The dance of the muon

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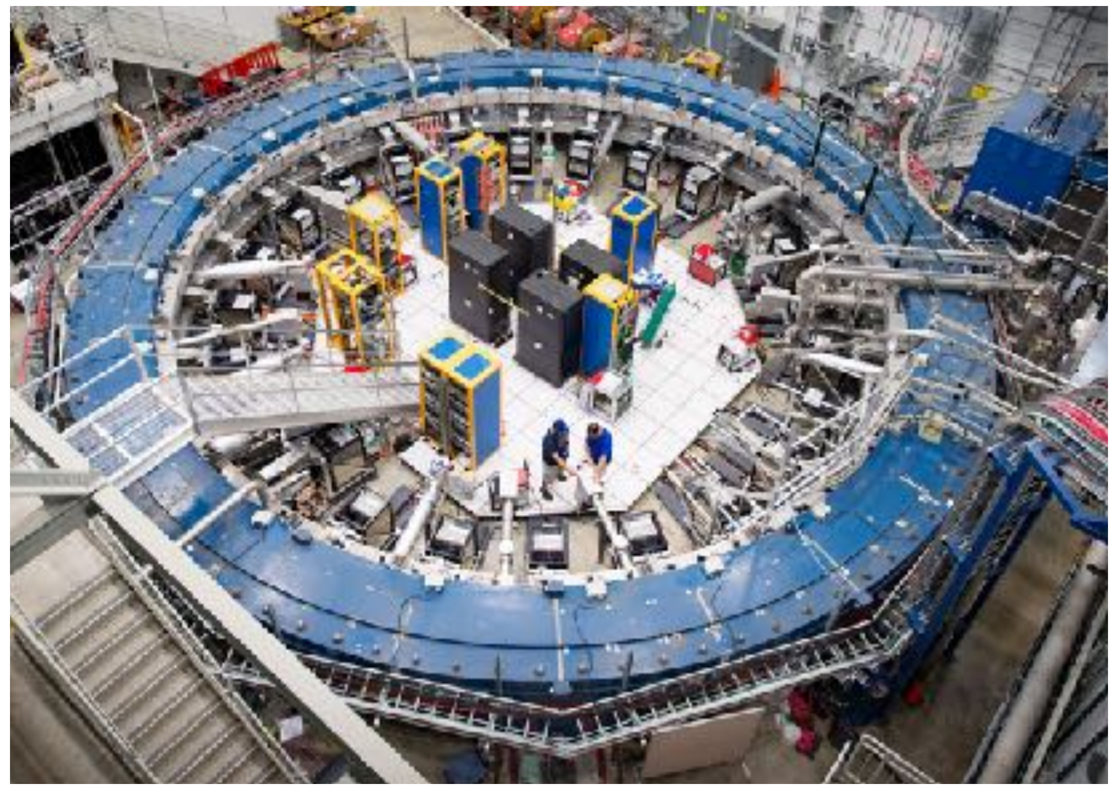
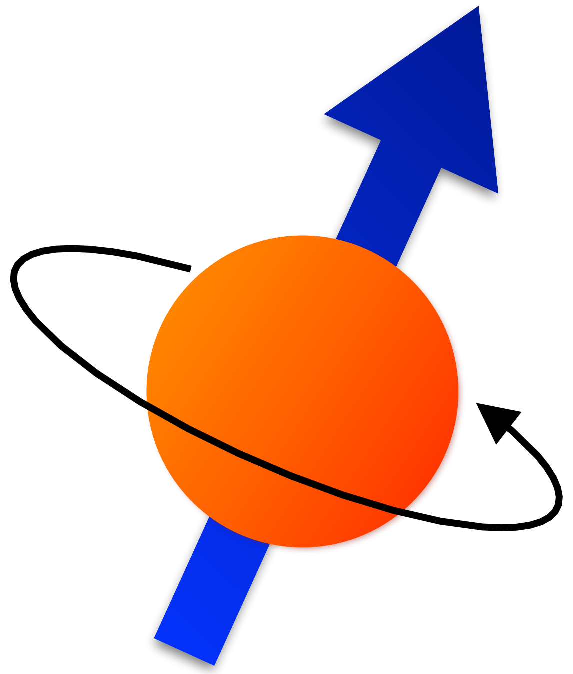


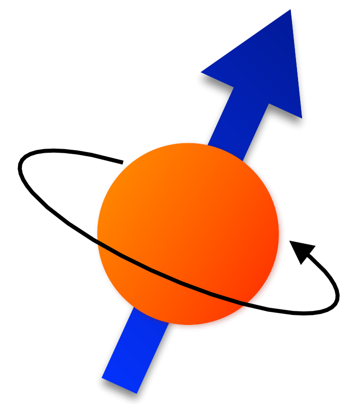
Theory Seminar
26 June 2023



Outline

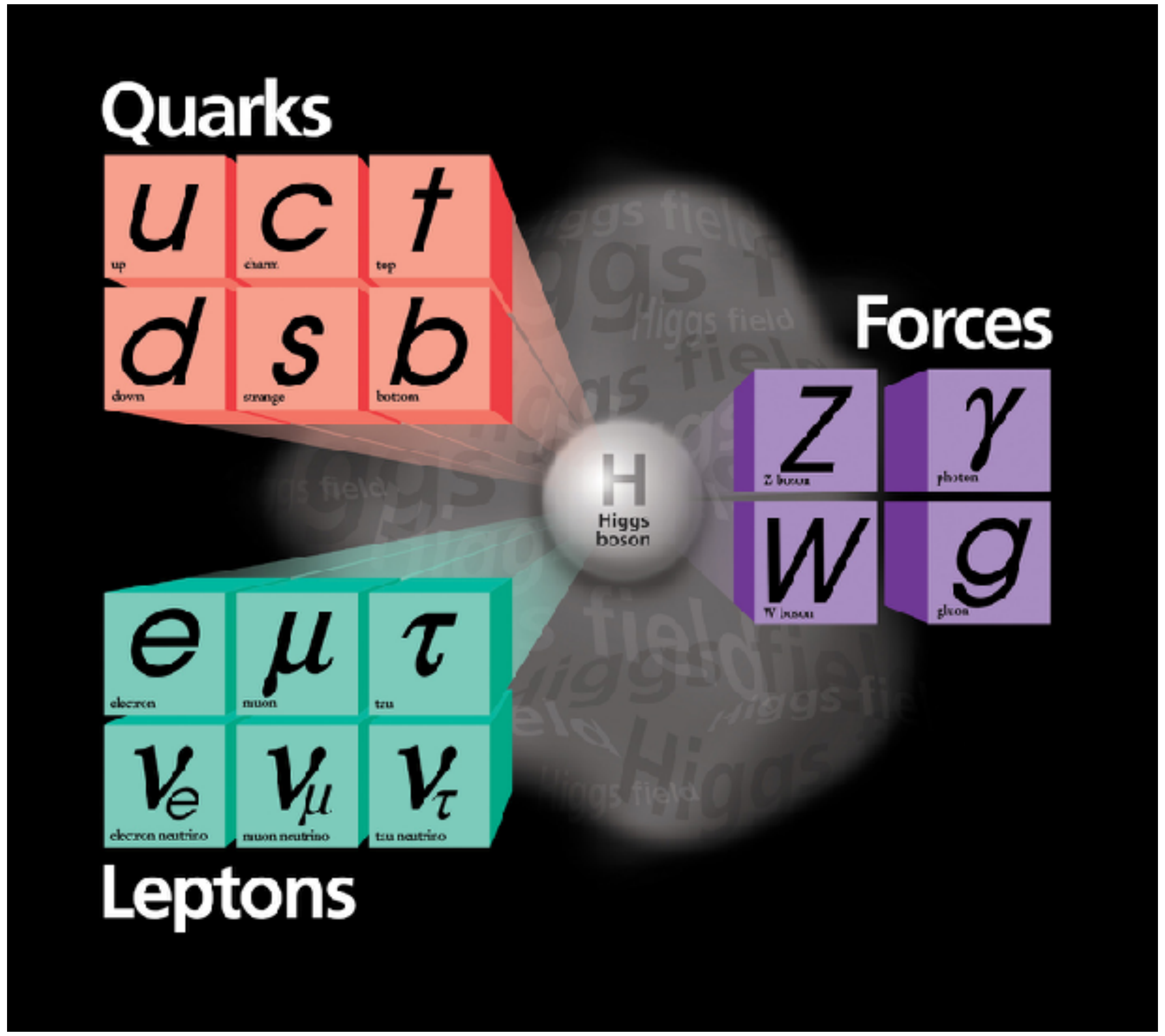
- Introduction
- Fermilab and JPARC muon g-2 experiments
- SM contribution to g-2
- Muon g-2 Theory Initiative
- dispersive/data driven methods
    - Hadronic Vacuum Polariztion (HVP)
    - Hadronic Light-by-Light (HLbL)
- lattice QCD
    - lattice HVP
    - lattice HLbL
- Connections
- Summary and Outlook

- ``The anomalous magnetic moment of the muon in the SM": 1st White Paper published in 2020 [T. Aoyama et al, arXiv:2006.04822, Phys. Repts. 887 (2020) 1-166.]
- ``Prospects for precise predictions of a_μ in the SM": 2022 Snowmass Summer Study, arXiv:2203.15810



The Standard Model of Particle Physics

The Higgs boson (discovered in 2012) completes the very successful SM



- ☆ The muon (μ) was the first **unexpected** discovery, prompting the question ``Who ordered that?", now phrased as: ``Why three generations?"
- ☆ Many other questions:
  - Dark matter
  - Dark energy
  - How (a lot) more matter than antimatter?
  - Why $m_{\text{Higgs}} \simeq 125$ GeV ?
  - …
- ☆ Answers to these questions will yield deeper insights and generically give rise to new particles and/or new forces.

