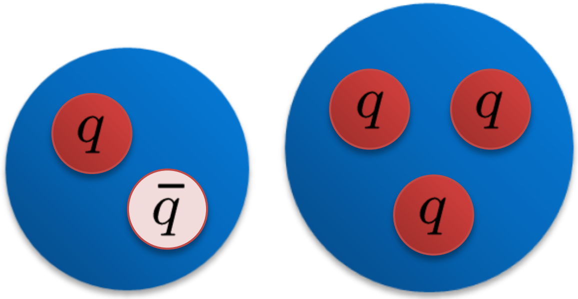
Hadron scattering, resonances and exotics from lattice QCD

Christopher Thomas, University of Cambridge

c.e.thomas@damtp.cam.ac.uk

JLab Theory Seminar (virtual),
12 December 2022

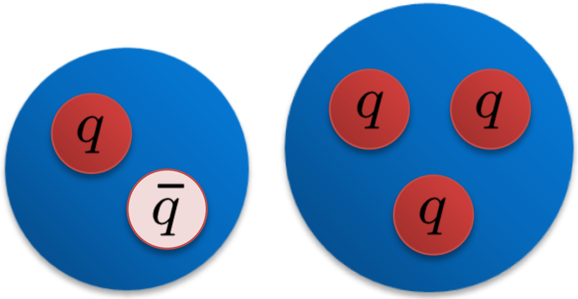
Hadron spectroscopy



Hadron spectroscopy

Intriguing observations, e.g. $X(3872)$, $Y(4260)$, $Z_c^+(4430)$, $Z_c^+(3900)$, Z_b^+, X(6900), X_{cc}, $D_{s0}(2317)$, charm-strange X(2900), light scalars, $\pi_1(1600)$ [$J^{PC} = 1^{-+}$], P_c , Roper, other baryon resonances

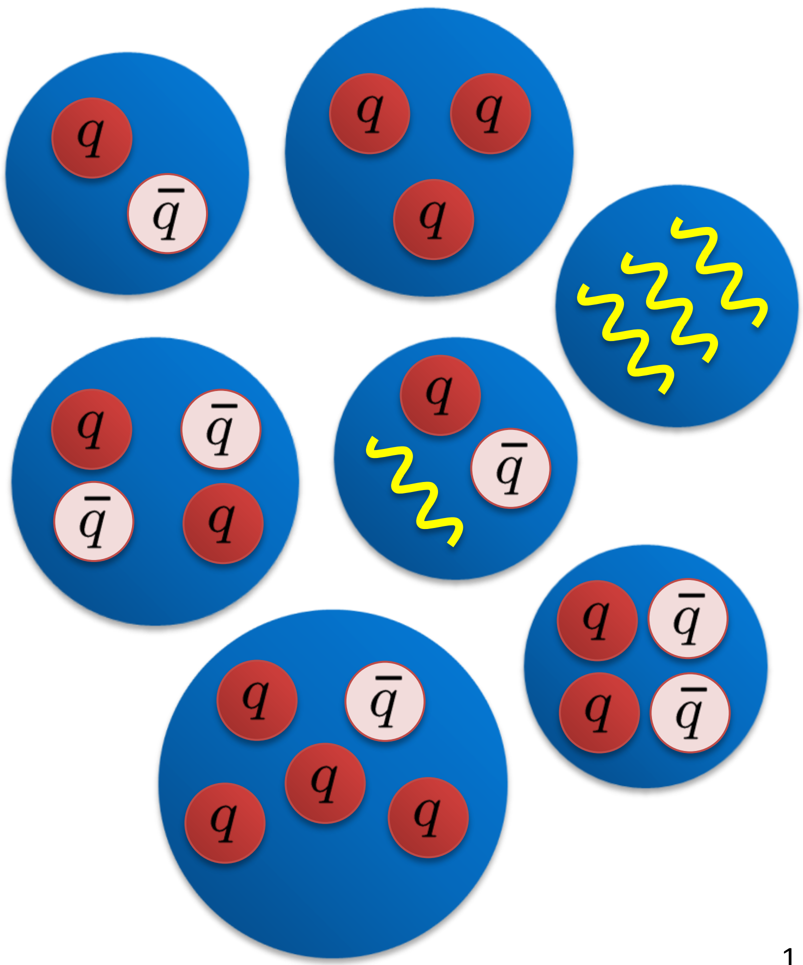
KLOE CLAS12



Hadron spectroscopy

Intriguing observations, e.g. $X(3872)$, $Y(4260)$, $Z_c^+(4430)$, $Z_c^+(3900)$, Z_b^+, X(6900), X_{cc}, $D_{s0}(2317)$, charm-strange X(2900), light scalars, $\pi_1(1600)$ [$J^{PC} = 1^{-+}$], P_c , Roper, other baryon resonances

KLOE CLAS12

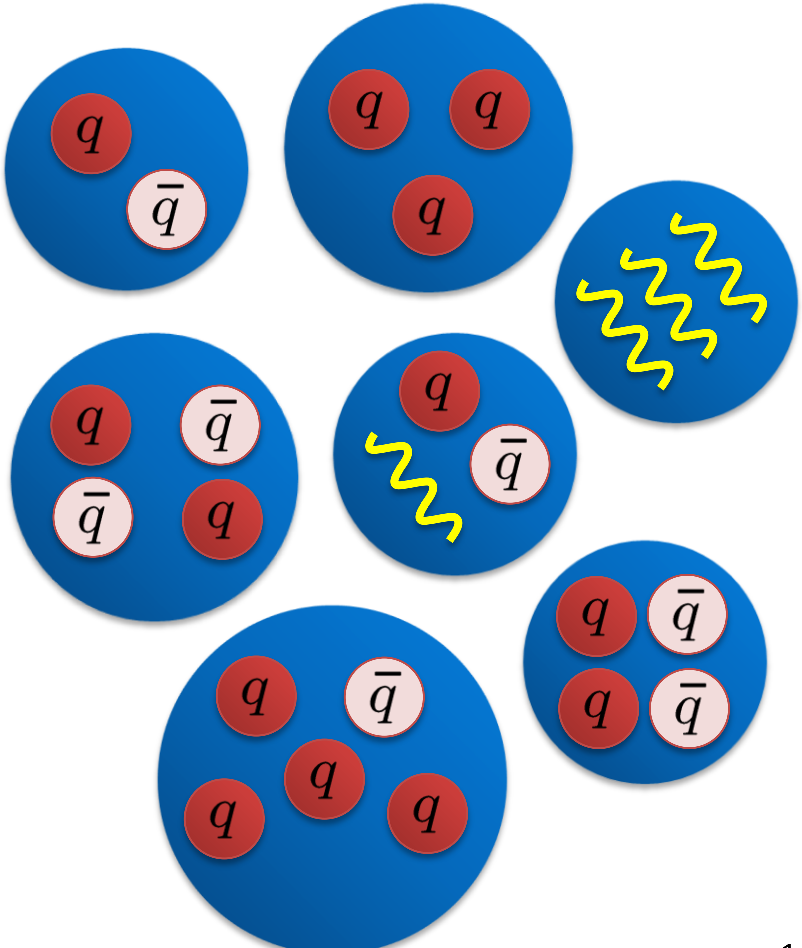


Hadron spectroscopy

Intriguing observations, e.g. $X(3872)$, $Y(4260)$, $Z_c^+(4430)$, $Z_c^+(3900)$, Z_b^+, X(6900), X_{cc}, $D_{s0}(2317)$, charm-strange X(2900), light scalars, $\pi_1(1600)$ [$J^{PC} = 1^{-+}$], P_c , Roper, other baryon resonances

Belle II BESIII ATLAS panda LHCb CMS

COMPASS KLOE CLAS12 GlueX



Exotic quantum numbers are particularly interesting, e.g. flavour or $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}$

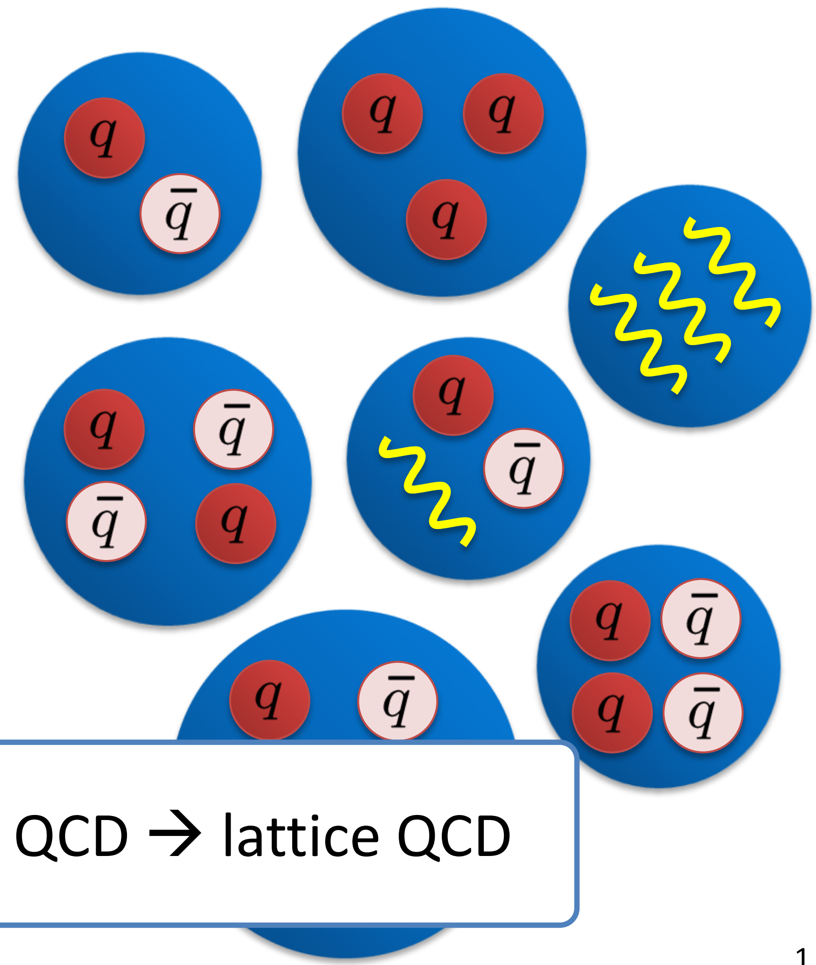
Hadron spectroscopy

Intriguing observations, e.g. $X(3872)$, $Y(4260)$, $Z_c^+(4430)$, $Z_c^+(3900)$, Z_b^+, X(6900), X_{cc}, $D_{s0}(2317)$, charm-strange X(2900), light scalars, $\pi_1(1600)$ [$J^{PC} = 1^{-+}$], P_c , Roper, other baryon resonances

Belle II BESIII ATLAS

panda LHCb CMS

COMPASS KLOE CLAS12 GlueX



Exotic

particu

e.g. fla

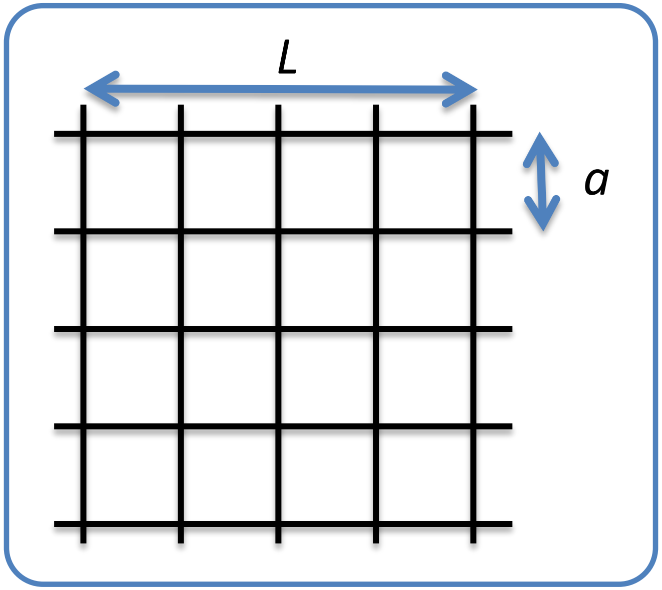
First-principles calculations in QCD → lattice QCD

Outline

- Introduction
- Charm mesons
  - $D\pi$ / DK ($J^P = 0^+$ $D_0^*(2300)$, $D_{s0}^*(2317)$)
  - $D^*\pi$ ($J^P = 1^+$ and 2^+)

Lattice QCD

Systematically-improvable
first-principles calculations

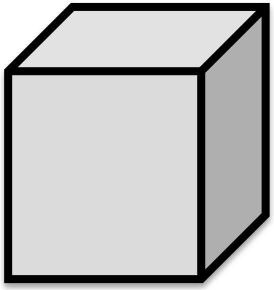


- **Discretise** spacetime in a **finite volume**
- Compute correlation fns. numerically (Euclidean time, $t \rightarrow i\,t$)

Note:

- Finite a and L
- Possibly heavy u, d quarks ($\rightarrow$ unphysical m_π)

Lattice QCD spectroscopy



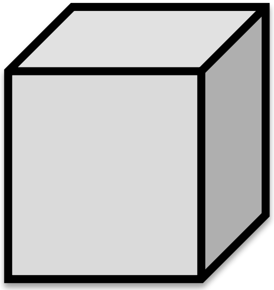
Finite-volume energy eigenstates from:

$$C_{ij}(t) = \left\langle 0 \left| \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) \right| 0 \right\rangle$$

$$= \sum_n \frac{e^{-E_n t}}{2\ E_n} \langle 0 | \mathcal{O}_i(0) | n \rangle \langle n | \mathcal{O}_j^\dagger(0) | 0 \rangle$$

Lower-lying hadrons in each flavour sector are well determined (also isospin breaking, QED).

Lattice QCD spectroscopy



Finite-volume energy eigenstates from:

$$C_{ij}(t) = \left\langle 0 \left| \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) \right| 0 \right\rangle$$

$$= \sum_n \frac{e^{-E_n t}}{2\ E_n} \langle 0|\mathcal{O}_i(0)|n\rangle\langle n|\mathcal{O}_j^\dagger(0)|0\rangle$$

Excited states: in each symmetry channel compute matrix of correlators for **large bases of interpolating operators** with appropriate variety of structures.

Variational method (generalised eigenvalue problem) → $\{E_n\}$

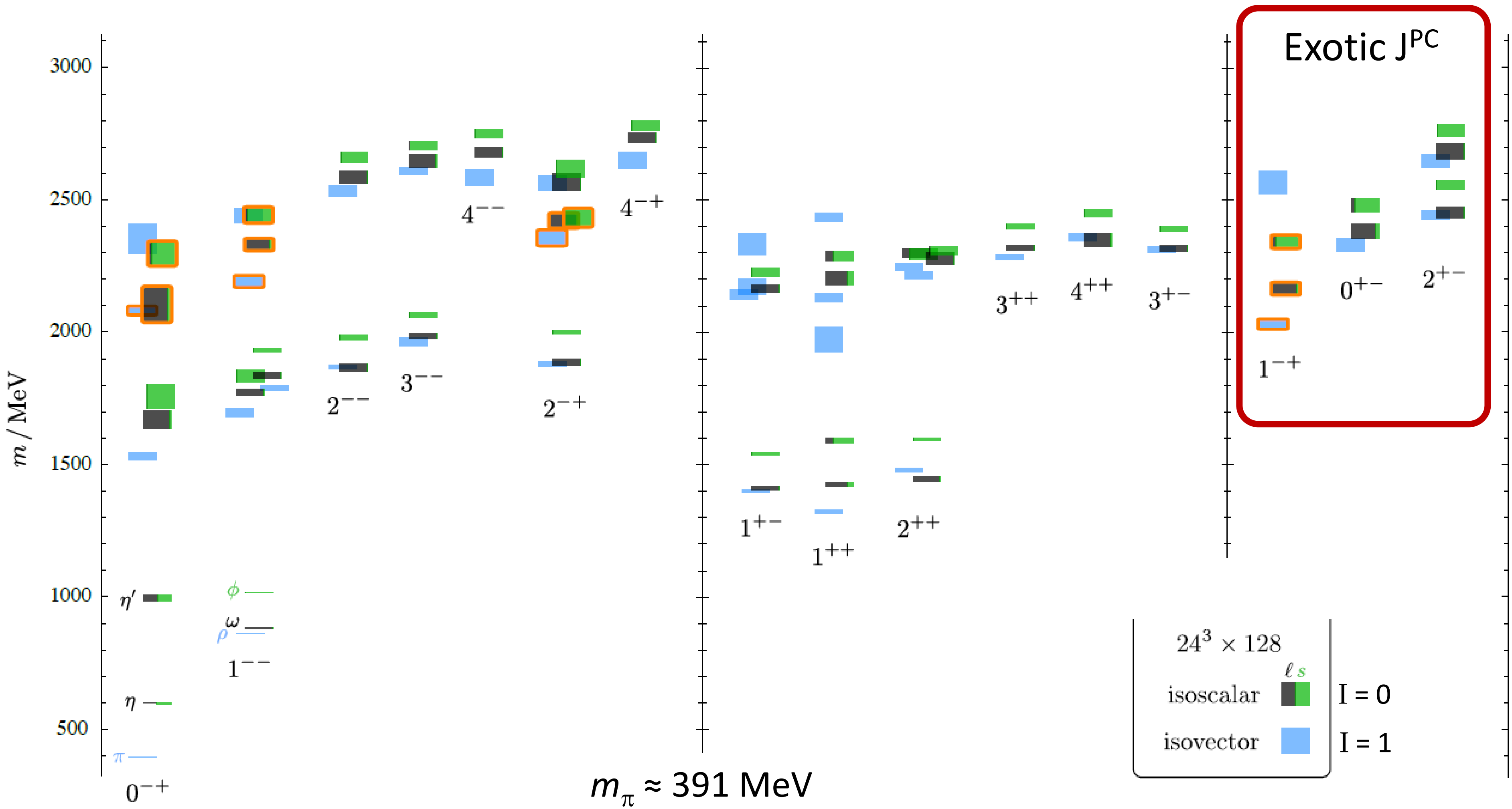
$$C_{ij}(t) v_j^{(n)} = \lambda^{(n)}(t) C_{ij}(t_0) v_j^{(n)}$$

$$\lambda^{(n)}(t) \sim e^{-E_n(t-t_0)} \qquad v_i^{(n)} \to Z_i^{(n)} \equiv \langle 0|\mathcal{O}_i|n\rangle$$

($t >> t_0$)

Light mesons (isospin = 0 and 1)

[Dudek, Edwards, Guo, CT, PR D88, 094505 (2013)]

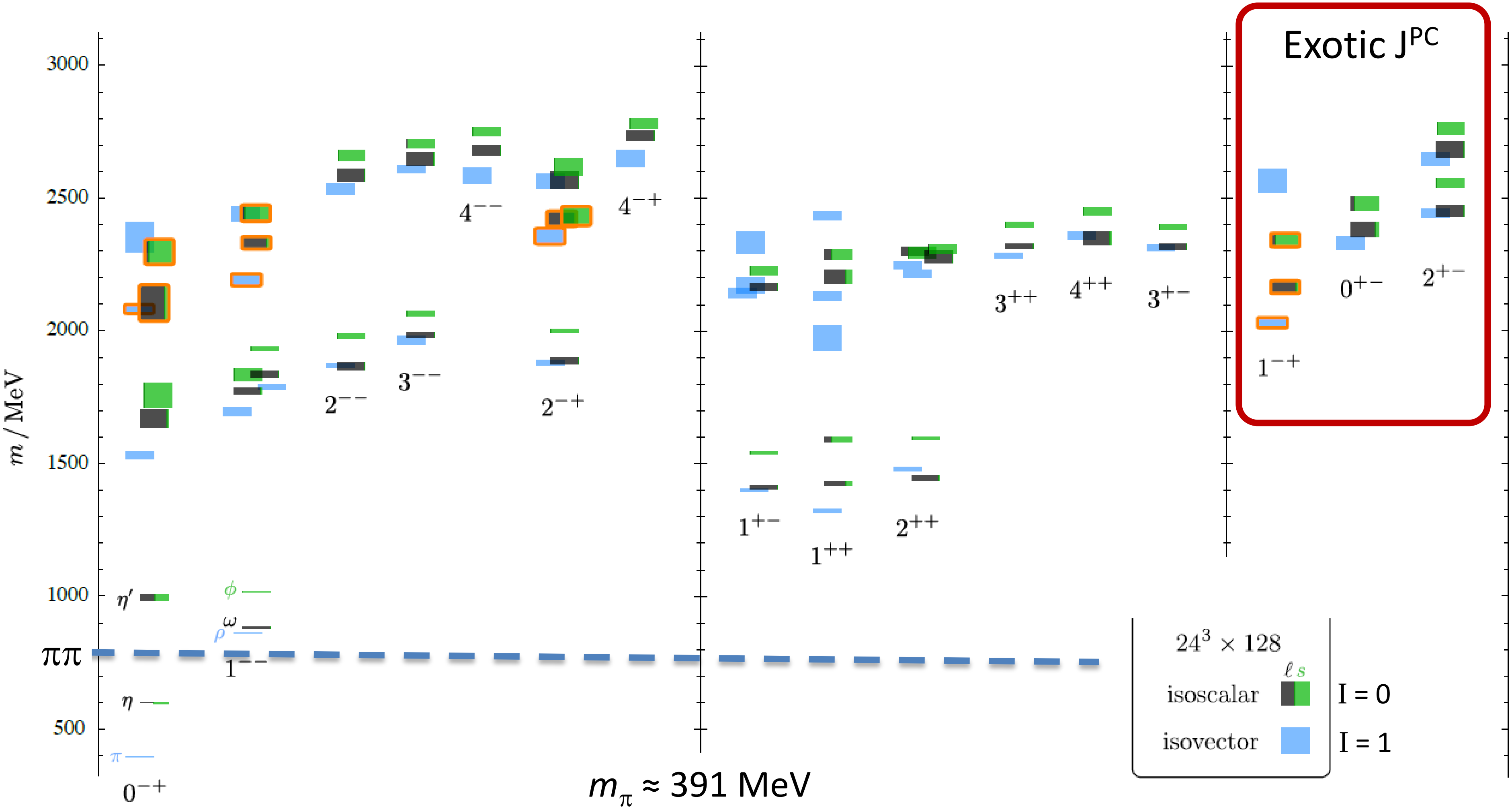


Large bases of only fermion-bilinear ops $\sim \bar{\psi}\Gamma D \ldots \psi$

(also other m_π and volumes)

