NORMAL SINGLE SPIN ASYMMETRIES IN ELECTRON SCATTERING
1/Nc EXPANSION APPROACH

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SINGLE SPIN ASYMMETRIES IN ELECTRON-NUCLEON SCATTERING

• Polarized beam or target
• Longitudinally polarized electron beam: test of Parity Violation (HERMES, HAPPEX, QWeak, SAMPLE)
• Transversely polarized target or beam (NSSA): test of spin effects at hadronic and partonic level: e.g. two photon exchange, Collins and Sivers functions
• SSAs in exclusive, semi-inclusive and inclusive processes
  • historical works that initiated the topic of SSAs: Barut & Fronsdal (1960); Leroy & Piketty; De Rujula, Kaplan and De Rafael; Gunther and Rodenberg; ...
  • numerous works since the 2000’s on TPE and SSA’s: Guichon & Vanderhaeghen; Blunden, Melnitchouk & Tjon; Gorchtein et al; Afanasev et al.; Myhrer et al; in DIS: Metz, Schlegel & Goeke; Afanasev, Strikman & Weiss; many more...
• NSSAs measured in high energy regime (Pol target: COMPASS, HERMES, JLab Hall A)
  (Pol beam: A4 at MAMI, JLab Hall C, SAMPLE at Bates). Very sparse kinematic ranges

\[ d\sigma_{SSA} \propto \vec{k} \cdot \vec{S} \]
\[ d\sigma_{SSA} \propto (\vec{k} \wedge \vec{k}') \cdot \vec{S} \]
OUTLINE

• Normal single spin asymmetries in electron scattering and two photon exchange - generalities

• Target normal SSA- 1/Nc expansion approach

• Results

• Comments and summary
**NORMAL SINGLE SPIN ASYMMETRIES - GENERALITIES**

**NSSA** linear in spin vector

Parity invariance then requires two momentum vectors

\[
d\sigma = d\sigma_U + 2\vec{e}_N \cdot \vec{S} \, d\sigma_N
\]

\[
\vec{e}_N = \frac{\vec{k}_f \times \vec{k}_i}{|\vec{k}_f \times \vec{k}_i|}
\]

Cases with more momenta (e.g., semi-inclusive scattering) will present more combinations of this type (e.g., for accessing Collins and Sivers functions)

**QED+QCD:** $T$ reversal symmetry forbids tree level SSA (Barut-Fronsdal (1960))

**SM:** has CP violation = $T$ violation: tree level possible but too small to be observable (Christ-Lee (1966))
T reversal symmetry allows SSA in the presence of absorptive part of scattering amplitude (Barut-Fronsdal (1960))

Absorptive part of amplitude results naturally from FSIs

In inclusive case, the FSI is EM: the absorptive part then starts at order $e^4$

Driven by TPE: absorptive part of box diagram

- Only states with invariant mass below $\sqrt{S}$ can contribute
- $d\sigma_N$ is free of collinear and IR divergencies: key role played by gauge invariance
- NSSA can be selected for exclusive and inclusive final states
- In principle electroproduction helicity amplitudes determine elastic NSSA – limited by range of experimentally established amplitudes
- Energy regimes:
  1) low/intermediate up to onset of second resonance region
  2) Resonance region: 1.5 to few GeV
  3) High energy region: e.g., partonic regime
- few degrees of freedom N, Δ, πN continuum
- unifying framework that treats this regime based on those dof and a rigorous
  QCD expansion: 1/Nc, and at low energy in addition Chiral Perturbation Theory
- implement 1/Nc expansion with N and Δ, and argue about the subleading role of the πN continuum
fundamental expansion in QCD: many phenomenological successes; LQCD also

recent tests in LQCD with Nc>3

can be implemented at hadronic level

particularly powerful for baryons:

at large Nc the baryon sector develops a spin-flavor SU(4) or SU(6) dynamical symmetry

N and Δ belong to an SU(4) multiplet: unification of properties

breaking of SU(4) by subleading orders in 1/Nc: formalism to implement higher orders 1/Nc

