Eletroweak Parton Distribution Functions & Applications at High-Energy Muon Colliders

Keping Xie
Pittsburgh Particle-physics, Astrophysics, and Cosmology Center, Department of Physics and Astronomy, University of Pittsburgh, PA 15260, USA

Jefferson Lab
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Based on work with T. Han and Y. Ma
2007.14300, 2103.09844, 2106.01393
Why muon colliders?

- **Leptons** are the ideal probes of short-distance physics
  - Cleaner background comparing to hadron colliders
  - High-energy physics probed with much smaller collider energy

- **Electron colliders**
  - A glorious past: discovery of charm, $\tau$, and gluon
  - Important future: Precision EW constraints on BSM physics, Higgs physics

- **Muon colliders**
  - A $s$-channel Higgs factory: Higgs production enhanced by $m^2_\mu/m^2_e \sim 40000$
  - Direct measurements on $y_\mu$ and $\Gamma_H$
  - Multi-TeV muon colliders: Less radiations then electron
  - Center of mass energy $3 - 15$ TeV and the more speculative $E_{cm} = 30$ TeV
  - New particle mass coverage $M \sim (0.5 - 1)E_{cm}$
  - Great accuracies for $WWH$, $WWHH$, $H^3$, $H^4$
  - [See Snowmass WPs, 2203.08033, 2203.07964, Report 2209.01318.]

**Muon Collider Physics Potential Pillars**

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<th>Direct search of heavy particles</th>
<th>High rate indirect probes</th>
<th>High energy probes</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUSY-inspired, WIMP, VBF production, 2-&gt;1</td>
<td>Higgs single and self-couplings, rare Higgs decays, exotic decays</td>
<td>difermion, diboson, EFT, Higgs compositeness</td>
</tr>
</tbody>
</table>
A possible high-energy muon collider

Size and Benchmarks [Ankenbrandt et al., arXiv:physics/9901022]

Integrated luminosity: $\mathcal{L} = \left(\frac{E_{\text{cm}}}{10 \text{ TeV}}\right)^2 \times 10 \text{ ab}^{-1}$ [The Muon Smasher’s Guide, 2103.14043]
Vector boson fusions vs. annihilations

General features:

- The annihilations decrease as $1/s$.
- ISR needs to be considered, which can give over 10% enhancement.
- The fusions increase as $\ln(s)$, which take over at high energies.
- The large collinear logarithm $\ln\left(\frac{Q^2}{m_\ell^2}\right)$ needs to be resummed.
- $W^+W^-$ as a reference to separate high-energy EW and low-energy QED/QCD

Q: How to treat parton properly at high energies when $W/Z$ become active?
EW physics at high energies

- At high energies, every particle become massless
  \[ \frac{v}{E} : \frac{v}{100 \text{ TeV}} \sim \frac{\Lambda_{\text{QCD}}}{100 \text{ GeV}}, \quad \frac{v}{E}, \frac{m_t}{E}, \frac{M_W}{E} \to 0! \]

- The splitting phenomena dominate due to large log enhancement

- The EW symmetry is restored: \( SU(2)_L \times U(1)_Y \) unbroken

- Goldstone Boson Equivalence:
  \[ \varepsilon^\mu_L(k) = \frac{E}{M_W}(\beta_W, \hat{k}) \simeq \frac{k^\mu}{M_W} + \mathcal{O}(\frac{M_W}{E}) \]
  The violation terms is power counted as \( v/E \to \text{QCD higher twist effects} \)

- We mainly focus on the splitting phenomena, which can be factorized and resummed as the EW PDFs in the ISR, and the Fragmentaions/Parton Shower in the FRS.

- Other interesting aspects: the polarized EW boson scattering, top-Yukawa coupling effect
Splitting phenomena

\[ d\sigma \simeq d\sigma_X \times d\mathcal{P}_{A\to B+C}, \quad E_B \approx zE_A, \quad E_C \approx \bar{z}E_A, \quad k_T \approx z\bar{z}E_A\theta_{BC} \]

\[
\frac{d\mathcal{P}_{A\to B+C}}{dzdk_T^2} \simeq \frac{1}{16\pi^2} \frac{z\bar{z}|\mathcal{M}^{(\text{split})}|^2}{(k_T^2 + \bar{z}m_B^2 + zm_C^2 - z\bar{z}m_A^2)^2}, \quad \bar{z} = 1 - z
\]

- The dimensional counting: \(|\mathcal{M}^{(\text{split})}|^2 \sim k_T^2 \) or \( m^2 \)
- To validate the factorization formalism
  - The observable \( \sigma \) should be infra-red safe
  - Leading behavior comes from the collinear splitting

[Ciafaloni et al., hep-ph/0004071; 0007096; Bauer, Webber et al., 1703.08562; 1808.08831]

[Manohar et al., 1803.06347; Han, Chen, Tweedie, 1611.00788]
EW Splitting functions

- Starting from the unbroken phase: all massless
  \[ \mathcal{L}_{SU(2)\times U(1)} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_\phi + \mathcal{L}_f + \mathcal{L}_{\text{Yukawa}} \]