If experimental measurements of the muon's anomalous magnetic moment disagree with Standard Model theory, this could be evidence for new particles and/or forces.

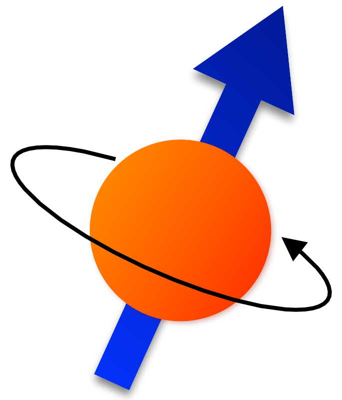
➠ discovery potential of precision measurements

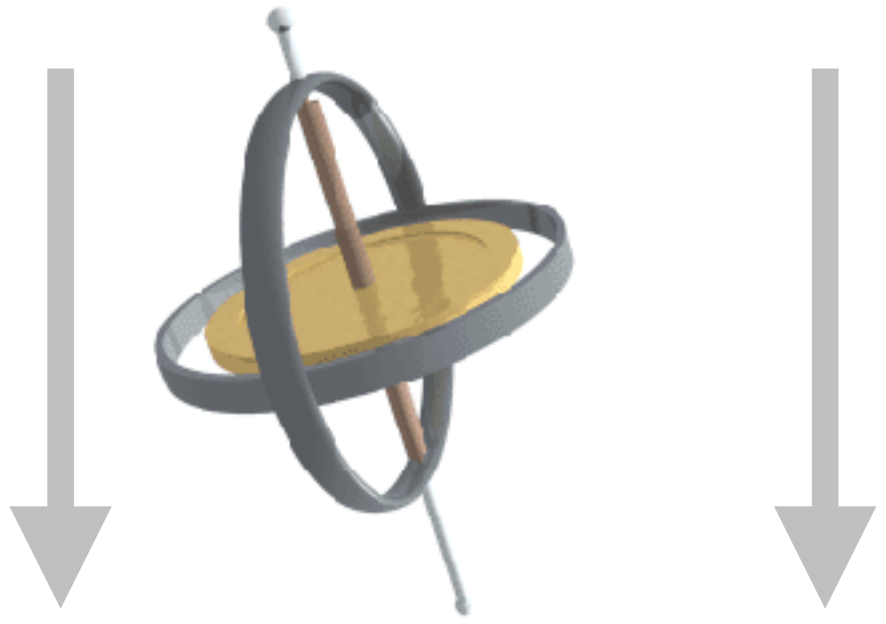
Introduction: magnetic moment

- ⭐ all leptons (electron, muon, tau-lepton, neutrinos) carry spin (intrinsic angular momentum), with spin = 1/2

  electric charge (e, μ, τ) + spin ➠ magnetic moment $\quad \vec{\mu} = g \frac{e}{2m} \vec{S}$

  ➠ a muon is a (tiny) magnetic dipole



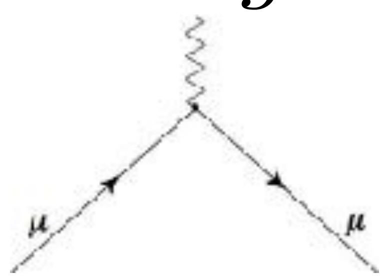


- ⭐ In a magnetic field it can precess similar to a spinning top
- ⭐ This precession can be measured very precisely.

Anomalous magnetic moment

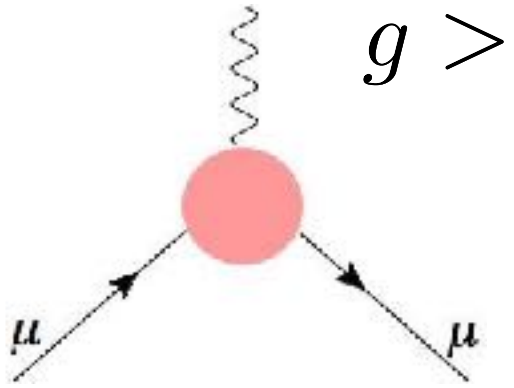
The magnetic moment of charged leptons (e, μ, τ): $\vec{\mu} = g \frac{e}{2m} \vec{S}$

Dirac: $g = 2$



quantum effects →

$g > 2$

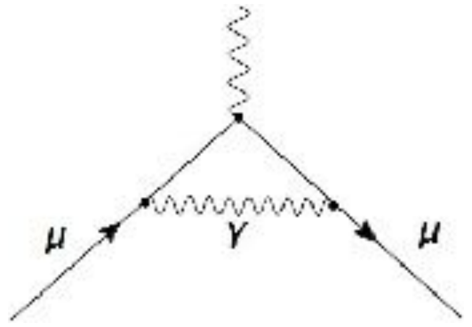


⭐ Quantum corrections change how a muon interacts with a magnetic field.

$$a \equiv \frac{g-2}{2} = 0.00116\ldots$$

⭐ All known particles contribute …

Julian Schwinger: [1948]



$$g = 2\left(1 + \frac{\alpha}{2\pi}\right) = 2(1 + 0.00116\ldots) \qquad \alpha \approx \frac{1}{137}$$

The Magnetic Moment of the Electron†

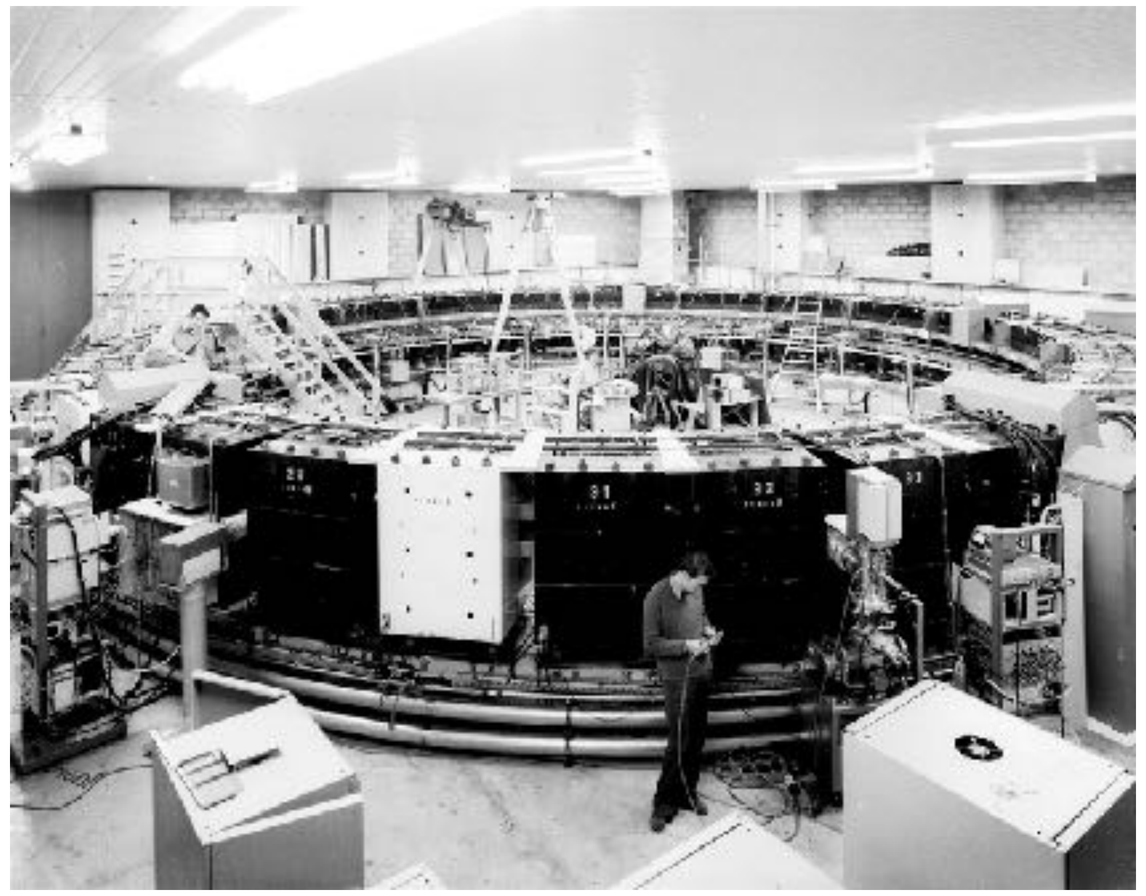
P. Kusch and H. M. Foley
Department of Physics, Columbia University, New York, New York
(Received April 19, 1948)

A comparison of the g_J values of Ga in the $^2P_{3/2}$ and $^2P_{1/2}$ states, In in the $^2P_{1/2}$ state, and Na in the $^2S_{1/2}$ state has been made by a measurement of the frequencies of lines in the *hfs* spectra in a constant magnetic field. The ratios of the g_J values depart from the values obtained on the basis of the assumption that the electron spin gyromagnetic ratio is 2 and that the orbital electron gyromagnetic ratio is 1. Except for small residual effects, the results can be described by the statement that $g_L = 1$ and $g_S = 2(1.00119 \pm 0.00005)$. The possibility that the observed effects may be explained by perturbations is precluded by the consistency of the result as obtained by various comparisons and also on the basis of theoretical considerations.

