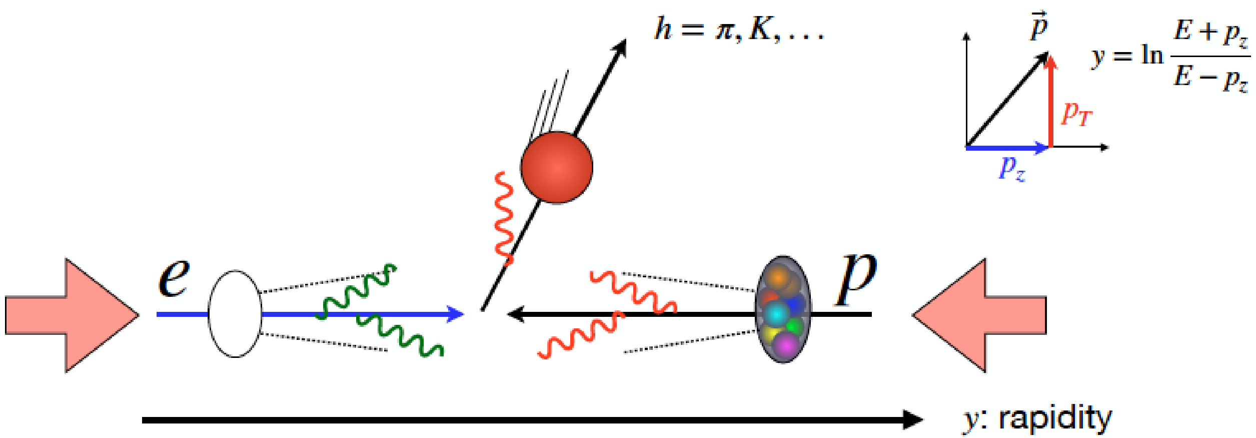
February 12, 2025

Joint QCD and QED Factorization for Single Hadron/Jet Production in Lepton-Hadron Collisions



- QCD, QCD factorization & Hadron Structure
- Role of Lepton-Hadron Collisions
- Collision Induces QCD & QED Radiations
- Need for Joint QCD and QED Factorization
- Prompt Single Hadron/Jet Production
- Summary and Outlook

In collaboration with K. Watanabe, T.B. Liu, …

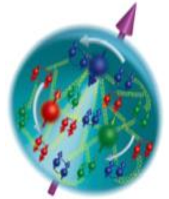
Jianwei Qiu
Jefferson Lab, Theory Center

Jefferson Lab

U.S. DEPARTMENT OF ENERGY | Office of Science

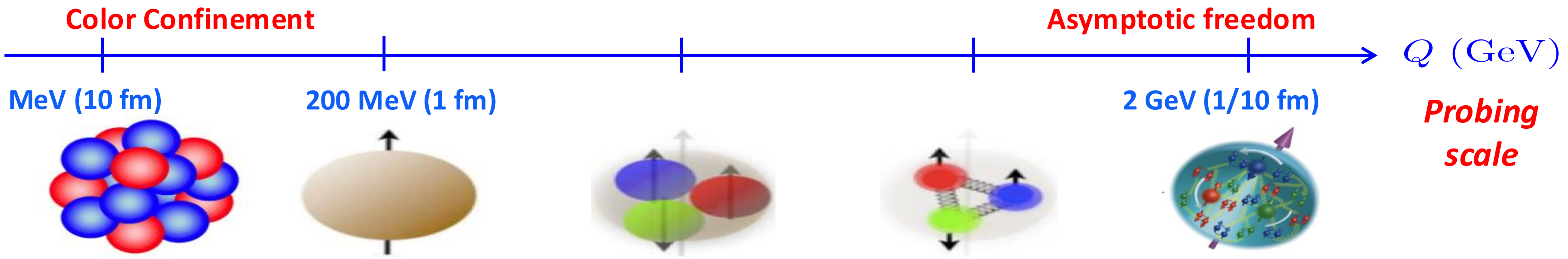
JSA

QCD Color is Fully Entangled



❑ QCD color confinement:

- *Do not see any quarks and gluons in isolation*
- *The structure of nucleons and nuclei – emergent properties of QCD*



QCD at the Fermi Scale: Femto-science (0.1-10 fm)

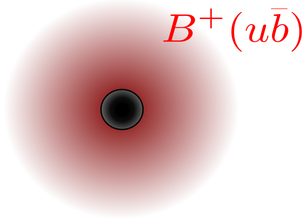
- *The most interesting, rich, and complex regime of the theory!*
- *All emergent phenomena depend on the scale at which we probe them!*

Asymptotic regime PQCD works beautifully!

❑ QCD is non-perturbative:

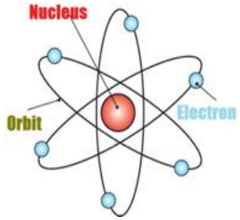
- *Any cross section/observable with identified hadron is NOT perturbatively calculable!*
- *Color is fully entangled!*

B-meson



Brown-Muck

Atomic structure



Quantum orbits

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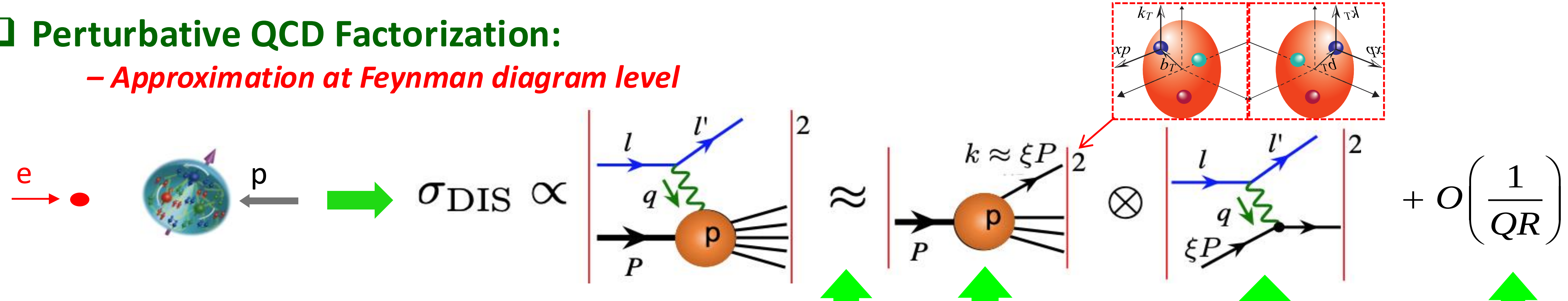
Theoretical Approaches – Approximations:

- **Perturbative QCD Factorization:**

  – Approximation at Feynman diagram level



Factorization Theory	Parton-distribution "Structure"	Hard-part Probe	Power corrections Approximation

- **Effective field theory (EFT):**

  – Approximation at the Lagrangian level

  Soft-collinear effective theory (SCET), Non-relativistic QCD (NRQCD), Heavy quark EFT, chiral EFT(s), …

- **Lattice QCD:**

  – Approximation for finite lattice spacing, finite box, lightest quark masses, … with Euclidean time formulation (removable with increased computational cost)

  Hadron structure, hadron spectroscopy, nuclear structure, phase shift, …

- **Other approaches:**

  Light-cone perturbation theory, Dyson-Schwinger Equations (DSE), Constituent quark models, AdS/CFT correspondence, …

Predictive Power of QCD Factorization

- **Universality of non-perturbative hadron structure + calculable matching coefficients:**
  - lepton-hadron reactions (COMPASS, JLab, **EIC**)

$$\sigma^{EXP}_{l+P\to l+X} = C_{l+k\to l+X} \otimes \mathrm{PDF}_P + O(Q_s^2/Q^2)$$

$$\sigma^{EXP}_{l+P\to l+H+X} = C_{l+k\to l+k+X} \otimes \mathrm{PDF}_P \otimes \mathrm{FF}_H + O(Q_s^2/Q^2)$$

  - hadron-hadron reactions (LHC)

$$\sigma^{EXP}_{P+P\to l+\bar{l}+X} = C_{k+k\to l+\bar{l}+X} \otimes \mathrm{PDF}_P \otimes \mathrm{PDF}_P + O(Q_s^2/Q^2)$$

  - lepton-lepton reactions (Belle)

$$\sigma^{EXP}_{l+\bar{l}\to H+X} = C_{l+\bar{l}\to k+X} \otimes \mathrm{FF}_H + O(Q_s^2/Q^2)$$

Plus other factorizable observables – Cross sections & Asymmetries
Plus LQCD calculable & factorizable hadron matrix elements

$$\sigma^{LQCD}_{n/P} = \langle P|\mathcal{O}_n|P\rangle Z^{-1}_{\mathcal{O}_n} = K_{\mathcal{O}_n} \otimes \mathrm{PDF}_P$$

- **Hadron structure = Theory + Experiment + Phenomenology:**
  - Factorization – Identify "Good" observables (Theory, including LQCD)
  - Measurement – Get "Reliable" data (Experiment + LQCD)
  - Global analysis – Extract "Universal" structure information (Phenomenology) by solving an inverse problem

QCD Global Analysis of Experimental Data (as well as Lattice Data)

❑ It is an inverse problem:

Experimental data

QCD Factorization Theorem

Future EIC Jefferson Lab, KEK, Brookhaven National Laboratory, DESY, CERN, Fermilab

Lattice data

= … Analytical Coefficient functions derived from QCD perturbation theory ⊗ Quantum Correlation Functions of quarks and gluons **Unknown** + Systematic **error** from theory

Flavors: u, d, s, …; g
Spin (quark, gluon hadron)
Momentum/position
(Before/after the collision)

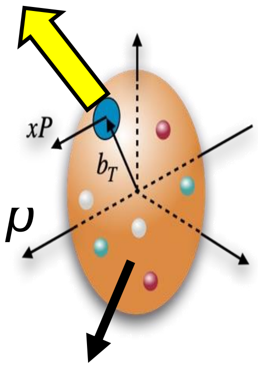
Locally, JAM, CJ, nCTEQ, …

Bayesian inference
Theory/Phenomenology
AI/ML, Computation

Extract QCFs

PDF_P, FF_H, …

Quark Polarization



Nucleon Polarization

❑ Input for QCD Global analysis/fitting:

PDFs, FFs at an input scale:

$$\phi_{f/h}(x, \mu_0^2, \{\alpha_j\})$$

Input scale ~ GeV

Fitting paramters

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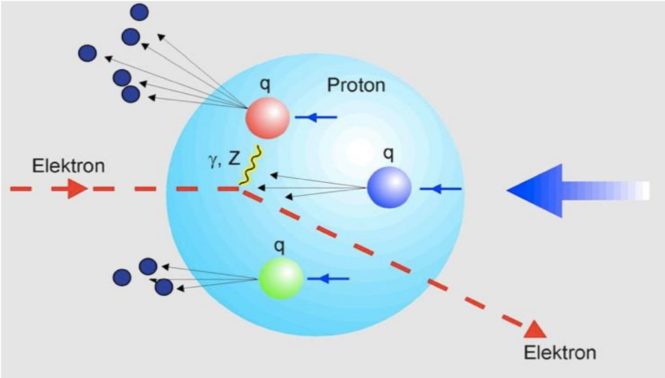
Lepton-Hadron Deep Inelastic Scattering (DIS)



$$\sigma^{\mathrm{DIS}}_{\ell p \to \ell' X(\mathrm{everything})}$$

Identified initial-state hadron – proton or nucleus!

