NORMAL SINGLE SPIN ASYMMETRIES IN ELECTRON SCATTERING
1/Nc EXPANSION APPROACH

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Phys Lett B835 (2022) 137580, and work in preparation

JLab, January 18, 2023  seminar

SINGLE SPIN ASYMMETRIES IN ELECTRON-NUCLEON SCATTERING

• Polarized beam or target

• Longitudinally polarized electron beam: test of Parity Violation (HERMES, HAPPEx, QWeak, SAMPLE)

• Transversely polarized target or beam (NSSA): test of spin effects at hadronic and partonic level: e.g. two photon exchange, Collins and Sivers functions

• SSAs in exclusive, semi-inclusive and inclusive processes

• historical works that initiated the topic of SSAs: Barut & Fronsdal (1960); Leroy & Piketty; De Rujula, Kaplan and De Rafael; Gunther and Rodenberg; ...

• numerous works since the 2000's on TPE and SSA's: Guichon & Vanderhaeghen; Blunden, Melnitchouk & Tjon; Gorchtein et al; Afanasev et al.; Myhrer et al; in DIS: Metz, Schlegel & Goeke; Afanasev, Strikman & Weiss; many more...

• NSSAs measured in high energy regime (Pol target: COMPASS, HERMES, JLab Hall A)

(Pol beam: A4 at MAMI, JLab Hall C, SAMPLE at Bates). Very sparse kinematic ranges

$$d\sigma_{SSA} \propto \vec{k} \cdot \vec{S} \qquad\qquad d\sigma_{SSA} \propto (\vec{k} \wedge \vec{k}') \cdot \vec{S}$$

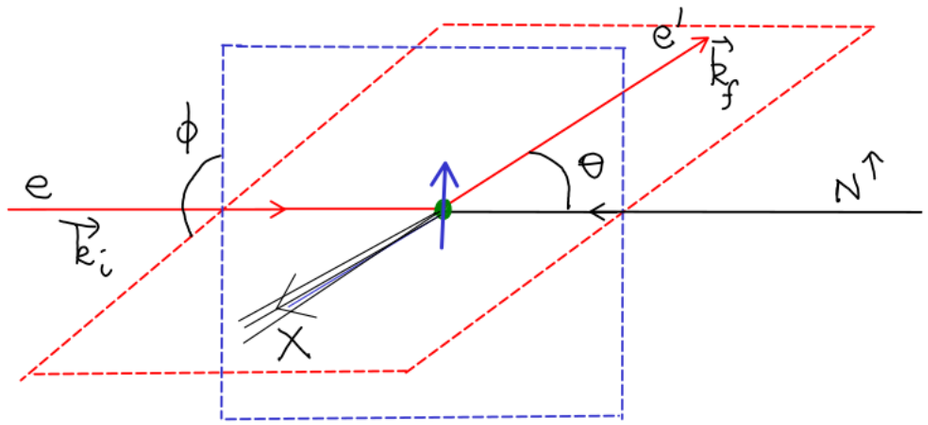
OUTLINE

- Normal single spin asymmetries in electron scattering and two photon exchange - generalities
- Target normal SSA- 1/Nc expansion approach
- Results
- Comments and summary

NORMAL SINGLE SPIN ASYMMETRIES - GENERALITIES

NSSA linear in spin vector

Parity invariance then requires two momentum vectors



$$d\sigma = d\sigma_U + 2\vec{e}_N \cdot \vec{S}\, d\sigma_N$$

$$\vec{e}_N = \frac{\vec{k}_f \times \vec{k}_i}{|\vec{k}_f \times \vec{k}_i|}$$

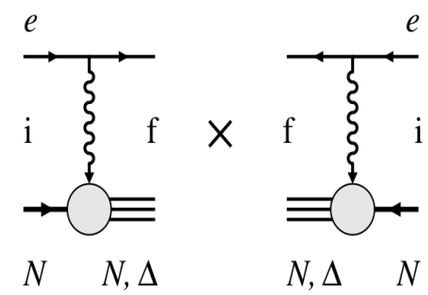
$$A_N = d\sigma_N / d\sigma_U$$

Cases with more momenta (eg semi-inclusive scattering) will present more combinations of this type (eg for accessing Collins and Sivers functions)

QED+QCD: T reversal symm forbids tree level SSA (Barut-Fronsdal (1960))

SM: has CP violation = T violation: tree level possible but too small to be observable (Christ-Lee (1966))

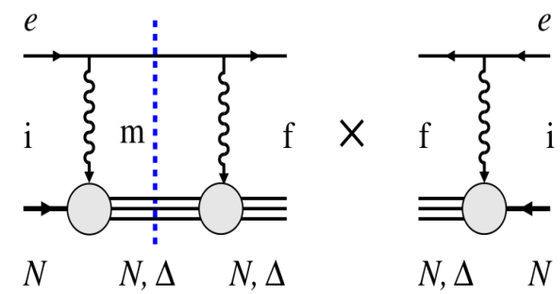
T reversal symmetry allows SSA in the presence of absorptive part of scattering amplitude (Barut-Fronsdal (1960))



