Energy-momentum tensor and generalized parton distributions in $N \to \Delta$ transitions

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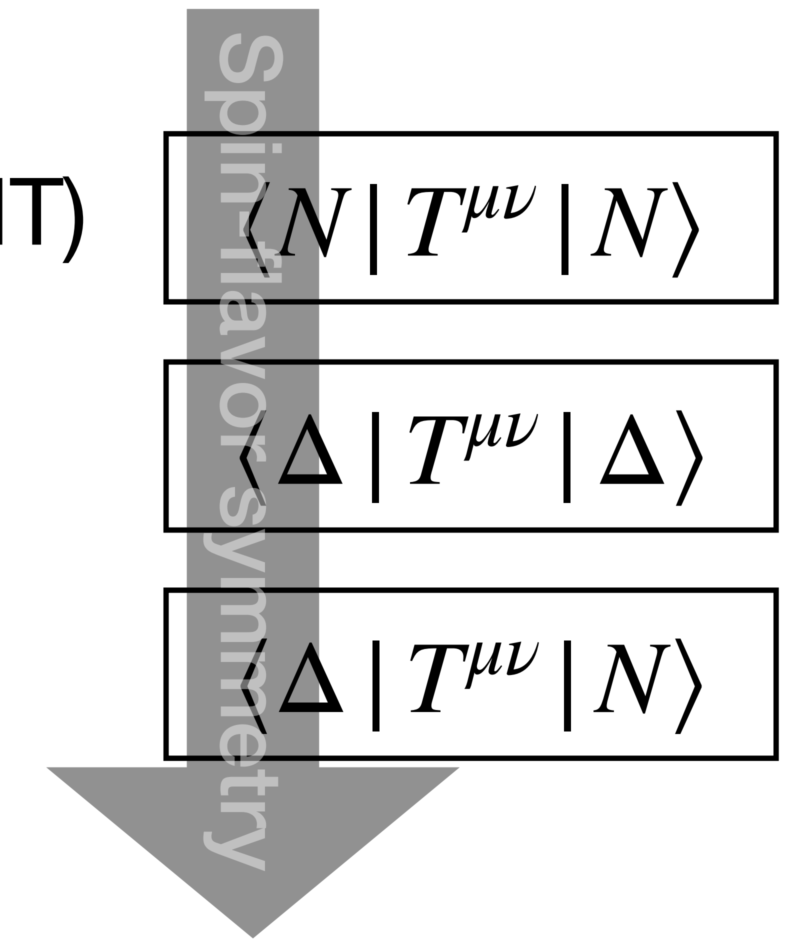
JLab Cake seminar

Outline

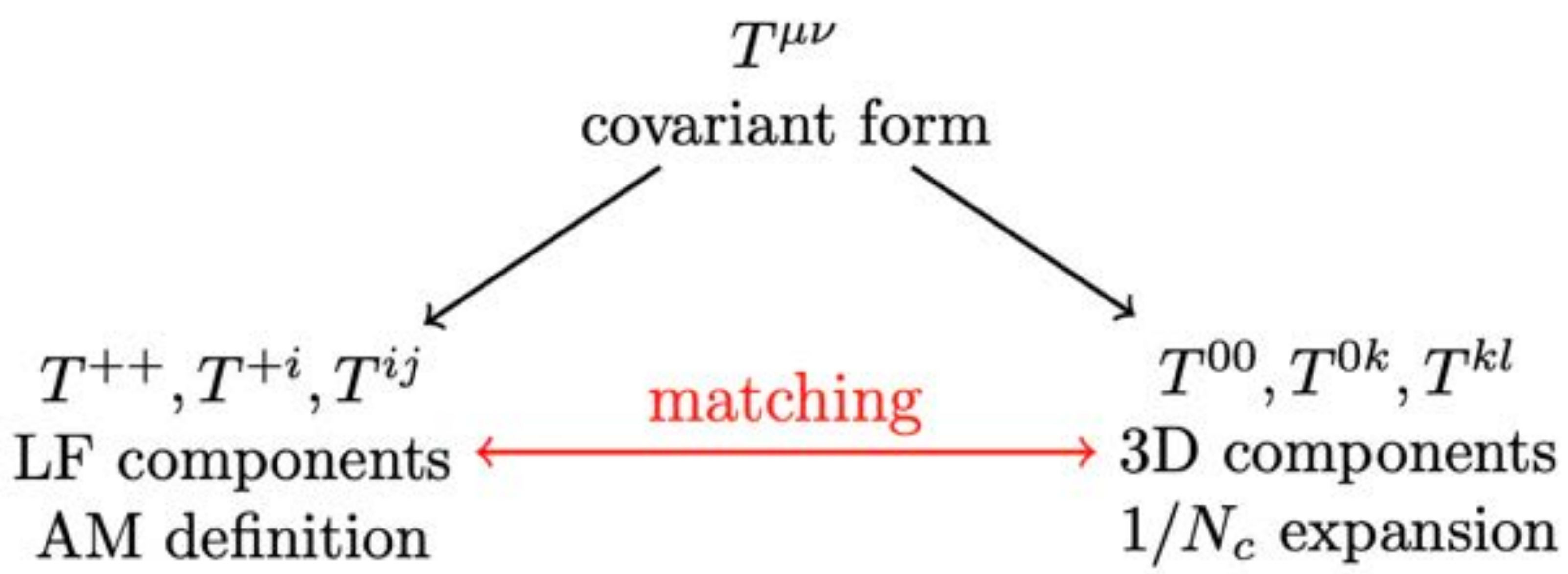
1. Model-independent formalism

- QCD energy-momentum tensor (EMT)
- Nucleon matrix element
- $N \to \Delta$ transition matrix element



2. Dynamical symmetry

- Spin-flavor symmetry in large N_c limit of QCD
- 2D LF interpretation in the large N_c limit of QCD



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3. Dynamics

- Chiral theory in the large N_c limit of QCD
- Interpolation between non-relativistic quark model and chiral perturbation theory in large N_c
- Chiral properties of the energy-momentum tensor
- Light-cone chiral quark-soliton model

4. Near future work

- Parity-odd EMT
- QCD operator vs effective operator

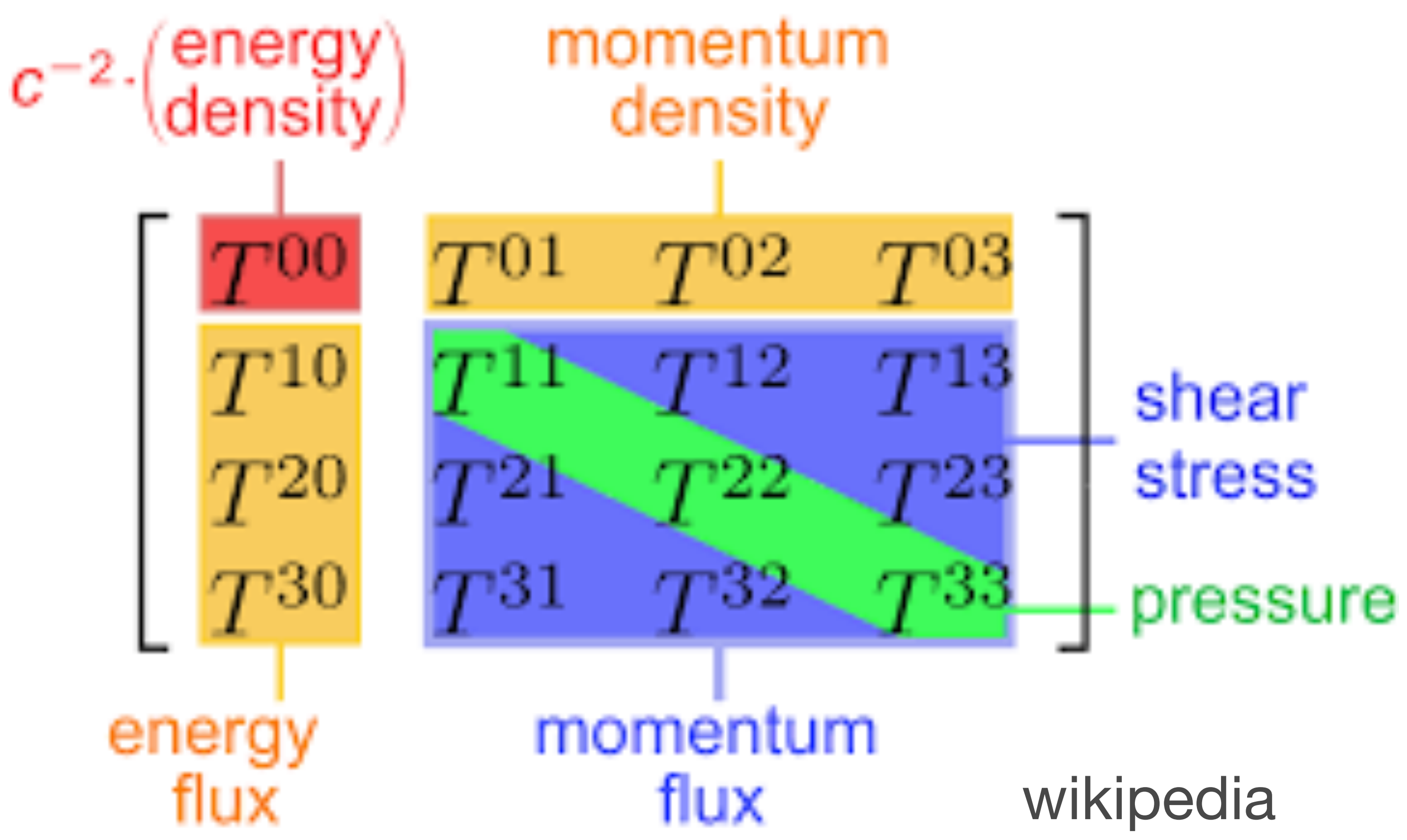
5. Summary

Model-independent formalism

QCD Energy-momentum tensor (EMT)

Nucleon energy-momentum tensor (EMT)

- EMT form factor was considered as an academic subject (weakly coupled nature of the graviton).
  I.Y. Kobzarev, et al (1962)/H. Pagels (1966)
- GPDs allow us to access the EMT form factors via the Mellin moments. K. Goeke, etal (2001)/ M. Diehl (2003)/ A. V. Belitsky et al (2005)
- The D-term form factor has been extracted from the deeply virtual Compton scattering (DVCS) data.
  K. Kumericki, etal (2016)/ V.D. Burkert (2018)
- EMT form factors encode information about the mechanical properties of the nucleon.
  M.V. Polyakov et al (2018)/ C. Lorcé etal (2018)/



wikipedia

QCD EMT current

$$\hat{T}_f^{\alpha\beta}(x) = i\bar{\psi}_f(x)\gamma^{\{\alpha}\overleftrightarrow{\nabla}^{\beta\}}\psi_f(x)$$

$$\{\alpha\beta\} \equiv \tfrac{1}{2}(\alpha\beta + \beta\alpha)$$

$$\overleftrightarrow{\nabla}^{\mu} \equiv \tfrac{1}{2}(\overrightarrow{\partial}^{\mu} - \overleftarrow{\partial}^{\mu}) - igA^{\mu}$$

- Current conservation

$$\hat{T}^{\mu\nu} = \sum_q \hat{T}_q^{\mu\nu} + \hat{T}_g^{\mu\nu}, \quad \partial_\mu \hat{T}^{\mu\nu} = 0$$

Matrix element of the EMT current

- 3D interpretation: intuitive, applicable to non-relativistic limit (3D multipole expansion, $1/N_c$ expansion) R.G. Sachs (1962)/M.V. Polyakov (2002)
- 2D LF interpretation: Unambiguous, loses longitudinal information
  M. Burkardt (2000) /G.Miller (2007)

$$\langle B'|\hat{T}_q^{\mu\nu}|B\rangle$$

3D ↙ ↘ 2D LF

$$\langle B'|\hat{T}_q^{00}, \hat{T}_q^{0k}, \hat{T}_q^{ij}|B\rangle \qquad \langle B'|\hat{T}_q^{++}, \hat{T}_q^{+k}, \hat{T}_q^{ij}|B\rangle$$

$$(B' = B) \quad \{q^0 = 0, \mathbf{P} = \mathbf{0}\} \qquad \{q^+ = 0, q^- = 0, \mathbf{P}_\perp = \mathbf{0}\}$$

$$(B' \neq B) \quad \{q^0 \neq 0, \mathbf{P} = \mathbf{0}\} \qquad \{q^+ = 0, q^- \neq 0, \mathbf{P}_\perp = \mathbf{0}\}$$

Nucleon matrix element of the QCD EMT current

EMT form factors in $N \to N$

- Discrete symmetries (DS), Current conservation (CC)

$$\langle N(p',\sigma')|\hat{T}^{\mu\nu}_a(0)|N(p,\sigma)\rangle = \overline{u}(p',\sigma')\Big[A^{N,a}(t)\frac{P^\mu P^\nu}{m_N} + J^{N,a}(t)\frac{i(P^\mu\sigma^{\nu\rho}+P^\nu\sigma^{\mu\rho})q_\rho}{2m_N} + D^{N,a}(t)\frac{q^\mu q^\nu - g^{\mu\nu}q^2}{4m_N} + m_N g^{\mu\nu}\bar{c}^{N,a}(t)\Big]u(p,\sigma),$$

conserved (blue) non-conserved (red)

I.Y. Kobzarev, et al (1962)/H. Pagels (1966)

$$q = p' - p, \quad P = (p'+p)/2, \quad t = q^2$$

Forward limit

M.V. Polyakov et al (2018)
C. Lorcé etal (2018)
C. Lorcé etal (2023)

- Light-front momentum (positivity)

$$\frac{\langle N|T^{++}_q|N\rangle}{2(P^+)^2} = A^q(0) = \int dx\, x f_1^q(x) = \langle x\rangle_q$$

- Mass decomposition

$$\frac{\langle N|T^{00}_q|N\rangle}{2P^0} = M_N[A^q(0) + \bar{c}^q(0)]$$

- Internal pressure

$$\frac{\langle N|T^{ii}_q|N\rangle}{2P^0} = -M_N\bar{c}^q(0)$$

(twist-4 GPDs)

Cosmological constant term

$$\langle 0|\hat{T}^{\mu\nu}|0\rangle = \Lambda g^{\mu\nu}$$

O. V. Teryaev, (2016)
K.-F. Liu (2021)

E.Leader (2013)/E.Leader, C.Lorcé (2014)

Non-forward matrix element

$$J^q = L^q + S^q$$

- Total angular momentum (AM)

Ji's relation X.D. Xi (1997)

$$2S^3_{\sigma'\sigma}J^{N,q}(0) = -i\epsilon^{3mk}\nabla^m_q \frac{\langle N|T^{\{+k\}}_{\text{kin},q}|N\rangle}{2P^+}\Big|_{\mathbf{q}=0}$$

Ji's sum rule (twist-2 GPDs)
X.D. Xi (1997)

- **Orbital angular momentum (OAM)** vs **Intrinsic spin** parts

$$2S^3_{\sigma'\sigma}L^{N,q}(0) = -i\epsilon^{3mk}\nabla^m_q \frac{\langle N|T^{+k}_{\text{kin},q}|N\rangle}{2P^+}\Big|_{\mathbf{q}=0}$$

OAM (twist-3 GPDs)
M.Penttinen, et al (2000)
Xi,Xiong, Yuan (2013) Hatta, Yoshida (2012)

$$-2S^3_{\sigma'\sigma}S^{N}(0) = -i\epsilon^{3mk}\nabla^m_q \frac{\langle N|T^{[+k]}_{\text{kin},q}|N\rangle}{2P^+}\Big|_{\mathbf{q}=0}$$