- Particle contents:
  - Chiral fermions \( f_{L,R} \)
  - Gauge bosons: \( B, W^0, \pm \)
  - Higgs \( H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} = \begin{pmatrix} \phi^+ \\ h - i\phi^0 \end{pmatrix} / \sqrt{2} \)

- Splitting functions [See Ciafaloni et al. hep-ph/0505047, Han et al. 1611.00788 for complete lists.]

\[
\frac{1}{8\pi^2} \frac{1}{k_T^2} \frac{1+z^2}{z} \quad \rightarrow \quad V_T f_s^{(l)} \quad [BW]_T^0 f_s \quad H^0(\ast) f_{-s} \quad \phi^\pm f_{-s}' \\
\frac{1}{8\pi^2} \frac{1}{k_T^2} \frac{z}{2} \\
\]

\( f_s = L, R \)

\( g_\nu^2 (Q_{f_s}^V)^2 \quad g_1 g_2 \ Y_{f_s} \quad T_{f_s}^3 \quad y_{f_R}^{(l)} \)

- Soft & collinear singularites \( (P_{gq}) \)
- Collinear singularity chirality-flip, Yukawa
Electroweak symmetry breaking

Goldstone Boson Equivalence Theorem (GBET)

[Lee, Quigg, Thacker (1977); Chanowitz & Gailard (1984)]

- At high energies $E \gg M_W$, the longitudinally polarized gauge bosons $V_L$ behave like the corresponding Goldstone bosons. They remember their origin!
- Scalarization of $V_L$

  $$\varepsilon^\mu_L(k) = \frac{E}{M_W}(\beta_W, \hat{k}) \approx \frac{k^\mu}{M_W} + \mathcal{O}(M_W/E)$$

- The GBET violation can be counted as power corrections $v/E$ → Higher twist effects in QCD ($\Lambda_{QCD}/Q$)

[Han et al. 1611.00788, Cuomo, Wulzer, 1703.08562; 1911.12366].
New splitting in a broken gauge theory

- Fermion splitting into longitudinal gauge boson $f \rightarrow V_L$

$$P \sim \frac{v^2}{k_T^2} \frac{d k_T^2}{k_T^2} \sim 1 - \frac{v^2}{Q^2}$$

- $V_L$ is of IR, $h$ has no IR [Han et al. 1611.00788]

The PDFs for $W_L/Z_L$ behaves as constants, which does not run at the leading log: “Bjorken scaling” restoration

$$f_{V_L/f}(x, Q^2) \sim \alpha \frac{1-x}{x}$$

Residuals of the EWSB, $v^2/E^2$, similar to higher-twist effects
Polarizations in the EW splittings

- The EW splittings must be polarized due to the chiral nature of the EW theory

\[
f_{V+/A+} \neq f_{V-/A-}, \quad f_{V+/A-} \neq f_{V-/A+},
\]

\[
\hat{\sigma}(V_+B_+) \neq \hat{\sigma}(V_-B_-), \quad \hat{\sigma}(V_+B_-) \neq \hat{\sigma}(V_-B_+)
\]

We are not able to factorize the cross sections in an unpolarized form.

\[
\sigma \neq f_{V/A} \hat{\sigma}(VB), \quad f_{V/A} = \frac{1}{2} \sum_{\lambda,s_1} f_{V_\lambda/A_{s_1}}, \quad \hat{\sigma}(VB) = \frac{1}{4} \sum_{\lambda,s_2} \hat{\sigma}(V_\lambda B_{s_2})
\]
Definition of (QCD) PDFs

- Fast moving proton in the $z$ direction $p^\mu = (E, 0, 0, p)$
  
  $n^\mu = (1, 0, 0, 1), \quad \bar{n} = (1, 0, 0, -1)$
  
  $n^2 = \bar{n}^2 = 0, \quad n \cdot \bar{n} = 2$

- Light-cone coordinates
  
  $p^\mu = \frac{1}{2} p^- n^\mu + \frac{1}{2} p^+ \bar{n}^\mu + p_\perp^\mu$, where
  
  $p^- = \bar{n} \cdot p = E + p_z \approx 2E, \quad p^+ = n \cdot p = E - p_z \approx \frac{m_p^2}{2E}$.

- Boost along $z$ axis,

  $p^+ \to \lambda p^+, \quad p^- \to p^- / \lambda, \quad p_\perp \to p_\perp$.

- Quark PDFs: light-cone Fourier transforms [Collins & Soper, 1982]

  $f_q(x, \mu) = \langle p | O_q(r^-) | p \rangle, \quad x = r^- / p^-$

  $O_q(r^-) = \frac{1}{4\pi} \int_{-\infty}^{\infty} d\xi e^{-i\xi r} [\bar{q}(\bar{n} \xi) \psi(\bar{n} \xi)] \bar{n} [\psi^\dagger(0) q(0)]$.

  Similar expressions for antiquark PDFs and gluon PDFs.

