The light-quark mass dependence of πK scattering

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hadspec
hadspec.org

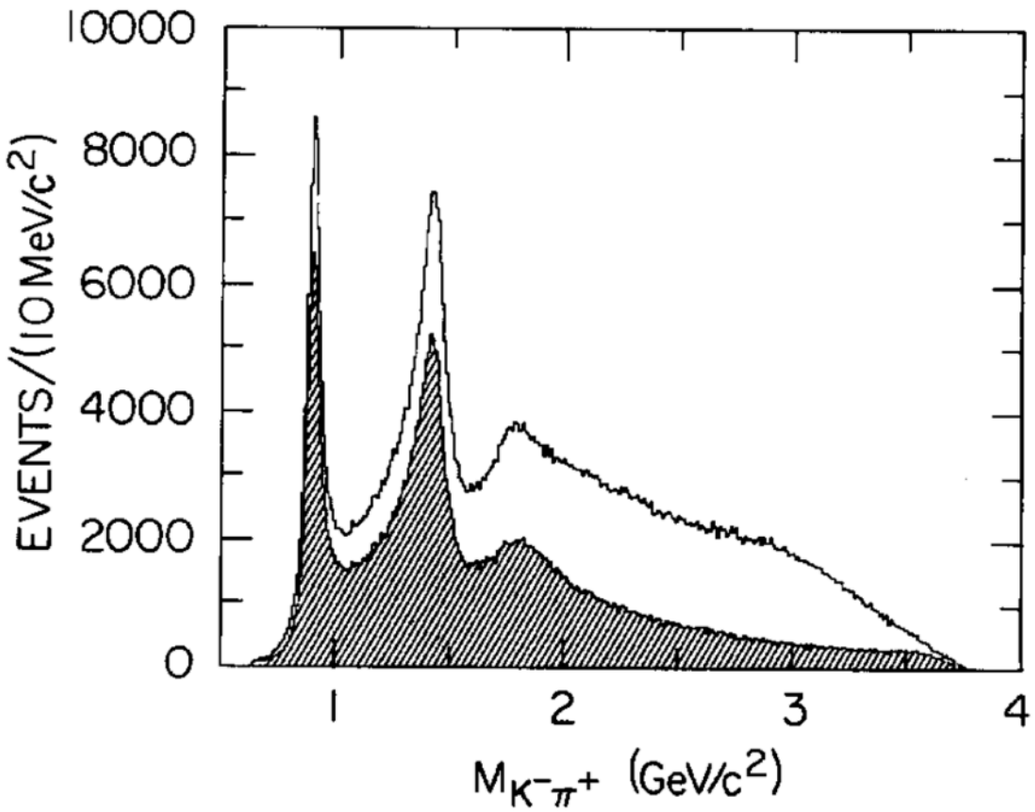
JLab theory seminar
Monday 1st February 2021

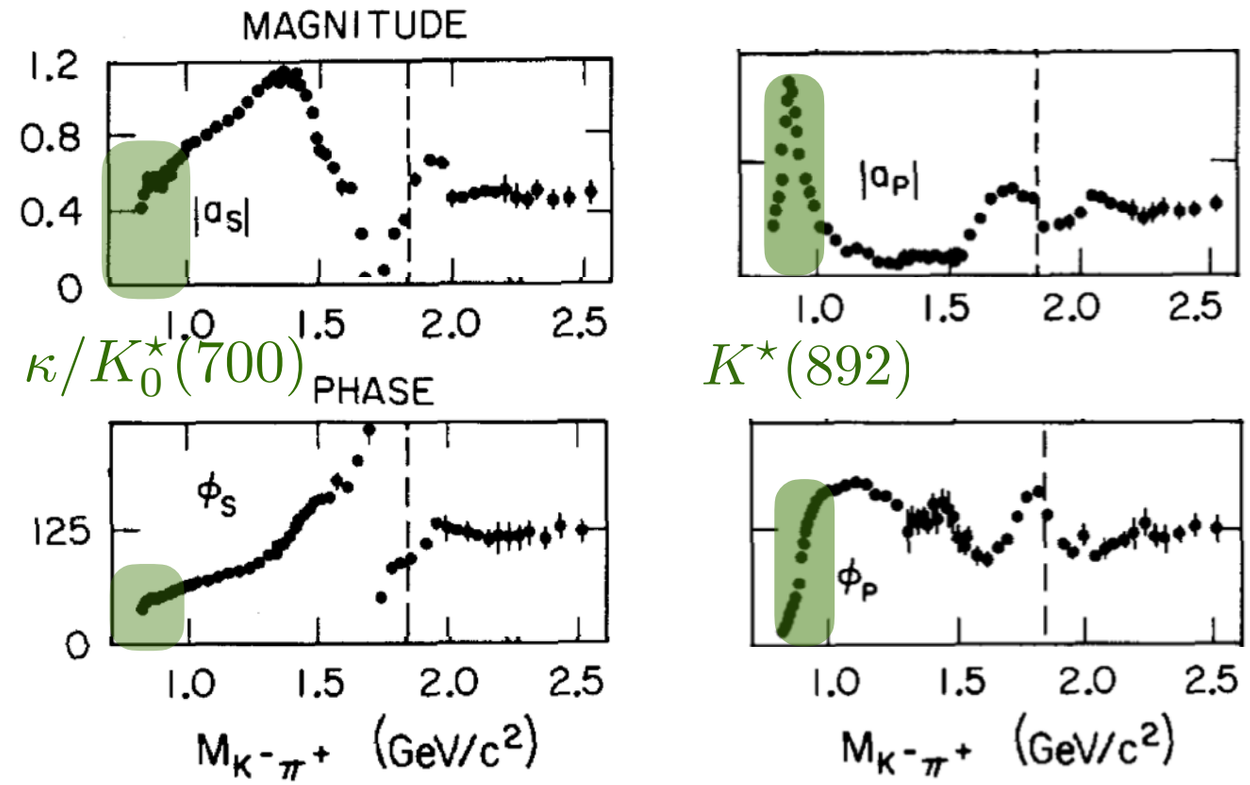
based on: Phys. Rev. Lett. 123 (2019) 4, 042002 (arXiv: 1904.03188)
D.J. Wilson, R.A. Briceno, J.J. Dudek, R.G. Edwards, C.E. Thomas