Absorptive part of amplitude results naturally from FSIs

In inclusive case, the FSI is EM: the absorptive part then starts at order e^4

Driven by TPE: absorptive part of box diagram



- Only states with invariant mass below $\sqrt{s}$ can contribute
- $d\sigma_N$ is free of collinear and IR divergencies: key role played by gauge invariance
- NSSA can be selected for exclusive and inclusive final states
- In principle electroproduction helicity amplitudes determine elastic NSSA – limited by range of experimentally established amplitudes
- Energy regimes:

  1) low/intermediate up to onset of second resonance region

  2) Resonance region: 1.5 to few GeV

  3) High energy region: e.g., partonic regime

FRAMEWORK FOR TSSA BELOW SECOND RESONANCE REGION

- few degrees of freedom N, Δ, πN continuum
- unifying framework that treats this regime based on those dof and a rigorous
  - QCD expansion: 1/Nc, and at low energy in addition Chiral Perturbation Theory
- implement 1/Nc expansion with N and Δ, and argue about the subleading role of the πN continuum

1/Nc EXPANSION

- fundamental expansion in QCD: many phenomenological successes; LQCD also
- recent tests in LQCD with Nc>3
- can be implemented at hadronic level
- particularly powerful for baryons:
- at large Nc the baryon sector develops a spin-flavor SU(4) or SU(6) dynamical symmetry
- N and Δ belong to an SU(4) multiplet: unification of properties
- breaking of SU(4) by subleading orders in 1/Nc: formalism to implement higher orders 1/Nc
- Nc scalings of basic observables:

$$m_{N,\Delta} = O(N_c) \qquad m_\Delta - m_N = O(1/N_c)$$
$$\Gamma_\Delta = O(1/N_c^2) \qquad g_{\pi N} = O(\sqrt{N_c})$$
$$F_\pi = O(\sqrt{N_c})$$

Kinematic range

	Energy regime	$1/N_c$ expansion regime	Channels open	Final states possible
I	$m_N < \sqrt{s} < m_\Delta$	$\sqrt{s} - m_N \sim N_c^{-1}, \quad k \sim N_c^{-1}$	N	elastic
II	$m_\Delta < \sqrt{s} \ll m_{N*}$	$\sqrt{s} - m_N \sim N_c^{-1}, \quad k \sim N_c^{-1}$	N, Δ	elastic or inelastic
III	$m_\Delta < \sqrt{s} \lesssim m_{N*}$	$\sqrt{s} - m_N \sim N_c^{0}, \quad k \sim N_c^{0}$	N, Δ, N^*(suppr)	elastic or inelastic

Calculation will cover the three regimes

Expansions

I & II - Low energy combined with $1/N_c$:

$O(p) = O(1/N_c) = O(\xi)$: BChPT x $1/N_c$

III - $1/N_c$ expansion only

SYSTEMATIC EXPANSION IN 1/Nc: LO AND NLO FOR NSSA

In regime of interest 1/Nc expansion implies NR expansion:
baryons have small velocities O(1/Nc) in CM frame

Expand the Baryon EM current to first subleading order in 1/Nc: O(Nc) & O(Nc^0)

$$J_{EM\,\mu} = \frac{1}{2} J^S_\mu + J^V_\mu$$

$$J^S_\mu = g_{\mu 0} G^S_E(t) - \frac{i}{\Lambda} G^S_M(t) \epsilon_{0\mu ij} q^i \hat{S}^j$$

$$J^V_\mu = g_{\mu 0} G^V_E(t) \hat{I}^3 - \frac{i}{\Lambda} \frac{6}{5} G^V_M(t) \epsilon_{0\mu ij} q^i \hat{G}^{j3}$$

$$\Lambda = m_N^{\rm phys} :$$

$$G^S_E = G^V_E = G^p_E$$

$$G^S_M = G^p_M + G^n_M$$

$$G^V_M = G^p_M - G^n_M$$

$\{\hat{S}^i, \hat{I}^a, \hat{G}^{ia}\}$ generators of SU(4)

G_{ia} spin-flavor generators of SU(4)- connect baryons of different spin

$$< B' | G_{ia} | B >= O(N_c)$$

EM current has O(Nc) term: isotriplet magnetic

Nc hierarchy in isosinglet vs isotriplet N magnetic moments

EM current has O(Nc) term: isotriplet magnetic

Nc hierarchy in isosinglet vs isotriplet N magnetic moments

$$\mu_0 = \frac{1}{2}(\mu_p + \mu_n) = 0.44\mu_N; \quad \mu_3 = \frac{1}{2}(\mu_p - \mu_n) = 2.35\mu_N$$