- Collinear PDFs are defined at $x^- = 0$ and $x_\perp = 0$, which are boost invariant.
**EW PDFs** (different from the QCD ones)

- Due to confinement, QCD observables are color invariant.

\[
O_q(r^-) = \frac{1}{4\pi} \int_{-\infty}^{\infty} d\xi \, e^{-i\xi r}[\bar{q}(\bar{n}\xi)\mathcal{W}(\bar{n}\xi)]\gamma^\mu \left[\frac{1}{T^a}\right] \mathcal{W}^+(0) q(0),
\]

\[
\langle p|\bar{q}\cdots q|p\rangle = f_q(x, \mu), \quad \langle p|\bar{q}\cdots T^a \cdots |p\rangle = 0.
\]

Equal probabilities to find the different colors, \(q_1, q_2, q_3\)

- EW symmetry is broken

\[
\langle p|\bar{q}\cdots t^a \cdots |p\rangle \neq 0
\]

That is, isospin charge is not invariant in a physical observable.

- Non-singlet PDFs \((I \neq 0)\)

\[
\langle p|\bar{q}_L\cdots t^3\cdots q_L|p\rangle = \frac{1}{2} [f_{u_L} - f_{d_L}] \neq 0, \quad f_{u_L} \neq f_{d_L},
\]

which gives non-zero non-singlet PDFs.

[Bauer et al., 1703.08562, 1712.07147; Manohar, Waalewijn, 1802.08687; Han, Ma, KX, 2007,14300, 2103.09844]
Factorization violation

- Recall the QCD collinear factorization [CSS, 80s’]
  - One-side QCD radiation (Drell-Yan, SIA, and DIS)
  - Sufficiently inclusive, i.e., $pp \rightarrow V + X$
    - Unitary condition $\sum_X |X\rangle\langle X| = 1 \implies$ Ward identity $\implies$ factorization

- Critical point: cancellation of Glauber gluon
  - $|p^+ p^-| \ll p_T^2 \ll Q^2$ [Rothstein, Stewart, 1601.04695]

- The Glauber non-cancellation leads to violation, e.g., $k_T$ in the back-to-back di-jet [Qiu, Collins 0705.2141, Collins 0708.4410]

- In the EW case, the factorization violation is everywhere [Rothstein et al., 1811.04120]

- Can we rescue it? $\implies$ Potentially!
  - Dealing with Glauber interaction
  - Deep $\leftrightarrow$ Shallow factorization [Sterman, 2207.06507]

- We need sufficiently inclusive observables, e.g., EW jets.

- New operators and formalism are needed, e.g. bared charges.
Shallow EW factorization

- A inclusive cross section can be factorized into hard, collinear (PDFs and/or FFs) and soft functions [Manohar, 1802.08687]

\[ \sigma(AB \rightarrow X) = \sum_{a,b} C_a/A C_b/B S_{ab} H_{ab}, \]

where the soft function

\[ S_{ab} = \langle 0 | S_1^\dagger t^a S_1 S_2^\dagger t^b S_2 \cdots | 0 \rangle. \]

- In the QCD case, \( t^a \rightarrow T^a \) vanish unless \( T^A = 1 \). \( S^\dagger S = 1 \) leaves a trivial soft function \( S_{ab} = 1 \).

- In the EW theory, \( S \) is not trivially identity, leads a angular dependence \( \rightarrow \) Rapidity divergence \( \Rightarrow \) Collins-Soper Equation [1981]
  \( \Leftrightarrow \) rapidity RGE in SCET [Chiu et al. 1104.0881, 1202.0814]

- EW PDFs/FFs involve both singlet and non-singlet components \( (I = 0, 1, 2) \).

- DGLAP equation in \( I \neq 0 \) sector will gives double-log evolution.
PDFs and Fragmentations (parton showers)

- **Initial state radiation (ISR): PDFs** [Bauer et al., 1703.08562; 1808.08831, Manohar et al., 1808.08831, Han, Ma, KX, 2007.14300],

  \[ f_{B}(z, \mu^2, \nu^2) = \sum_{A} \int_{z}^{1} \frac{d\xi}{\xi} f_{A}(\xi, \mu^2, \nu^2) \int_{m^2}^{\mu^2} dk_{T}^{2} \mathcal{P}_{A \rightarrow B + C}(z/\xi, k_{T}^{2}, \nu^2) \]

  \[ \frac{df_{B}(z, \mu^2, \nu^2)}{d \ln \mu^2} = \sum_{A} \int_{z}^{1} \frac{d\xi}{\xi} \frac{d\mathcal{P}_{A \rightarrow B + C}(z/\xi, \mu^2, \nu^2)}{dzdk_{T}^{2}} f_{A}(\xi, \mu^2, \nu^2) \]

  \[ \frac{df_{B}^{(I \neq 0)}(z, \mu^2, \nu^2)}{d \ln \nu^2} = \hat{\gamma}_{V} f_{B}^{(I \neq 0)}(z, \mu^2, \nu^2) \]

- **The leading order splitting gives the effective \( W \) approximation (EWA)** [Kane, Repko, Rolnick, PLB1984, Dawson, NPB1985, Chanowitz, Gaillard, NPB1985]

  \( W_{L}(Z_{L}) \) capture the remnants of EWSB, governed by power correction \( \mathcal{O}(M_{Z}^{2}/Q^{2}) \) to the Goldstone Equivalence.