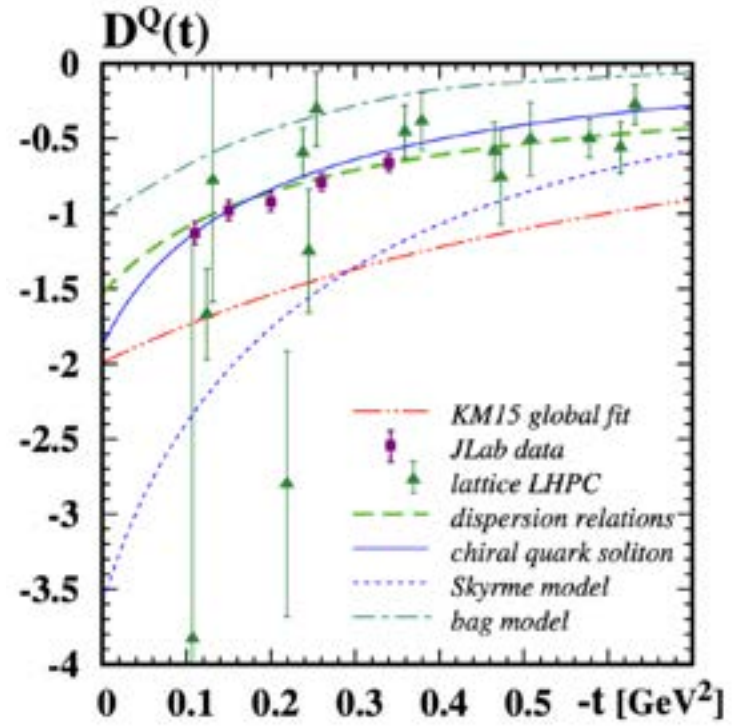
Axial vector charge (twist-2 GPDs)

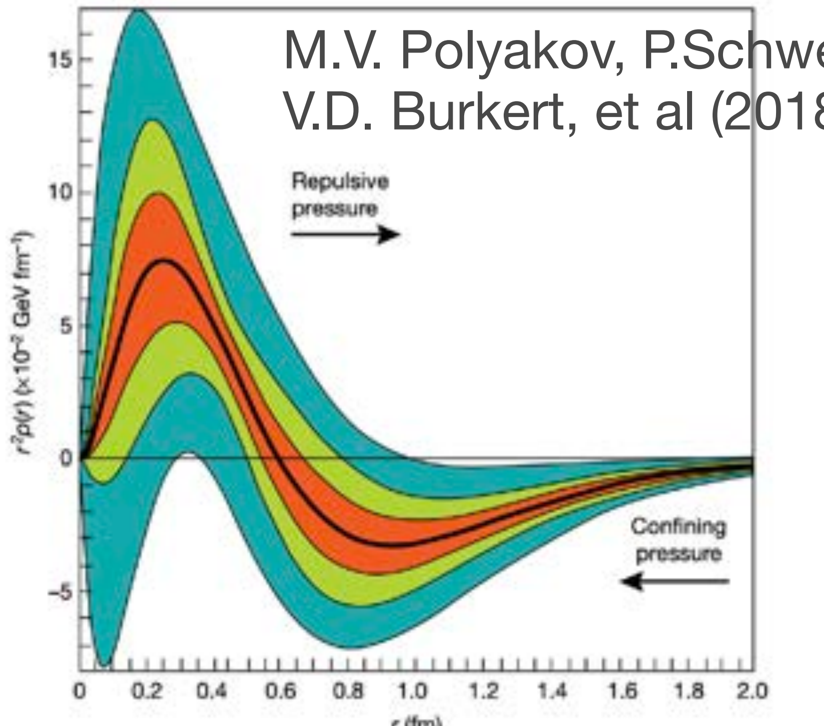
$$\overline{\psi}(x)\gamma^{[\alpha}i\overleftrightarrow{D}^{\beta]}\psi(x) = -\varepsilon^{\alpha\beta\mu\lambda}\partial_\mu\left[\overline{\psi}(x)\gamma_\lambda\gamma_5\psi(x)\right]$$

- Polyakov & Weiss D-term M.V. Polyakov, C.Weiss (1999)

carries information about the Internal forces in the nucleon.

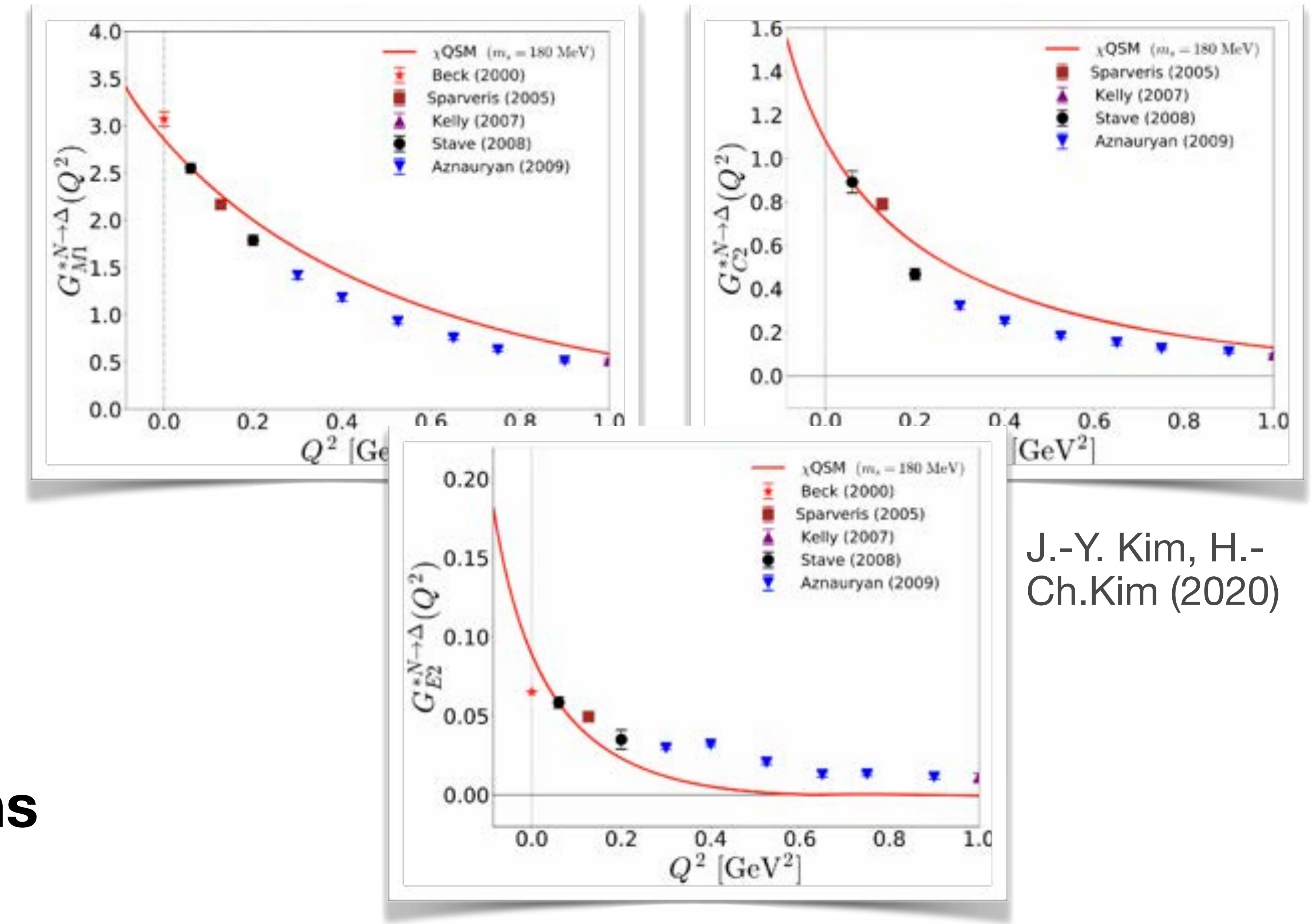
extracted from the DVCS data, assuming "flavor blindness" inspired by the large N_c limit.





M.V. Polyakov, P.Schweitzer, (2018)
V.D. Burkert, et al (2018)

$N \to \Delta$ transition matrix element ?



J.-Y. Kim, H.-Ch.Kim (2020)

Electromagnetic properties in $N \to \Delta$ transitions

- extensively studied both theoretically and experimentally.

CLAS collaboration, V. Pascalutsa, et al (2006)

- We were restricted to the EM form factors.

Transition generalized parton distributions (GPDs) in $N \to \Delta$ transitions

- Recently, the first measurement of the hard exclusive $\pi^- \Delta^{++}$ electroproduction beam-spin asymmetries off the proton has been made.

S. Diehl, etal (2023)

- Related theoretical works on hard exclusive scattering in the $N \to \Delta$ transitions have been carried out.

P. Kroll, etal (2023)/ K. M. Semenov-Tian-Shansky, etal (2023)

- x dependence of the transition GPDs?

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Energy-momentum tensor in $N \to \Delta$ transitions

→Induced from the charges of the hadron states

- Parametrization of the matrix element?

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- Connection to the GPDs?
- Mechanical interpretations of the transition EMT?

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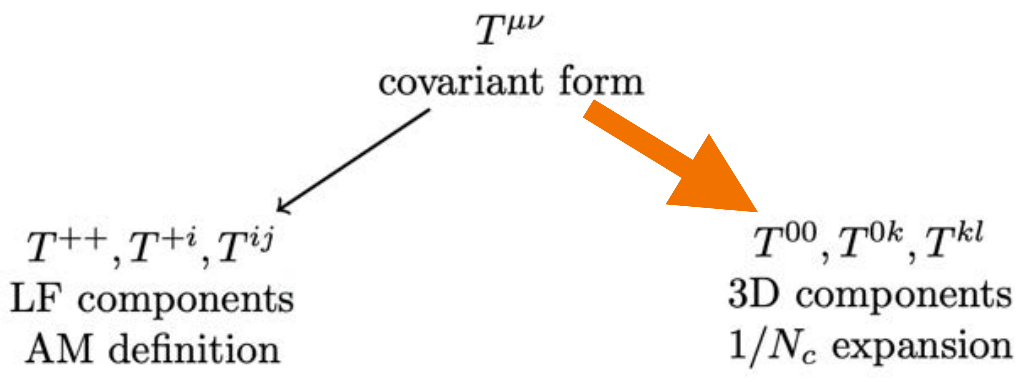

Parametrization of the matrix element

EMT form factors in $N \to \Delta$ transition

- Non-diagonal matrix element → constraints from the discrete symmetries are relaxed. $q \cdot P \neq 0$
- Isospin difference $|I' - I| = 1 \to$ isoscalar current is not allowed.

$$\langle \Delta(p', \sigma') | \hat{T}^{\mu\nu}_a(0) | N(p, \sigma) \rangle = \bar{u}^\alpha(p', \sigma') (\Gamma^{N\to\Delta})^{\mu\nu}_\alpha \gamma_5 u(p, \sigma),$$

$$(\Gamma^{N\to\Delta})^{\mu\nu}_\alpha = F_1^{N\Delta,a}(t) g_\alpha^{\{\mu} P^{\nu\}} + \frac{F_2^{N\Delta,a}(t)}{4\bar{m}^2} P^\mu P^\nu q_\alpha + \frac{F_3^{N\Delta,a}(t)}{4\bar{m}^2} q^\mu q^\nu q_\alpha + 2\bar{m} F_4^{N\Delta,a}(t) \gamma^{\{\mu} g_\alpha^{\nu\}} + \frac{F_5^{N\Delta,a}(t)}{2\bar{m}} \gamma^{\{\mu} P^{\nu\}} q_\alpha + C_1^{N\Delta,a}(t) g^{\mu\nu} q_\alpha + \frac{C_2^{N\Delta,a}(t)}{4\bar{m}^2} q^{\{\mu} P^{\nu\}} q_\alpha + \frac{C_3^{N\Delta,a}(t)}{2\bar{m}} \gamma^{\{\mu} q^{\nu\}} q_\alpha + C_4^{N\Delta,a}(t) g_\alpha^{\{\mu} q^{\nu\}}.$$

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3D components

$$q = p' - p, \quad P = (p' + p)/2, \quad \bar{m} = (m_\Delta + m_N)/2 \qquad \delta_{N\Delta} = m_\Delta - m_N$$

- Constraint on the EMT form factors

$$\sum_{a=q,g} F_i^{N\Delta,a} \; (i = 1,2,3,4,5) = 0 \qquad \sum_{a=q,g} C_i^{N\Delta,a} \; (i = 1,2,3,4) = 0$$

3D multipole expansion of the EMT

- **Breit frame** vs **Generalized Breit frame**

$$P = (E_B, \mathbf{0}), \quad q = (0, \mathbf{q}), \qquad P = \left(\frac{E_\Delta + E_N}{2}, \mathbf{0}\right), \quad q = (E_\Delta - E_N, \mathbf{q})$$

- N-rank irreducible tensors

$$Y_0(\Omega_q) = 1, \quad Y_1^i(\Omega_q) = \frac{q^i}{q}, \quad Y_2^{ij}(\Omega_q) = \frac{q^i q^j}{q^2} - \frac{1}{3}\delta^{ij}$$

- Transition multipole-spin tensors

$$\tilde{S}^i_{\sigma'\sigma} := \sqrt{J(J+1)} \sqrt{\frac{2J+1}{2J'+1}} \sum_{\lambda'} C^{\frac{3}{2}\sigma'}_{1\lambda' \frac{1}{2}\sigma} \epsilon^{*i}_{\lambda'} \qquad \tilde{Q}^{ij}_{\sigma'\sigma} := \sqrt{\frac{3}{2}} \sum_{\lambda' s'} C^{\frac{3}{2}\sigma'}_{1\lambda' \frac{1}{2}s'} \hat{S}^i_{s'\sigma} \epsilon^{*j}_{\lambda'}$$

- 3D multipole expansion

$$\langle \Delta(p', \sigma') | \hat{T}^{00}_a(0) | N(p, \sigma) \rangle = 2\mathcal{E}_2^{N\Delta,a}(t) \boldsymbol{q}^2 Y_2^{ij} \tilde{Q}^{ij}_{\sigma'\sigma},$$

$$\langle \Delta(p', \sigma') | \hat{T}^{0k}_a(0) | N(p, \sigma) \rangle = 2\bar{m} \left[-i\epsilon^{kij} q^i \tilde{S}^j_{\sigma'\sigma} \mathcal{J}_1^{N\Delta,a}(t) + q^i \tilde{Q}^{\{ki\}}_{\sigma'\sigma} \mathcal{J}_2^{N\Delta,a}(t) + \frac{q^i q^j}{4\bar{m}^2} q^k \tilde{Q}^{ji}_{\sigma'\sigma} \mathcal{J}_3^{N\Delta,a}(t) \right],$$

$$\langle \Delta(p', \sigma') | \hat{T}^{ij}_a(0) | N(p, \sigma) \rangle = 2 \Big[4\bar{m}^2 \tilde{Q}^{\{ij\}}_{\sigma'\sigma} \mathcal{D}_{2,0}^{N\Delta,a}(t) + \tilde{Q}^{kl} Y^{kl} \delta^{ij} \boldsymbol{q}^2 \mathcal{D}_{2,1}^{N\Delta}(t) + (Y^{li}\tilde{Q}^{\{lj\}} + Y^{lj}\tilde{Q}^{\{li\}}) \boldsymbol{q}^2 \mathcal{D}_{2,2}^{N\Delta,a}(t) + (Y^{li}\tilde{Q}^{[lj]} + Y^{lj}\tilde{Q}^{[li]}) \boldsymbol{q}^2 \mathcal{D}_{2,3}^{N\Delta,a}(t) + \frac{\boldsymbol{q}^4}{4\bar{m}^2} Y^{ijkl} \tilde{Q}^{kl}_{\sigma'\sigma} \mathcal{D}_{3,0}^{N\Delta,a}(t) \Big].$$

