# The Impact of QED Effects in SIDIS

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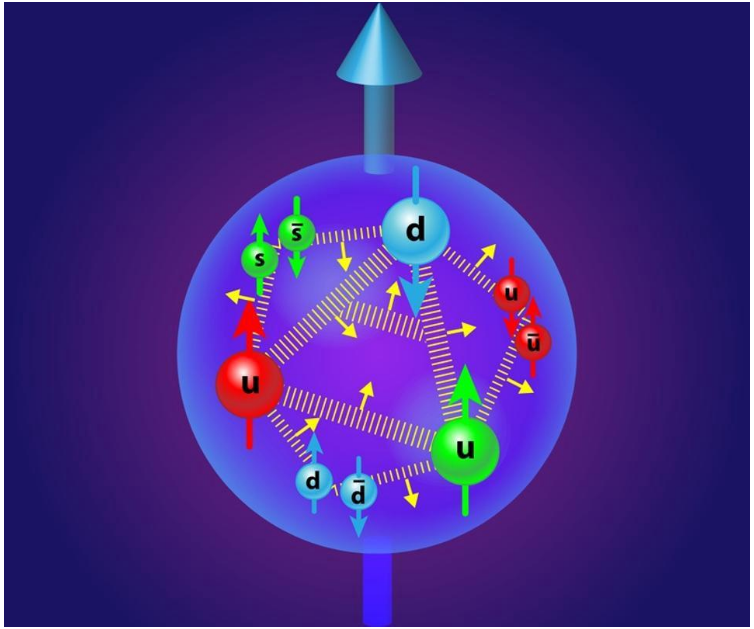
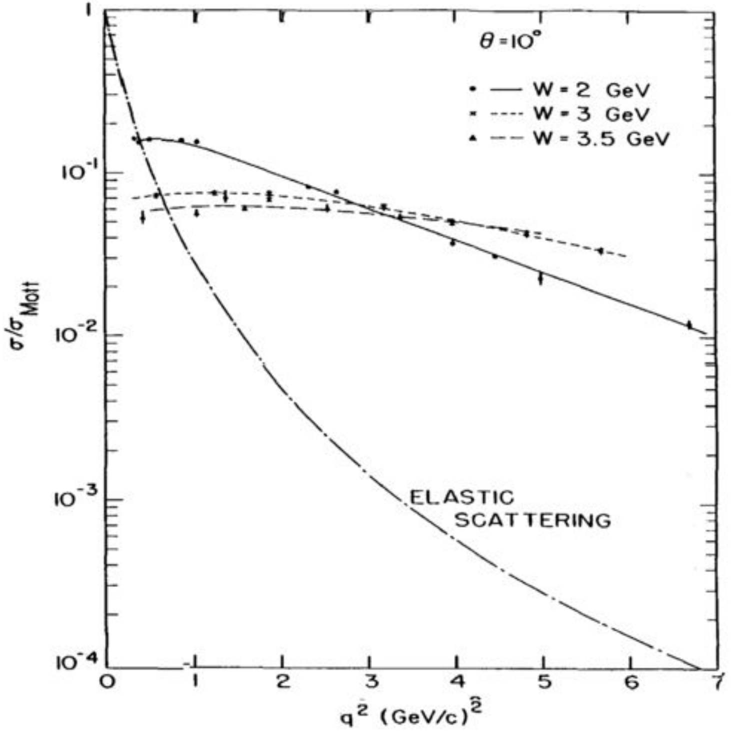
# Outline

- **Motivation**
- Semi-Inclusive Deep Inelastic Scattering
  - Standard Approach
  - Puzzles from Standard Approach
- QED Effects
  - Derivation of Perturbative Coefficients
  - Cross Section Comparison

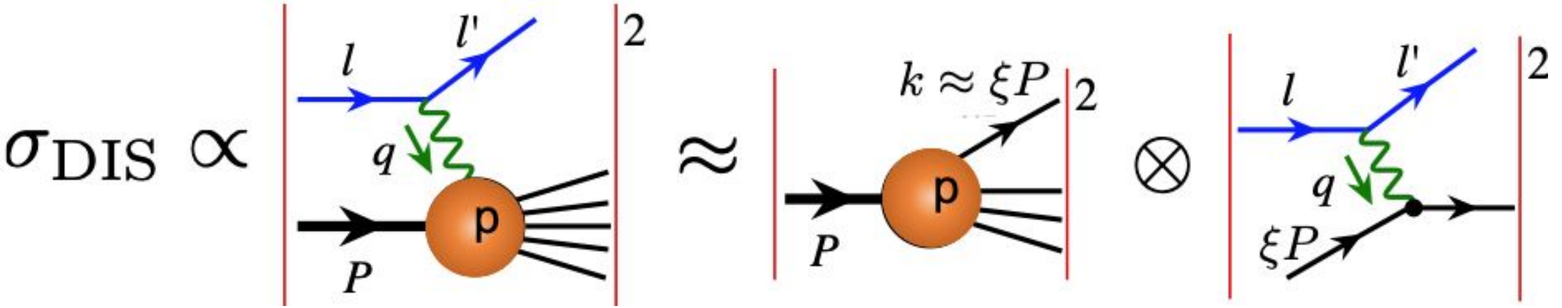


# Structure of the Proton

- Early SLAC experiments performed DIS measurements
- Similar to Rutherford scattering, results were not consistent with uniform distribution inside proton
- Hadrons composed of point-like partons (Feynman 1969), later determined to be quarks and gluons
- Several classes of functions defined to characterize behavior of partons



# DIS Factorization




$$E' \frac{d\sigma_{ep\to e'X}}{d^3l'} \approx \sum_i \int d\xi\, f_{i/p}(\xi)\, E' \frac{d\hat{\sigma}_{ei\to e'X}}{d^3l'} = \sum_i e_i^2 \left\{ \frac{2\alpha^2}{Q^2 s} \left[ \frac{1+(1-y)^2}{y^2} \right] \right\} f_{i/p}(x)$$

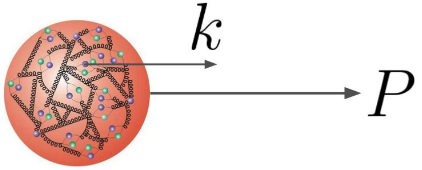
- Separates process into non-perturbative QCD effects (PDFs) and perturbative hard scattering process

# Parton Distribution Functions

- Parton Distribution Functions (PDFs): describe probability of finding parton with given momentum fraction inside hadron

$$f_i(\xi) = \int \frac{\mathrm{d}w^-}{4\pi} e^{-i\xi p^+ w^-} \langle N|\bar{\psi}_i(0, w^-, \mathbf{0}_\mathrm{T})\gamma^+\psi_i(0)|N\rangle$$

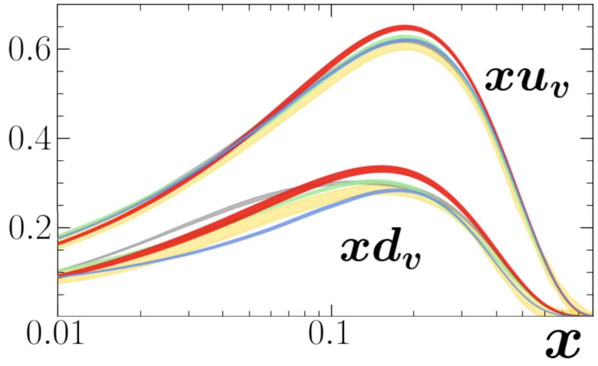
$$\xi = \frac{k^+}{P^+}$$




- In the free field approximation:

$$\psi_i(x) = \sum_{k,\alpha} b_{k,\alpha}(x^+)u_{k,\alpha}e^{-ik^+x^- + ik_\mathrm{T}\cdot x_\mathrm{T}} + d^\dagger_{k,\alpha}(x^+)u_{k,-\alpha}e^{ik^+x^- - ik_\mathrm{T}\cdot x_\mathrm{T}}$$

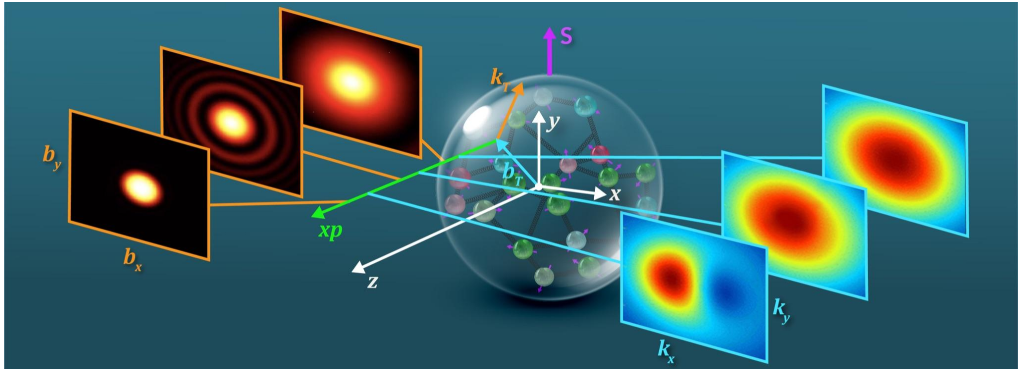
$$f_i(\xi) \sim \sum_\alpha \int \mathrm{d}^2k_\mathrm{T} \langle N| \underbrace{b^\dagger_{k,\alpha}b_{k,\alpha}(\xi p^+, k_\mathrm{T}, \alpha)}_{\text{number operator}} |N\rangle$$