UNIVERSITY OF CAMBRIDGE

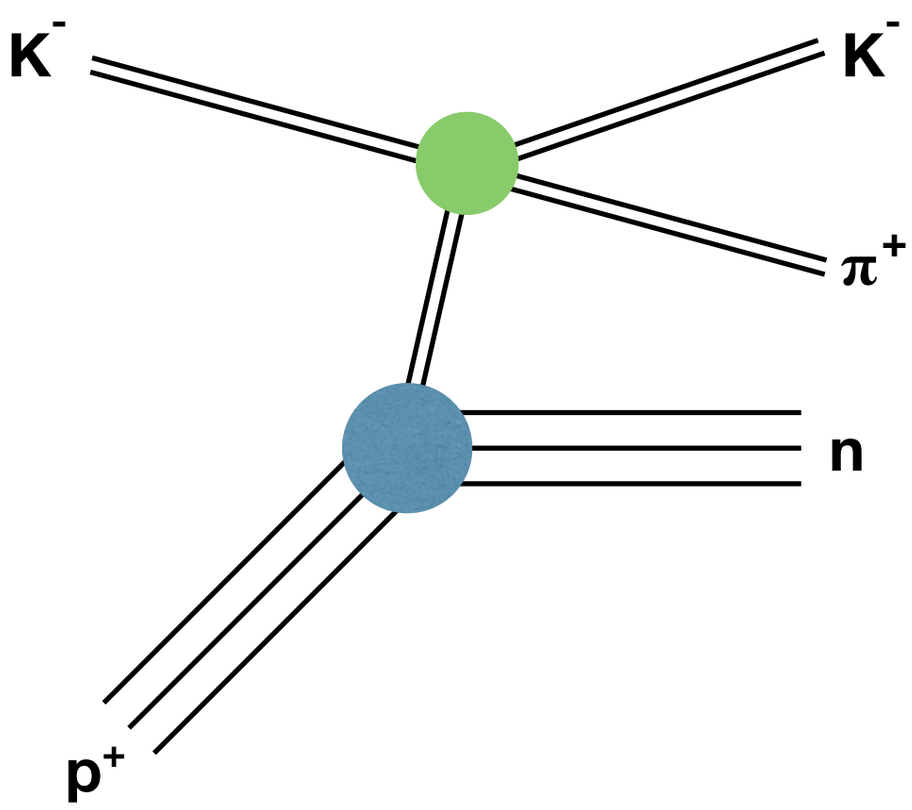
THE ROYAL SOCIETY

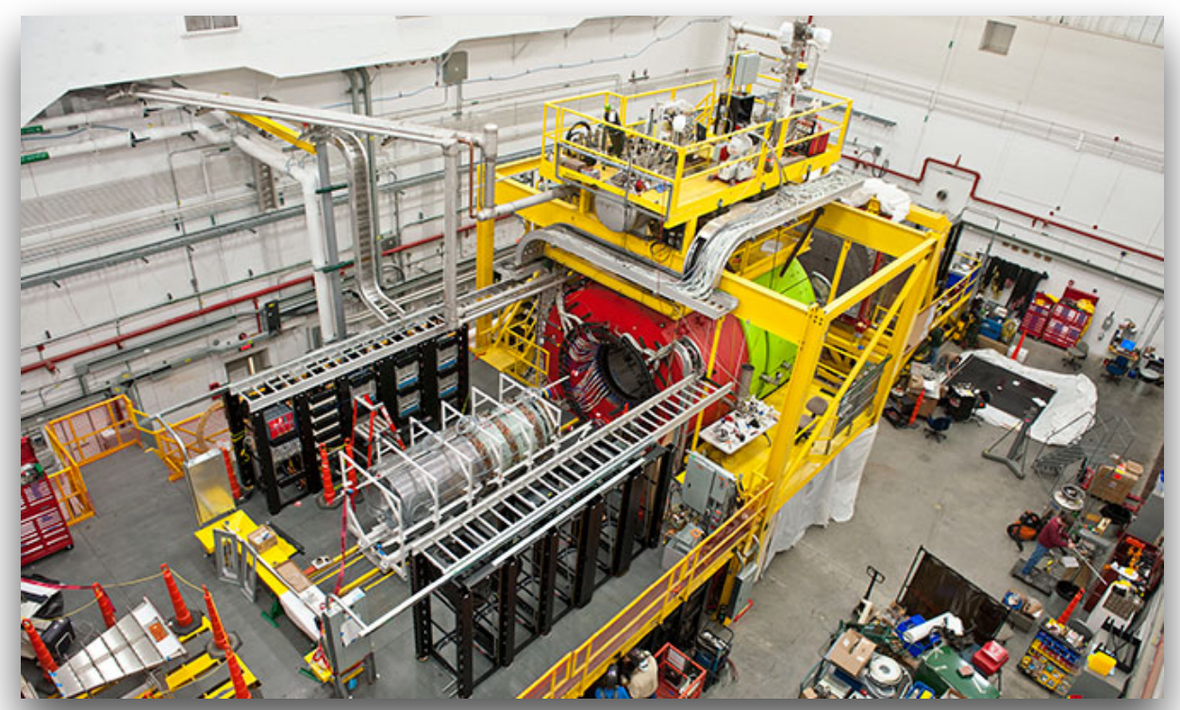
πK scattering in experiment





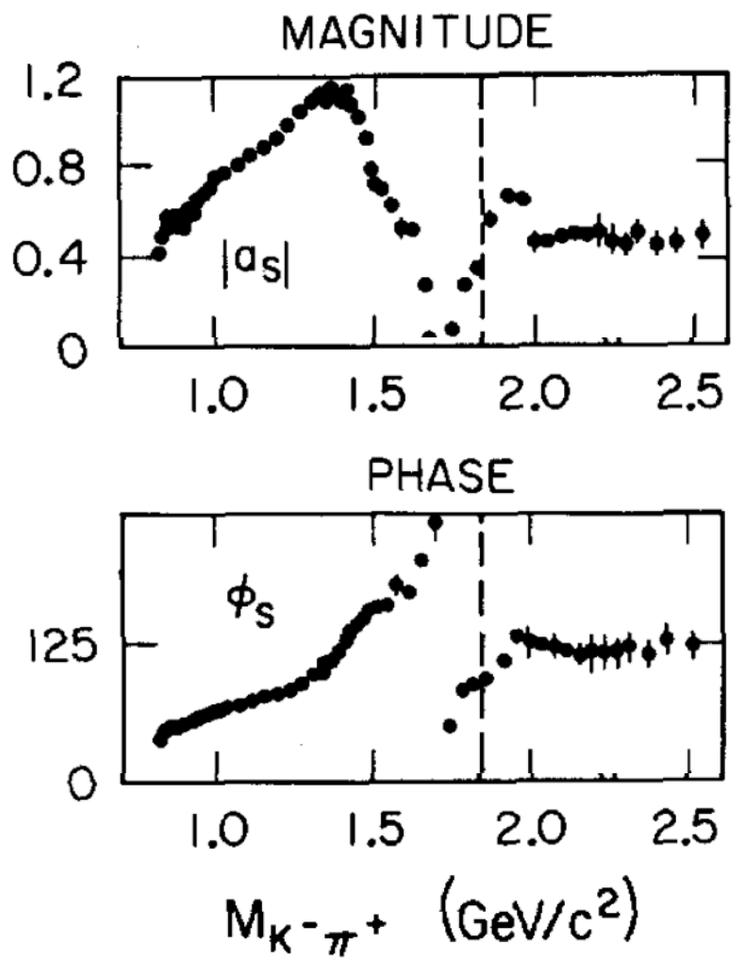
LASS experiment at SLAC $E_K = 11$ GeV



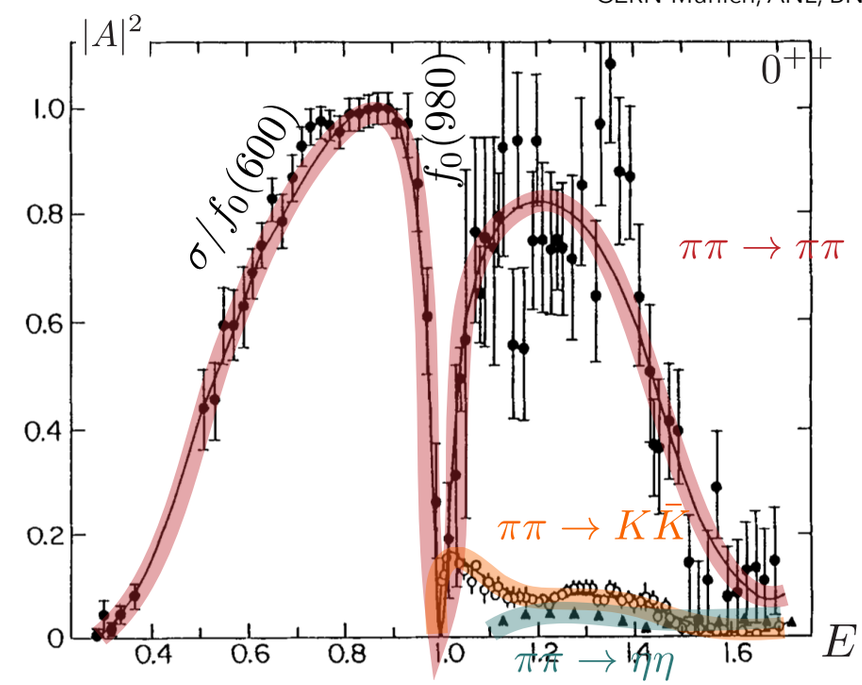


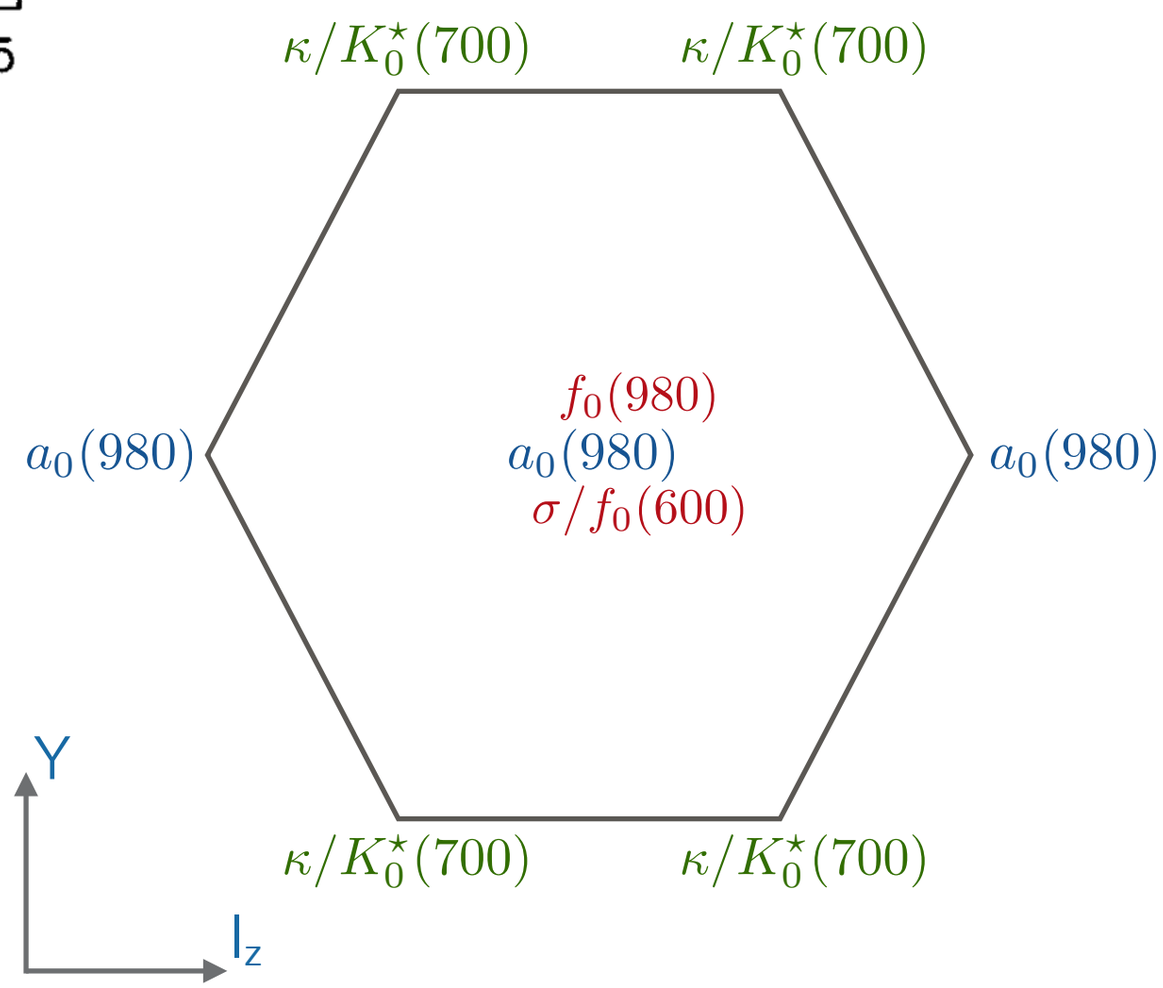
πK and ππ scattering - scalars

LASS experiment at SLAC $E_K = 11$ GeV

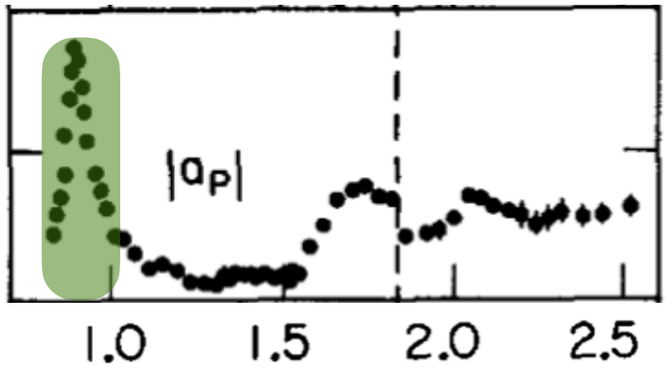


CERN-Munich, ANL, BNL

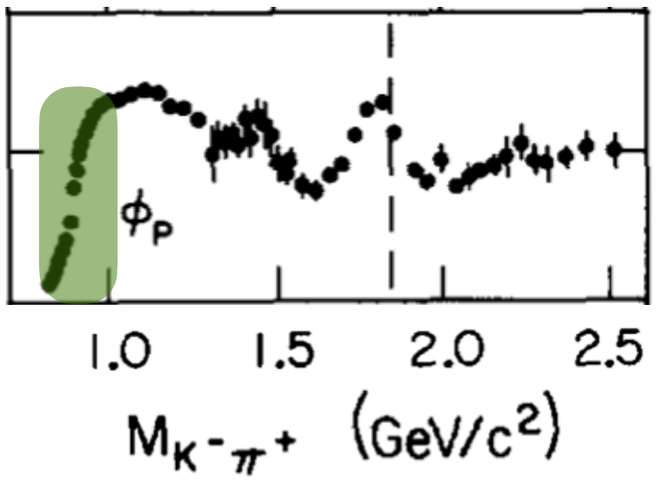




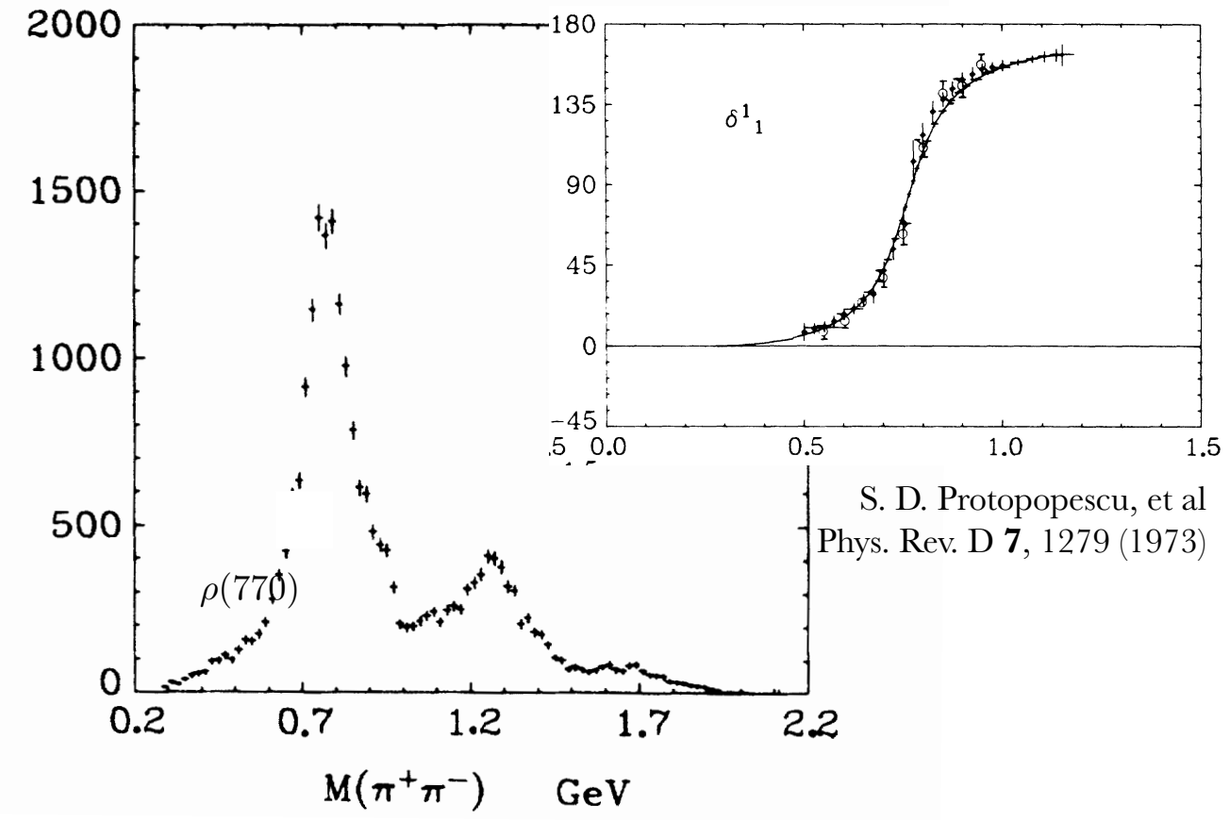
πK and ππ scattering - vectors



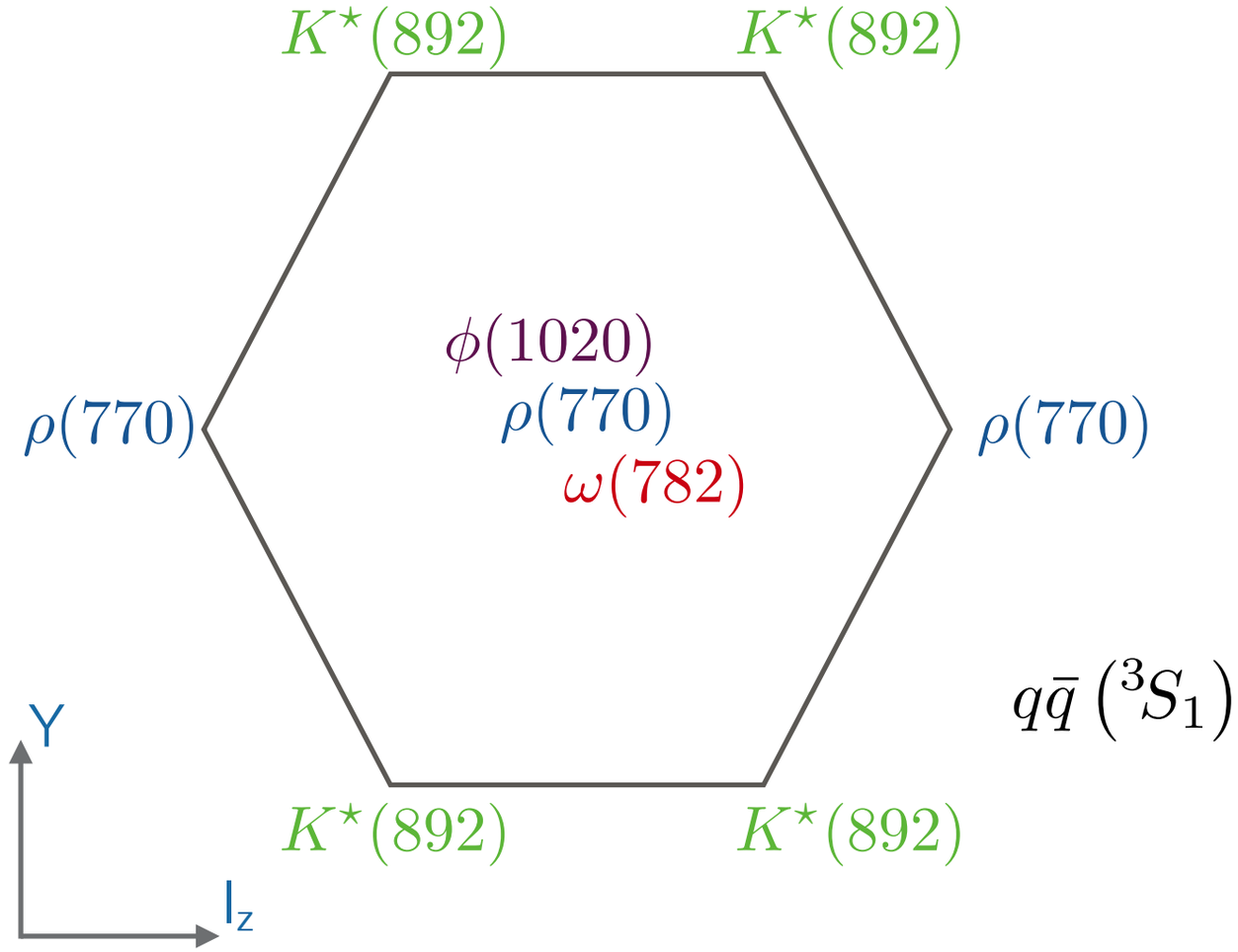
$K^\star(892)$



LASS experiment at SLAC $E_K = 11$ GeV



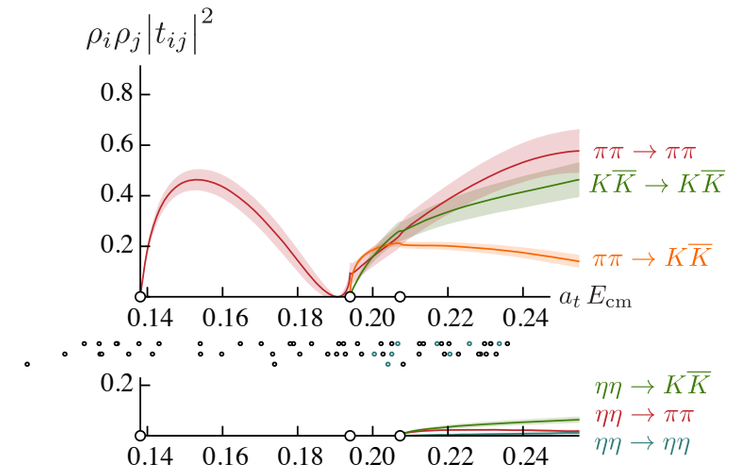
S. D. Protopopescu, et al
Phys. Rev. D **7**, 1279 (1973)



$q\bar{q}\,(^3S_1)$

two-to-two scattering of (coupled) pseudoscalars

- straightforward to extract 3x3 scattering matrices, eg a_0, f_0, $D\pi$
- straightforward to extract the first few partial waves, *SPD*
- straightforward to also consider hadrons with spin

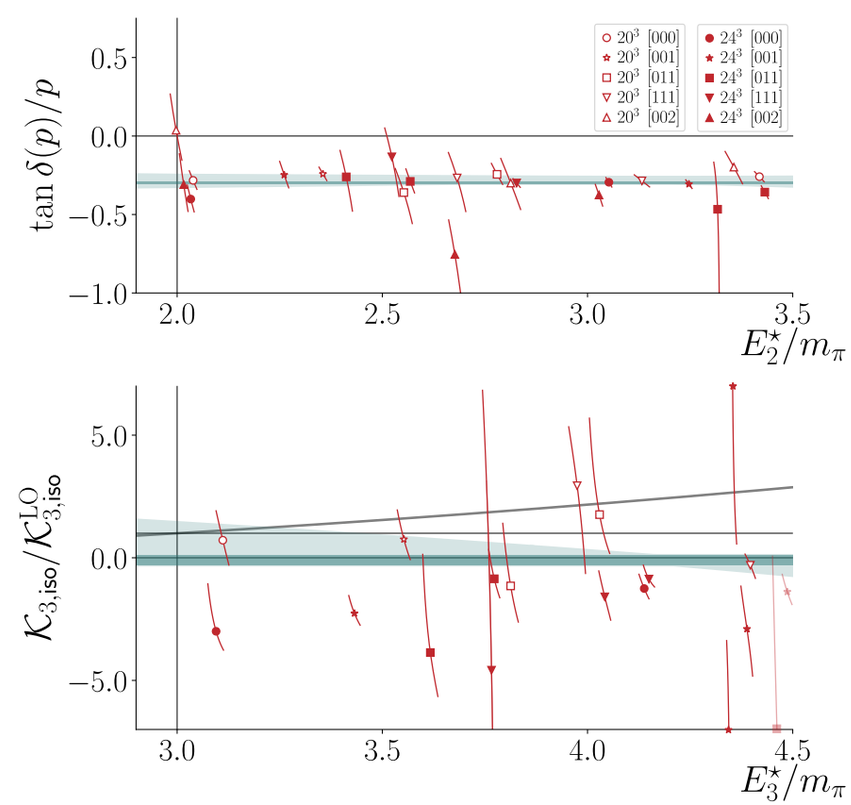


typically heavier-than-physical pion masses

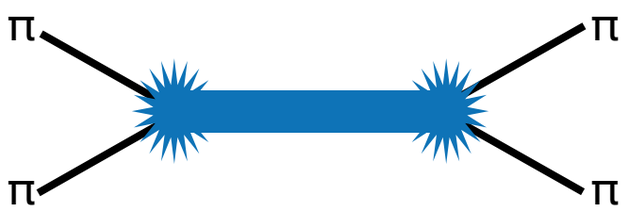
- choice to avoid many-body effects
- useful tool to probe light quark mass dependence
- helpful for near-threshold states

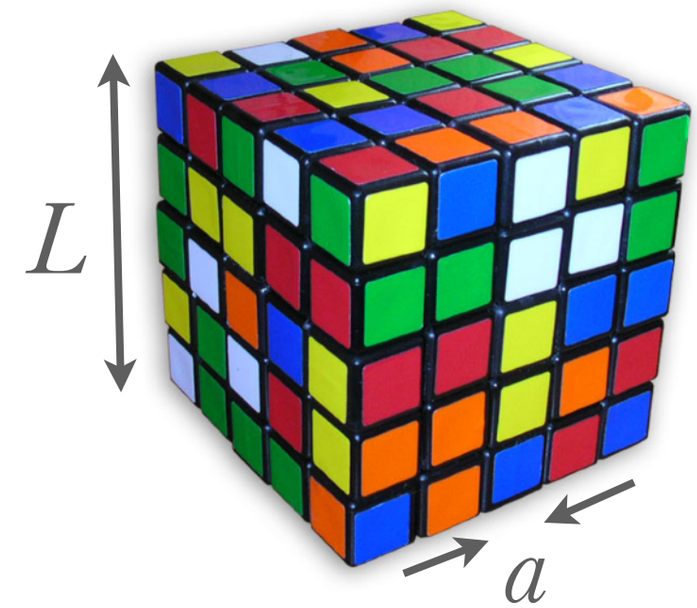
three-body formalism is rapidly maturing

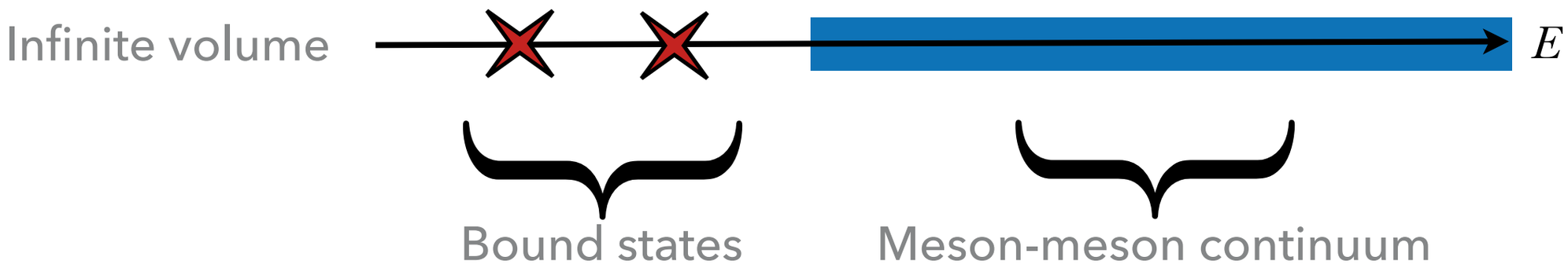
- to allow for lighter pions, higher resonances
- several recent applications with 3 identical particles
- see e.g. Hansen et al arXiv:2009.04931
- recent proposal for non-identical particles: Blanton & Sharpe, arXiv:2011.05520



Scattering in a finite volume?

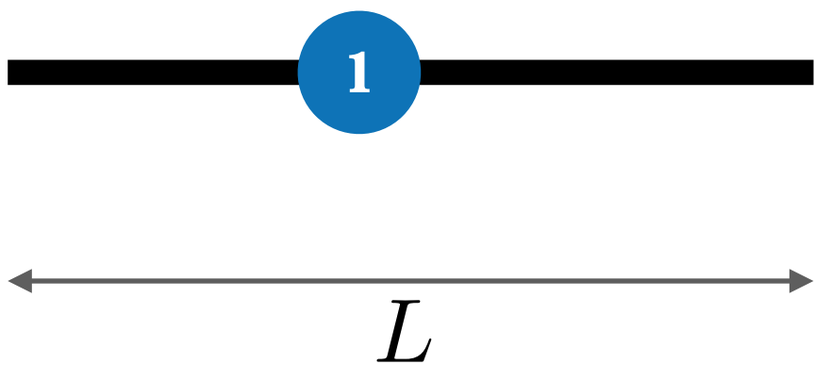


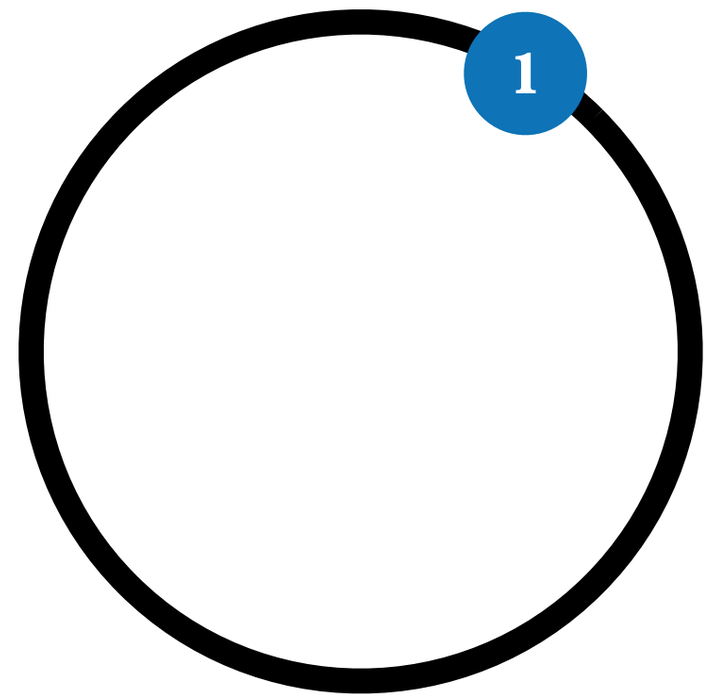




Momentum is quantised - no continuum $\vec{p} = \frac{2\pi}{L}\vec{n}$

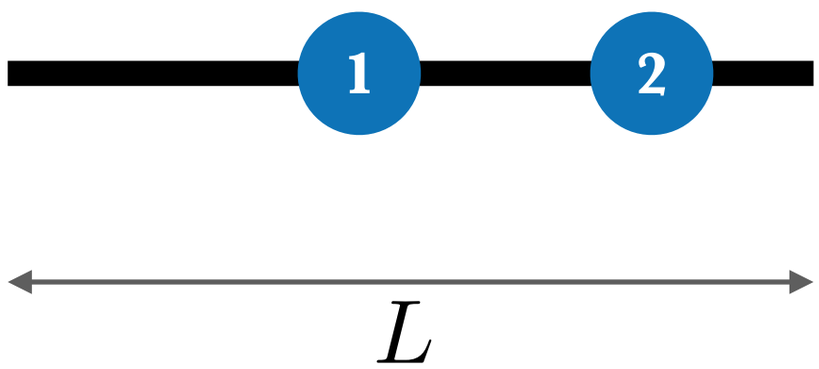
1-dimensional QM, periodic BC, single particle:

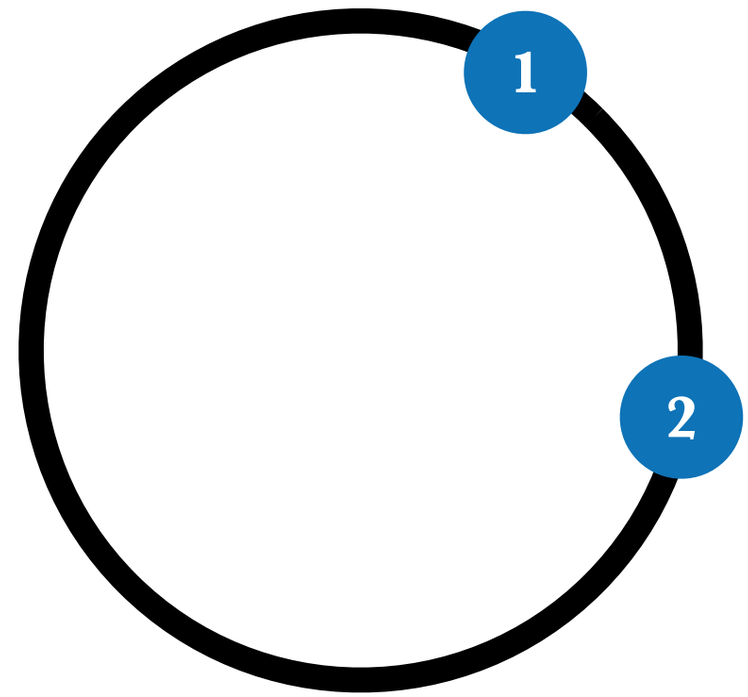




momentum is quantised: $p = \frac{2\pi n}{L}$

1-dimensional QM, periodic BC, two particles, no interactions





momentum is quantised: $p_i = \dfrac{2\pi n_i}{L}$

two particle energies are discrete:

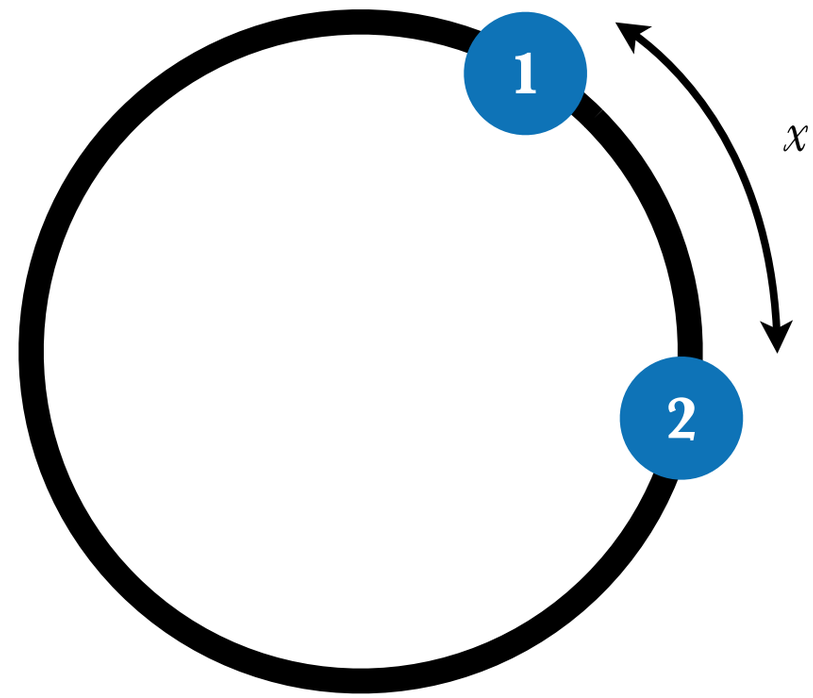
$$E = (p_1^2 + m_1^2)^{\frac{1}{2}} + (p_2^2 + m_2^2)^{\frac{1}{2}}$$

1-dimensional QM, periodic BC, two interacting particles: $V(x_1 - x_2) \neq 0$

$$\psi(0) = \psi(L), \quad \left.\frac{\partial\psi}{\partial x}\right|_{x=0} = \left.\frac{\partial\psi}{\partial x}\right|_{x=L}$$

$$\sin\left(\frac{pL}{2} + \delta(p)\right) = 0$$

$$p = \frac{2\pi n}{L} - \frac{2}{L}\delta(p)$$



Phase shifts via Lüscher's method:

$$\tan\delta_1 = \frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1; q^2)}$$

$$\mathcal{Z}_{00}(1; q^2) = \sum_{n \in \mathbb{Z}^3} \frac{1}{|\vec{n}|^2 - q^2}$$