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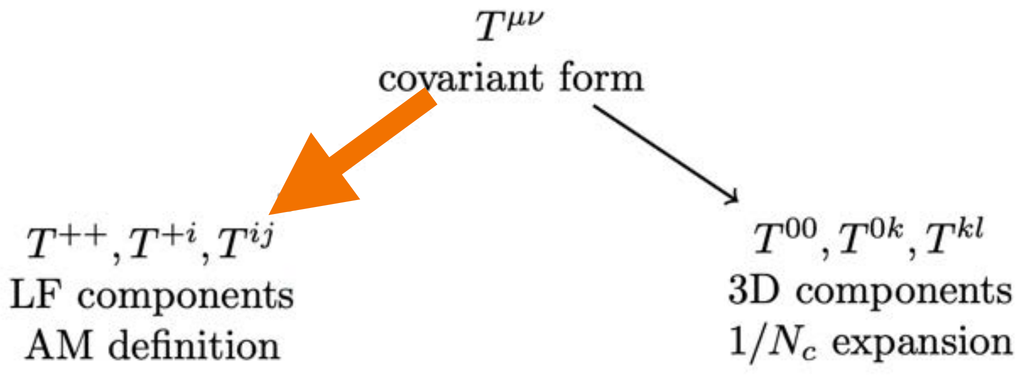
Mechanical interpretation of the EMT form factors



- Mechanical interpretation in terms of the 3D components brings about **ambiguous relativistic corrections.**

2D light-front (LF) AM

$$2\tilde{S}^3_{\sigma'\sigma}\mathcal{J}^{N\Delta,\mathrm{V}}_{\mathrm{LF}} = -i\epsilon^{3mk}\nabla^m_\Delta \frac{\langle \Delta(p',\sigma')|T^{+k}|N(p,\sigma)\rangle}{2P^+}\bigg|_{\Delta=0}$$
$$= 2\tilde{S}^3_{\sigma'\sigma}\sqrt{\frac{2}{3}}\left[-\frac{1}{2}F_1^{N\Delta,\mathrm{V}}(0) + \frac{1}{2}F_5^{N\Delta,\mathrm{V}}(0) + \frac{2\bar{m}}{m_\Delta}F_4^{N\Delta,\mathrm{V}}(0) + \frac{\delta_{N\Delta}}{2m_\Delta}F_5^{N\Delta,\mathrm{V}}(0)\right],$$

- Ambiguous relativistic corrections are kinematically suppressed.

Helicity form factors

- "**good**" component of the EMT T^{++} → transition GPDs

$$\langle \Delta(p',\sigma')|\hat{T}^{++}|N(p,\sigma)\rangle = 2(P^+)^2 e^{i(\sigma-\sigma')\theta_q} A_{\sigma'\sigma}(t),$$

$$\boldsymbol{q} = q(\cos\theta_q \hat{\boldsymbol{x}} + \sin\theta_q \hat{\boldsymbol{y}})$$

$$A_{\frac{3}{2}\frac{1}{2}} = A_{-\frac{3}{2}-\frac{1}{2}} = -\frac{\sqrt{-t}}{2\sqrt{2}\bar{m}}\left(-\frac{\delta_{N\Delta}}{4\bar{m}}F_2^{N\Delta,\mathrm{V}}(t) - 2F_5^{N\Delta,\mathrm{V}}(t)\right),$$
$$A_{\frac{3}{2}-\frac{1}{2}} = -A_{-\frac{3}{2}\frac{1}{2}} = \frac{t}{2\sqrt{2}\bar{m}^2}\left(\frac{1}{4}F_2^{N\Delta,\mathrm{V}}(t)\right),$$
$$A_{\frac{1}{2}\frac{1}{2}} = -A_{-\frac{1}{2}-\frac{1}{2}} = \frac{1}{\sqrt{6}}\left(\frac{2\delta_{N\Delta}}{m_\Delta}F_1^{N\Delta,\mathrm{V}}(t) + \frac{\delta_{N\Delta}(2\bar{m}\delta_{N\Delta}+t)}{8\bar{m}^2 m_\Delta}F_2^{N\Delta,\mathrm{V}}(t)\right.$$
$$\left. + \frac{8\bar{m}}{m_\Delta}F_4^{N\Delta,\mathrm{V}}(t) + \frac{(2\bar{m}\delta_{N\Delta}+t)}{\bar{m}m_\Delta}F_5^{N\Delta,\mathrm{V}}(t)\right) + \frac{t}{2\sqrt{6}\bar{m}^2}\frac{1}{4}F_2(t),$$
$$A_{\frac{1}{2}-\frac{1}{2}} = A_{-\frac{1}{2}\frac{1}{2}} = \frac{\sqrt{-t}}{\sqrt{6}\bar{m}}\left(-\frac{2\bar{m}}{m_\Delta}F_1^{N\Delta,\mathrm{V}}(t) - \frac{2\bar{m}\delta_{N\Delta}+t}{8m_\Delta\bar{m}}F_2(t) - \frac{\delta_{N\Delta}}{8\bar{m}}F_2^{N\Delta,\mathrm{V}}(t) - F_5^{N\Delta,\mathrm{V}}(t)\right).$$

- **Angular condition** : 8 helicity FF → 4 helicity FF

J.-Y. Kim, H.-Y. Won, J. L. Goity, C. Weiss in preparation

Transversely polarized LF momentum distribution

- LF momentum distribution (2D Fourier transform of the FFs)

$$\varepsilon_T^{N\Delta}(\boldsymbol{b}, s'_x = 1/2, s_x = 1/2) = \frac{\bar{m}}{P^+}\int \frac{d^2\Delta}{(2\pi)^2} e^{-i\boldsymbol{b}\cdot\boldsymbol{\Delta}}\left[\frac{\langle \Delta(p', s'_x = \frac{1}{2})|T^{++}|N(p, s_x = \frac{1}{2})\rangle}{2P^+}\right]$$

- Multipole patterns in the LF momentum distribution are quantified by (gravitational) multipole moments:

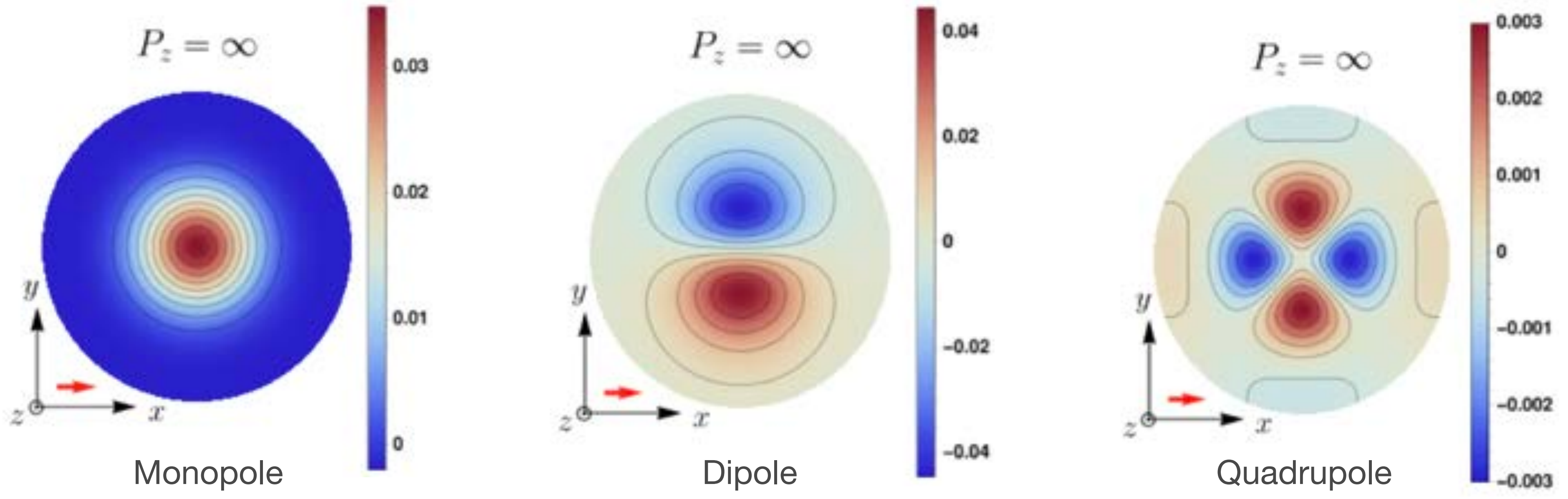
$$m^{N\Delta} = \int d^2b\, \varepsilon_T^{N\Delta}(\boldsymbol{b}, s'_x, s_x), \quad d^{N\Delta} = \int d^2b\, b_y \varepsilon_T^{N\Delta}(\boldsymbol{b}, s'_x, s_x), \quad Q^{N\Delta} = \int d^2b\, (b_x^2 - b_y^2)\varepsilon_T^{N\Delta}(\boldsymbol{b}, s'_x, s_x).$$

- The multipole moments are given by EMT form factors:

$$m^{N\Delta} = \sqrt{\frac{2}{3}}\frac{\bar{m}}{4}\left(\frac{2\delta_{N\Delta}}{m_\Delta}F_1^{N\Delta,\mathrm{V}}(0) + \frac{\delta^2_{N\Delta}}{4\bar{m}m_\Delta}F_2^{N\Delta,\mathrm{V}}(0) + \frac{8\bar{m}}{m_\Delta}F_4^{N\Delta,\mathrm{V}}(0) + \frac{2\delta_{N\Delta}}{m_\Delta}F_5^{N\Delta,\mathrm{V}}(0)\right)$$

$$d^{N\Delta} = \frac{1}{\sqrt{6}}\left(\frac{\bar{m}}{m_\Delta}F_1^{N\Delta,\mathrm{V}}(0) - F_5^{N\Delta,\mathrm{V}}(0) + \frac{(\bar{m}-m_\Delta)\delta_{N\Delta}}{m_\Delta\bar{m}}F_2^{N\Delta,\mathrm{V}}(0)\right)$$

$$Q^{N\Delta} = \frac{3}{4\sqrt{6}\bar{m}}\left(F_2^{N\Delta,\mathrm{V}}(0)\right)$$



Vector GPDs in $N \to \Delta$ transitions

Parametrization of the transition GPDs

$$\begin{aligned}\mathrm{GPDs}^{[\gamma^\mu]}(P,\Delta,\xi) = \bar{u}^\alpha(p',\lambda')[&a_1 g^\mu_\alpha + a_2 P^\mu \Delta_\alpha + a_3 \Delta^\mu \Delta_\alpha + a_4 P^\mu n_\alpha + a_5 \Delta^\mu n_\alpha + a_6 n^\mu \Delta_\alpha + a_7 n^\mu n_\alpha \\ &+ b_1 i\sigma^{\mu\Delta}\Delta_\alpha + b_2 i\sigma^{\mu\Delta} n_\alpha + b_3 i\sigma^{n\Delta} g^\mu_\alpha + b_4 i\sigma^{n\Delta} P^\mu \Delta_\alpha + b_5 i\sigma^{n\Delta}\Delta^\mu \Delta_\alpha \\ &+ b_6 i\sigma^{n\Delta} P^\mu n_\alpha + b_7 i\sigma^{n\Delta}\Delta^\mu n_\alpha + b_8 i\sigma^{n\Delta} n^\mu \Delta_\alpha + b_9 i\sigma^{n\Delta} n^\mu n_\alpha \\ &+ c_1 i\sigma^{\mu n}\Delta_\alpha + d_1 i\sigma^{\mu n} n_\alpha]\gamma^5 u(p,\lambda).\end{aligned}$$

- Leading-twist GPDs:

$$\mathrm{GPDs}^{[\gamma^+]}(P,\Delta,\xi) = \frac{1}{2P^+}\bar{u}^\alpha(p',\lambda')\left[F_{2,1}P^+ n_\alpha + F_{2,2}\frac{P^+\Delta_\alpha}{4\bar{m}^2} \boxed{+ 2\bar{m}F_{2,3}\gamma^+ n_\alpha} + F_{2,4}\frac{\gamma^+\Delta_\alpha}{2\bar{m}}\right]\gamma^5 u(p,\lambda),$$

- We found an additional Lorentz structure that is independent of the other structures.

- GPDs in $N \to \Delta$ transitions

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x}\langle \Delta(p',\sigma')|\bar{\psi}(-\lambda n/2)\not{n}\tau^3\psi(\lambda n/2)|N(p,\sigma)\rangle = \sum_{I=M,E,C,X}\sqrt{\frac{2}{3}}\bar{u}_\alpha(p',\sigma')H_I(x,\xi,t)\mathcal{K}^{\alpha\mu}_I n_\mu u(p,\sigma),$$

K. Goeke, et al (2002)/H.F. Johns, M. D. Scadron (1973)

- Kinematical factors:

$$\mathcal{K}^{\alpha\mu}_M = -i\frac{6\bar{m}}{2m_N[4\bar{m}^2 - t]}\varepsilon^{\alpha\mu\sigma\tau}P_\sigma q_\tau, \quad \mathcal{K}^{\alpha\mu}_E = -\mathcal{K}^{\alpha\mu}_M - \frac{12\bar{m}}{m_N Z(t)}\varepsilon^{\alpha\sigma\nu\gamma}P_\nu q_\gamma \varepsilon^{\mu\rho\delta}_\sigma P_\rho q_\delta \gamma_5,$$

$$\mathcal{K}^{\alpha\mu}_C = -\frac{6\bar{m}}{m_N Z(t)}q^\alpha[tP^\mu - q\cdot Pq^\mu]\gamma_5, \quad \boxed{\mathcal{K}^{\alpha\mu}_X = 2\bar{m}n^\alpha\gamma^\mu\gamma_5.}$$

$$Z(t) = [4\bar{m}^2 - t][\delta^2_{N\Delta} - t]$$

- First Mellin moments of the transition GPDs:

$$\int_{-1}^{1} dx\, H_{M,E,C}(x,\xi,t) = 2G^*_{M,E,C}(t), \quad \int_{-1}^{1} dx\, H_X(x,\xi,t) = 0.$$