Nc scalings of basic observables:

\[
m_{N,\Delta} = O(N_c) \quad m_{\Delta} - m_N = O(1/N_c) \\
\Gamma_{\Delta} = O(1/N_c^2) \quad g_{\pi N} = O(\sqrt{N_c}) \\
F_\pi = O(\sqrt{N_c})
\]
Kinematic range

<table>
<thead>
<tr>
<th>Energy regime</th>
<th>1/$N_c$ expansion regime</th>
<th>Channels open</th>
<th>Final states possible</th>
</tr>
</thead>
<tbody>
<tr>
<td>I $m_N &lt; \sqrt{s} &lt; m_\Delta$</td>
<td>$\sqrt{s} - m_N \sim N_c^{-1}$, $k \sim N_c^{-1}$</td>
<td>$N$</td>
<td>elastic</td>
</tr>
<tr>
<td>II $m_\Delta &lt; \sqrt{s} \ll m_{N^*}$</td>
<td>$\sqrt{s} - m_N \sim N_c^{-1}$, $k \sim N_c^{-1}$</td>
<td>$N, \Delta$</td>
<td>elastic or inelastic</td>
</tr>
<tr>
<td>III $m_\Delta &lt; \sqrt{s} \lesssim m_{N^*}$</td>
<td>$\sqrt{s} - m_N \sim N_c^0$, $k \sim N_c^0$</td>
<td>$N, \Delta, N^*(\text{suppr})$</td>
<td>elastic or inelastic</td>
</tr>
</tbody>
</table>

Calculation will cover the three regimes

Expansions
I & II - Low energy combined with 1/$N_c$:

\[ O(p) = O(1/N_c) = O(\xi) : \text{BChPT x 1/N}_c \]

III - 1/$N_c$ expansion only
SYSTEMATIC EXPANSION IN 1/Nc: LO AND NLO FOR NSSA

In regime of interest 1/Nc expansion implies NR expansion:

baryons have small velocities O(1/Nc) in CM frame

Expand the Baryon EM current to first subleading order in 1/Nc: O(Nc) & O(Nc^0)

\[ J_{EM \mu} = \frac{1}{2} J^S_\mu + J^V_\mu \]

\[ J^S_\mu = g_\mu 0 G^S_E(t) - \frac{i}{\Lambda} G^S_M(t) \epsilon_{0 \mu ij} q^i \hat{S}^j \]

\[ J^V_\mu = g_\mu 0 G^V_E(t) \hat{I}^3 - \frac{i}{\Lambda} \frac{6}{5} G^V_M(t) \epsilon_{0 \mu ij} q^i \hat{G}^j \]

\[ \{ \hat{S}^i, \hat{I}^a, \hat{G}^{ia} \} \] generators of SU(4)

\[ G_{ia} \] spin-flavor generators of SU(4)- connect baryons of different spin

\[ < B' | G_{ia} | B > = O(N_c) \]

EM current has O(Nc) term: isotriplet magnetic

Nc hierarchy in isosinglet vs isotriplet N magnetic moments
EM current has $O(N_c)$ term: isotriplet magnetic

$N_c$ hierarchy in isosinglet vs isotriplet $N$ magnetic moments

$$\mu_0 = \frac{1}{2}(\mu_p + \mu_n) = 0.44\mu_N; \quad \mu_3 = \frac{1}{2}(\mu_p - \mu_n) = 2.35\mu_N$$

Subleading terms in current

convection current: $$(\vec{p}_i + \vec{p}_f)/2m_N = O(1/N_c)$$

electric quadrupole current: $O(1/N_c^2)$

neglected term:
$$\frac{1}{m_N\Lambda} g^{\mu\nu} \epsilon^{ijk} q^i (p_i + p_f)^j G^{k\alpha} = O(N_c^0)$$
$$= O(\xi^3) \text{ for } q = O(1/N_c)$$
CALCULATION OF THE TARGET NSSA

Leptonic tensor

\[ L^{\mu \nu \rho}(k_i, k_f, k_n) = \text{Tr}(k_i \gamma^\mu k_f \gamma^\nu k_n \gamma^\rho) \]