Lüscher 1986, 1991

generalisation to a 3-dimensional strongly-coupled QFT

→ powerful non-trivial mapping from finite vol spectrum to infinite volume phase

Direct extension of the elastic quantization condition

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot\boldsymbol{t}(E)\cdot\left(\mathbf{1} + i\boldsymbol{\mathcal{M}}(E,L)\right)\right] = 0$$

phase space

infinite volume scattering *t*-matrix

known finite-volume functions

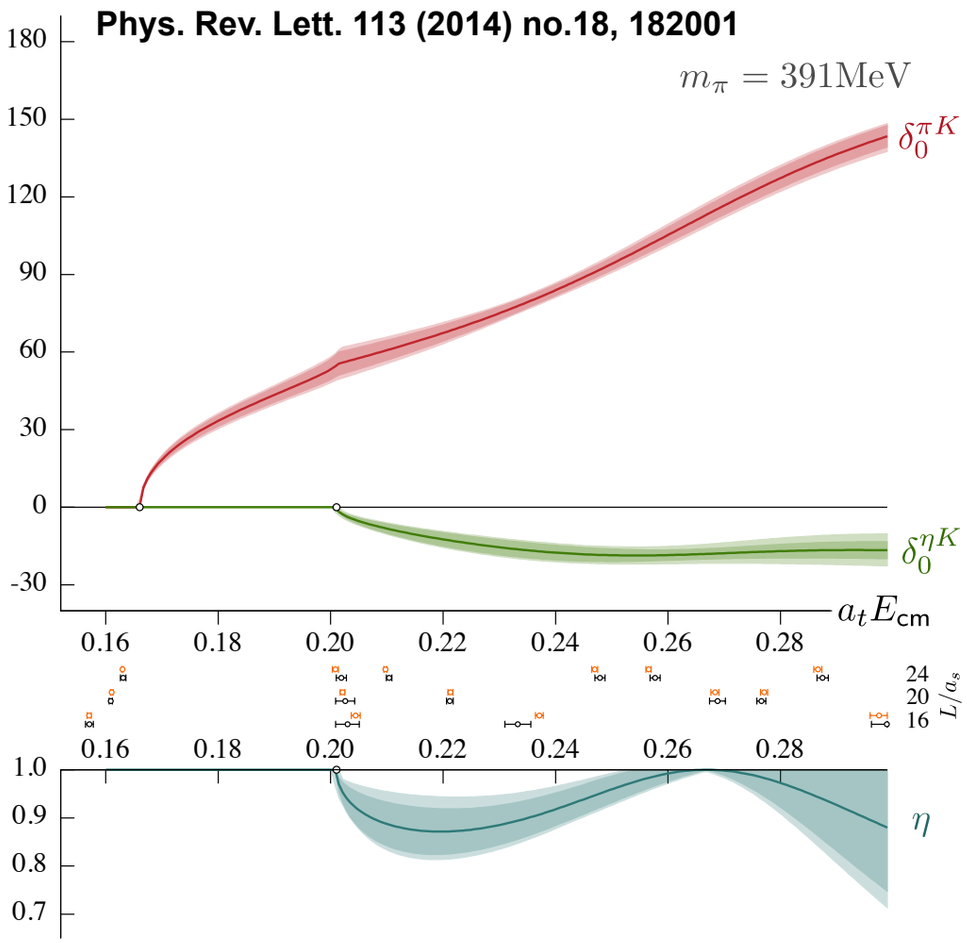
Elastic scattering: Lüscher 1986,1991
Generalised to moving frames: Gottlieb, Rummukainen 1995
Unequal masses: Prelovsek, Leskovec 2012

Many derivations of the coupled-channel extension, **all in agreement**:
He, Feng, Liu 2005 - two channel QM, strong coupling
Hansen & Sharpe 2012 - field theory, multiple two-body channels
Briceño & Davoudi 2012 - strongly-coupled Bethe-Salpeter amplitudes
Guo et al 2012 - Hamiltonian & Lippmann-Schwinger

Also derivations in specific channels, or for a specific parameterization of the interactions like NREFT, chiral PT, Finite Volume Hamiltonian, etc.

Briceño 2014 - Generalised to scattering of particles with non-zero spin, and spin-½.

developments towards a general 3-body quantization condition are being made



we investigate elastic pi-K scattering using two new lattices generated with L/a_s=24 and an existing lattice with L/a_s=32.

pion masses: **239, 284, 327** (& 391) MeV

anisotropic action (3.5 finer spacing in time)
Wilson-Clover fermions

operators used:

$\bar{\psi}\Gamma \overleftrightarrow{D} \ldots \overleftrightarrow{D}\ \psi$ local qq-like constructions

$$\sum_{\vec{p}_1+\vec{p}_2\in\vec{p}} C(\vec{p}_1,\vec{p}_2;\vec{p})\Omega_\pi(\vec{p}_1)\ \Omega_\pi(\vec{p}_2)$$

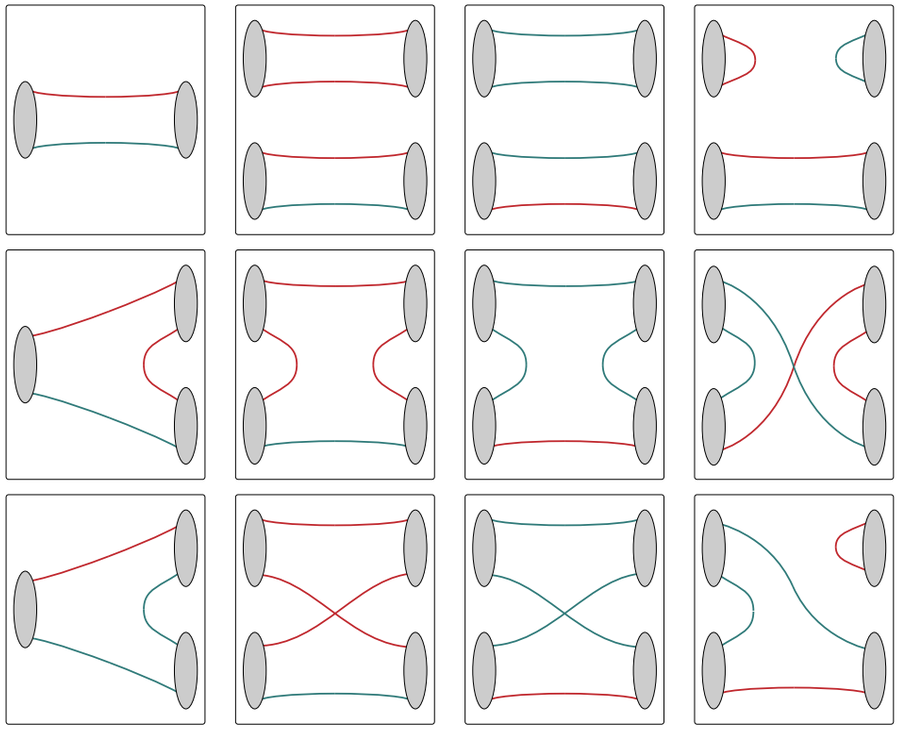
two-hadron constructions

$$\Omega_\pi^\dagger = \sum_i v_i \mathcal{O}_i^\dagger$$

uses the eigenvector from the variational method performed in e.g. pion quantum numbers

using *distillation* (Peardon *et al* 2009)
compute a large correlation matrix &
use GEVP to extract energies

many wick contractions, pi-K, eta-K & qq operators:



+ …

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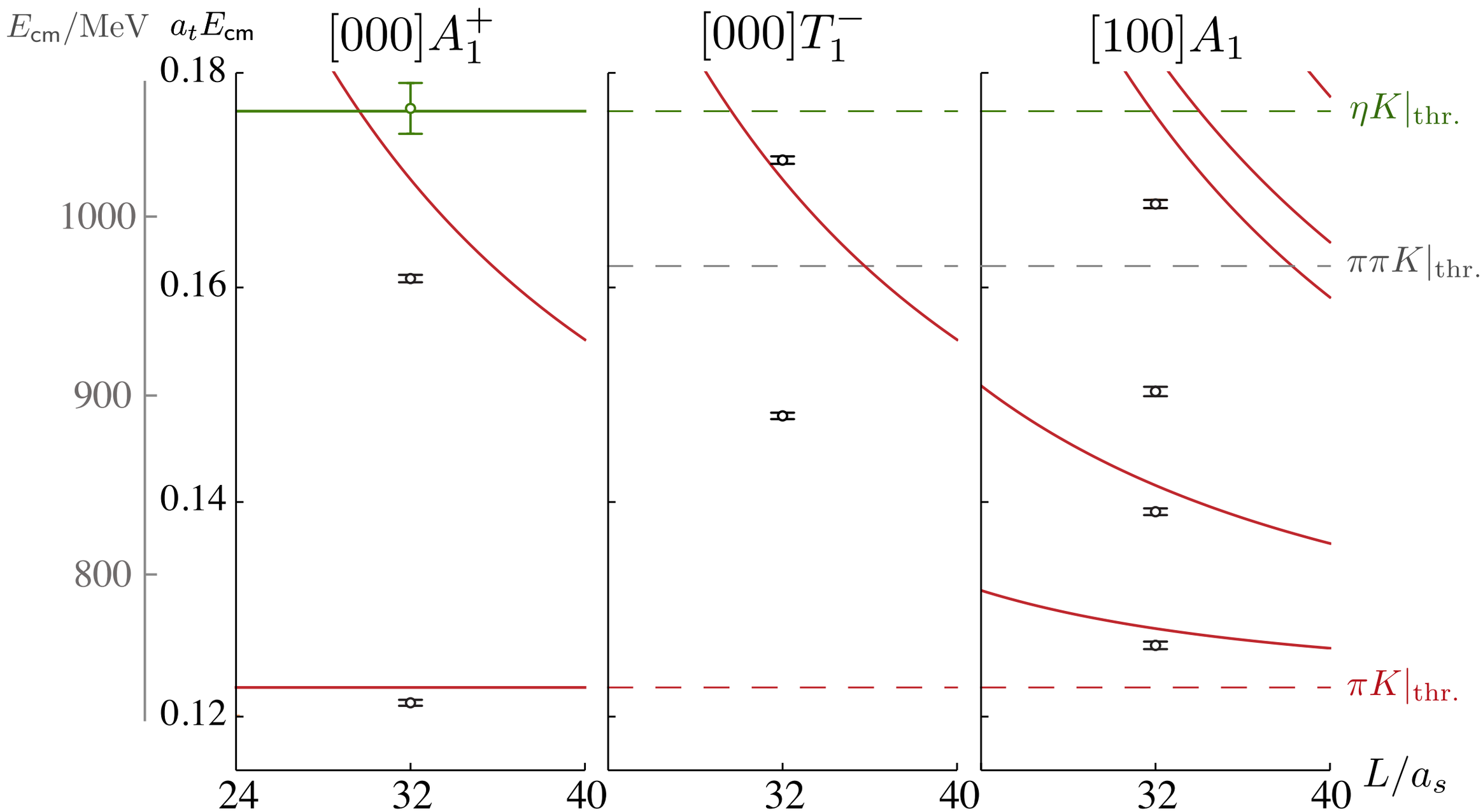
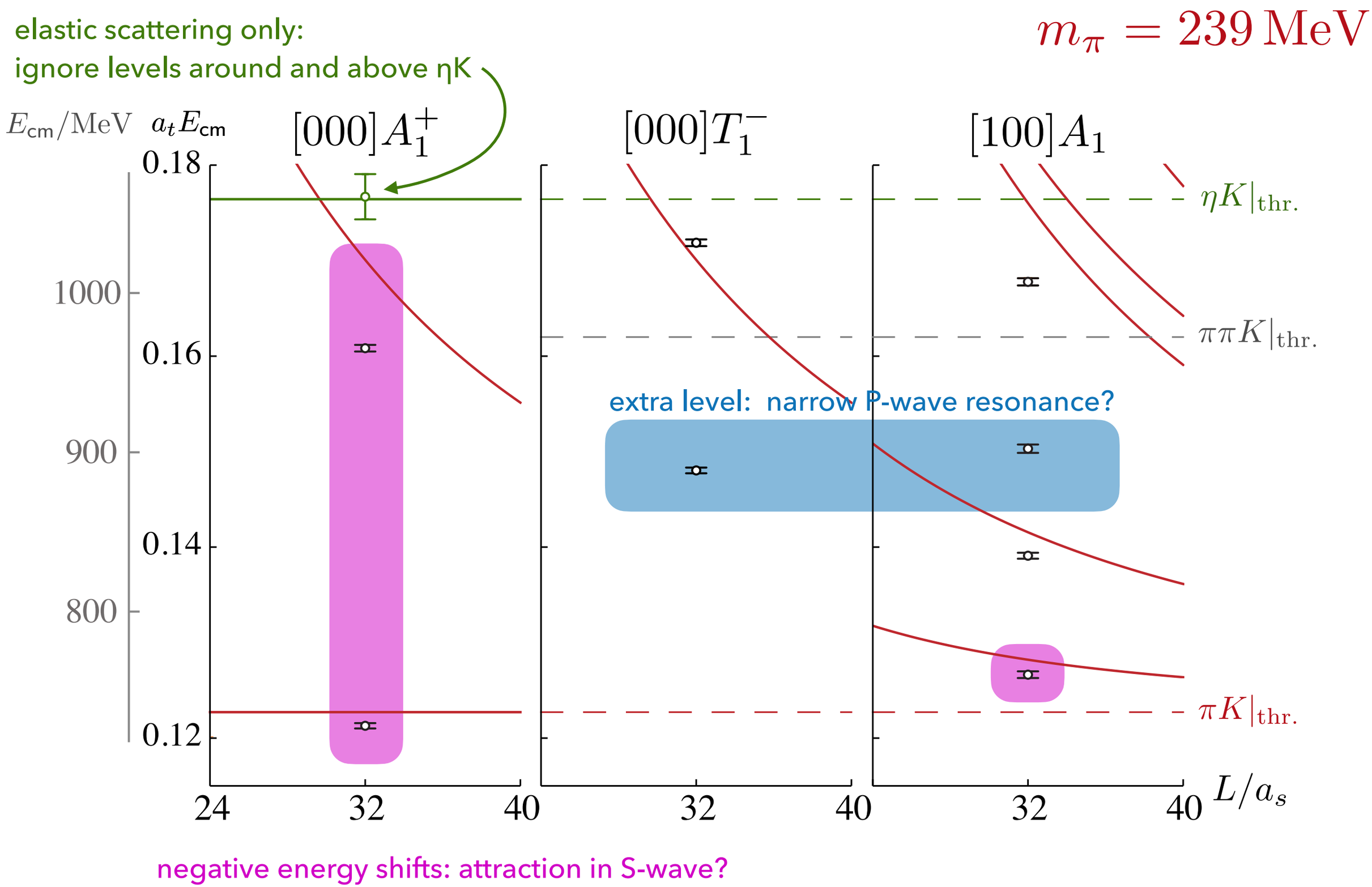
$m_\pi = 239\,\mathrm{MeV}$




elastic scattering only:
ignore levels around and above ηK
$m_\pi = 239\,\mathrm{MeV}$
$E_{\mathrm{cm}}/\mathrm{MeV}$
$a_t E_{\mathrm{cm}}$
$[000]A_1^+$
$[000]T_1^-$
$[100]A_1$
$\eta K|_{\mathrm{thr.}}$
$\pi\pi K|_{\mathrm{thr.}}$
$\pi K|_{\mathrm{thr.}}$
extra level: narrow P-wave resonance?
L/a_s
negative energy shifts: attraction in S-wave?

S-wave amplitude:

$$t^{\ell=0} = \frac{K(s)}{1 + I(s)K(s)}, \quad K(s) = \gamma_0 + \gamma_1 s$$

$$I(s) = I(s_0) - \frac{s - s_0}{\pi} \int_{s_0}^{\infty} ds' \, \frac{\rho(s')}{(s' - s)(s' - s_0)}, \quad \rho = 2k_{\text{cm}}/\sqrt{s}$$

"Chew-Mandelstam" phase space
- once-subtracted dispersion relation
- produces real log from the imaginary part due to unitarity

- respects s-channel unitarity
- ignores left-hand cuts
- can produce subthreshold ("Adler"-like) zeros
- some forms of K(s) can produce physical sheet poles

which are foribben by analyticity - we check for these

on the real axis in the physical scattering region,
this is a lot like the familiar effective range parameterisation: $k_{\text{cm}} \cot \delta_0 = \frac{1}{a} + \frac{1}{2} r k_{\text{cm}}^2$

P-wave amplitude:

relativistic Breit-Wigner, coupling g_R, mass m_R

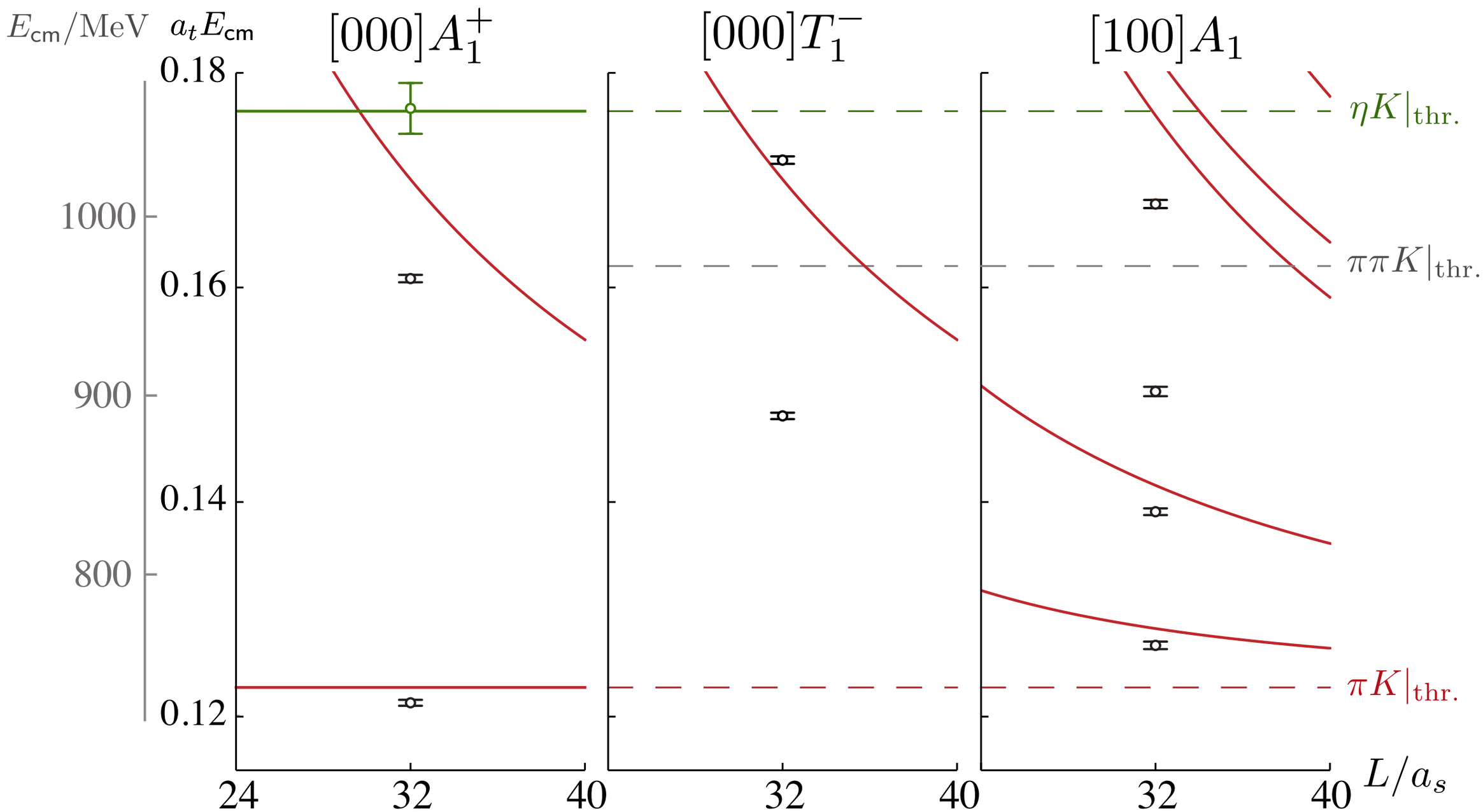
$$t^{(\ell=1)}(s) = \frac{1}{\rho(s)} \frac{\sqrt{s}\,\Gamma_{\ell=1}(s)}{m_R^2 - s - i\sqrt{s}\,\Gamma_{\ell=1}(s)},$$

$$\Gamma_{\ell=1}(s) = \frac{g_R^2}{6\pi} \frac{k_{\text{cm}}^3}{s}$$

can describe narrow and broad resonance poles

minimise g_R, m_R along with S-wave parameters until spectra are well-described

$m_\pi = 239\,\mathrm{MeV}$



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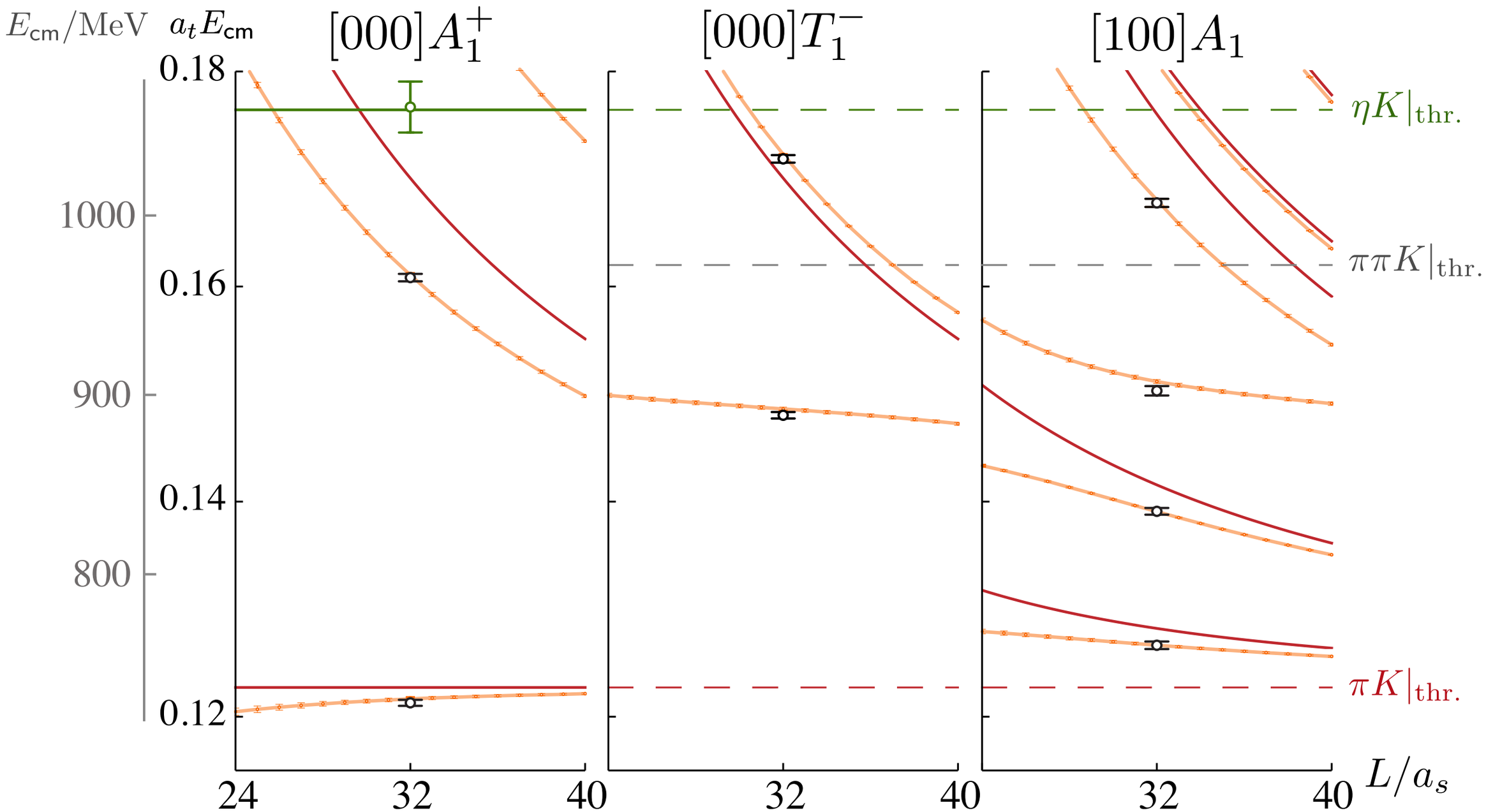