crucial test of QED and perturbation theory

Muon g-2: history of experiment vs theory

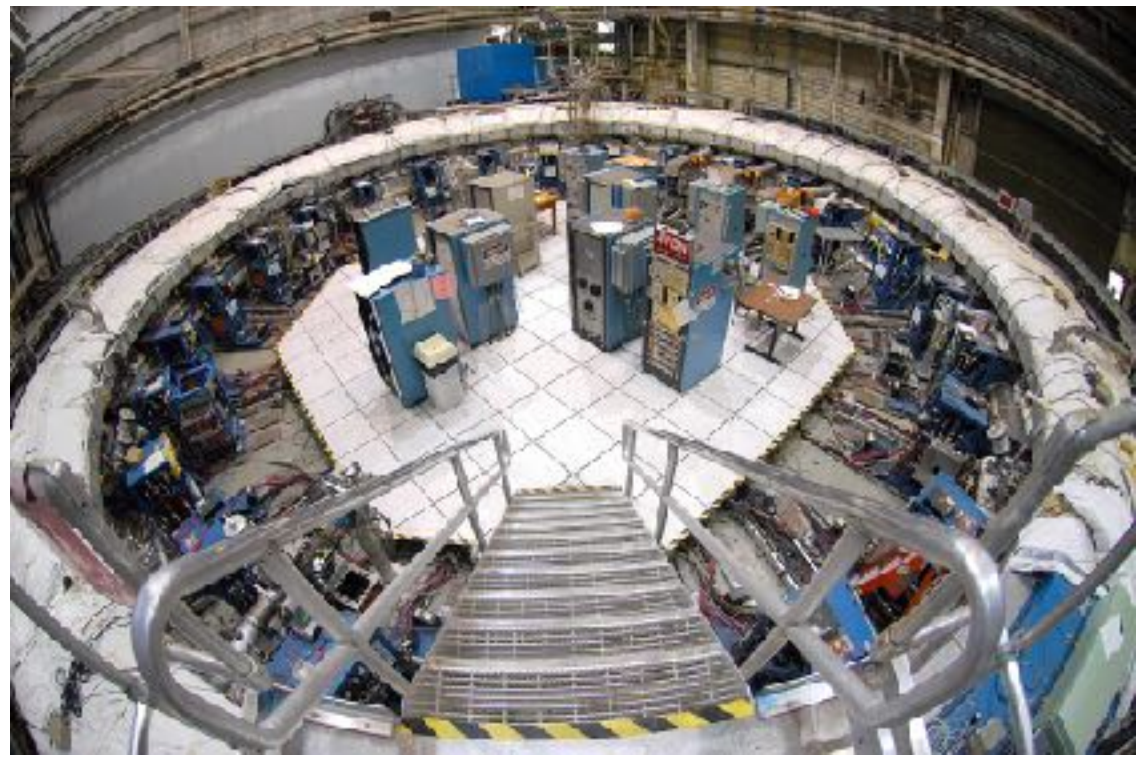
☆ 1959: start of first muon g-2 experiments at CERN

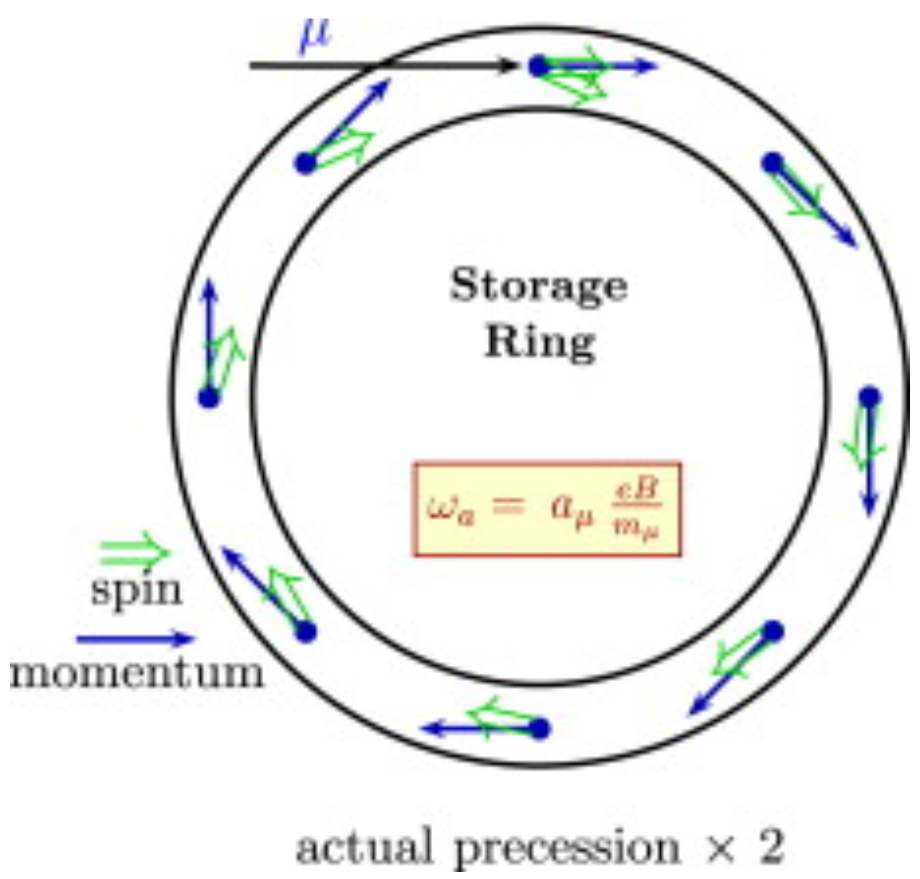


https://cds.cern.ch/record/41876

- Experimental measurements with increasing precision, down to ~10ppm
- Required more precise theoretical calculations
- Good agreement on final results