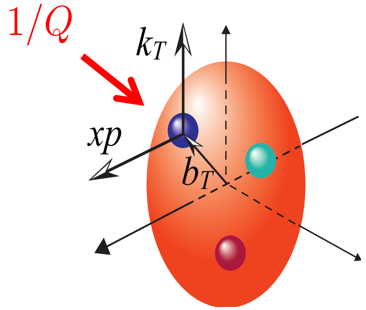
- **Localized probe:**

  $$Q^2 = -(l - l')^2 \gg 1\ \mathrm{fm}^{-2} \;\Rightarrow\; \frac{1}{Q} \ll 1\ \mathrm{fm}$$

- **Two variables (hadron rest frame):**

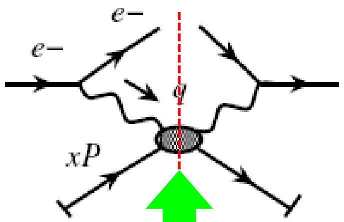
  $$Q^2 = 4EE' \sin^2(\theta/2) \qquad y = \frac{s}{x_B Q^2}$$

  $$x_B = \frac{Q^2}{2m_N \nu} \qquad \nu = E - E'$$



❑ QCD factorization (approximation!)

$$\sigma^{\mathrm{DIS}}_{\ell p \to \ell' X(\mathrm{everything})} =$$



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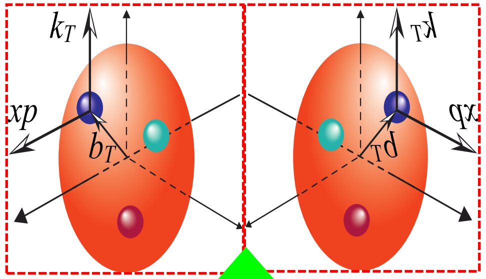
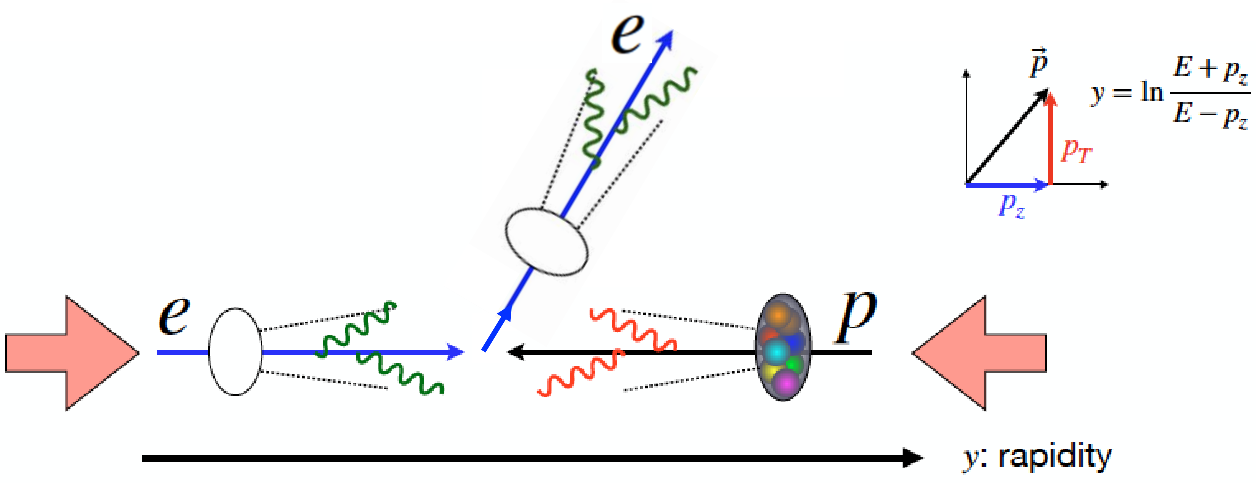


$$+\; O\left(\frac{1}{QR}\right)$$

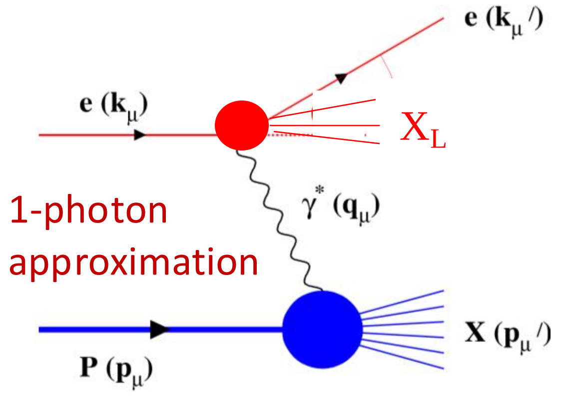
Physical Observable (x_B, Q^2) **Controllable Probe** (x_B, Q^2) **Quantum Probabilities Structure** **Color entanglement Approximation**

- **Played a key role in discovering quarks (the SLAC experiment) and the development of QCD**
- **Has been considered as the best clean probe for extracting hadron structure (JLab and future EIC)**

Hard Collision Induces both QCD and QED Radiations

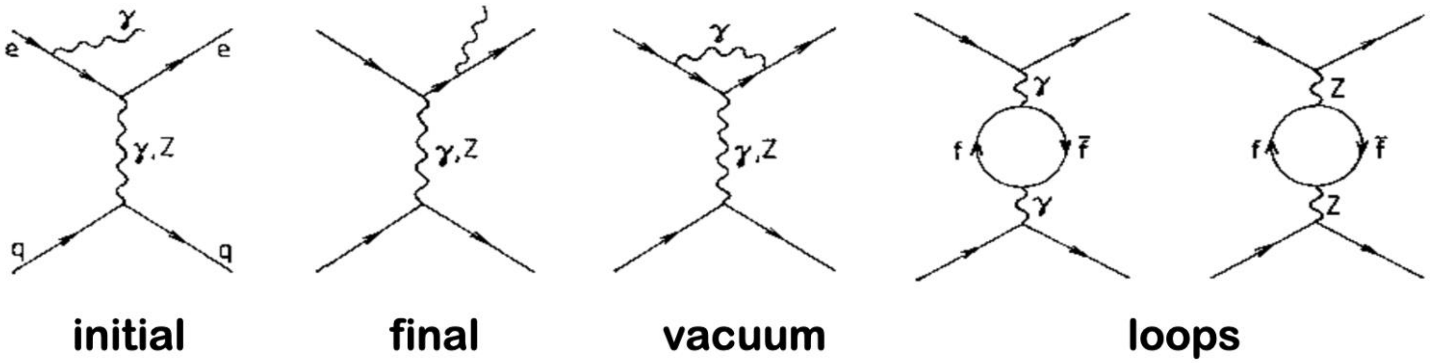


Historically, 1-photon approximation
+ radiative corrections (radiation from leptons)



$$(x_B, Q^2) \rightarrow (\hat{x}_B, \widehat{Q}^2)$$

"We know how to calculate QED radiation perturbatively!"

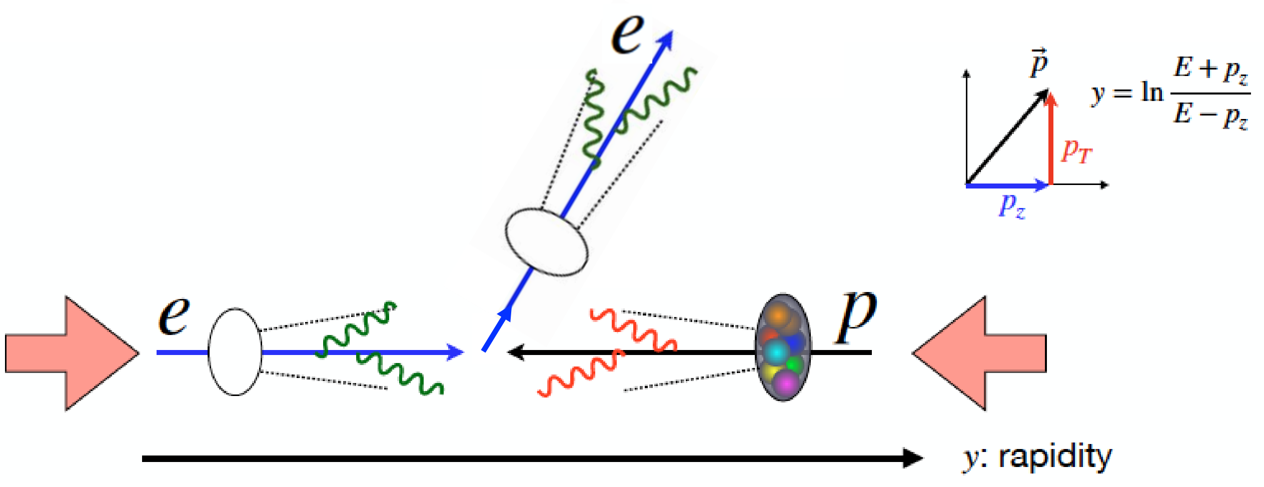


+ two-photon channel + …

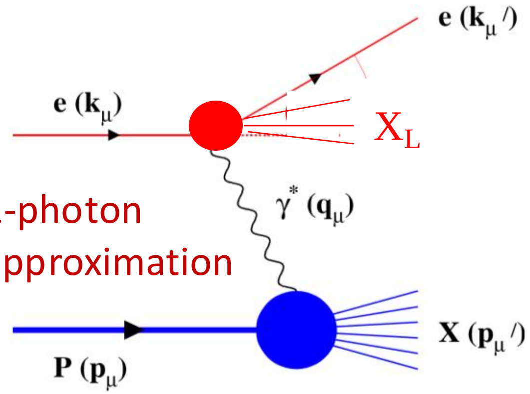
MC program(s) for the RC with "cutoff(s)"

Always keep the γ* virtual!

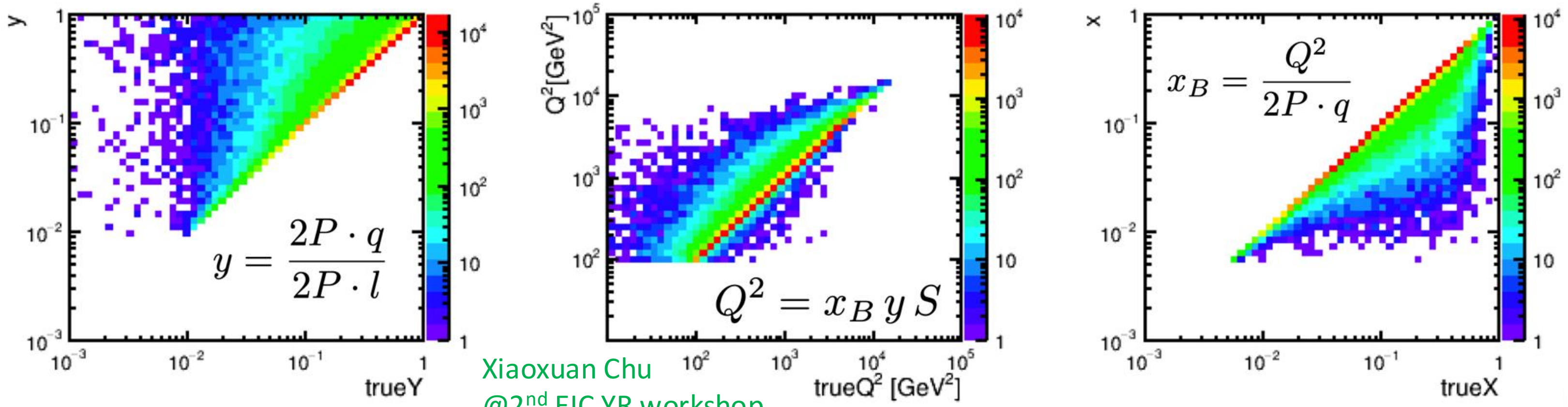
Hard Collision Induces both QCD and QED Radiations



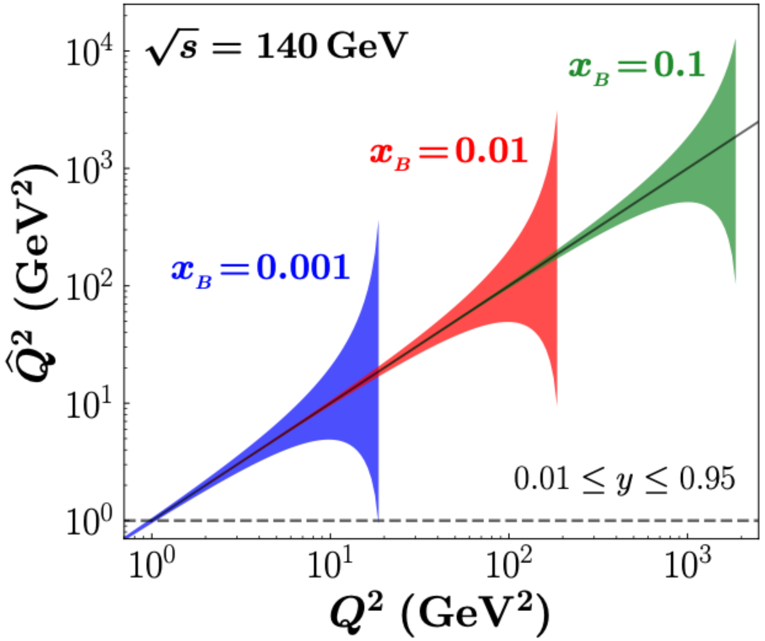
Historically, 1-photon approximation + radiative corrections (radiation from leptons)



"We know how to calculate QED radiation perturbatively!"



Xiaoxuan Chu @2nd EIC YR workshop

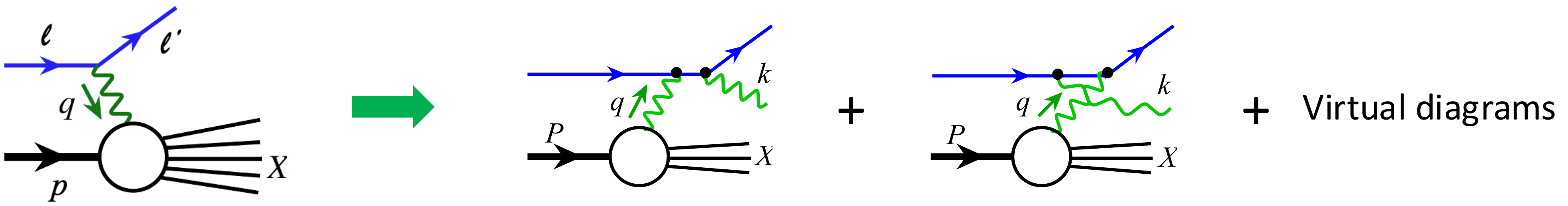


$$x_B \to \hat{x}_B \in [x_B, 1] \qquad \hat{Q}^2_{\min} = Q^2\,\frac{(1-y)}{(1-x_B\,y)} \qquad \hat{Q}^2_{\max} = Q^2\,\frac{1}{(1-y+x_B\,y)}$$

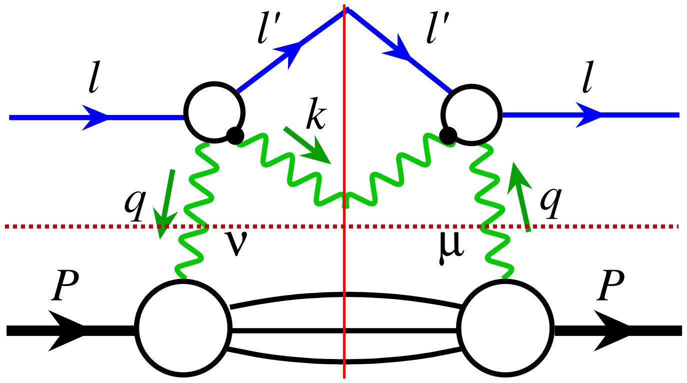
Why We Need Parameter(s) for the Traditional RC Approach?