- Second Mellin moments of the transition GPDs:

$$\sqrt{\frac{2}{3}}\int dx\, xH_M(x,\xi,t) = -\frac{2m_N}{3\bar{m}}F_1^{N\Delta,V}(t) + \frac{m_N(4\bar{m}^2 - t)}{6m_\Delta\bar{m}^2}F_5^{N\Delta,V}(t) - \frac{\xi m_N(4\bar{m}^2 - t)}{3m_\Delta\bar{m}^2}C_3^{N\Delta,V}(t) + \frac{4\xi m_N}{3\bar{m}}C_4^{N\Delta,V}(t),$$

$$\sqrt{\frac{2}{3}}\int dx\, xH_E(x,\xi,t) = -\frac{2m_N}{3\bar{m}}F_1^{N\Delta,V}(t) + \frac{m_N(t - 4\bar{m}^2)}{6m_\Delta\bar{m}^2}F_5^{N\Delta,V}(t) + \frac{\xi m_N(4\bar{m}^2 - t)}{3m_\Delta\bar{m}^2}C_3^{N\Delta,V}(t) + \frac{4\xi m_N}{3\bar{m}}C_4^{N\Delta,V}(t),$$

$$\begin{aligned}\sqrt{\frac{2}{3}}\int dx\, xH_C(x,\xi,t) = &-\frac{2m_N((3\xi+1)m_\Delta^2 - (\xi-1)(t - m_N^2))}{3\bar{m}(2\bar{m}\delta_{N\Delta}\xi + t)}F_1^{N\Delta,V}(t) \\ &- \frac{m_N(-2m_\Delta^2(m_N^2 + t) + m_\Delta^4 + (t - m_N^2)^2)}{24\bar{m}^3(2\bar{m}\delta_{N\Delta}\xi + t)}F_2^{N\Delta,V}(t) \\ &- \frac{\xi^2 m_N(-2m_\Delta^2(m_N^2 + t) + m_\Delta^4 + (t - m_N^2)^2)}{6\bar{m}^3(2\bar{m}\delta_{N\Delta}\xi + t)}F_3^{N\Delta,V}(t) \\ &- \frac{m_N((\xi+1)m_\Delta + (\xi-1)m_N)(4\bar{m}^2 - t)}{3\bar{m}^2(2\bar{m}\delta_{N\Delta}\xi + t)}F_5^{N\Delta,V}(t) \\ &+ \frac{\xi m_N(-2m_\Delta^2(m_N^2 + t) + m_\Delta^4 + (t - m_N^2)^2)}{6\bar{m}^3(2\bar{m}\delta_{N\Delta}\xi + t)}C_2^{N\Delta,V}(t) \\ &+ \frac{2\xi m_N((\xi+1)m_\Delta + (\xi-1)m_N)(4\bar{m}^2 - t)}{3\bar{m}^2(2\bar{m}\delta_{N\Delta}\xi + t)}C_3^{N\Delta,V}(t) \\ &+ \frac{4\xi m_N((3\xi+1)m_\Delta^2 - (\xi-1)(t - m_N^2))}{3\bar{m}(2\bar{m}\delta_{N\Delta}\xi + t)}C_4^{N\Delta,V}(t),\end{aligned}$$

$$\sqrt{\frac{2}{3}}\int dx\, xH_X(x,\xi,t) = 2F_4^{N\Delta,V}(t).$$

J.-Y. Kim, H.-Y. Won, J. L. Goity, C. Weiss in preparation

- Kinematical constraints on the GPDs

$$\int dx\, xH_E(x,\xi,\delta^2_{N\Delta}) = \frac{\delta_{N\Delta}}{2m_\Delta}\int dx\, xH_C(x,\xi,\delta^2_{N\Delta}). \qquad \text{at } t = \delta^2_{N\Delta}$$

$$\int dx\, xH_M(x,\xi,4\bar{m}^2) = \int dx\, xH_E(x,\xi,4\bar{m}^2) = \frac{\bar{m}}{m_\Delta}\int dx\, xH_C(x,\xi,4\bar{m}^2), \qquad \text{at } t = 4\bar{m}^2$$

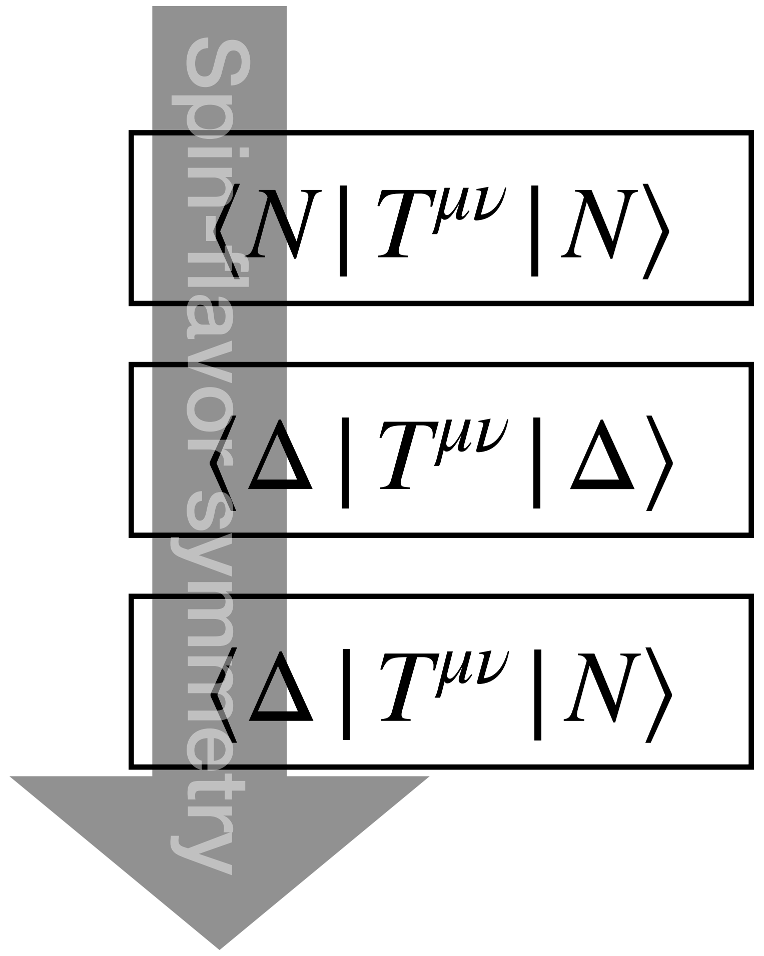
Dynamical symmetry

- Spin-flavor symmetry in large N_c limit of QCD
- 2D LF interpretation in the large N_c limit of QCD

Spin-flavor symmetry in the large N_c limit of QCD

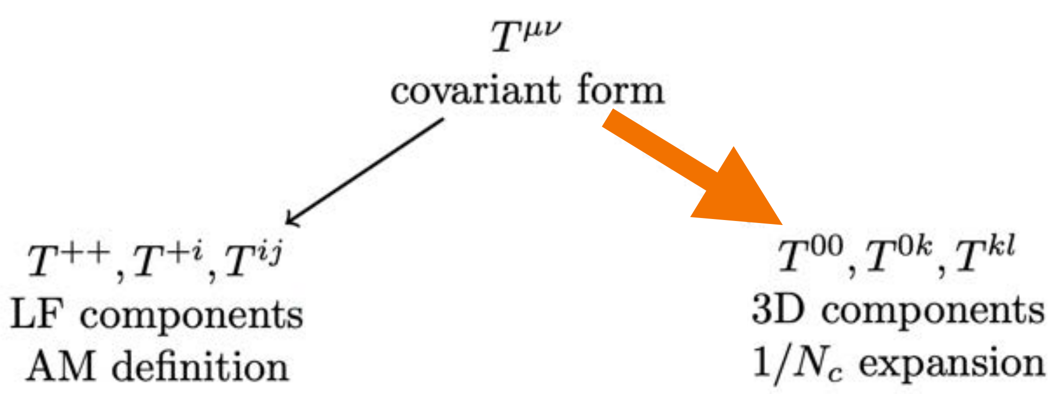
- The spin-flavor symmetry in the large N_c limit of QCD is a powerful tool to investigate the $N \to \Delta$ matrix element.

$$F^N(t) = aF^{\Delta}(t) = bF^{N\Delta}(t) = cF_{\mathrm{sol}}(t)$$



Chiral soliton approach

- One of the realization of the spin-flavor symmetry in the large N_c limit of QCD
- Classical nucleon - non-relativistic object (3D interpretation)
- Access to the EMT form factors through the 3D components



N_c-scaling behavior of the EMT form factors

$$\mathcal{E}_2^{N\Delta,\mathrm{V}} \sim \mathcal{O}(N_c^1), \quad \mathcal{J}_1^{N\Delta,\mathrm{V}} \sim \mathcal{O}(N_c^1),$$
$$\mathcal{J}_{2,3}^{N\Delta,\mathrm{V}} = 0, \quad \mathcal{D}_{2,0}^{N\Delta,\mathrm{V}} \sim \mathcal{O}(N_c^{-1}),$$
$$\mathcal{D}_{2,1}^{N\Delta,\mathrm{V}} \sim \mathcal{O}(N_c^1), \quad \mathcal{D}_{2,2}^{N\Delta,\mathrm{V}} \sim \mathcal{O}(N_c^1),$$
$$\mathcal{D}_{2,3}^{N\Delta,\mathrm{V}} \sim \mathcal{O}(N_c^1), \quad \mathcal{D}_{3,0}^{N\Delta,\mathrm{V}} \sim \mathcal{O}(N_c^3).$$

Isoscalar S= (u+d), isovector V= (u-d)

Angular momentum (AM) form factor

- The AMs for both the nucleon and the Δ baryon are found to be the same.

$$\mathcal{J}_1^{N,\mathrm{S}}(t) = \mathcal{J}_1^{\Delta,\mathrm{S}}(t) = \mathcal{J}_{1,\mathrm{sol}}^{\mathrm{S}}(t)$$

- $N \to \Delta$ transition AM is not allowed

$$\mathcal{J}_1^{N\Delta,S}(t) = 0 \quad \longleftrightarrow \quad \mathcal{J}_1^{N\Delta,u}(t) = -\mathcal{J}_1^{N\Delta,d}(t)$$

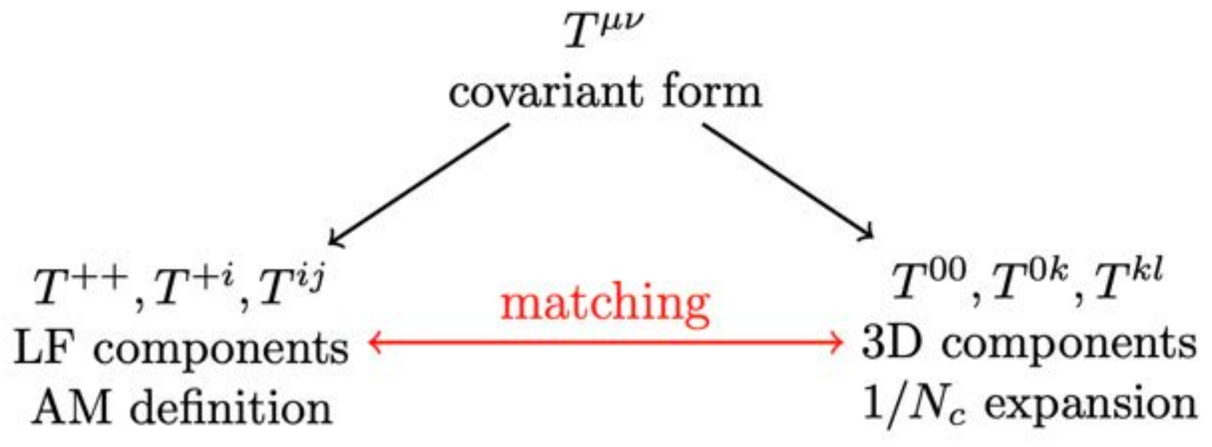
Isovector AM form factor

- The isovector AM for the nucleon is related to that for the Δ baryon or for the $N \to \Delta$ transition.

$$\mathcal{J}_1^{p,\mathrm{V}}(t) = 5\mathcal{J}_1^{\Delta^+,\mathrm{V}}(t) = \frac{1}{\sqrt{2}}\mathcal{J}_1^{p\Delta^+,\mathrm{V}}(t) = -\frac{2}{3}\mathcal{J}_{1,\mathrm{sol}}^{\mathrm{V}}(t).$$

- These large N_c relations between all the EMT form factors for the nucleon, the Δ baryon, and the $N \to \Delta$ transitions are obtained.
- However, the mechanical interpretation of the EMT form factors in the $N \to \Delta$ transitions is not obvious.

2D LF interpretation in the large N_c limit of QCD



Using covariant form

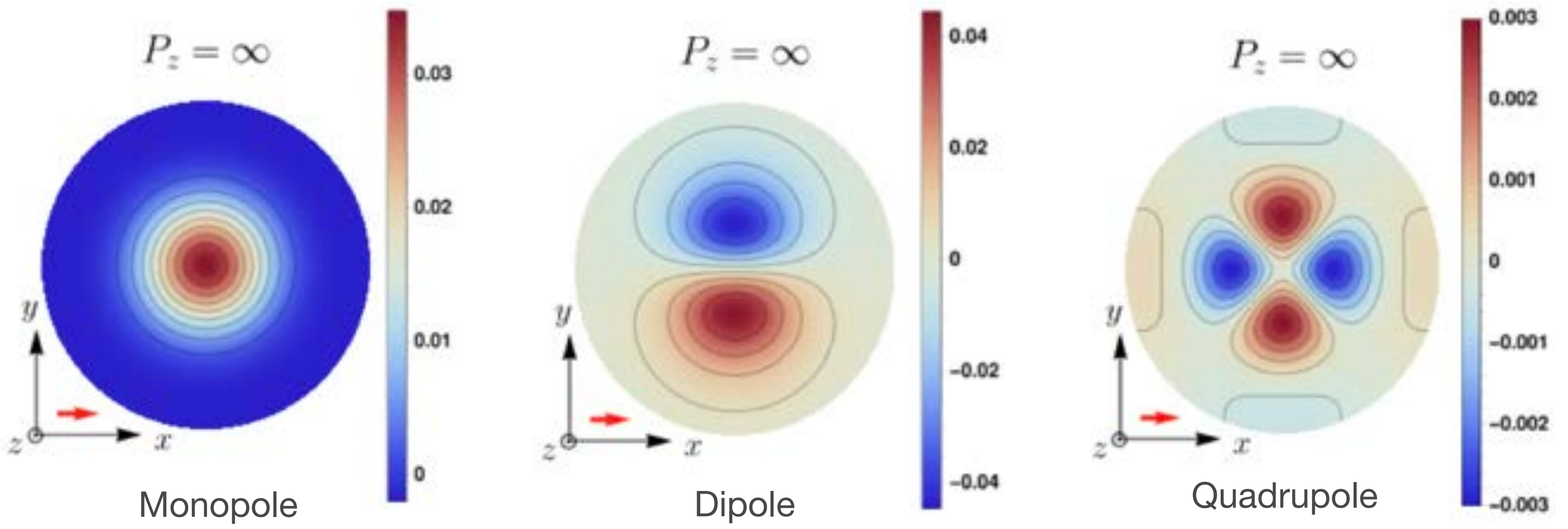
- Linear combination of the 3D multipole form factor → 2D LF multipole form factors.

- 3D AM is equivalent to the 2D LF AM in $1/N_c$ expansion

$$\underset{T^{0k}}{\mathcal{J}_1^{p\Delta^+,V}} = \underset{T^{+k}}{\mathcal{J}_{\rm LF}^{p\Delta^+,V}} = \underset{T^{++}}{-d^{p\Delta^+,V}}$$

- **Hierarchy** of the multipole moments in the $1/N_c$ expansion

$$\underbrace{d^{N\Delta}}_{\mathcal{O}(N_c^1)} > \underbrace{m^{N\Delta}}_{\mathcal{O}(N_c^0)} \sim \underbrace{Q^{N\Delta}}_{\mathcal{O}(N_c^0)}.$$



- **Numerical results of $N \to \Delta$ transition AM**

Lattice QCD	$J^S_{p\to p}$	$J^S_{\Delta^+\to\Delta^+}$	$J^V_{p\to p}$	$J^V_{p\to\Delta^+}$	$J^V_{\Delta^+\to\Delta^+}$
[9] $\mu^2 = 4\,\mathrm{GeV}^2$	0.33*	0.33	0.41*	0.58	0.08
[10] $\mu^2 = 4\,\mathrm{GeV}^2$	0.21*	0.21	0.22*	0.30	0.04
[11] $\mu^2 = 4\,\mathrm{GeV}^2$	0.24*	0.24	0.23*	0.33	0.05
[12] $\mu^2 = 1\,\mathrm{GeV}^2$	–	–	0.23*	0.33	0.05
[13] $\mu^2 = 4\,\mathrm{GeV}^2$	–	–	0.17*	0.24	0.03

[9] M.Göckeler, etal (2004)
[10] P. Häger, etal (2008)
[11] J.D. Bratt, etal (2010)
[12] G.S. Bali, etal (2019)
[13] C. Alexandrou, etal (2020)

Matching

- Without knowing the explicit expressions of the matrix element, one can easily map the 3D components of the EMT to the 2D LF ones.