Light mesons (isospin = 0 and 1)

[Dudek, Edwards, Guo, CT, PR D88, 094505 (2013)]

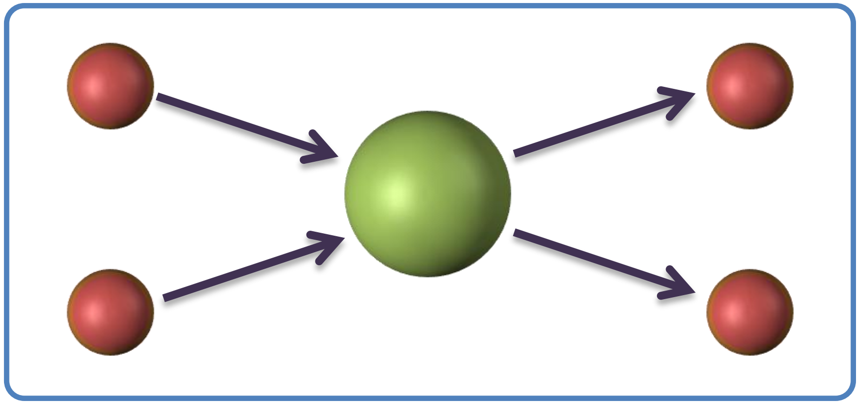


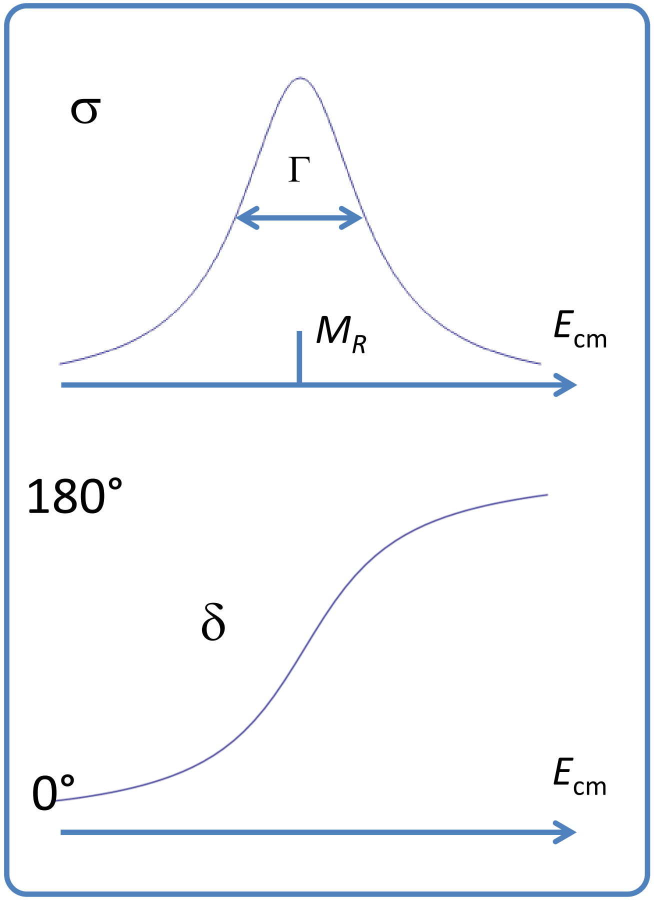
Large bases of only fermion-bilinear ops $\sim \bar{\psi}\Gamma D \ldots \psi$

(also other m_π and volumes)

Scattering and resonances

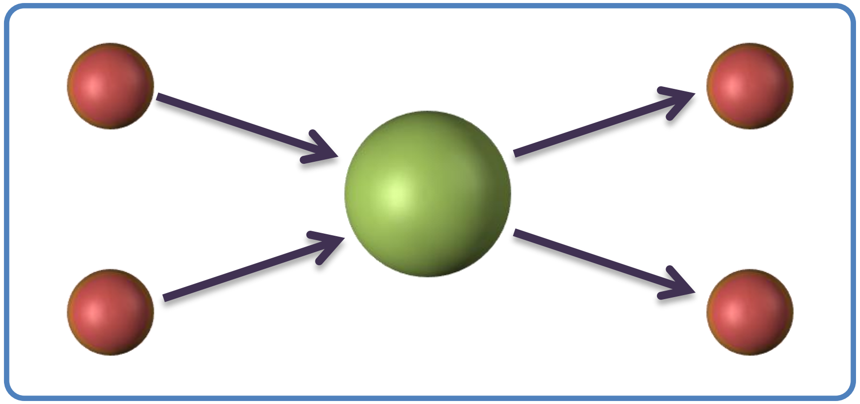
Most hadrons appear as resonances in scattering of lighter hadrons



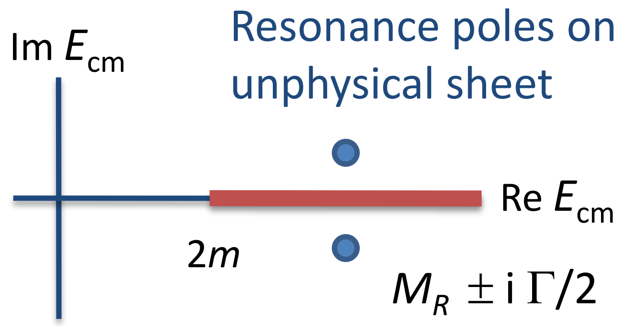


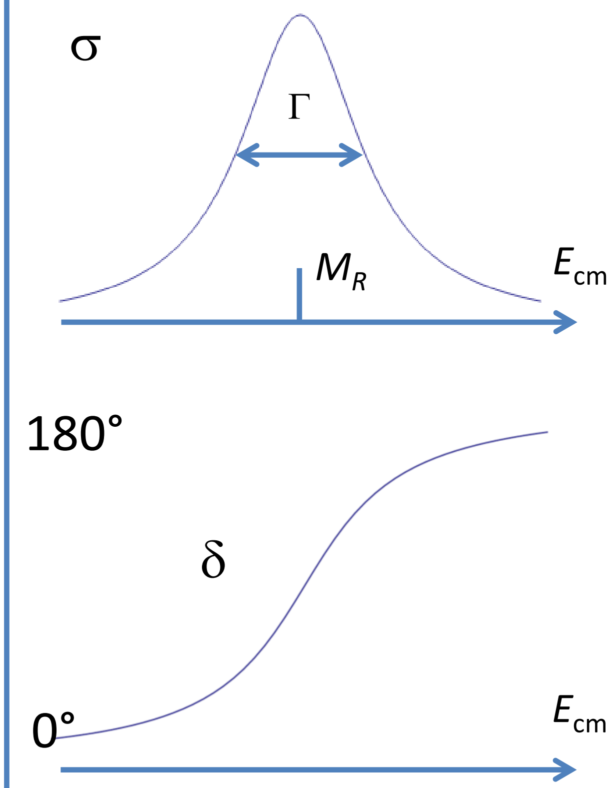
Scattering and resonances

Most hadrons appear as resonances in scattering of lighter hadrons



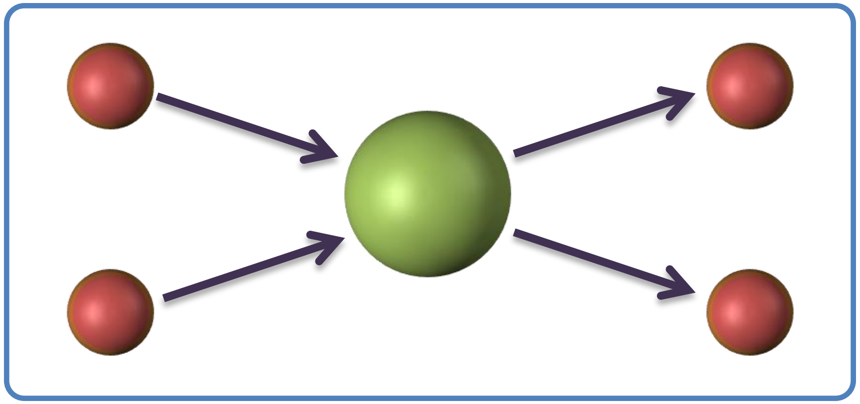
Singularity structure of scattering matrix (poles → state content)



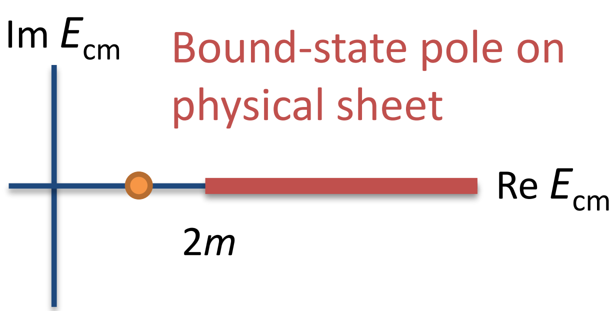


Scattering and resonances

Most hadrons appear as resonances in scattering of lighter hadrons



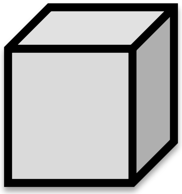
Singularity structure
of scattering matrix



Scattering and resonances in lattice QCD

Can't directly compute scattering amplitudes in lattice QCD

Lüscher method [NP B354, 531 (1991)] and extensions: relate discrete set of **finite-volume energy levels** $\{E_{\text{cm}}\}$ to **infinite-volume scattering *t*-matrix**.

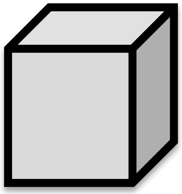


$$\vec{p} = \frac{2\pi}{L}(n_x, n_y, n_z)$$

Scattering and resonances in lattice QCD

Can't directly compute scattering amplitudes in lattice QCD

Lüscher method [NP B354, 531 (1991)] and extensions: relate discrete set of **finite-volume energy levels** $\{E_{\mathrm{cm}}\}$ to **infinite-volume scattering *t*-matrix**.



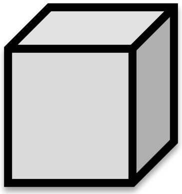
$$\vec{p} = \frac{2\pi}{L}(n_x, n_y, n_z)$$

c.f. 1-dim: $k = \frac{2\pi}{L}n + \frac{2}{L}\delta(k)$

Scattering and resonances in lattice QCD

Can't directly compute scattering amplitudes in lattice QCD

Lüscher method [NP B354, 531 (1991)] and extensions: relate discrete set of **finite-volume energy levels** $\{E_{\text{cm}}\}$ to **infinite-volume scattering *t*-matrix**.



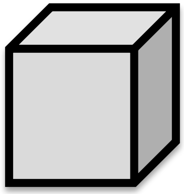
$$\vec{p} = \frac{2\pi}{L}(n_x, n_y, n_z)$$

$$\det\left[1 + i\,\boldsymbol{\rho}(E_{\text{cm}})\,\boldsymbol{t}(E_{\text{cm}})\,\left(1 + i\,\boldsymbol{\mathcal{M}}^{\vec{P}}(E_{\text{cm}}, L)\right)\right] = 0$$

Scattering and resonances in lattice QCD

Can't directly compute scattering amplitudes in lattice QCD

Lüscher method [NP B354, 531 (1991)] and extensions: relate discrete set of **finite-volume energy levels** $\{E_{\rm cm}\}$ to **infinite-volume scattering *t*-matrix**.



$$\vec{p} = \frac{2\pi}{L}(n_x, n_y, n_z)$$

$$\det\left[1 + i\,\boldsymbol{\rho}(E_{\rm cm})\,\boldsymbol{t}(E_{\rm cm})\,\left(1 + i\,\boldsymbol{\mathcal{M}}^{\vec{P}}(E_{\rm cm}, L)\right)\right] = 0$$

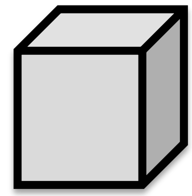
Elastic scattering: one-to-one mapping $E_{\rm cm} \leftrightarrow t(E_{\rm cm})$

[Complication: reduced sym. of lattice vol. → mixing of partial waves]

Scattering and resonances in lattice QCD

Can't directly compute scattering amplitudes in lattice QCD

Lüscher method [NP B354, 531 (1991)] and extensions: relate discrete set of **finite-volume energy levels** $\{E_{\rm cm}\}$ to **infinite-volume scattering *t*-matrix**.



$$\vec{p} = \frac{2\pi}{L}(n_x, n_y, n_z)$$

$$\det\left[1 + i\,\rho(E_{\rm cm})\,t(E_{\rm cm})\,\left(1 + i\,\mathcal{M}^{\vec{P}}(E_{\rm cm}, L)\right)\right] = 0$$

Elastic scattering: one-to-one mapping $E_{\rm cm} \leftrightarrow t(E_{\rm cm})$

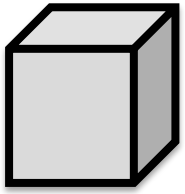
Coupled channels: under-constrained problem
(each $E_{\rm cm}$ constrains *t*-matrix at that $E_{\rm cm}$)
Param. $t(E_{\rm cm})$ using various forms (*K*-matrix forms, …)
[see e.g. review Briceño, Dudek, Young, Rev. Mod. Phys. 90, 025001 (2018)]

[Complication: reduced sym. of lattice vol. → mixing of partial waves]

Scattering and resonances in lattice QCD

Can't directly compute scattering amplitudes in lattice QCD

Lüscher method [NP B354, 531 (1991)] and extensions: relate discrete set of **finite-volume energy levels** $\{E_{\mathrm{cm}}\}$ to **infinite-volume scattering *t*-matrix**.



$$\vec{p} = \frac{2\pi}{L}(n_x, n_y, n_z)$$

$$\det\left[1 + i\,\rho(E_{\mathrm{cm}})\,\boldsymbol{t}(E_{\mathrm{cm}})\,\left(1 + i\,\boldsymbol{\mathcal{M}}^{\vec{P}}(E_{\mathrm{cm}}, L)\right)\right] = 0$$

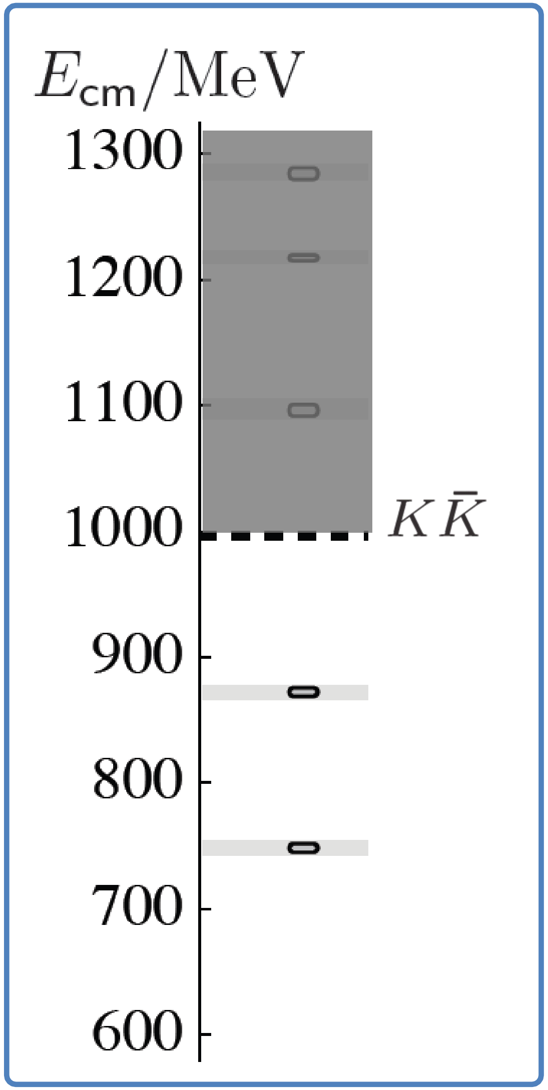
Elastic scattering: one-to-one mapping $E_{\mathrm{cm}} \leftrightarrow t(E_{\mathrm{cm}})$

Coupled channels: under-constrained problem
(each E_{cm} constrains *t*-matrix at that E_{cm})
Param. $t(E_{\mathrm{cm}})$ using various forms (*K*-matrix forms, …)

Analytically continue $t(E_{\mathrm{cm}})$ in complex E_{cm} plane, look for poles.