PDFs from JAM Collaboration

# 3D Structure of the Proton

- Transverse Momentum Dependent PDFs: Intrinsic transverse motion of parton considered $f(\xi, k_\mathrm{T})$
- Generalized Parton Distribution Functions: Position space distribution compared to center
- Need two-scale observables

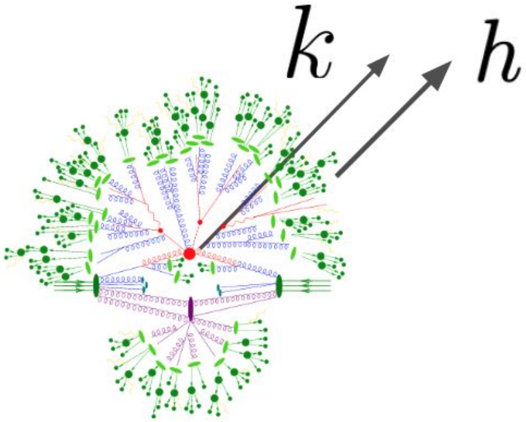


# Hadron Formation

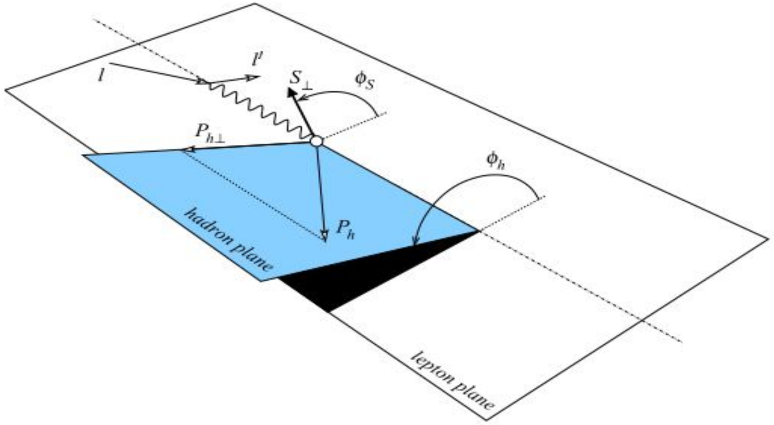
- Fragmentation Function (FFs): describe probability of hadron is formed by a parton given momentum fraction

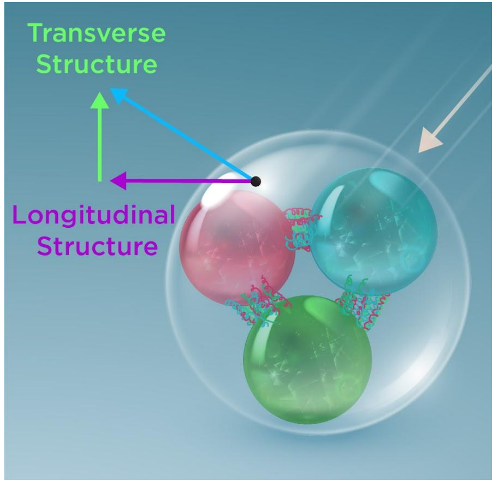
$$\zeta = \frac{p_h^+}{k^+}$$

$$d_{h/j}(\zeta) = \frac{\mathrm{Tr}_{\text{color,Dirac}}}{4N_{c,j}} \sum_X \zeta \int \frac{dw^+}{2\pi} e^{i(p_h^-/\zeta)w^+}$$
$$\times \gamma^- \langle 0 \left| \bar{\psi}_j(0, w^+, \mathbf{0}_\mathrm{T}) \right| p_h, X \rangle \langle p_h, X | \, \psi_j(0) | \, 0 \rangle$$



# Semi-Inclusive DIS: Frontier of Hadron Structure

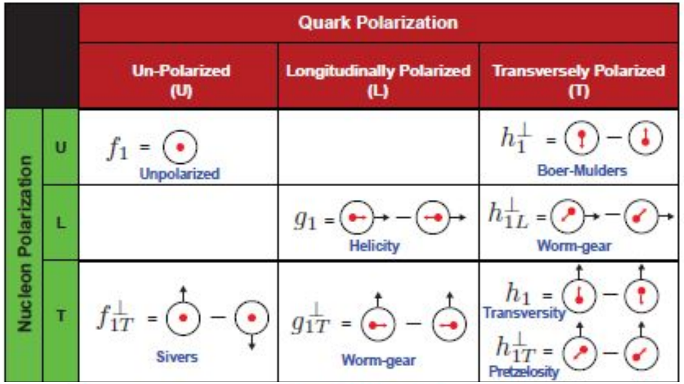


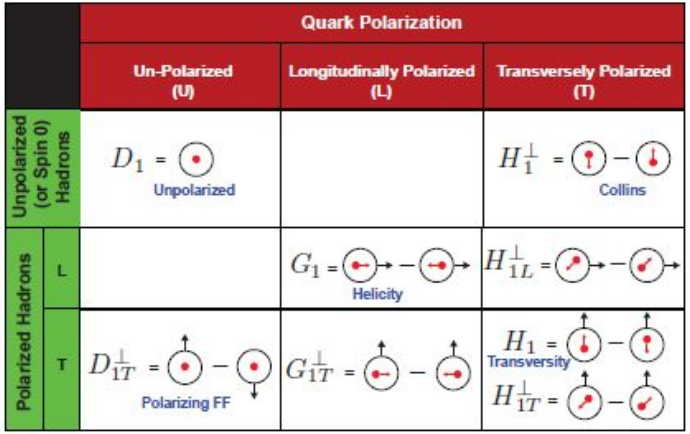




- Observation of two particles allows consideration of two scales of observables: $P_{h\perp}$ and $Q^2$

# SIDIS Cross Section

$$
\begin{aligned}
\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} =\; & \frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} \\
& +\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} \\
& +S_\parallel\Big[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\Big] \\
& +S_\parallel\lambda_e\Big[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\Big] \\
& +|\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\Big(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\Big) \\
& \quad+\varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} \\
& \quad+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Big] \\
& +|\boldsymbol{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} \\
& \quad+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\},
\end{aligned}
$$




Bacchetta et al., 2004

$x \Leftrightarrow x_B$

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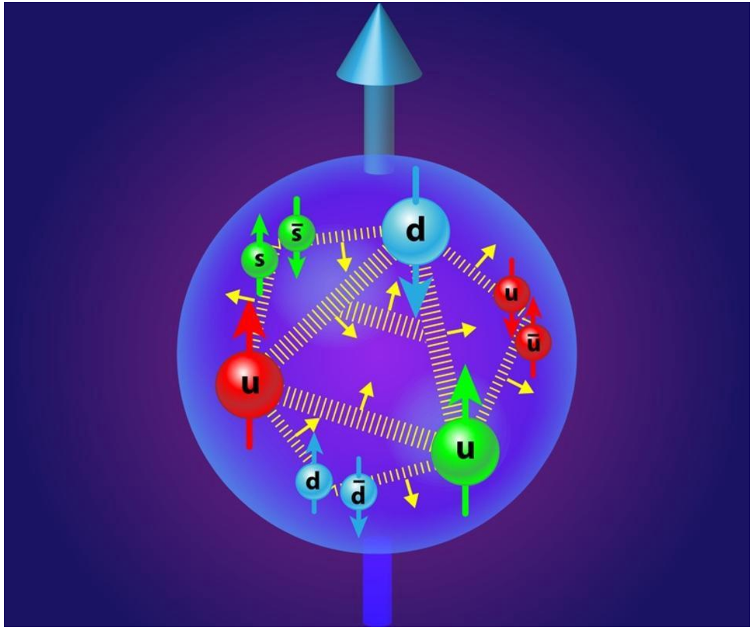
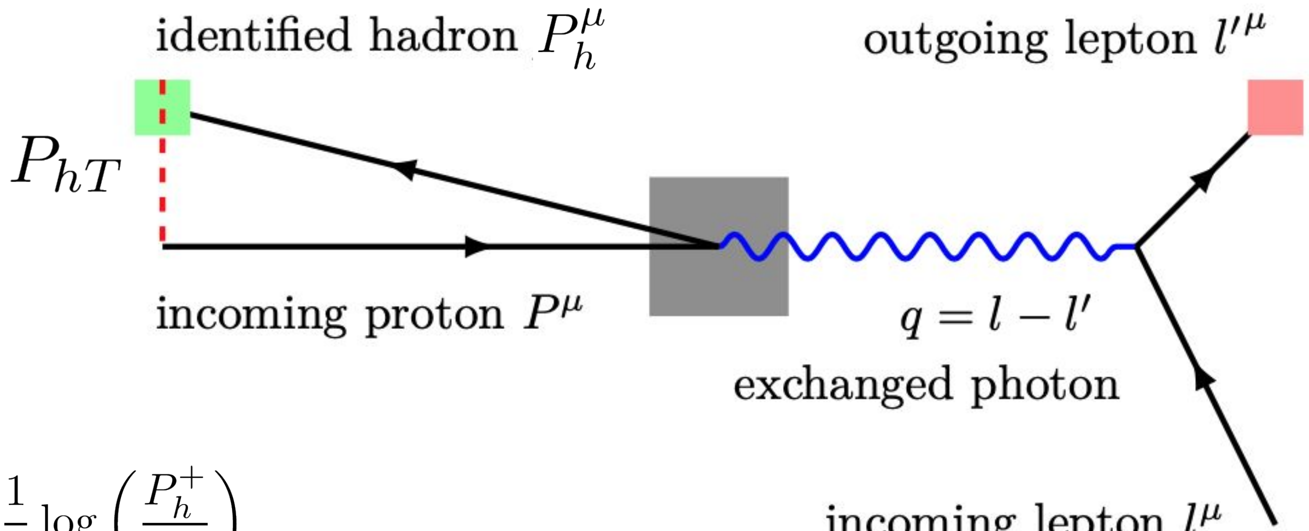
  - Effects on Kinematic Variables
  - Cross Section Comparison