- This "matching" procedure is greatly simplified in the large N_c limit of QCD.

Matching

Λ : Lorentz boost

$D^j(p,\Lambda)$: Wigner rotation matrix

$$\langle \Delta(\tilde{p},\tilde{\sigma}')|\hat{T}^{\mu\nu}|N(\tilde{p},\tilde{\sigma})\rangle = \sum_{\sigma',\sigma} D^{*(\frac{3}{2})}_{\tilde{\sigma}'\sigma'}(p',\Lambda) D^{(\frac{1}{2})}_{\tilde{\sigma}\sigma}(p,\Lambda) \Lambda^{\mu}_{\alpha}\Lambda^{\nu}_{\beta} \langle \Delta(p',\sigma')|\hat{T}^{\alpha\beta}|N(p,\sigma)\rangle.$$

Wigner rotation (Melosh rotation)

- In the large N_c limit of QCD (where the parametric regime $|\mathbf{p}'|, |\mathbf{p}| \sim O(N_c^0)$), the Wigner rotations are suppressed.

- Light-cone helicity states = canonical spin state at rest frame

Admixture of the 3D components under the Lorentz boost

- 2D LF components with respect to 3D components

$$T^{++} = \frac{1}{2}\left(T^{00} + T^{\{03\}} + T^{33}\right), \quad T^{+k} = \frac{1}{\sqrt{2}}(T^{0k} + T^{3k})$$

J.-Y. Kim, H.-Y. Won, J. L. Goity, C. Weiss in preparation

Transition GPDs in large N_c limit of QCD

Second Mellin moments of the GPDs in the large N_c limit of QCD

$$\lim_{N_c\to\infty,t\to 0}\int_{-1}^{1} dx\, x H_M(x,\xi,t) = 2\mathcal{J}_1^{N\Delta,\mathrm{V}}(0),$$

$$\lim_{N_c\to\infty,t\to 0}\int_{-1}^{1} dx\, x H_E(x,\xi,t) = -4\xi D_{2,2}^{N\Delta,\mathrm{V}}(0),$$

$$\lim_{N_c\to\infty,t\to 0}\int_{-1}^{1} dx\, x H_C(x,\xi,t) = -8\frac{m}{\delta_{N\Delta}}\xi D_{2,2}^{N\Delta,\mathrm{V}}(0),$$

$$\lim_{N_c\to\infty,t\to 0}\int_{-1}^{1} dx\, x H_X(x,\xi,t) = 3D_{2,0}^{N\Delta,\mathrm{V}}(0).$$

: leading in Nc

: subleading in Nc

J.-Y. Kim, H.-Y. Won, J. L. Goity, C. Weiss (2023)

- N_c scalings of the GPDs can be deduced from those of the EMT form factors.

$$H_M(x,\xi,t) \sim N_c^3 \times \mathrm{function}(N_c x, N_c\xi, t)$$

$$H_E(x,\xi,t) \sim N_c^2 \times \mathrm{function}(N_c x, N_c\xi, t),$$
$$H_C(x,\xi,t) \sim N_c^4 \times \mathrm{function}(N_c x, N_c\xi, t),$$
$$H_X(x,\xi,t) \sim N_c^1 \times \mathrm{function}(N_c x, N_c\xi, t).$$

Incomplete

- To have reliable results in the large N_c limit of QCD, the subleading order in the $1/N_c$ expansion is necessary.

- Nevertheless we are still able to obtain a relation between the second Mellin moments of the GPDs:

$$\lim_{N_c\to 0,t\to 0}\int_{-1}^{1} dx\, x H_E(x,\xi,t) = \lim_{N_c\to 0,t\to 0}\int_{-1}^{1} dx\, x \left(\frac{\delta_{N\Delta}}{2m} H_C(x,\xi,t)\right).$$

- Similar large N_c relations have been derived between quadrupole EM form factors:

$$G_E^*(0) = \frac{\delta_{N\Delta}}{2m} G_C^*(0).$$

↕

$$\lim_{N_c\to 0,t\to 0}\int_{-1}^{1} dx H_E(x,\xi,t) = \lim_{N_c\to 0,t\to 0}\int_{-1}^{1} dx \left(\frac{\delta_{N\Delta}}{2m} H_C(x,\xi,t)\right)$$

Dynamics

- Chiral theory in the large N_c limit of QCD
- Interpolation between non-relativistic quark model and chiral perturbation theory in large N_c
- Chiral properties of the energy-momentum tensor
- Light-cone chiral quark-soliton model

Chiral theory in the large N_c limit of QCD

- One of the realizations of the spin-flavor symmetry in the large N_c limit of QCD
  E. Witten (1979), R.F. Dashen et al (1994)
- Inspired by QCD instanton vacuum → chiral symmetry and its spontaneous breakdown
  D. Diakonov, V.Y. Petrov(1986)/ D. Diakonov, M.V. Polyakov, C. Weiss (1996)

Chiral soliton approach

D.Diakonov, et al (1988)/ C. V. Christov, et al (1998)

- Effective Action $S_{\rm eff}(\bar{\psi},\psi,U)=\int \mathrm{d}^4x\,\bar{\psi}\,(i\partial\!\!\!/ - M\,U^{\gamma_5}-m)\psi$
- **Hedgehog symmetry** (spin-flavor symmetry) $U(r)=e^{iP(r)\vec{r}\cdot\vec{\tau}}$
- Correlation function

$$\Pi_N(T)=\int dU\mathcal{F}[U]e^{-S_{\rm eff}[U]} \qquad \lim_{T\to\infty}\Pi_N(T)\sim e^{-M_N T}$$

- **Saddle point approximation** and saddle point equation $MR\sim 1$

$$\frac{\delta S_{\rm eff}[U]}{\delta U}\bigg|_{U_c}=0 \qquad M_N\sim 1.2\ \mathrm{GeV} \qquad P(r)\sim -2\arctan[R^2/r^2]$$

- (Translational, rotational) Zero-mode quantization→Assign quantum number
- These corrections are generally suppressed in the $1/N_c$ expansion.

$$\sim\frac{\mathbf{P^2}}{2M_N},\frac{J^2}{2I} \qquad \text{where} \qquad M_N\sim\mathcal{O}(N_c),\ I\sim\mathcal{O}(N_c)$$

Matrix element of the EMT current

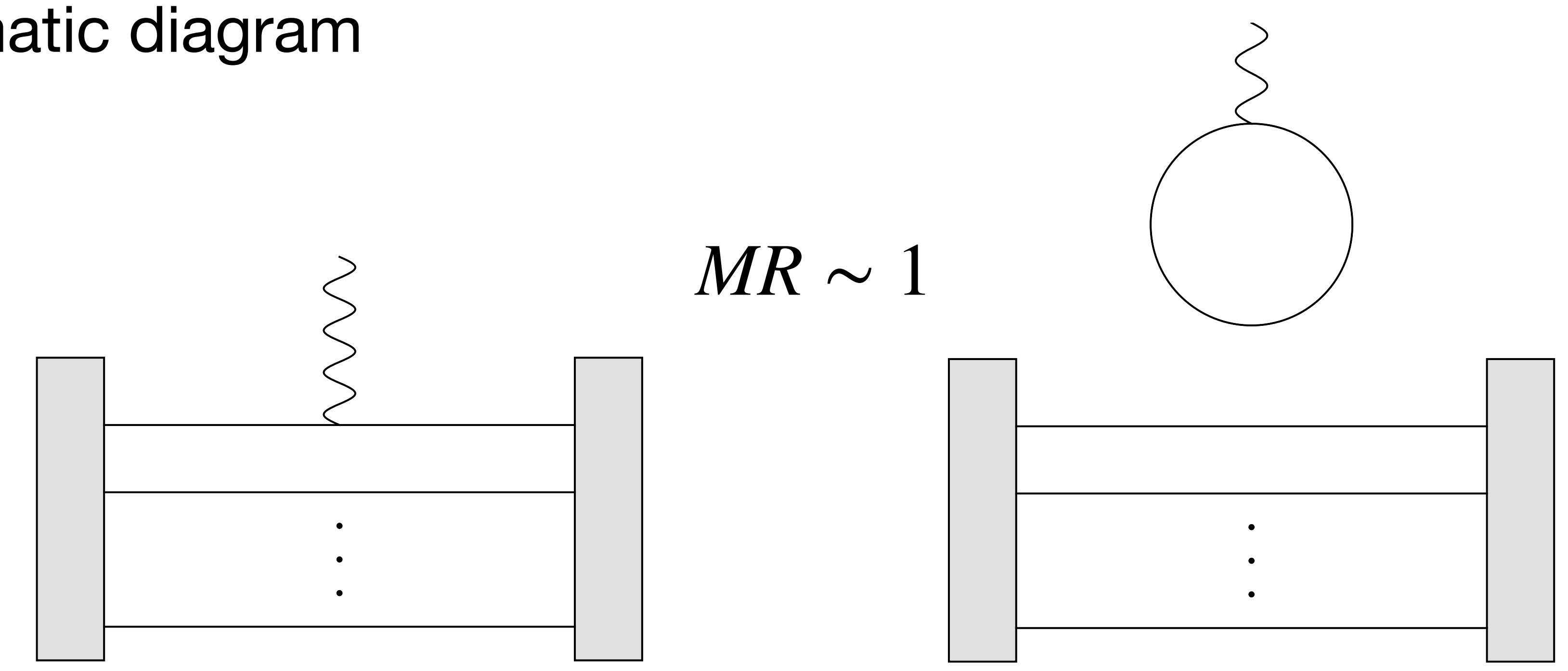
- Effective operator

$$\hat{T}^{\rm eff}_{\mu\nu}(x)=\frac{i}{4}\bar{\psi}(x)\left(\gamma_\mu\overrightarrow{\partial}_\nu+\gamma_\nu\overrightarrow{\partial}_\mu-\gamma_\mu\overleftarrow{\partial}_\nu-\gamma_\nu\overleftarrow{\partial}_\mu\right)\tau\psi(x),$$

- Correlation function K. Goeke, et al (2007)

$$\langle B',p'|(\hat{T}^{S,V}_{\rm eff})^{\mu\nu}(0)|B,p\rangle=\lim_{T\to\infty}\frac{1}{Z_{\rm eff}}\mathcal{N}^*(p')\mathcal{N}(p)e^{ip_4\frac{T}{2}-ip'_4\frac{T}{2}}\int d^3\boldsymbol{x}\,d^3\boldsymbol{y}\,e^{(-i\boldsymbol{p}'\cdot\boldsymbol{y}+i\boldsymbol{p}\cdot\boldsymbol{x})}$$
$$\times\int\mathcal{D}\psi\mathcal{D}\psi^\dagger\mathcal{D}U\,J_{B'}(\boldsymbol{y},T/2)(\hat{T}^{S,V}_{\rm eff,E})^{\mu\nu}(0)J^\dagger_B(\boldsymbol{x},-T/2)\exp[-S_{\rm eff}],$$

- Schematic diagram



Valence quark

$$\sim\langle y|\frac{1}{i\partial\!\!\!/+MU^{\gamma_5}}|x\rangle\Gamma^{\mu\nu}\langle x|\frac{1}{i\partial\!\!\!/+MU^{\gamma_5}}|x'\rangle+\ldots$$

Sea quark

$$\sim\mathrm{Tr}[\Gamma^{\mu\nu}\langle x|\frac{1}{i\partial\!\!\!/+MU^{\gamma_5}}|x\rangle]$$

- Spectral representation (exact calculation in a finite box)

$$\langle t',\mathbf{x}'|\frac{i}{i\partial_t-H}|t,\mathbf{x}\rangle_{\sigma\tau}=\Theta(t-t')\sum_{E_n>0}e^{-iE_n(t'-t)}\Phi^{(n)}_\sigma(\mathbf{x}')\Phi^{(n)*}_\tau(\mathbf{x})$$
$$-\Theta(t'-t)\sum_{E_n<0}e^{-iE_n(t'-t)}\Phi^{(n)}_\sigma(\mathbf{x}')\Phi^{(n)*}_\tau(\mathbf{x})$$

Interpolation between NQM and ChPT in large N_c

- This chiral theory naturally interpolates the NQM and ChPT in the large N_c limit of QCD.

$R \to 0$	$MR \sim 1$	$R \to \infty$
NQM	Chiral theory	ChPT Skyrmion

Non-relativistic quark model (NQM)

- $R \to 0$, all the pion cloud effects are suppressed and the valence quark contribution becomes dominant (NQM).

$$g_A^{u+d} = 1; \quad g_A^{u-d} = \frac{N_c + 2}{3}$$

Chiral perturbation theory (ChPT)

- Gradient expansion — slowly varying chiral field ($R \to \infty$) allows us to expand the quark propagator in the pion momentum ($\partial_\mu U$).

D.Diakonov, et al (1988)

- It generates an infinite power of the pion momentum contributions.