Demonstrated in calcs. of ρ, light scalars, b_1, charm mesons, …

The ρ resonance: **elastic** P-wave $\pi\pi$ scattering



$m_\pi \approx 236$ MeV

Experimentally

$\mathrm{BR}(\rho \to \pi\pi) \sim 100\%$

Use many different operators

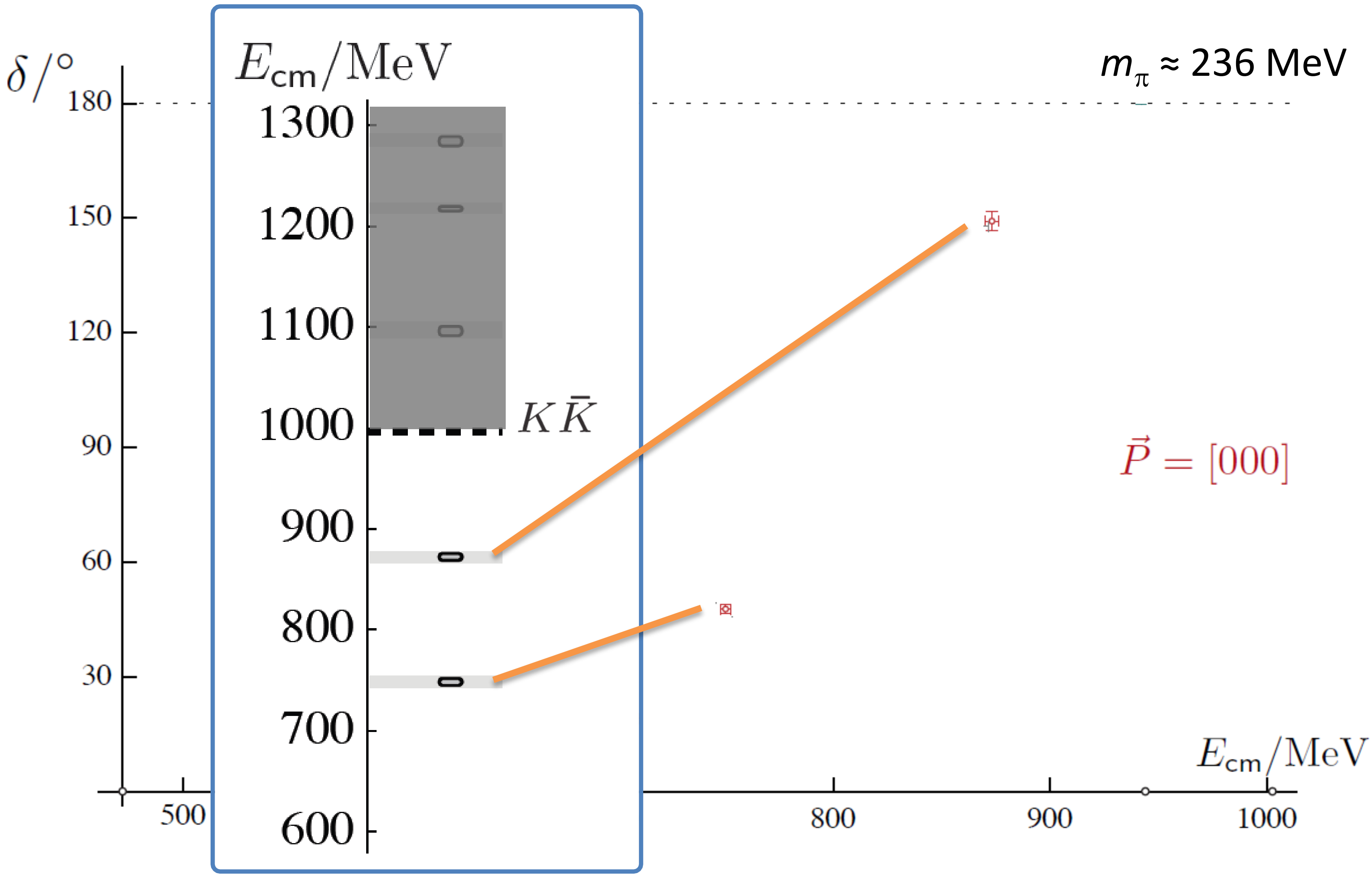
$$\bar{\psi}\Gamma D \ldots \psi$$

$$\sum_{\vec{p}_1,\vec{p}_2} C(\vec{P},\vec{p}_1,\vec{p}_2)\pi(\vec{p}_1)\pi(\vec{p}_2)$$

…

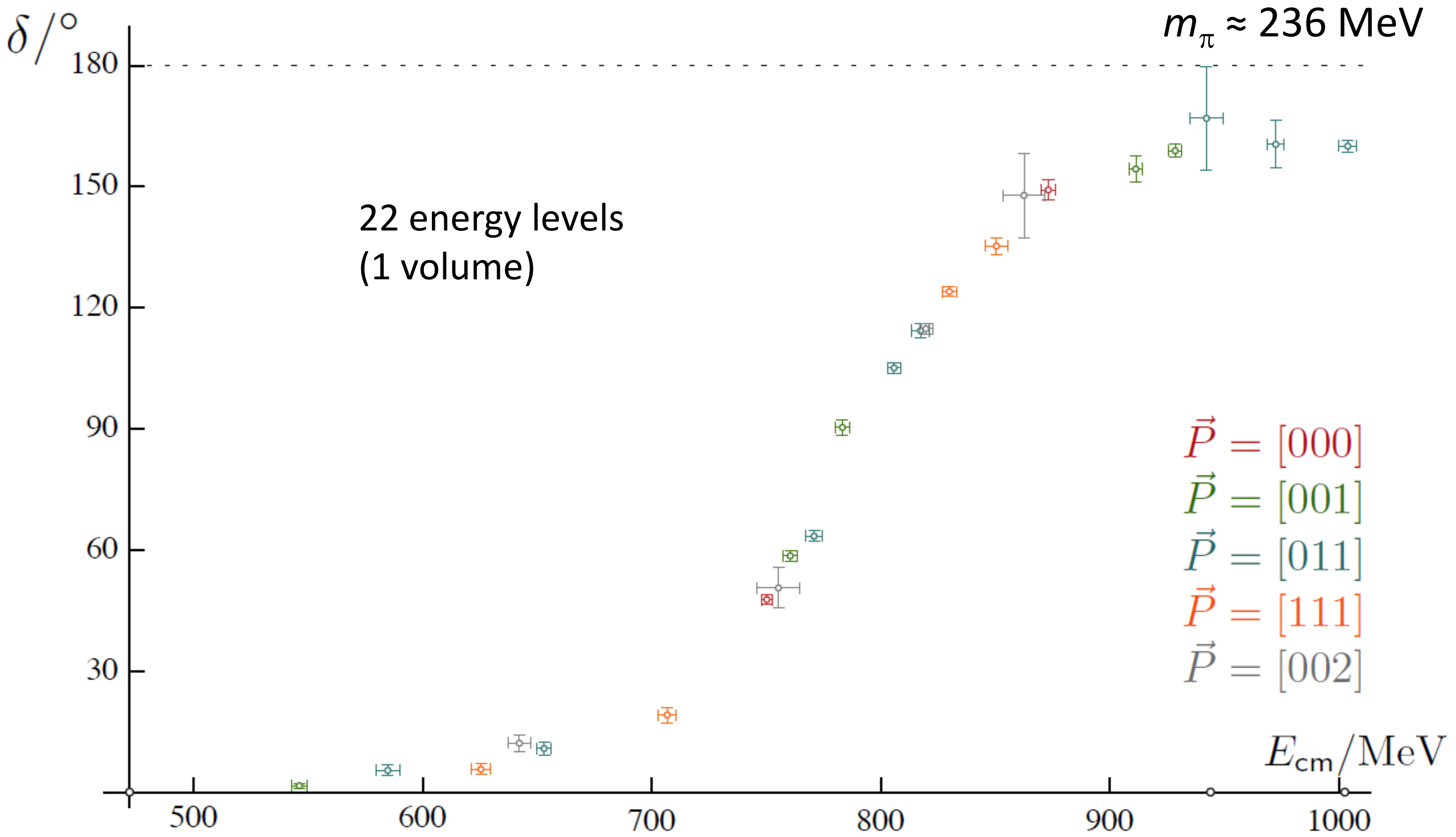
(HadSpec) [PR D87, 034505 (2013); PR D92, 094502 (2015)]

The ρ resonance: **elastic** P-wave ππ scattering



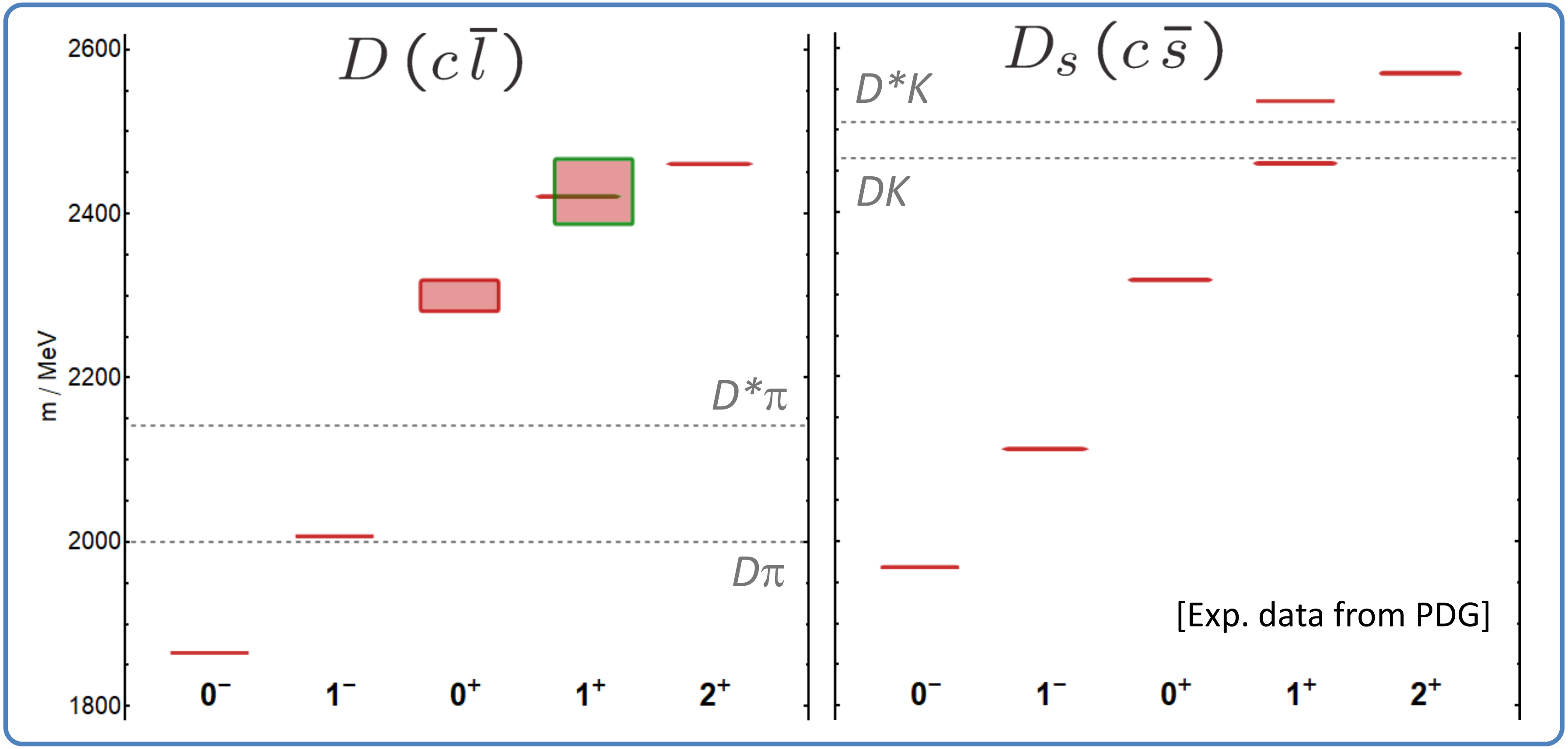
(HadSpec) [PR D87, 034505 (2013); PR D92, 094502 (2015)]

The ρ resonance: **elastic** P-wave ππ scattering

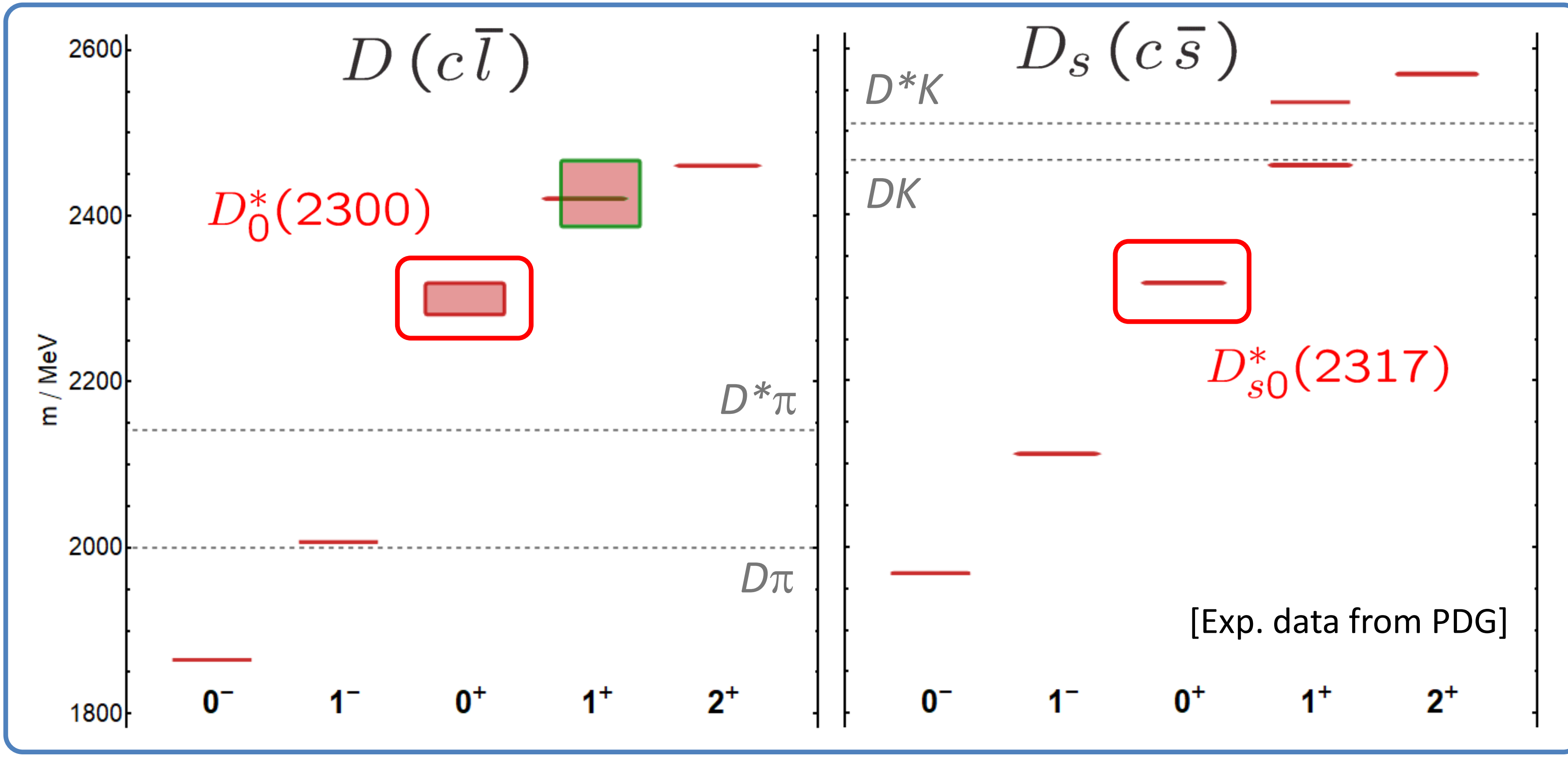


(HadSpec) [PR D87, 034505 (2013); PR D92, 094502 (2015)]

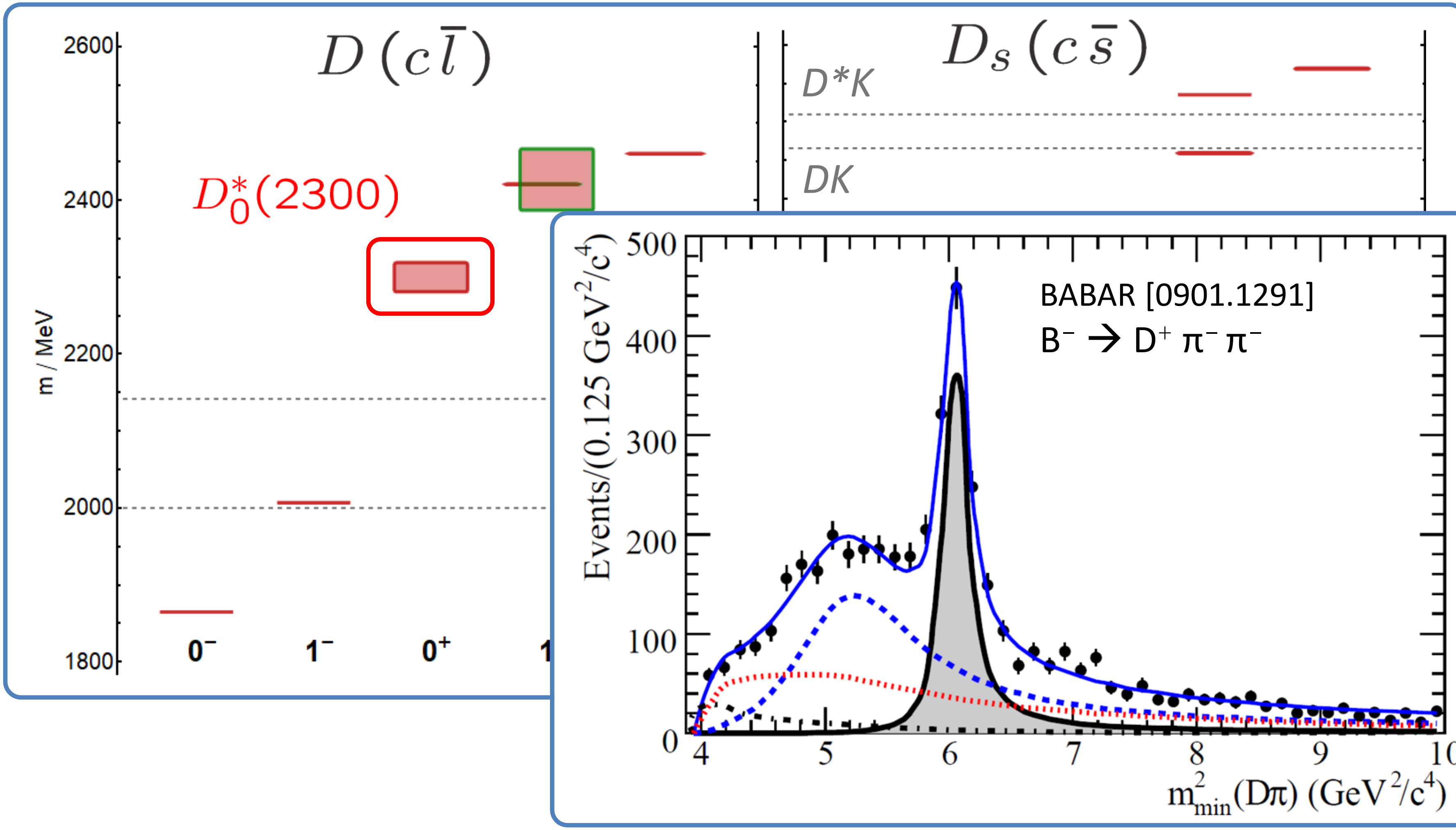
Charm (D) and charm-strange (D_s) mesons



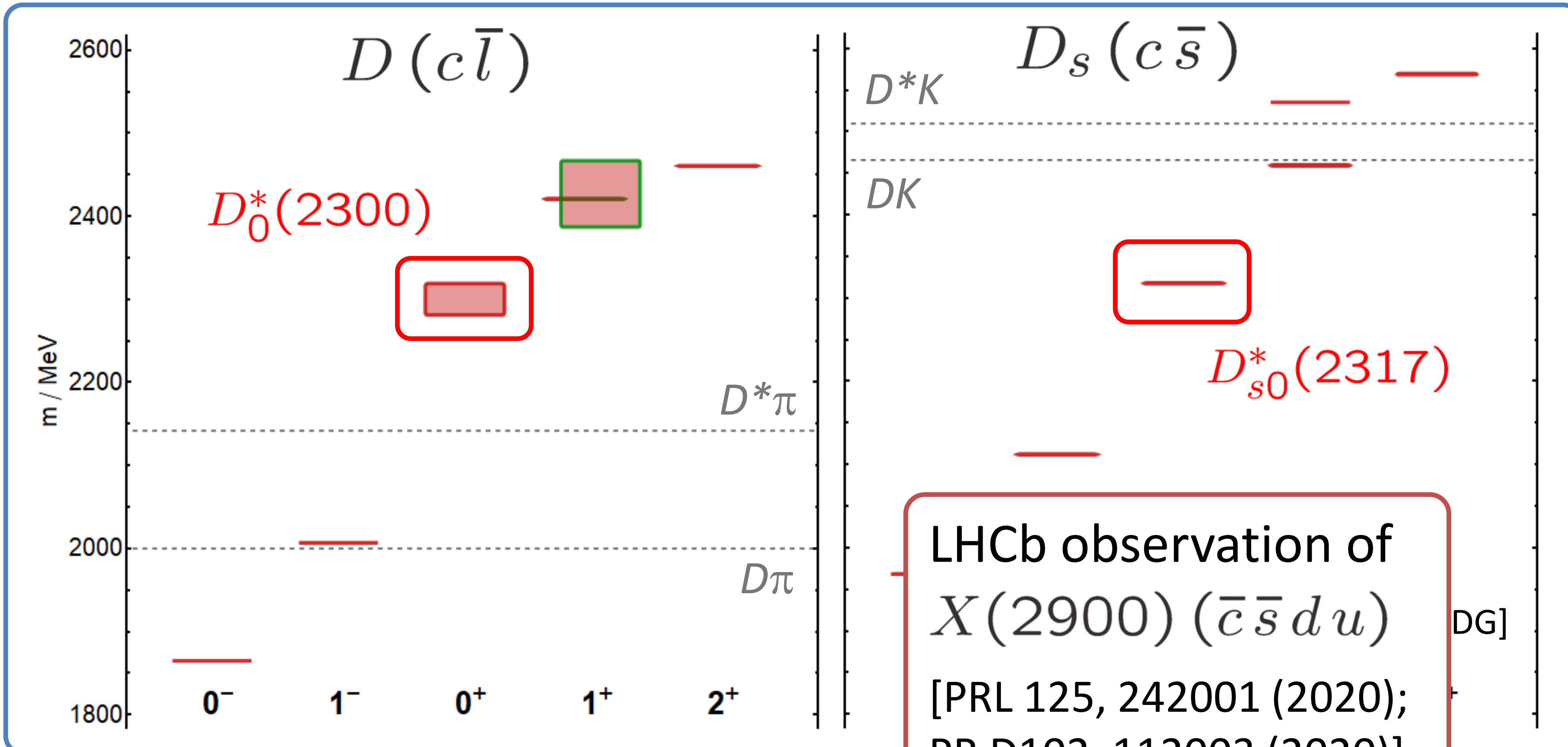
Charm (D) and charm-strange (D_s) mesons



Charm (D) and charm-strange (D_s) mesons



Charm (D) and charm-strange (D_s) mesons



LHCb observation of
$X(2900)\;(\bar{c}\,\bar{s}\,d\,u)$
[PRL 125, 242001 (2020);
PR D102, 112003 (2020)]

Other calculations

Some other lattice QCD work on $D\,K$ and/or $D\,\pi$ scattering:

- Mohler *et al* [PR D87, 034501 (2013), 1208.4059];
- Liu *et al* [PR D87, 014508 (2013), 1208.4535];
- Mohler *et al* [PRL 111, 222001 (2013), 1308.3175];
- Lang *et al* [PR D90, 034510 (2014), 1403.8103];
- Bali *et al* (RQCD) [PR D96, 074501 (2017), 1706.01247];
- Alexandrou *et al* (ETM) [PR D101 034502 (2020), 1911.08435];
- Gregory *et al* [2106.15391]

Also:

- Martínez Torres *et al* [JHEP 05 (2015) 153, 1412.1706];
- Albaladejo *et al* [PL B767, 465 (2017), 1610.06727];
- Du *et al* [PR D98, 094018 (2018), 1712.07957];
- Guo *et al* [*PR D*98 014510 (2018), 1801.10122];
- Guo *et al* [EPJ C79, 13 (2019), 1811.05585]
- Lutz, Guo, Heo, Korpa [2209.10601]

DK (isospin=0)

[Cheung, CT, Wilson, Moir, Peardon, Ryan (HadSpec), JHEP 02 (2021) 100, arXiv:2008.06432]

Anisotropic lattices,
$a_s/a_t \approx 3.5$, $a_s \approx 0.12$ fm,
various volumes.