- **Final state radiation (FSR): Fragmentations** [Bauer et al., 1806.10157; Han, Ma, KX, 2203.11129] or parton showers [Han et al., 1611.00788]

  \[ \Delta_{A}(t) = \exp \left[ -\sum_{B} \int_{t}^{t} dt' \int dz \frac{d\mathcal{P}_{A \rightarrow B + C}(z, t')}{dzdt'} \right] \]
Parton inside of a lepton

**Equivalent photon approximation (EPA)** [Fermi, Z. Phys. 29, 315 (1924), von Weizsacker, Z. Phys. 88, 612 (1934)]

Treat photon as a parton constituent in the lepton

\[ \sigma(\ell^- + a \rightarrow \ell^- + X) = \int dx f_{\gamma/\ell} \hat{\sigma}(\gamma a \rightarrow X) \]

\[ f_{\gamma/\ell,\text{EPA}}(x_\gamma, Q^2) = \frac{1}{2\pi} \frac{1 + (1 - x_\gamma)^2}{x_\gamma} \ln \frac{Q^2}{m_\ell^2} \]

Extra terms to Improve: [Budnev, Ginzburg, Meledin, Serbo, Phys. Rept. (1975)], [Frixione, Mangano, Nason, Ridolfi, 9310350]

Photon fusions and annihilations with initial state radiations

**Effective \(W\) approximation (EWA)** [Kane, Repko, Rolnick, PLB1984, Dawson, NPB1985, Chanowitz, Gaillard, NPB1985]
The novel features of the EWA

- The EW PDFs must be polarized due to the chiral nature of the EW theory
  \[ f_{V+/A+} \neq f_{V-/A-}, \quad f_{V+/A-} \neq f_{V-/A+}, \]
  \[ \hat{\sigma}(V_+ B_+) \neq \hat{\sigma}(V_- B_-), \quad \hat{\sigma}(V_+ B_-) \neq \hat{\sigma}(V_- B_+) \]
  We are not able to factorize the cross sections in an unpolarized form.

  \[ \sigma \neq f_{V/A} \hat{\sigma}(VB), \quad f_{V/A} = \frac{1}{2} \sum_{\lambda, s_1} f_{V\lambda/A{s_1}}, \quad \hat{\sigma}(VB) = \frac{1}{4} \sum_{\lambda, s_2} \hat{\sigma}(V\lambda B{s_2}) \]

- The interference gives the mixed PDFs
  \[ f_{\gamma Z} \sim A^{\mu\nu} Z_{\mu\nu} + \text{h.c.}, \quad f_{hZ_L} \sim hZ_L \]

- Bloch-Nordsieck theorem violation due to the non-cancelled divergence in \( f \rightarrow f'V \):
  fully inclusive observables [Manohar, 1802.08687]
  \[ f_1 = \frac{1}{2} (f_\nu + f_e) \sim \frac{\alpha_W}{2\pi} \log, \]
  \[ f_3 = \frac{1}{2} (f_\nu - f_e) \sim \frac{\alpha_W}{2\pi} \log^2 \]

- Numerical small \( \Rightarrow \) cutoff \( M_V/Q \) [Bauer et al., 1703.08562, Han, Ma, KX, 2007.14300]
Go beyond the EPA/EWA

We have been doing:

- $\ell^+\ell^-$ annihilation
- EPA and ISR
- “Effective $W$ Approx.” (EWA)

```
\[ \ell^+ \ell^- \quad \text{annihilation} \]

\[ \gamma \]

\[ \ell^+ \quad \ell^- \]

\[ \ell^+ \quad \ell^- \]

\[ \ell^+ \quad \ell^- \]

\[ \ell^+ \quad \ell^- \]

\[ \ell^+ \quad \ell^- \]

\[ \ell^+ \quad \ell^- \]

\[ \ell^+ \quad \ell^- \]

\[ \ell^+ \quad \ell^- \]

\[ \ell^+ \quad \ell^- \]

We complete:

- Above $\mu_{QCD}$: QED$\otimes$QCD
  - $q, g$ become active [Han, Ma, KX, 2103.09844]

```

\[ \ell^+ \quad \ell^- \quad \gamma \quad f \quad \bar{f} \]

\[ \ell^+ \quad \ell^- \quad \gamma \quad f \quad \bar{f} \]

\[ \ell^+ \quad \ell^- \quad \gamma \quad f \quad \bar{f} \]

\[ \ell^+ \quad \ell^- \quad \gamma \quad f \quad \bar{f} \]