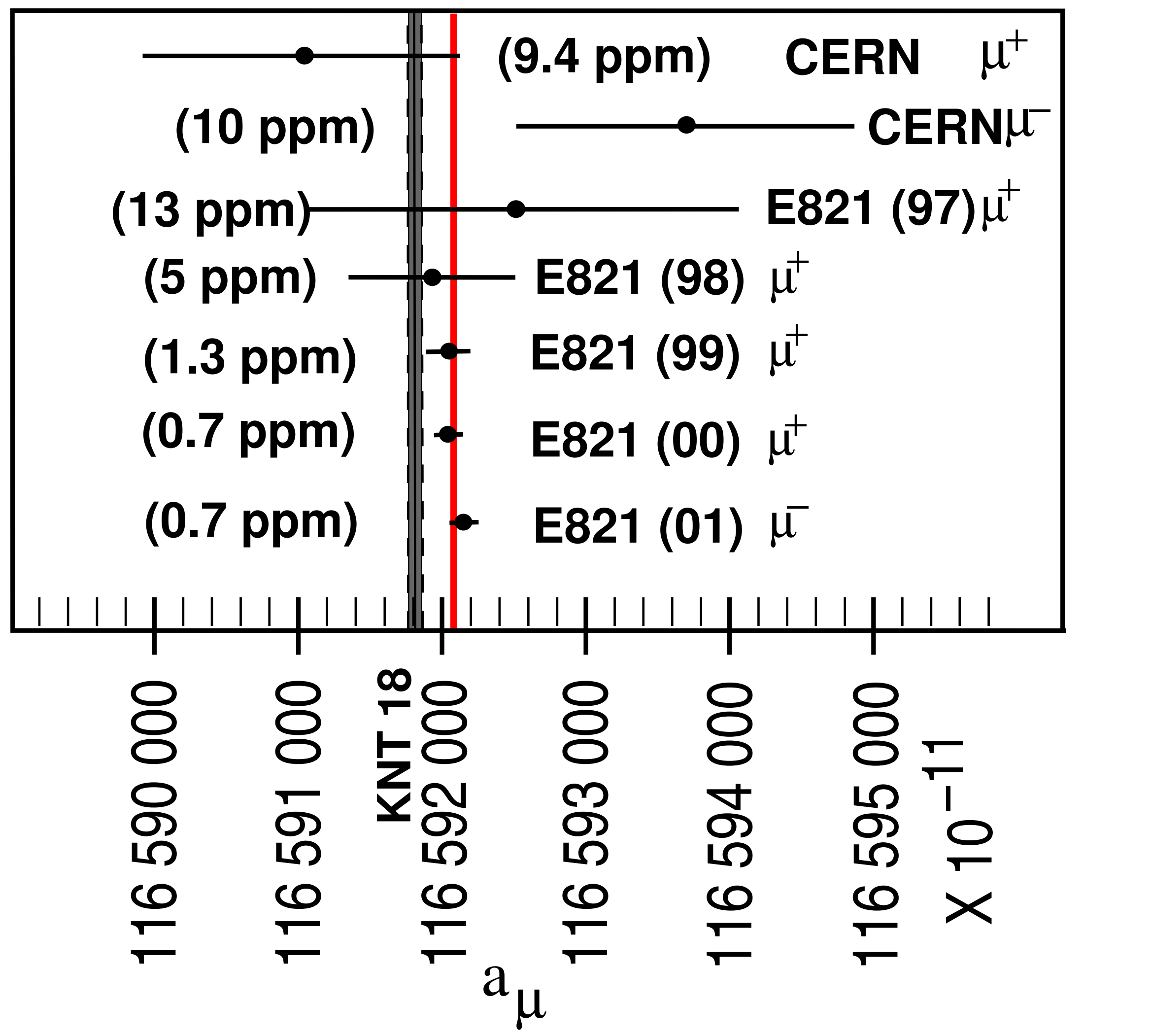
☆ 1984: start of Brookhaven experiment

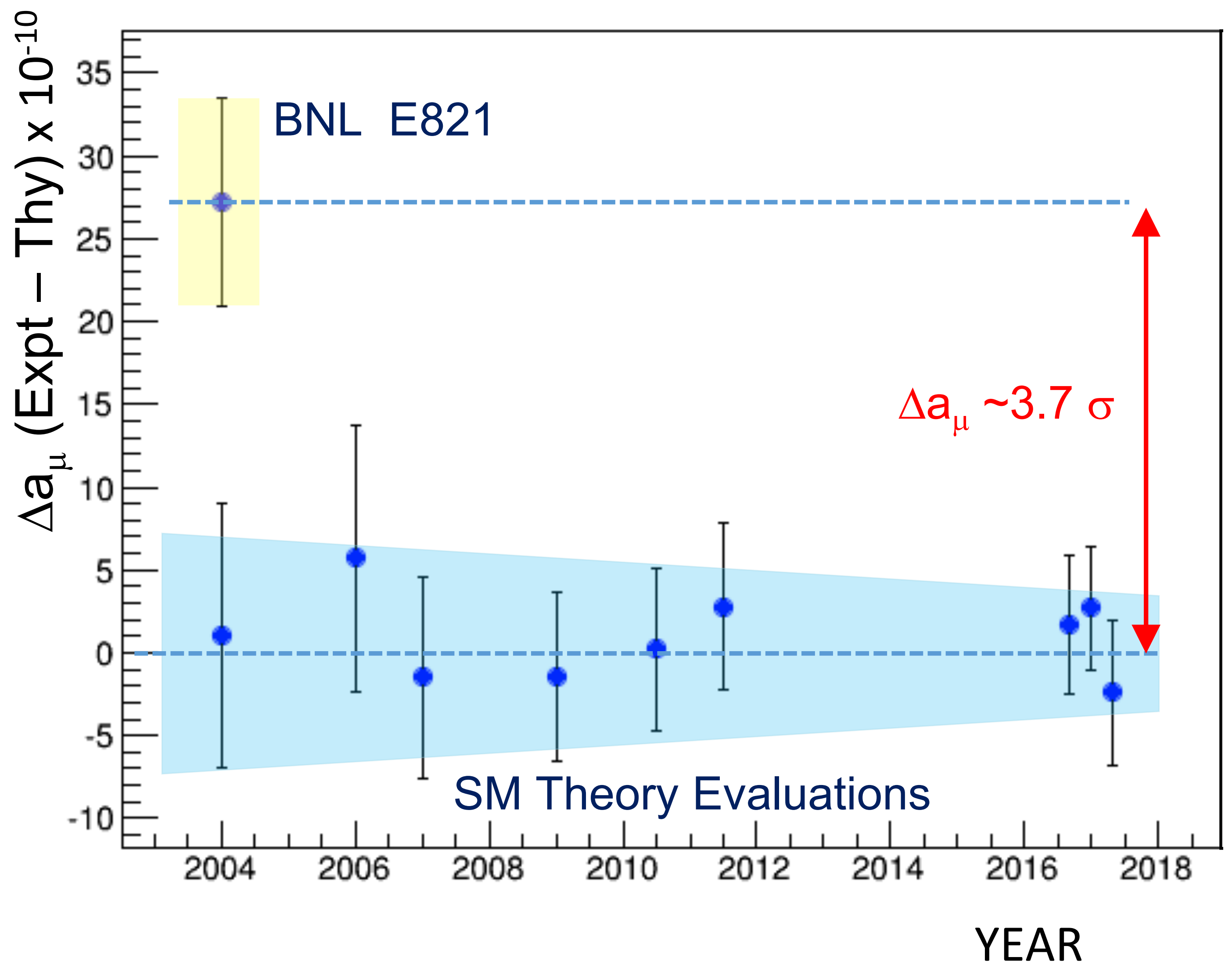




☆ By 2000: very precise measurements which disagree with theory, but not yet significant enough…

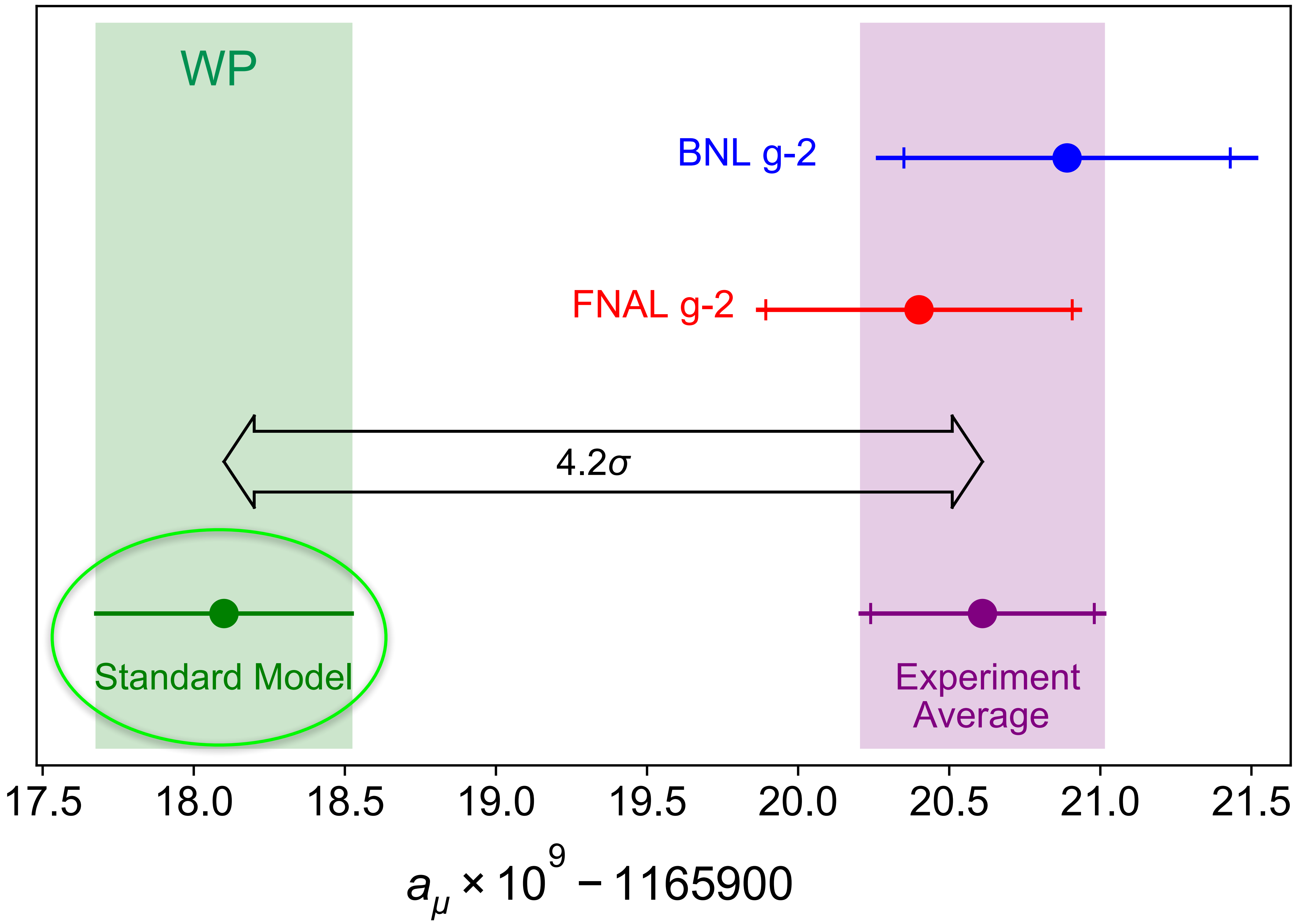
[L. Roberts, arXiv:1811.06974, SciPost Phys. Proc.]