# Breit Frame




$$y_h = \frac{1}{2}\log\left(\frac{P_h^+}{P_h^-}\right)$$

# Kinematic Variables Appearing in Full SIDIS Cross Section

$$x_B = \frac{Q^2}{2P\cdot q}, \quad y = \frac{P\cdot q}{P\cdot l}, \quad z = \frac{P\cdot P_h}{P\cdot q}, \quad \gamma = \frac{2Mx_B}{Q}$$

$$\varepsilon = \frac{1 - y - \frac{1}{4}\gamma^2 y^2}{1 - y + \frac{1}{2}y^2 + \frac{1}{4}\gamma^2 y^2}$$

$$\cos(\phi_h) = -\frac{l_\mu P_{h\nu} g_\perp^{\mu\nu}}{\sqrt{l_\perp^2 P_{h\perp}^2}}, \quad \cos(\phi_S) = -\frac{l_\mu S_\nu g_\perp^{\mu\nu}}{\sqrt{l_\perp^2 S_\perp^2}},$$

$$\sin(\phi_h) = -\frac{l_\mu P_{h\nu} \epsilon_\perp^{\mu\nu}}{\sqrt{l_\perp^2 P_{h\perp}^2}}, \quad \sin(\phi_S) = -\frac{l_\mu S_\nu \epsilon_\perp^{\mu\nu}}{\sqrt{l_\perp^2 S_\perp^2}}$$

$$S^\mu = S_\parallel \frac{P^\mu - q^\mu M^2/(P\cdot q)}{M\sqrt{1+\gamma^2}} + S_\perp^\mu$$

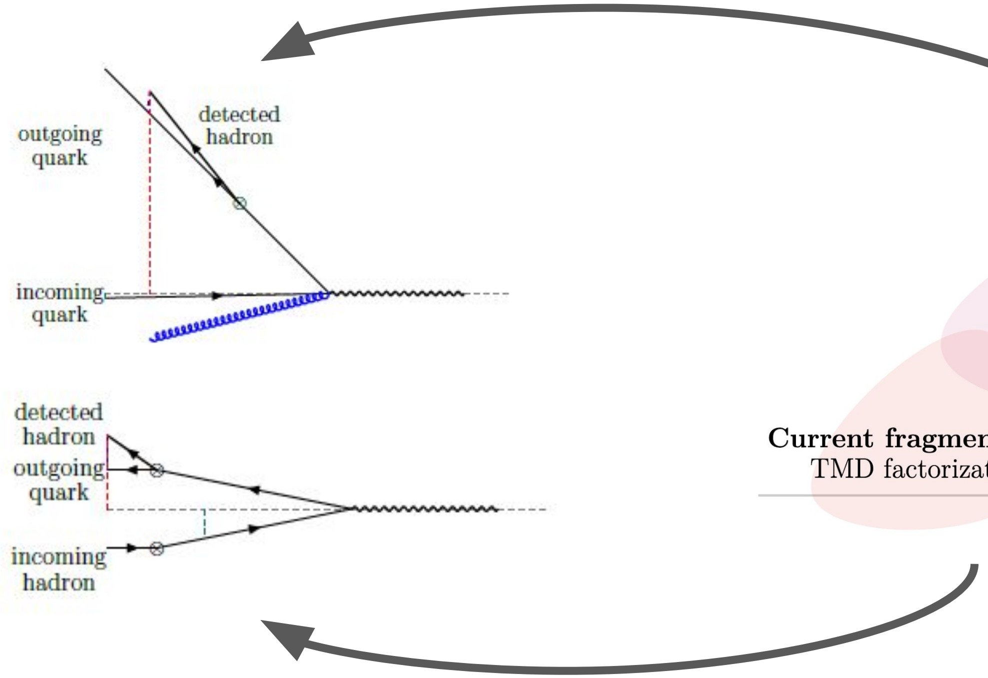
$$l_\perp^\mu = g_\perp^{\mu\nu} l_\nu, \quad P_{h\perp}^\mu = g_\perp^{\mu\nu} P_{h\nu}$$

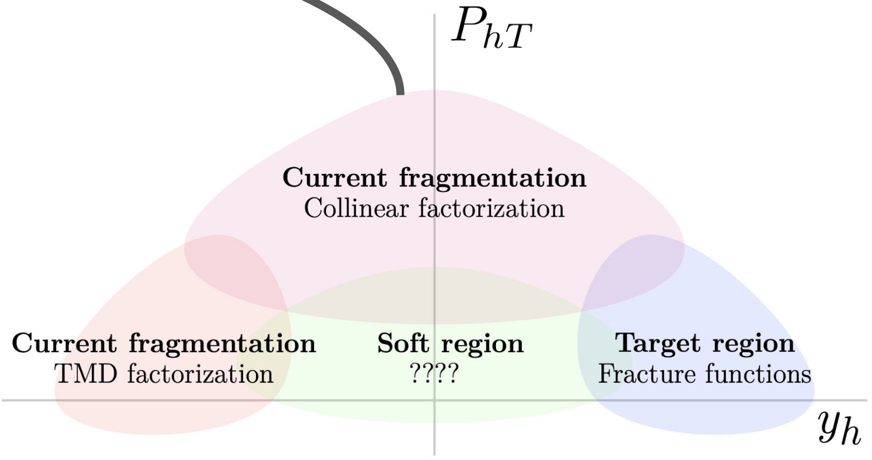
$$S_\parallel = \frac{S\cdot q}{P\cdot q}\frac{M}{\sqrt{1+\gamma^2}}, \quad S_\perp^\mu = g_\perp^{\mu\nu} S_\nu$$

$$g_\perp^{\mu\nu} = g_{\mu\nu} - \frac{q^\mu P^\nu + q^\nu P^\mu}{P\cdot q(1+\gamma^2)} + \frac{\gamma^2}{1+\gamma^2}\left(\frac{q^\mu q^\nu}{Q^2} - \frac{P^\mu P^\nu}{M^2}\right)$$

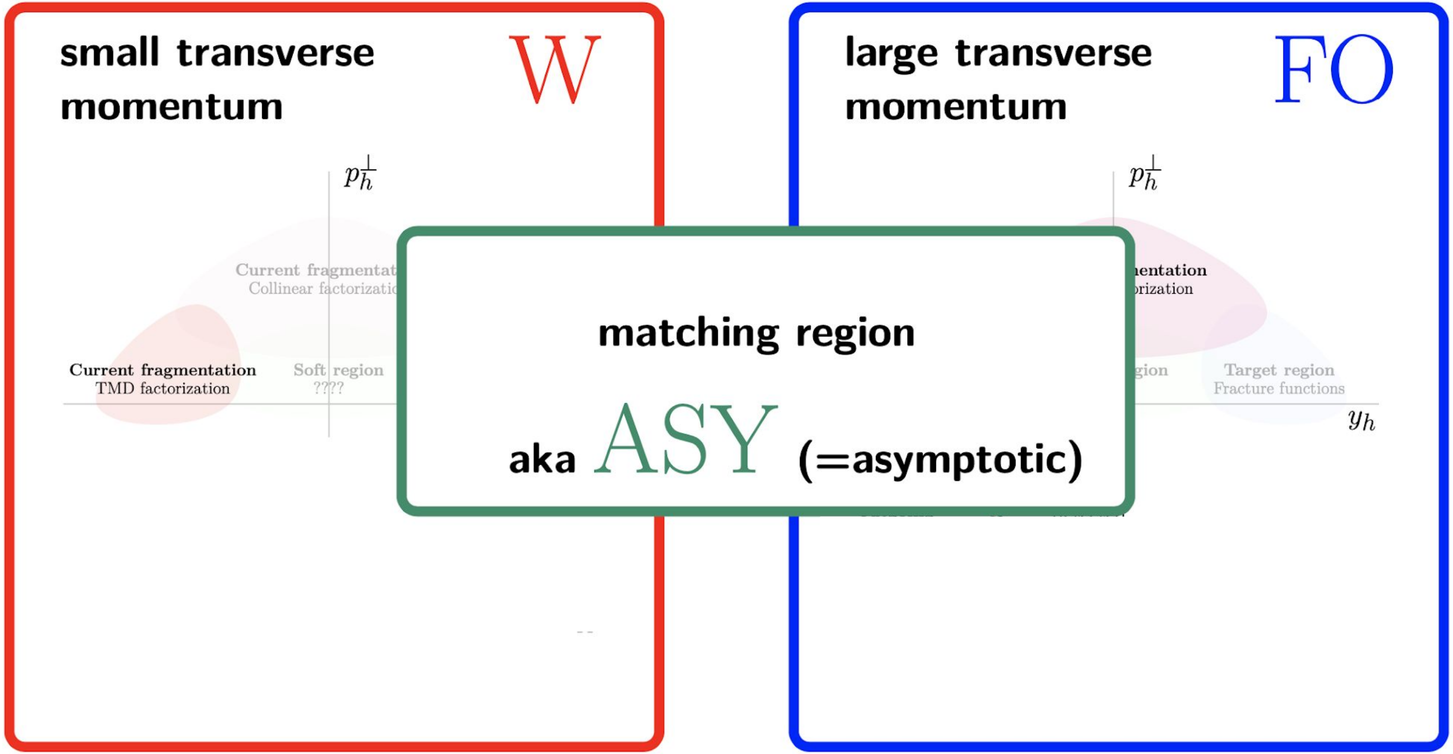
$$\epsilon_\perp^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}\frac{P_\rho q_\sigma}{P\cdot q\sqrt{1+\gamma^2}}$$