- Above $\mu_{EW} = M_Z$: EW$\otimes$QCD
  - EW partons emerge [Han, Ma, KX, 2007.14300]

```

\[ \ell^+ \quad \ell^- \quad \gamma/Z/W \quad \gamma/Z/W \quad \ell^+ \]

\[ \ell^+ \quad \ell^- \quad \gamma/Z/W \quad \gamma/Z/W \quad \ell^+ \]

\[ \ell^+ \quad \ell^- \quad \gamma/Z/W \quad \gamma/Z/W \quad \ell^+ \]

\[ \ell^+ \quad \ell^- \quad \gamma/Z/W \quad \gamma/Z/W \quad \ell^+ \]

In the end, every content is a parton, i.e. the full SM PDFs.
The PDF evolution: DGLAP

- The DGLAP equations

\[
\frac{d f_i}{d \log Q^2} = \sum_I \frac{\alpha_I}{2\pi} \sum_j P_{ij}^I \otimes f_j
\]

- The initial conditions

\[f_\ell/\ell(x, m_\ell^2) = \delta(1 - x)\]

- Three regions and two matchings
  - \(m_\ell < Q < \mu_{\text{QCD}}\): QED
  - \(Q = \mu_{\text{QCD}} \lesssim 1 \text{ GeV}\): \(f_q \propto P_q \gamma \otimes f_\gamma, f_g = 0\) [Simplified Non-pert. parameterization.]
  - \(\mu_{\text{QCD}} < Q < \mu_{\text{EW}}\): QED\otimes\text{QCD}
  - \(Q = \mu_{\text{EW}} = M_Z\): \(f_\nu = f_t = f_W = f_Z = f_\gamma Z = 0\)
  - \(Q > \mu_{\text{EW}}\): EW\otimes\text{QCD}.

\[
\begin{pmatrix}
  f_B \\
  f_{W^3} \\
  f_{BW^3}
\end{pmatrix}
= \begin{pmatrix}
  c_W^2 & s_W^2 & -2c_W s_W \\
  s_W^2 & c_W^2 & 2c_W s_W \\
  c_W s_W & -c_W s_W & c_W^2 - s_W^2
\end{pmatrix}
\begin{pmatrix}
  f_\gamma \\
  f_Z \\
  f_\gamma Z
\end{pmatrix}
\]

- We work in the \((B, W)\) basis [See backup for details.]

- Double logs are retained through [Bauer, Ferland, Webber, 1703.08562.]

\[
f_3 = \frac{\alpha_W}{2\pi} \log \int_x^{1 - M_V/Q} \frac{dz P_{ff} \otimes (f_\nu - f_e)}{x} \sim \frac{\alpha_W}{2\pi} \log^2
\]

Same physics as the Rapidity RGE [Manohar, Waalewijn 1802.08687]
The QED⊗QCD PDFs for lepton colliders

**Electron beam:**
- Scale unc. 10% for $f_{g/e}$ [2103.09844]
- $\mu_{QCD}$ unc. 15%
- The averaged momentum fractions $\langle x_i \rangle = \int x f_i(x) dx \ [%]$

<table>
<thead>
<tr>
<th>$Q(e^\pm)$</th>
<th>$e_{val}$</th>
<th>$\gamma$</th>
<th>$\ell_{sea}$</th>
<th>$q$</th>
<th>$g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 GeV</td>
<td>96.6</td>
<td>3.20</td>
<td>0.069</td>
<td>0.080</td>
<td>0.023</td>
</tr>
<tr>
<td>50 GeV</td>
<td>96.5</td>
<td>3.34</td>
<td>0.077</td>
<td>0.087</td>
<td>0.026</td>
</tr>
<tr>
<td>$M_Z$</td>
<td>96.3</td>
<td>3.51</td>
<td>0.085</td>
<td>0.097</td>
<td>0.028</td>
</tr>
</tbody>
</table>

**Muon beam:**
- Scale unc. 20% for $f_{g/\mu}$ [2103.09844]
- $\mu_{QCD}$ unc. 5% [2106.01393]

<table>
<thead>
<tr>
<th>$Q(\mu^\pm)$</th>
<th>$\mu_{val}$</th>
<th>$\gamma$</th>
<th>$\ell_{sea}$</th>
<th>$q$</th>
<th>$g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 GeV</td>
<td>98.2</td>
<td>1.72</td>
<td>0.019</td>
<td>0.024</td>
<td>0.0043</td>
</tr>
<tr>
<td>50 GeV</td>
<td>98.0</td>
<td>1.87</td>
<td>0.023</td>
<td>0.029</td>
<td>0.0051</td>
</tr>
<tr>
<td>$M_Z$</td>
<td>97.9</td>
<td>2.06</td>
<td>0.028</td>
<td>0.035</td>
<td>0.0062</td>
</tr>
</tbody>
</table>
**EWPDFs of a lepton**

- The sea leptonic and quark PDFs

\[ \nu = \sum_i (\nu_i + \bar{\nu}_i), \ \ell_{\text{sea}} = \ell_{\text{val}} + \sum_{i \neq \ell_{\text{val}}} (\ell_i + \bar{\ell}_i), \ q = \sum_{i = d}^t (q_i + \bar{q}_i) \]

Even neutrino becomes active.

- All SM particles are partons [Han, Ma, KX, 2007.14300]