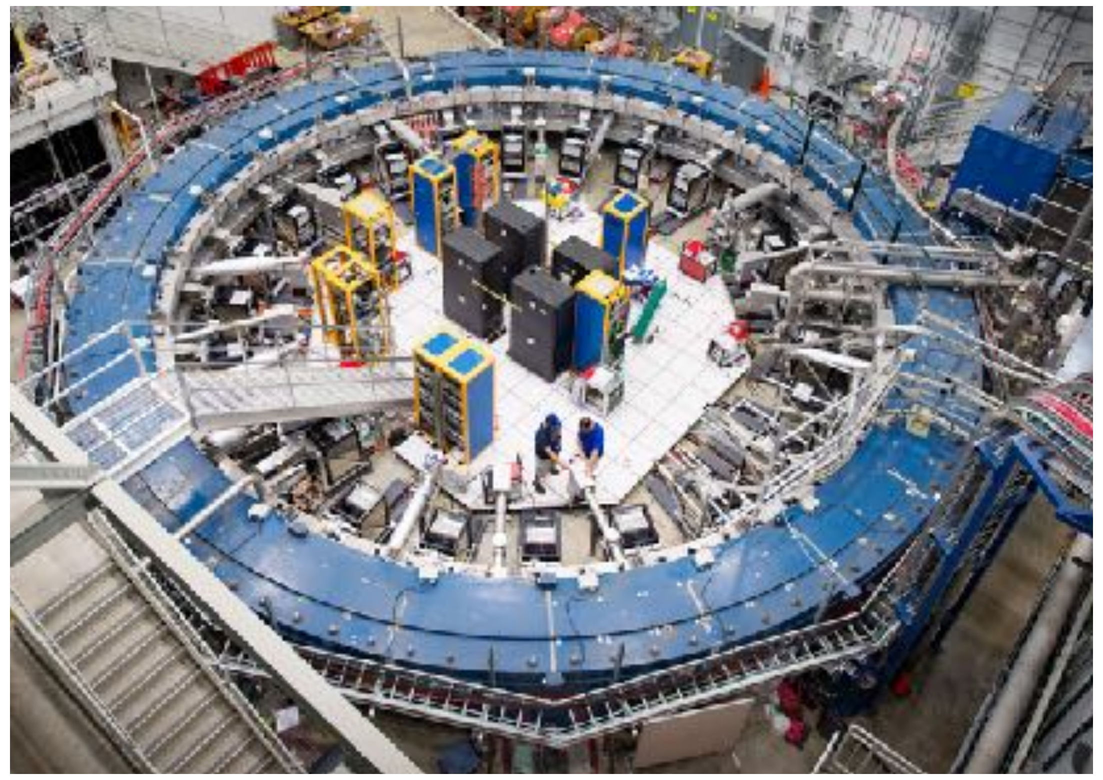


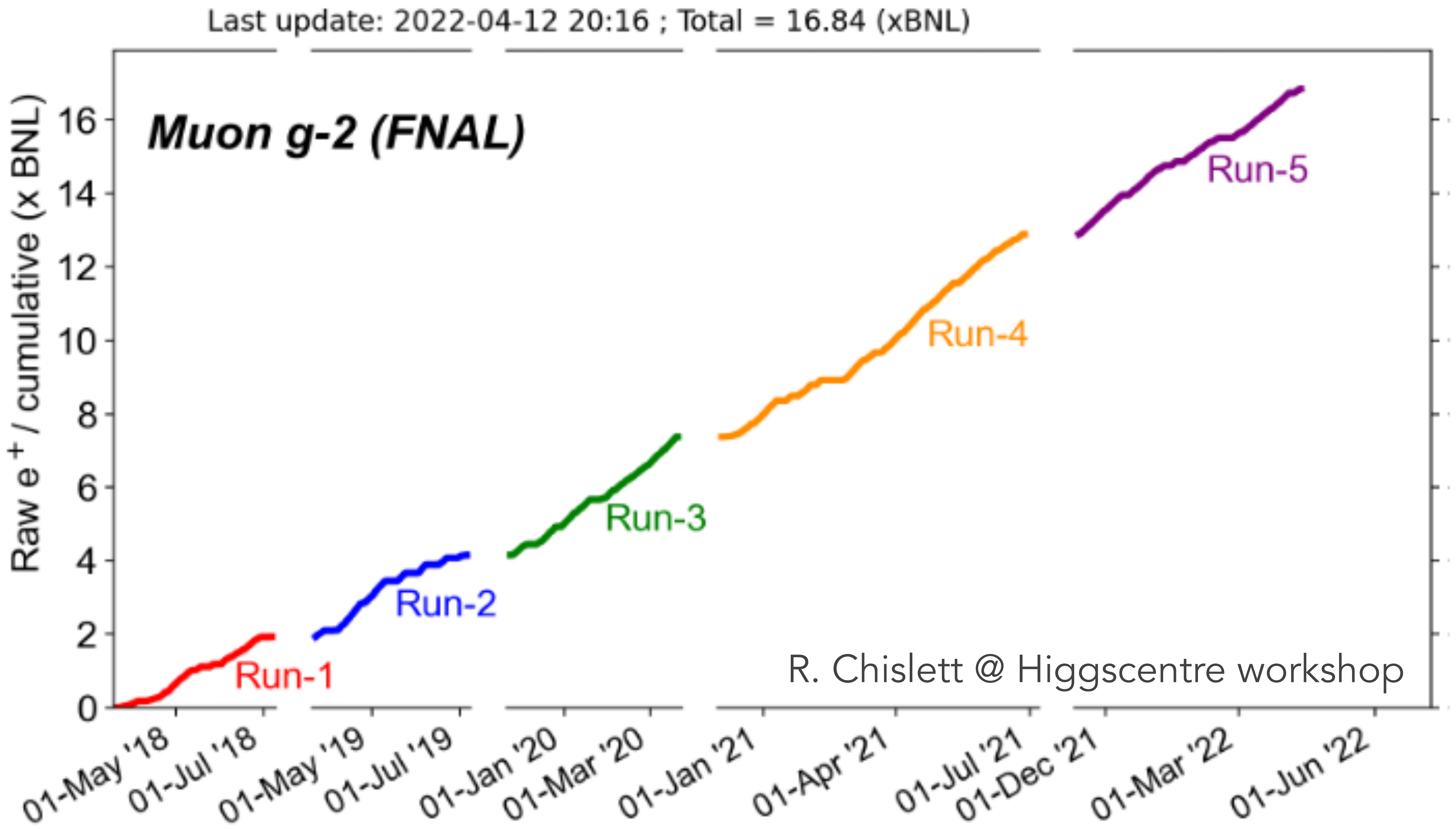


Fermilab muon g-2 experiment

- The Fermilab experiment released the measurement result from their run 1 data on 7 April 2021. [B. Abi et al, Phys. Rev. Lett. 124, 141801 (2021)]
- Runs 2 and 3 measurement: August 2023; Run 6 completed Spring 2023.

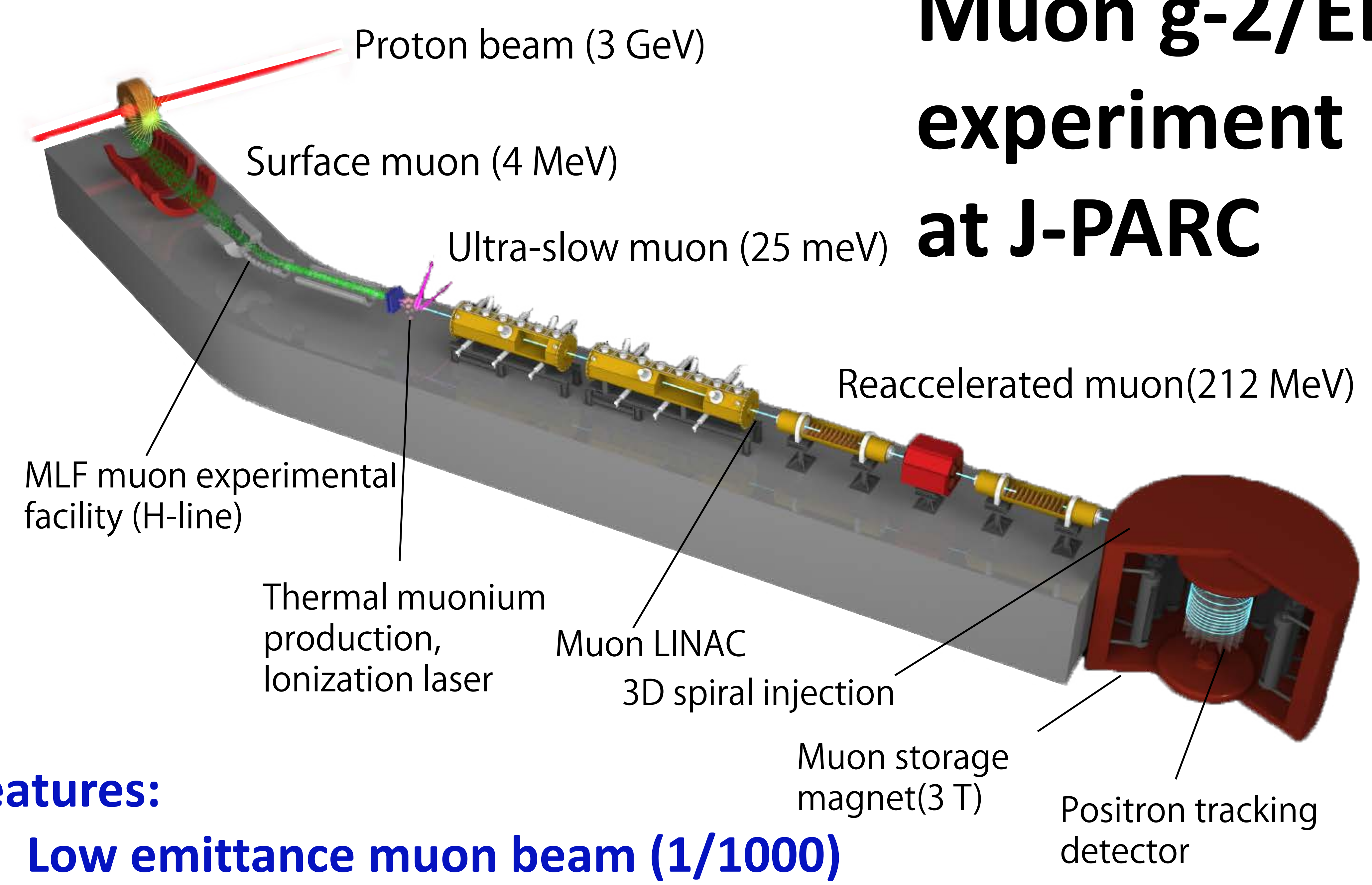






JPARC experiment

T. Mibe for E34 @ INT g-2 workshop



Muon g-2/EDM experiment at J-PARC

Features:

- **Low emittance muon beam (1/1000)**
- **No strong focusing (1/1000) & good injection eff. (x10)**
- **Compact storage ring (1/20)**
- **Tracking detector with large acceptance**
- **Completely different from BNL/FNAL method**

- 2018: Stage II approval by IPNS and IMSS directors.
- March 2019: Endorsed by KEK-SAC as a near-term priority
- 2020: Funding request
- 2024+: data taking runs

Muon g-2: SM contributions

$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$

QED

$$a_\mu(\text{QED}) = A_1 + A_2\left(\frac{m_\mu}{m_e}\right) + A_2\left(\frac{m_\mu}{m_\tau}\right) + A_3\left(\frac{m_\mu}{m_e}, \frac{m_\mu}{m_\tau}\right)$$