Subleading terms in current

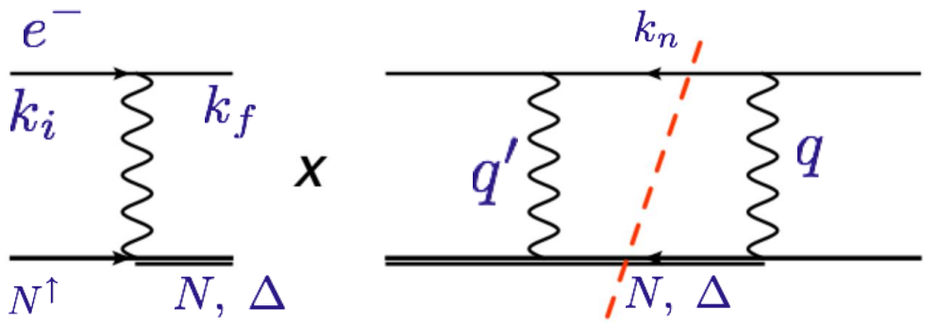
convection current: $(\vec{p}_i + \vec{p}_f)/2m_N = O(1/N_c)$

electric quadrupole current: $O(1/N_c^2)$

neglected term: $\frac{1}{m_N\Lambda} g^{\mu 0}\epsilon^{0ijk} q^i (p_i + p_f)^j G^{ka} = O(N_c^0)$

$$= O(\xi^3) \text{ for } q = O(1/N_c)$$

CALCULATION OF THE TARGET NSSA



$$e_N^\mu a_\mu \frac{d\sigma_{N_{fi,n}}}{d\Omega} = \frac{\alpha^3}{16\pi}\frac{k_f}{k_i}\frac{m_N m_{B_f} m_n}{t s^{3/2} k_i k_f k_n}\,\mathrm{Im}\Big(\int d\Omega_{\hat{\mathbf{k}}_\mathbf{n}} \frac{L_{\mu\nu\rho}(k_\mathrm{i},k_\mathrm{f},k_n) H^{\mu\nu\rho}_{\mathrm{fi,n}}(k_\mathrm{i},k_\mathrm{f},k_n)}{(1-\hat{\mathbf{k}}_\mathrm{i}\cdot\hat{\mathbf{k}}_\mathbf{n})(\mathbf{1}-\hat{\mathbf{k}}_\mathrm{f}\cdot\hat{\mathbf{k}}_\mathbf{n})}\Big)$$

Leptonic tensor

$$L^{\mu\nu\rho}(k_\mathrm{i},k_\mathrm{f},k_n) = Tr(\not{k}_\mathrm{i}\gamma^\mu \not{k}_\mathrm{f}\gamma^\nu \not{k}_n\gamma^\rho)$$

Hadronic tensor

$$H^{\mu\nu\rho}_{\mathrm{fi},n}(k_\mathrm{i},k_\mathrm{f},k_n) = \langle B_\mathrm{i} \mid (J^\mu_{\mathrm{EM}}(k_\mathrm{i}-k_\mathrm{f}))^\dagger \mid B_\mathrm{f}\rangle$$
$$\times \langle B_\mathrm{f} \mid J^\nu_{\mathrm{EM}}(k_n-k_\mathrm{f})\mathcal{P}_n J^\rho_{\mathrm{EM}}(k_\mathrm{i}-k_n) \mid B_\mathrm{i}\rangle$$

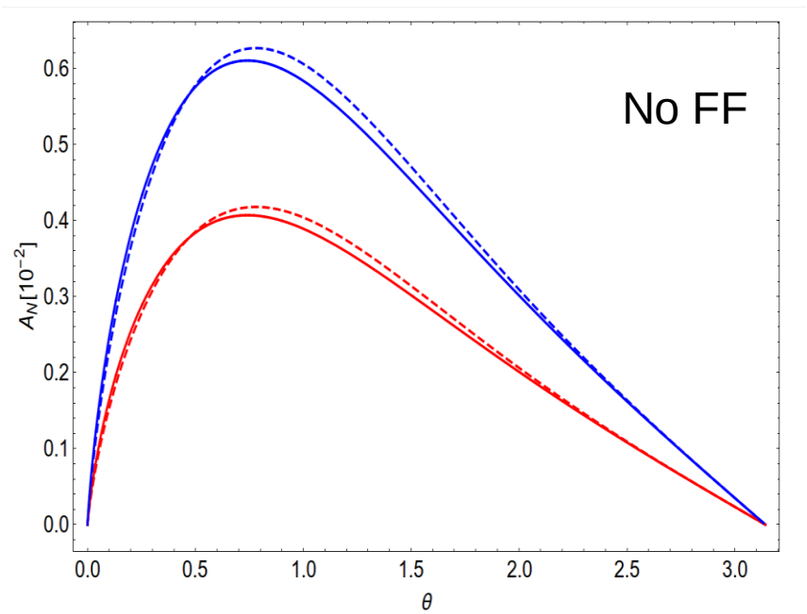
- SSA needs t-channel J=1 components of hadronic tensor, I=0, 1
- Box has t-channel J_{Box}=0,1 for B_f=N, and J_{Box}=1,2 for B_f=Δ
- Strict large Nc limit: only isotriplet spin current contributes: LO given by