- \( W_L(Z_L) \) does not evolve: **Bjorken-scaling restoration**: \( f_{W_L}(x) = \frac{\alpha_2}{4\pi} \frac{1-x}{x} \).
- The EW correction can be large: \( \sim 50\% \ (100\%) \) for \( f_{d/e} \ (f_{d/\mu}) \) due to the relatively **large SU(2) gauge coupling**. [Han, Ma, KX et. al, 2106.01393]
- Scale uncertainty: \( \sim 15\% \ (20\%) \) between \( Q = 3 \) TeV and \( Q = 5 \) TeV
Parton luminosities at high-energy lepton colliders

Consider a 3 TeV $e^+e^-$ machine and a 10 TeV $\mu^+\mu^-$ machine

- Partonic luminosities for $\ell^+\ell^-$, $\gamma\ell$, $\gamma\gamma$, $qq$, $\gamma q$, $\gamma g$, $gg$, and $gg$

- The partonic luminosity of $\gamma g + \gamma q$ is $\sim 50\% \ (20\%)$ of the $\gamma\gamma$ one
- The partonic luminosities of $qq$, $gq$, and $gg$ are $\sim 2\% \ (0.5\%)$ of the $\gamma\gamma$ one
- Given the large strong coupling, sizable QCD cross sections are expected.
- Scale unc. are $\sim 20\% \ (50\%)$ for photon (gluon) luminosities
Di-jet production at possible lepton colliders

- Low-$p_T$ range is dominated by non-perturbative hadron production [backup for details]
- High-$p_T$ range $p_T > (4 + \sqrt{s}/3 \text{ TeV})$ GeV: perturbatively computable
- Threshold cut: $\hat{s} > 20$ GeV
- Detector angle: $\theta_{\text{cut}} = 5^\circ (10^\circ) \Leftrightarrow |\eta| < 3.13 (2.44)$

Including the QCD contribution leads to much larger total cross section.
- $gg$ initiated cross sections are large for its large multiplicity
- $gq$ initiated cross sections are large for its large luminosity.
- $\gamma\gamma$ initiated cross sections are slightly smaller than the EPA estimations.
- scale variation $Q : \sqrt{\hat{s}}/2 \to \sqrt{\hat{s}}$ brings a $6\% - 15\% (30\% - 40\%)$ enhancement
Kinematic distributions

A conservative acceptance cut: \(10^\circ < \theta < 170^\circ \Leftrightarrow |\eta| < 2.44\)

Two different mechanisms: \(\mu^+\mu^-\) annihilation v.s. fusion processes

- Annihilation is more than 2 orders of magnitude smaller than fusion process.
- Annihilation peaks at \(m_{ij} \sim \sqrt{s}\);
- Fusion processes peak near \(m_{ij}\) threshold.
- Annihilation sharply peaked around \(y_{ij} \sim 0\), spread out due to ISR;
- Fusion processes spread out, especially for \(\gamma q\) and \(\gamma g\) initiated ones.
Jet production dominates over $WW$ production until $p_T \gtrsim 60$ GeV or $E_j \gtrsim 200$ GeV.

QCD contributions are mostly forward-backward; $\gamma\gamma$, $\gamma q$, and $\gamma g$ initiated processes are more isotropic.
A high-energy muon collider

- All SM particles are partons at high energies: \( \langle x_i \rangle = \int x f_i(x) dx \) [%]

<table>
<thead>
<tr>
<th>( Q )</th>
<th>( \mu_{\text{val}} )</th>
<th>( \gamma, Z, \gamma Z )</th>
<th>( W^\pm )</th>
<th>( \nu )</th>
<th>( \ell_{\text{sea}} )</th>
<th>( q )</th>
<th>( g )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_Z )</td>
<td>97.9</td>
<td>2.06</td>
<td>0</td>
<td>0</td>
<td>0.028</td>
<td>0.035</td>
<td>0.0062</td>
</tr>
<tr>
<td>3 TeV</td>
<td>91.5</td>
<td>3.61</td>
<td>1.10</td>
<td>3.59</td>
<td>0.069</td>
<td>0.13</td>
<td>0.019</td>
</tr>
<tr>
<td>5 TeV</td>
<td>89.9</td>
<td>3.82</td>
<td>1.24</td>
<td>4.82</td>
<td>0.077</td>
<td>0.16</td>