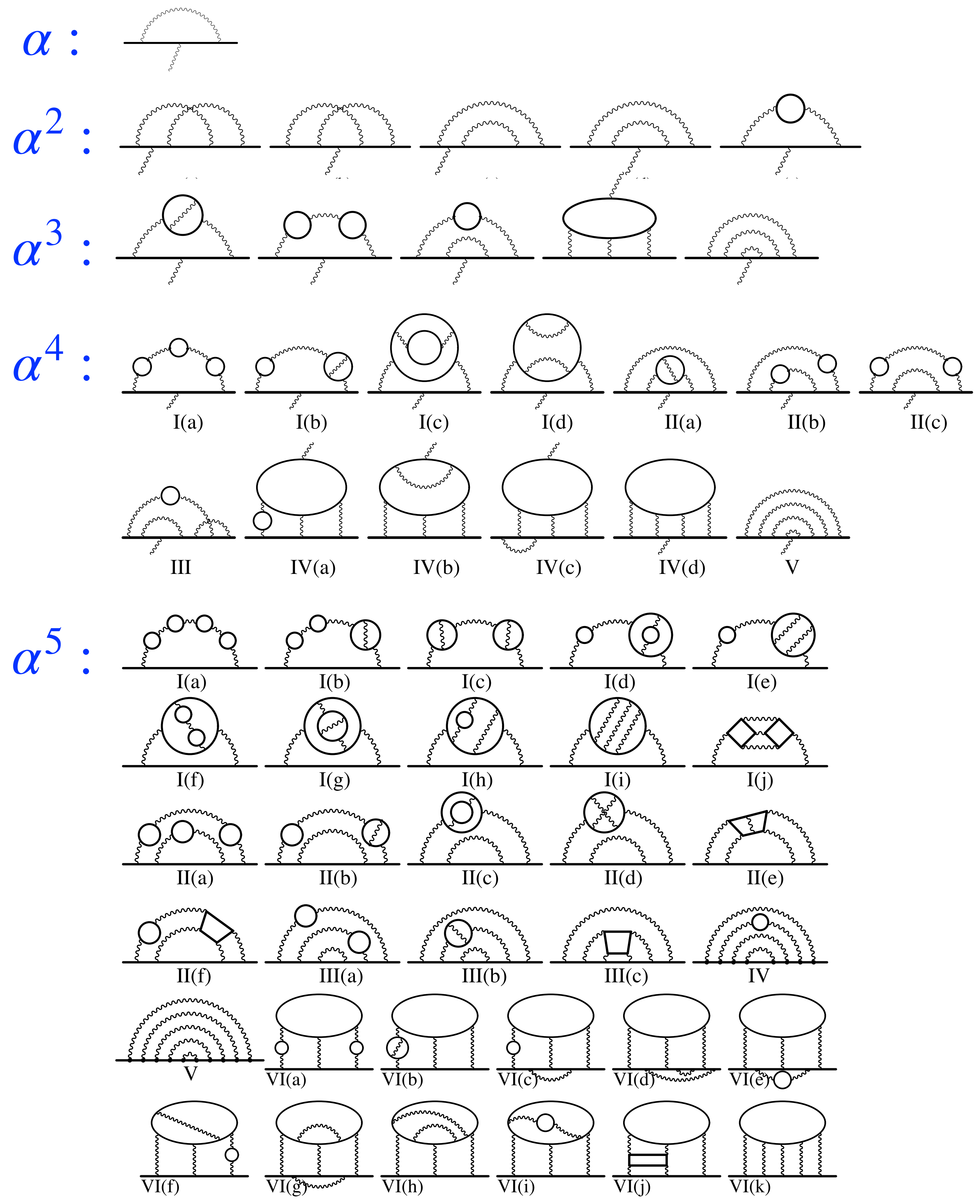
$$A_i = \sum_{n=0} \left(\frac{\alpha}{\pi}\right)^n A_i^{2n}$$

n	# of diagrams	Contribution x 10^{11}
1	1	116140973.32
2	7	413 217.63
3	71	30141.90
4	891	381.00
5	12672	5.08

$$a_\mu(\text{QED}) = 116\,584\,718.9\,(1) \times 10^{-11}$$

[T. Aoyama et al, arXiv:1205.5370, PRL;
T. Aoyama, T. Kinoshita, M. Nio, Atoms 7 (1) (2019) 28]

4.8σ difference for A_1 at 5th order in α. [Volkov, 2019]

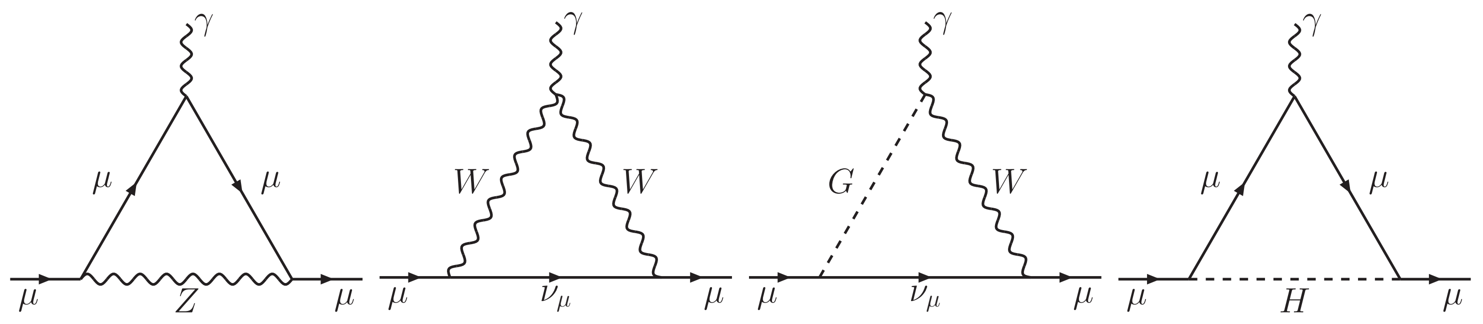


Muon g-2: SM contributions

$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$

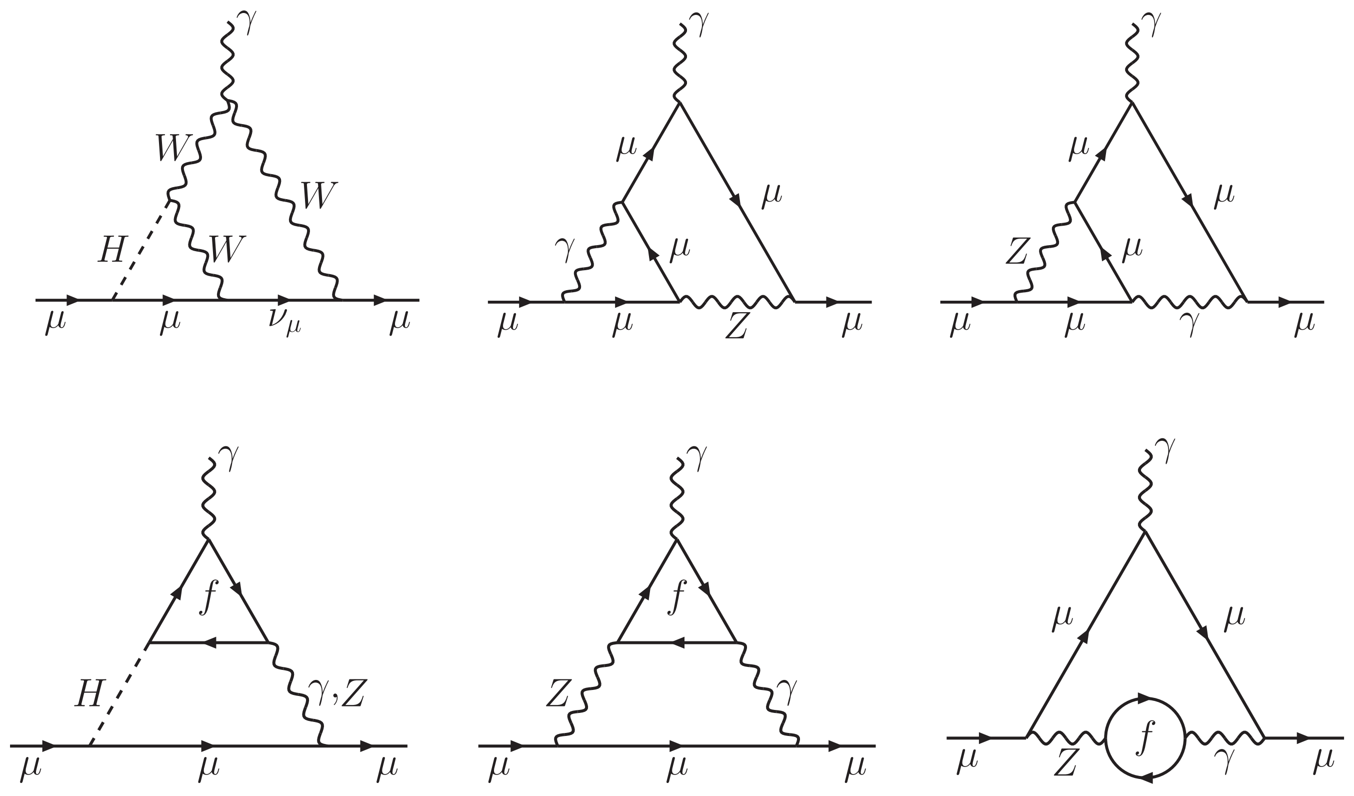
Electroweak
(contributions from W,Z,H bosons)