- If one is satisfied with the leading order in $1/N_c$ expansion, all the low-energy constants in the ChPT are dynamically generated.

$$F_\pi^2 \mathcal{O}(p^2) + L\mathcal{O}(p^4) + \ldots$$

H-A. Choi, H-Ch. Kim (2003)/ K.Goeke, et al (2007)

- Truncated version of the gradient expansion → Skyrme model

$$M_N = -\frac{f_\pi^2}{2}\int d^3x \mathrm{tr}[L_i L_i] \qquad g_A^{u-d} \sim f_\pi^2 \int d^3x\ \mathrm{tr}[(L_i + R_i)\tau^a]$$

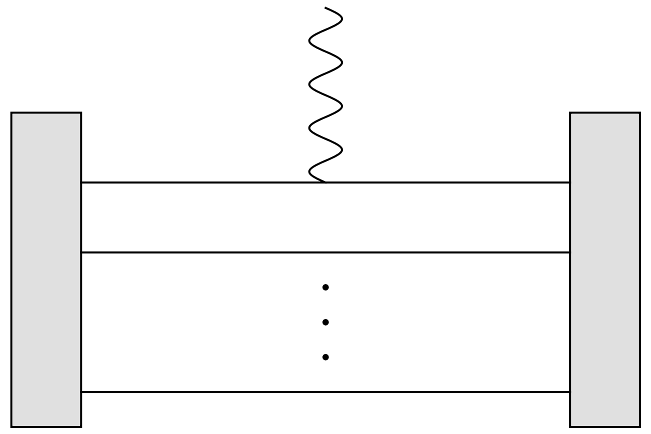
$$B = -\frac{1}{24\pi^2}\int d^3x \varepsilon_{ijk} \mathrm{tr}_f \left[U^\dagger(\partial_i U)U^\dagger(\partial_j U)U^\dagger(\partial_k U)\right]$$

G.S. Adkins (1983)/ C. V. Christov, et al (1998)

$L^\mu = U^\dagger \partial^\mu U; \ R^\mu = U\partial^\mu U^\dagger$

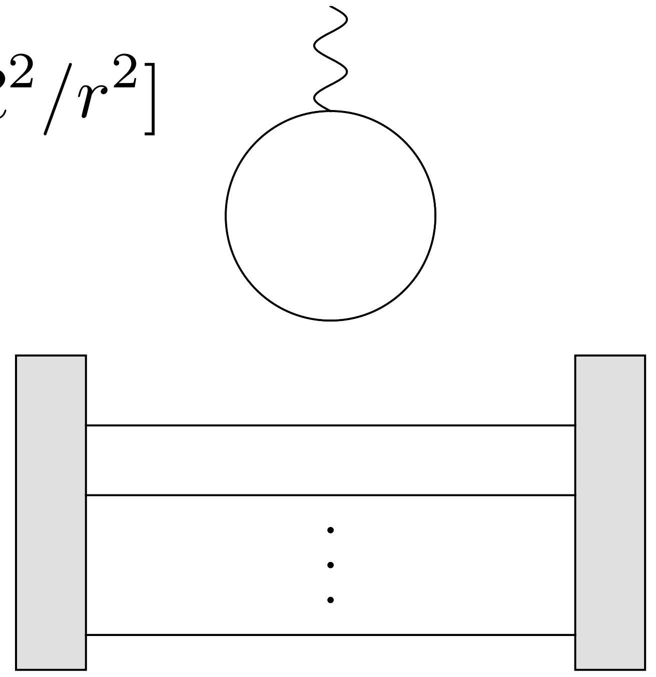
- By controlling the size of the soliton, one can interpolate the NQM and the ChPT in the large N_c limit of QCD.

$$P(r) \sim -2\arctan[R^2/r^2]$$



Valence quark

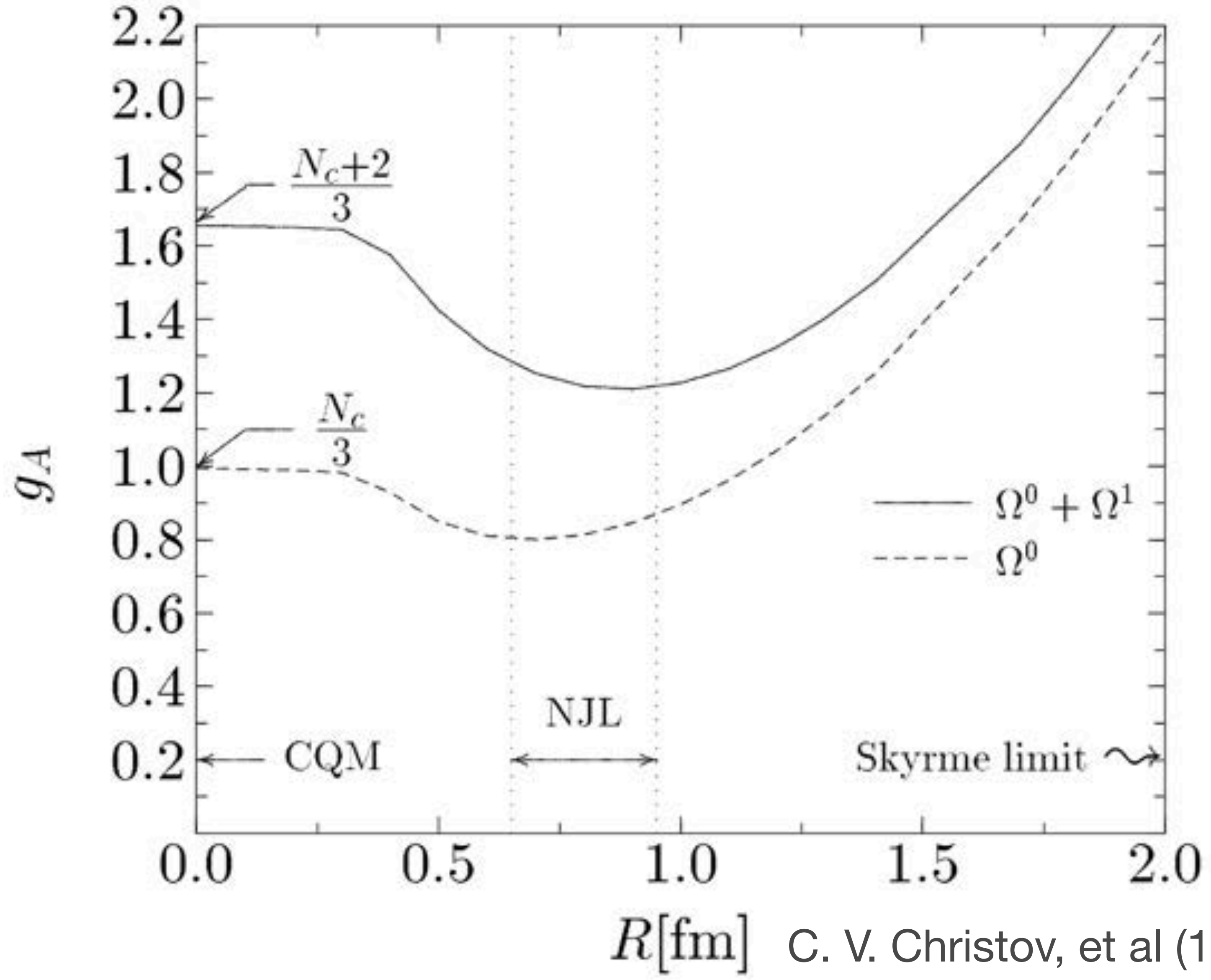
$$\sim \langle y|\frac{1}{i\partial\!\!\!/ + MU^{\gamma_5}}|x\rangle\Gamma^{\mu\nu}\langle x|\frac{1}{i\partial\!\!\!/ + MU^{\gamma_5}}|x'\rangle + \ldots$$



Sea quark

$$\sim \mathrm{Tr}[\Gamma^{\mu\nu}\langle x|\frac{1}{i\partial\!\!\!/ + MU^{\gamma_5}}|x\rangle]$$

$MR \sim 1$



C. V. Christov, et al (1998)

Chiral properties of the EMT

$$\sim \mathrm{Tr}[\Gamma^{\mu\nu}\langle x|\frac{1}{i\partial\!\!\!/ + MU^{\gamma_5}}|x\rangle]$$

J.-Y. Kim, J. L. Goity, C. Weiss in preparation

	Isoscalar	Isovector (Unexplored sector; Essential to $N \to \Delta$ transition)
$T^{00}(A)$	$\mathcal{O}(N_c^2), \mathcal{O}(p^2)$ $-M_N\delta_{B'B}f_\pi^2\int d^3x\, e^{i\mathbf{\Delta}\cdot\mathbf{x}}\mathrm{tr}[L_aL_a]+\ldots$	$\mathcal{O}(N_c^1), \mathcal{O}(p^3)$ $\frac{M_NN_c}{12I}\frac{1}{192\pi^2}\int d^3x e^{i\mathbf{\Delta}\cdot\mathbf{x}}\epsilon^{abc}\mathrm{tr}\Big[34L_aL_bL_c+2L_aL_bL_c\{\tau^l[U^\dagger,\tau^l]U+U^\dagger\tau^l[U,\tau^l]\}\ldots\Big]$
$T^{0k}(J)$	$\mathcal{O}(N_c^1), \mathcal{O}(p^1)$ $-i\frac{f_\pi^2}{2}\frac{M_N}{I}S^l\int d^3 e^{i\mathbf{\Delta}\cdot\mathbf{x}}\mathrm{tr}[L_k\tau^l+R_k\tau^l]+\ldots$	$\mathcal{O}(N_c^2), \mathcal{O}(p^4)$ $-2M_NN_c\langle D^{3l}\rangle\frac{i}{4}\int d^3x\, e^{i\mathbf{\Delta}\cdot\mathbf{x}}\frac{\epsilon^{abc}}{96\pi^2}\mathrm{tr}[\tau^l(\partial_kR_aR_bR_c+2R_a\partial_kR_bR_c+3R_aR_b\partial_kR_c)]\ldots$
$T^{ij}(D,\bar{c})$	$\mathcal{O}(N_c^2), \mathcal{O}(p^2)$ $-\frac{f_\pi^2}{4}M_N\delta_{B'B}\int d^3xe^{i\mathbf{\Delta}\cdot\mathbf{x}}\mathrm{tr}[2L_iL_j-\delta^{ij}L_aL_a]+(i\leftrightarrow j)+\ldots$	$\mathcal{O}(N_c^1), \mathcal{O}(p^3)$ $\mathcal{O}(p^3)$

- Large distance behavior of the isoscalar EMT distributions (T^{00} and T^{ij}) are equivalent to the prediction from ChPT.
- $L^{u+d}=J^{u+d},\ S^{u+d}=0$ at $\mathcal{O}(p^2)$ C. Granados, C.Weiss (2019)
- $S^{u-d}=-L^{u-d}$ at $\mathcal{O}(p^2)$ $g_A^{u-d}\sim f_\pi^2\int d^3x\ \mathrm{tr}[(L_i+R_i)\tau^a]$

JYKim, C.Weiss, J.Goity, in progress

$$L^\mu = U^\dagger\partial^\mu U;\ R^\mu = U\partial^\mu U^\dagger$$

$$\rho_E(r)=\frac{f_\pi^2}{4}\mathrm{tr}_F\left[\nabla^kU(\mathbf{r})\nabla^kU^\dagger(\mathbf{r})\right]+\ldots \overset{r\to\text{large}}{\longrightarrow} 3\left(\frac{3g_A}{8\pi f_\pi}\right)^2\frac{1}{r^6}$$

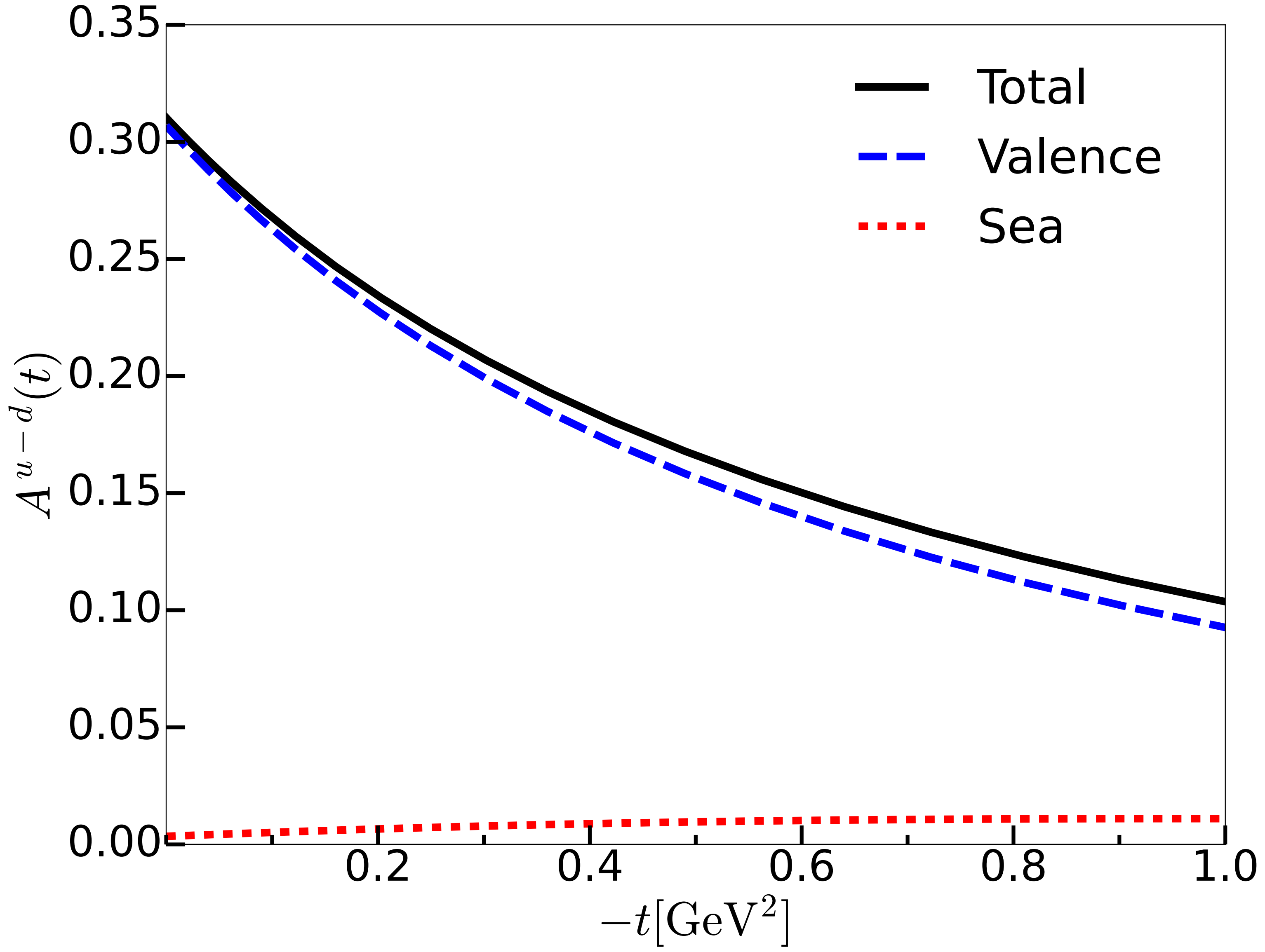
$$p(r)=-\left(\frac{3g_A}{8\pi f_\pi}\right)^2\frac{1}{r^6}\quad\text{and}\quad s(r)=3\left(\frac{3g_A}{8\pi f_\pi}\right)^2\frac{1}{r^6}\quad\text{at large } r$$