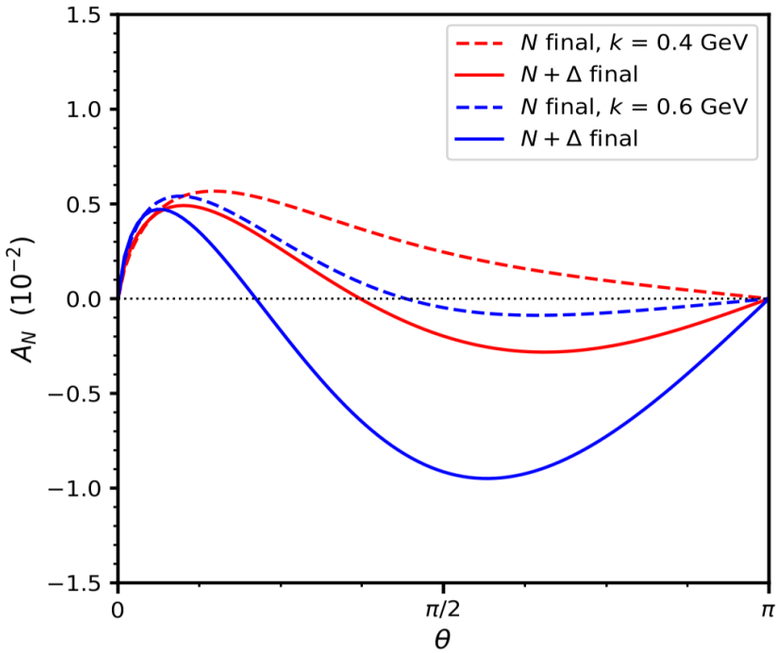
  J=I and $J_{Box}=I_{Box}$ (I=J rule of large Nc)
- Subleading corrections: I≠J with LO current and subleading currents
- Q^2 range up to $4E_e^2$ - need to include FFs in calculation: common dipole form for E and M components
- Integrals of box have IR and/or collinear divergencies: they cancel for each combination of terms of the hadronic current and each for the corresponding J_{Box} and I_{Box} and for different B_f and B_n – good check for calculation
- Calculation checked with known purely elastic case

A FIRST CALCULATION: LO IN 1/Nc – LARGE Nc LIMIT

JLG, Weiss, Willemyns
Phys Lett B835 (2022) 137580

- given in terms of the isovector magnetic current
- match magnetic coupling at Nc=3 to physical one
- pick LO terms in hadronic tensor: LO given by I=J t-channel rule
- simple results without t-dependence of FF – shows dominance of the elastic SSA with intermediate state contribution of N equal ½ of the Δ
- Inclusion of FF makes major difference: inelastic channel, Bf=Δ, becomes large and of opposite sign to elastic – non-trivial FF interplay in box





NLO CALCULATION

Weiss, Willemyns, JLG in preparation

- decompose hadronic tensor into t-channel irreducible angular momentum and isospin components: SU(4) algebra organizes contributions in powers of 1/Nc
- separate effects by N and Δ channels in the box and final state
- consistent cancellations of IR and collinear divergences for each channel and for gauge invariant combinations of EM current components
- validated with NR expansion of the known purely elastic case
- calculation without t-dependence in FFs
- inclusion of FFs and Δ width

Results without FFs

$$\frac{d\sigma_N}{d\Omega}(N_\mathrm{i}, N, N) = \frac{1}{4}k^2 m_N^2 \Big(k(G_M^{I_3}(N_c+7) - G_M^{-I_3}(N_c-3)) - 10 G_E^{I_3}\Lambda\Big)$$
$$\times \Big(G_M^{I_3}(N_c+7) - G_M^{-I_3}(N_c-3)\Big)^2 \Big((3+x)\log\frac{1-x}{2} - 2(1+x)\Big)\,\mathbf{X}$$

$$\frac{d\sigma_N}{d\Omega}(N_\mathrm{i}, N, \Delta) = \theta(k_\Delta) k_\Delta m_\Delta m_N^3 (N_c+5)(N_c-1)(G_M^p - G_M^n)^2$$
$$\times \Big(k(G_M^{I_3}(N_c+7) - G_M^{-I_3}(N_c-3)) + 5 G_E^{I_3}\Lambda\Big)$$
$$\times \Big(\Big(\frac{\mathbf{Y}}{2m_N^4} - \frac{k}{m_N^2}(1+x)\Big)\log\frac{1-x}{2} - \frac{k^2}{\mathbf{Y}}(1+x)\Big)\,\mathbf{X}$$

$$\frac{d\sigma_N}{d\Omega}(N_\mathrm{i}, \Delta, N) = \theta(k_\Delta)\frac{1}{32}\frac{m_\Delta}{m_N}(N_c+5)(N_c-1)(G_M^p - G_M^n)^2\mathbf{Y}$$
$$\times \Big((1+x)\Big((G_M^{I_3}(N_c+7) - G_M^{-I_3}(N_c-3))(11k^2 - kk_\Delta + 4k_\Delta^2) + 4G_E^{I_3}(k-k_\Delta)\Lambda\Big)$$
$$+ 2\Big(\Big(G_M^{I_3}(N_c+7) - G_M^{-I_3}(N_c-3)\Big)(2k^2 + 2(2-x)k_\Delta^2 + 3(1-x)kk_\Delta)$$
$$+ 20 G_E^{I_3}(2k - (1+x)k_\Delta)\Lambda\Big)\log\frac{1-x}{2}\Big)\mathbf{X}$$