1-loop



Compared to QED, suppressed by $\sim \dfrac{m_\mu^2}{M_W^2} \sim 10^{-6}$

2-loop



$$a_\mu(\text{EW}) = 153.6\,(1.0) \times 10^{-11}$$

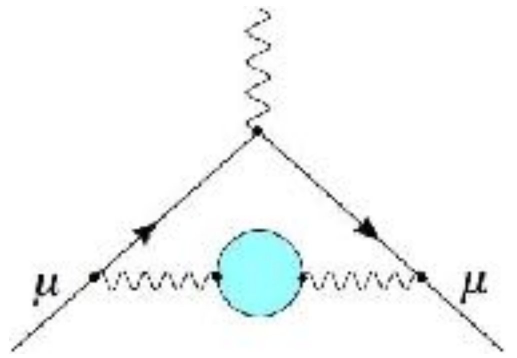
[A. Czarnecki et al, hep-ph/0212229, PRD;
C. Gnendinger et al, arXiv:1306.5546, PRD]

Muon g-2: SM contributions

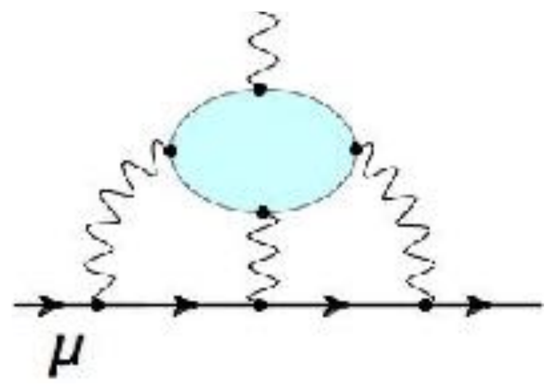
$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$

leading hadronic

α^2



α^3

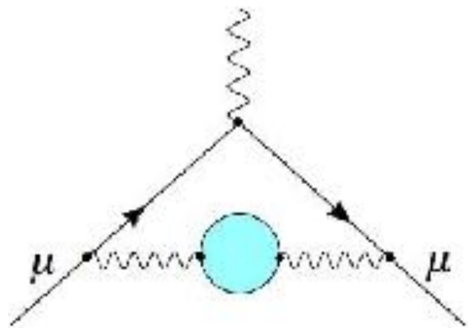


✦ The hadronic contributions are written as:

$$a_\ell(\text{hadronic}) = a_\ell^{\text{HVP, LO}} + a_\ell^{\text{HVP, NLO}} + a_\ell^{\text{HVP, NNLO}} + \ldots$$
$$+ a_\ell^{\text{HLbL}} + a_\ell^{\text{HLbL, NLO}} + \ldots$$

α^2 α^3 α^4

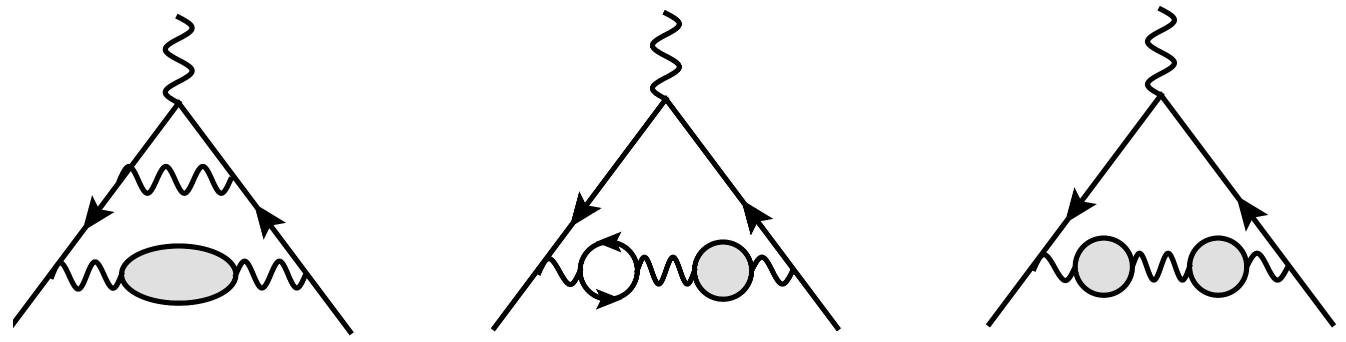
$\sim 10^{-7}$



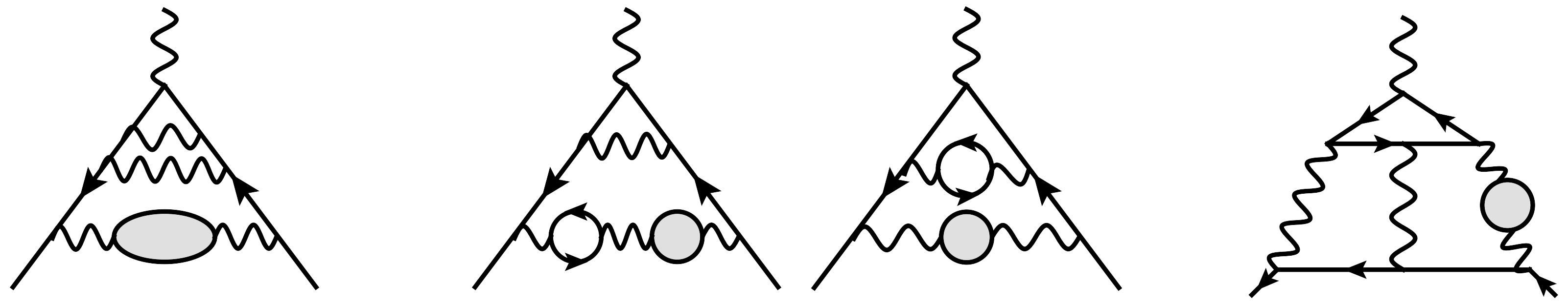
HVP: higher order (NLO, NNLO)

Figures by T. Teubner

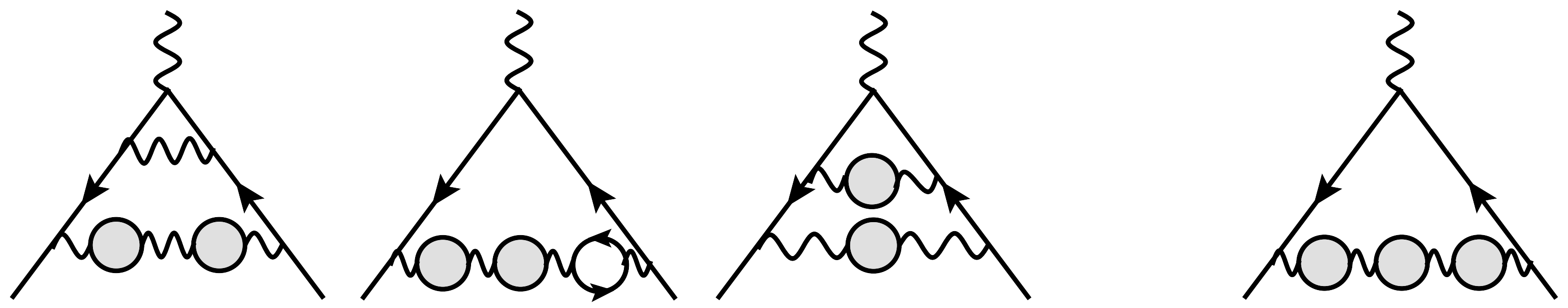
$a_\mu^{\text{HVP,NLO}} = -9.83(7) \times 10^{-10}$ [based on KNT 2019]



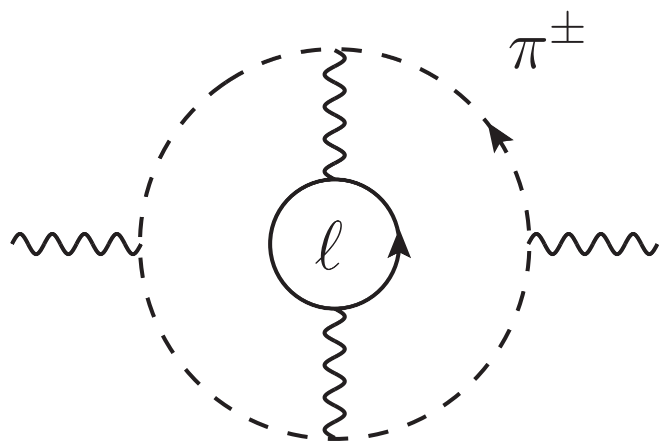
$a_\mu^{\text{HVP,NNLO}} = 1.24(1) \times 10^{-10}$ [Kurz et al, arXiv:1403.6400, PLB 2014]



space-like NLO and NNLO HVP kernels for LQCD evaluations and MUonE
[Balsani et al, arXiv:2112.05704; Nesterenko, arXiv:2209.03217, arXiv: 2112.05009]



mixed leptonic, hadronic (double bubble) contributions to a_μ are $< 10^{-11}$
[Hoferichter + Teubner, arXiv:2112.06929]