$N_f = 2+1$,
Wilson-clover fermions,
$m_\pi \approx 239$ MeV & 391 MeV.

Use many different fermion-bilinear

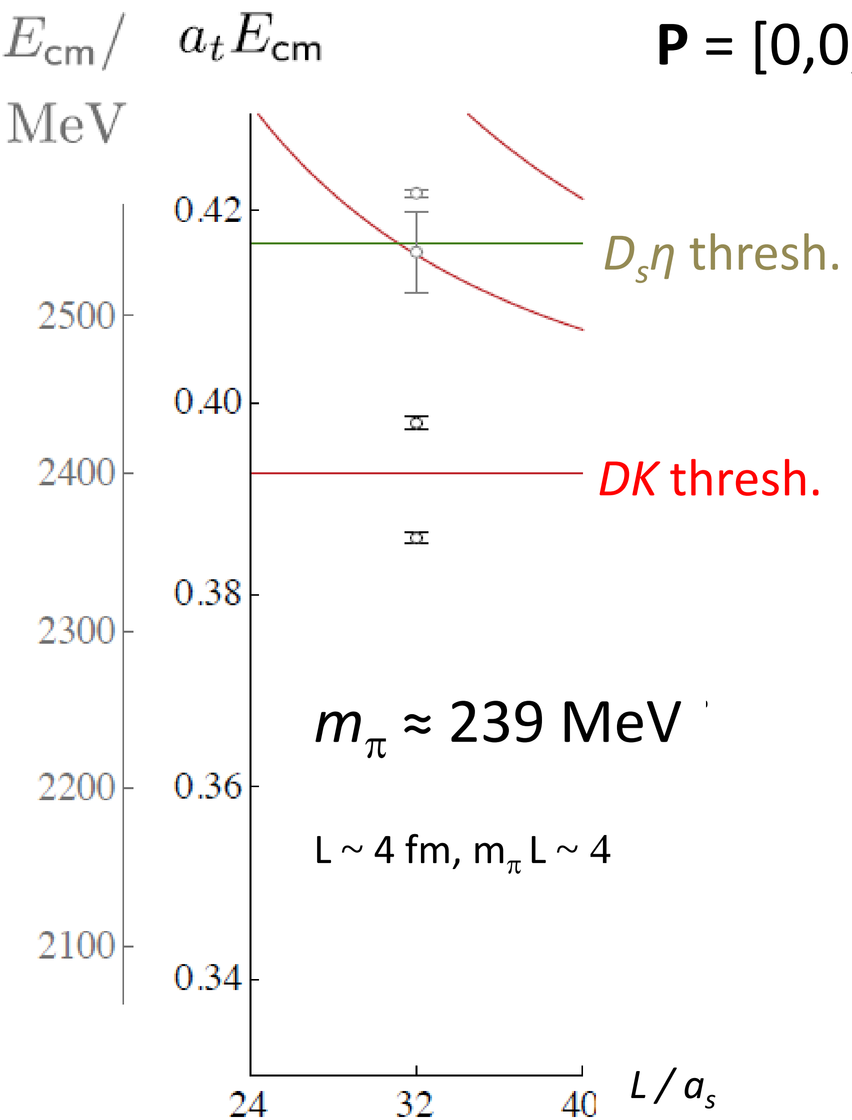
$$\sim \bar{\psi}\Gamma D \ldots \psi$$

and *DK*, ... operators

DK (isospin=0)

[Cheung, CT, Wilson, Moir, Peardon, Ryan (HadSpec), JHEP 02 (2021) 100, arXiv:2008.06432]

P = [0,0,0] $J^P = 0^+, (4^+, \ldots)$



Use many different fermion-bilinear

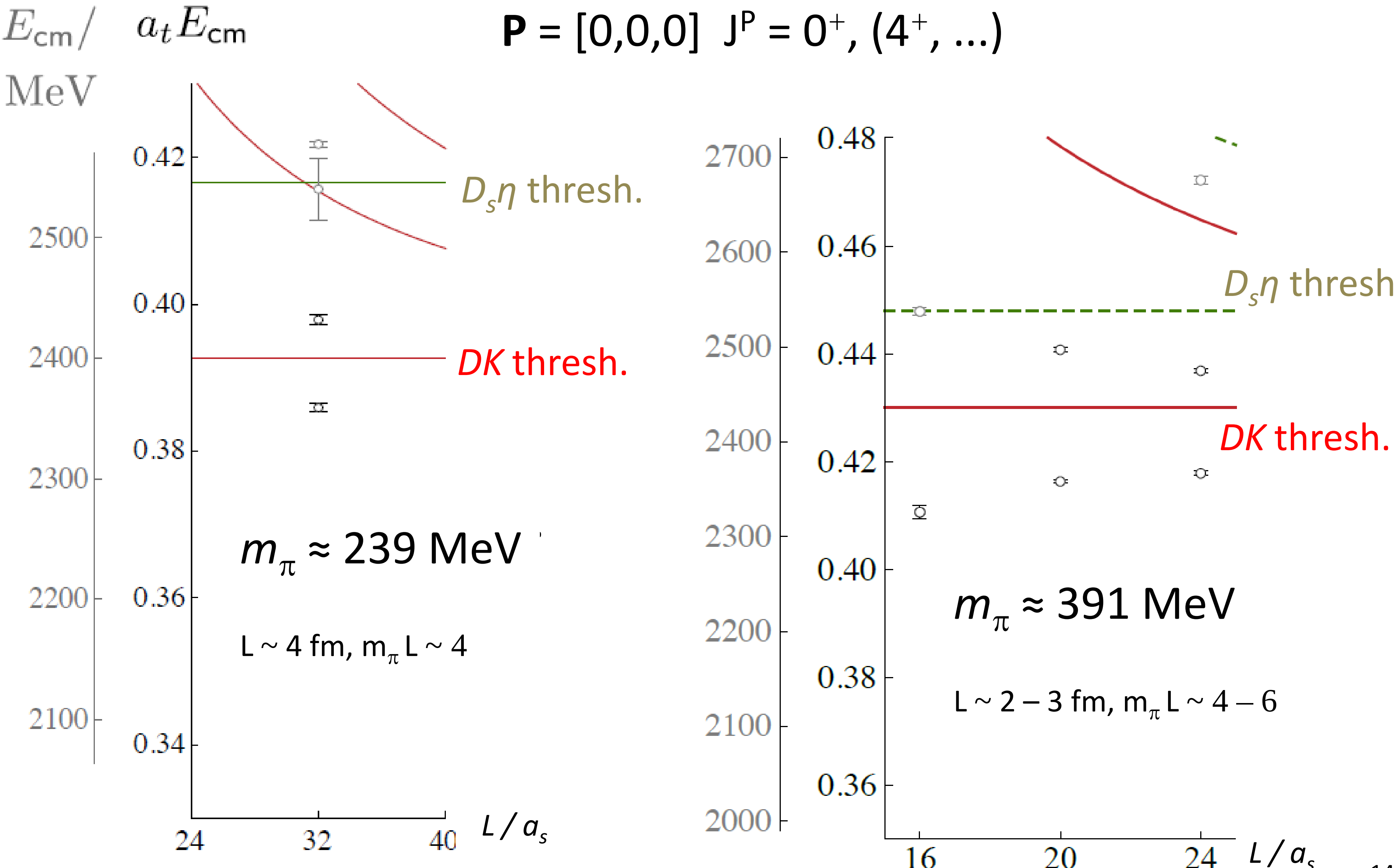
$$\sim \bar{\psi}\Gamma D \ldots \psi$$

and *DK*, ... operators

DK (isospin=0)

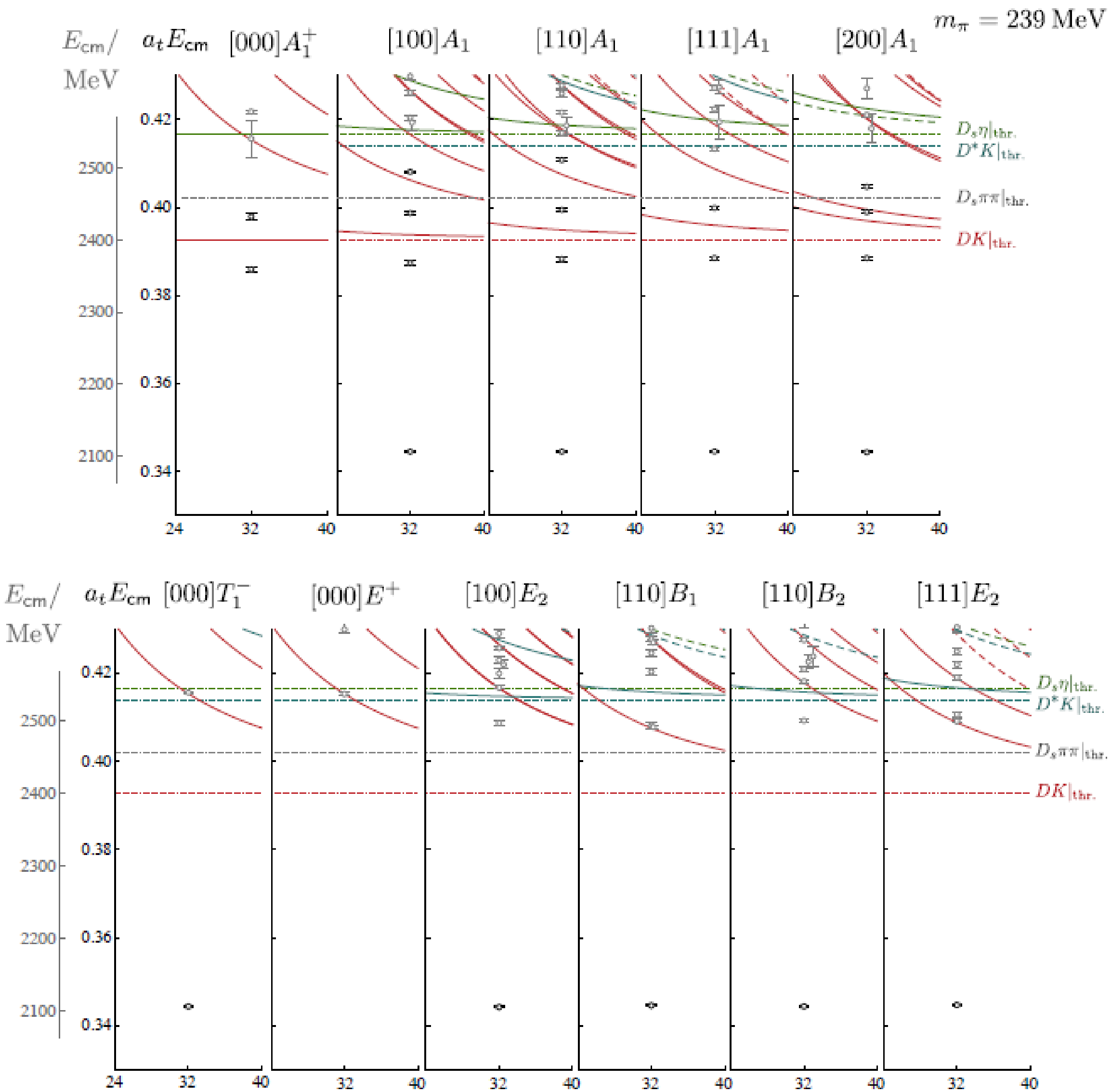
[Cheung, CT, Wilson, Moir, Peardon, Ryan (HadSpec), JHEP 02 (2021) 100, arXiv:2008.06432]

P = [0,0,0] $J^P = 0^+, (4^+, ...)$



DK (isospin=0) – spectra

[JHEP 02 (2021) 100]

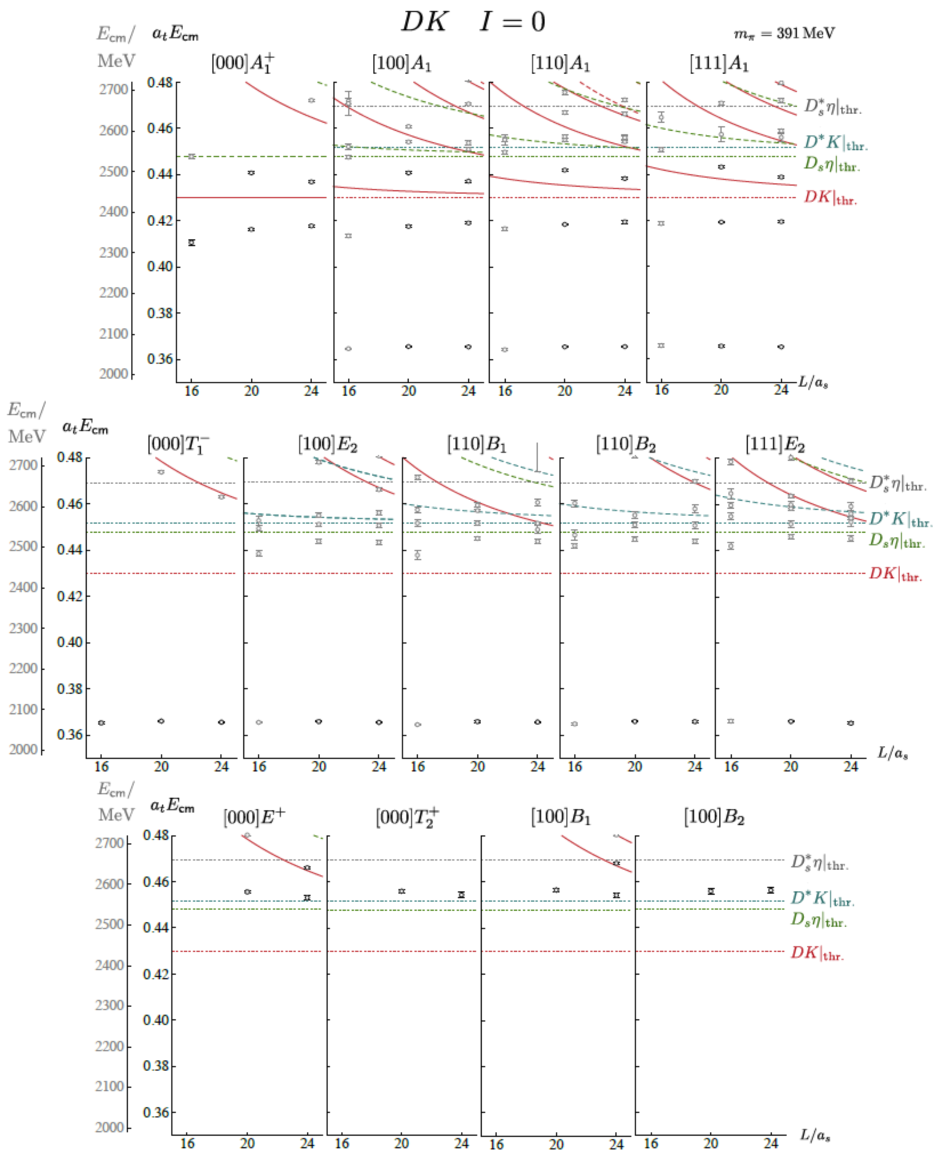


$m_\pi \approx 239$ MeV

Use 22 energy levels for ℓ = 0, 1

DK (isospin=0) – spectra

[JHEP 02 (2021) 100]



$m_\pi \approx 391$ MeV

Use 34 energy levels for ℓ = 0, 1

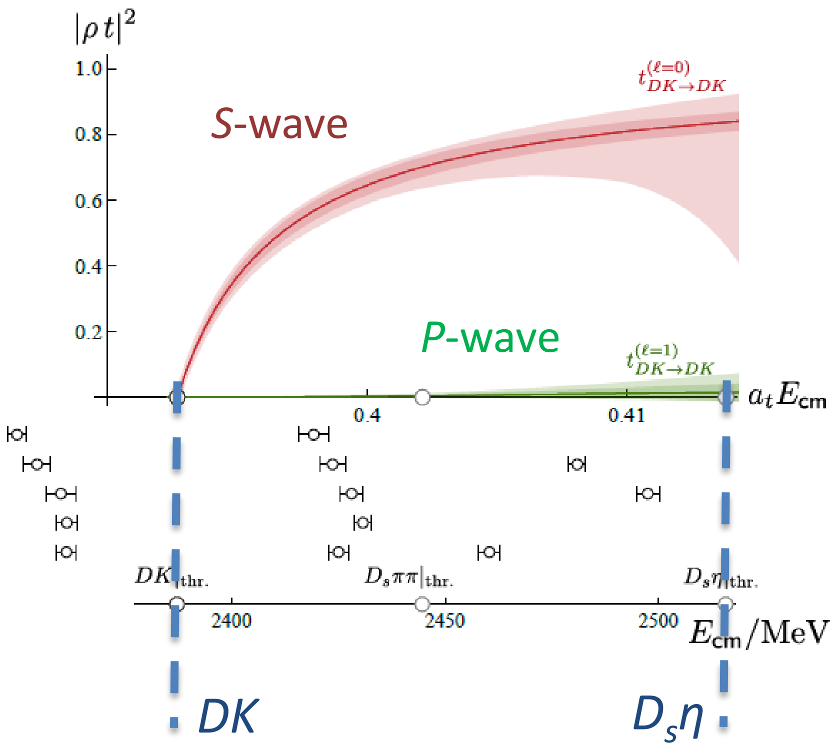
DK (isospin=0) – amplitudes

[JHEP 02 (2021) 100]

$m_\pi \approx 239$ MeV

(22 energy levels)

~ |amp|^2



Elastic DK scattering in S and P-wave

Sharp turn-on in S-wave at threshold

DK (isospin=0) – amplitudes

[JHEP 02 (2021) 100]

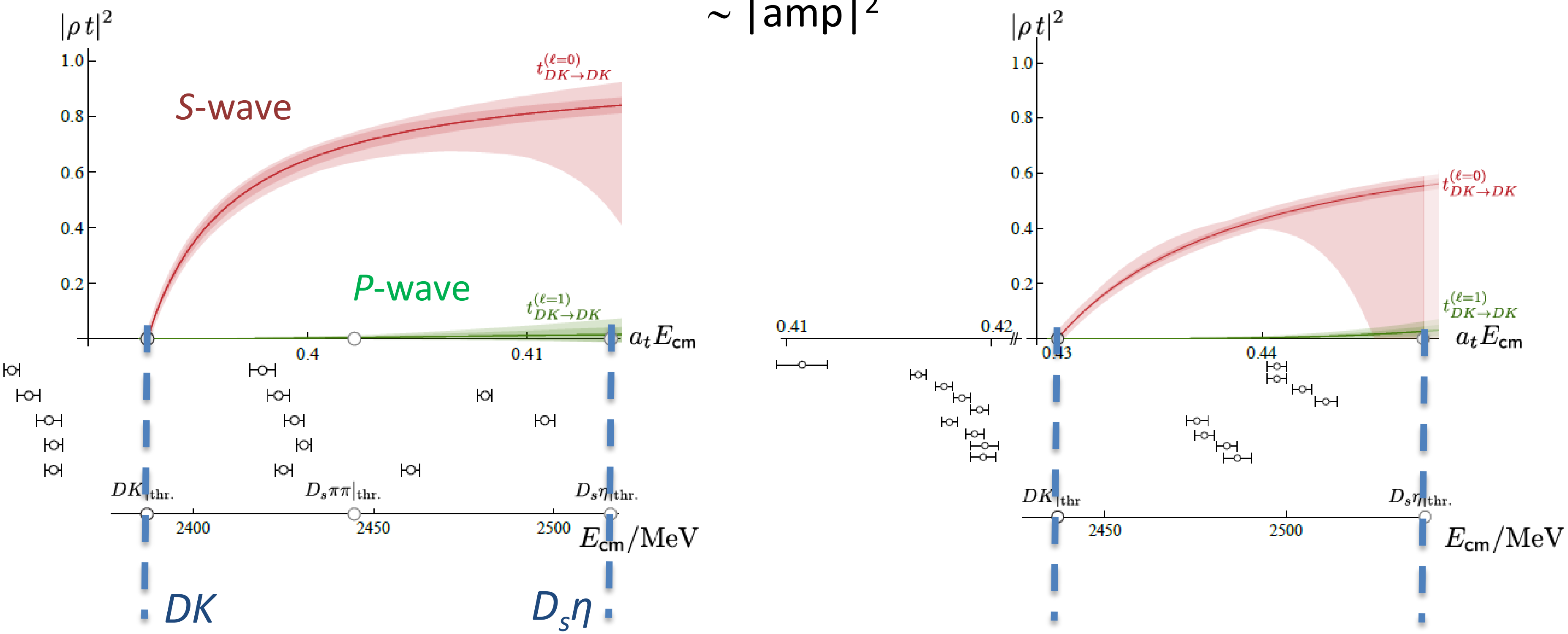
$m_\pi \approx 239$ MeV

(22 energy levels)