<td>0.022</td>
</tr>
</tbody>
</table>

- We need polarized PDFs due to the chiral nature of EW theory
- The EW parton luminosities [Han, Ma, KX, 2007.14300]
EW semi-inclusive processes

Just like in hadronic collisions:

\[ \mu^+ \mu^- \rightarrow \text{exclusive particles} + \text{remnants} \]
$t\bar{t}$ production at a muon collider
$W^+ W^-$, $ZH$ production

$\mu^+ \mu^- \rightarrow W^+ W^-$

$\mu^+ \mu^- \rightarrow ZH$

$\sqrt{s} = 14$ TeV

$\sqrt{s} = 14$ TeV

[Han, Ma, KX et al., 2106.01393]
Summary and prospects

- A high-energy muon collider is a dream machine for new physics search, both for energy and precision frontiers

**The parton picture play an important role**
- At very high energies, the collinear splittings dominate. **All SM particles should be treated as partons that described by EW PDFs.**
- The large collinear logarithm needs to be resummed via solving the DGLAP equations, so the **QCD partons (quarks and gluons) emerge.**
- When $Q > \mu_{EW}$, the EW splittings are activated: the EW partons appear, and the existing QED$\otimes$QCD PDFs may receive big corrections.

**A high-energy muon collider is an EW version HE LHC**
- Two classes of processes: $\mu^+\mu^-$ annihilation v.s. VBF [Han, Ma, KX, 2007.14300]
- Quark and gluon initiated jet production dominates [Han, Ma, KX, 2103.09844]
- EW PDFs are essential for high-energy muon colliders [Han, Ma, KX, 2007.14300, 2106.01393]
The QED⊗QCD DGLAP evolution

- The singlets and gauge bosons

\[
\frac{d}{d \log Q^2} \begin{pmatrix} f_L \\ f_U \\ f_D \\ f_\gamma \\ f_g \end{pmatrix} = \begin{pmatrix} P_{\ell\ell} & 0 & 0 & 2N_\ell P_{\ell\gamma} & 0 \\ 0 & P_{uu} & 0 & 2N_u P_{u\gamma} & 2N_u P_{ug} \\ 0 & 0 & P_{dd} & 2N_d P_{d\gamma} & 2N_d P_{dg} \\ P_{\gamma\ell} & P_{\gamma u} & P_{\gamma d} & P_{\gamma\gamma} & 0 \\ 0 & P_{gu} & P_{gd} & 0 & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} f_L \\ f_U \\ f_D \\ f_\gamma \\ f_g \end{pmatrix}
\]

- The non-singlets

\[
\frac{d}{d \log Q^2} f_{NS} = P_{ff} \otimes f_{NS}.
\]

- The averaged momentum fractions of the PDFs: \( f_{\ell\text{val}}, f_\gamma, f_{\ell\text{sea}}, f_q, f_g \)

\[
\langle x_i \rangle = \int x f_i(x) dx, \quad \sum_i \langle x_i \rangle = 1
\]

\[
\frac{\langle x_q \rangle}{\langle x_{\ell\text{sea}} \rangle} \gtrsim \frac{N_c \left[ \sum_i (e_{u_i}^2 + \bar{e}_{u_i}^2) + \sum_i (e_{d_i}^2 + \bar{e}_{d_i}^2) \right]}{e_{\ell\text{val}}^2 + \sum_{i \neq \text{val}} (e_{\ell_i}^2 + e_{\bar{\ell}_i}^2)} = \frac{22/3}{5}
\]
The EW isospin (T) and charge-parity (CP) basis

- The leptonic doublet and singlet in the (T,CP) basis
  \[ f_0^{0\pm} = \frac{1}{4} \left[ (f_{\nu_L} + f_{\ell_L}) \pm (f_{\bar{\nu}_L} + f_{\bar{\ell}_L}) \right], \quad f_1^{1\pm} = \frac{1}{4} \left[ (f_{\nu_L} - f_{\ell_L}) \pm (f_{\bar{\nu}_L} - f_{\bar{\ell}_L}) \right]. \]
  \[ f_0^{0\pm} = \frac{1}{2} [f_{e_R} \pm f_{\bar{e}_R}] \]

- Similar for the quark doublet and singlets.

- The bosonic
  \[ f_B^{0\pm} = f_{B_+} \pm f_{B_-}, \quad f_W^{1\pm} = f_{BW_+} \pm f_{BW_-}, \]
  \[ f_W^{0\pm} = \frac{1}{3} \left[ (f_{W_+} + f_{W_-} + f_{W_3^0}) \pm (f_{W_+} + f_{W_-} + f_{W_3^0}) \right], \]
  \[ f_W^{1\pm} = \frac{1}{2} \left[ (f_{W_+} - f_{W_-}) \mp (f_{W_+} - f_{W_-}) \right], \]