$$\frac{d\sigma_N}{d\Omega}(N_\mathrm{i}, \Delta, \Delta) = \theta(k_\Delta)\frac{1}{160}\frac{k_\Delta m_\Delta^2}{k\, m_N^2}(N_c+5)(N_c-1)\mathbf{Y}(G_M^p - G_M^n)^2$$
$$\times \Big((1+x)\Big((23k_\Delta - 9k)(G_M^{I_3}(N_c+27) - G_M^{-I_3}(N_c-23)) + 200 G_E^{I_3}(k-k_\Delta)\Lambda\Big)$$
$$+ 2\Big((G_M^{I_3}(N_c+27) - G_M^{-I_3}(N_c-23))(6(k^2+k_\Delta^2) - (3+5x)kk_\Delta)$$
$$+ 100 G_E^{I_3}((1+x)k - 2k_\Delta)\Lambda\Big)\log\frac{1-x}{2}\Big)\,\mathbf{X}$$

$$x = \cos\theta$$

$$\frac{1-x}{2} = \sin^2\frac{\theta}{2}$$

$$\mathbf{X} = \frac{m_N \alpha^3}{1000\, s^{3/2} t(1+x)\Lambda^3}\hat{\mathbf{k}}_\mathrm{f} \times \hat{\mathbf{k}}_\mathrm{i}\cdot\mathbf{S}$$

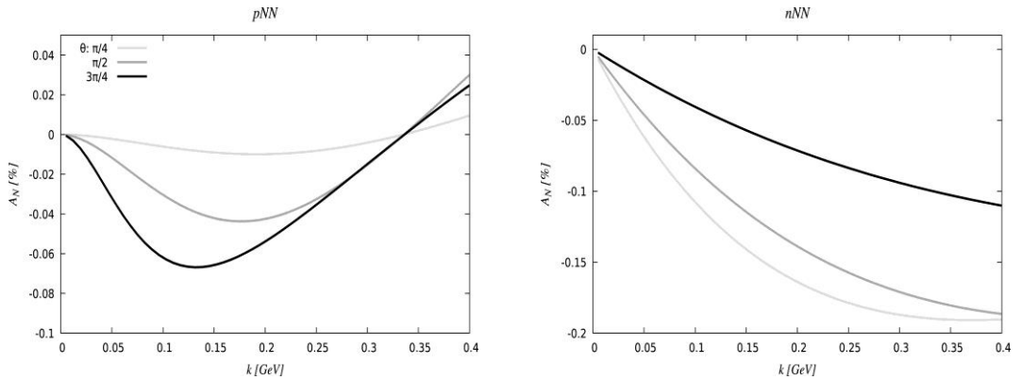
$$\mathbf{Y} = \sqrt{k^2 + m_N^2}\,(m_N^2 - m_\Delta^2) + k\,(m_N^2 + m_\Delta^2)$$

$$A_N = \frac{d\sigma_N}{d\sigma_{U\,\mathrm{elastic}}}$$

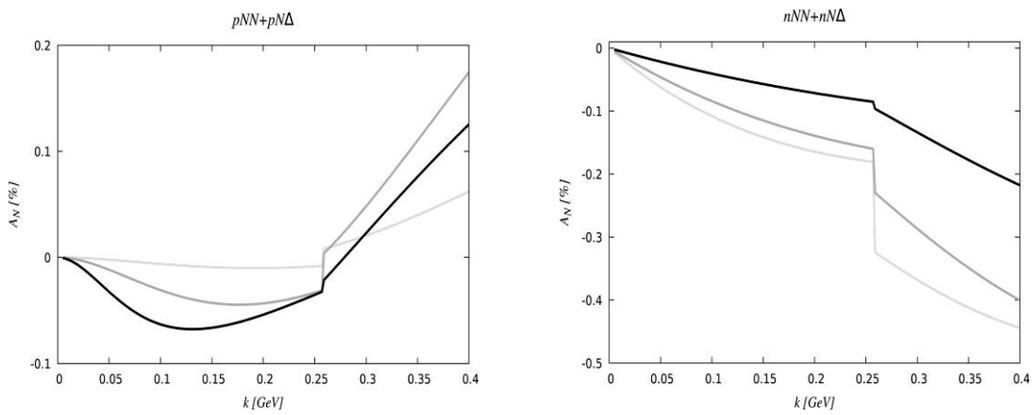
no FF, vanishing Γ_Δ

- Electric component of current only appears linearly
- Large Nc limit checked
- Dominance of elastic asymmetry
- Very small inelastic asymmetry
- Kinematics taken exactly
- Threshold enhancement at Δ mass
- Δ in box gives important contribution ~ twice of N