$m_\pi \approx 391$ MeV

(34 energy levels)

~ |amp|^2



Elastic *DK* scattering in *S* and *P*-wave

Sharp turn-on in *S*-wave at threshold

DK (isospin=0) – *S*-wave poles

[JHEP 02 (2021) 100]

Bound-state pole strongly coupled to *S*-wave *DK*

ΔE = 25(3) MeV for $m_\pi \approx$ 239 MeV

ΔE = 57(3) MeV for $m_\pi \approx$ 391 MeV

DK (isospin=0) – *S*-wave poles

[JHEP 02 (2021) 100]

Bound-state pole strongly coupled to *S*-wave *DK*

ΔE = 25(3) MeV for $m_\pi \approx$ 239 MeV

ΔE = 57(3) MeV for $m_\pi \approx$ 391 MeV

c.f. experiment $\Delta E \approx$ 45 MeV (decays to $D_s \pi^0$)

DK (isospin=0) – *S*-wave poles

[JHEP 02 (2021) 100]

Bound-state pole strongly coupled to *S*-wave *DK*

$\Delta E = 25(3)$ MeV for $m_\pi \approx 239$ MeV $Z \lesssim 0.11$

$\Delta E = 57(3)$ MeV for $m_\pi \approx 391$ MeV $Z \approx 0.13(6)$

c.f. experiment $\Delta E \approx 45$ MeV (decays to $D_s \pi^0$)

Weinberg [PR 137, B672 (1965)] compositeness, $0 \leq Z \leq 1$ (assuming binding is sufficiently weak)

DK (isospin=0) – S-wave poles

[JHEP 02 (2021) 100]

Bound-state pole strongly coupled to *S*-wave *DK*

$\Delta E = 25(3)$ MeV for $m_\pi \approx 239$ MeV $Z \lesssim 0.11$

$\Delta E = 57(3)$ MeV for $m_\pi \approx 391$ MeV $Z \approx 0.13(6)$

c.f. experiment $\Delta E \approx 45$ MeV (decays to $D_s \pi^0$)

Weinberg [PR 137, B672 (1965)] compositeness, $0 \leq Z \leq 1$ (assuming binding is sufficiently weak)

Also deeply bound state in *P*-wave, D_s^* , but doesn't strongly influence *DK* scattering at these energies

$D\bar{K}$ (isospin=0,1)

Exotic flavour $(\bar{l}\,\bar{l}\,c\,s)$

[JHEP 02 (2021) 100]

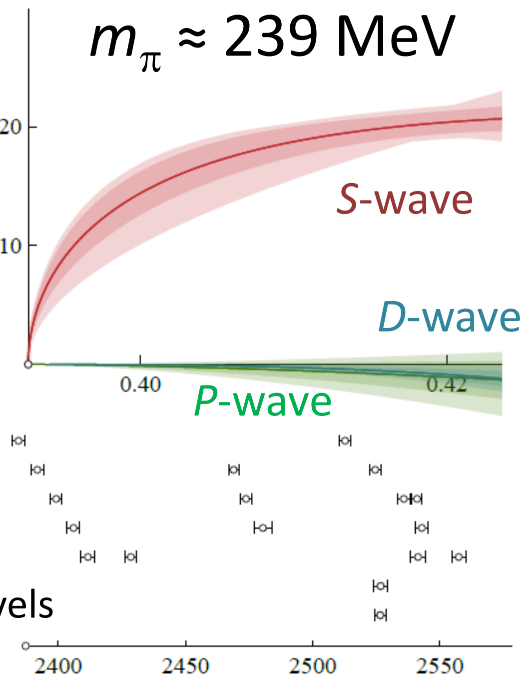
$D\bar{K}$ (isospin=0,1)

Exotic flavour $(\bar{l}\,\bar{l}\,c\,s)$

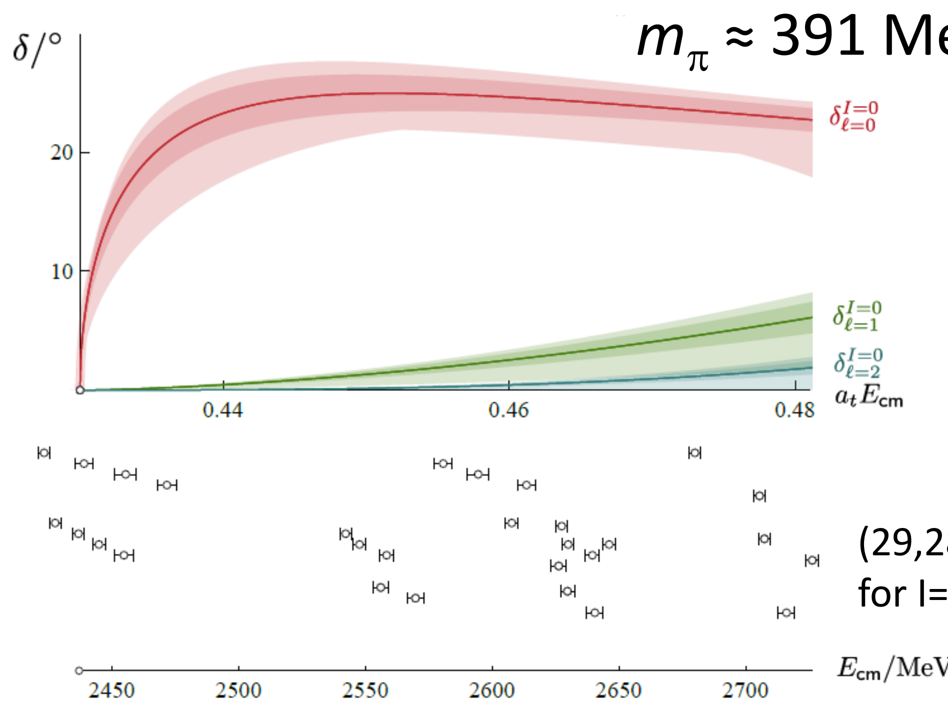
[JHEP 02 (2021) 100]

δ / °

I = 0



(18, 18 levels for I=0,1)



(29,28 levels for I=0,1)

$D\bar{K}$ (isospin=0,1)

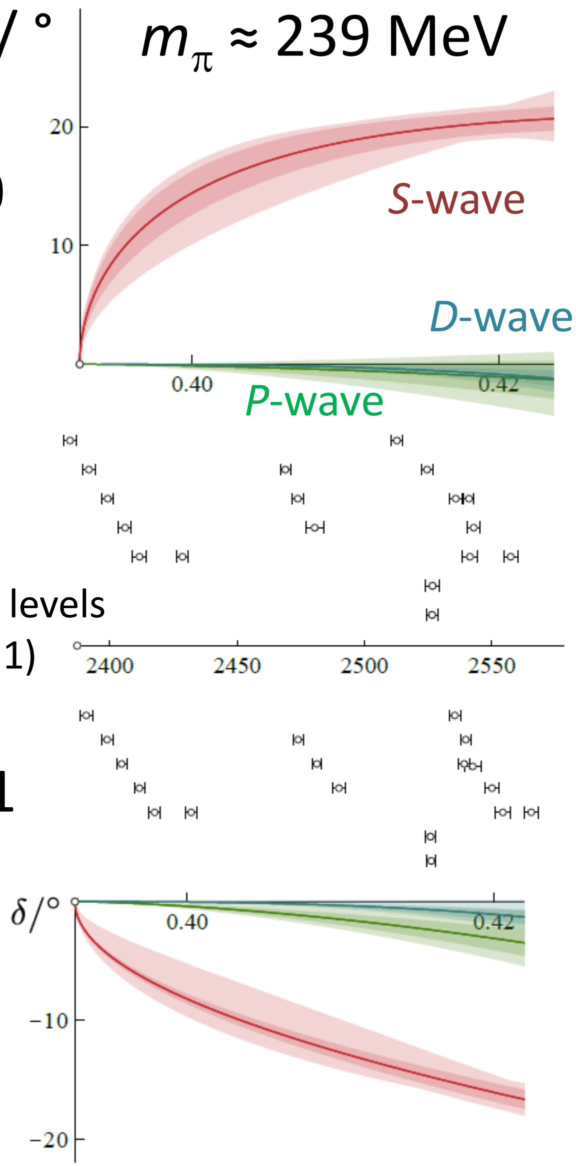
Exotic flavour $(\bar{l}\,\bar{l}\,c\,s)$

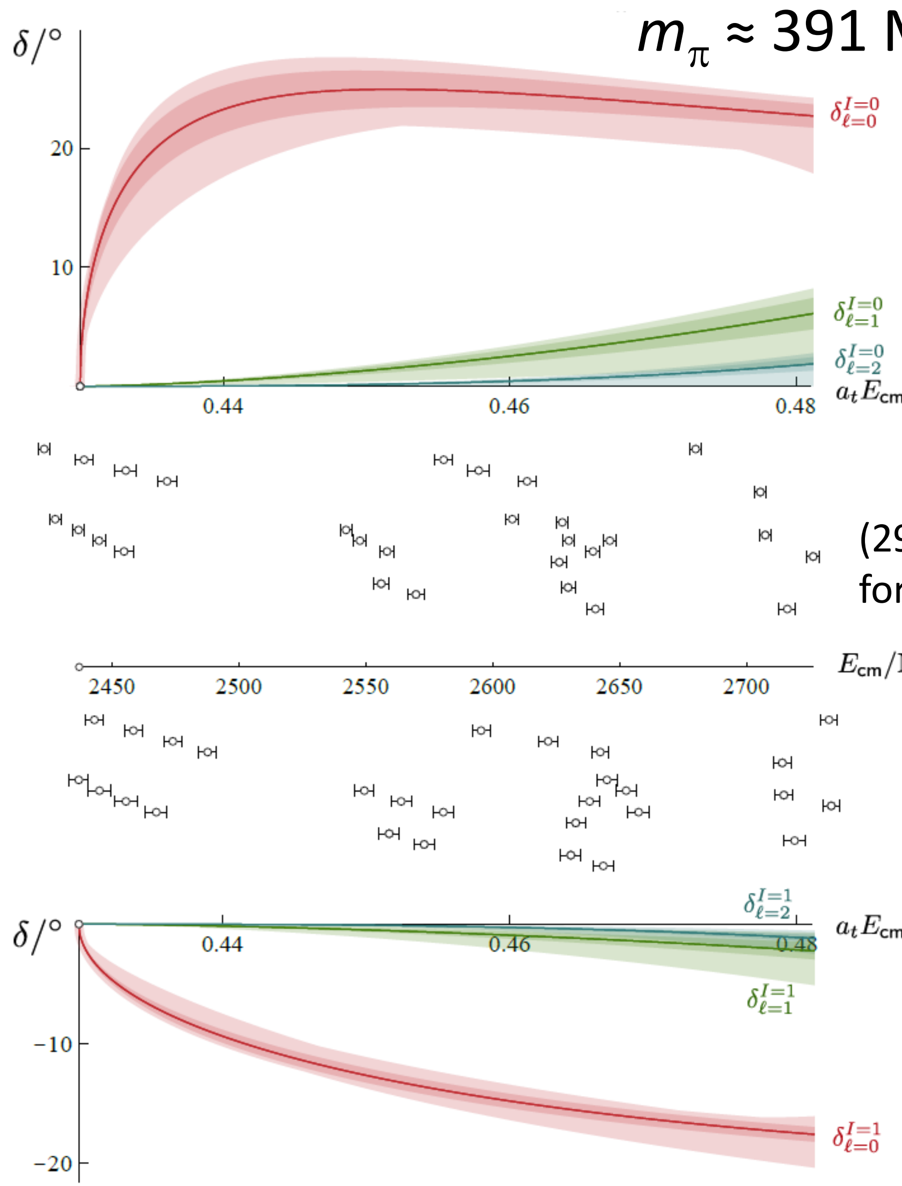
[JHEP 02 (2021) 100]

I = 0

I = 1

(18, 18 levels for I=0,1)



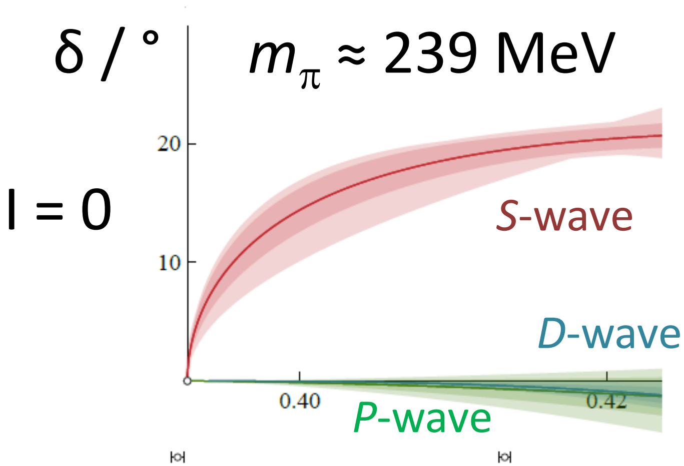


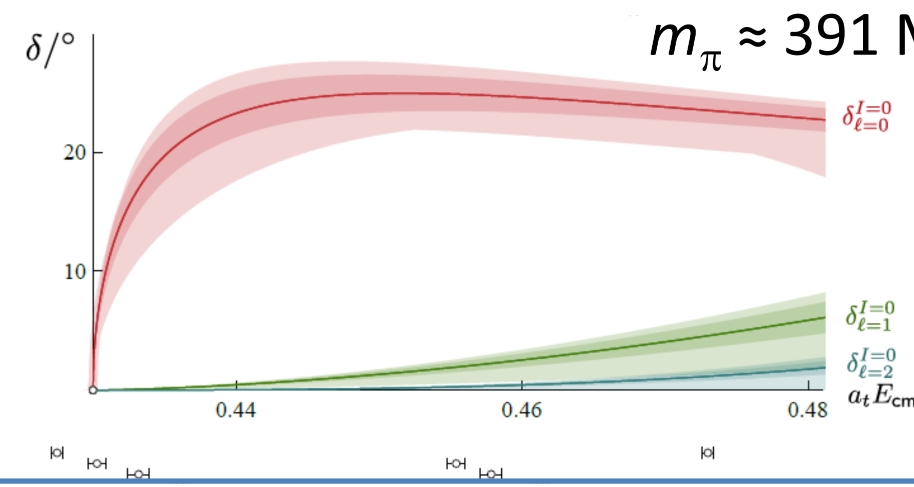
(29,28 levels for I=0,1)

$D\bar{K}$ (isospin=0,1)

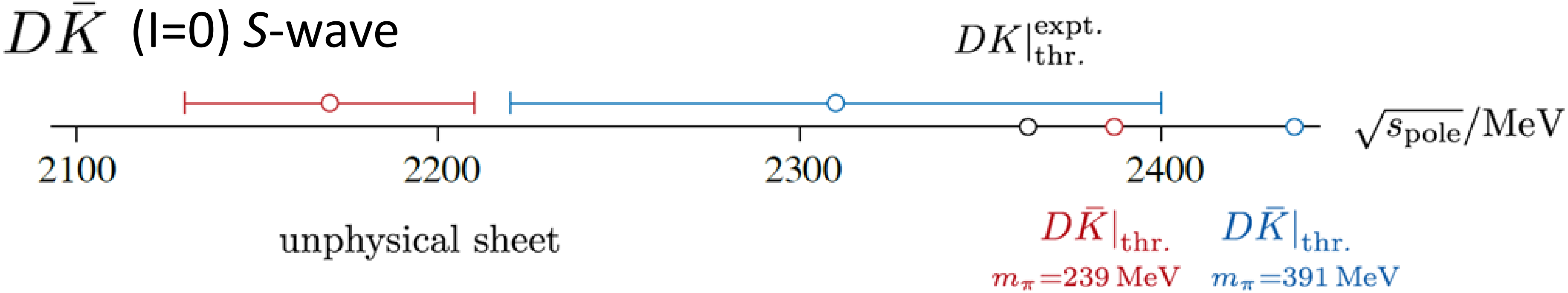
Exotic flavour $(\bar{l}\bar{l}cs)$

[JHEP 02 (2021) 100]

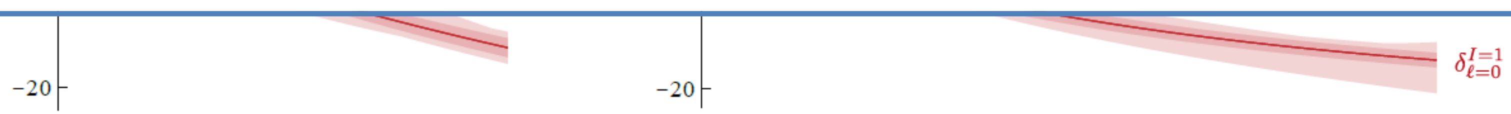




$D\bar{K}$ (I=0) *S*-wave



Suggestion of a **virtual bound-state** pole (**exotic flavour**)



$D\pi$ (isospin=1/2) – S-wave

[Gayer, Lang, Ryan, Tims, CT, Wilson (HadSpec), JHEP 07 (2021) 123]

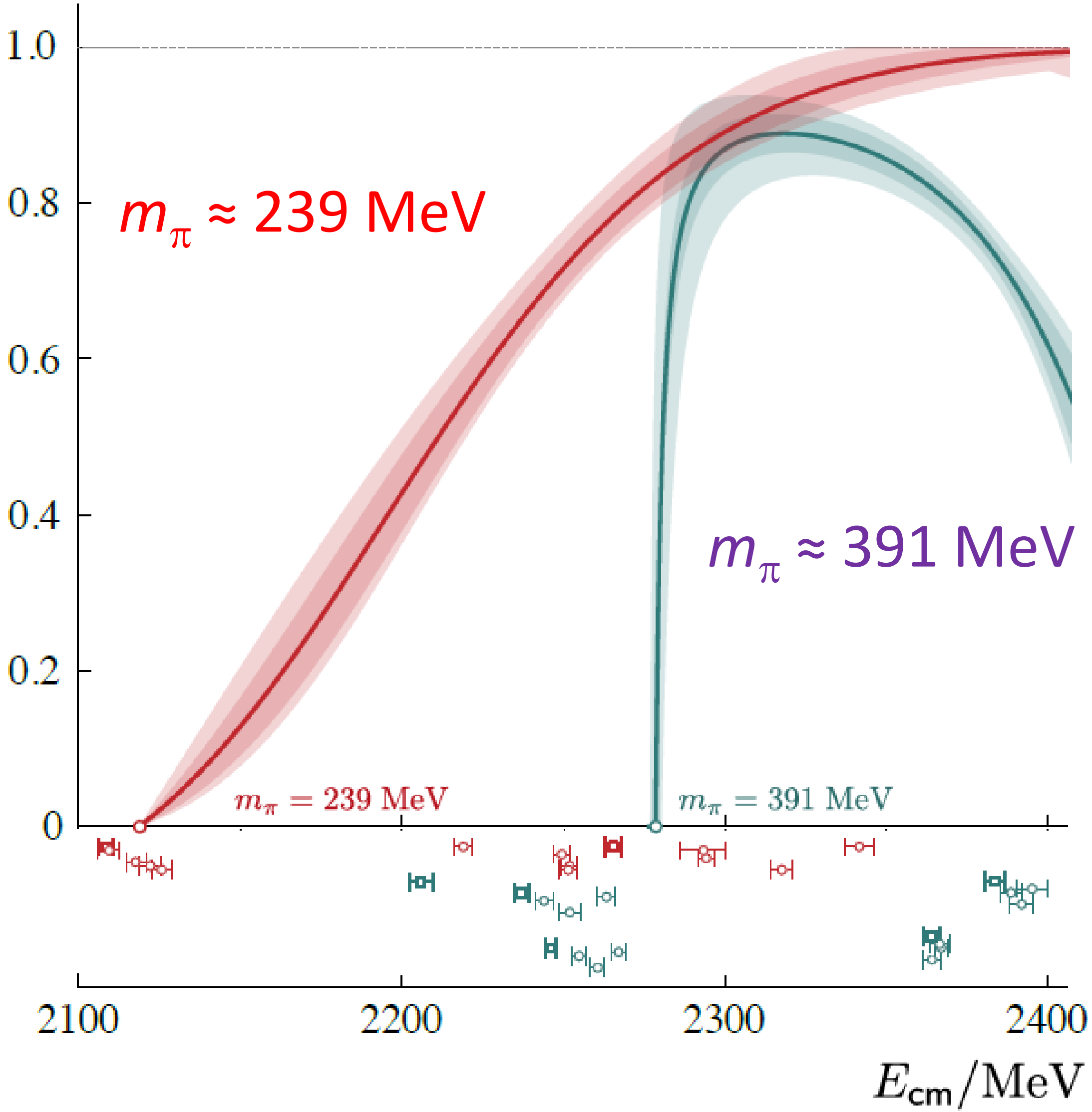
[Moir, Peardon, Ryan, CT, Wilson (HadSpec) JHEP 10 (2016) 011]