❑ Leading Order Radiative Correction:

Cammarota, Qiu, Watanabe, Zhang
arXiv: 2408.08377, in preparation



Contribution to the cross section – cut diagram notation:



The virtuality of the exchange photon q^2 depends on the phase-space of unobserved photon of momentum k

$$E'\frac{d\sigma^{\rm RC}_{eh\to eX}}{d^3\ell'} \propto \int d^4q \left[W^{\mu\nu}(P,q)\frac{1}{q^2+i\epsilon}L^{(1)}_{\mu\nu}(\ell,\ell',q)\frac{1}{q^2-i\epsilon}\right]$$

$$\to \infty$$

The q^2 is perturbatively pinched to $q^2=0$ if the phase space allows!

➡ **Impose "cutoff/limit" on the radiated and un-observed photon(s) to keep it "virtual"– parameter(s)**

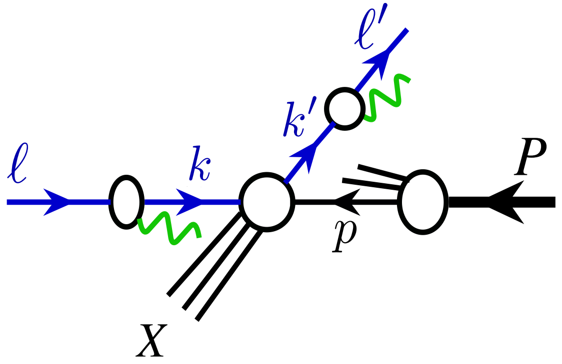
The parameter(s) must be collision energy dependent – tuning parameter(s) in MC for RC
– Predictive power?

Joint QCD and QED Factorization for Deep Inelastic Scattering (DIS)

Liu, Melnitchouk, Qiu, Sato, Phys.Rev.D 104 (2021) 094033
JHEP 11 (2021) 157
Cammarota, Qiu, Watanabe, Zhang [2408.08377]

❑ Without the "one-photon" approximation:

~ Inclusive single lepton production at high transverse momentum



$$E_{k'}\frac{d\sigma_{kP\to k'X}}{d^3k'} = \frac{1}{2s}\sum_{i,j,a}\int_{\zeta_{\min}}^{1}\frac{d\zeta}{\zeta^2}\int_{\xi_{\min}}^{1}\frac{d\xi}{\xi}\,D_{e/j}(\zeta,\mu^2)\,f_{i/e}(\xi,\mu^2)$$
$$\times\int_{x_{\min}}^{1}\frac{dx}{x}\,f_{a/N}(x,\mu^2)\,\widehat{H}_{ia\to jX}(\xi k, xP, k'/\zeta,\mu^2) + \cdots$$

LFFs → $D_{e/j}$; LDFs → $f_{i/e}$; PDFs → $f_{a/N}$

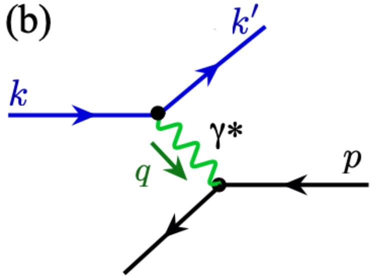
No structure functions, but have PDFs, LDFs, LFFs, …

❑ Hard parts in power of $\alpha^m\alpha_s^n$ – $\widehat{H}^{(m,n)}_{ia\to jX}$:

Hard parts are not sensitive to long-distance physics or the colliding states

- **LO – Let h(P) to be q(P):**

(b)



$$\widehat{H}^{(2,0)}_{eq\to eX} = (2\hat{s})\left[E_{k'}\frac{d\sigma^{(\mathrm{LO})}_{eq\to eq}}{d^3k'}\right] = e_q^2\,(4\alpha_{em}^2)\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right]\delta\left(\hat{s}+\hat{t}+\hat{u}\right)$$
$$= e_q^2\,(4\alpha_{em}^2)\,\frac{x^2\zeta\left[(\zeta\xi s)^2+u^2\right]}{(\xi t)^2\,(\zeta\xi s+u)}\,\delta(x-x_{\min})$$

$$x_{\min} = \frac{\xi\, x_B\, y}{\xi\zeta+y-1},$$
$$\xi_{\min} = \frac{1-y}{\zeta-x_B y},$$
$$\zeta_{\min} = 1-(1-x_B)y,$$

$$s=(\ell+P)^2,$$
$$u=(\ell'-P)^2=(y-1)s,$$
$$t=(\ell-\ell')^2=-Q^2$$

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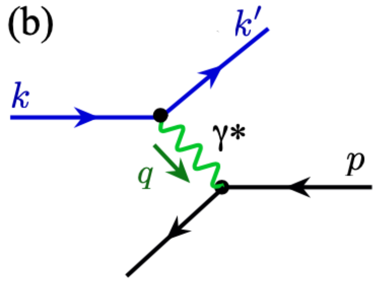
❑ Keep the "one-photon" approximation – resum collinear radiation:



$$\frac{d^2\sigma_{\ell P\to\ell' X}}{dx_B dy} \approx \int_{\zeta_{\min}}^1 \frac{d\zeta}{\zeta^2}\int_{\xi_{\min}}^1 d\xi\, D_{e/e}(\zeta,\mu^2)\, f_{e/e}(\xi,\mu^2)\left[\frac{Q^2}{x_B}\frac{\hat{x}_B}{\widehat{Q}^2}\right]$$

$$\times\frac{4\pi\alpha^2}{\hat{x}_B\,\hat{y}\,\widehat{Q}^2}\left[\hat{x}_B\hat{y}^2F_1(\hat{x}_B,\widehat{Q}^2)+\left(1-\hat{y}-\frac{1}{4}\hat{y}^2\hat{\gamma}^2\right)F_2(\hat{x}_B,\widehat{Q}^2)\right]$$

(the bracketed second line is marked as) $\frac{d^2\sigma_{kP\to k'X}}{d\hat{x}_B d\hat{y}}$

When: $D_{e/e}(\zeta)=\delta(1-\zeta)\,\delta^{ee}$

$f_{e/e}(\xi)=\delta(1-\xi)\,\delta^{ee}$

➡ Recover the Born expression

❑ Next-to-Leading Order (NLO) to the leading subprocess: $e(k)+q(p)\to e(k')+X$

- **NLO – Let h(P) to be q(P):**

$$\sigma^{(3,0)}_{eq\to eX} = (2s)E'\frac{d\sigma^{(3,0)}_{eq\to eX}}{d^3\ell'}$$

$$= D^{(0)}_{e/e}\otimes_\zeta f^{(0)}_{e/e}\otimes_\xi \widehat{H}^{(3,0)}_{eq\to eX}\otimes_x f^{(0)}_{q/q} + D^{(1)}_{e/e}\otimes_\zeta f^{(0)}_{e/e}\otimes_\xi \widehat{H}^{(2,0)}_{eq\to eX}\otimes_x f^{(0)}_{q/q}$$

$$+ D^{(0)}_{e/e}\otimes_\zeta f^{(1)}_{e/e}\otimes_\xi \widehat{H}^{(2,0)}_{eq\to eX}\otimes_x f^{(0)}_{q/q} + D^{(0)}_{e/e}\otimes_\zeta f^{(0)}_{e/e}\otimes_\xi \widehat{H}^{(2,0)}_{eq\to eX}\otimes_x f^{(1)}_{q/q}$$

$$+ D^{(0)}_{e/e}\otimes_\zeta f^{(0)}_{e/e}\otimes_\xi \widehat{H}^{(2,0)}_{e\gamma\to eX}\otimes_x f^{(1)}_{\gamma/q}$$

- **LO LFF, LDF, PDF:**

$$D^{(0)}_{e/e}(\zeta)=\delta(1-\zeta)$$

$$f^{(0)}_{e/e}(\xi)=\delta(1-\xi)$$

$$f^{(0)}_{q/q}(x)=\delta(1-x)$$

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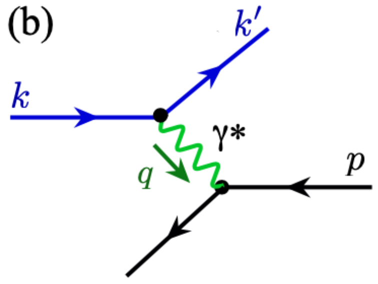

$$\frac{d^2\sigma_{\ell P\to\ell' X}}{dx_B dy} \approx \int_{\zeta_{\min}}^{1} \frac{d\zeta}{\zeta^2} \int_{\xi_{\min}}^{1} d\xi\, D_{e/e}(\zeta,\mu^2)\, f_{e/e}(\xi,\mu^2) \left[\frac{Q^2}{x_B}\frac{\hat{x}_B}{\widehat{Q}^2}\right]$$

$$\times \underbrace{\frac{4\pi\alpha^2}{\hat{x}_B\,\hat{y}\,\widehat{Q}^2}\left[\hat{x}_B\hat{y}^2 F_1(\hat{x}_B,\widehat{Q}^2) + \left(1-\hat{y}-\frac{1}{4}\hat{y}^2\hat{\gamma}^2\right)F_2(\hat{x}_B,\widehat{Q}^2)\right]}_{\frac{d^2\sigma_{kP\to k'X}}{d\hat{x}_B d\hat{y}}}$$

When: $D_{e/e}(\zeta) = \delta(1-\zeta)\,\delta^{ee}$

$f_{e/e}(\xi) = \delta(1-\xi)\,\delta^{ee}$

➡ Recover the Born expression

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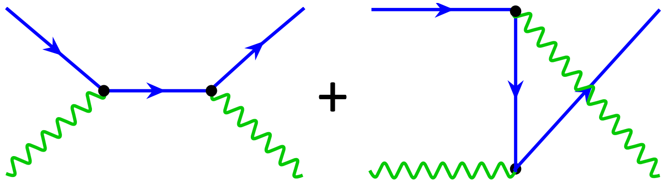
- **NLO – Let h(P) to be q(P):**

$$\widehat{H}^{(3,0)}_{eq\to eX} = \sigma^{(3,0)}_{eq\to eX} - D^{(1)}_{e/e} \otimes_\zeta \widehat{H}^{(2,0)}_{eq\to eX} - f^{(1)}_{e/e} \otimes_\xi \widehat{H}^{(2,0)}_{eq\to eX}$$
$$- f^{(1)}_{q/q} \otimes_x \widehat{H}^{(2,0)}_{eq\to eX} - \boxed{f^{(1)}_{\gamma/q} \otimes_x \widehat{H}^{(2,0)}_{e\gamma\to eX}}$$



Similar to the Bethe-Heitler subprocess for Exclusive Processes

Completely UV, IR, CO safe!
No need for any free parameter other than the standard factorization scale

Calculation of NLO QED Contributions – beyond 1-vector boson exchange

- **Calculation of $\sigma^{(3,0)}_{eq\to eX}$, $D^{(1)}_{e/e}, f^{(1)}_{e/e}, f^{(1)}_{q/q}$, and $f^{(1)}_{\gamma/q}$:**

