{P}^T_{\mu\sigma}(k^f) - \left[ Q_\sigma + k^i_\sigma \frac{Q^2}{2m^2_{(qq,1+)}} \right] \mathcal{P}^T_{\mu\rho}(k^i), \]

\[ T^3_{\mu,\rho\sigma}(K, Q) = \frac{K_\mu}{m^2_{(qq,1+)}} \left[ Q_\rho - k^i_\rho \frac{Q^2}{2m^2_{(qq,1+)}} \right] \left[ Q_\sigma + k^i_\sigma \frac{Q^2}{2m^2_{(qq,1+)}} \right], \]

\[ P^T_{\rho\sigma}(p) = \delta_{\rho\sigma} - p_{\rho} p_{\sigma}/p^2. \]

• Transition:

\[ \Gamma^{10}_{\rho\mu}(k_2, k_1) = \Gamma^{01}_{\rho\mu}(-k_2, k_1) = \Gamma^{01}_{\mu\rho}(k_1, k_2), \]

\[ \Gamma^{01}_{\mu\rho}(k_1, k_2) = \frac{g^{01}}{m_{(qq,1+)}} \epsilon_{\mu\rho\sigma\lambda} k_1\lambda k_2\beta G^{01}(Q^2). \]

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