Hadronic tensor

\[ H^{\mu \nu \rho}_{\text{fi,n}}(k_i, k_f, k_n) = \langle B_i \mid (J_{\text{EM}}^\mu(k_i - k_f))^\dagger \mid B_f \rangle \]
\[ \times \langle B_f \mid J_{\text{EM}}^\nu(k_n - k_f) P_n J_{\text{EM}}^\rho(k_i - k_n) \mid B_i \rangle \]
- SSA needs t-channel $J=1$ components of hadronic tensor, $I=0,1$
- Box has t-channel $J_{\text{Box}}=0,1$ for $B_{f}=N$, and $J_{\text{Box}}=1,2$ for $B_{f}=\Delta$
- Strict large $N_c$ limit: only isotriplet spin current contributes: LO given by $J=I$ and $J_{\text{Box}}=I_{\text{Box}}$ ($I=J$ rule of large $N_c$)
- Subleading corrections: $I \neq J$ with LO current and subleading currents
- $Q^2$ range up to $4E_e^2$ - need to include FFs in calculation: common dipole form for $E$ and $M$ components
- Integrals of box have IR and/or collinear divergencies: they cancel for each combination of terms of the hadronic current and each for the corresponding $J_{\text{Box}}$ and $I_{\text{Box}}$ and for different $B_{f}$ and $B_{n}$ – good check for calculation
- Calculation checked with known purely elastic case
A FIRST CALCULATION: LO IN $1/N_c$ – LARGE $N_c$ LIMIT

- given in terms of the isovector magnetic current
- match magnetic coupling at $N_c=3$ to physical one
- pick LO terms in hadronic tensor: LO given by $I=J$ t-channel rule
- simple results without $t$-dependence of FF – shows dominance of the elastic SSA
  with intermediate state contribution of $N$ equal $\frac{1}{2}$ of the $\Delta$
- Inclusion of FF makes major difference: inelastic channel, $Bf=\Delta$, becomes large and of opposite sign to elastic – non-trivial FF interplay in box
- decompose hadronic tensor into $t$-channel irreducible angular momentum and isospin components: SU(4) algebra organizes contributions in powers of $1/N_c$
- separate effects by $N$ and $\Delta$ channels in the box and final state
- consistent cancellations of IR and collinear divergences for each channel and for gauge invariant combinations of EM current components
- validated with NR expansion of the known purely elastic case
- calculation without $t$-dependence in FFs
- inclusion of FFs and $\Delta$ width
Results without FFs

\[
\frac{d\sigma_N}{d\Omega}(N_i, N, N) = \frac{1}{4} k^2 m_N^2 \left( k(G_M^f(N_c + 7) - G_M^{-f}(N_c - 3)) - 10G_E^f \Lambda \right) \\
\times \left( G_M^f(N_c + 7) - G_M^{-f}(N_c - 3) \right)^2 \left( (3 + x) \log \frac{1 - x}{2} - 2(1 + x) \right) \mathbf{X}
\]

\[
\frac{d\sigma_N}{d\Omega}(N_i, N, \Delta) = \theta(k_{\Delta})k_{\Delta} m_{\Delta} m_N^3 (N_c + 5)(N_c - 1)(G_M^p - G_M^n)^2 \\
\times \left( k(G_M^f(N_c + 7) - G_M^{-f}(N_c - 3)) + 5G_E^f \Lambda \right) \\
\times \left( \left( \frac{Y}{2m_N^2} - \frac{k}{m_N^2} (1 + x) \right) \log \frac{1 - x}{2} - \frac{k^2}{Y} (1 + x) \right) \mathbf{X}
\]

\[
\frac{d\sigma_N}{d\Omega}(N_i, \Delta, N) = \theta(k_{\Delta}) \frac{1}{32} \frac{m_{\Delta}}{m_N^2} (N_c + 5)(N_c - 1)(G_M^p - G_M^n)^2 Y \\
\times \left( 1 + x \right) \left( (G_M^f(N_c + 7) - G_M^{-f}(N_c - 3))(11k^2 - kk_{\Delta} + 4k_{\Delta}^2) + 4G_E^f (k - k_{\Delta}) \Lambda \right) \\
+ 2 \left( (G_M^f(N_c + 7) - G_M^{-f}(N_c - 3))(2k^2 + 2(2 - x)k_{\Delta}^2 + 3(1 - x)kk_{\Delta}) \right) \\
+ 20G_E^f (2k - (1 + x)k_{\Delta}) \Lambda \right) \log \frac{1 - x}{2} \mathbf{X}
\]