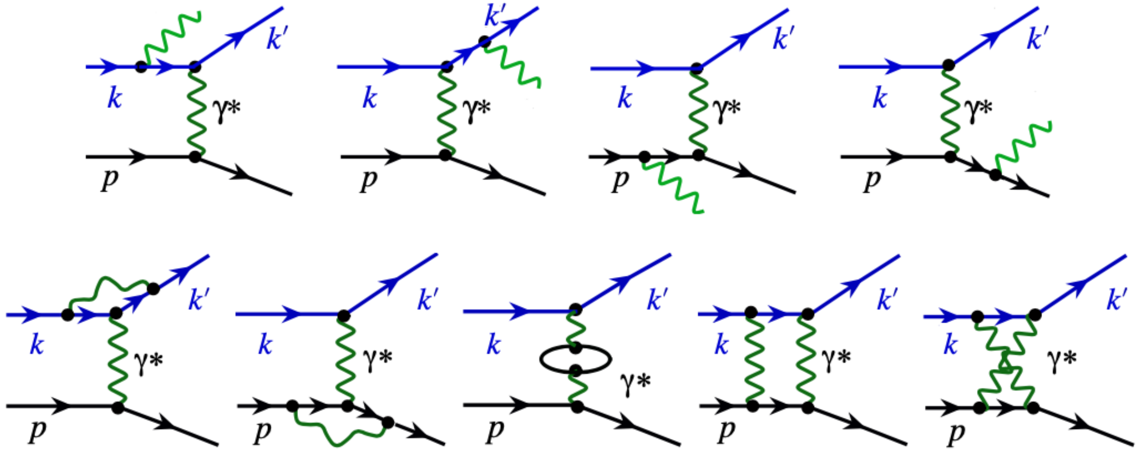
Cammarota, Qiu, Watanabe, Zhang [2408.08377]
In preparation

$$\widehat{H}^{(3,0)}_{eq\to eX} = \sigma^{(3,0)}_{eq\to eX} - D^{(1)}_{e/e} \otimes_\zeta \widehat{H}^{(2,0)}_{eq\to eX} - f^{(1)}_{e/e} \otimes_\xi \widehat{H}^{(2,0)}_{eq\to eX} - f^{(1)}_{q/q} \otimes_x \widehat{H}^{(2,0)}_{eq\to eX} - f^{(1)}_{\gamma/q} \otimes_x \widehat{H}^{(2,0)}_{e\gamma\to eX}$$

$$\sigma^{(3,0)}_{eq\to eX} = (2s)E' \frac{d\sigma^{(3,0)}_{eq\to eX}}{d^3\ell'}$$

➡ Perturbatively divergent due to collinear singularities along external observed momenta

➡ Dimensional regularization are used



$$f^{(1)}_{i/j}(z)_{\overline{\rm MS}} = \left(-\frac{1}{\epsilon}\right)_{\rm CO} (4\pi)^\epsilon \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} P_{i/j}(z)$$

$$P_{e/e}(z) = \frac{1}{e_q^2} P_{q/q}(z) = \frac{\alpha}{2\pi}\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right]$$

$$P_{\gamma/q}(z) = \frac{\alpha}{2\pi} e_q^2 \left[\frac{1+(1-z)^2}{z}\right]$$

NLO QED contributions – beyond 1-vector boson exchange

❑ NLO QED contribution:

$$\hat{H}^{(3,0)}_{eq\to eX} \propto \alpha^3\, e_q^2 \left\{ e_l^2\, \frac{2(1+\hat{v}^2)}{9\hat{v}} \left[3\ln\frac{(1-\hat{v})s}{\mu^2} - 5\right] \delta(1-\hat{w}) \right.$$

$$+ e_q \left[a_1 \delta(1-\hat{w}) + \frac{a_7}{(1-\hat{w})_+} + a_6\right]$$

$$+ e_q^2 \left[b_1 \delta(1-\hat{w}) + b_2 \left(\frac{1}{1-\hat{w}}\right)_+ + b_3 \left(\frac{\ln(1-\hat{w})}{1-\hat{w}}\right)_+ + b_4\right]$$

$$\left. + c_1 \delta(1-\hat{w}) + c_2 \left(\frac{1}{1-\hat{w}}\right)_+ + c_3 \left(\frac{\ln(1-\hat{w})}{1-\hat{w}}\right)_+ + c_4 \right\}$$

$$\hat{v} = 1 - \frac{x_B}{x}\frac{y}{\zeta}$$

$$\hat{w} = \frac{1-y}{\xi\,(\zeta - (x_B/x)\,y)}$$

a_1, a_6, a_7
b_1, b_2, b_3, b_4
c_1, c_2, c_3, c_4
are analytic functions of $\hat{v}$ and $\hat{w}$.

$$e_l^2 = \sum_f N_c^f\, e_f^2$$ Sum over the flavors appeared in the photon vacuum polarization

Completely IR and CO safe! Only depends on factorization scale μ, same in all partonic scattering channels
No need for any "cut-off" parameter(s) in the traditional "Radiative Correction"

➡ **In joint QCD & QED factorization: Lepton-distributions are not pure QED !**
Hadron's parton distributions are not pure QCD !

Impact of Factorized QED Contribution to Lepton-Hadron Scattering

Perturbative lepton distributions:

$$f_{e/e}^{(\mathrm{NLO})}(\xi,\mu^2) = \delta(1-\xi) + \frac{\alpha_{em}}{2\pi}\left[\frac{1+\xi^2}{1-\xi}\ln\frac{\mu^2}{(1-\xi)^2 m_e^2}\right]_+$$

$$D_{e/e}^{(\mathrm{NLO})}(\zeta,\mu^2) = \delta(1-\zeta) + \frac{\alpha_{em}}{2\pi}\left[\frac{1+\zeta^2}{1-\zeta}\ln\frac{\zeta^2\mu^2}{(1-\zeta)^2 m_e^2}\right]_+$$

Electron mass to regularize the CO divergence
– Only defined under the integration.

Model distributions:

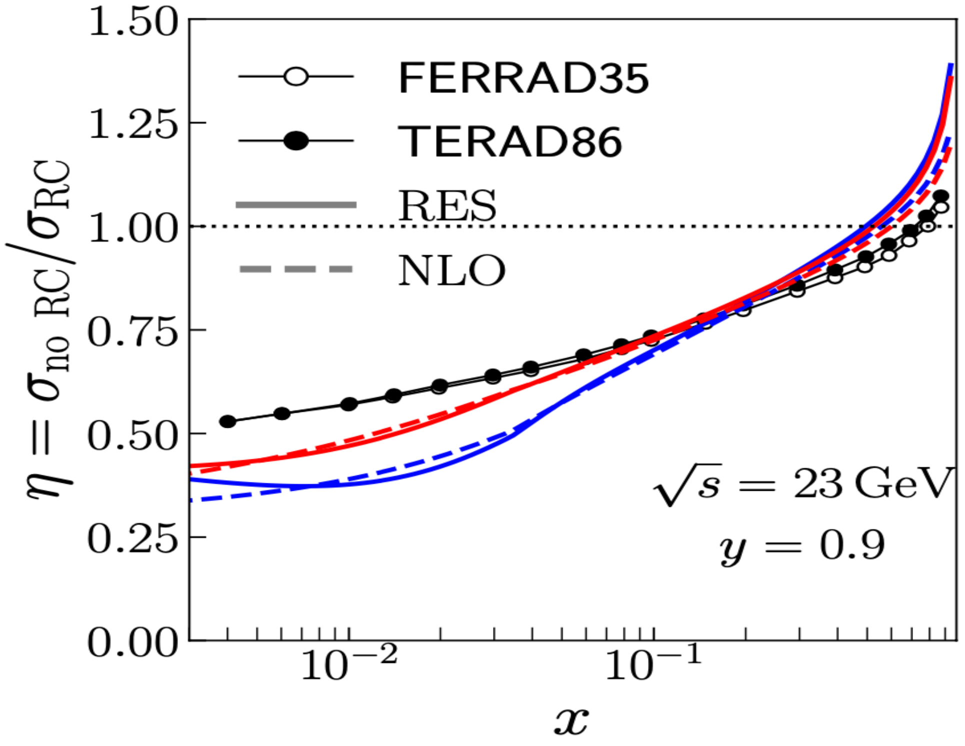
LDFs: Very different from PDFs, peaked
LFFs: at larger momentum fraction

$$f_{e/e}(x) \approx D_{e/e}(x) = N_e \frac{x^\alpha (1-x)^\beta}{B(1+\alpha, 1+\beta)}$$

with $N_e = 1$

$$(\alpha,\beta) = (5, 1/2),\ (50, 1/8)$$

PDFs: CTEQ CT18
not much difference from using other set of PDFs



Liu, Melnitchouk, Qiu, Sato, JHEP 11 (2021) 157

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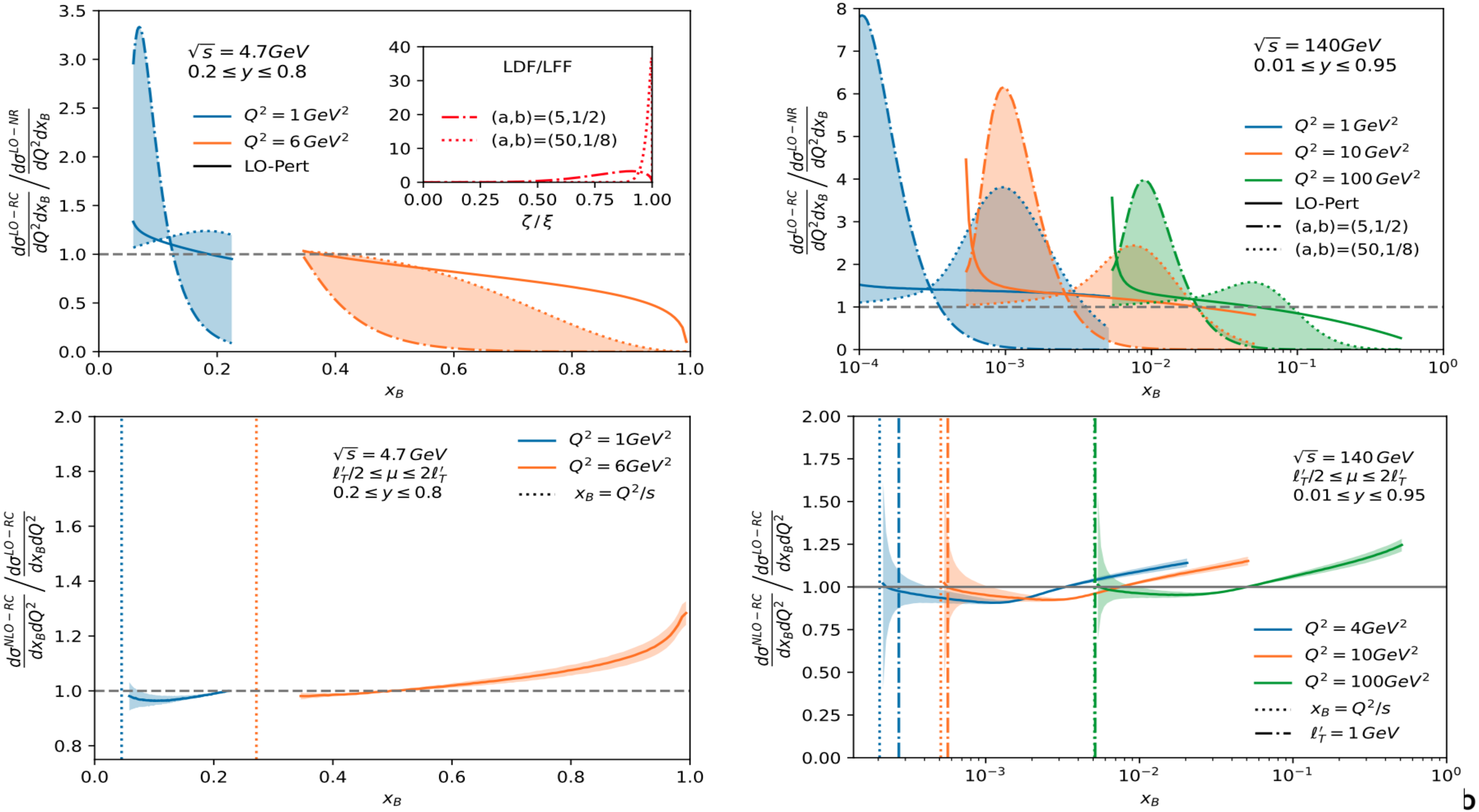
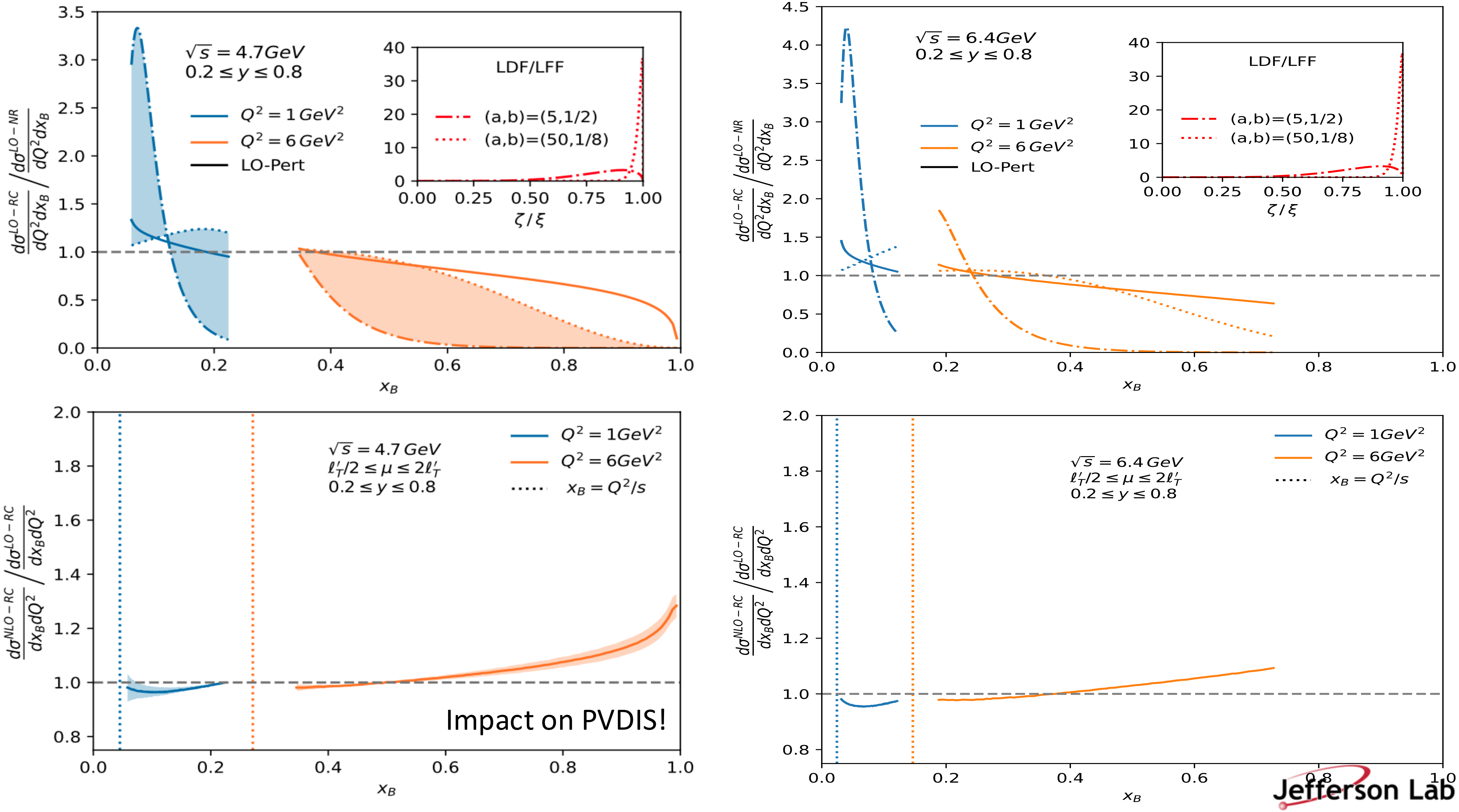
Impact of Factorized QED Contribution to Lepton-Hadron Scattering