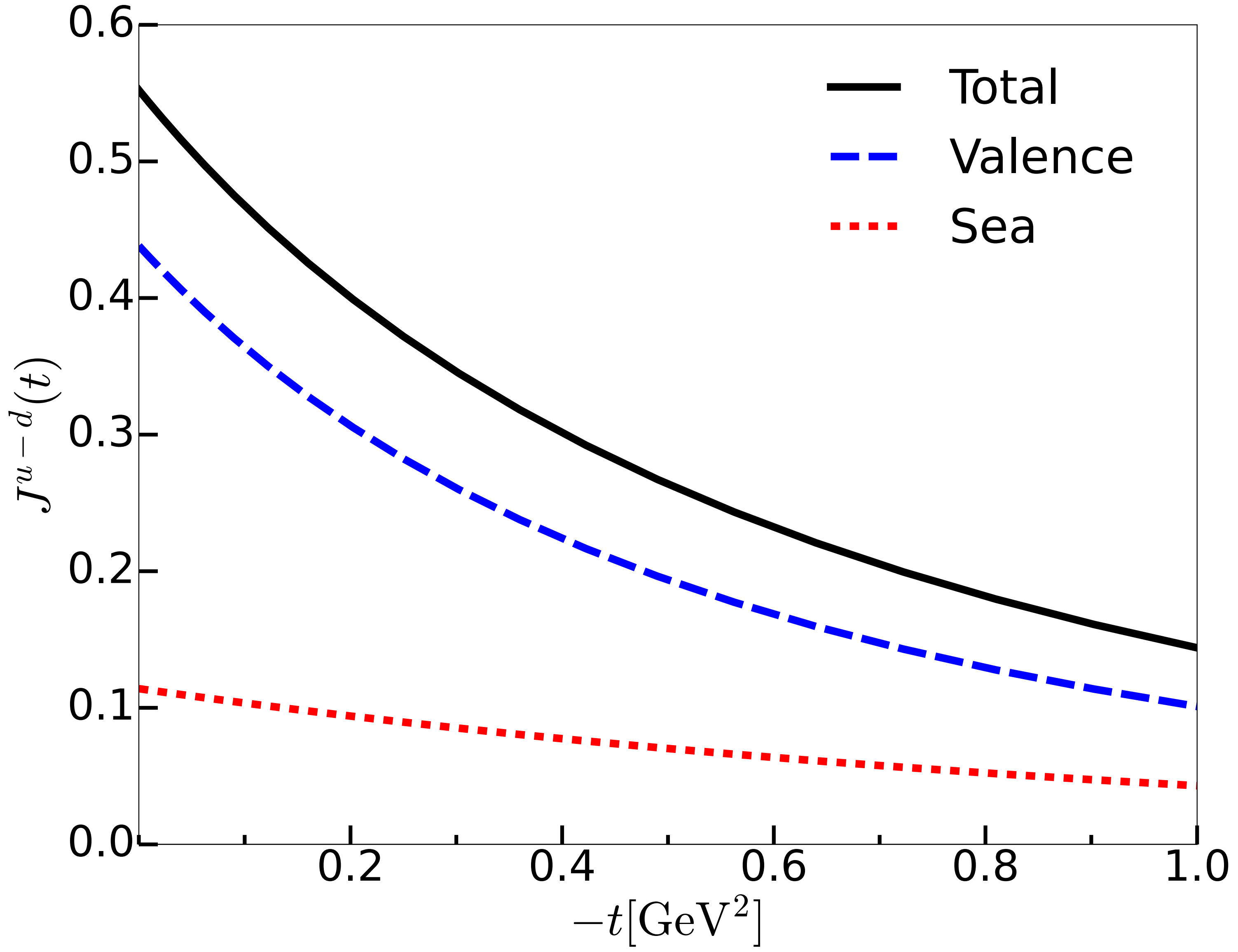
K. Goeke, et al (2007)

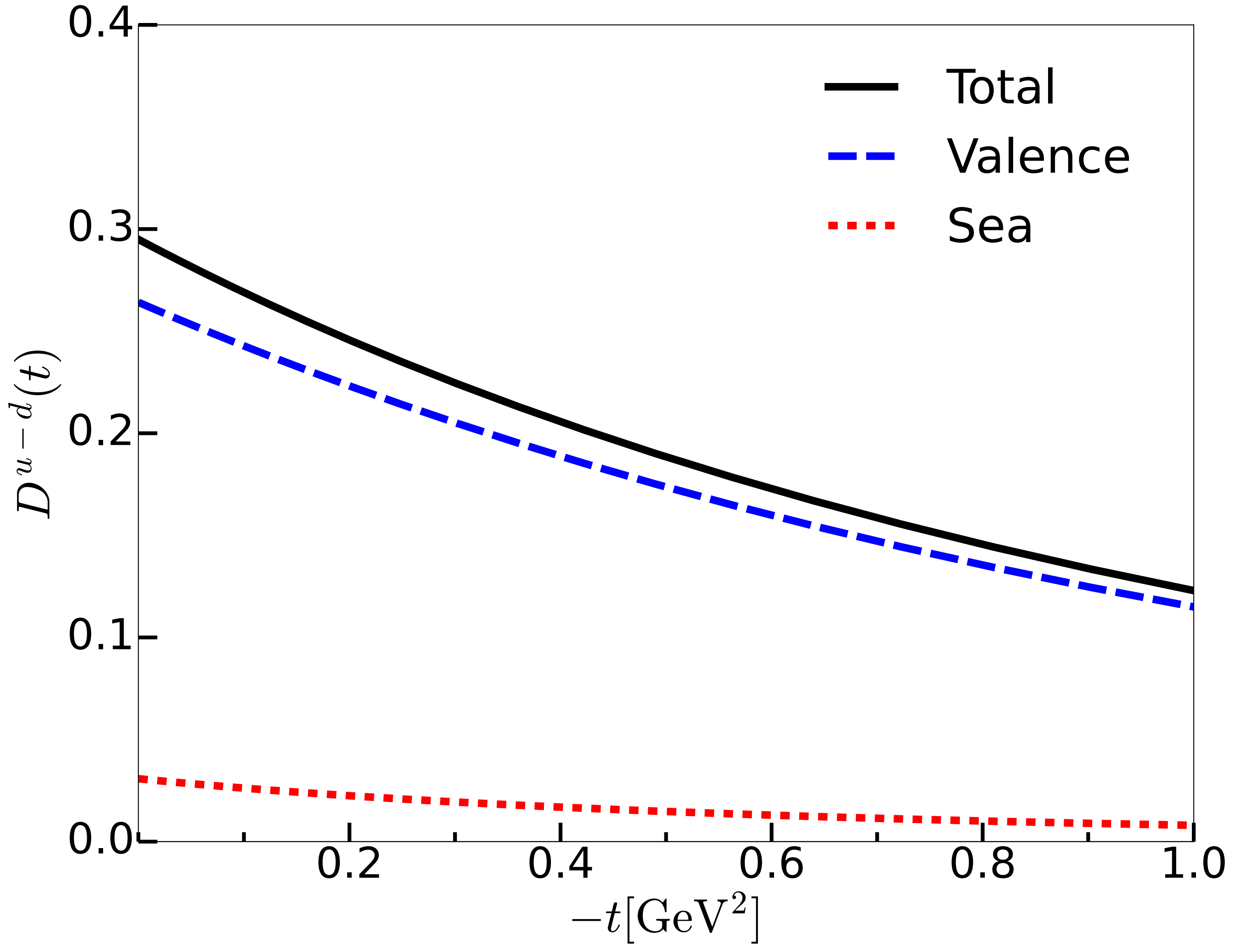
Numerical estimation

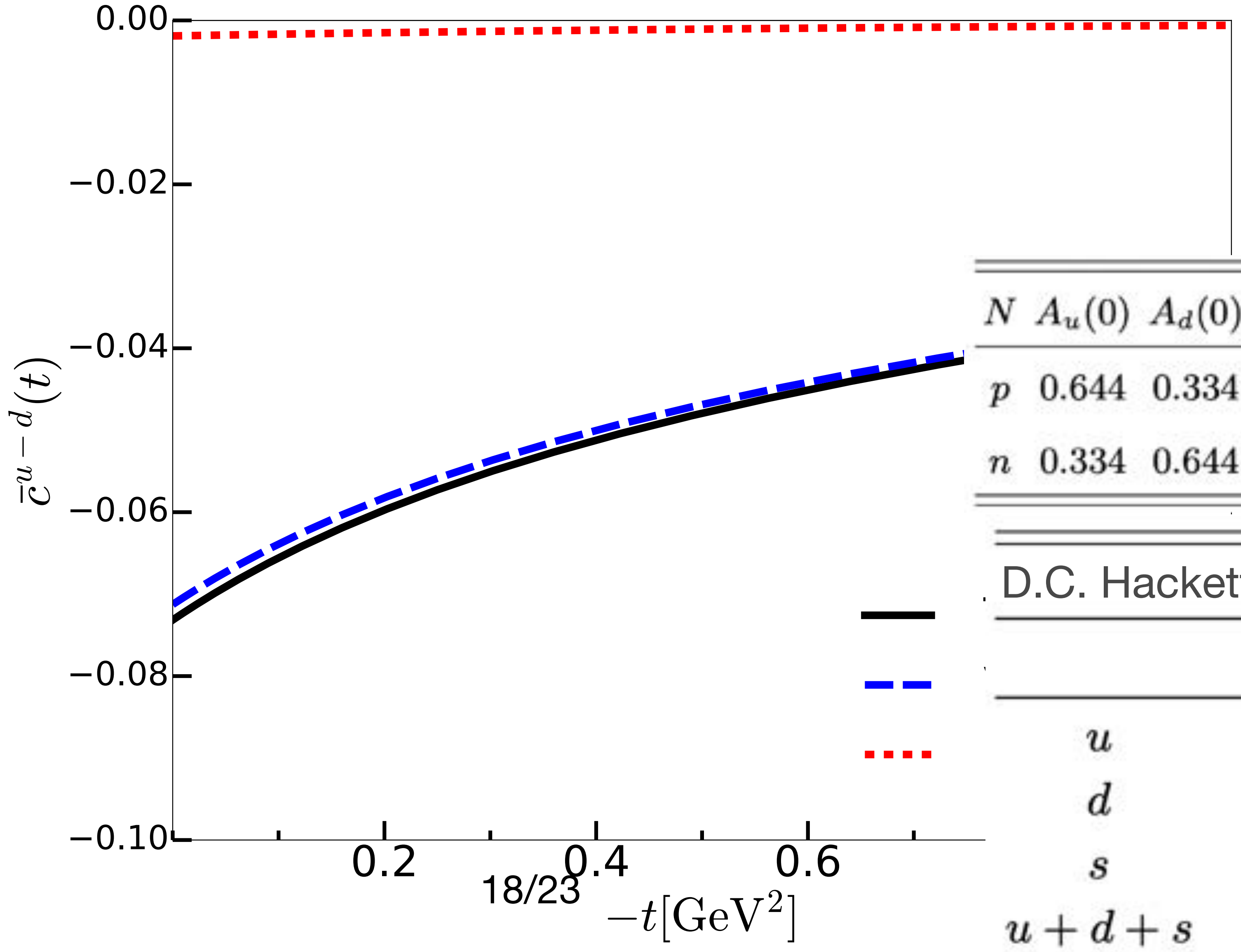
- Isoscalar EMT: $A^{u+d}(0)=1,\ J^{u+d}(0)=1/2,\ D^{u+d}(0)=-7.4$
- Isovector EMT: $A^{u-d}(0)=0.0025,\ J^{u-d}(0)=0.35,\ D^{u-d}(0)=0.064,\ \bar{c}^{u-d}(0)=-0.0008$

Exact calculation









Gradient expansion

$$A^{u-d}(0) = 0.0025,\ J^{u-d}(0) = 0.35,\ D^{u-d}(0) = 0.064,\ \bar{c}^{u-d}(0) = -0.0008$$

- Valence contributions are dominant in the isovector EMT form factors.
- All the sea quark contributions are consistent with the results from the gradient expansion.
- If one requires an accurate value of the D-term extracted from the DVCS, “flavor blindness” should be revisited.

$$D^{u+d} \sim -2 \text{ vs } D^{u-d} \sim 0.3$$

HYWon, H-Ch.Kim, JYKim, 2302.02974

- We extend it to the flavor SU(3) sector (by performing a trivial embedding).

HYWon, H-Ch.Kim, JYKim, 2307.00740
HYWon, H-Ch.Kim, JYKim, 2310.04670

N	$A_u(0)$	$A_d(0)$	$A_s(0)$	$J_u(0)$	$J_d(0)$	$J_s(0)$	$D_u(0)$	$D_d(0)$	$D_s(0)$	$\bar{c}_u(0)$	$\bar{c}_d(0)$	$\bar{c}_s(0)$
p	0.644	0.334	0.022	0.519	−0.056	0.037	−1.016	−1.077	−0.438	−0.048	0.013	0.035
n	0.334	0.644	0.022	−0.056	0.519	0.037	−1.077	−1.016	−0.438	0.013	−0.048	0.035

D.C. Hackett. et al, 2310.08484

	Dipole		
	A_i	J_i	D_i
u	0.3255(92)	0.2213(85)	−0.56(17)
d	0.1590(92)	0.0197(85)	−0.57(17)
s	0.0257(95)	0.0097(82)	−0.18(17)
$u+d+s$	0.510(25)	0.251(21)	−1.30(49)

Light-cone chiral quark-soliton model

- Boosting the system to the infinite momentum frame, we obtain the light-cone wave functions. V. Y. Petrov and M. V. Polyakov (2002) D. Diakonov and V. Petrov (2004) C. Lorce (2006)/(2008)/(2009)
- Infinite tower of the quark-antiquark pair wave functions produces the higher-Fock components.
- Overlap representation of the LCWFs → spin-flavor symmetry and dynamical information of the EMT.
- It is straightforward to explore the x dependence of the AM.

Spin-flavor symmetry (LCWFs)

- Spin

$$\Delta u_{p\to\Delta^+}(x) = -\frac{1}{\sqrt{2}}\Delta u_p(x) = -\sqrt{2}\Delta u_{\Delta^+}(x),$$
$$\Delta d_{p\to\Delta^+}(x) = -2\sqrt{2}\Delta d_p(x) = 2\sqrt{2}\Delta d_{\Delta^+}(x).$$

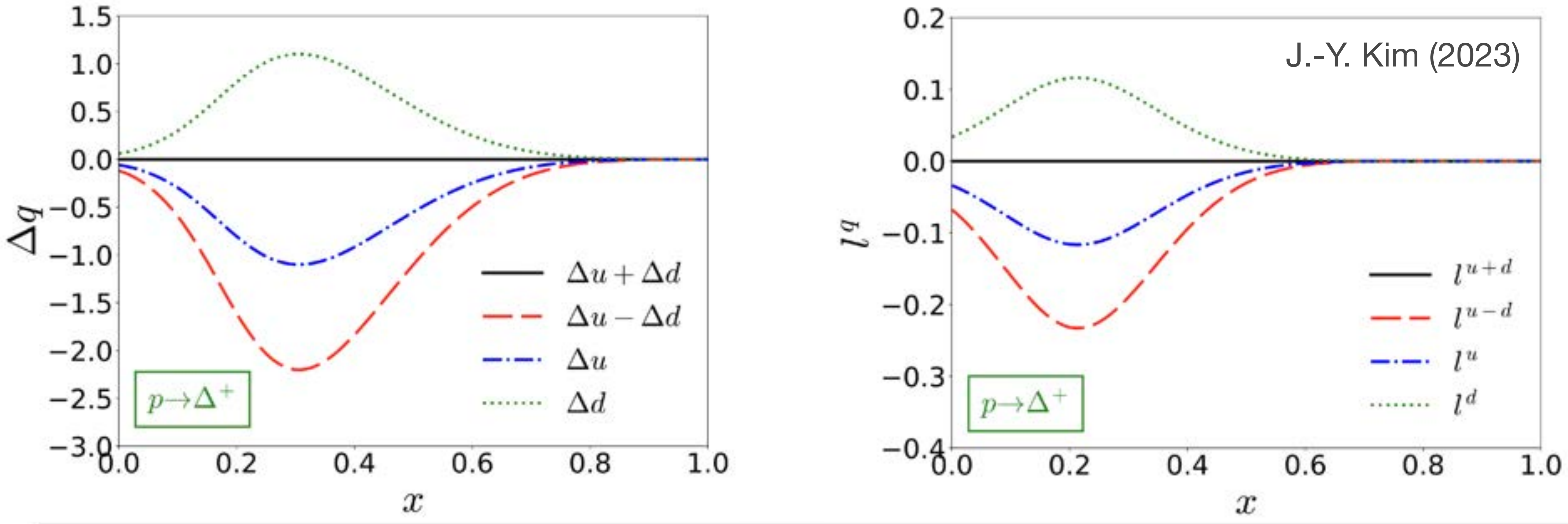
- OAM

$$l^u_{p\to\Delta^+}(x) = -\frac{\sqrt{2}}{3}\left(l^u_{\Delta^+}(x) + l^u_p(x)\right) = -\frac{\sqrt{2}}{9}\left(2l^u_n(x) + 5l^u_p(x)\right),$$
$$l^d_{p\to\Delta^+}(x) = \frac{\sqrt{2}}{3}\left(5l^d_{\Delta^+}(x) - l^d_p(x)\right) = \frac{\sqrt{2}}{9}\left(5l^d_n(x) + 2l^d_p(x)\right),$$

- Forward limit of the axial-vector is related to the N\to \Delta transition PDF.
- Transition OAM GPDs has not been identified (parametrization of the higher-twist transition GPDs are under investigation).