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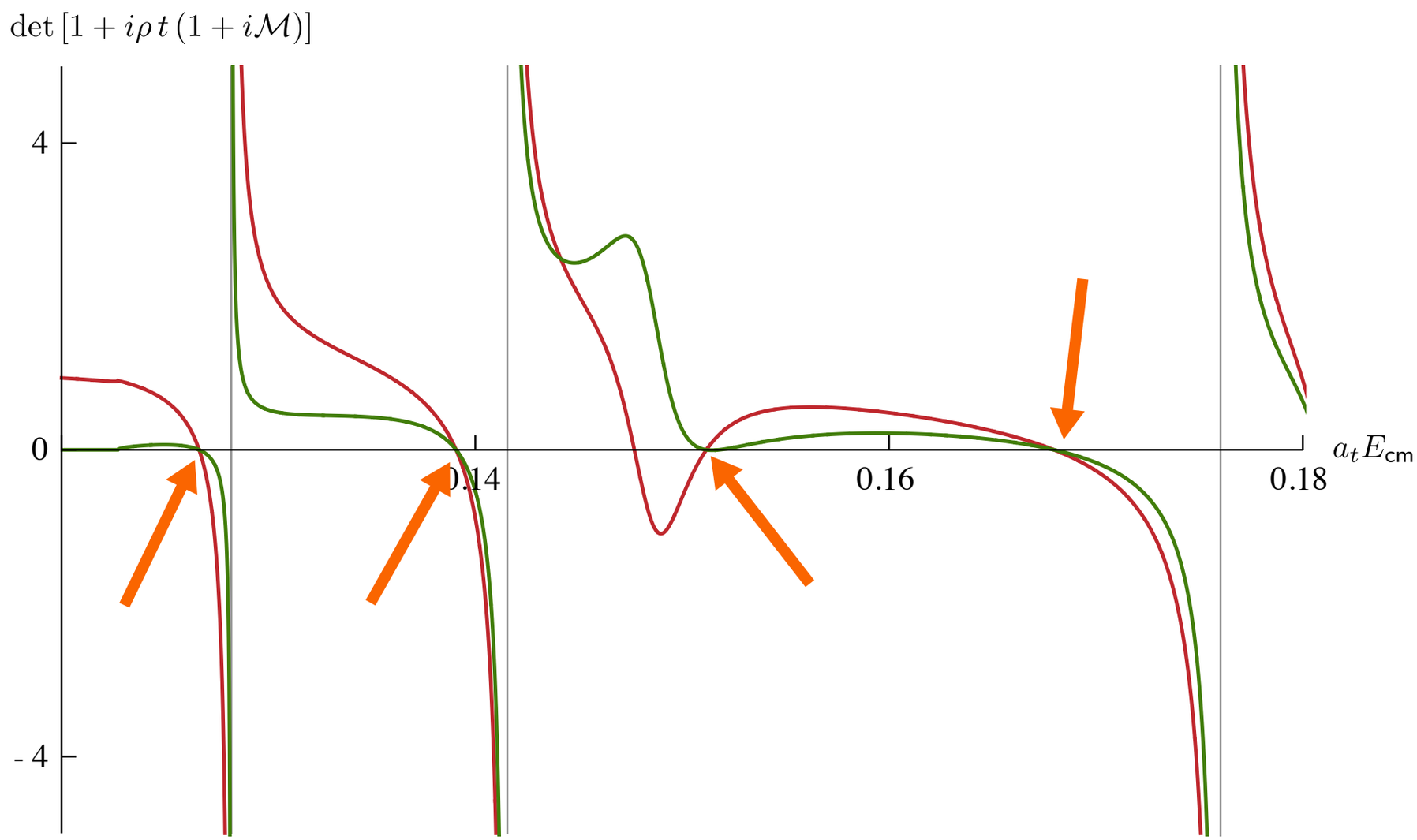

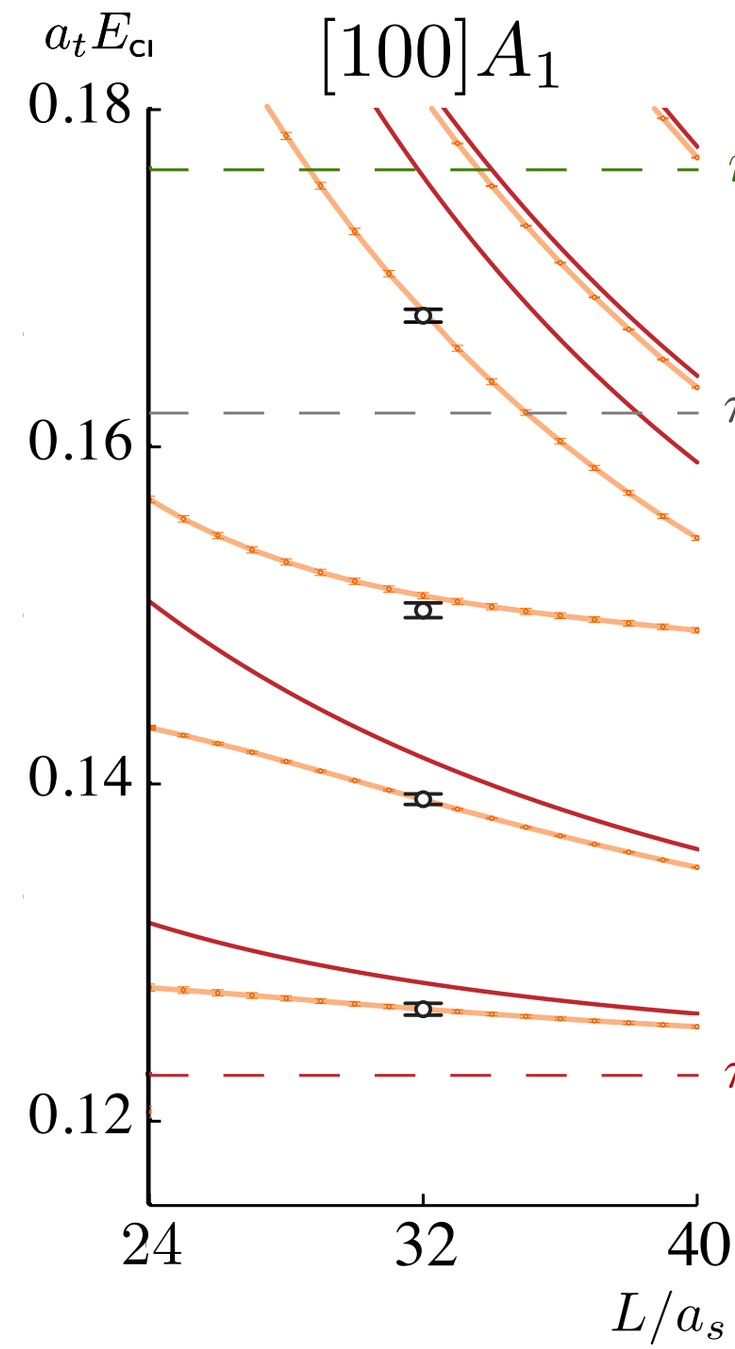

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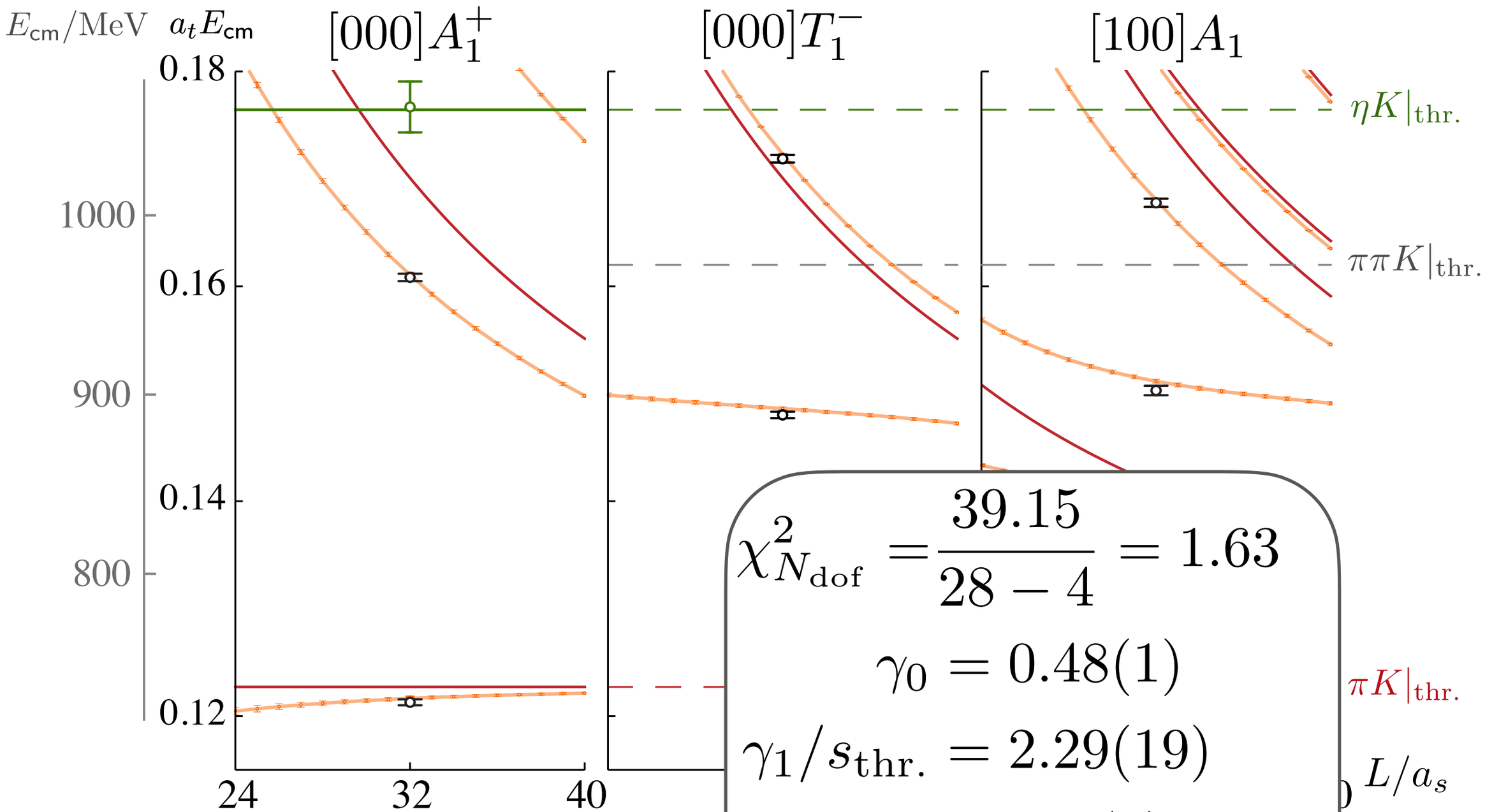


$$\chi^2_{N_{\mathrm{dof}}} = \frac{39.15}{28-4} = 1.63$$

$$\gamma_0 = 0.48(1)$$

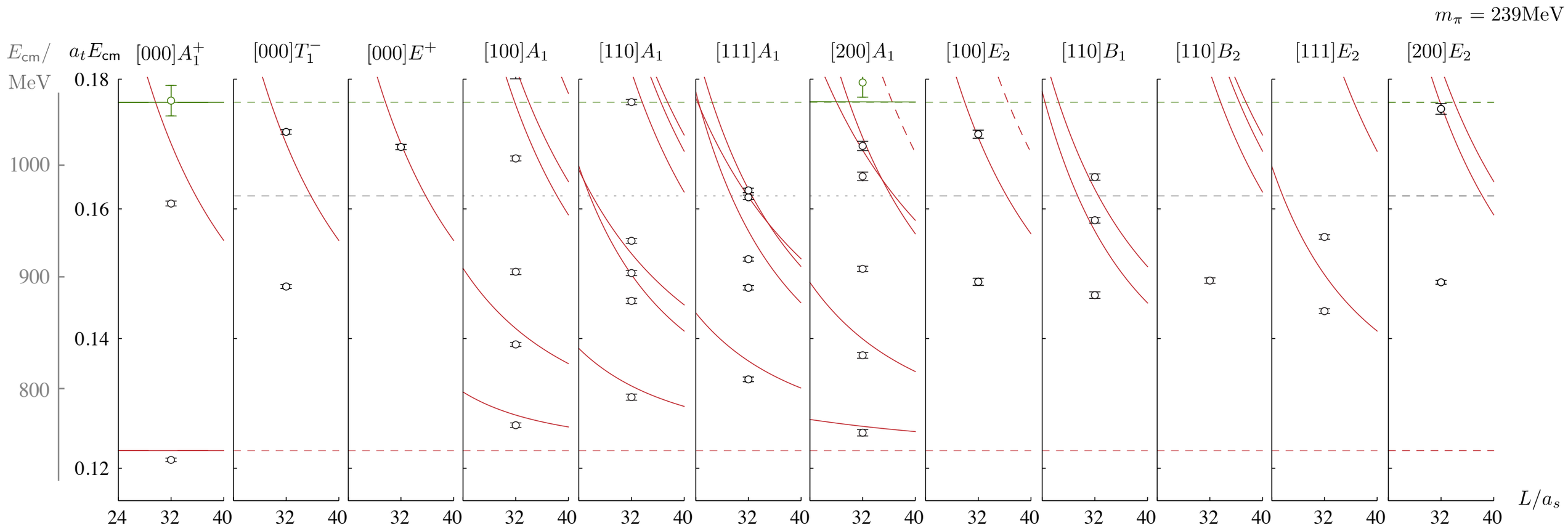
$$\gamma_1/s_{\mathrm{thr.}} = 2.29(19)$$

$$m_R = 904(1)\ \mathrm{MeV}$$

$$g_R = 4.81(7)$$

this analysis: m_π~239, 284, 327 & 391 MeV

4 light quark masses



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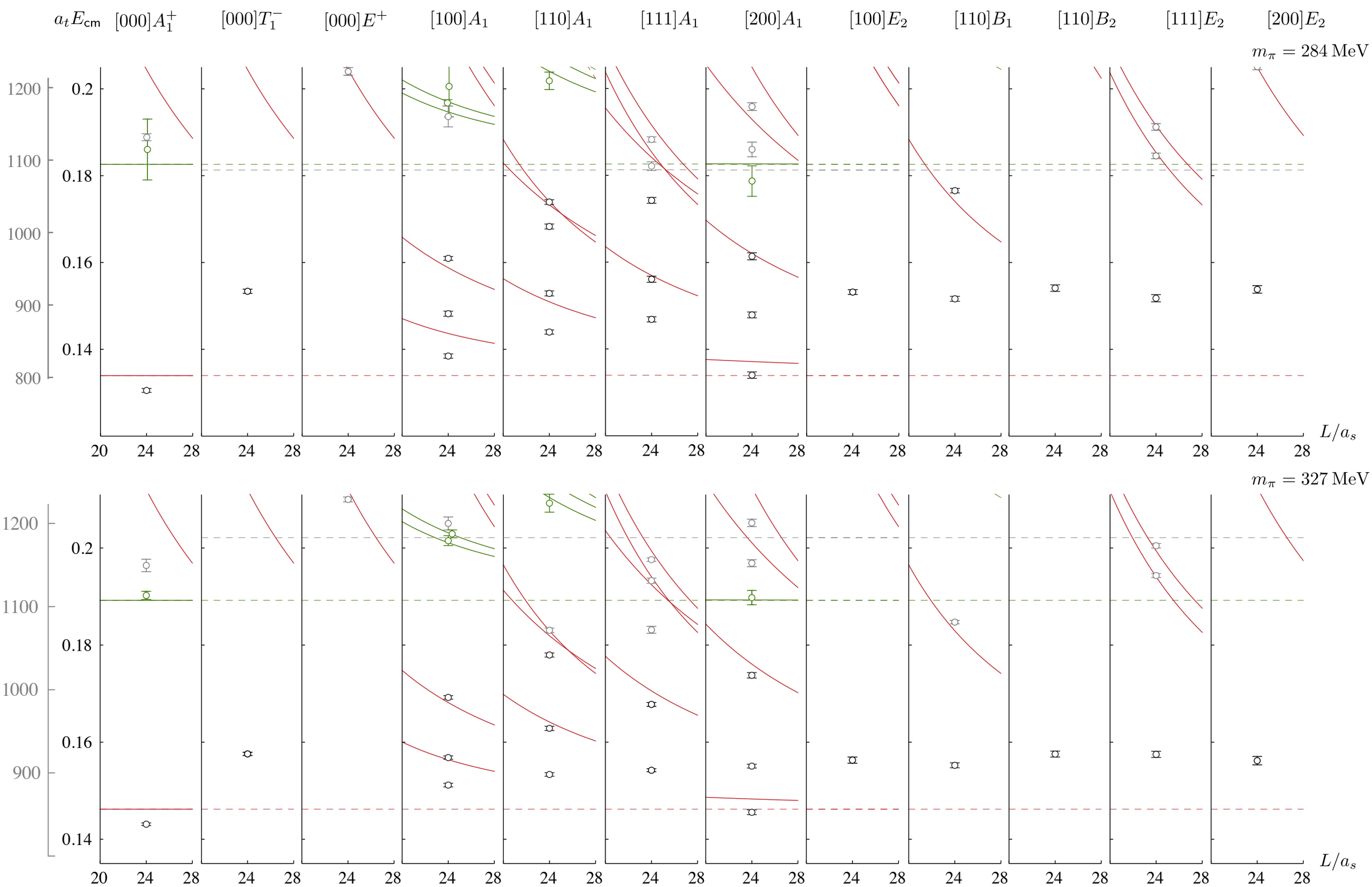
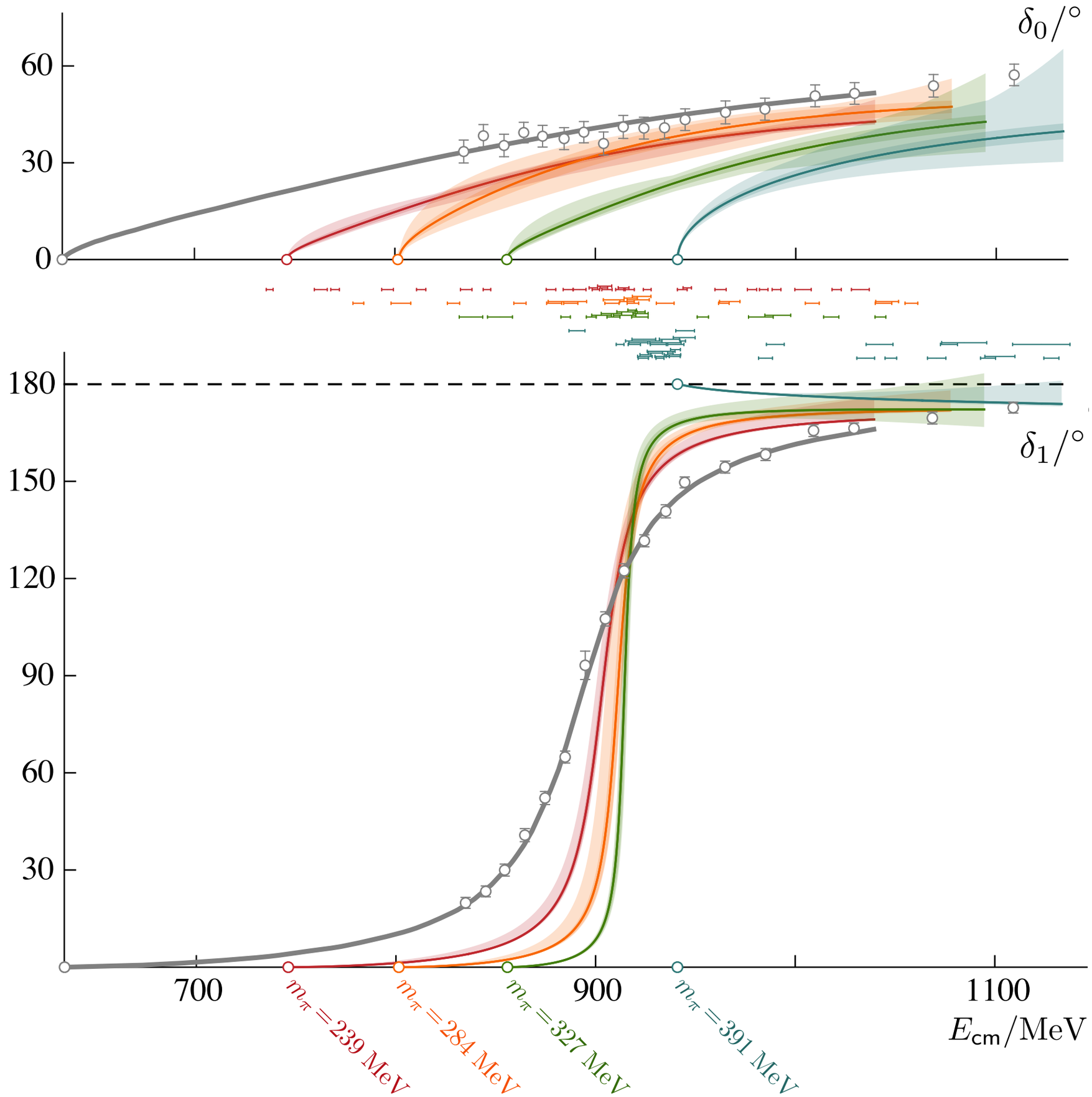
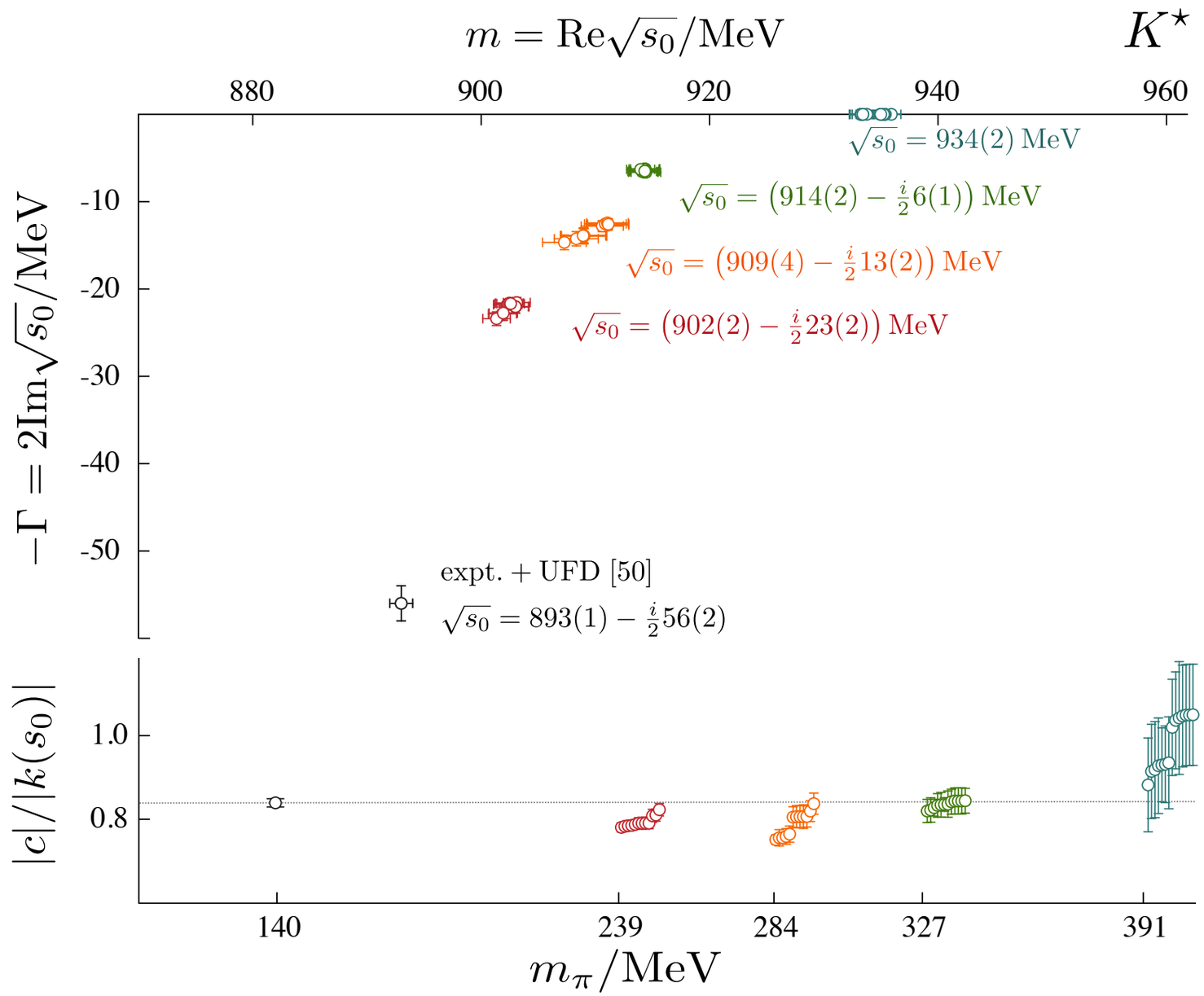