Impact of Factorized QED Contribution to Lepton-Hadron Scattering



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Impact of Factorized QED Contribution to Lepton-Hadron Scattering

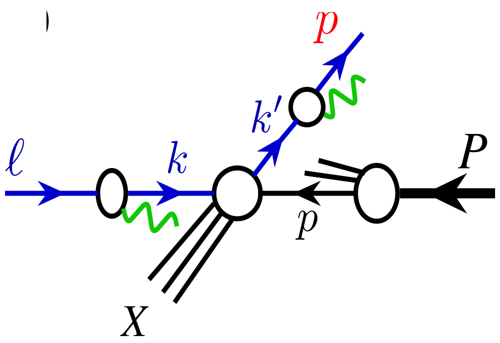
- **Q^2 is NOT an ideal hard scale at small x_B and/or beyond LO in QED:**

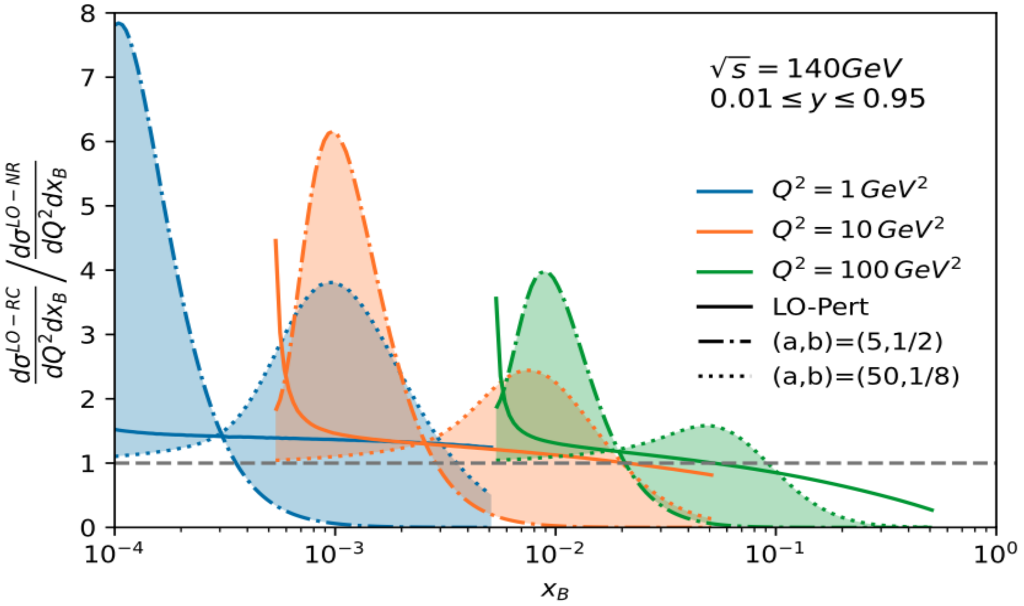
$$p_T^2 = Q^2(1-y) = Q^2\left(1 - \frac{Q^2}{x_B s}\right)$$

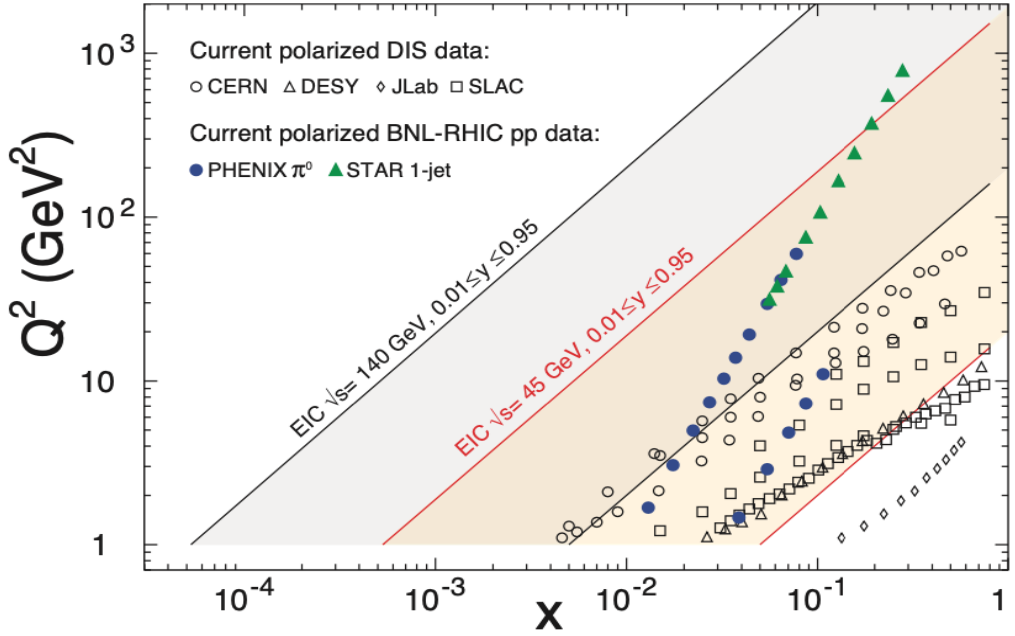
For EIC: $y \leq 0.95$

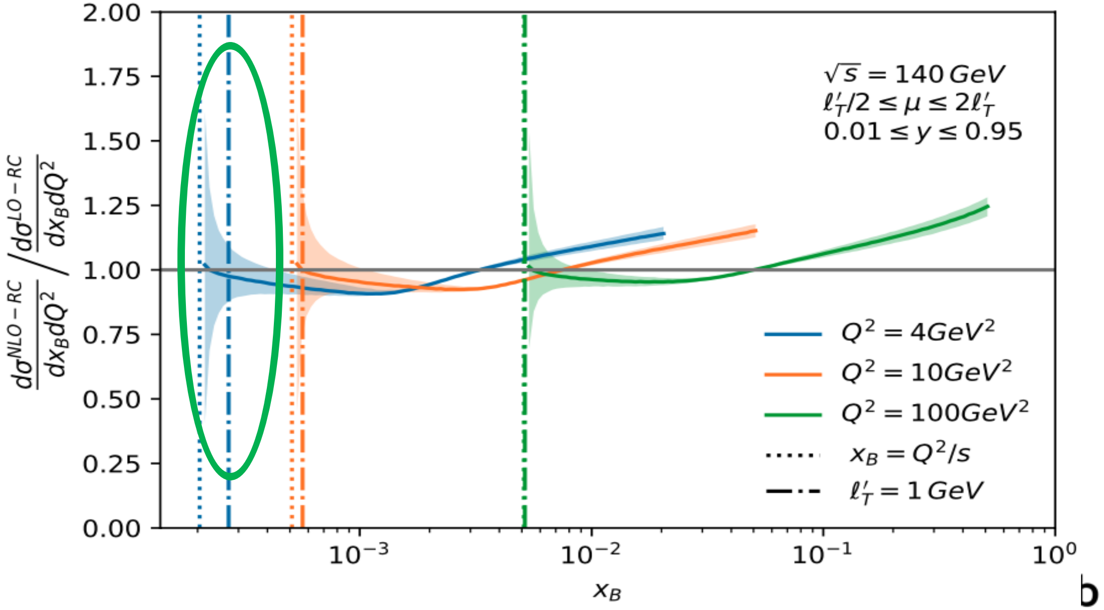
$$p_{T_{\min}}^2 = Q^2(1-y_{\max}) = \frac{Q^2}{20}!!!$$

Factorization does not work if p_T^2 is too small!









Impact on Extracting Non-perturbative Hadron Structure

❑ Additional universal & non-perturbative LDFs and LFFs:

- lepton-hadron reactions (COMPASS, JLab, **EIC**)

$$\sigma^{EXP}_{l+P\,!\;l+X} = C_{l+k\,!\;l+X} \otimes \mathrm{PDF}_P \otimes \mathrm{LDF}_e \otimes \mathrm{LFF}_e + \mathcal{O}(Q_s^2/Q^2)$$

$$\sigma^{\mathrm{EXP}}_{l+P\to l+H+X} = C_{l+k\to l+k+X} \otimes \mathrm{PDF}_P \otimes \mathrm{FF}_H \otimes \mathrm{LDF}_e \otimes \mathrm{LFF}_e + \mathcal{O}(Q_s^2/Q^2)$$

- hadron-hadron reactions (LHC)

$$\sigma^{\mathrm{EXP}}_{P+P\to l+\bar{l}+X} = C_{k+k\to l+\bar{l}+X} \otimes \mathrm{PDF}_P \otimes \mathrm{PDF}_P + O(Q_s^2/Q^2)$$

- lepton-lepton reactions (Belle)

$$\sigma^{\mathrm{EXP}}_{l+\bar{l}\to H+X} = C_{l+\bar{l}\to k+X} \otimes \mathrm{FF}_H \otimes \mathrm{LDF}_e \otimes \mathrm{LDF}_{\bar{e}} + \mathcal{O}(Q_s^2/Q^2)$$

❑ Additional physical processes sensitive to LDFs and/or LFFs:

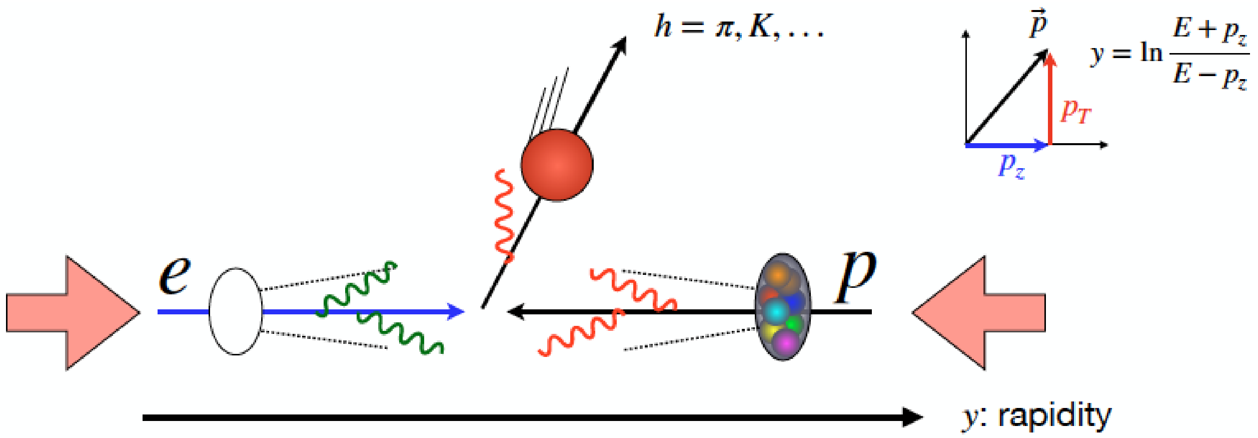
- lepton-hadron reactions (COMPASS, JLab, **EIC**)

$$\sigma^{\mathrm{Exp}}_{l+P\to H+X} = C_{l+k\to k+X} \otimes \mathrm{LDF}_s \otimes \mathrm{PDF}_P \otimes \mathrm{FF}_H + \mathcal{O}(Q_s^2/Q^2)$$

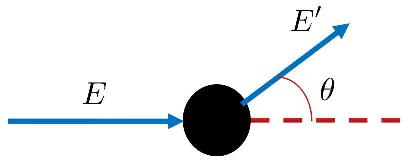
Prompt Single Hadron/Jet Production:

❑ Single hadron (Jet) production in lepton-hadron scattering:

Kang, Meta, Qiu, Zhou, PRD 2011
Hinderer, Schlegel, Vogelsang, PRD 2015, 2016
Abelof, Boughezal, Liu, Petriello, PLB, 2016
Qiu, Wang, Xing, CPL, 2021
Qiu, Watanabe, in preparation



JLab kinematics – fixed target:



$$p_T = E' \sin(\theta) \gtrsim 1 \text{ GeV?}$$

Factorization:

$$E\frac{d\sigma_{\ell P \to pX}}{d^3p} \approx \frac{1}{2s}\sum_{i,a,b}\int_{z_{\min}}^{1}\frac{dz}{z^2}\int_{\xi_{\min}}^{1}\frac{d\xi}{\xi}D_{h/b}(z,\mu^2)\,f_{i/e}(\xi,\mu^2)$$
$$\times \int_{x_{\min}}^{1}\frac{dx}{x}f_{a/N}(x,\mu^2)\,\widehat{H}_{ia\to bX}(\xi\ell, xP, p/z, \mu^2) + (1/p_T)^{\alpha}$$

Single hard scale
Collinear factorization

Nayak, Qiu, Sterman, PRD72, 114012 (2005)

Hard parts are valid at NLO
Partially available at NNLO
in QCD

The new unknown is $f_{i/e}(\xi,\mu^2)$
lepton distribution functions (LDFs)

Hadron fragmentation functions (FFs):

Known, but, limited knowledge, in particular, at large z !

