The light-quark mass dependence of $\pi K$ scattering

David Wilson

JLab theory seminar
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D.J. Wilson, R.A. Briceno, J.J. Dudek, R.G. Edwards, C.E. Thomas
\[ K^- \rightarrow \pi^+ K^- + p^+ n \]
LASS experiment at SLAC $E_K = 11$ GeV

$\pi K$ and $\pi\pi$ scattering - scalars

$|q_s|$ and $\phi_s$ as functions of $M_{K^-\pi^+}$ (GeV/c²)

$|A|^2$ plots for various channels, including $\pi\pi \rightarrow \pi\pi$ and $\pi\pi \rightarrow K\bar{K}$.
\( K^*(892) \)

FIG. 750-3000

LASS experiment at SLAC \( E_K = 11 \text{ GeV} \)

\( \phi_p \)

\( M_{K^-\pi^+} \) (GeV/c²)

\( \rho(770) \)

\( \phi(1020) \)

\( \omega(782) \)

\( q\bar{q} \left( ^3S_1 \right) \)
two-to-two scattering of (coupled) pseudoscalars
- straightforward to extract 3x3 scattering matrices, eg a_0, f_0, D_\pi
- straightforward to extract the first few partial waves, SPD
- straightforward to also consider hadrons with spin

typically heavier-than-physical pion masses
- choice to avoid many-body effects
- useful tool to probe light quark mass dependence
- helpful for near-threshold states

three-body formalism is rapidly maturing
- to allow for lighter pions, higher resonances
- several recent applications with 3 identical particles
- see e.g. Hansen et al arXiv:2009.04931
- recent proposal for non-identical particles:
Scattering in a finite volume?

Infinite volume

- Bound states
- Meson-meson continuum

Finite volume

Momentum is quantised - no continuum

\[ \vec{p} = \frac{2\pi}{L} \hat{n} \]
1-dimensional QM, periodic BC, single particle:

\[ p = \frac{2\pi n}{L} \]
1-dimensional QM, periodic BC, two particles, no interactions

momentum is quantised: 
\[ p_i = \frac{2\pi n_i}{L} \]

two particle energies are discrete: 
\[ E = (p_1^2 + m_1^2)^{\frac{1}{2}} + (p_2^2 + m_2^2)^{\frac{1}{2}} \]
1-dimensional QM, periodic BC, two interacting particles: $V(x_1 - x_2) \neq 0$

\[
\psi(0) = \psi(L), \quad \left. \frac{\partial \psi}{\partial x} \right|_{x=0} = \left. \frac{\partial \psi}{\partial x} \right|_{x=L}
\]

\[
\sin \left( \frac{pL}{2} + \delta(p) \right) = 0
\]

\[
p = \frac{2\pi n}{L} - \frac{2}{L} \delta(p)
\]

Phase shifts via Lüscher’s method:

\[
\tan \delta_1 = \frac{\pi^{3/2} q}{Z_{00}(1; q^2)}
\]

\[
Z_{00}(1; q^2) = \sum_{n \in \mathbb{Z}^3} \frac{1}{|\vec{n}|^2 - q^2}
\]

Lüscher 1986, 1991

generalisation to a 3-dimensional strongly-coupled QFT

→ powerful non-trivial mapping from finite vol spectrum to infinite volume phase
Direct extension of the elastic quantization condition

\[
\text{det} \left[ 1 + i \rho(E) \cdot t(E) \cdot (1 + i M(E, L)) \right] = 0
\]

phase space
infinite volume scattering
\(t\)-matrix
known finite-volume functions

Generalised to moving frames: Gottlieb, Rummukainen 1995
Unequal masses: Prelovsek, Leskovec 2012

Many derivations of the coupled-channel extension, all in agreement:
He, Feng, Liu 2005 - two channel QM, strong coupling
Hansen & Sharpe 2012 - field theory, multiple two-body channels
Briceño & Davoudi 2012 - strongly-coupled Bethe-Salpeter amplitudes
Guo et al 2012 - Hamiltonian & Lippmann-Schwinger

Also derivations in specific channels, or for a specific parameterization of the interactions like NREFT, chiral PT, Finite Volume Hamiltonian, etc.