# Separation of Kinematic Regions



# Standard Approach



# Scale Separation of Cross Section

$$\frac{d\sigma}{dx\,dQ^2\,dz\,dP_{hT}} = \mathrm{W} + \mathrm{FO} - \mathrm{ASY} + \mathcal{O}(\Lambda^2_{QCD}/Q^2)$$

where

$$P_{hT} = q_T z_h$$

$$x \Leftrightarrow x_B$$

$$\sim \mathrm{W} \quad \text{for } q_\mathrm{T} \ll Q$$

$$\sim \mathrm{FO} \quad \text{for } q_\mathrm{T} \sim Q$$

- W term has resummed logarithms
- Asymptotic term perturbatively expands W term to fixed order
- Covers matching region between W and fixed order and removes non-dominant contributions

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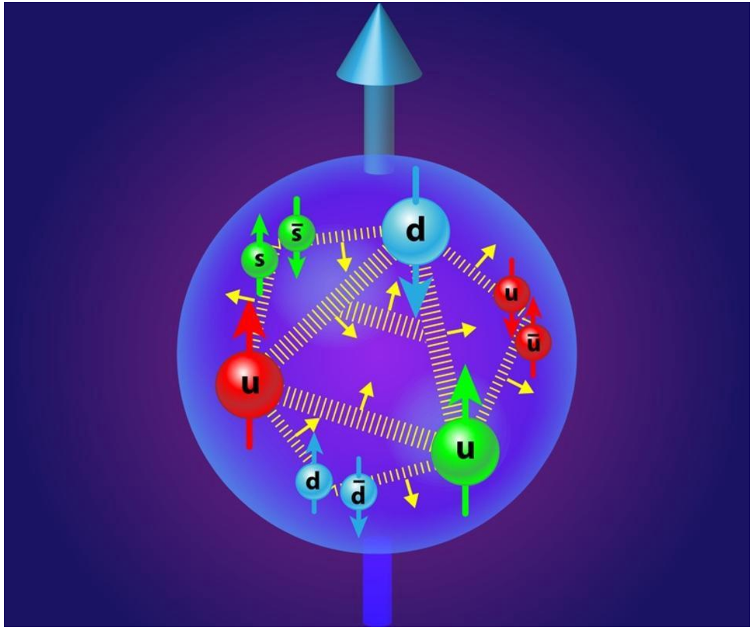
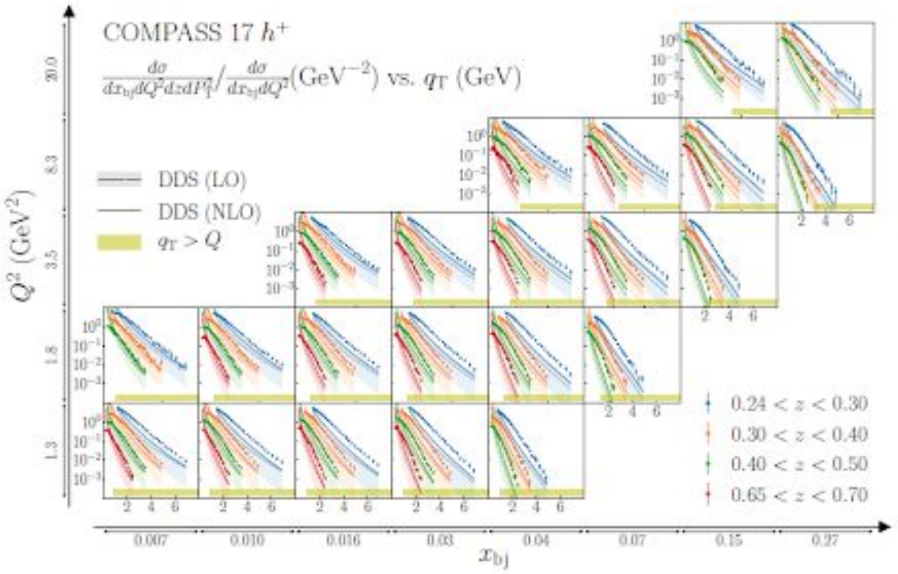


# Inconsistency with Data

- Large discrepancy with data (example from COMPASS 2017 run, Aghasyan et al., 2018)
- Possible Solutions
  - Higher twist corrections
  - Better constraints on PDFs and FFs
  - NLO corrections
  - Power corrections



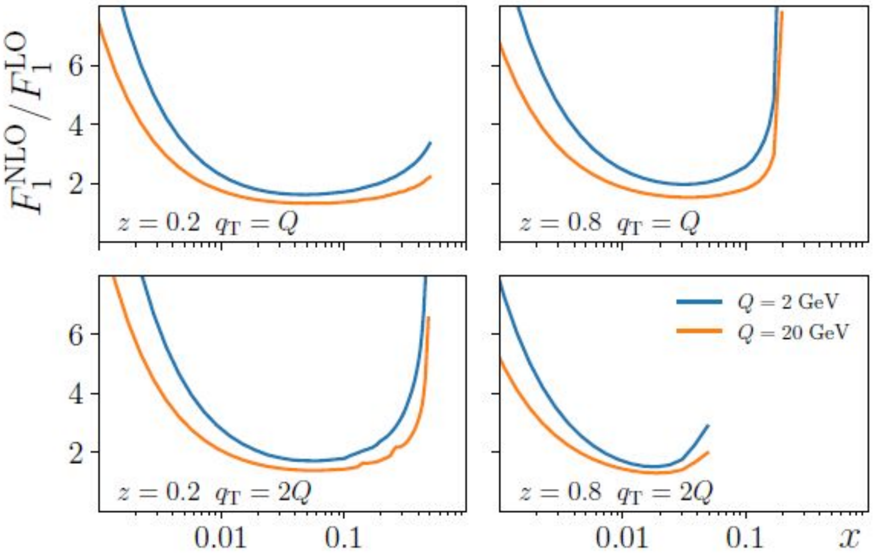

Gonzalez-Hernandez et al., 2018

# Beyond Leading Order Corrections

- Order of $\alpha_s^{\ 2}$ contributions calculated in Wang et al., 2019
- Compared *K* factor ratio for $F_1$
- Important at large *x*
  - Soft gluon effects near threshold
- Important at small *x*
  - Large $(P+q)^2$

$$P_1^{\mu\nu} W_{\mu\nu} = F_1$$

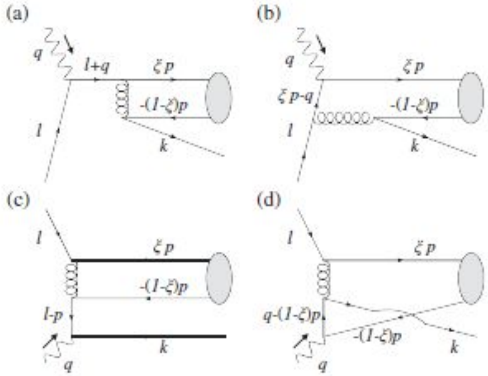
$$P_1^{\mu\nu} = \frac{\left(2(\hat{x}_B/x)^2\right) p^\mu p^\nu}{\hat{Q}^2} - \frac{1}{2} g^{\mu\nu}$$

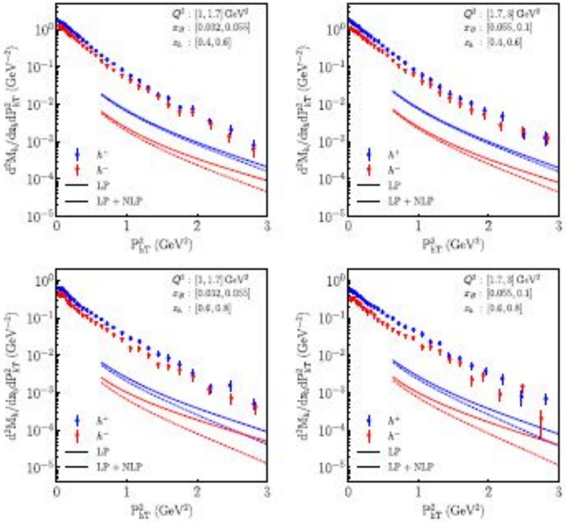



$$x \Leftrightarrow x_B$$

# Power Corrections

- Considered by Liu and Qiu, 2020
- Typical subleading power corrections are suppressed by large $P_{h\perp}$
- Enhancement from hadronization and edge of phase space
- Requires consideration of quark-antiquark FFs
  - Hadronization of a pair (i.e. $u\bar{d} \to \pi^+$)