Could be a pre-requisite for applying the factorization for SIDIS - validity of fragmentation

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Evolution of lepton distribution functions (LDFs)

❑ Modified DGLAP equation for LDFs:

Qiu, Watanabe
In preparation

$$\frac{\partial}{\partial \ln \mu^2}\begin{pmatrix} f_{e/e}(\xi,\mu^2) \\ f_{\bar{e}/e}(\xi,\mu^2) \\ f_{\gamma/e}(\xi,\mu^2) \\ f_{q/e}(\xi,\mu^2) \\ f_{\bar{q}/e}(\xi,\mu^2) \\ f_{g/e}(\xi,\mu^2) \end{pmatrix} = \left(\begin{array}{ccc|ccc} P_{ee}^{(1,0)} & P_{e\bar{e}}^{(2,0)} & P_{e\gamma}^{(1,0)} & P_{eq}^{(2,0)} & P_{e\bar{q}}^{(2,0)} & P_{eg}^{(2,1)} \\ P_{\bar{e}e}^{(2,0)} & P_{\bar{e}\bar{e}}^{(1,0)} & P_{\bar{e}\gamma}^{(1,0)} & P_{\bar{e}q}^{(2,0)} & P_{\bar{e}\bar{q}}^{(2,0)} & P_{\bar{e}g}^{(2,1)} \\ P_{\gamma e}^{(1,0)} & P_{\gamma\bar{e}}^{(1,0)} & P_{\gamma\gamma}^{(1,0)} & P_{\gamma q}^{(1,0)} & P_{\gamma\bar{q}}^{(1,0)} & P_{\gamma g}^{(1,1)} \\ \hline P_{qe}^{(2,0)} & P_{q\bar{e}}^{(2,0)} & P_{q\gamma}^{(1,0)} & P_{qq}^{(0,1)} & P_{q\bar{q}}^{(0,2)} & P_{qg}^{(0,1)} \\ P_{\bar{q}e}^{(2,0)} & P_{\bar{q}\bar{e}}^{(2,0)} & P_{\bar{q}\gamma}^{(1,0)} & P_{\bar{q}q}^{(0,2)} & P_{\bar{q}\bar{q}}^{(0,1)} & P_{\bar{q}g}^{(0,1)} \\ P_{ge}^{(2,1)} & P_{g\bar{e}}^{(2,1)} & P_{g\gamma}^{(1,1)} & P_{gq}^{(0,1)} & P_{g\bar{q}}^{(0,1)} & P_{gg}^{(0,1)} \end{array}\right) \otimes \begin{pmatrix} f_{e/e}(\xi,\mu^2) \\ f_{\bar{e}/e}(\xi,\mu^2) \\ f_{\gamma/e}(\xi,\mu^2) \\ f_{q/e}(\xi,\mu^2) \\ f_{\bar{q}/e}(\xi,\mu^2) \\ f_{g/e}(\xi,\mu^2) \end{pmatrix}$$

- **Factorization scale:**

  $$\mu^2 \sim m_c^2$$

- **Input LDFs at μ^2:**
  - **Perturbatively generated by solving QED evolution from lepton mass threshold**
  - **With perturbatively calculated fixed-order MSbar LDFs**
  - **Test the size of non-perturbative hadronic contribution**
  - …

Evolution kernels in both QCD and QED:

$$P_{ij}(\xi,\mu^2) = \sum_{n,m=0}^{\infty}\left(\frac{\alpha_{em}(\mu^2)}{2\pi}\right)^n \left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^m \hat{P}_{ij}^{(n,m)}(\xi) = \sum_{n,m=0}^{\infty} P_{ij}^{(n,m)}(\xi,\mu^2)$$

with $\quad P_{ij}^{(0,0)} = 0\,,\quad N_F\,,\quad N_l$

The QCD/QED mixing is critically important for canceling all collinear divergence between LO!

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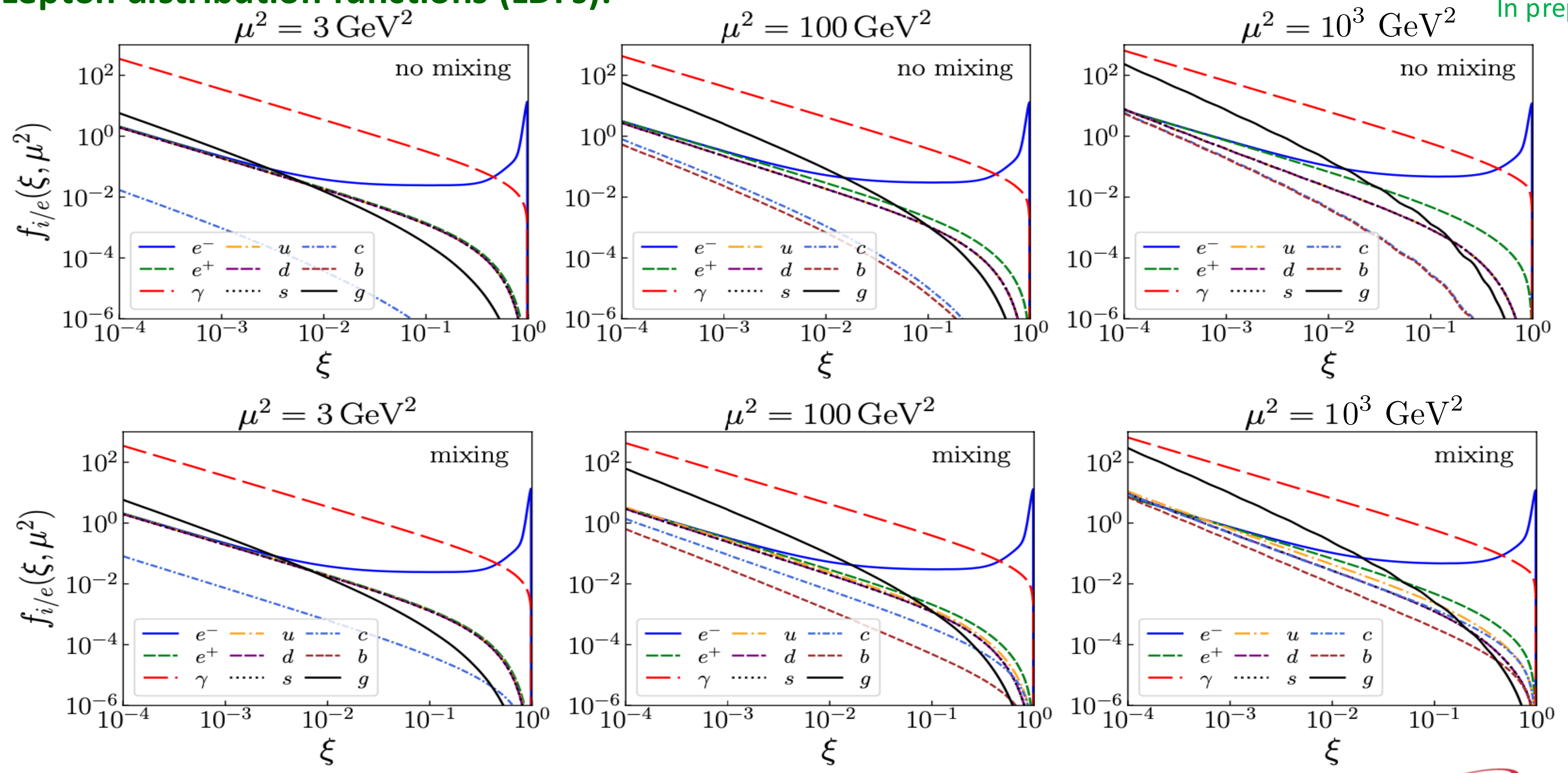
Evolution of lepton distribution functions (LDFs)

Qiu, Watanabe
In preparation

❑ Lepton distribution functions (LDFs):



With LDFs, we calculated single hadron production, including J/ψ production at the EIC

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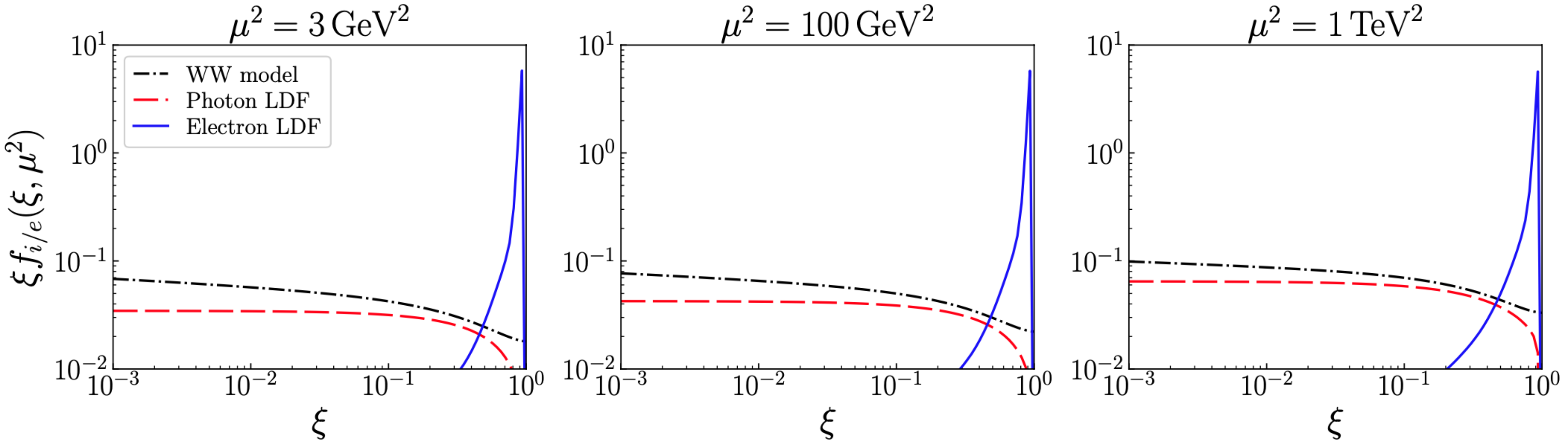
Evolution of lepton distribution functions (LDFs)

Qiu, Watanabe
In preparation

❑ Photon distribution of the electron:

- **Weizsa¨cker-William photon distribution:**

$$f^{\mathrm{WW}}_{\gamma/e}(\xi,\mu^2) = \frac{\alpha_{em}(\mu^2)}{2\pi} P_{\gamma e}(\xi)\left[\ln\left(\frac{\mu^2}{\xi^2 m_e^2}\right) - 1\right]$$



- **LDFs are not purely perturbative in QED or perturbative – need global analysis!**
  - **Precision measurements for BSM physics at the EIC needs reliable lepton distributions**
  - **Joint global analysis of lepton and hadron distribution functions should be carried out.**
  - **Impact on searching BSM at ILC or CEPC, FCC, …**