$\delta_0/°$
60
30
0
180
150
120
90
60
30
$\delta_1/°$
700
900
1100
$E_{\mathsf{cm}}/\mathrm{MeV}$
$m_\pi = 239$ MeV
$m_\pi = 284$ MeV
$m_\pi = 327$ MeV
$m_\pi = 391$ MeV

scattering-amplitude poles ⇔ spectroscopic content

$$t \sim \frac{c^2}{s_0 - s}$$

P-wave is easy:



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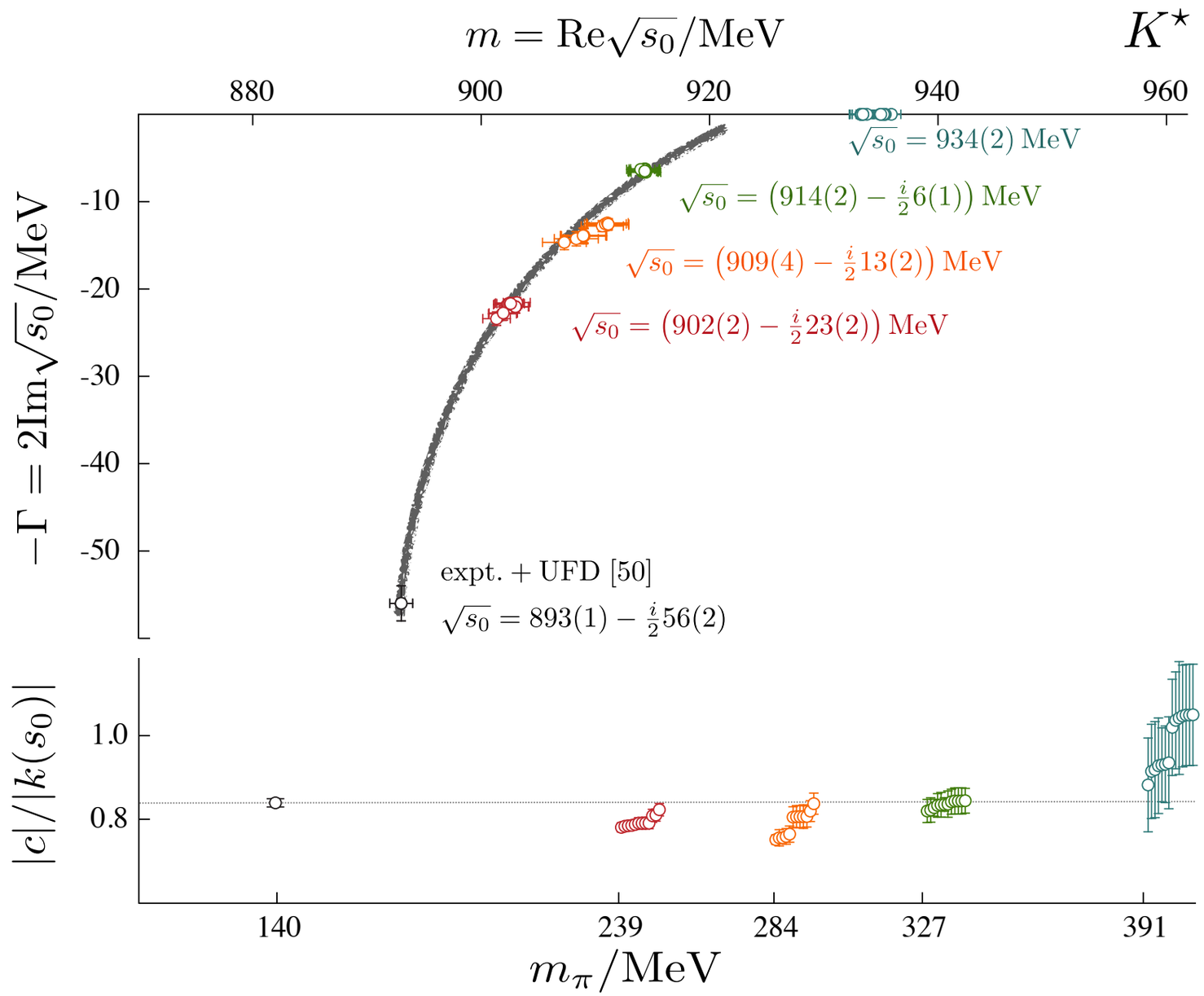
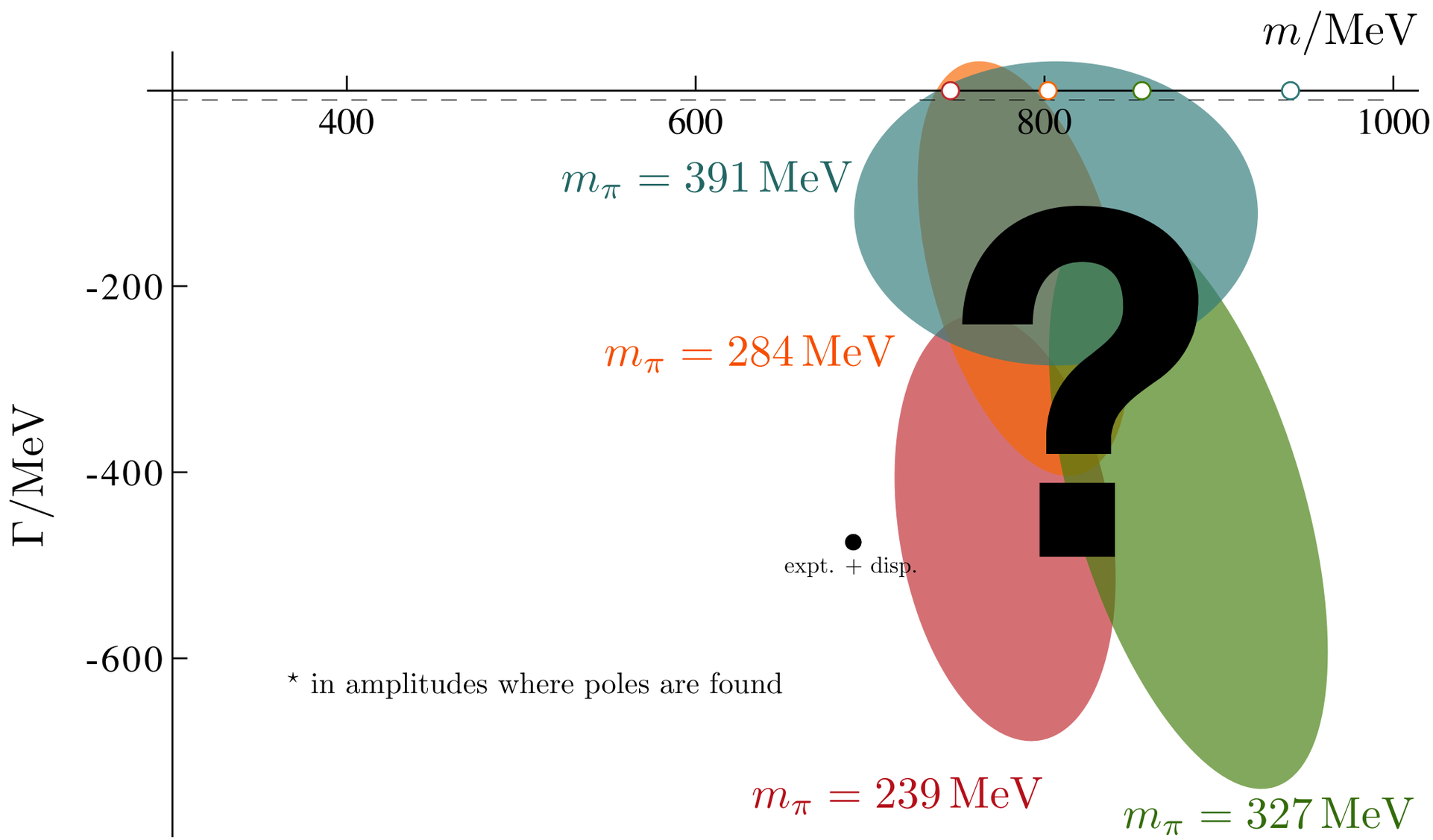


scattering-amplitude poles ⇔ spectroscopic content

$$t \sim \frac{c^2}{s_0 - s}$$

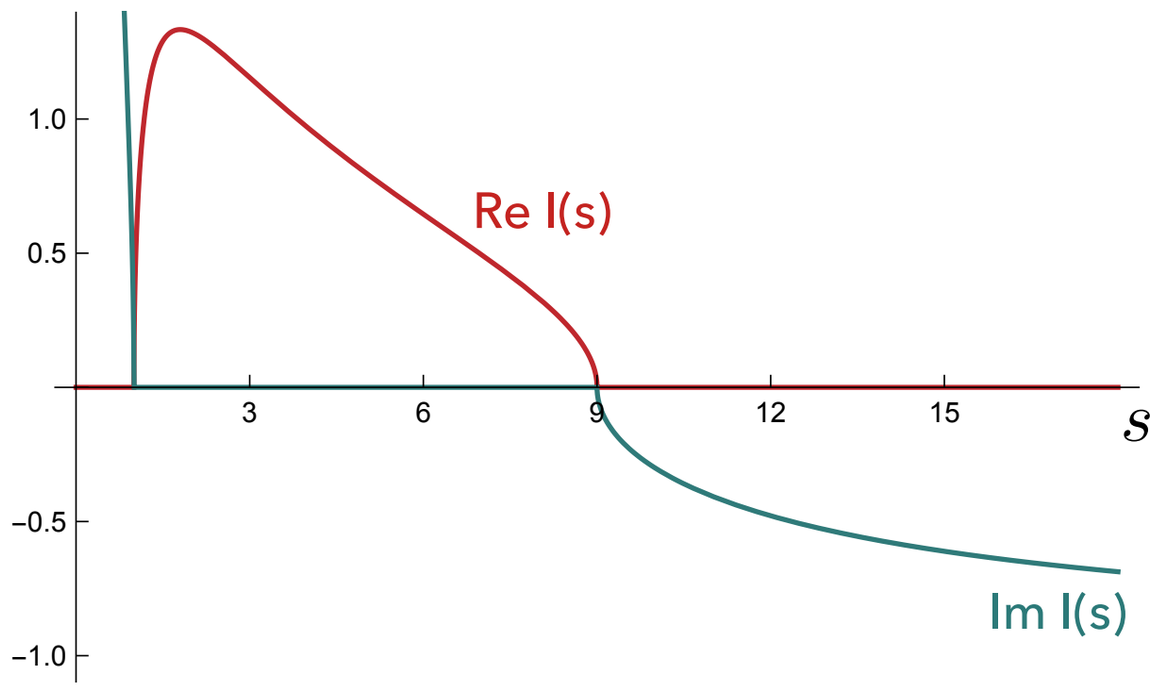
S-wave is inconclusive

- many amplitudes have poles, but not all
- broad spread in position possible at all 4 masses
- uncertainties from different parameterisations often don't overlap
- some amplitudes have no nearby poles: cannot claim a robust determination



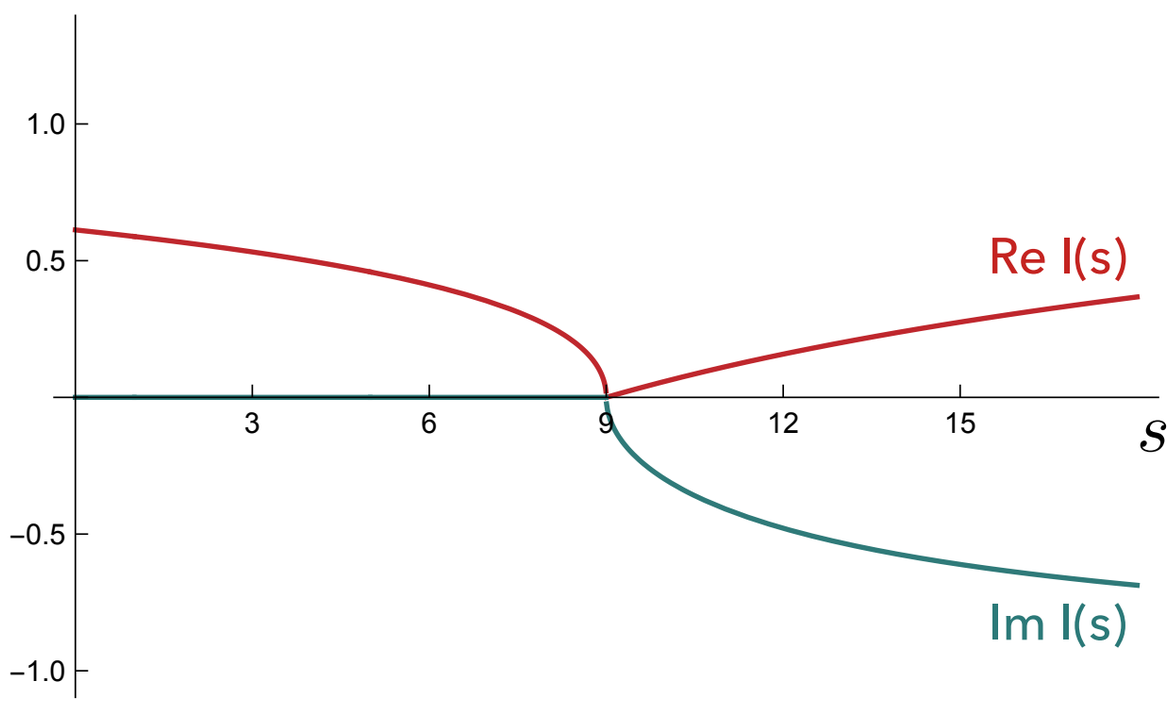
"simple" phase space

unequal masses: m_1=1, m_2=2



$$I(s) = -i\rho(s)$$
$$= -\frac{i}{s}\left(s-(m_1+m_2)^2\right)^{\frac{1}{2}}\left(s-(m_2-m_1)^2\right)^{\frac{1}{2}}$$