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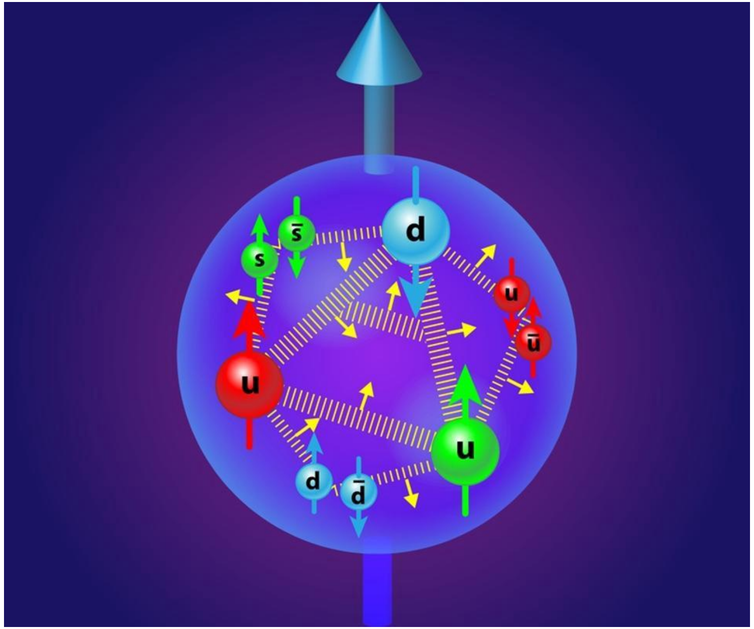
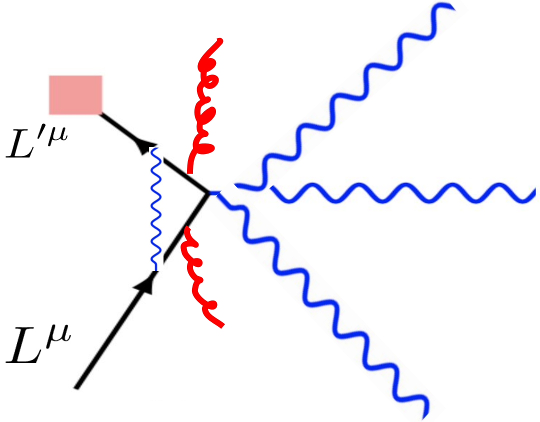
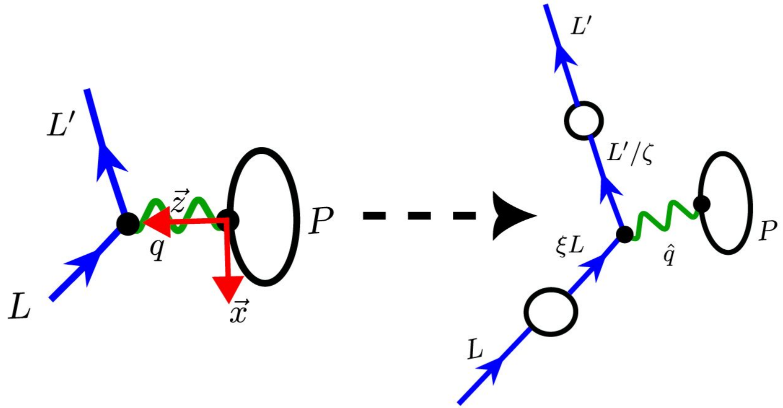


# QED Effects

- Radiation of photon from initial state changes the direction of the exchanged photon
- Unobserved lepton radiation introduces ambiguity, as the radiation changes the unobserved photon's momentum (Liu et al., 2021)
- Hybrid factorization method to address these issues



# Collision Induced QED Radiation Effects

- Previous approaches focused on a radiation correction factor to scattering without radiation
  - Translated $x_B, Q^2 \Rightarrow x_{eff}, Q^2_{eff}$
  - Virtual photon still fully determined
- Needed hadronic information form "effective" variables, so difficult to extract hadronic information precisely from this approach
- New QED factorization approach takes

$$x_B, Q^2 \Rightarrow \hat{x}_B, \hat{Q}^2 \in \{[x_B, 1], [Q^2_{\text{min}}, Q^2_{\text{max}}]\}$$

# Kinematic Variables with QED effects Highlighted

$$x_B = \frac{Q^2}{2P \cdot q}, \quad y = \frac{P \cdot q}{P \cdot l}, \quad z = \frac{P \cdot P_h}{P \cdot q}, \quad \gamma = \frac{2Mx_B}{Q}$$

$$\varepsilon = \frac{1 - y - \frac{1}{4}\gamma^2 y^2}{1 - y + \frac{1}{2}y^2 + \frac{1}{4}\gamma^2 y^2}$$

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$$S^\mu = S_\parallel \frac{P^\mu - q^\mu M^2/(P \cdot q)}{M\sqrt{1+\gamma^2}} + S_\perp^\mu$$

$$l_\perp^\mu = g_\perp^{\mu\nu} l_\nu, \quad P_{h\perp}^\mu = g_\perp^{\mu\nu} P_{h\nu}$$

$$S_\parallel = \frac{S \cdot q}{P \cdot q} \frac{M}{\sqrt{1+\gamma^2}}, \quad S_\perp^\mu = g_\perp^{\mu\nu} S_\nu$$

$$g_\perp^{\mu\nu} = g_{\mu\nu} - \frac{q^\mu P^\nu + q^\nu P^\mu}{P \cdot q(1+\gamma^2)} + \frac{\gamma^2}{1+\gamma^2}\left(\frac{q^\mu q^\nu}{Q^2} - \frac{P^\mu P^\nu}{M^2}\right)$$

$$\epsilon_\perp^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} \frac{P_\rho q_\sigma}{P \cdot q\sqrt{1+\gamma^2}}$$

# Hybrid Factorization Approach

- Leading collinear radiative corrections resummed into lepton distribution functions and lepton fragmentation functions
- Can still extract PDFs and TMDs from scattering cross sections

$$E_{P_h} E_{L'} \frac{\mathrm{d}^6\sigma^{LA\to L'hX}}{\mathrm{d}^3\vec{P}_h \mathrm{d}^3\vec{L}'} \approx \sum_{i,j} \int \frac{\mathrm{d}\zeta}{\zeta^2} \frac{\mathrm{d}\xi}{\xi} D_{e/j}(\zeta) f_{i/e}(\xi)$$

$$\times \left[ E_{k'} E_{P_h} \frac{\mathrm{d}^6\sigma^{kA\to k'hX}}{\mathrm{d}^3\vec{P}_h \mathrm{d}^3\vec{k}'} \right]_{k=\xi L, k'=L'/\zeta}$$

# Explicit vs Structure Function Expansion

- Explicit Perturbative Expansion

$$E_{k'} E_{P_h} \frac{\mathrm{d}^6 \sigma^{kA \to k' h X}}{\mathrm{d}^3 \vec{P}_h \mathrm{d}^3 \vec{k'}} \approx \frac{1}{2S} \sum_{n,m} \int \frac{\mathrm{d}z}{z^2} \frac{\mathrm{d}x}{x} \tilde{D}_{h/n}(z) \tilde{f}_{m/h}(x) \hat{H}^{im \to jn}(k, k', p, p') \Big|_{p = xP, p' = P_h/z}$$

$$\hat{H}^{im \to jn}(k, k', p, p') = \sum_{s,r} \hat{H}^{(s,r)}_{im \to jn}(k, k', p, p') \alpha^s \alpha_S^r + \mathcal{O}\left( \frac{\Lambda^2_{QCD}}{Q^2}, \frac{m_e^2}{Q^2} \right)$$

- Structure Function Expansion

$$E_{k'} E_{P_h} \frac{\mathrm{d}^6 \sigma^{kA \to k' h X}}{\mathrm{d}^3 \vec{P}_h \mathrm{d}^3 \vec{k'}} = \left( \frac{4 \hat{x}_B}{\hat{Q}^2} \sqrt{\hat{z}_h^2 - (\hat{\gamma} \hat{P}_{h\perp} / \hat{Q})^2} \right) \frac{\hat{x}_B}{x_B \xi \zeta}$$

$$\times \left[ \frac{\alpha^2}{\hat{x}_B \hat{y} \hat{Q}^2} \frac{\hat{y}}{2(1 - \hat{\epsilon})} \left( 1 + \frac{\hat{\gamma}^2}{2 \hat{x}_B} \right) \sum_n \hat{w}_n F_n^h \left( \hat{x}_B, \hat{Q}^2, \hat{z}_h, \hat{P}^2_{h\perp} \right) \right]$$

# Lepton Distribution and Fragmentation Functions

- At NLO, leading logarithmically resummed universal functions
- Lepton Distribution Functions: radiation prior to the collision
- Lepton Fragmentation Functions: radiation after the collision
- Follow DGLAP evolution kernel
- Perturbatively calculable (up to hadronic contributions)

$$f_{e/e}^{(1)}\left(\xi,\mu^2\right) = \frac{\alpha}{2\pi}\left[\frac{1+\xi^2}{1-\xi}\ln\frac{\mu^2}{(1-\xi)^2 m_e^2}\right]_+$$

$$D_{e/e}^{(1)}\left(\zeta,\mu^2\right) = \frac{\alpha}{2\pi}\left[\frac{1+\zeta^2}{1-\zeta}\ln\frac{\zeta^2\mu^2}{(1-\zeta)^2 m_e^2}\right]_+$$

# Endpoint Numerical Issues

- General form of the cross section:

$$\sigma = \int_{\zeta_{\min}}^{1} d\zeta \int_{\xi_{\min}(\zeta)}^{1} d\xi f(\xi) D(\zeta) H(\xi, \zeta)$$