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x}\langle \Delta^+(p',S_3')|\bar{\psi}(-z/2)\not{n}\gamma^5\tau^3\psi(z/2)|p(p',S_3')\rangle = \bar{u}^\beta(p',S_3')\left[C_1(x,\xi,t)n_\beta + \ldots\right]u(p,S_3)$$

$$[\Delta u(x) - \Delta d(x)]_{p\to\Delta^+} = \sqrt{\frac{2}{3}}C_1(x,0,0)$$



Contents		l^q				Δq				J^q			
q		u	d	$u-d$	$u+d$	u	d	$u-d$	$u+d$	u	d	$u-d$	$u+d$
NR	$p\to p$	0	0	0	0	2/3	−1/6	5/6	1/2	2/3	−1/6	5/6	1/2
	$\Delta^+\to\Delta^+$	0	0	0	0	1/3	1/6	1/6	1/2	1/3	1/6	1/6	1/2
	$p\to\Delta^+$	0	0	0	0	$-\sqrt{2}/3$	$\sqrt{2}/3$	$-2\sqrt{2}/3$	0	$-\sqrt{2}/3$	$\sqrt{2}/3$	$-2\sqrt{2}/3$	0
Rel.	$p\to p$	0.073	−0.003	0.076	0.070	0.574	−0.144	0.718	0.431	0.647	−0.147	0.794	0.5
	$\Delta^+\to\Delta^+$	0.046	0.023	0.023	0.070	0.287	0.144	0.144	0.431	0.333	0.167	0.167	0.5
	$p\to\Delta^+$	−0.037	0.037	−0.074	0	−0.406	0.406	−0.812	0	−0.443	0.443	−0.887	0

J.-Y. Kim (2023)

Near future work

- Parity-odd EMT
- QCD operator vs effective operator

Parity-odd EMT

- Parity-odd EMT current $\hat{T}^{\mu\nu}_{5,q} = \frac{1}{2}\bar{\psi}_q i\gamma^\mu\gamma_5 \overleftrightarrow{D}^\nu \psi_q$
- Spin-orbit correlation, second Mellin moments of the axial-vector GPDs

Matrix element C. Lorce (2014)

$$\langle N(p', S_3')|\hat{T}^{\mu\nu}_{5,q}|N(p, S_3)\rangle = \bar{u}(p', S_3')\Gamma_{5,q} u(p, S_3)$$

$$\Gamma_{5,q} = \frac{P^{[\mu}\gamma^{\nu]}\gamma_5}{2}\tilde{A}^q(t) + \frac{P^{[\mu}\Delta^{\nu]}\gamma_5}{4M_N}\tilde{B}^q(t) + \frac{P^{[\mu}\gamma^{\nu]}\gamma_5}{2}\tilde{C}^q(t) + \frac{P^{[\mu}\Delta^{\nu]}\gamma_5}{4M_N}\tilde{D}^q(t) + M_N i\sigma^{\mu\nu}\gamma_5\tilde{F}^q(t).$$

Spin-orbit correlation (sym+antisy)

- Alignment of the spin and OAM

$$\hat{C}^q_z = \int d^3x\, \frac{1}{2}\bar{\psi}_q(x)\gamma^+\gamma^5(\boldsymbol{r}\times i\overleftrightarrow{\boldsymbol{D}})_z\psi_q(x). \quad C^q_z = -\frac{1}{2}\underbrace{\left[F^q_1(0) - \frac{m}{2M_N}H^q_1(0)\right]}_{\text{antisym}} + \underbrace{\frac{1}{2}\int_{-1}^{1} dx\, x\tilde{H}^q(x,0,0).}_{\text{sym}}$$

- **Antisymmetric part** → antisym
- Reduced to the known quantities (tensor and vector form factors)

$$\bar{\psi}_q\gamma^{[\mu}\gamma_5 i\overleftrightarrow{D}^{\nu]}\psi = 2m\bar{\psi}_q i\sigma^{\mu\nu}\gamma_5\psi_q - \epsilon^{\mu\nu\alpha\beta}\partial_\alpha(\bar{\psi}_q\gamma_\beta\psi_q).$$

- **Symmetric part (second Mellin moments of the GPDs)** → sym

$$\int dx\, x\tilde{H}_q(x,\xi,t) = \tilde{A}_q(t),$$
$$\int dx\, x\tilde{E}_q(x,\xi,t) = \tilde{B}_q(t),$$

Large N_c limit of QCD $T^{\mu\nu}_{\text{eff},5,q} = \frac{1}{2}\bar{\psi}i\gamma^\mu\gamma_5\overleftrightarrow{\partial}^\nu\psi$

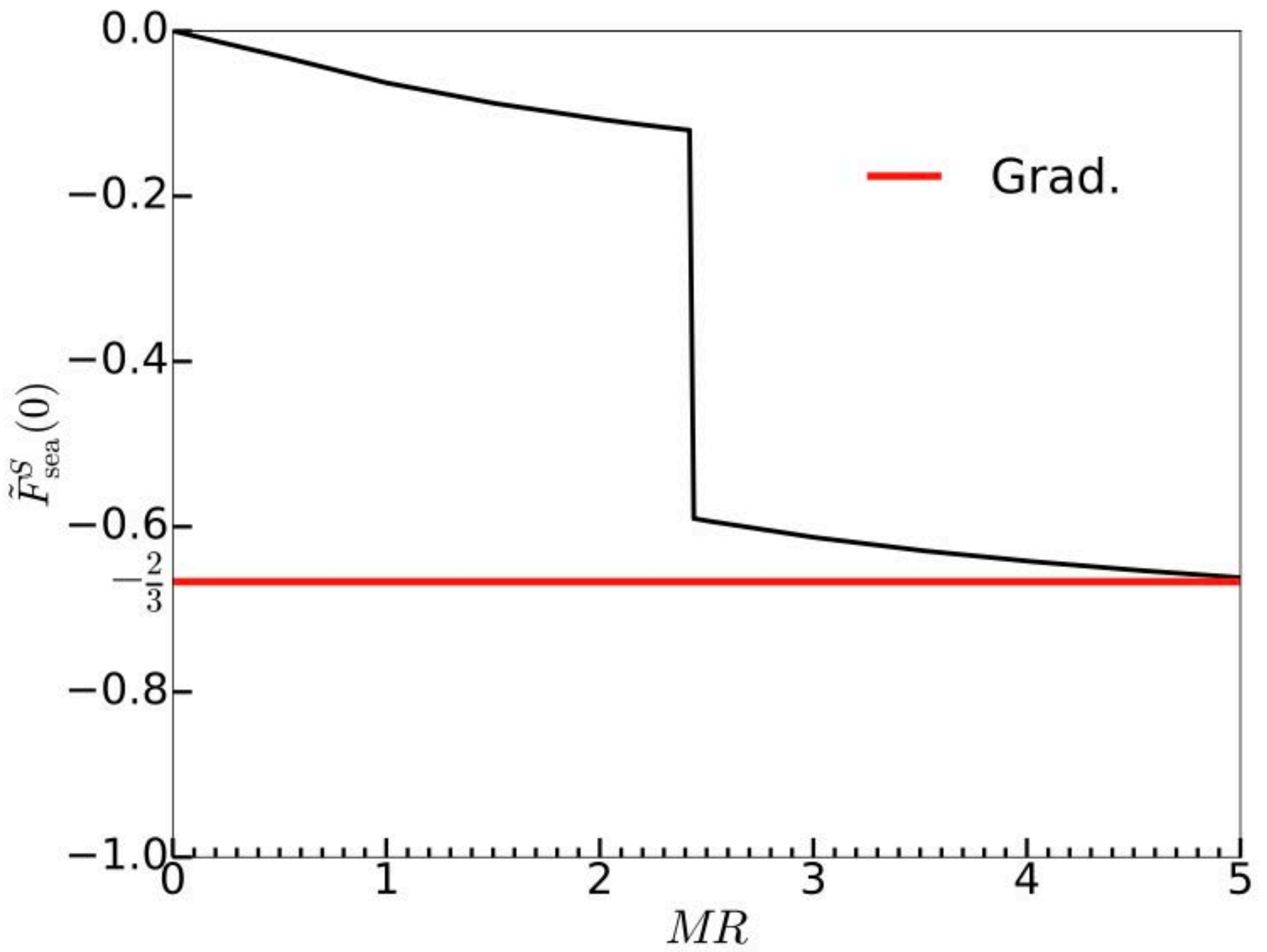
- Isoscalar component: $C^q_z = -\frac{1}{2}\underbrace{\left[F^q_1(0) - \frac{m}{2M_N}H^q_1(0)\right]}_{\mathcal{O}(N_c^1)\ \text{antisym}} + \underbrace{\frac{1}{2}\int_{-1}^{1} dx\, x\tilde{H}^q(x,0,0).}_{\mathcal{O}(N_c^{-1})\ \text{sym}}$

Expected from the QCD relation

$$C^{u+d}_z = \tilde{F}^{u+d}(0) = -\frac{1}{2}F^{u+d}_1(0) = -\frac{3}{2}$$

Effective model results

$$C^{u+d}_z = \tilde{F}^{u+d}(0) = -\frac{1}{2}F^{u+d}_1(0) + \delta F^{u+d}_1(0) = -\frac{2}{3}$$ **?**



J.-Y. Kim, H.-Y. Won, H-Ch.Kim, C. Weiss in preparation;
C. Weiss, J.-Y. Kim in preparation

QCD operator vs effective operator

- We actually use the Jaffe/Manohar type operator in the estimation of the EMT/parity-odd EMT. $\overleftrightarrow{D}^{\mu} \approx \overrightarrow{\partial}^{\mu} - \overleftarrow{\partial}^{\mu}$
- The gluon contributions are parametrically suppressed in the leading-twist observables. J. Balla, M. V. Polyakov, C. Wiess (1998)
- Most of dynamical models (e.g., light-cone wave function model) employ "Jaffe-Manohar operator".

QCD	vs	Effective theory
$\hat{T}^{\mu\nu}_{5,q} = \frac{1}{2}\bar{\psi}_q i\gamma^{\mu}\gamma_5 \overleftrightarrow{D}^{\nu}\psi_q$		$T^{\mu\nu}_{\mathrm{eff},5,q} = \frac{1}{2}\bar{\psi} i\gamma^{\mu}\gamma_5 \overleftrightarrow{\partial}^{\nu}\psi$

- In twist-3, the gauge-dependent part in the covariant derivative could play an important role.

- We have recently found that the gluon contribution to the twist-3 operator has the same parametric order as the naive quark operator.

$$\overleftrightarrow{D} = \overrightarrow{\partial}^{\mu} - \overleftarrow{\partial}^{\mu} - 2igA^{\mu} \xrightarrow{\text{Instanton}} MU^{\gamma_5} \text{ (Chiral field)} \sim \bar{\mathcal{O}}^{\alpha\beta}_{\mathrm{eff}}, \mathcal{O}^{\mu\nu}_{\mathrm{eff}}$$

- Indeed, the gluon contributions are found to be larger than the quark contributions (in the chiral theory in the large N_c limit).

$$\langle N|\bar{\psi}\overleftrightarrow{\partial}^{[\mu}\gamma^{\nu]}\gamma_5\psi|N\rangle < \langle N|\bar{\psi}2igA^{[\mu}\gamma^{\nu]}\gamma_5\psi|N\rangle$$

-Talk by C.Weiss

C. Weiss, J.-Y. Kim in preparation

Decomposition of the AM (twist-3) M. Wakamatus, el al (2005)/(2006)

$$J^V = L^V + S^V + \delta J^V \longrightarrow$$

QCD relation

$$\overline{\psi}(x)\gamma^{[\alpha}i\overleftrightarrow{D}^{\beta]}\psi(x) = -\varepsilon^{\alpha\beta\mu\lambda}\partial_\mu\left[\overline{\psi}(x)\gamma_\lambda\gamma_5\psi(x)\right]$$

Effective operator

$$\bar{\psi}\left\{\gamma^{[\alpha}i\overleftrightarrow{\partial}^{\beta]} + \bar{\mathcal{O}}^{\alpha\beta}_{\mathrm{eff}}\right\}\psi = -\epsilon^{\alpha\beta\mu\lambda}\partial_\mu(\bar{\psi}\gamma_\lambda\gamma_5\psi),$$

Spin-orbit correlation (twist-3)

$$C^S_z = -\frac{1}{2}F^S_1(0) + \delta F^S_1(0) \longrightarrow$$

QCD relation

$$\bar{\psi}_q\gamma^{[\mu}\gamma_5 i\overleftrightarrow{D}^{\nu]}\psi = 2m\bar{\psi}_q i\sigma^{\mu\nu}\gamma_5\psi_q - \epsilon^{\mu\nu\alpha\beta}\partial_\alpha(\bar{\psi}_q\gamma_\beta\psi_q)$$

Effective operator

$$\bar{\psi}\left\{\gamma^{[\mu}\gamma_5 i\overleftrightarrow{\partial}^{\nu]} + \mathcal{O}^{\mu\nu}_{\mathrm{eff}}\right\}\psi = 2m\bar{\psi}i\sigma^{\mu\nu}\gamma_5\psi - \epsilon^{\mu\nu\alpha\beta}\partial_\alpha(\bar{\psi}\gamma_\beta\psi),$$

Chiral-odd twist-3 quark distribution functions

P. Schweitzer (2003)/ M. Wakamatus, el al (2005)

$$\int_{-1}^{1} dx\, x\, e^S(x) = \frac{m}{M_N}N_c + \frac{M}{M_N}\beta,$$

C. Weiss, J.-Y. Kim in progress

Summary

Model independent

- Parametrization of the $N \to \Delta$ transition matrix element of the EMT current; 3D multipole expansion; mechanical interpretations;
- We provide the connection between the EMT form factors and the transition GPDs.

Large N_c limit of QCD

- Using the spin-flavor symmetry in the large N_c limit of QCD, we relate the EMT form factors for the nucleon, the Δ baryon and the $N \to \Delta$ transitions.
- In the large N_c limit of QCD, we find that the 3D AM is equivalent to both the 2D LF AM and the gravitational dipole moment.

$$\underset{T^{0k}}{\mathcal{J}_1^{p\Delta^+,V}} = \underset{T^{+k}}{\mathcal{J}_{\mathrm{LF}}^{p\Delta^+,V}} = \underset{T^{++}}{-d^{p\Delta^+,V}}$$

- This AM or gravitational dipole moment can be extracted from the second Mellin moment of the GPD.

$$\lim_{N_c\to\infty,t\to 0}\int_{-1}^{1} dx\, xH_M(x,\xi,t) = 2\mathcal{J}_1^{N\Delta,\mathrm{V}}(0),$$

Dynamics

- In the chiral soliton approach, we see that this model naturally interpolates NQM and ChPT.
- We see the cancellation between the spin and OAM in the isovector component.

$$S^{u-d} = -L^{u-d} \quad \text{at} \quad \mathcal{O}(p^2)$$

- We estimate the EMT form factors in the chiral soliton approach and observe D-term “flavor (u,d) blindness” in the flavor SU(3).

$D_u(0)$	$D_d(0)$	$D_s(0)$
-1.016	-1.077	-0.438

Near future works

- Estimation of the parity-odd EMT
- Matching the twist-3 QCD operator with the effective operator

Thank you very much!