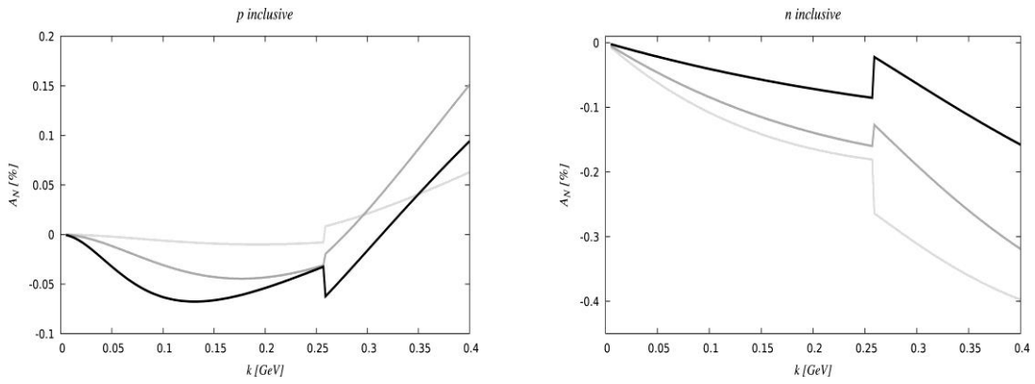
Elastic with only N in box



Elastic with N and Δ in box

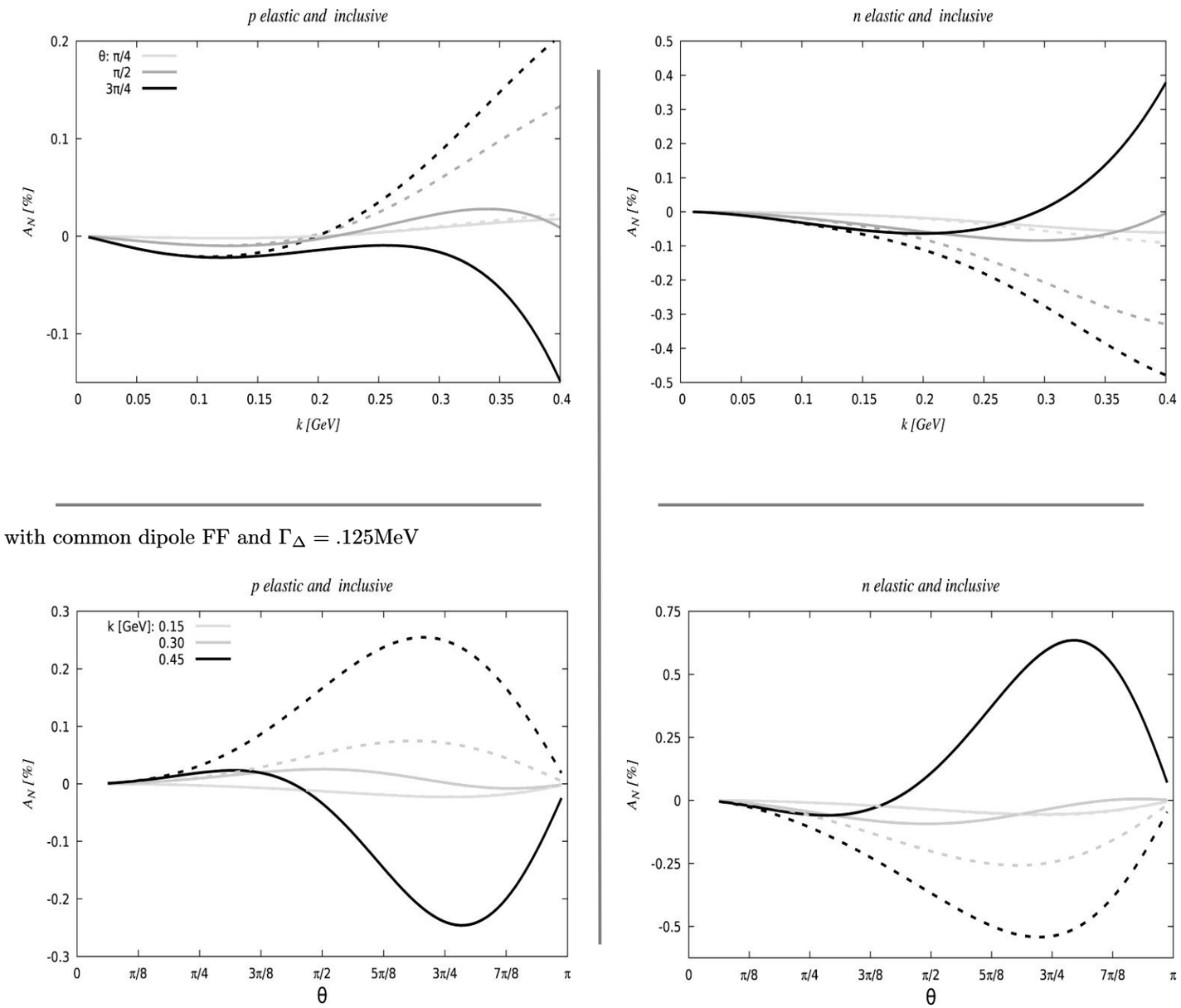


Full inelastic



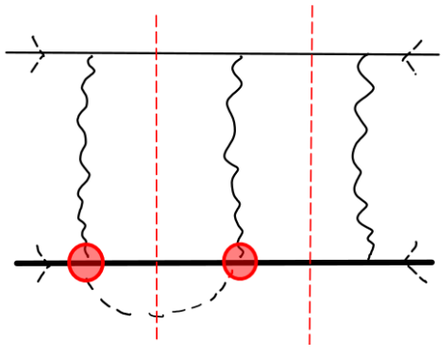
A_N with common dipole FF and $\Gamma_\Delta = 125\text{MeV}$