- Poles at endpoint, requires mathematical tricks to separate problematic region

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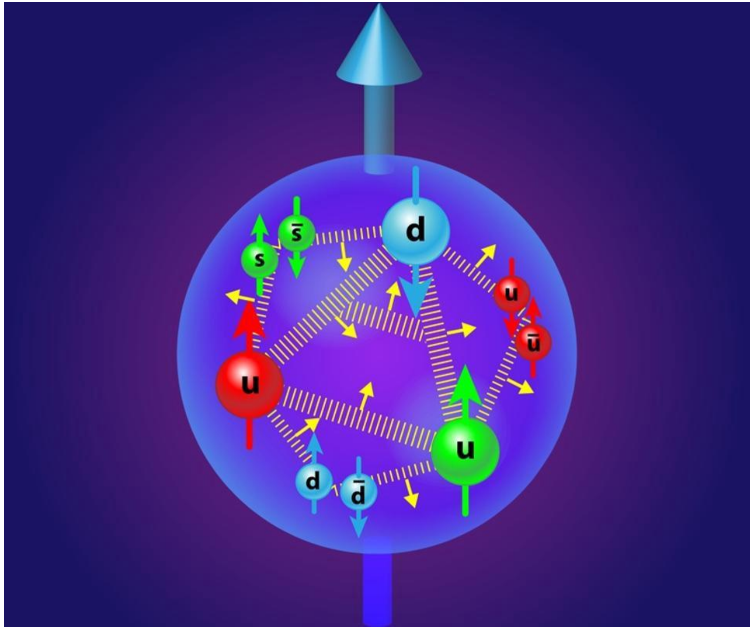


# Helicity Basis

- Working with helicity-based hadronic structure functions (similar to lepton case in Liu et al., 2021)
- Vectors defined as in Ji et al., 2006

$$\hat{W}^{\mu\nu} = \underbrace{\frac{1}{2}\left(\hat{X}^\mu\hat{X}^\nu + \hat{Y}^\mu\hat{Y}^\nu\right)\hat{H}_{TT}}_{\text{Transverse}} + \underbrace{\left(\hat{T}^\mu\hat{T}^\nu\hat{H}_L + \left(\hat{T}^\mu\hat{X}^\nu + \hat{X}^\mu\hat{T}^\nu\right)\hat{H}_\Delta + \dots\right)}_{\text{other, }\mathcal{O}\left(\alpha_S^2\right)}$$

$$Z^\mu = -\frac{\hat{q}^\mu}{\hat{Q}}$$

$$T^\mu = \left(\frac{1}{\hat{Q}}\right)\left(\hat{q}^\mu + 2\hat{x}_B P^\mu\right)$$

$$X^\mu = \left(\frac{1}{\vec{\hat{q}}_T}\right)\left(\frac{(P')^\mu}{z_B} - \hat{q}^\mu - \left(1 + \frac{\vec{\hat{q}}_T^2}{\hat{Q}^2}\right)\hat{x}_B P^\mu\right)$$

$$Y^\mu = \epsilon^{\mu\nu\alpha\beta} Z_\nu T_\alpha X_\beta$$

# Perturbative Coefficients in Helicity Basis

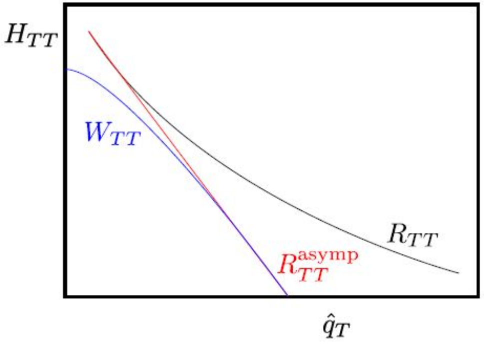
- Fixed order projection onto partonic states
- Separate into real and virtual terms
- Separate based on momentum scale

$$H_{TT} = \frac{1}{2}(X^\mu X^\nu + Y^\mu Y^\nu) W_{\mu\nu}$$

$$W_{\mu\nu} = R_{\mu\nu} + V_{\mu\nu}$$

$$H_{TT}(\vec{\hat{q}}_T) = W_{TT}(\text{small } \vec{\hat{q}}_T) + Y_{TT}(\text{large } \vec{\hat{q}}_T)$$

$$Y_{TT} = R_{TT} - R_{TT}^{\text{asymp}}$$



# CSS-like Formalism

- Similar process as in CSS formalism (Collins et al., 1985)

$$\tilde{W}_{TT}(\vec{b}_T, Q) = \int \mathrm{d}^2\vec{\hat{q}}_T e^{i\vec{\hat{q}}_T\cdot\vec{b}_T} W_{TT}(\vec{\hat{q}}_T, Q) = e^{-S}[C_f \otimes f] \otimes [C_D \otimes D]$$

- Expanding in powers of $\alpha_S$

$$\tilde{W}_{TT} = C_f^{(1)} C_D^{(0)} S^{(0)} + C_f^{(0)} C_D^{(1)} S^{(0)} + C_f^{(0)} C_D^{(0)} S^{(1)}$$

$$S = 1 - \frac{\alpha}{\pi}\left[\frac{1}{2}A^{(1)} \ln^2\frac{\nu_Q^2}{\mu_b^2} + B^{(1)} \ln\frac{\nu_Q^2}{\mu_b^2}\right]$$

# NLO Perturbative Coefficients

$$\tilde{W}_{TT} = C_f^{(1)} C_D^{(0)} S^{(0)} + C_f^{(0)} C_D^{(1)} S^{(0)} + C_f^{(0)} C_D^{(0)} S^{(1)}$$

- Comparing the fixed order calculation with the above perturbative expansion:

$$A^{(1)} = 1$$

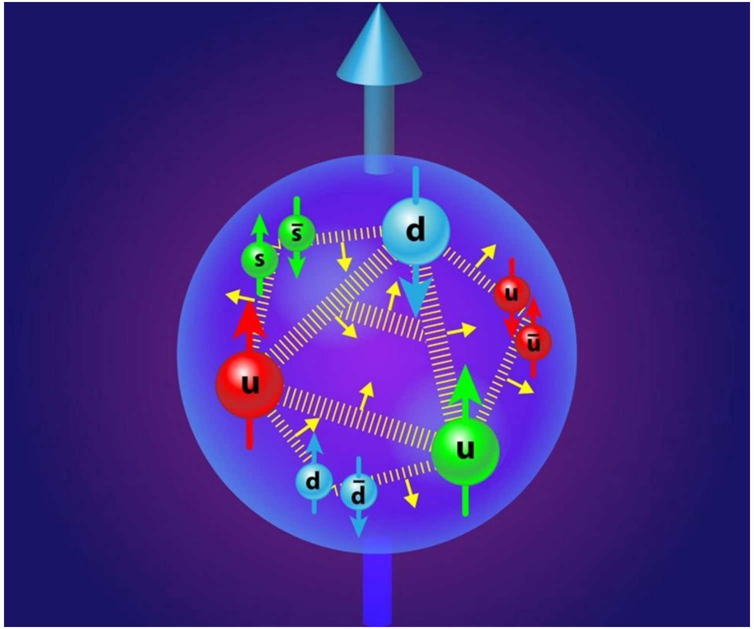
$$B^{(1)} = -\frac{3}{2}$$

$$C_f^{(1)}(\lambda) = \frac{1}{2\lambda}(1-2\lambda) - \frac{1}{\lambda}\left(\frac{\lambda^2+1}{1-\lambda}\right)_+ \ln\frac{\mu_{\bar{M}S}}{\mu_b} - \delta(1-\lambda)$$

$$C_D^{(1)}(\eta) = \frac{1}{2\eta}(1-2\eta) - \frac{1}{\eta}\left(\frac{\eta^2+1}{1-\eta}\right)_+ \ln\frac{\mu_{\bar{M}S}}{\mu_b} - \delta(1-\eta)$$

# Outline

- Motivation
- Semi-Inclusive Deep Inelastic Scattering
  - Standard Approach
  - Puzzles from Standard Approach
- QED Effects
  - Derivation of Perturbative Coefficients
  - **Cross Section Comparison**

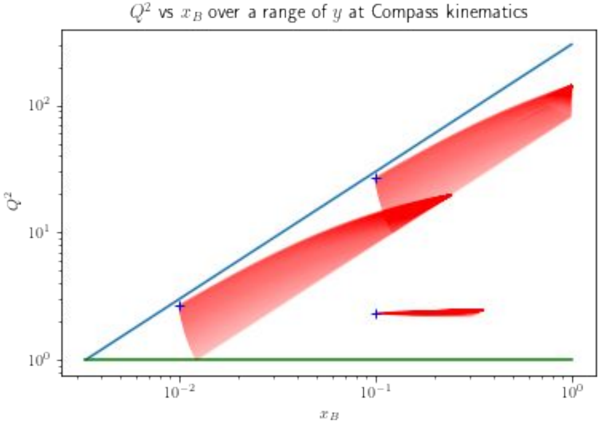


# QED Effects on Relevant Kinematic Variables

- Explicit dependence of major kinematic variables on radiation parameters $\xi$,$\zeta$

$$\hat{x}_B = \frac{\xi y x_B}{\xi\zeta - 1 + y} \qquad \hat{z}_h = \frac{\zeta y z_h}{\xi\zeta - 1 + y}$$

$$\hat{Q}^2 = \frac{\xi}{\zeta} Q^2 \qquad \hat{y} = \frac{\hat{Q}^2}{\xi S \hat{x}_B}$$