Briceño 2014 - Generalised to scattering of particles with non-zero spin, and spin-\(\frac{1}{2}\).

developments towards a general 3-body quantization condition are being made
we investigate elastic pi-K scattering using two new lattices generated with $L/a_s=24$ and an existing lattice with $L/a_s=32$.

pion masses: 239, 284, 327 (& 391) MeV

anisotropic action (3.5 finer spacing in time) Wilson-Clover fermions

operators used:

\[ \bar{\psi} \Gamma \bar{D} \cdots \bar{D} \psi \quad \text{local qq-like constructions} \]

\[ \sum_{\bar{p}_1+\bar{p}_2=\bar{p}} C(\bar{p}_1, \bar{p}_2; \bar{p}) \Omega_\pi(\bar{p}_1) \Omega_\pi(\bar{p}_2) \quad \text{two-hadron constructions} \]

uses the eigenvector from the variational method performed in e.g. pion quantum numbers

using *distillation* (Peardon et al 2009)

compute a large correlation matrix & use GEVP to extract energies

many wick contractions, pi-K, eta-K & qq operators:
$m_\pi = 239$ MeV
amplitude selection

elastic scattering only:
ignore levels around and above $\eta K$

$m_\pi = 239$ MeV

$E_{cm}/\text{MeV}$

0.18

[000] $A_1^+$

0.16

1000

900

800

700

600

500

400

300

200

100

0.14

0.12

[000] $T_1^-$

extra level: narrow P-wave resonance?

[100] $A_1$

$\eta K|_{\text{thr.}}$

$\pi\pi K|_{\text{thr.}}$

negative energy shifts: attraction in S-wave?
S-wave amplitude:

\[ t_{\ell=0} = \frac{K(s)}{1 + I(s)K(s)}, \quad K(s) = \gamma_0 + \gamma_1 s \]

\[ I(s) = I(s_0) - \frac{s - s_0}{\pi} \int_{s_0}^{\infty} ds' \frac{\rho(s')}{(s' - s)(s' - s_0)}, \quad \rho = 2k_{cm}/\sqrt{s} \]

"Chew-Mandelstam” phase space
- once-subtracted dispersion relation
- produces real log from the imaginary part due to unitarity
- respects s-channel unitarity
- ignores left-hand cuts
- can produce subthreshold ("Adler"-like) zeros
- some forms of K(s) can produce physical sheet poles
  which are foribben by analyticity - we check for these

on the real axis in the physical scattering region,
this is a lot like the familiar effective range parameterisation:

\[ k_{cm} \cot \delta_0 = \frac{1}{a} + \frac{1}{2} r k_{cm}^2 \]
P-wave amplitude:

relativistic Breit-Wigner, coupling $g_R$, mass $m_R$

$$t^{(\ell=1)}(s) = \frac{1}{\rho(s)} \frac{\sqrt{s} \Gamma_{\ell=1}(s)}{m_R^2 - s - i\sqrt{s} \Gamma_{\ell=1}(s)},$$

$$\Gamma_{\ell=1}(s) = \frac{g_R^2}{6\pi} \frac{k_{cm}^3}{s}$$

can describe narrow and broad resonance poles

minimise $g_R, m_R$ along with S-wave parameters until spectra are well-described
spectrum fitting

\[ m_\pi = 239 \text{ MeV} \]
$m_\pi = 239 \text{ MeV}$
spectrum fitting

\[ m_\pi = 239 \text{ MeV} \]
$m_{\pi} = 239$ MeV
4 light quark masses

\[ m_\pi \sim 239, 284, 327 \text{ & } 391 \text{ MeV} \]
this analysis: $m_\pi \sim 239, 284 & 327$ MeV
scattering-amplitude poles $\leftrightarrow$ spectroscopic content

**P-wave** is easy:

\[ m = \text{Re}\sqrt{s_0}/\text{MeV} \]

\[ t \sim \frac{c^2}{s_0 - s} \]

-20 -10 0 10 20

\[ \sqrt{s_0} = 934(2) \text{ MeV} \]
\[ \sqrt{s_0} = (914(2) - \frac{i}{2}6(1)) \text{ MeV} \]
\[ \sqrt{s_0} = (909(4) - \frac{i}{2}13(2)) \text{ MeV} \]
\[ \sqrt{s_0} = (902(2) - \frac{i}{2}23(2)) \text{ MeV} \]

\[ \sqrt{s_0} = 893(1) - \frac{i}{2}56(2) \]

expt. + UFD [50]
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\[ \text{expt. + UFD [50]} \]
scattering-amplitude poles ↔ spectroscopic content