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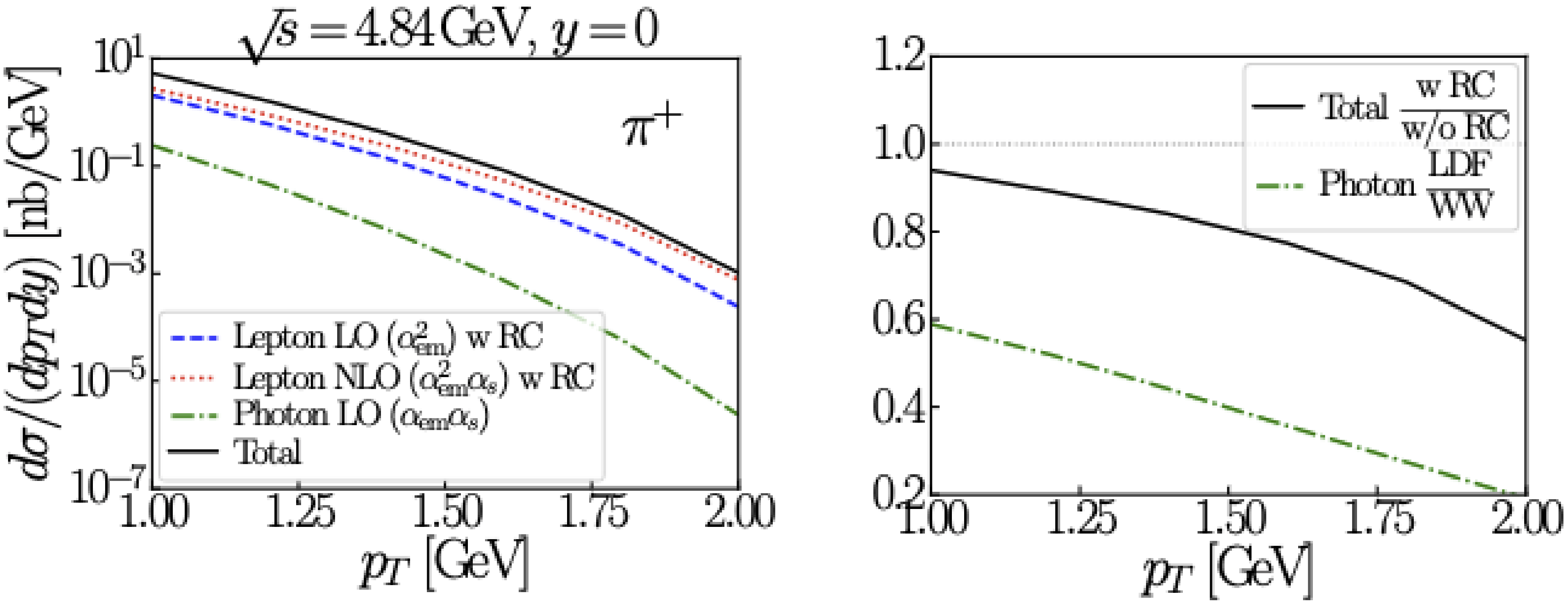
Calculation for JLab 12 GeV Energy

- **FFs – JAM20:**

Qiu, Watanabe
In preparation

Preliminary

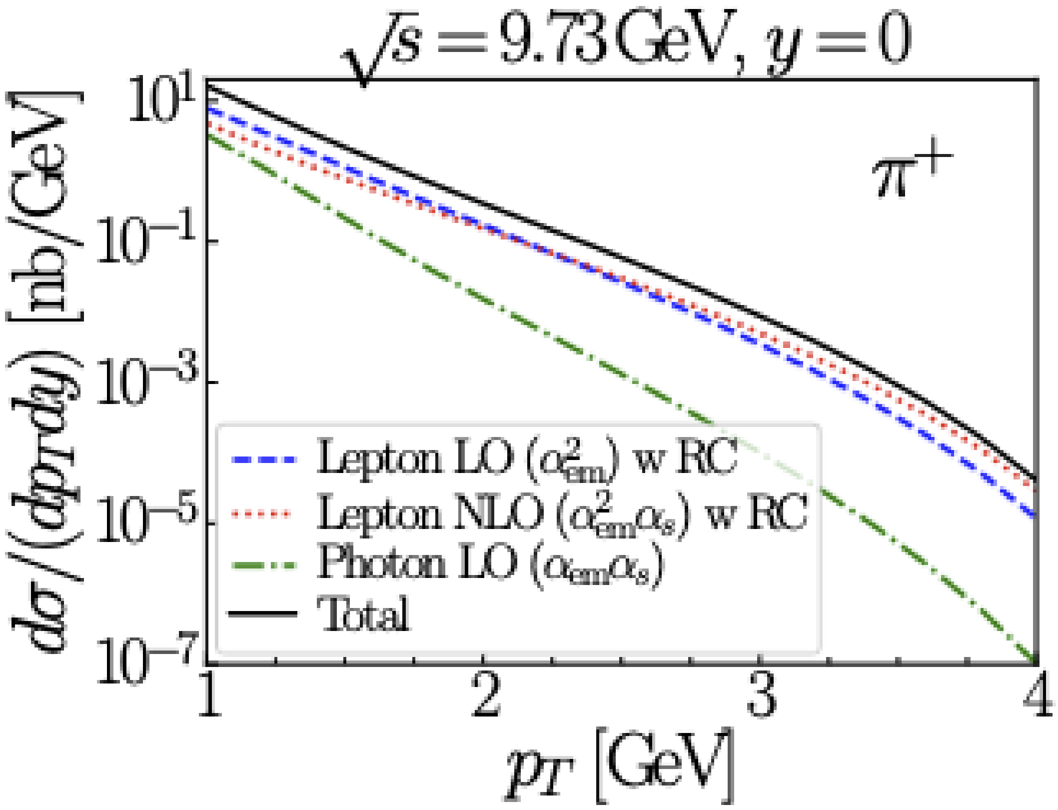


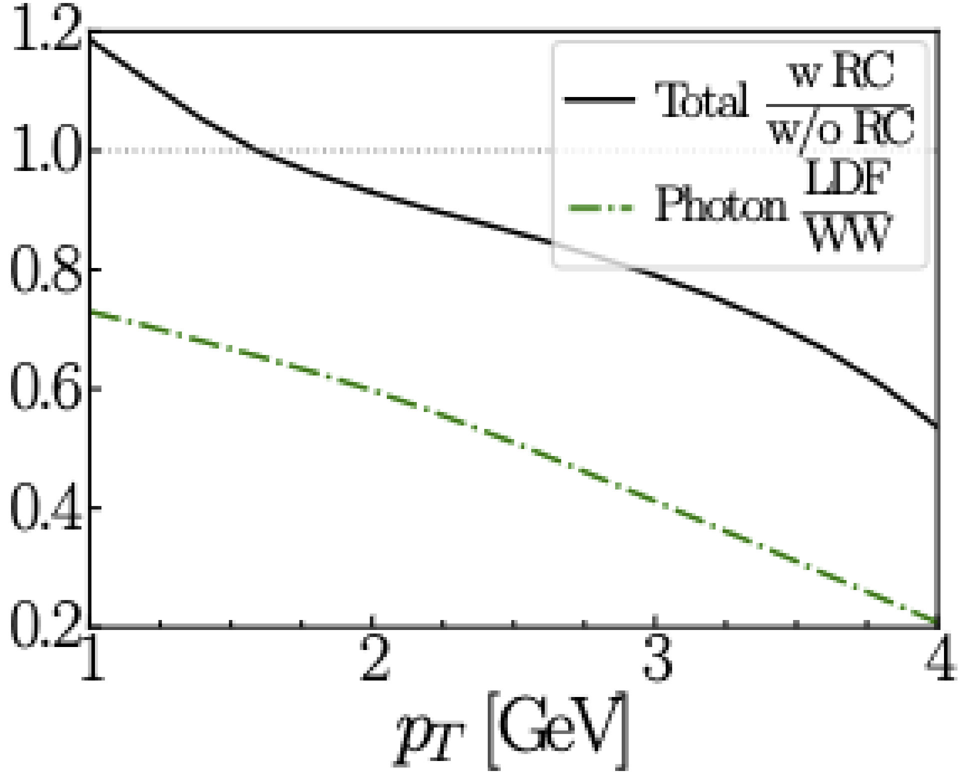
Calculations for SLAC Energy

- **FFs – JAM20:**

Qiu, Watanabe
In preparation

Preliminary



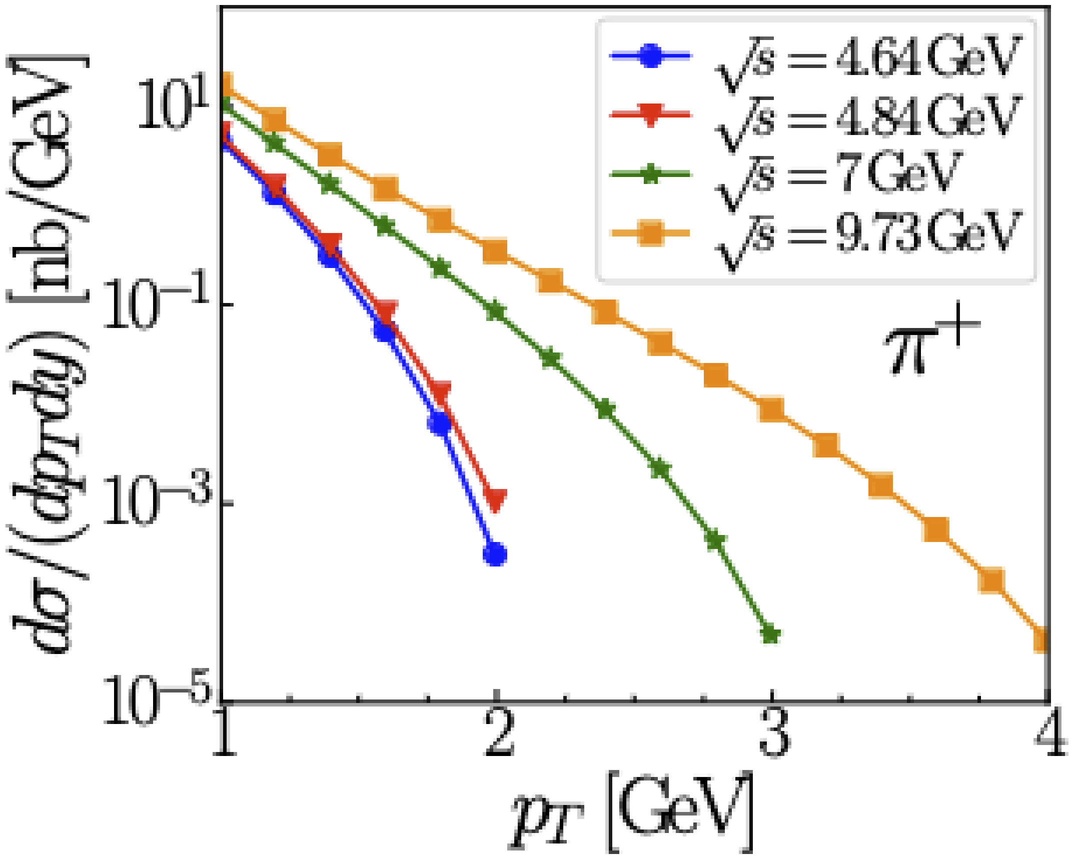


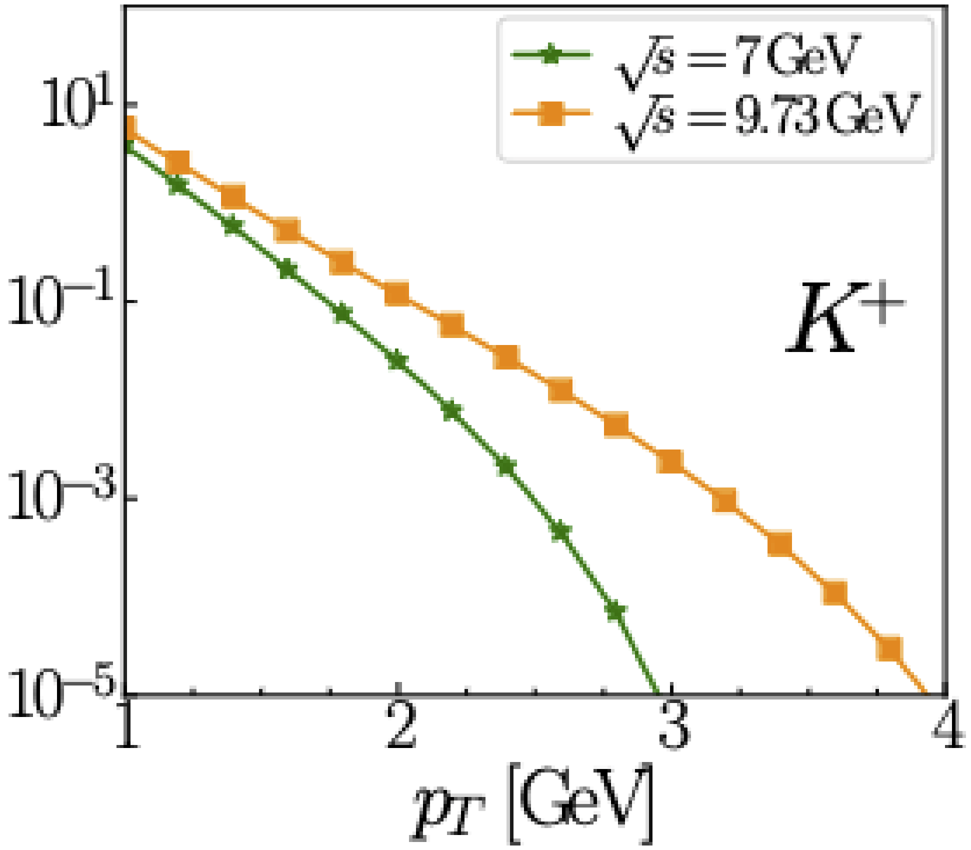
Calculations for various Fixed Target Energies