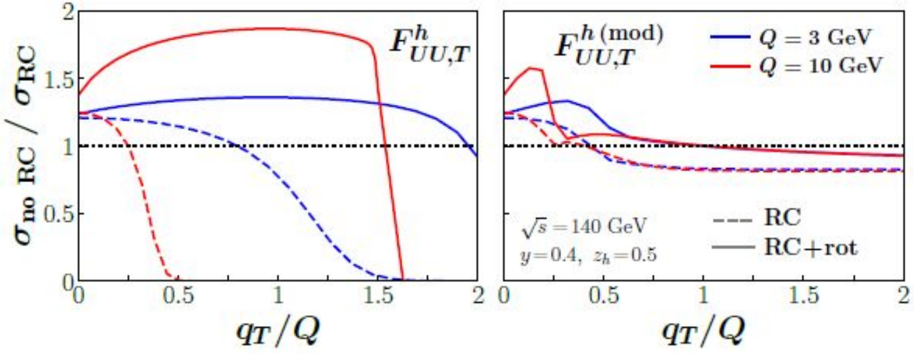



$$\hat{P}^2_{h\perp}(\xi,\zeta,\phi_h,y,z_h,Q,P_{h\perp}) = \frac{-2\cos\phi_h Q^5 z_h(\zeta\xi-1)P_{h\perp}\sqrt{1-y}(\zeta\xi+y-1) + Q^4 P^2_{h\perp}(\zeta\xi+y-1)^2 - Q^6(y-1)z_h^2(\zeta\xi-1)^2}{Q^4(\zeta\xi+y-1)^2}$$

# Previous Comparison of Unpolarized Cross Sections

- Liu et al., 2020 and 2021 established new factorization approach
- Showed effects of radiative corrections to Gaussian approximation of W term with artificial fixed order tail

$$F^h_{UU,\text{Gaussian}}(x_B, y, z_h, Q^2, q_\perp) = \sum_q e_q^2 f_{q/h}(x_B, Q^2) D_{h'/q}(z_h, Q^2) \frac{e^{-(z_h q_\perp)^2/\langle (z_h q_\perp)^2 \rangle}}{\pi \langle (z_h q_\perp)^2 \rangle}$$

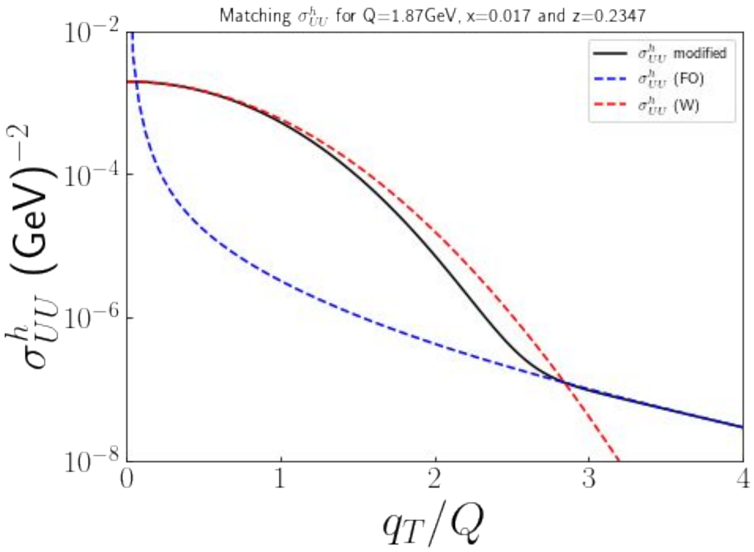


# Setup

- Fixed order calculation comes from Nadolsky et al., 1999 (at finite $q_T \sim Q$ )
- To match W to FO, toy scheme used in this work (and in Liu et al., 2021):

$$\sigma^h_{UU,\text{Modified}} = \textcolor{red}{\text{W}} R + (1 - R)\textcolor{blue}{\text{FO}}$$

$$R = e^{-A(q_T/Q)^B}$$



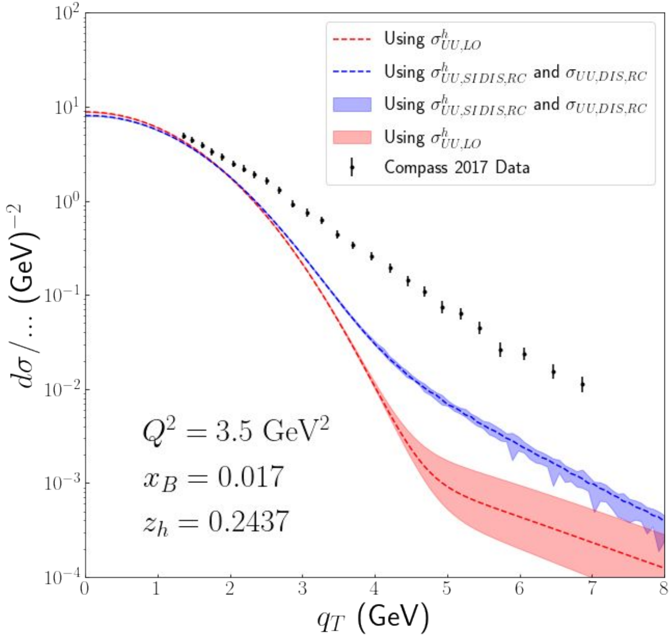

Before QED corrections

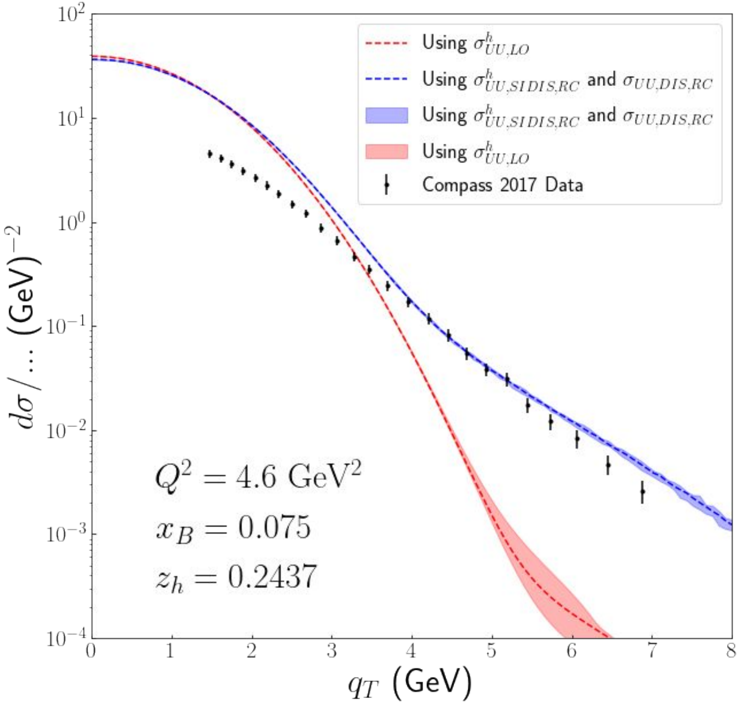
# Structure Functions Considered

- 4 unpolarized structure functions in SIDIS cross section
- 2 depend on hadronic angle
    - Possible to include from fixed order calculations
    - Ignored in this study as contributions are small (~1%)

| Structure Function | W term | FO term |
|---|---|---|
| $F_{UU,T}$ | Yes | Yes |
| $F_{UU,L}$ | No | Yes |
| $F_{UU}^{\cos\phi_h}$ | No | Possible |
| $F_{UU}^{\cos 2\phi_h}$ | No | Possible |

# Results




$$\frac{d\sigma}{dx_B\, dQ^2\, dz_h\, dP_{hT}^2} \Big/ \frac{d\sigma}{dx_B\, dQ^2}$$

$$P_{hT} = q_T z_h$$

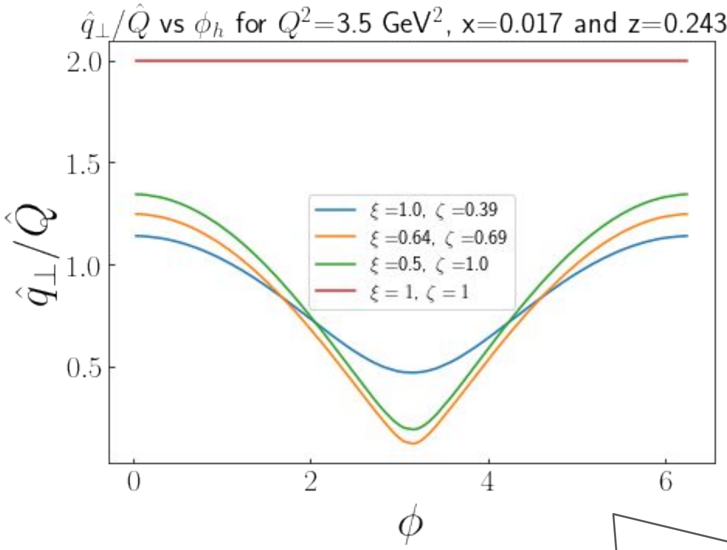
# Role of Angular Effects on $\hat{P}_{hT}$