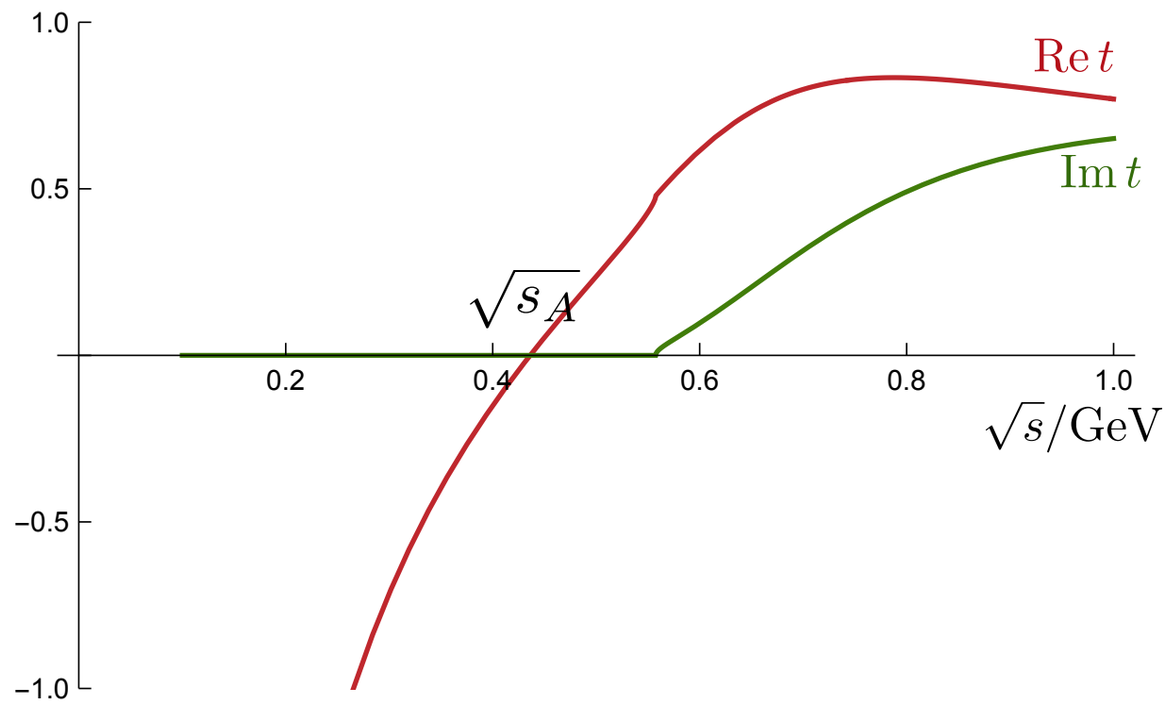
"Chew-Mandelstam" phase space



$$I(s) = I(s_0) - \frac{s-s_0}{\pi}\int_{s_0}^{\infty} ds' \, \frac{\rho(s')}{(s'-s)(s'-s_0)}$$

$$\mathrm{Im} I(s) = -\rho(s)$$

a zero in t below pi-K threshold



from fit using 239 MeV spectra

amplitude has CM phase space, linear K(s), zero arises from minimisation

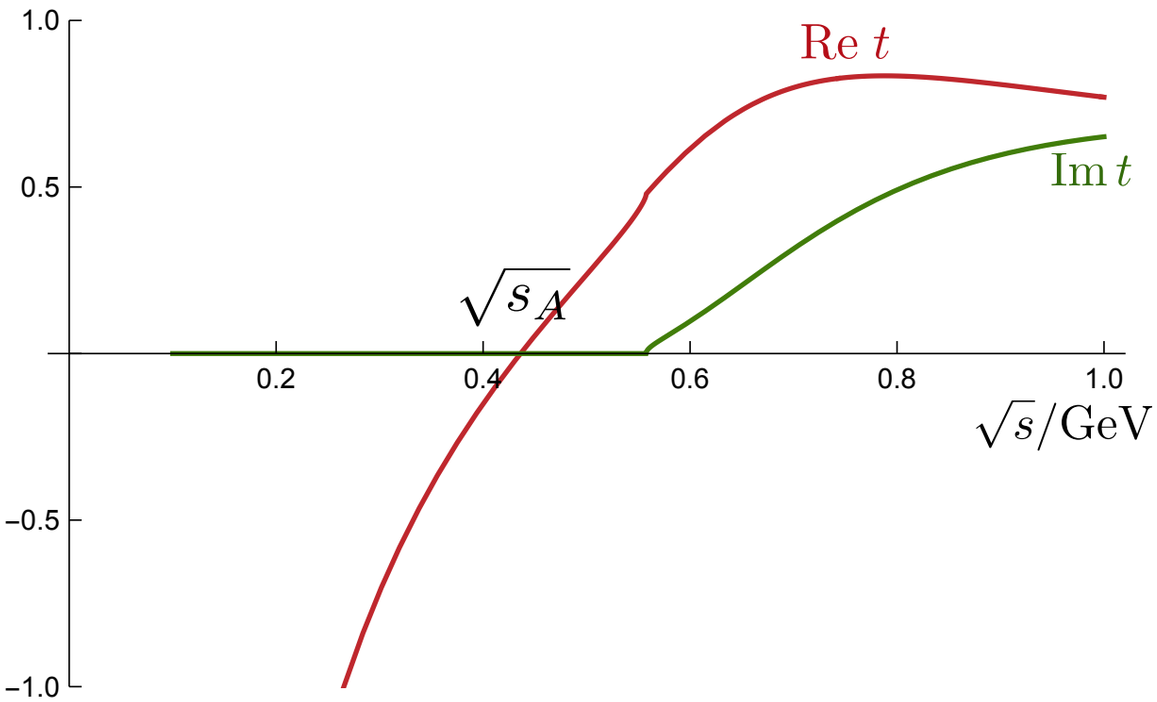
often argued as an essential feature
arises in chiral perturbation theory, at leading order:

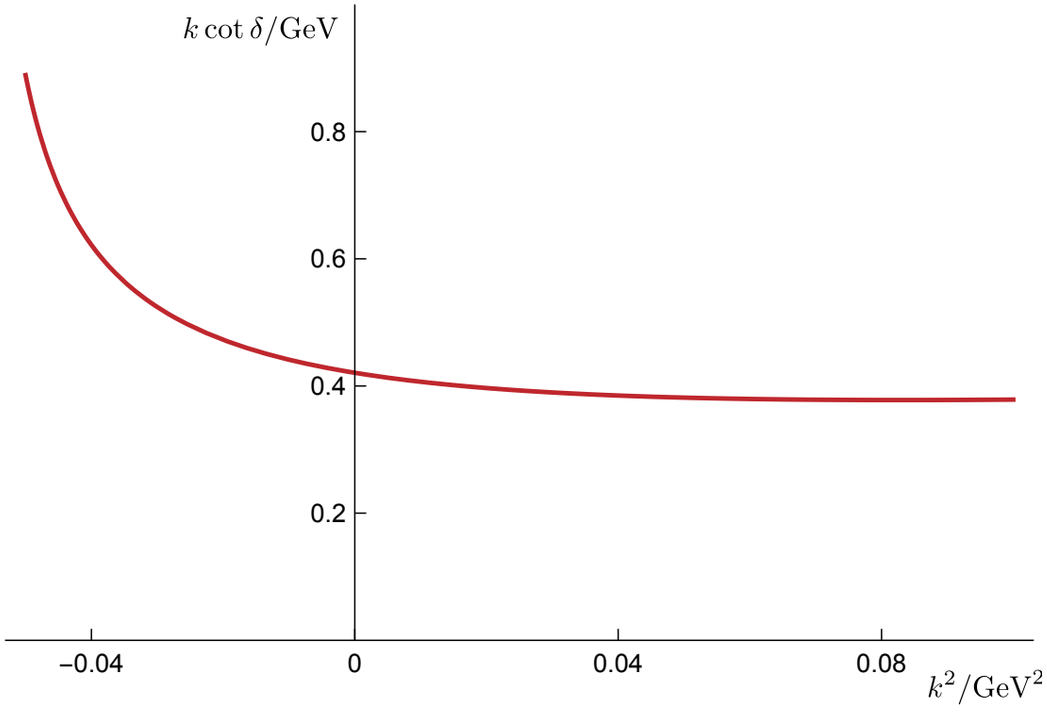
$$s_A = \frac{1}{5}\left(m_K^2 + m_\pi^2 \pm 2\left(4m_\pi^4 - 7m_K^2 m_\pi^2 + 4m_K^4\right)^{\frac{1}{2}}\right)$$