Dπ (isospin=1/2) – S-wave

[Gayer, Lang, Ryan, Tims, CT, Wilson (HadSpec), JHEP 07 (2021) 123]

[Moir, Peardon, Ryan, CT, Wilson (HadSpec) JHEP 10 (2016) 011]

$\rho^2|t|^2 \sim |\text{amp}|^2$



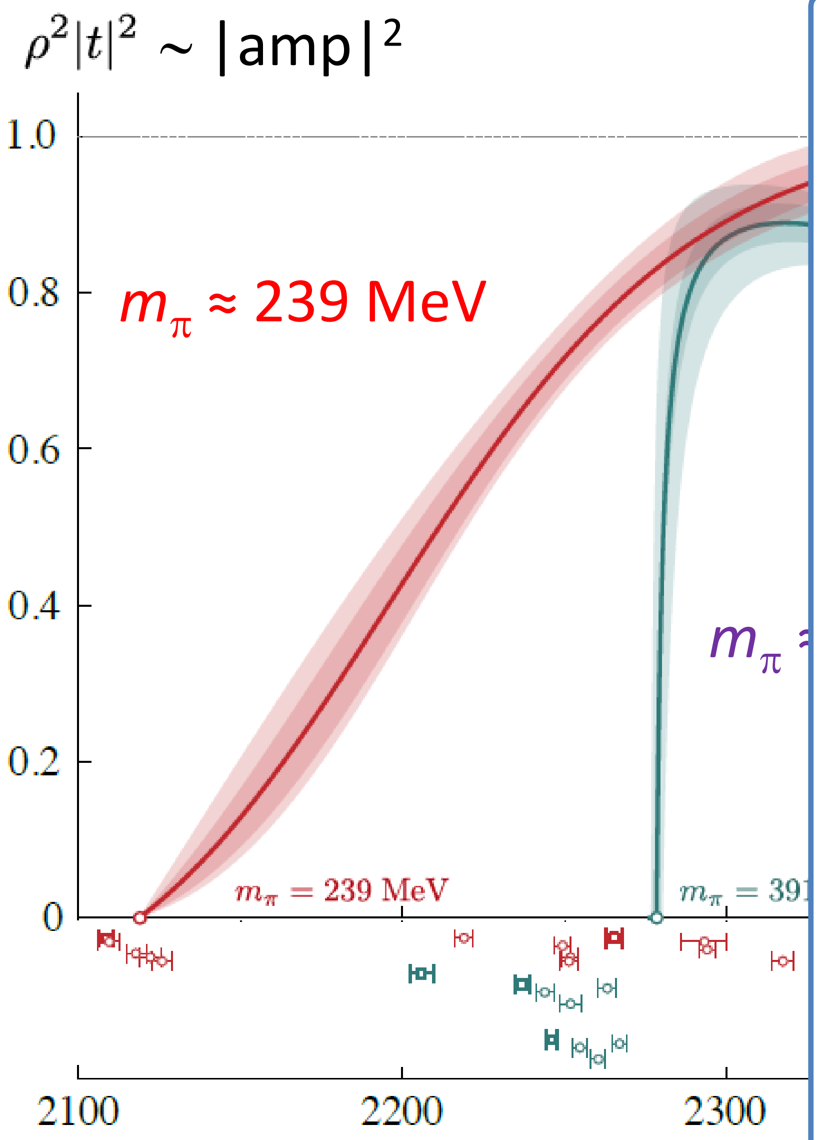
$m_\pi \approx 239$ MeV
29 energy levels
(1 volume)

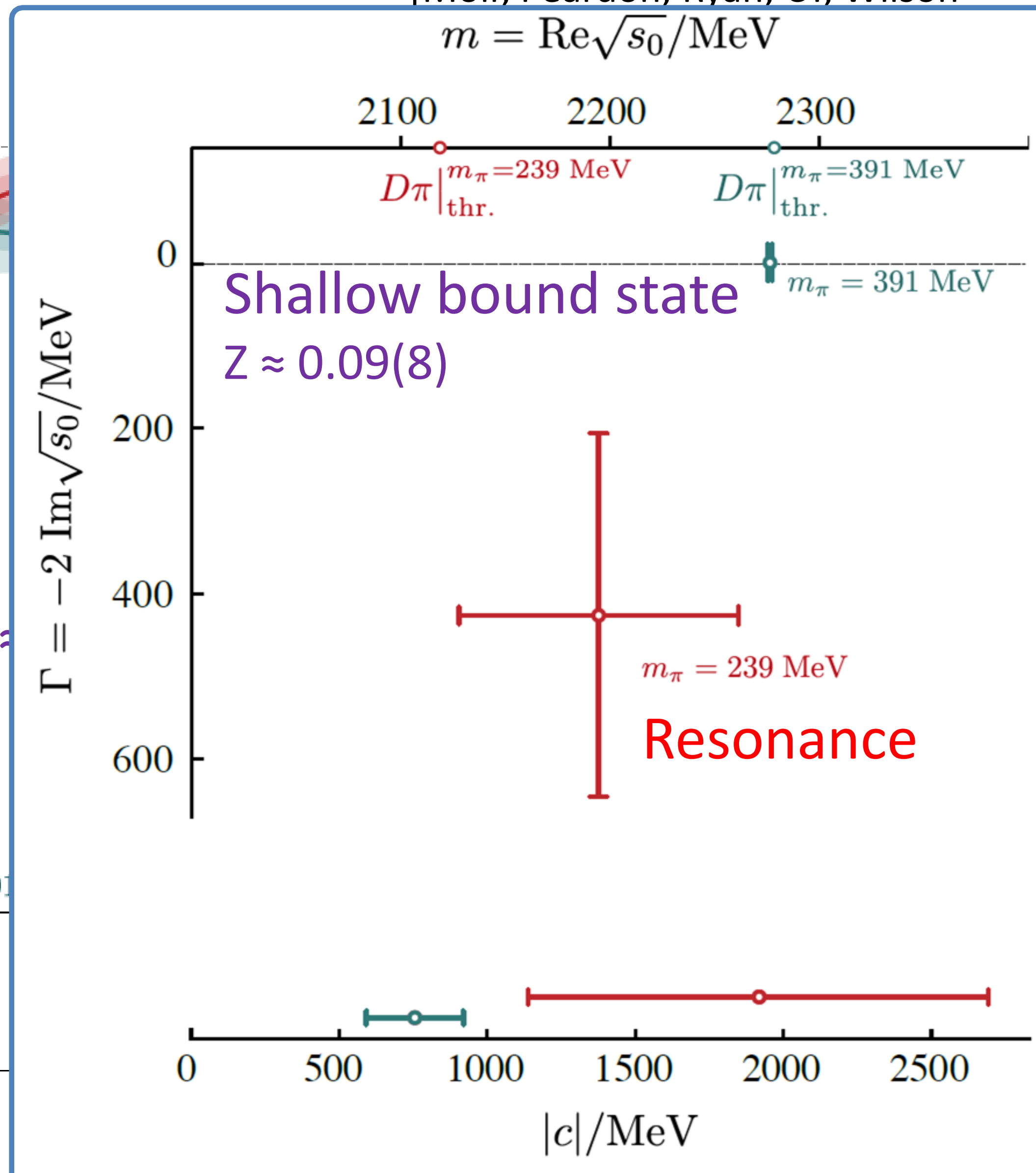
$m_\pi \approx 391$ MeV
47 energy levels
(3 volumes)

Dπ (isospin=1/2) – S-wave

[Gayer, Lang, Ryan, Tims, CT, Wilson (HadSpec), JHEP 07 (2021) 123]

[Moir, Peardon, Ryan, CT, Wilson

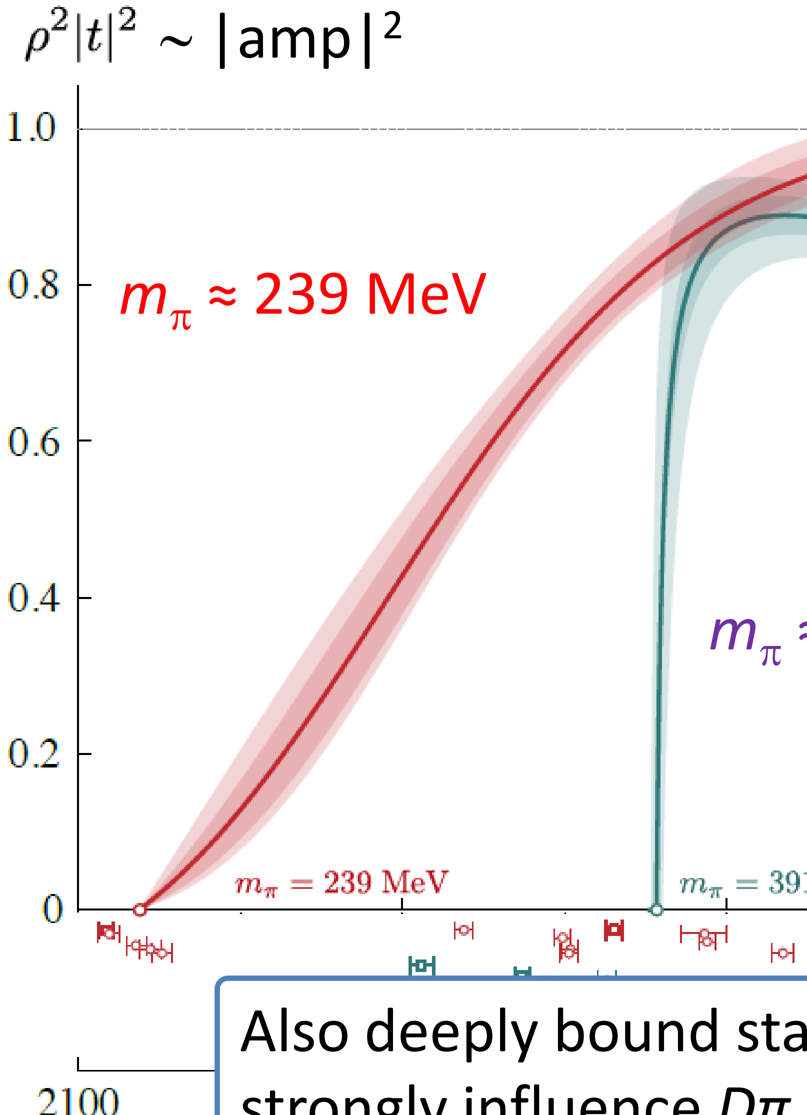


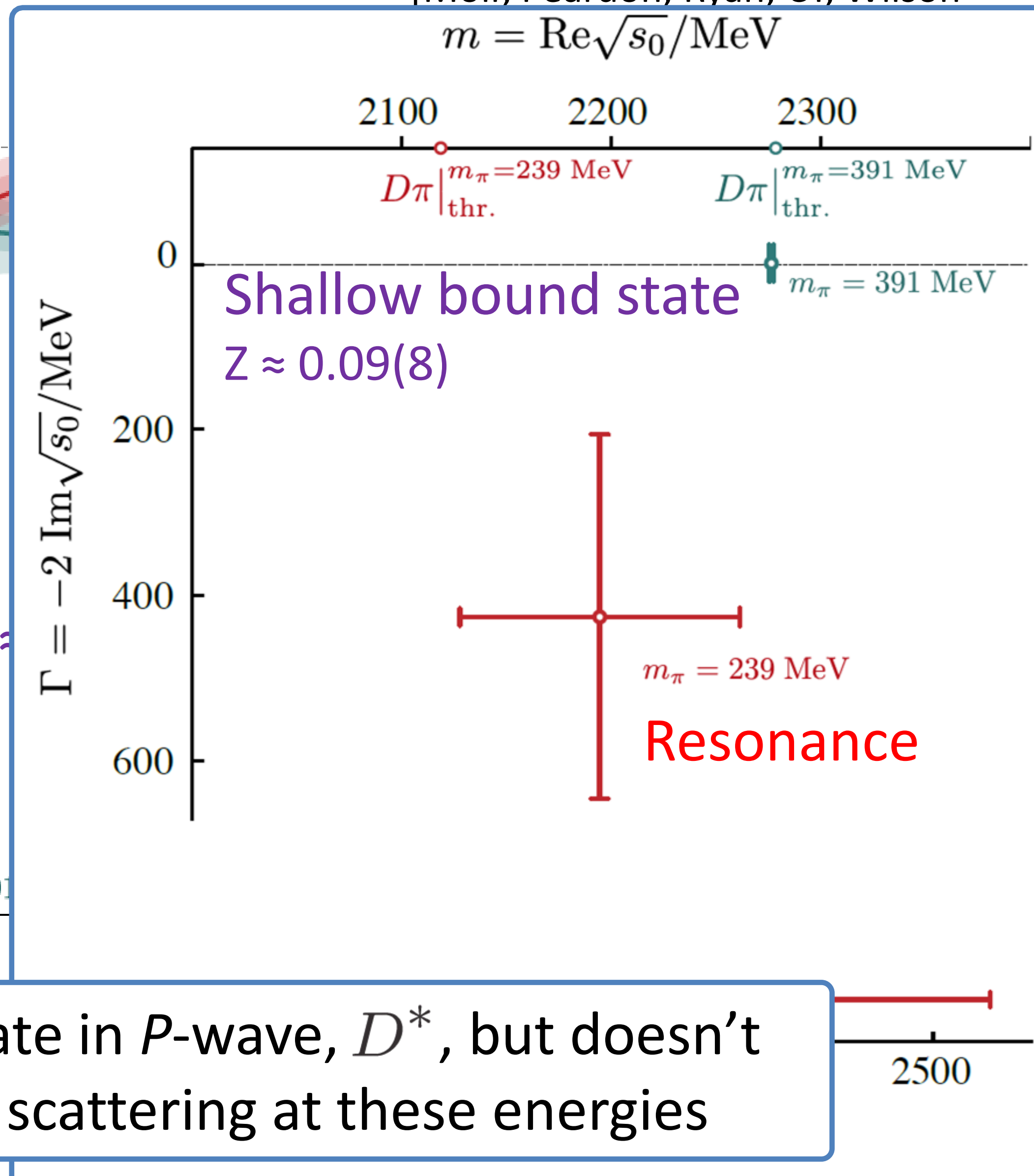


Dπ (isospin=1/2) – S-wave

[Gayer, Lang, Ryan, Tims, CT, Wilson (HadSpec), JHEP 07 (2021) 123]

[Moir, Peardon, Ryan, CT, Wilson





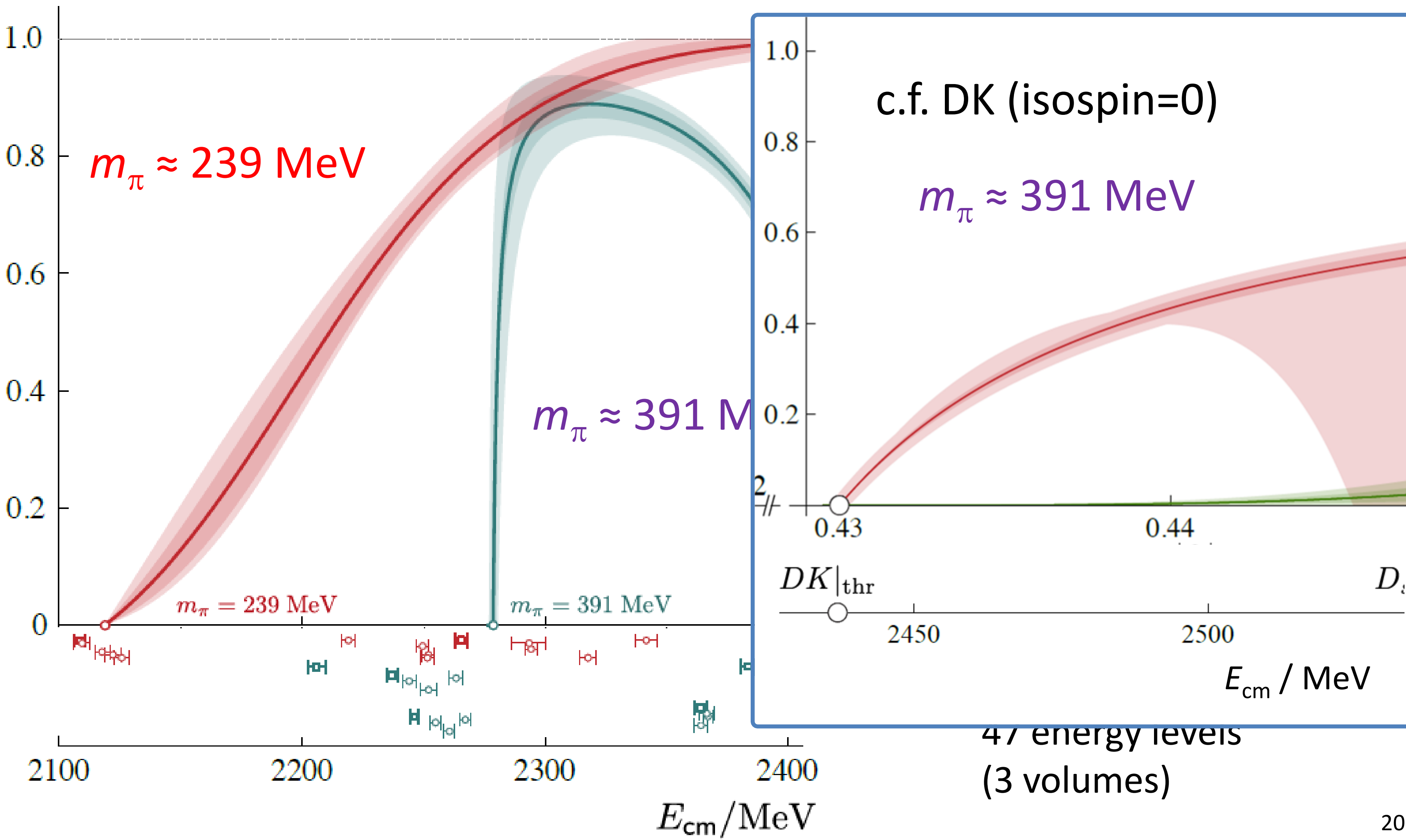
Also deeply bound state in *P*-wave, D^*, but doesn't strongly influence *Dπ* scattering at these energies

Dπ (isospin=1/2) – S-wave

[Gayer, Lang, Ryan, Tims, CT, Wilson (HadSpec), JHEP 07 (2021) 123]

[Moir, Peardon, Ryan, CT, Wilson (HadSpec) JHEP 10 (2016) 011]