- **FFs – JAM20:**

Qiu, Watanabe
In preparation

Preliminary





Explore Uncertainties of Fragmentation Functions

Qiu, Watanabe
In preparation

- **Parameterize the JAM20 FFs:**

Preliminary

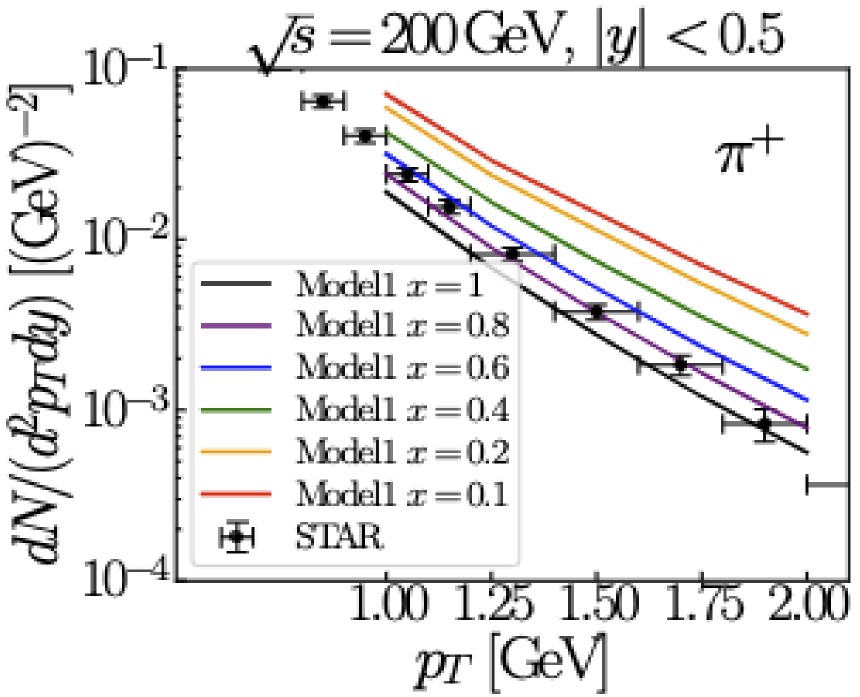
$$(\text{Model1}): zD_i^{\pi^+}(z, x) = N_i \frac{z^{\alpha_i}(1-z)^{\beta_i x}}{B[1+\alpha_i, 1+\beta_i x]},$$

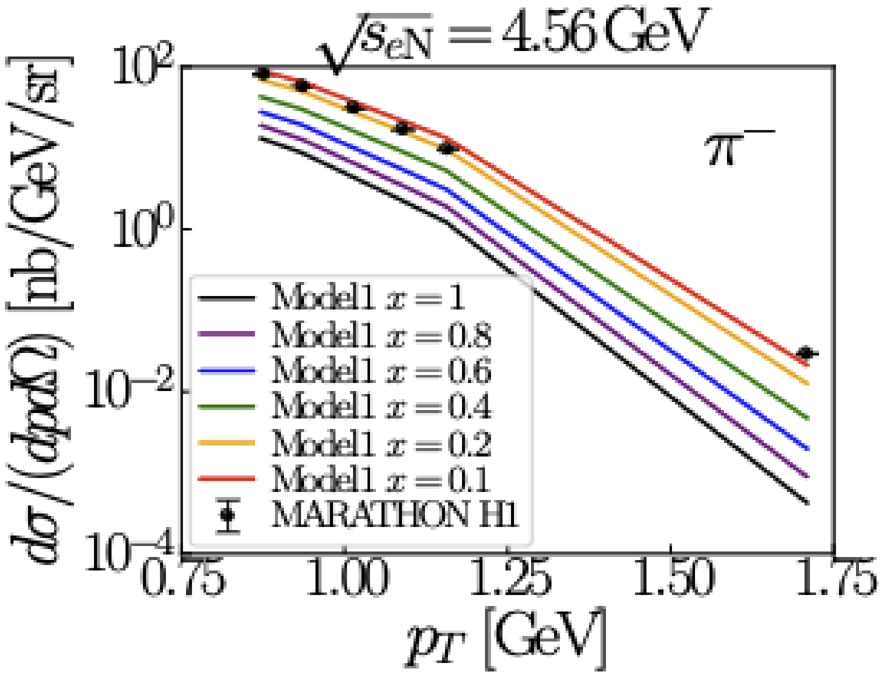
$$(\text{Model2}): zD_i^{\pi^+}(z, x) = N_i \frac{z^{\alpha_i}(1-z)^{\beta_i x}(1+c_i z^{\gamma_i})}{B[1+\alpha_i, 1+\beta_i x] + c_i B[1+\alpha_i+\gamma_i, 1+\beta_i x]}.$$

With a parameter: $x \in [0, 1]$

- Smaller x = Larger contribution from large z region
- e+e- has weak constraint on large z

- **Compare with data:**





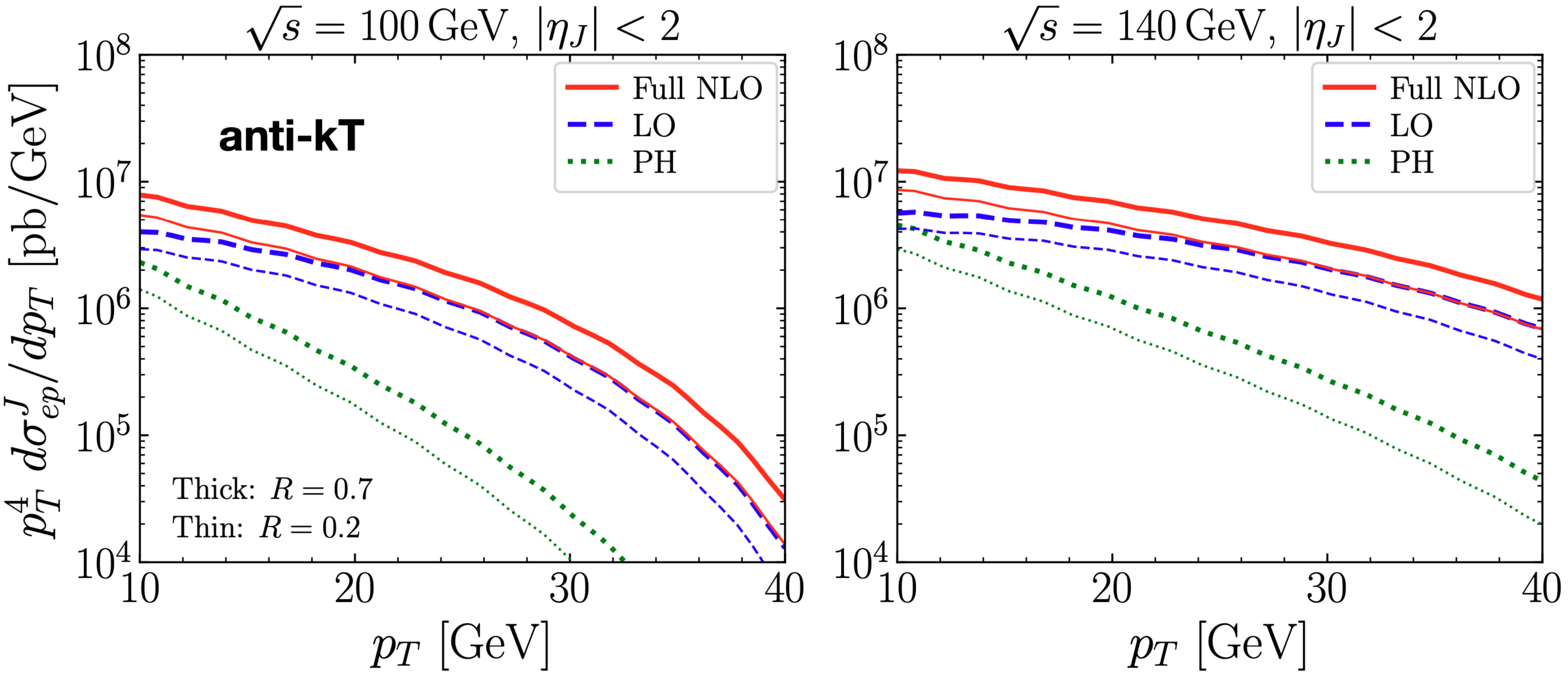
Role of power corrections

Calculation for Jet Production at the EIC

❑ Jet production – pT dependence:

Qiu, Watanabe
In preparation

Preliminary

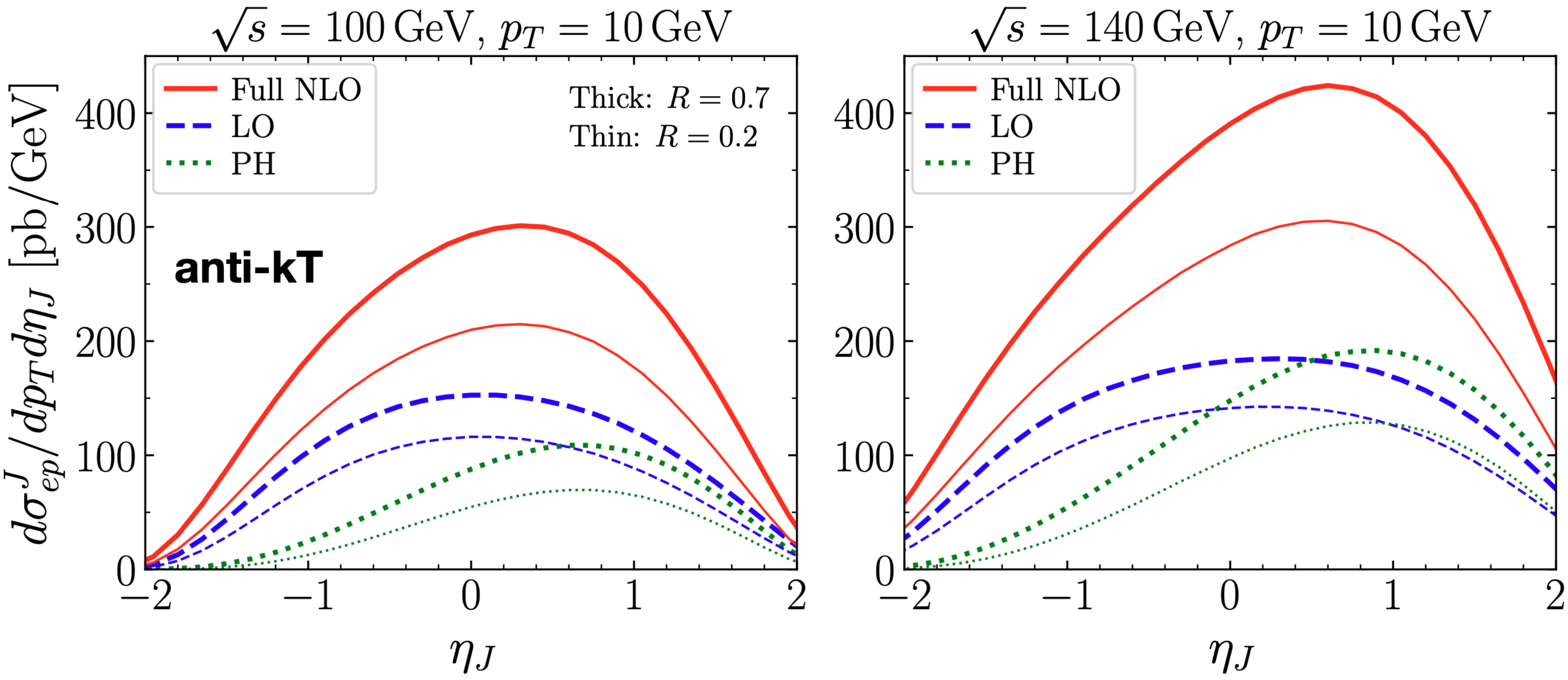


Calculation for Jet Production at the EIC

❑ Jet production – η dependence:

Qiu, Watanabe
In preparation

Preliminary



Summary and Outlook – Thank you!

- Collision induced QED radiation is an integrated part of the lepton-hadron collision
  - Radiative correction approach is difficult for a consistent treatment beyond the inclusive DIS
  - No well-defined photon-hadron frame, if we cannot recover all QED radiation
  - Radiative corrections are more important for events with high momentum transfers and large phase space to shower – such as those at the EIC

  Factorization approach to include both QCD and QED radiative contributions provides a consistent and controllable approximation to high-energy lepton-hadron scattering processes

- Proposed to use the prompt single hadron inclusive production to test the fragmentation picture in particular at JLab energy – Prerequisite to fitting SIDIS data with current factorization formalism
  - Current fragmentation functions with the NLO perturbative coefficients can fit the RHIC data
  - But, have a difficulty to fit the JLab data, which could have included "non-prompt" pions (those from rho decay, …)
  - Same problem is much less important due the more steep falling spectrum of rho and other VMs.

  Need to consistently subtract the "background" from "non-prompt" source of "leading hadron" when studying SIDIS at JLab energies