- FF play crucial role – enhances inelastic channel
- inelastic channel gives opposite sign asymmetry to elastic one: changes sign of inclusive asymmetry
- sensitive energy dependence of asymmetry – we keep kinematic factors exact



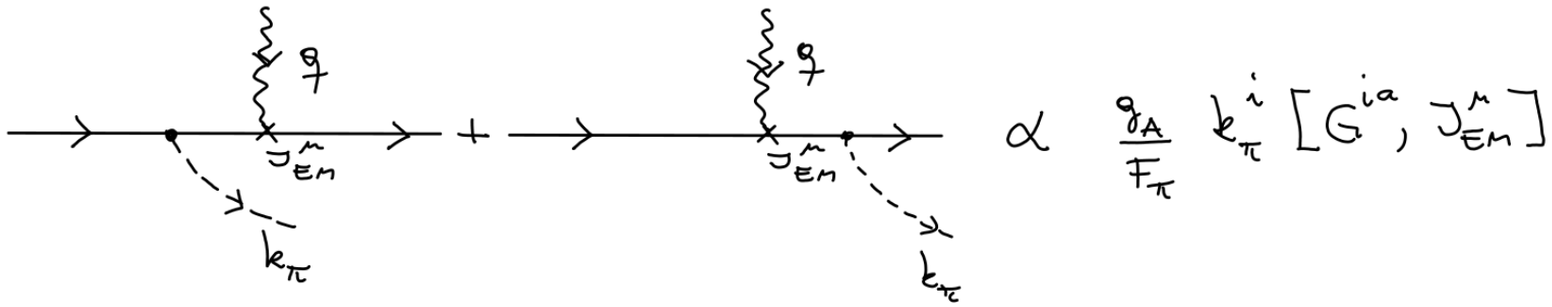
Other TSSA contributions

- πN Continuum



$$\pi \text{ baryon coupling: } \propto \frac{g_A}{F_\pi} k_\pi^i G^{ia} = O(\sqrt{N_c})$$

$$[G^{ia}, G^{ib}] = O(N_c^0); \quad [G^{ia}, S^i] = O(N_c); \quad [G^{ia}, I^b] = O(N_c)$$

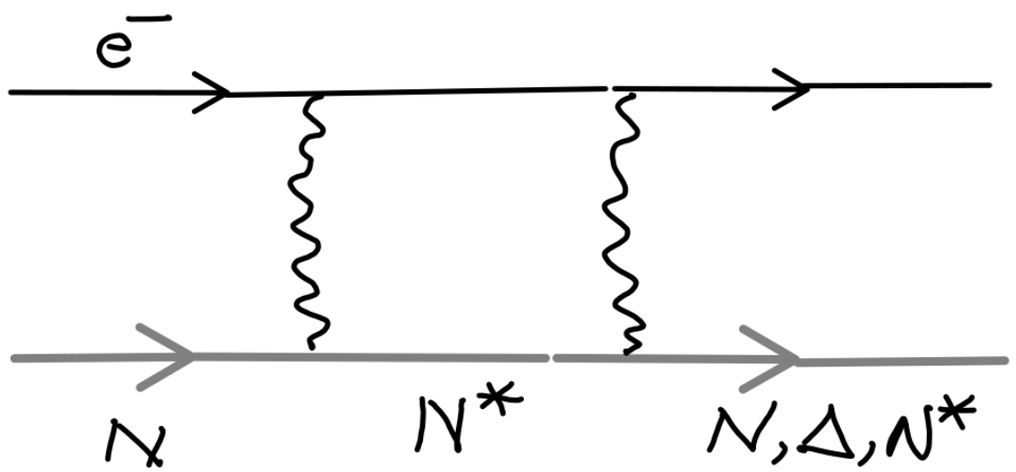


O($1/N_c$) wrt LO, and O(ξ^3) wrt LO term in ξ expansion domain

- N* resonances

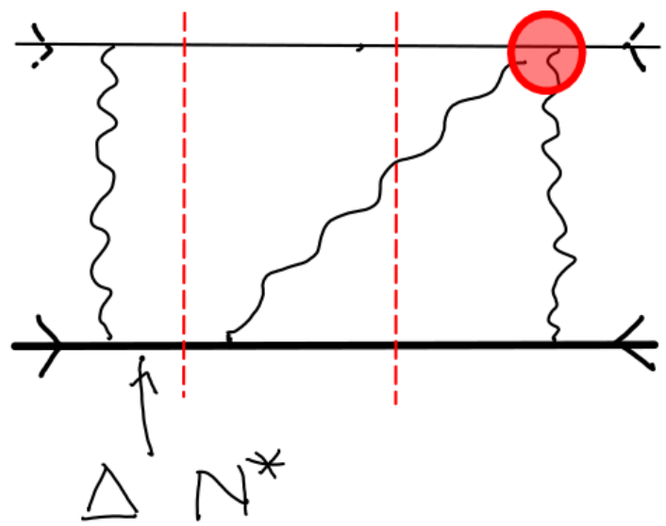
EM N-N* couplings carry extra factor $1/\sqrt{N_c}$