~(480 MeV)^2 at all mass points considered

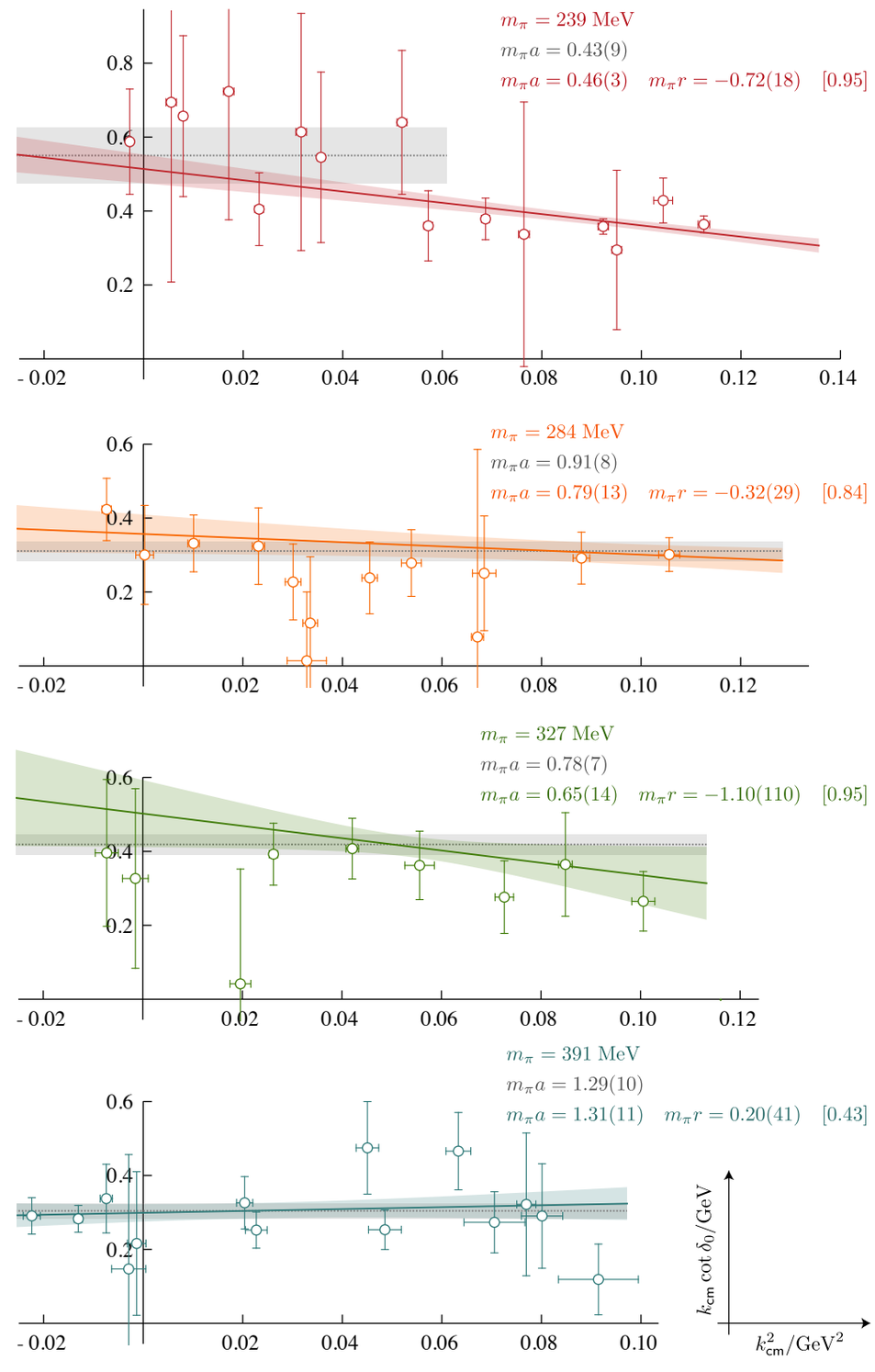
SU(3) ChiPT, I=1/2, S-wave projection

strong correlation with extracted pole position



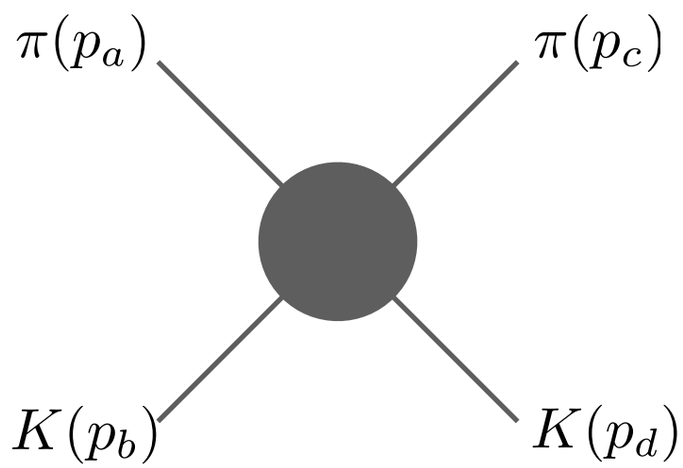


zero in t below threshold
⇒ pole in $k\cot\delta$ at negative k^2

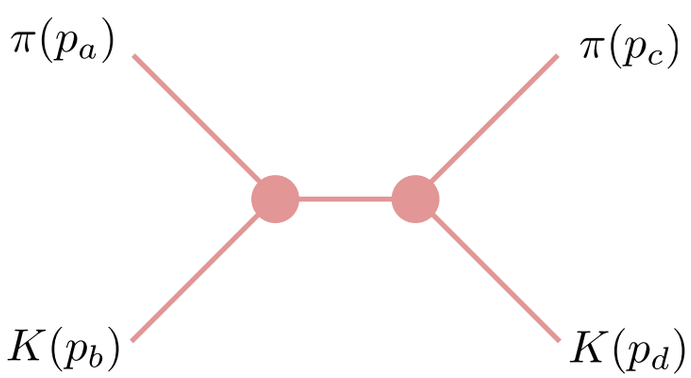


no clear sign from lattice spectra

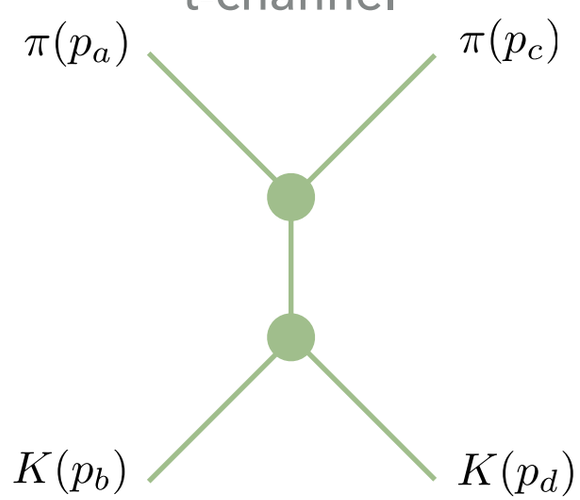
cross-channel scattering



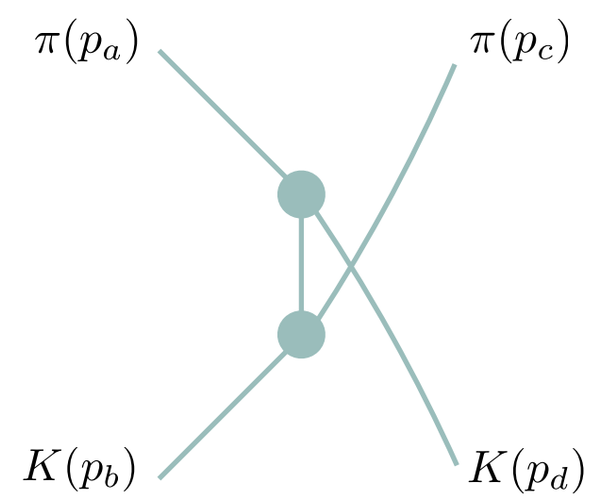
s-channel



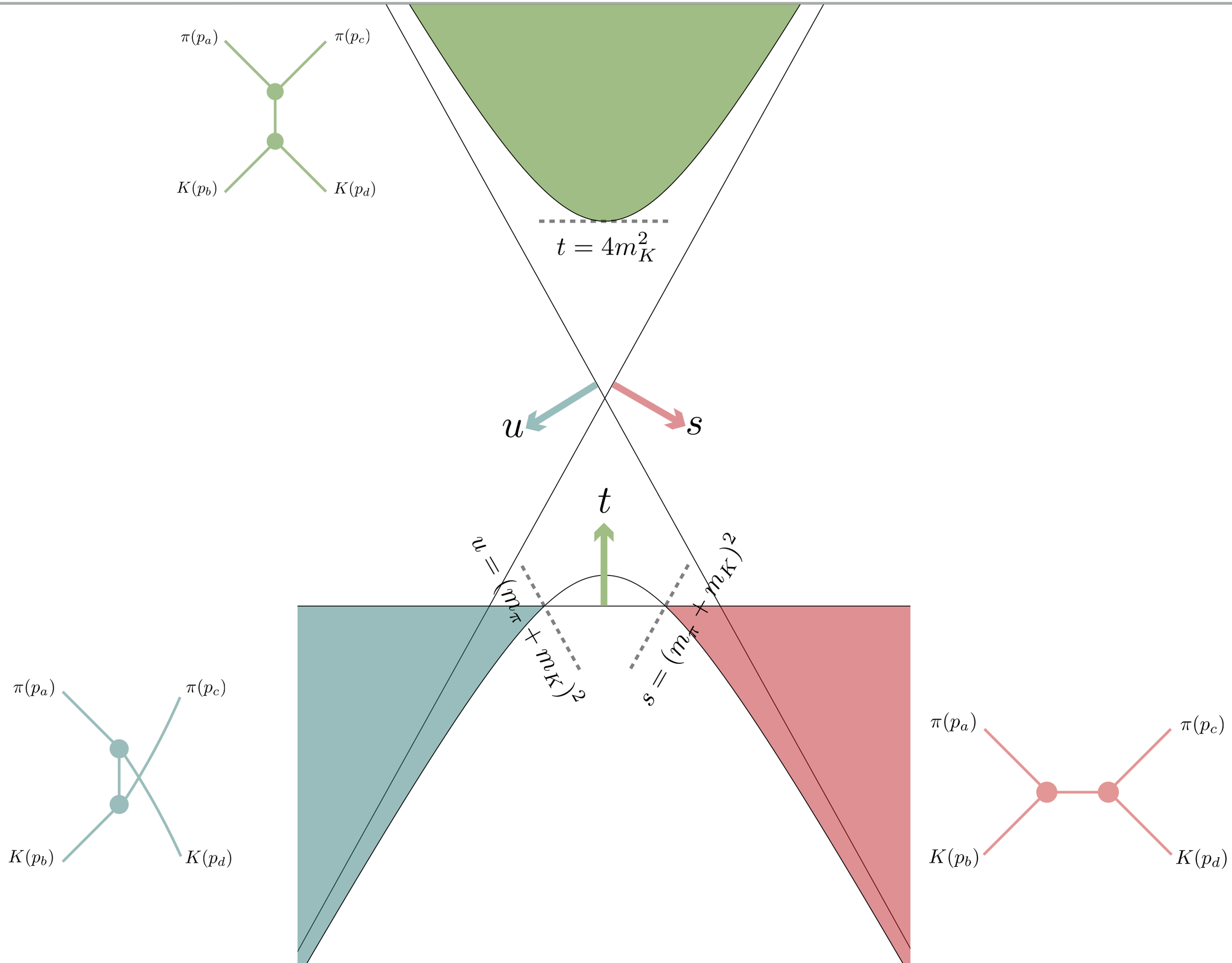
t-channel



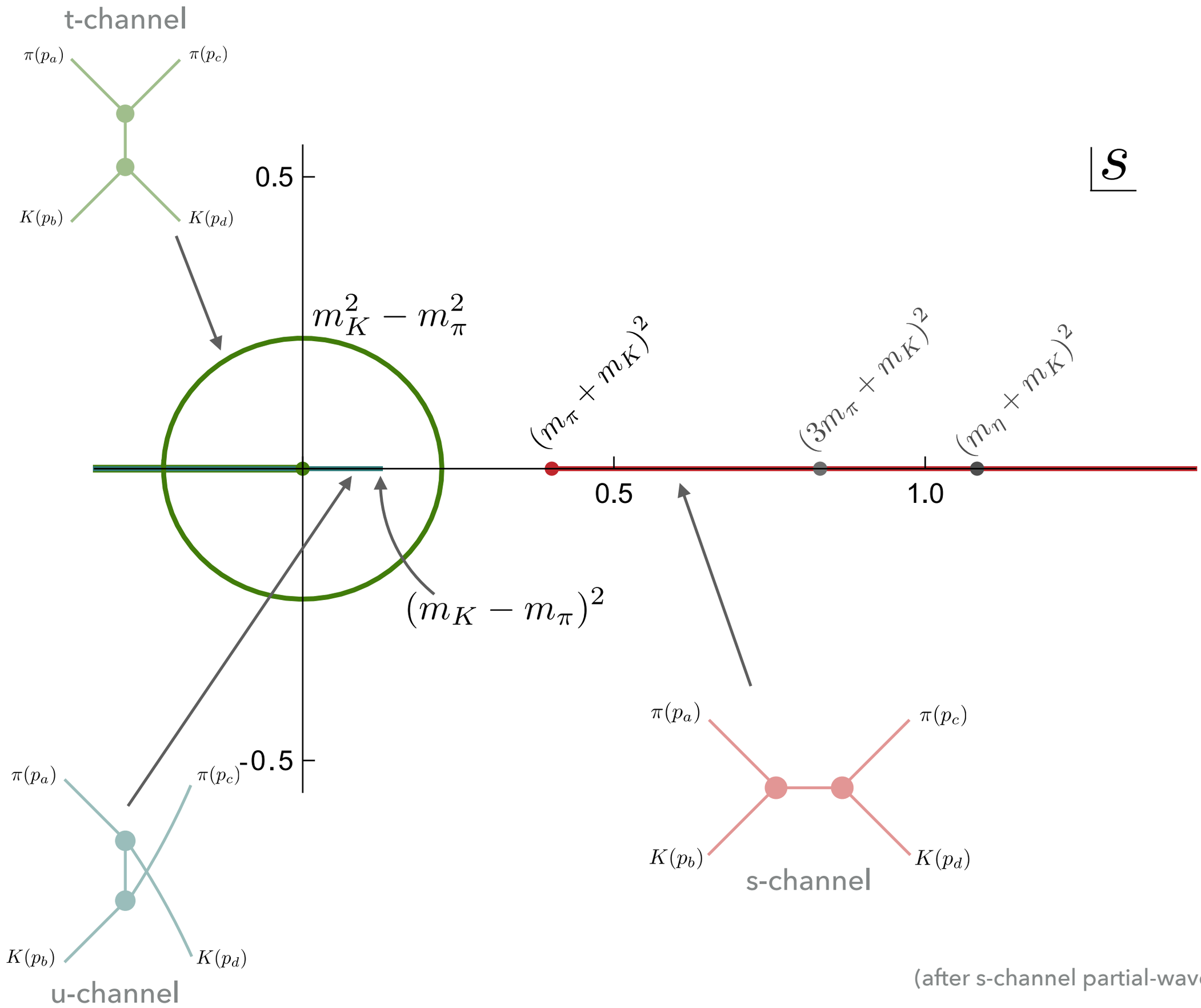
u-channel



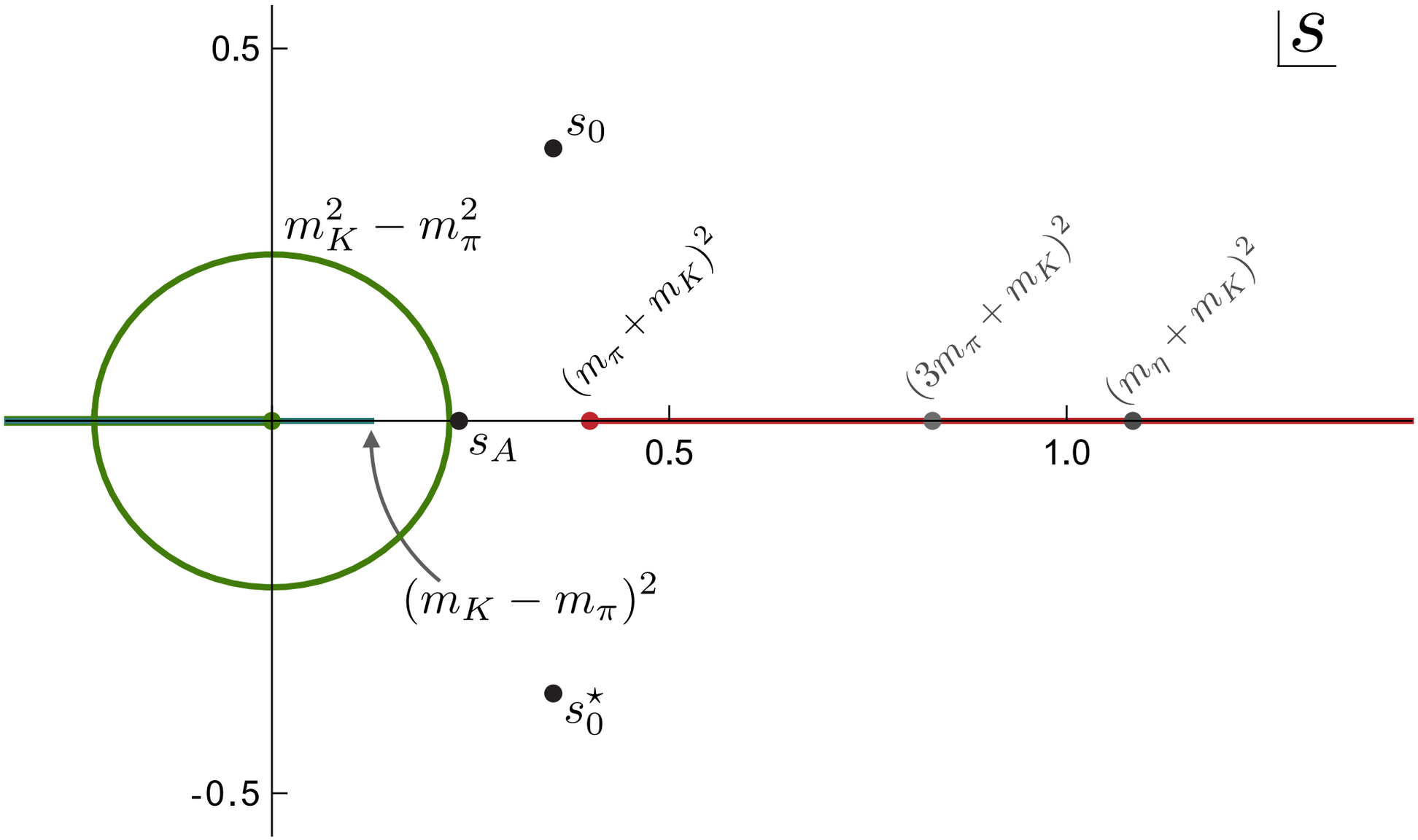
cross-channel scattering



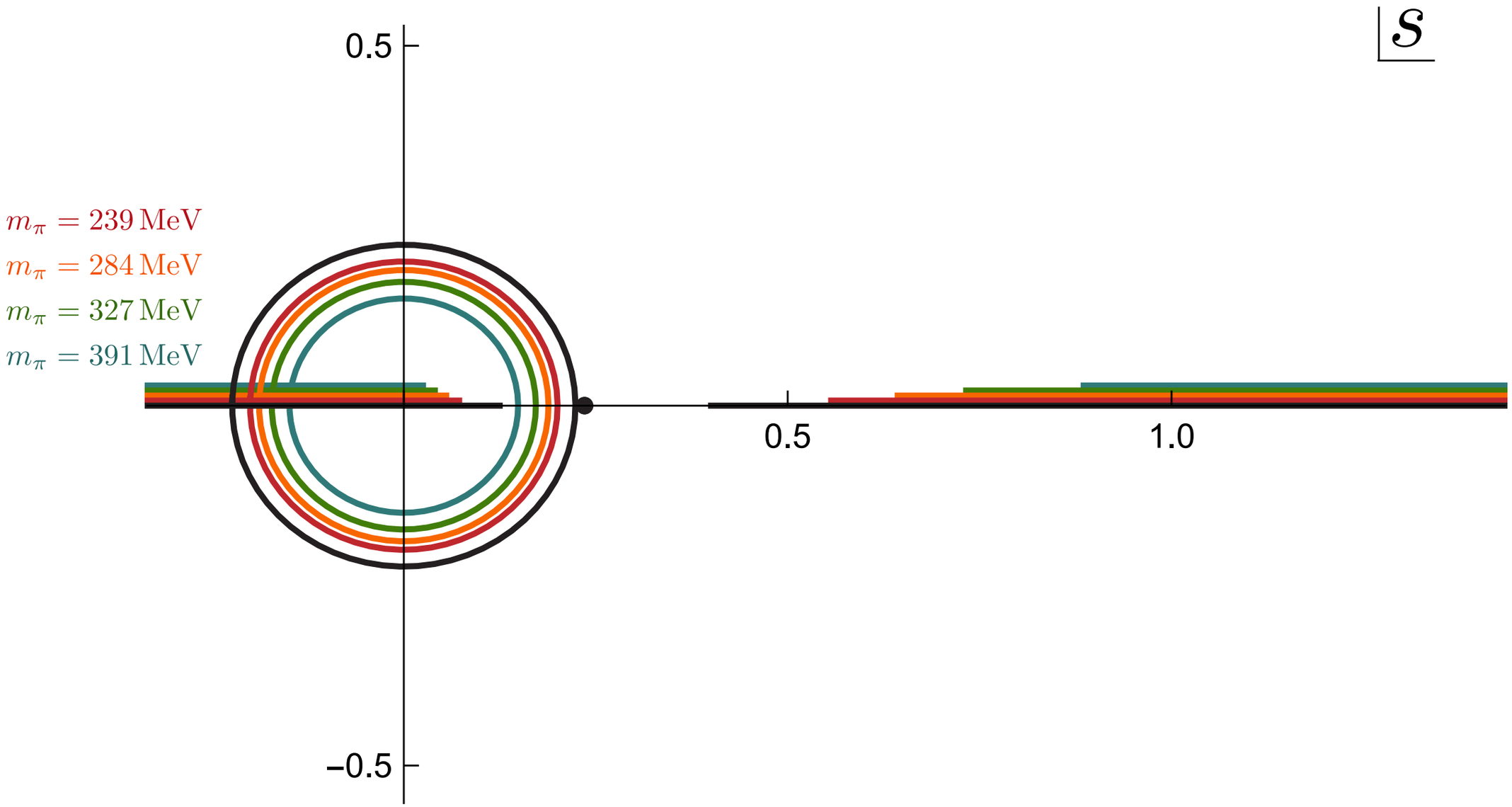
cross-channel scattering



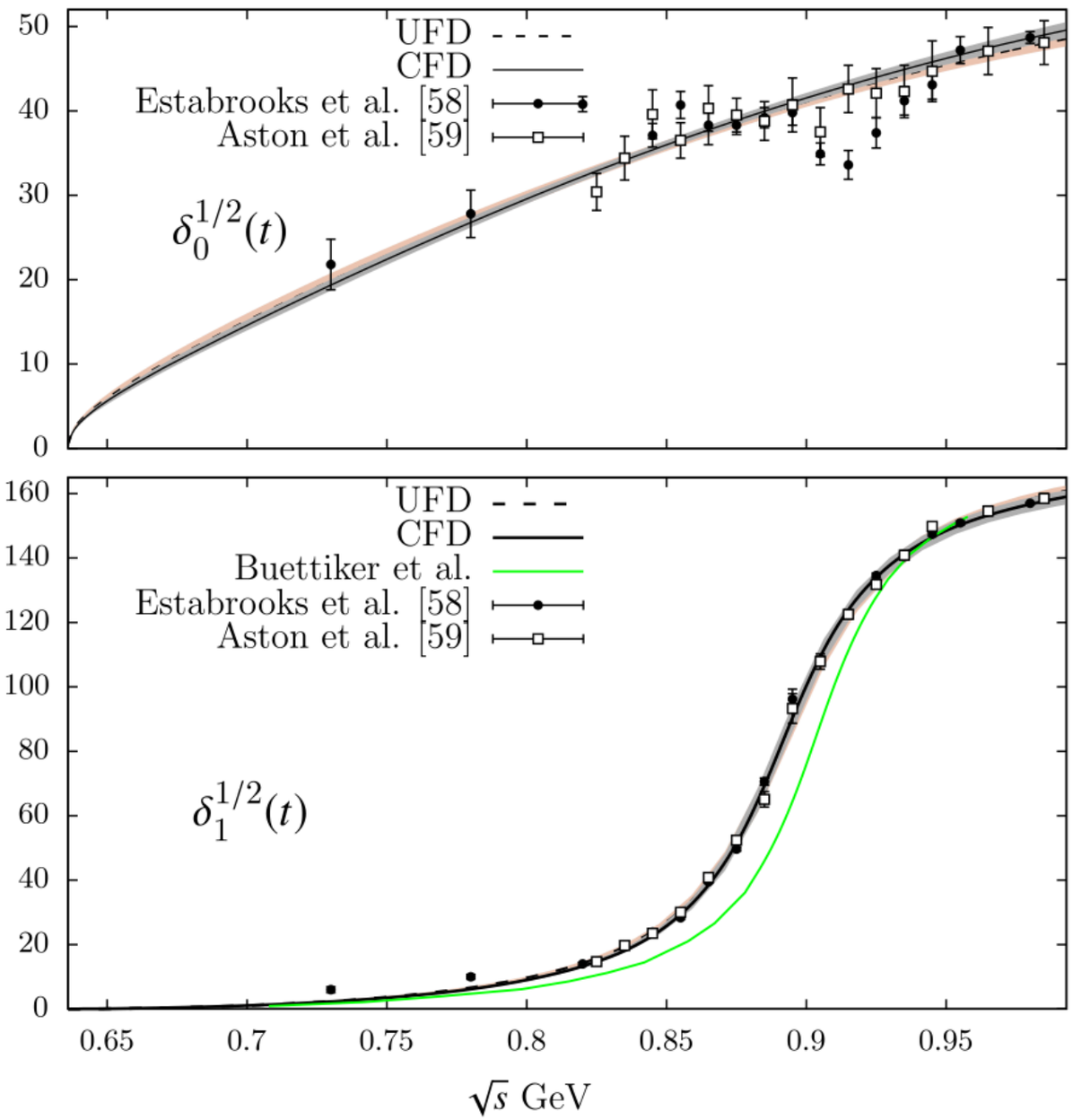
(after s-channel partial-wave projection)

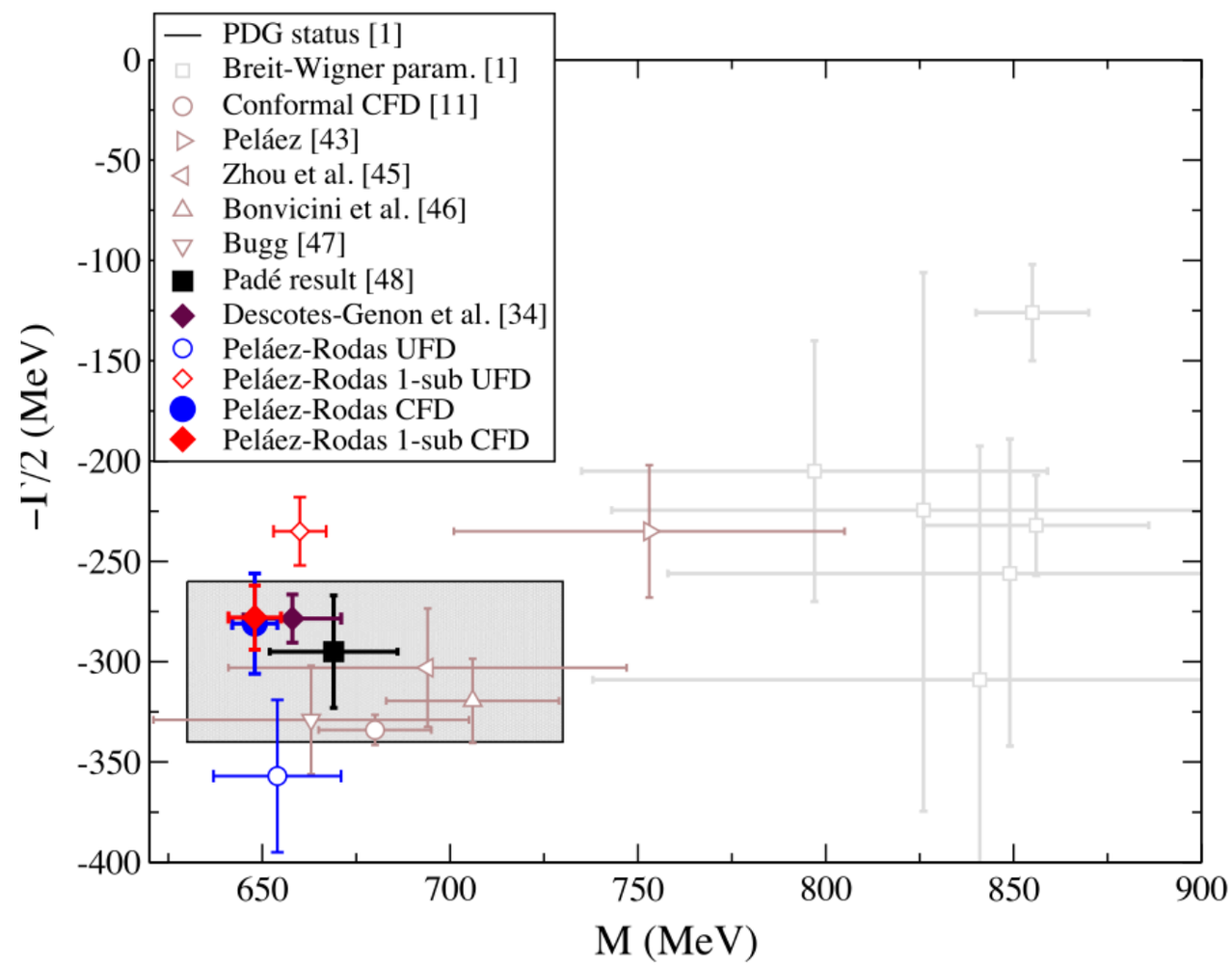


hard to argue for zero at s_A without including left cuts

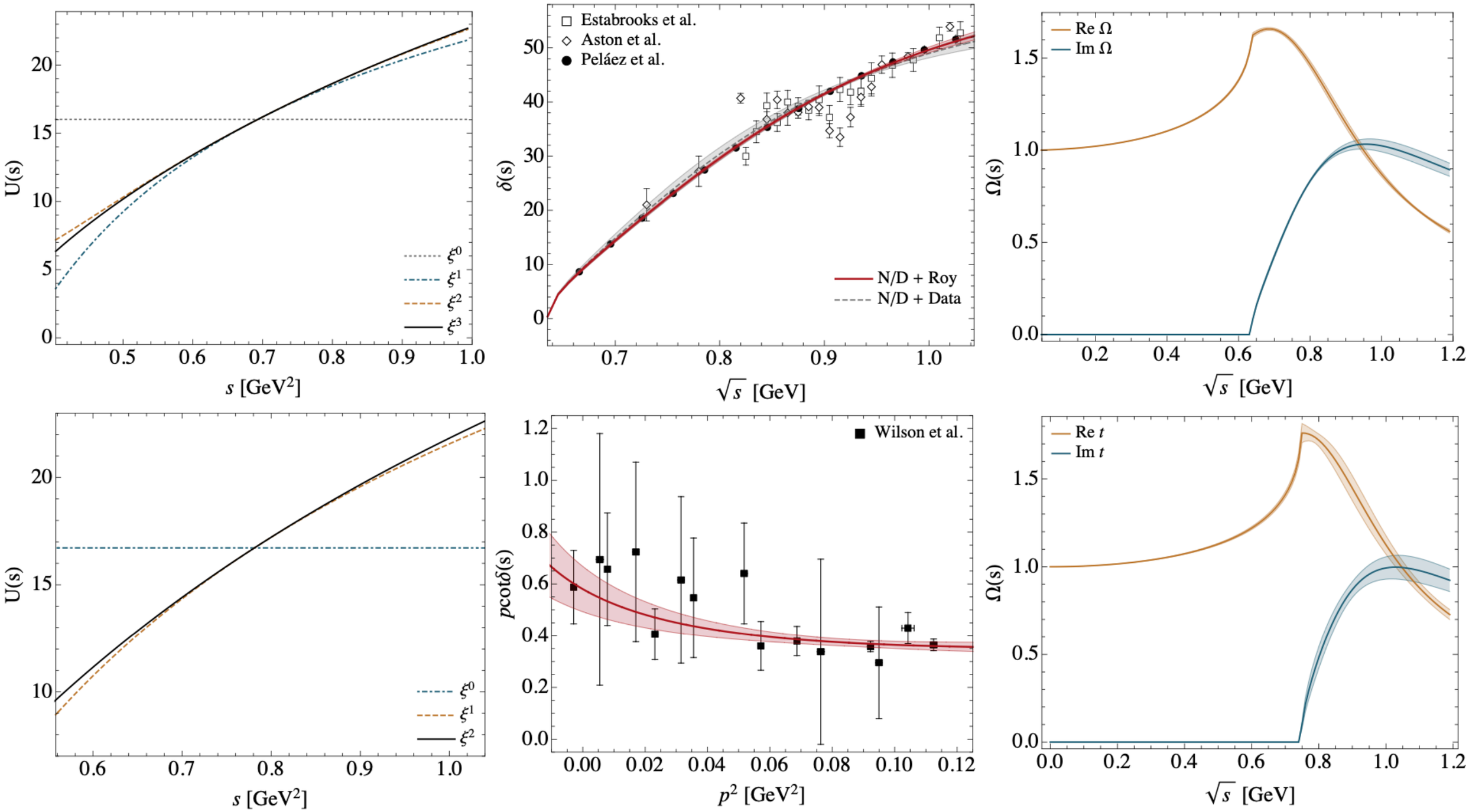


hard to argue for zero at s_A without including left cuts


UFD
CFD
Estabrooks et al. [58]
Aston et al. [59]
$\delta_0^{1/2}(t)$
UFD
CFD
Buettiker et al.
Estabrooks et al. [58]
Aston et al. [59]
$\delta_1^{1/2}(t)$
$\sqrt{s}$ GeV

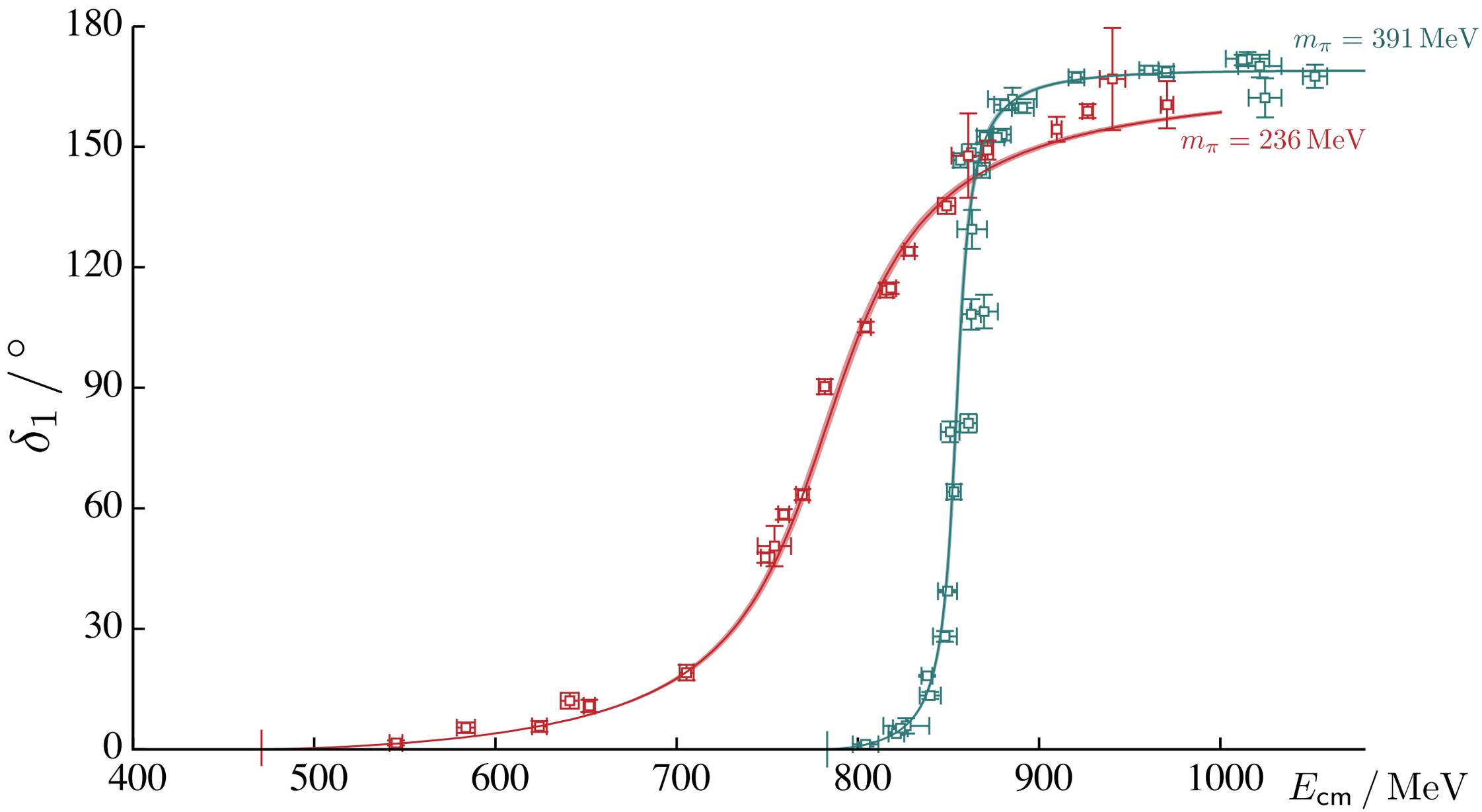

PDG status [1]
Breit-Wigner param. [1]
Conformal CFD [11]
Peláez [43]
Zhou et al. [45]
Bonvicini et al. [46]
Bugg [47]
Padé result [48]
Descotes-Genon et al. [34]
Peláez-Rodas UFD
Peláez-Rodas 1-sub UFD
Peláez-Rodas CFD
Peláez-Rodas 1-sub CFD
$-\Gamma/2$ (MeV)
M (MeV)