  \[ f_W^{2\pm} = \frac{1}{6} \left[ (f_{W_+} + f_{W_-} - 2f_{W_3^0}) \pm (f_{W_+} + f_{W_-} - 2f_{W_3^0}) \right]. \]
The EW PDFs in the singlet/non-singlet basis

Construct the singlets and non-singlets

- **Singlets**
  
  \[ f_L^{0,1\pm} = \sum_i f_{\ell}^{0,1\pm}, \quad f_E^{0\pm} = \sum_i f_e^{0\pm}, \]

- **Non-singlets**
  
  \[ f_{L,NS}^{0,1\pm} = f_{\ell_1}^{0,1\pm} - f_{\ell_2}^{0,1\pm}, \quad f_{E,NS}^{0\pm} = f_e^{0\pm} - f_e^{0\pm} \]

- **The trivial non-singlets**
  
  \[ f_{L,23}^{0,1\pm} = f_{E,23}^{0\pm} = 0 \]

Reconstruct the PDFs for each flavors

- **The leptonic PDFs**
  
  \[ f_{\ell_1}^{0,1\pm} = \frac{f_L^{0,1\pm} + (N_g - 1) f_{L,NS}^{0,1\pm}}{N_g}, \quad f_{\ell_2}^{0,1\pm} = f_{\ell_3}^{0,1\pm} = \frac{f_L^{0,1\pm} - f_{L,NS}^{0,1\pm}}{N_g}, \]
  \[ f_e^{0\pm} = \frac{f_E^{0\pm} + (N_g - 1) f_{E,NS}^{0\pm}}{N_g}, \quad f_{e_2}^{0\pm} = f_{e_3}^{0\pm} = \frac{f_E^{0\pm} - f_{E,NS}^{0\pm}}{N_g}. \]

- **The quark components can be constructed as singlets/non-singlets, and reconstructed correspondingly as well.**
The DGLAP in the singlet and non-singlet basis

\[
\frac{d}{dL} \begin{pmatrix}
    f_L^{0\pm} \\
    f_Q^{0\pm} \\
    f_E^{0\pm} \\
    f_U^{0\pm} \\
    f_D^{0\pm} \\
    f_B^{0\pm} \\
    f_W^{0\pm} \\
    f_g^{0\pm}
\end{pmatrix} = 
\begin{pmatrix}
    P_{LL}^{0\pm} & 0 & 0 & 0 & 0 & P_{LB}^{0\pm} & P_{LW}^{0\pm} & 0 \\
    0 & P_{QQ}^{0\pm} & 0 & 0 & 0 & P_{QB}^{0\pm} & P_{QW}^{0\pm} & P_{Qg}^{0\pm} \\
    0 & 0 & P_{EE}^{0\pm} & 0 & 0 & P_{EB}^{0\pm} & 0 & 0 \\
    0 & 0 & 0 & P_{UU}^{0\pm} & 0 & P_{UB}^{0\pm} & 0 & P_{Ug}^{0\pm} \\
    0 & 0 & 0 & 0 & P_{DD}^{0\pm} & P_{DB}^{0\pm} & 0 & P_{Dg}^{0\pm} \\
    P_{BL}^{0\pm} & P_{BQ}^{0\pm} & P_{BE}^{0\pm} & P_{BU}^{0\pm} & P_{BB}^{0\pm} & 0 & 0 & 0 \\
    P_{WL}^{0\pm} & P_{WQ}^{0\pm} & 0 & 0 & 0 & P_{WW}^{0\pm} & 0 & 0 \\
    0 & P_{gQ}^{0\pm} & 0 & P_{gU}^{0\pm} & P_{gD}^{0\pm} & 0 & 0 & P_{gg}^{0\pm}
\end{pmatrix} \otimes
\begin{pmatrix}
    f_L^{0\pm} \\
    f_Q^{0\pm} \\
    f_E^{0\pm} \\
    f_U^{0\pm} \\
    f_D^{0\pm} \\
    f_B^{0\pm} \\
    f_W^{0\pm} \\
    f_g^{0\pm}
\end{pmatrix}
\]

\[
\frac{d}{dL} \begin{pmatrix}
    f_L^{1\pm} \\
    f_Q^{1\pm} \\
    f_W^{1\pm} \\
    f_B^{1\pm} \\
\end{pmatrix} = 
\begin{pmatrix}
    P_{LL}^{1\pm} & 0 & P_{LL}^{1\pm} & P_{LM}^{1\pm} \\
    0 & P_{QQ}^{1\pm} & P_{QW}^{1\pm} & P_{QM}^{1\pm} \\
    P_{WL}^{1\pm} & P_{WQ}^{1\pm} & P_{WW}^{1\pm} & 0 \\
    P_{ML}^{1\pm} & P_{MQ}^{1\pm} & 0 & P_{MM}^{1\pm}
\end{pmatrix} \otimes
\begin{pmatrix}
    f_L^{1\pm} \\
    f_Q^{1\pm} \\
    f_W^{1\pm} \\
    f_B^{1\pm}
\end{pmatrix}
\]

\[
\frac{d}{dL} f_W^{2\pm} = P_{WW}^{2\pm} \otimes f_W^{2\pm}
\]

The splitting functions can be constructed in terms of Refs. [Han et al. 1611.00788, Bauer et al. 1703.08562, 1808.08831]
Large photon induced non-perturbative hadronic production


- $\sigma_{\gamma\gamma \rightarrow \text{hadrons}}$ may reach micro-barns level at TeV c.m. energies
- $\sigma_{\ell\ell \rightarrow \text{hadrons}}$ may reach nano-barns, after folding in the $\gamma\gamma$ luminosity

• The events populate at low $p_T$ regime
  So we can separate from this non-perturbative range via a $p_T$ cut.

[T. Barklow, D. Dannheim, M. O. Sahin, and D. Schulte, LCD-2011-020]