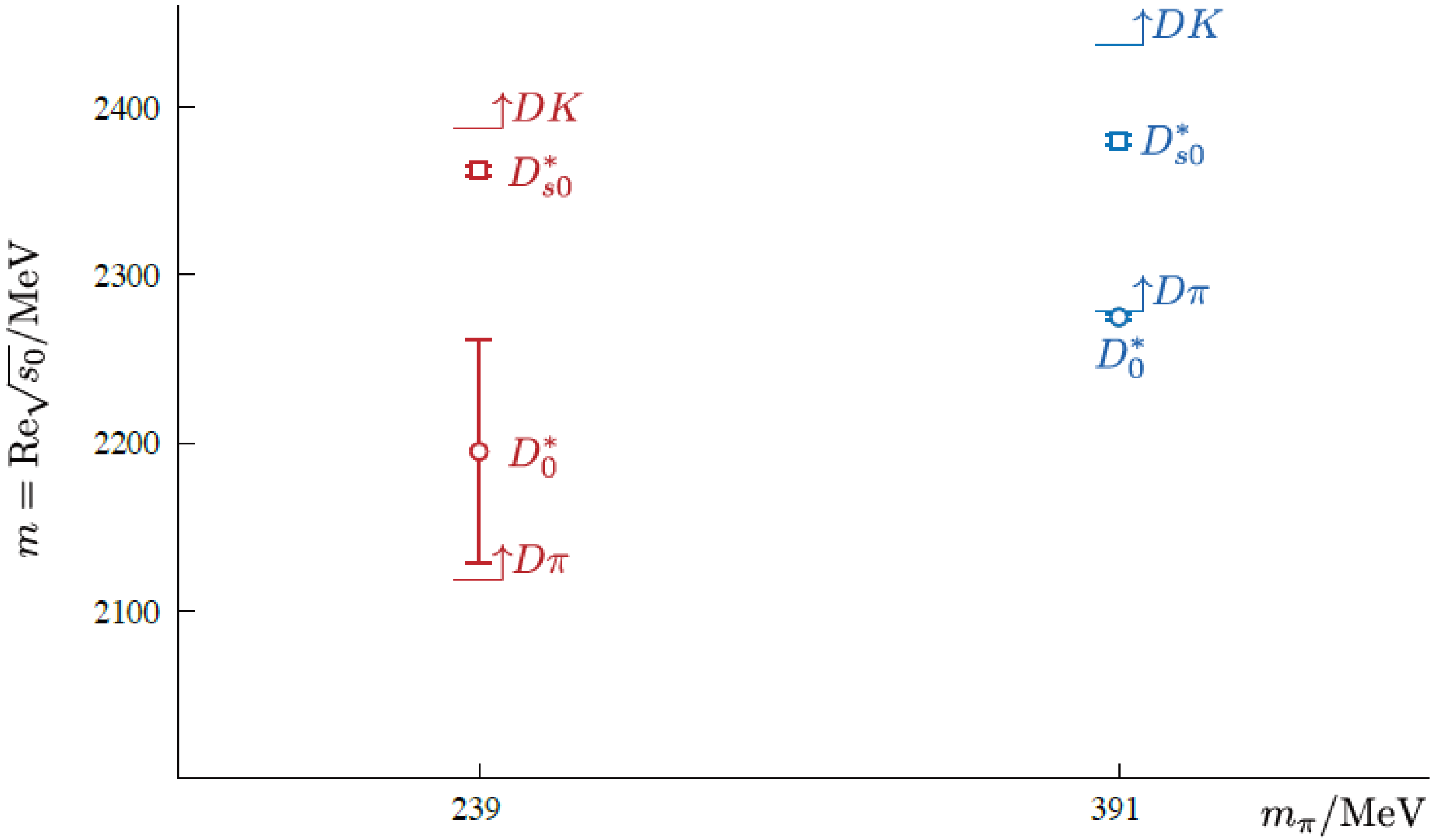
$\rho^2|t|^2 \sim |\text{amp}|^2$



47 energy levels
(3 volumes)

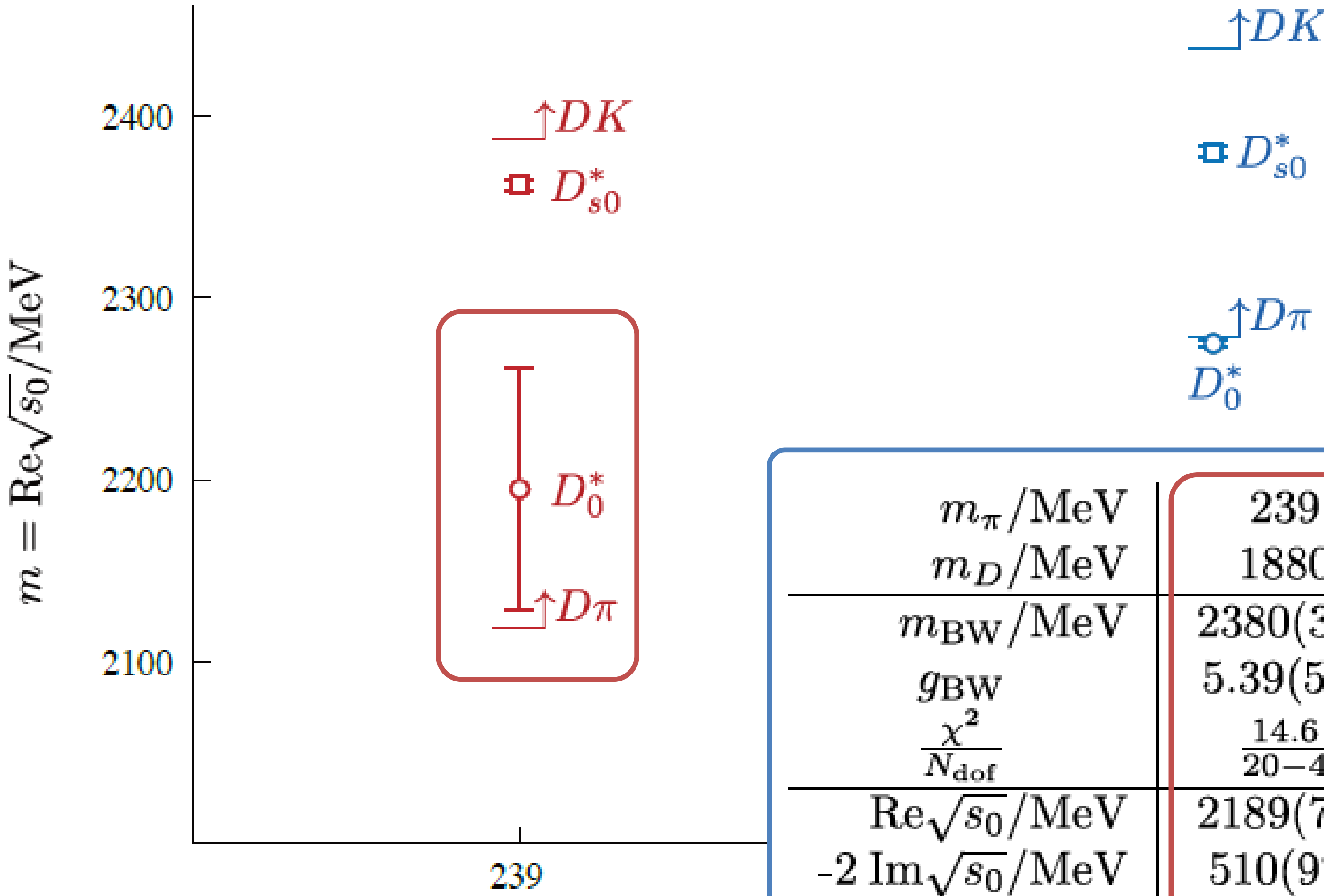
DK and Dπ – S-wave poles

[JHEP 07 (2021) 123, JHEP 02 (2021) 100,
JHEP 10 (2016), 011]



DK and Dπ – S-wave poles

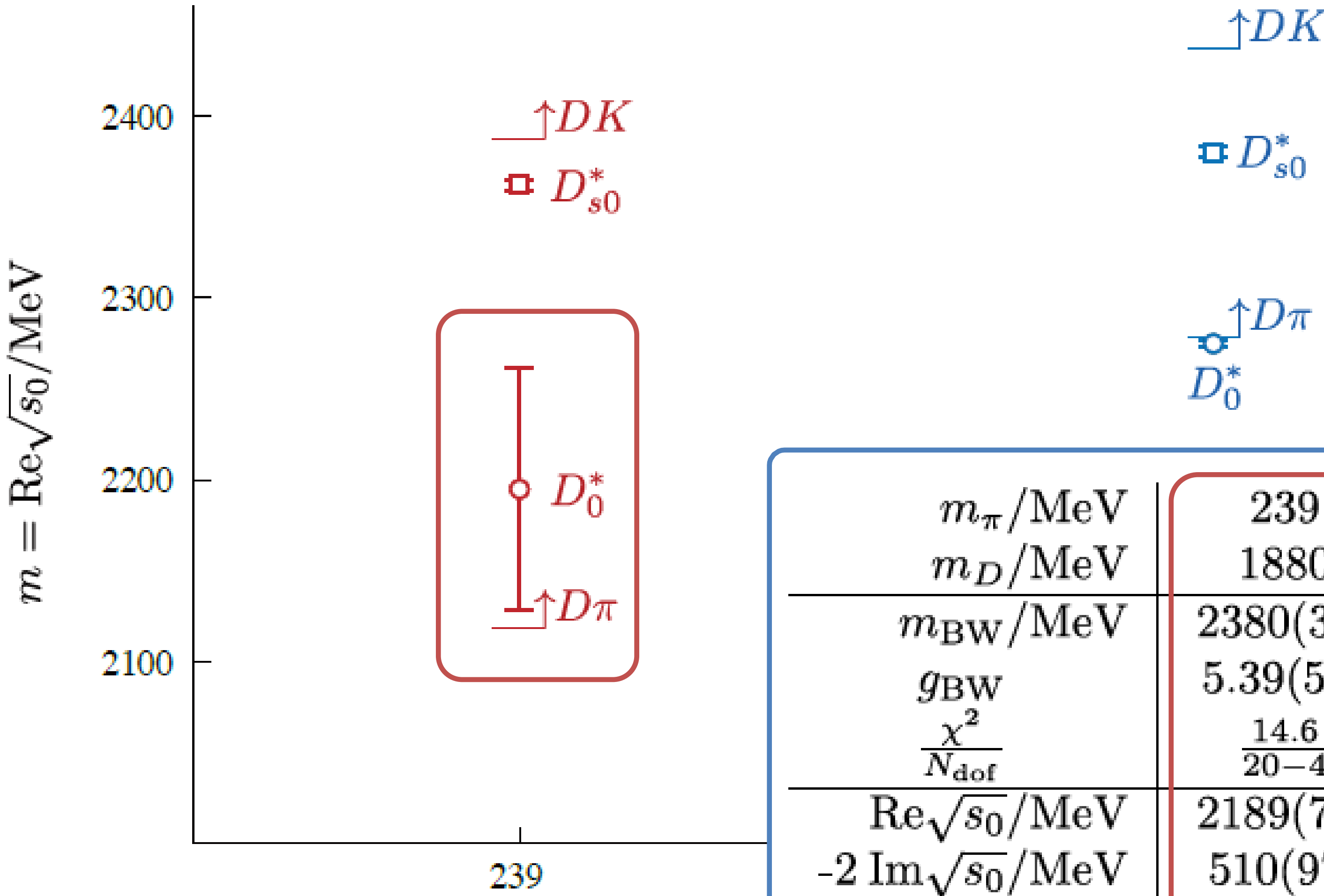
[JHEP 07 (2021) 123, JHEP 02 (2021) 100, JHEP 10 (2016), 011]



m_π/MeV	239	391
m_D/MeV	1880	1887
$m_{\mathrm{BW}}/\mathrm{MeV}$	2380(36)	2206(32)
g_{BW}	5.39(56)	7.62(75)
$\frac{\chi^2}{N_{\mathrm{dof}}}$	$\frac{14.6}{20-4}$	$\frac{36.0}{29-5}$
$\mathrm{Re}\sqrt{s_0}/\mathrm{MeV}$	2189(72)	2275(1)
$-2\,\mathrm{Im}\sqrt{s_0}/\mathrm{MeV}$	510(97)	-
$\lvert c\rvert/\mathrm{MeV}$	2391(411)	826(133)

DK and Dπ – S-wave poles

[JHEP 07 (2021) 123, JHEP 02 (2021) 100, JHEP 10 (2016), 011]



m_π/MeV	239	391
m_D/MeV	1880	1887
m_{BW}/MeV	2380(36)	2206(32)
g_{BW}	5.39(56)	7.62(75)
$\frac{\chi^2}{N_{\mathrm{dof}}}$	$\frac{14.6}{20-4}$	$\frac{36.0}{29-5}$
$\mathrm{Re}\sqrt{s_0}$/MeV	2189(72)	2275(1)
-2 $\mathrm{Im}\sqrt{s_0}$/MeV	510(97)	-

D_0^* pole position may be lower than currently reported exp. mass.
(See also Du *et al*, PRL 126, 192001 (2021), 2012.04599)

SU(3) flavour symmetry

[JHEP 02 (2021) 100]

SU(3) flavour symmetry

[JHEP 02 (2021) 100]

SU(3) multiplets:

$D_{(s)}$ $\overline{\mathbf{3}}$ Light/strange meson $\mathbf{8}$ or $\mathbf{1}$

$$\overline{\mathbf{3}} \otimes \mathbf{8} \rightarrow \overline{\mathbf{3}} \oplus \mathbf{6} \oplus \overline{\mathbf{15}}, \quad \overline{\mathbf{3}} \otimes \mathbf{1} \rightarrow \overline{\mathbf{3}}$$

SU(3) flavour symmetry

[JHEP 02 (2021) 100]

SU(3) multiplets:

$D_{(s)}$ $\overline{\mathbf{3}}$ Light/strange meson $\mathbf{8}$ or $\mathbf{1}$

$$\overline{\mathbf{3}} \otimes \mathbf{8} \to \overline{\mathbf{3}} \oplus \mathbf{6} \oplus \overline{\mathbf{15}}, \quad \overline{\mathbf{3}} \otimes \mathbf{1} \to \overline{\mathbf{3}}$$

$(I=0)$ DK-$D_s\eta$: $\overline{\mathbf{3}} \oplus \overline{\mathbf{15}}$

$(I=\frac{1}{2})$ $D\pi$-$D\eta$-$D_s\bar{K}$: $\overline{\mathbf{3}} \oplus \mathbf{6} \oplus \overline{\mathbf{15}}$

$(I=1)$ DK-$D_s\pi$: $\mathbf{6} \oplus \overline{\mathbf{15}}$

$(I=0)$ $D\bar{K}$: $\mathbf{6}$

$(I=\frac{1}{2})$ D_sK, $(I=1)$ $D\bar{K}$, $(I=\frac{3}{2})$ $D\pi$: $\overline{\mathbf{15}}$

SU(3) flavour symmetry

[JHEP 02 (2021) 100]

SU(3) multiplets:

$D_{(s)}$ $\overline{\mathbf{3}}$ Light/strange meson $\mathbf{8}$ or $\mathbf{1}$

$$\overline{\mathbf{3}} \otimes \mathbf{8} \to \overline{\mathbf{3}} \oplus \mathbf{6} \oplus \overline{\mathbf{15}}, \quad \overline{\mathbf{3}} \otimes \mathbf{1} \to \overline{\mathbf{3}}$$

$(I=0)$ DK-$D_s\eta$: $\overline{\mathbf{3}} \oplus \overline{\mathbf{15}}$

$(I=1)$ DK-$D_s\pi$: $\mathbf{6} \oplus \overline{\mathbf{15}}$

$(I=\frac{1}{2})$ $D\pi$-$D\eta$-$D_s\bar{K}$: $\overline{\mathbf{3}} \oplus \mathbf{6} \oplus \overline{\mathbf{15}}$

$(I=0)$ $D\bar{K}$: $\mathbf{6}$

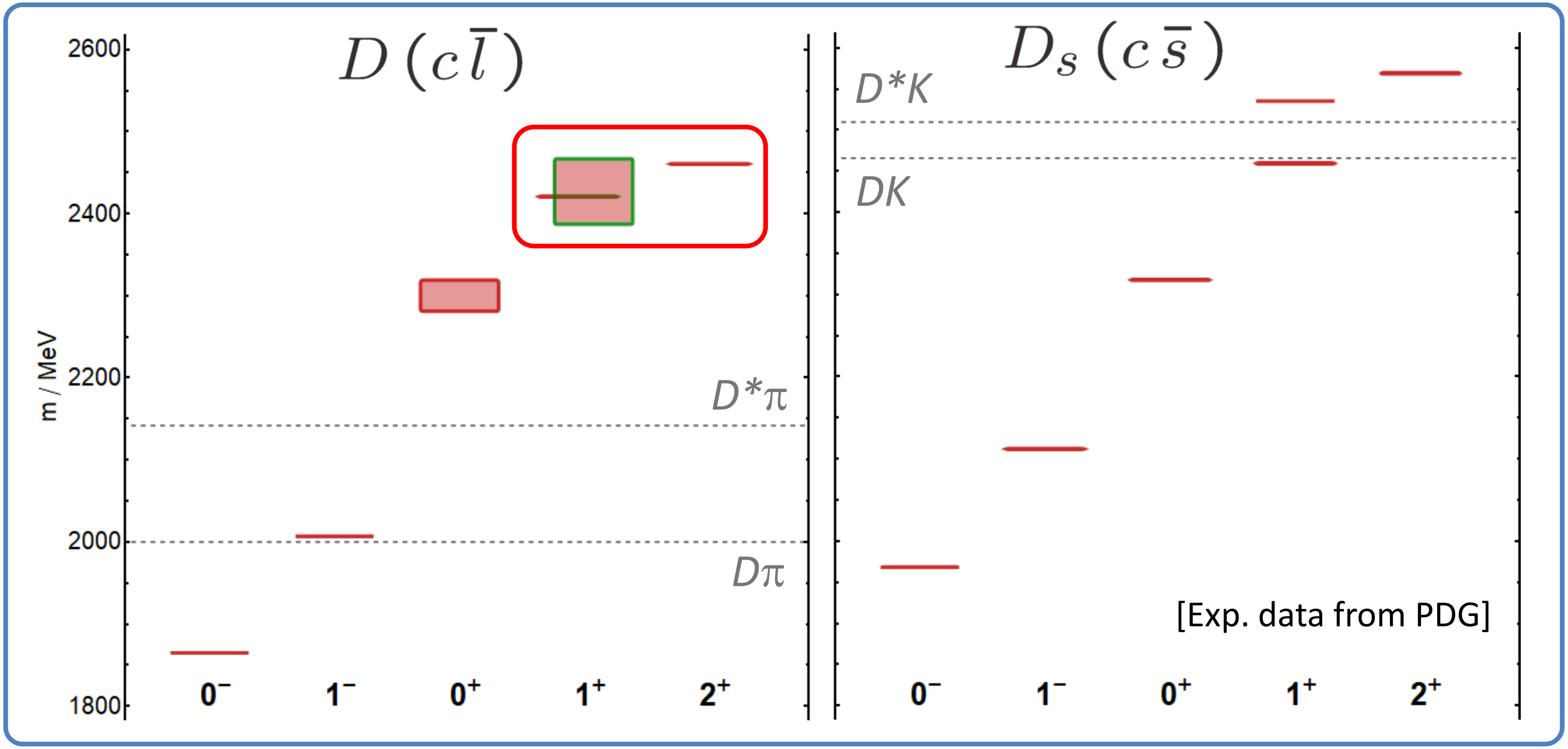
$(I=\frac{1}{2})$ D_sK, $(I=1)$ $D\bar{K}$, $(I=\frac{3}{2})$ $D\pi$: $\overline{\mathbf{15}}$

S-wave results [broken SU(3)] suggest:

- $\overline{\mathbf{3}}$ resonance/bound state
- $\mathbf{6}$ virtual bound state
- $\overline{\mathbf{15}}$ weak repulsion

[See also PR D87, 014508 (2013) (1208.4535); PL B767, 465 (2017) (1610.06727); PR D98, 094018 (2018) (1712.07957); *PR D98* 014510 (2018) (1801.10122); *EPJ C79, 13 (2019)* (1811.05585); arXiv:2106.15391]

Charm (D) and charm-strange (D_s) mesons



$D^* \pi$ (isospin=1/2)

[**Lang**, Wilson (HadSpec), arXiv:2205.05026]

Scattering involving non-zero spin hadrons [see also Woss, CT, Dudek, Edwards, Wilson, arXiv:1802.05580 (JHEP)]

$J = \ell \otimes S$ and different partial waves with the same J^P can mix dynamically,