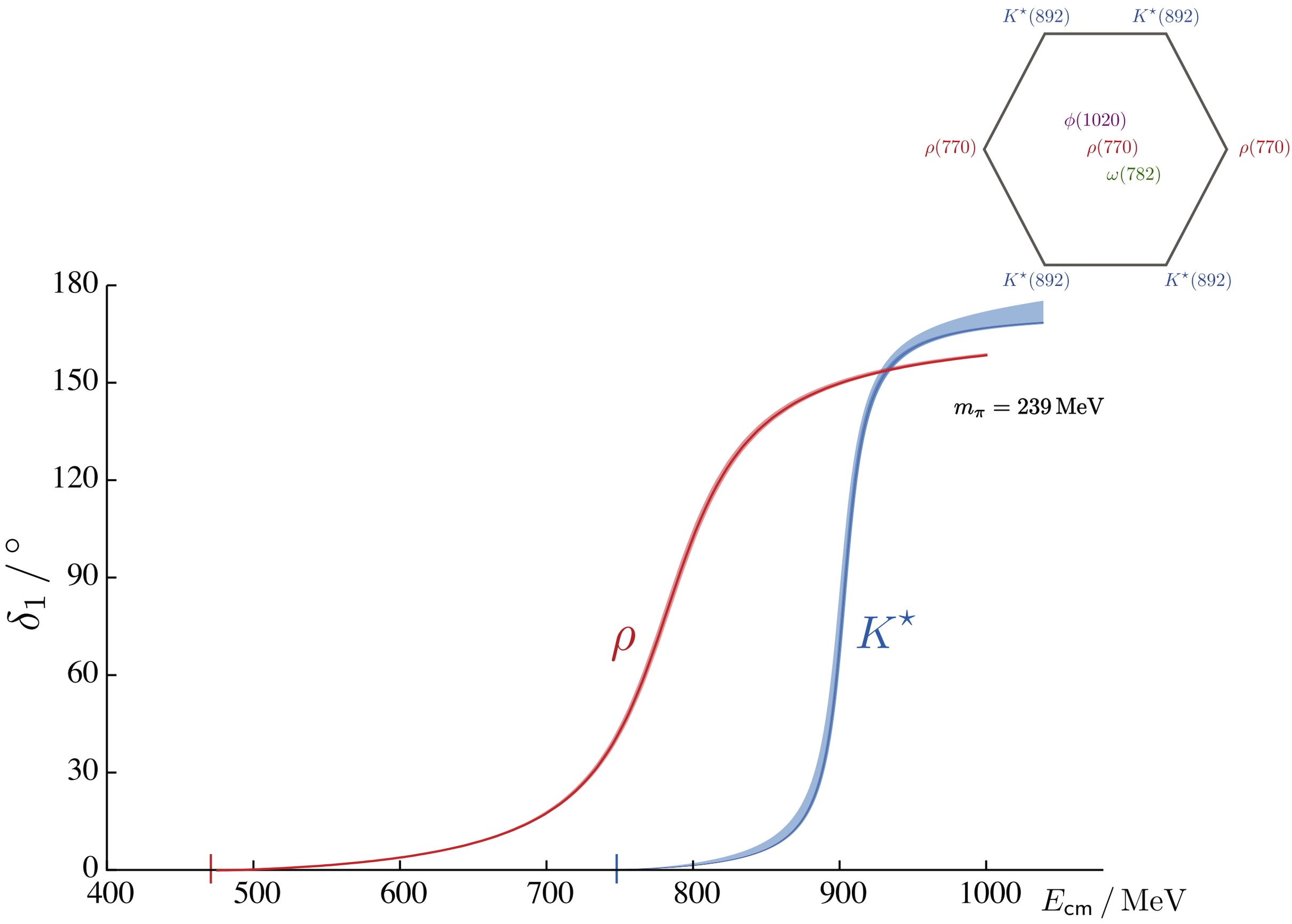
N/D-like approach

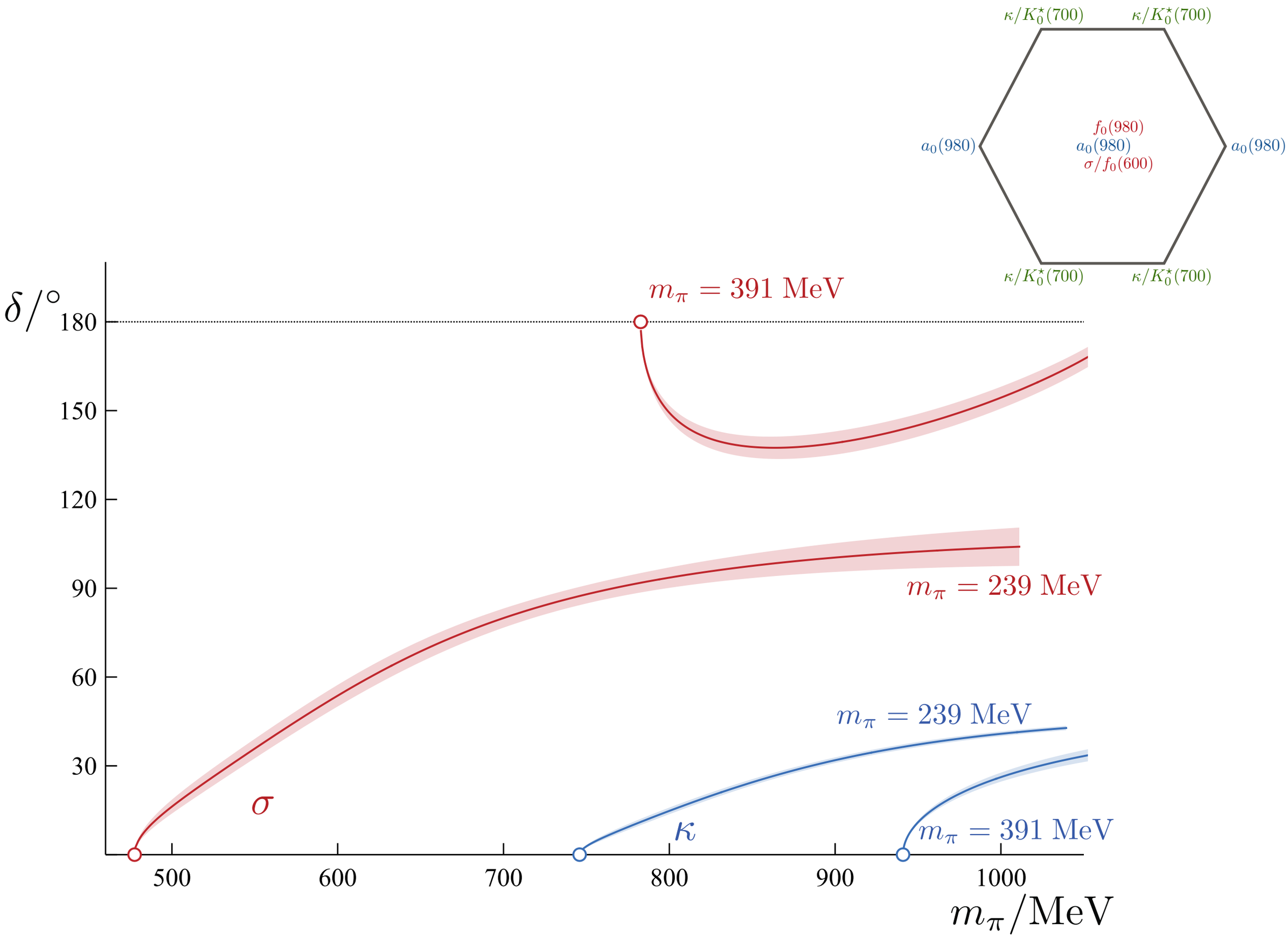


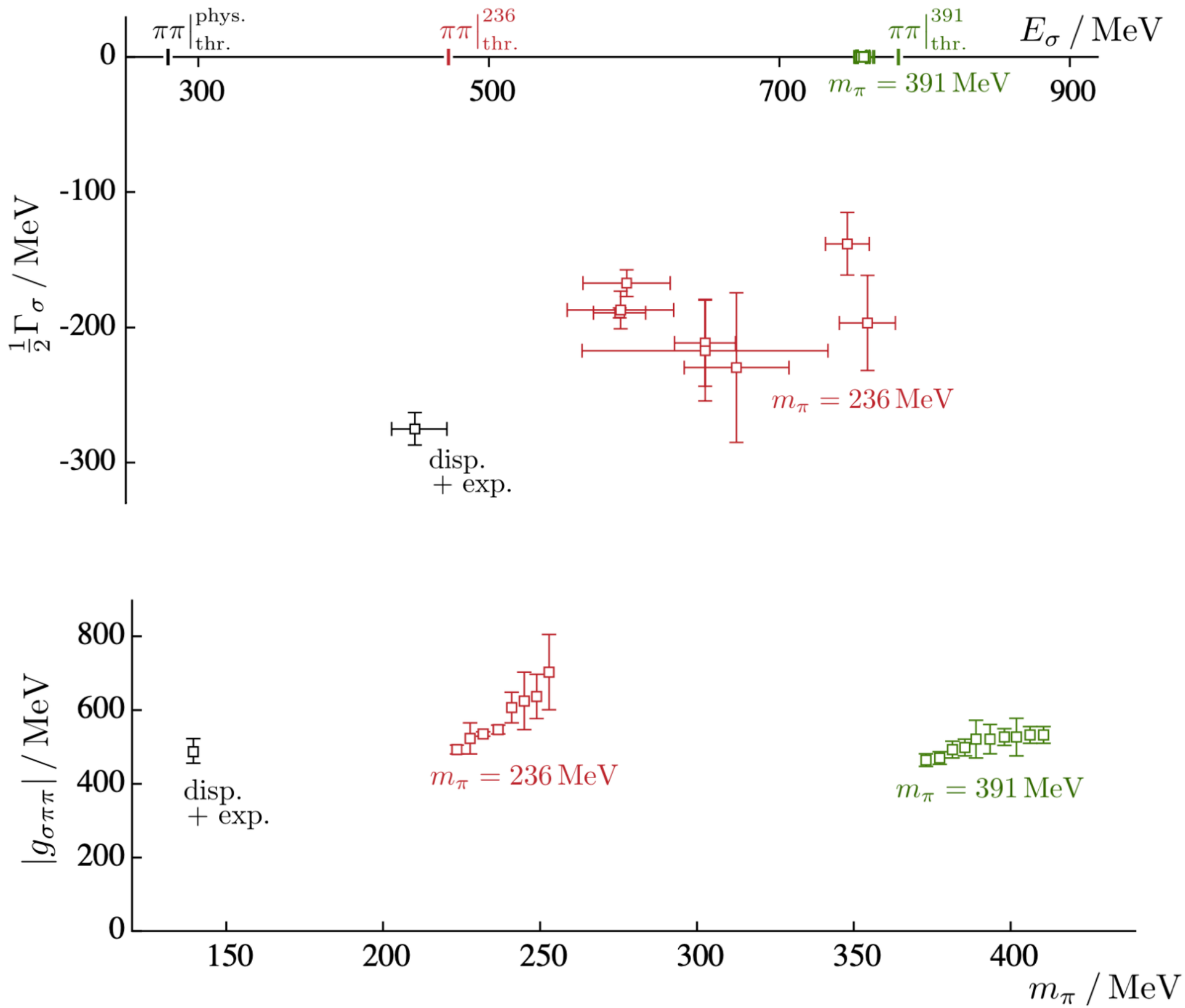
	Our results	
	$\sqrt{s_p}$, MeV	$\lvert g_{pa}\rvert/\sqrt{N_{aa}}$, GeV
$\kappa/K_0^*(700)$		
Exp. SC	$701(12) - i\,287(17)$	πK : 4.16(14)
Lattice, $m_\pi = 239$ MeV	$749(38) - i\,265(16)$	πK : 4.18(18)

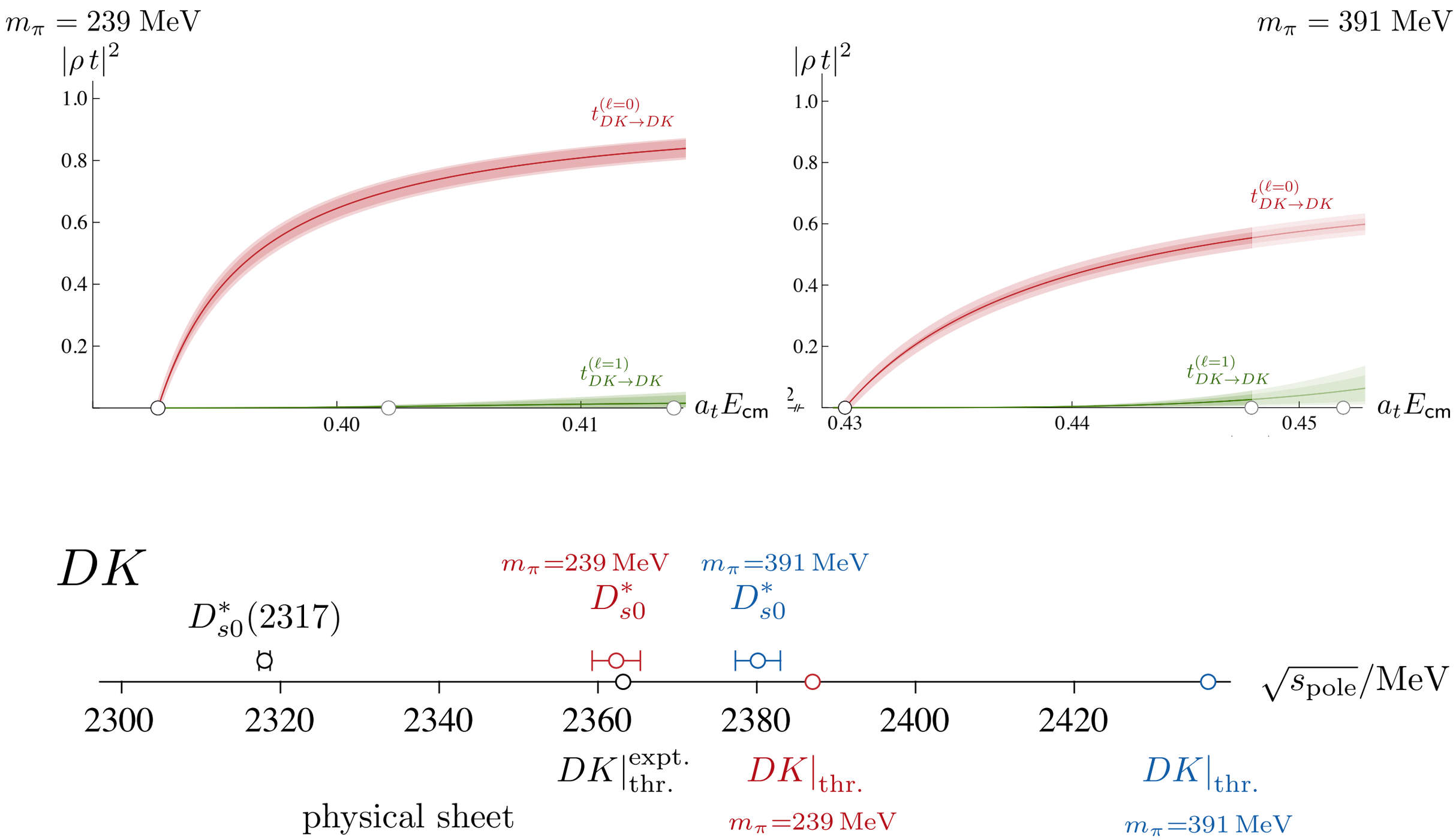
Phys.Rev.D 87 (2013) 3, 034505, arXiv:1212.0830
Phys.Rev.D 92 (2015) 9, 094502, arXiv:1507.02599




$K^\star(892)$
$K^\star(892)$
$\phi(1020)$
$\rho(770)$
$\rho(770)$
$\omega(782)$
$\rho(770)$
$K^\star(892)$
$K^\star(892)$
$m_\pi = 239\,\mathrm{MeV}$
ρ
$K^\star$
δ_1 / °
180
150
120
90
60
30
0
400
500
600
700
800
900
1000
E_{cm} / MeV


$\kappa/K_0^\star(700)$
$\kappa/K_0^\star(700)$
$a_0(980)$
$f_0(980)$
$a_0(980)$
$\sigma/f_0(600)$
$a_0(980)$
$\kappa/K_0^\star(700)$
$\kappa/K_0^\star(700)$
$\delta/^\circ$
180
150
120
90
60
30
$m_\pi = 391$ MeV
$m_\pi = 239$ MeV
$m_\pi = 239$ MeV
$m_\pi = 391$ MeV
σ
κ
500
600
700
800
900
1000
m_π/MeV


$\pi\pi|^{\text{phys.}}_{\text{thr.}}$
$\pi\pi|^{236}_{\text{thr.}}$
$\pi\pi|^{391}_{\text{thr.}}$
E_σ / MeV
0
300
500
700
900
$m_\pi = 391$ MeV
-100
-200
-300
$\frac{1}{2}\Gamma_\sigma$ / MeV
$m_\pi = 236$ MeV
disp.
+ exp.
800
600
400
200
0
$|g_{\sigma\pi\pi}|$ / MeV
disp.
+ exp.
$m_\pi = 236$ MeV
$m_\pi = 391$ MeV
150
200
250
300
350
400
m_π / MeV



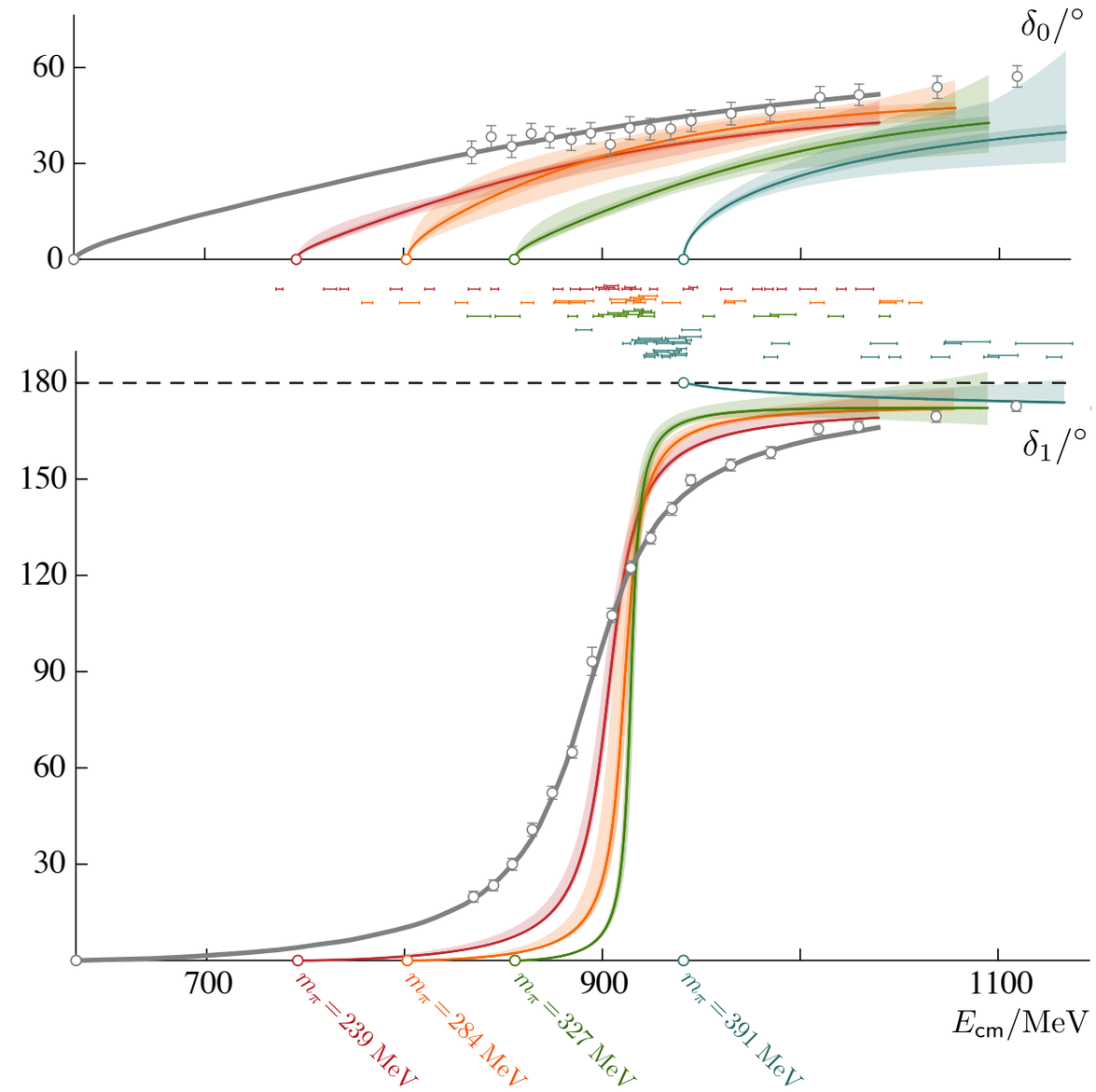
+D-pi coming soon...

Lattice QCD provides a first-principles tool to do hadron spectroscopy

Many S-wave resonances remain puzzling, in cases like the κ(700), left cut physics is important

Lots of other stuff - see hadspec.org

These methods are widely applicable,
- coupled-channel scattering
- baryons
- charm quarks
- form factors

…

Control of 3+ body effects needed for
- lighter pion masses
- higher resonances