Individual resonances contribute $\mathrm{O}(1/N_c)$ wrt N and Δ



Case of a few resonances was studied (Vanderhaeghen et al) - still a lot to be done

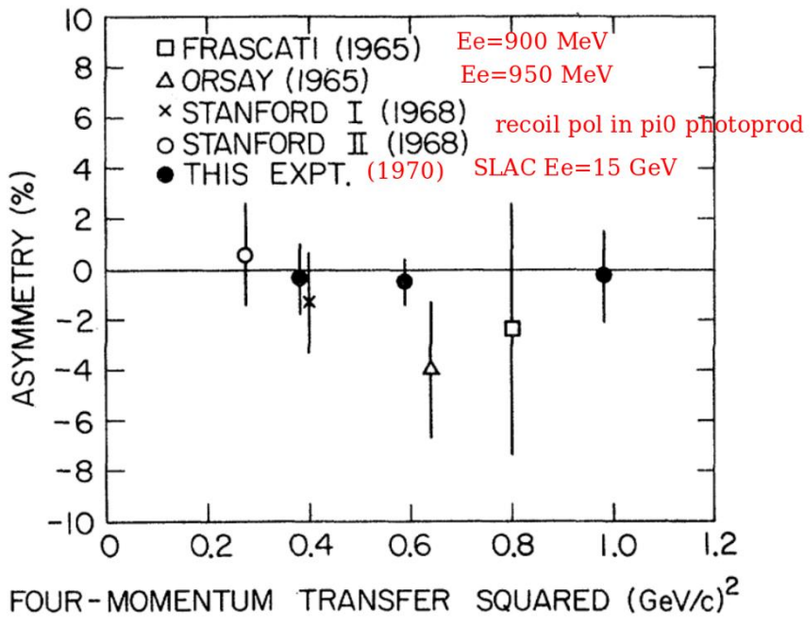
Photon emission

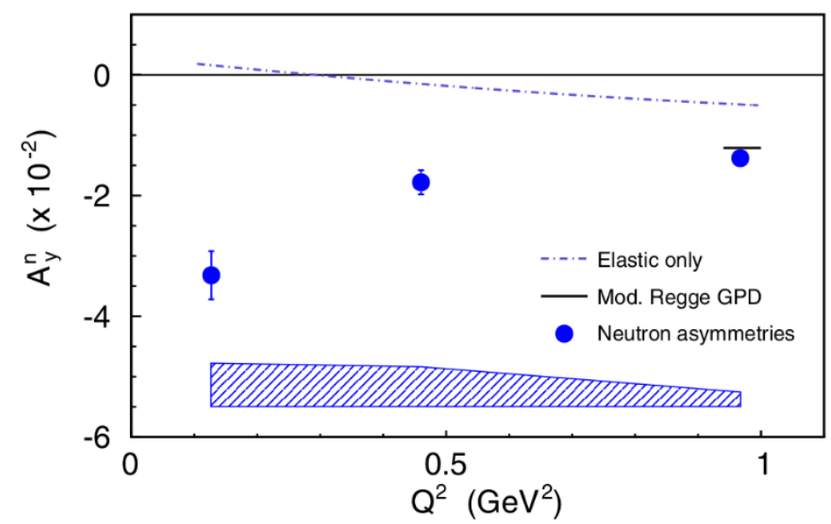


Δ: $k_\gamma = O(1/N_c)$ - phase space suppression - NLO

for $\sqrt{s} > m_{N^*}$, N^*N EM current $O(1/\sqrt{N_c})$ suppression - NLO

THE EXPERIMENTAL SIDE





JLab Hall A ^{3}He target

- old expts at relatively low energy - errors too large, no definite signal of asymmetry
- JLab Hall A SIDIS (^{3}He target): E_e=1.2, 2.4 & 3.6 GeV - energy a bit high for direct comparison consistent with magnitude – more complicated target
- HERMES on proton: SIDIS with e- and e+ beam – E_e=27.6 GeV
- new proposal for JLab Hall A (Grauvogel et al): E_e>2.2 GeV (may use e+)
- only e- beam energy in region of our interest is MAMI @ Mainz: A4 experiment

  measures normal beam SSAs on various targets
- important evolution of target NSSA from low to intermediate energies: first significant measurements would be very important and interesting!

COMMENTS & SUMMARY

- SSAs are important tool to study baryon structure in all energy regimes
- 1/Nc expansion provides one systematic approach in the energy range below second resonance region with N and Δ as effective dof
- it organizes the contributions by ordering in powers of 1/Nc; helps sort out the physics in more detail
- LO qualitatively OK, but NLO is important to describe the transition from purely elastic to inclusive
- very important role played by form factors: big enhancement of Δ final state asymmetry - changes of inclusive asymmetry
- The resonance region needs further theoretical exploration – interesting and important problem
- Lack of experimental results from low energy to onset of partonic regime: need for experiments eN from ~.3 to few GeV -