**S-wave is inconclusive**
- many amplitudes have poles, but not all
- broad spread in position possible at all 4 masses
- uncertainties from different parameterisations often don't overlap
- some amplitudes have no nearby poles: cannot claim a robust determination

\[ t \sim \frac{c^2}{s_0 - s} \]
unequal masses: $m_1=1$, $m_2=2$

$$I(s) = -i\rho(s)$$
$$= -\frac{i}{s} \left(s - (m_1 + m_2)^2\right)^{\frac{1}{2}} \left(s - (m_2 - m_1)^2\right)^{\frac{1}{2}}$$

"Chew-Mandelstam" phase space

$$I(s) = I(s_0) - \frac{s - s_0}{\pi} \int_{s_0}^{\infty} ds' \frac{\rho(s')}{(s' - s)(s' - s_0)}$$

$$\text{Im}I(s) = -\rho(s)$$
a zero in $t$ below $\pi$-$K$ threshold

often argued as an essential feature
arises in chiral perturbation theory, at leading order:

$$s_A = \frac{1}{5} \left( m_K^2 + m_\pi^2 \pm 2 \left( 4m_\pi^4 - 7m_K^2m_\pi^2 + 4m_K^4 \right)^{\frac{1}{2}} \right)$$

$\sim (480 \text{ MeV})^2$ at all mass points considered

SU(3) ChiPT, $I=1/2$, S-wave projection

strong correlation with extracted pole position
zero in $t$ below threshold  
⇒ pole in $k \cot \delta$ at negative $k^2$  

no clear sign from lattice spectra
cross-channel scattering

\[
\begin{align*}
\pi(p_a) & \quad \pi(p_c) \\
K(p_b) & \quad K(p_d)
\end{align*}
\]
cross-channel scattering

\[ u = (m^2 - m_K^2)^{\frac{1}{2}} \]

\[ s = (m^2 + m_K^2)^{\frac{1}{2}} \]

\[ t = 4m_K^2 \]
cross-channel scattering

\[ m_K^2 - m_\pi^2 \]

(s-channel)

\[ (m_K - m_\pi)^2 \]

(t-channel)

\[ (m_\pi + m_K)^2 \]

(u-channel)

\[ (3m_\pi + m_K)^2 \]

\[ (m_\pi + m_K)^2 \]

s-channel

(after s-channel partial-wave projection)
hard to argue for zero at $s_A$ without including left cuts
cross-channel scattering

$m_\pi = 239\text{ MeV}$
$m_\pi = 284\text{ MeV}$
$m_\pi = 327\text{ MeV}$
$m_\pi = 391\text{ MeV}$

hard to argue for zero at $s_A$ without including left cuts
N/D-like approach

<table>
<thead>
<tr>
<th>$\kappa/K^*_0(700)$</th>
<th>$\sqrt{s_p}$, MeV</th>
<th>$g_{pa}/\sqrt{N_{aa}}$, GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp. SC</td>
<td>701(12) – 287(17)</td>
<td>$\pi K : 4.16(14)$</td>
</tr>
<tr>
<td>Lattice, $m_\pi = 239$ MeV</td>
<td>749(38) – 265(16)</td>
<td>$\pi K : 4.18(18)$</td>
</tr>
</tbody>
</table>
σ poles

\[ \frac{1}{2} \Gamma_{\sigma} / \text{MeV} \]

\[ |g_{\sigma\pi}| / \text{MeV} \]

E\_\sigma / MeV

\[ m_\pi = 391 \text{ MeV} \]

\[ m_\pi = 236 \text{ MeV} \]

\[ m_\pi / \text{MeV} \]
**DK**

\[ m_\pi = 239 \text{ MeV} \]

\[ m_\pi = 391 \text{ MeV} \]

\[ |\rho t|^2 \]

\[ a_t E_{cm} \]

\[ \text{DK}^{\text{expt.}} |_{m_\pi=239 \text{ MeV}} \]

\[ \text{DK}^{\text{thr.}} |_{m_\pi=239 \text{ MeV}} \]

\[ \text{DK}^{\text{thr.}} |_{m_\pi=391 \text{ MeV}} \]

+D-pi coming soon…
Lattice QCD provides a first-principles tool to do hadron spectroscopy

Many S-wave resonances remain puzzling, in cases like the $\kappa(700)$, left cut physics is important

Lots of other stuff - see hadspec.org

These methods are widely applicable,
- coupled-channel scattering
- baryons
- charm quarks
- form factors
...

Control of 3+ body effects needed for
- lighter pion masses
- higher resonances