\[
x = \cos \theta
\]

\[
\frac{1 - x}{2} = \sin^2 \frac{\theta}{2}
\]

\[
\mathbf{X} = \frac{m_N \alpha^3}{1000 s^{3/2} t(1 + x)^3} \mathbf{k}_t \times \mathbf{k}_i \cdot \mathbf{S}
\]

\[
Y = \sqrt{k^2 + m_N^2 (m_N^2 - m_{\Delta}^2) + k(m_N^2 + m_{\Delta}^2)}
\]
\[ A_N = \frac{d\sigma_N}{d\sigma_U \text{elastic}} \]

no FF, vanishing \( \Gamma_\Delta \)

- Electric component of current only appears linearly
- Large \( Nc \) limit checked
- Dominance of elastic asymmetry
- Very small inelastic asymmetry
- Kinematics taken exactly
- Threshold enhancement at \( \Delta \) mass
- \( \Delta \) in box gives important contribution \( \sim \) twice of \( N \)
$A_N$ with common dipole FF and $\Gamma_\Delta = 125\text{MeV}$

- FF play crucial role – enhances inelastic channel
- inelastic channel gives opposite sign asymmetry to elastic one: changes sign of inclusive asymmetry
- sensitive energy dependence of asymmetry – we keep kinematic factors exact
Other TSSA contributions

- $\pi N$ Continuum

$\pi$ baryon coupling: $\propto \frac{g_A}{F_\pi} k_\pi^i G^{ia} = O(\sqrt{N_c})$

$[G^{ia}, G^{ib}] = O(N_c^0)$; $[G^{ia}, S^i] = O(N_c)$; $[G^{ia}, I^b] = O(N_c)$

$O(1/N_c)$ wrt LO, and $O(\xi^3)$ wrt LO term in $\xi$ expansion domain
- \(N^*\) resonances

EM N-N* couplings carry extra factor \(1/\sqrt{N_c}\)

Individual resonances contribute \(O(1/N_c)\) wrt N and \(\Delta\)

Case of a few resonances was studied (Vanderhaeghen et al) - still a lot to be done
Photon emission

\[ \Delta: \ k_\gamma = O(1/N_c) - \text{phase space suppression - NLO} \]

for \( \sqrt{s} > m_{N^*} \), \( N^* N \) EM current \( O(1/\sqrt{N_c}) \) suppression - NLO
THE EXPERIMENTAL SIDE

- Old expts at relatively low energy - errors too large, no definite signal of asymmetry
- JLab Hall A SIDIS ($^3$He target): $E_e=1.2, 2.4 \& 3.6$ GeV - energy a bit high for direct comparison consistent with magnitude – more complicated target
- HERMES on proton: SIDIS with e- and e+ beam – $E_e=27.6$ GeV
- New proposal for JLab Hall A (Grauvogel et al): $E_e>2.2$ GeV (may use e+)
- Only e- beam energy in region of our interest is MAMI @ Mainz: A4 experiment measures normal beam SSAs on various targets
- Important evolution of target NSSA from low to intermediate energies: first significant measurements would be very important and interesting!
• SSAs are important tool to study baryon structure in all energy regimes

• 1/Nc expansion provides one systematic approach in the energy range below second resonance region with N and Δ as effective dof

• it organizes the contributions by ordering in powers of 1/Nc; helps sort out the physics in more detail

• LO qualitatively OK, but NLO is important to describe the transition from purely elastic to inclusive

• very important role played by form factors: big enhancement of Δ final state asymmetry - changes of inclusive asymmetry

• The resonance region needs further theoretical exploration – interesting and important problem

• Lack of experimental results from low energy to onset of partonic regime: need for experiments eN from ~.3 to few GeV -