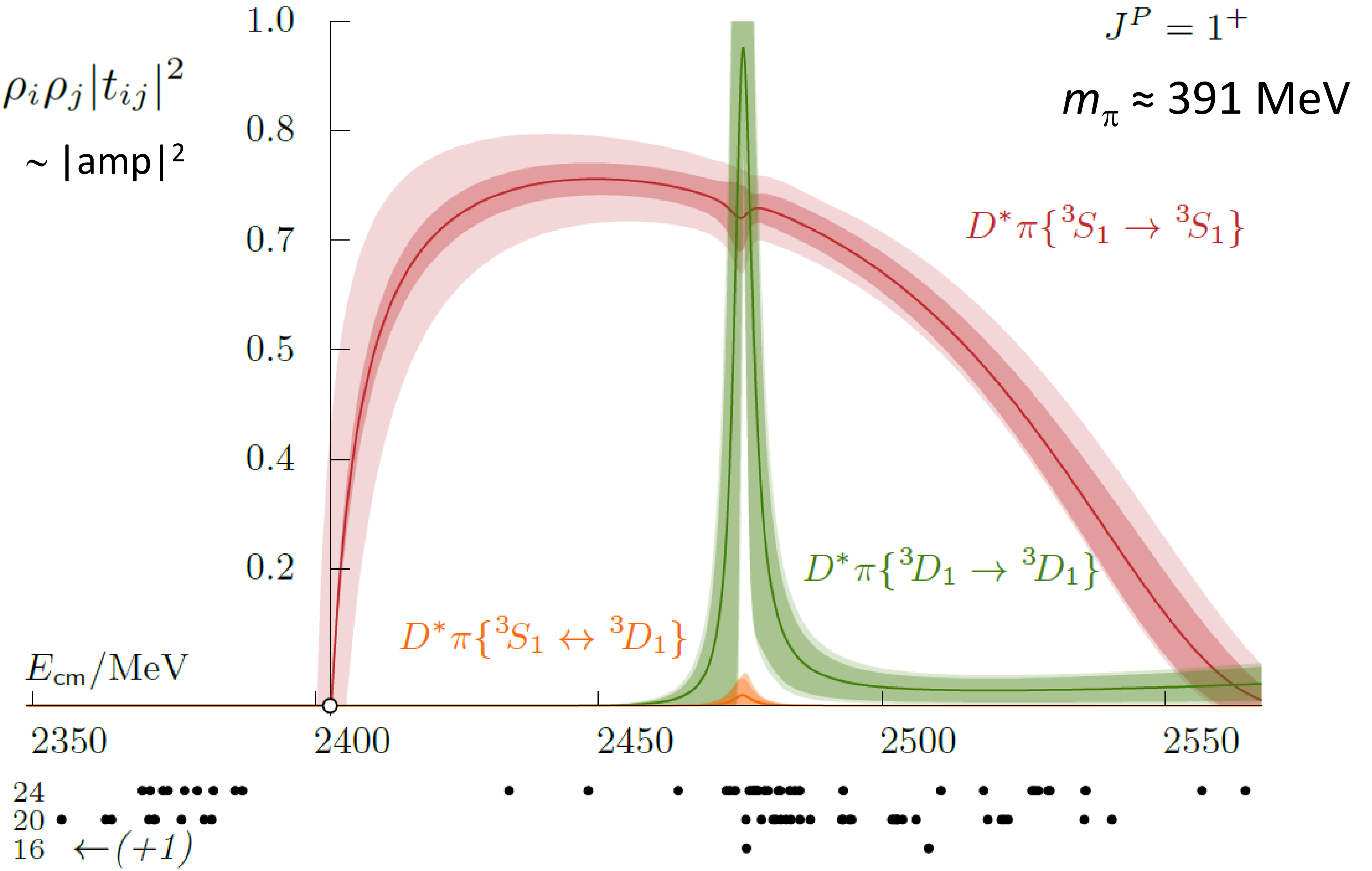
e.g. $J^P = 1^+$ $({}^{2S+1}\ell_J = {}^3S_1, {}^3D_1)$

$$\mathbf{t} = \begin{bmatrix} t({}^3S_1|\,{}^3S_1) & t({}^3S_1|\,{}^3D_1) \\ t({}^3S_1|\,{}^3D_1) & t({}^3D_1|\,{}^3D_1) \end{bmatrix}$$

Finite-volume lattice QCD: reduced sym → additional 'mixing'

D* π (isospin=1/2)

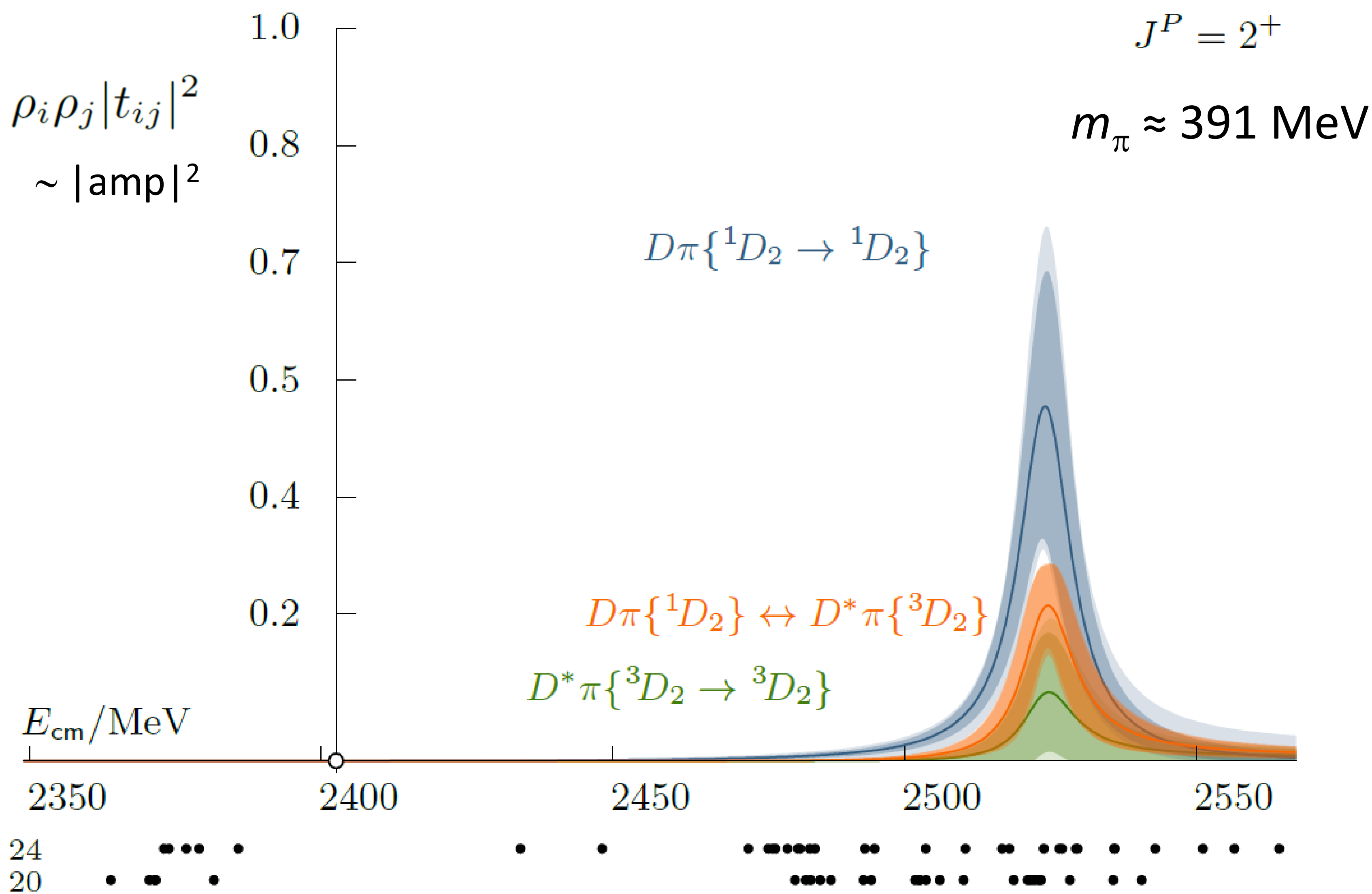
[arXiv:2205.05026]



94 energies to constrain $J^P = 1^+, 2^+, 0^-, 1^-, 2^-$

D* π (isospin=1/2)

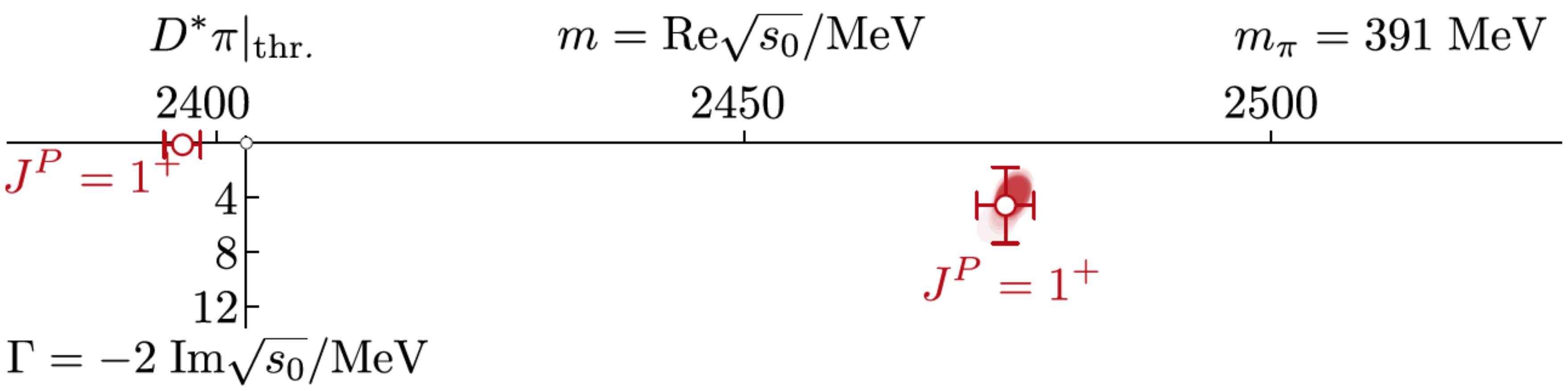
[arXiv:2205.05026]



94 energies to constrain $J^P = 1^+, 2^+, 0^-, 1^-, 2^-$

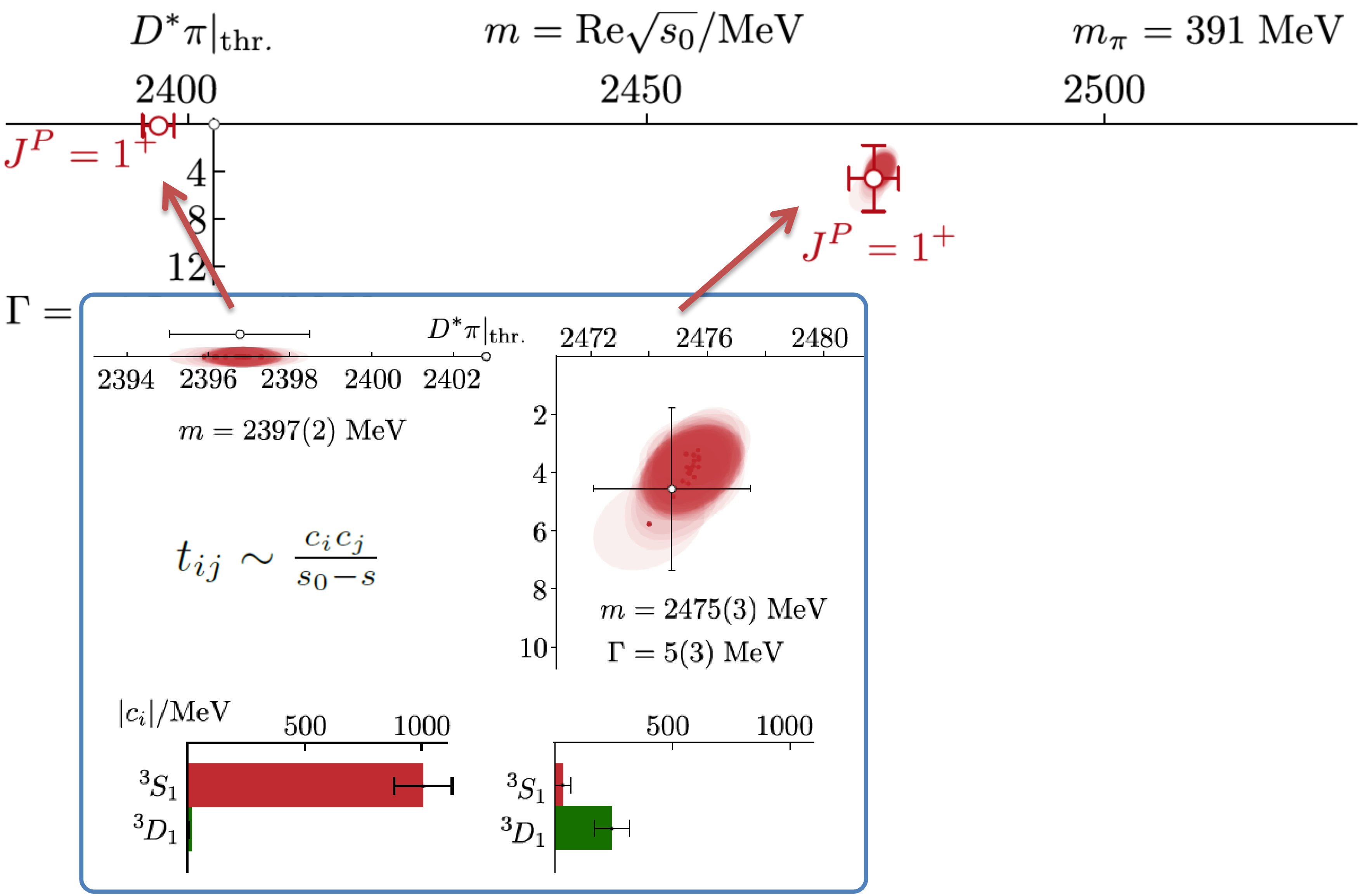
D* π (isospin=1/2) – poles

[arXiv:2205.05026]



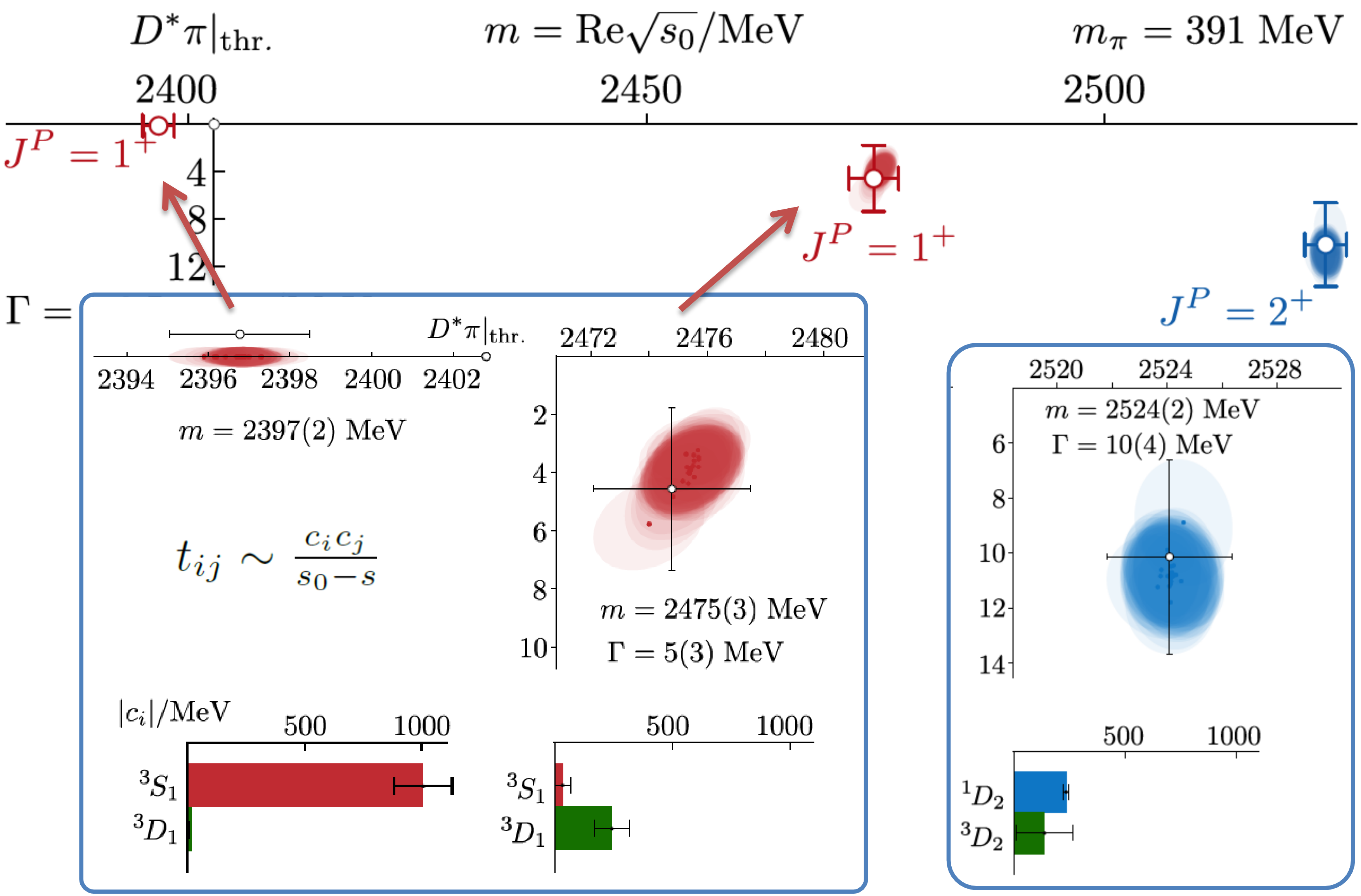
D* π (isospin=1/2) – poles

[arXiv:2205.05026]



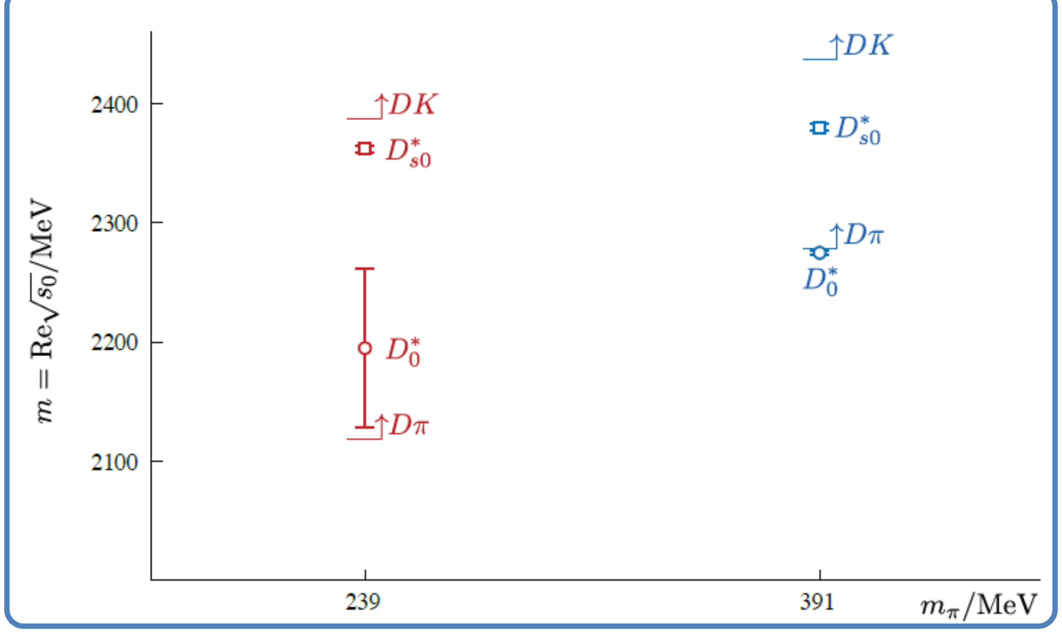
D* π (isospin=1/2) – poles

[arXiv:2205.05026]



Summary

- **Mapping out energy-dependence** of **scattering amplitudes** using lattice QCD. A few examples.
- *DK*, *Dπ*, **exotic-flavour** isospin-0 $D\bar{K}$, *D*π*
- Lighter (or heavier) light quarks? With SU(3) flavour sym?
- Further up in energy, inelastic scattering (3-meson scattering)



Acknowledgements

UNIVERSITY OF CAMBRIDGE

Science and Technology Facilities Council

DiRAC

Hadron Spectrum Collaboration

[*www.hadspec.org*]

hadspec

Jefferson Lab and surroundings, USA:

JLab: Robert Edwards, Jie Chen, Frank Winter, Arkaitz Rodas

W&M: Jozef Dudek[1], F*elipe Ortega;* ODU: Raúl Briceño[1], Andrew Jackura

ORNL: Bálint Joó ([1] and Jefferson Lab)

Trinity College Dublin, Ireland: Michael Peardon, Sinéad Ryan, *Nicolas Lang*

UK: University of Cambridge: CT, David Wilson, *Daniel Yeo, James Delaney*

Edinburgh: Max Hansen; Southampton: Bipasha Chakraborty

Tata Institute, India: Nilmani Mathur

Ljubljana, Slovenia: Luka Leskovec