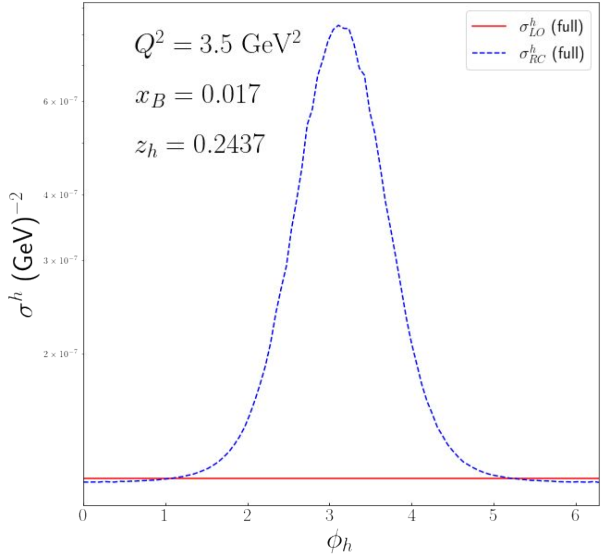
- Standard approach assumes separable angular dependence
- Lepton radiation introduces internal angular dependence
- Increases weight of back-to-back region

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_B \mathrm{d}z_h \mathrm{d}Q^2 \mathrm{d}P_\perp^2} \propto F_{UU,T} + F_{UU,T} + \cos\phi_h F_{UU}^{\cos\phi_h} + \cos(2\phi_h) F_{UU}^{\cos 2\phi_h}$$

$$\hat{P}_{h\perp}^2(\xi, \zeta, \phi_h, y, z_h, Q, P_{h\perp}) \propto \frac{\cos\phi_h(\zeta\xi - 1)P_{h\perp}}{(\zeta\xi + y - 1)} + P_{h\perp}^2$$

# Importance of Angular Effects

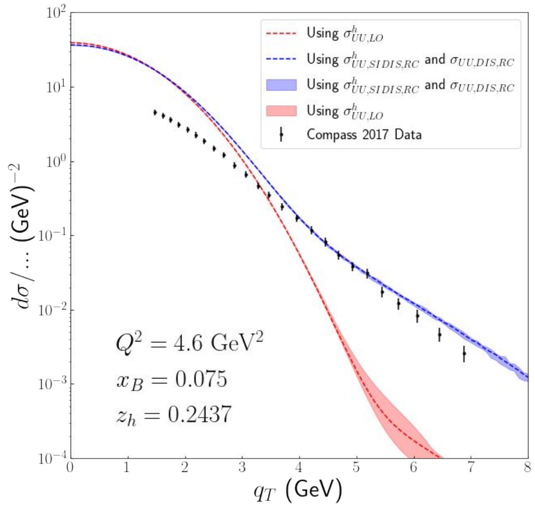


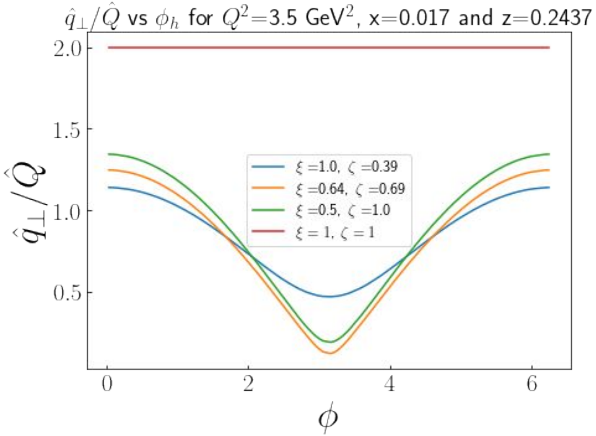




Angular modulation on internal transverse momentum induced by QED effects!

# Conclusion

- Joint QED + QCD factorization scheme for SIDIS process showed significant effects on cross section
- Full transverse momentum spectrum required to address QED effects
- Potential resolution to discrepancies between previous theory and COMPASS data



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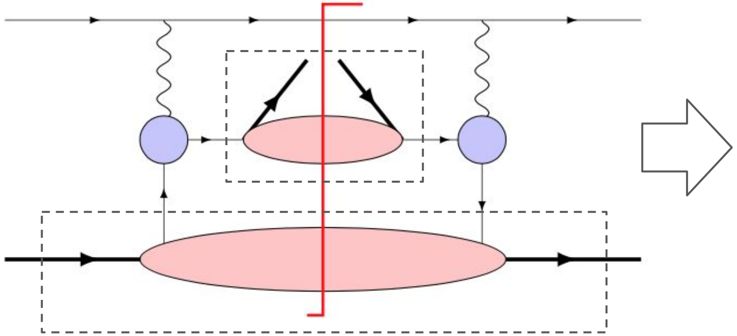
# Backup Slides

# Factorization of SIDIS



$$\mathrm{d}\sigma = \sum_{ij} H_{ij}^{\mathrm{SIDIS}} \otimes f_i \otimes d_j$$

- Up to soft gluon exchanges, provides predictive power
  - Short distance portion calculable
  - PDFs, TMDs, and FFs are universal, nonperturbative functions

# Cross Section Expression

$$E_{L'}E_{P'}\frac{\mathrm{d}\sigma}{\mathrm{d}^3\vec{L}'\,\mathrm{d}^3\vec{P}'} = \sum_q \int_{\zeta_{\min},\xi_{\min},z_{\min},0}^{1} \frac{\mathrm{d}\zeta}{\zeta^2}\frac{\mathrm{d}\xi}{\xi}\frac{\mathrm{d}z}{z^2}\frac{\mathrm{d}x}{x}\tilde{D}_{L/e}(\zeta)\tilde{f}_{e/L}(\xi)$$

$$\times\,\tilde{D}_{h/q}(z)\tilde{f}_{q/h}(x)\left(\frac{1}{4(2\pi)^6}\right)\frac{-8e^4\,e_q^2\,g^2\zeta}{3\xi Q^2 Sxz(\zeta\xi S+U')(S'+\zeta\xi U)}$$

$$\Big(\xi^2\left(2Q^4z^2+2\zeta Q^2Uz+\zeta^2(S^2x^2z^2+U^2)\right)+2\xi S'\big(Q^2z+\zeta(U-U')\big)$$

$$-2\xi U'\big(Q^2z(\xi\zeta-xz)+\zeta^2\xi U\big)+(S')^2+(U')^2\big(2\zeta^2\xi^2+x^2z^2\big)\Big)$$

$$\times\,\frac{(2\pi)}{(1/\zeta)U'+\xi S+(1/z)T'}\delta\left(x-\frac{(\xi/\zeta)Q^2-(1/z\zeta)S'+(\xi/z)U}{(1/\zeta)U'+\xi S+(1/z)T'}\right)$$

# Transformation Between Lab Frame and Virtual Breit Frame

- Traditionally Breit frame is photon-hadron frame
- Lepton radiation makes frame determination ambiguous
- All historical factorization formula defined in photon hadron frame
- Introduce **virtual** photon-hadron frame which is determined by a given pair of $\xi$,$\zeta$ under one-photon exchange approximation

$$x^{\mu} = (x^{+}, x^{-}, \vec{x}_{\perp} = (x^{1}, x^{2}))$$

$$\tilde{x}^{\sigma} = R^{\sigma}_{\nu} \Lambda^{\nu}_{\mu} x^{\mu}$$

$$\theta_{\text{No Radiation}} = \arctan\left(\frac{-E' \sin\theta_{L,L'}}{E - E'\cos\theta_{L,L'}}\right)$$

$$y \Rightarrow \tilde{y} = y$$

$$\theta_{\text{With Radiation}} = \arctan\left(\frac{-E' \sin\theta_{L,L'}}{(\xi\zeta)E - E'\cos\theta_{L,L'}}\right)$$

$$\begin{pmatrix} x \\ z \end{pmatrix} \Rightarrow \begin{pmatrix} \tilde{x} \\ \tilde{z} \end{pmatrix} = R(\theta(\xi,\zeta)) \begin{pmatrix} x \\ z \end{pmatrix}$$

$$\phi = \tanh^{-1}\left(-\sqrt{\frac{S}{4m^2 + S}}\right)$$

# Full W term

$$\tilde{W}_{TT} = \frac{4e_Q^2}{3}\left(\frac{\alpha_S}{\pi}\right)\Bigg[\delta(1-\lambda)\delta(1-\eta)\left(-\frac{1}{2}\left(\ln^2\frac{\nu_Q^2}{\mu_b^2} - 3\ln\frac{\nu_Q^2}{\mu_b^2}\right)\right)$$
$$-\ln\frac{\mu_{\bar{M}S}}{\mu_b}\left(\frac{1}{\lambda}\left(\frac{\lambda^2+1}{1-\lambda}\right)_+\delta(1-\eta) + \frac{1}{\eta}\left(\frac{\eta^2+1}{1-\eta}\right)_+\delta(1-\lambda)\right)$$
$$+\frac{1}{2}\left(\left(\frac{1-2\lambda}{\lambda}\right)\delta(1-\eta) + \left(\frac{1-2\eta}{\eta}\right)\delta(1-\lambda)\right) - 2\delta(1-\lambda)\delta(1-\eta)$$
$$-\frac{1}{2\epsilon}\left(\frac{1}{\lambda}\left(\frac{\lambda^2+1}{1-\lambda}\right)_+\delta(1-\eta) + \frac{1}{\eta}\left(\frac{\eta^2+1}{1-\eta}\right)_+\delta(1-\lambda) + \delta(1-\lambda)\delta(1-\eta)\right)\Bigg]$$

# Subtraction Method

- Endpoints not well behaved when computing evolution
- Separate out endpoint and use Mellin transformation to calculate those regions

$$\sigma(y) = \int_y^1 \mathrm{d}x f(x) \left( \frac{H\left(\frac{y}{x}\right)}{x} - H(y) \right) + H(y) y \frac{1}{\pi} \mathrm{Im} \left( \int_0^{c+i\infty} \mathrm{d}N y^{-N} \frac{\tilde{f}(N)}{N-1} \right)$$