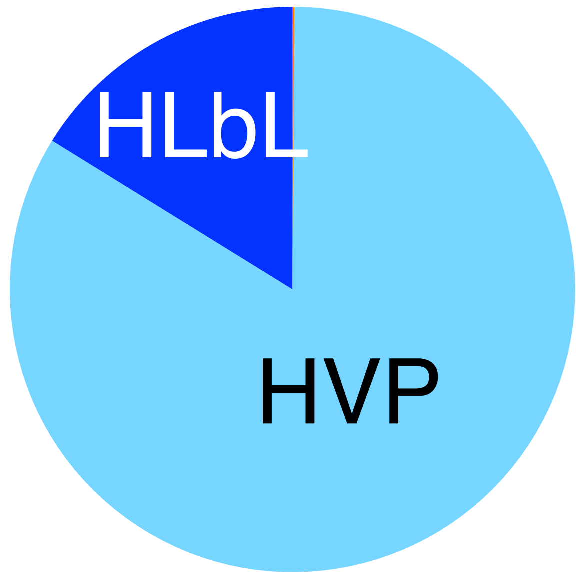
Muon g-2: SM contributions

$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$

contribution

- QED
- EW
- HVP
- HLbL

error²

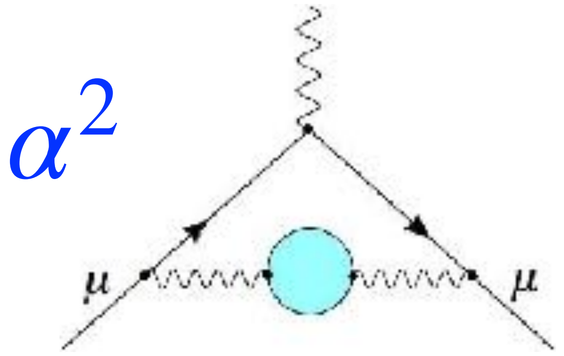


$116\,584\,718.9\,(1) \times 10^{-11}$ 0.001 ppm

$153.6\,(1.0) \times 10^{-11}$ 0.01 ppm

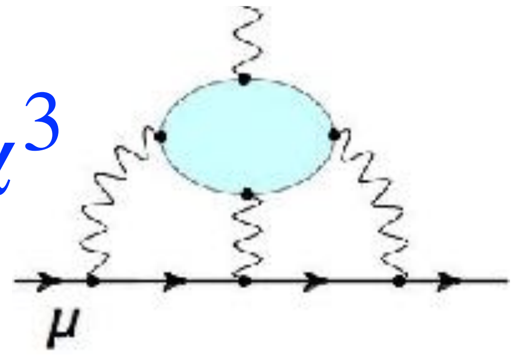
Hadronic corrections

HVP α^2

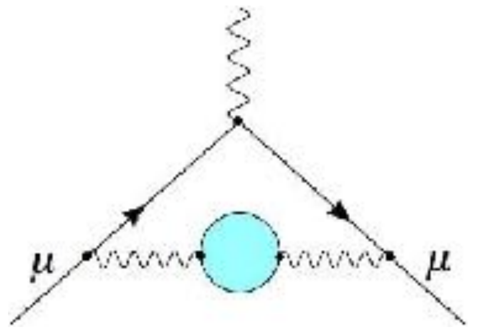


+… (NNLO) $6845\,(40) \times 10^{-11}$ [0.6%] 0.34 ppm

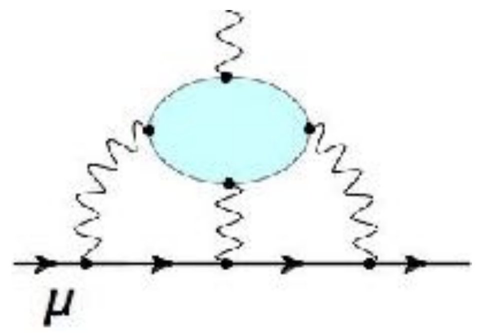
HLbL α^3



+… (NLO) $92\,(18) \times 10^{-11}$ [20%] 0.15 ppm



Hadronic Corrections



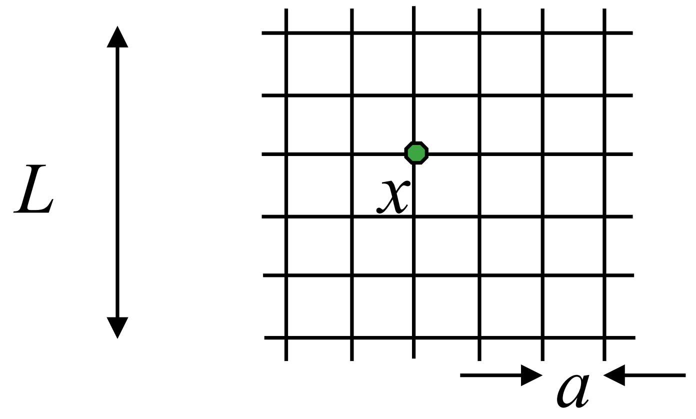
Two different, independent strategies:

- **For HVP:** use dispersion relations to rewrite integral in terms of hadronic cross section:

Many experiments (over 20+ years) have measured the e^+e^-cross sections for the different channels over the needed energy range with increasing precision.

For HLbL: new dispersive approach

- Direct calculation using Euclidean Lattice QCD



Approximations:
discrete space-time (spacing a)
finite spatial volume (L), and time extent (T)
…

Integrals are evaluated numerically using Monte Carlo methods.

- *ab-initio* method to quantify QCD effects
- already used for simple hadronic quantities with high precision
- requires large-scale computational resources
- allows for entirely SM theory based evaluations

Muon g-2 Theory Initiative

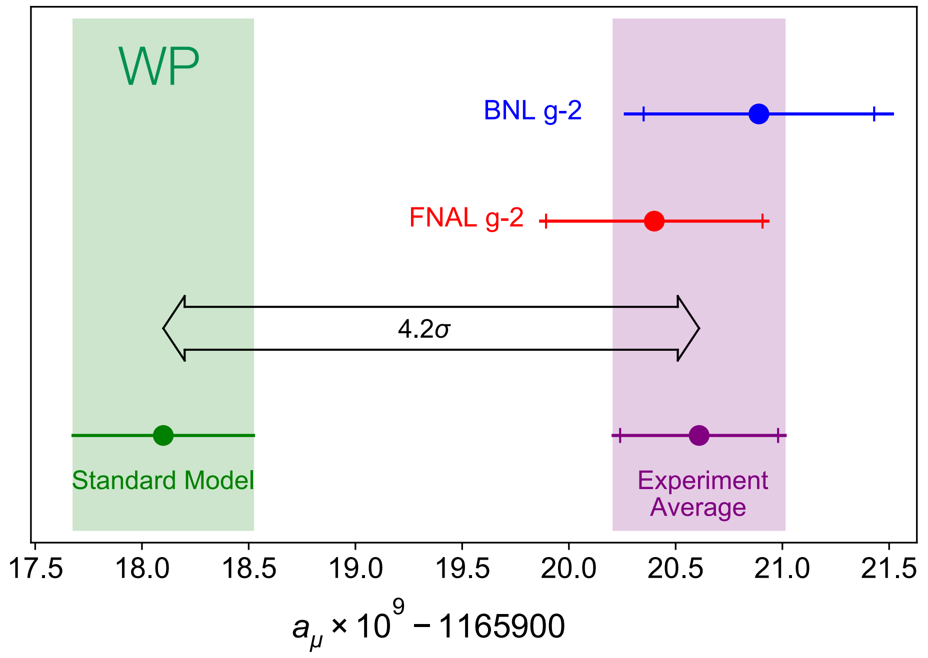
Steering Committee

- Gilberto Colangelo (Bern)
- Michel Davier (Orsay) co-chair
- Aida El-Khadra (UIUC & Fermilab) chair
- Martin Hoferichter (Bern)
- Christoph Lehner (Regensburg University) co-chair
- Laurent Lellouch (Marseille)
- Tsutomu Mibe (KEK) J-PARC Muon g-2/EDM experiment
- Lee Roberts (Boston) Fermilab Muon g-2 experiment
- Thomas Teubner (Liverpool)
- Hartmut Wittig (Mainz)

https://muon-gm2-theory.illinois.edu

- Maximize the impact of the Fermilab and J-PARC experiments
  ➠ quantify and reduce the theoretical uncertainties on the hadronic corrections
- summarize the theory status and assess reliability of uncertainty estimates
- organize workshops to bring the different communities together:
  First plenary workshop @ Fermilab: 3-6 June 2017
  HVP workshop @ KEK: 12-14 February 2018
  HLbL workshop @ U Connecticut: 12-14 March 2018
  Second plenary workshop @ HIM (Mainz): 18-22 June 2018
  Third plenary workshop @ INT (Seattle): 9-13 September 2019
  Lattice HVP at high precision workshop (virtual): 16-20 November 2020
  Fourth plenary workshop @ KEK (virtual): 28 June - 02 July 2021
  Fifth plenary workshop @ Higgs Centre (Edinburgh): 5-9 September 2022
  Sixth plenary workshop @ University of Bern: 4-8 September 2023
  Seventh plenary workshop @ KEK (Japan): June 2024

Near-term Timeline

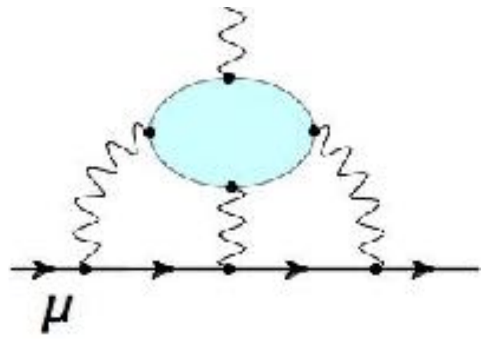


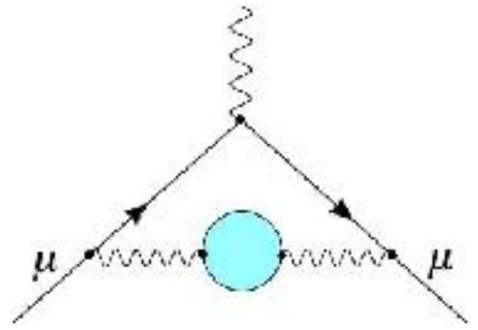
Theory Initiative:

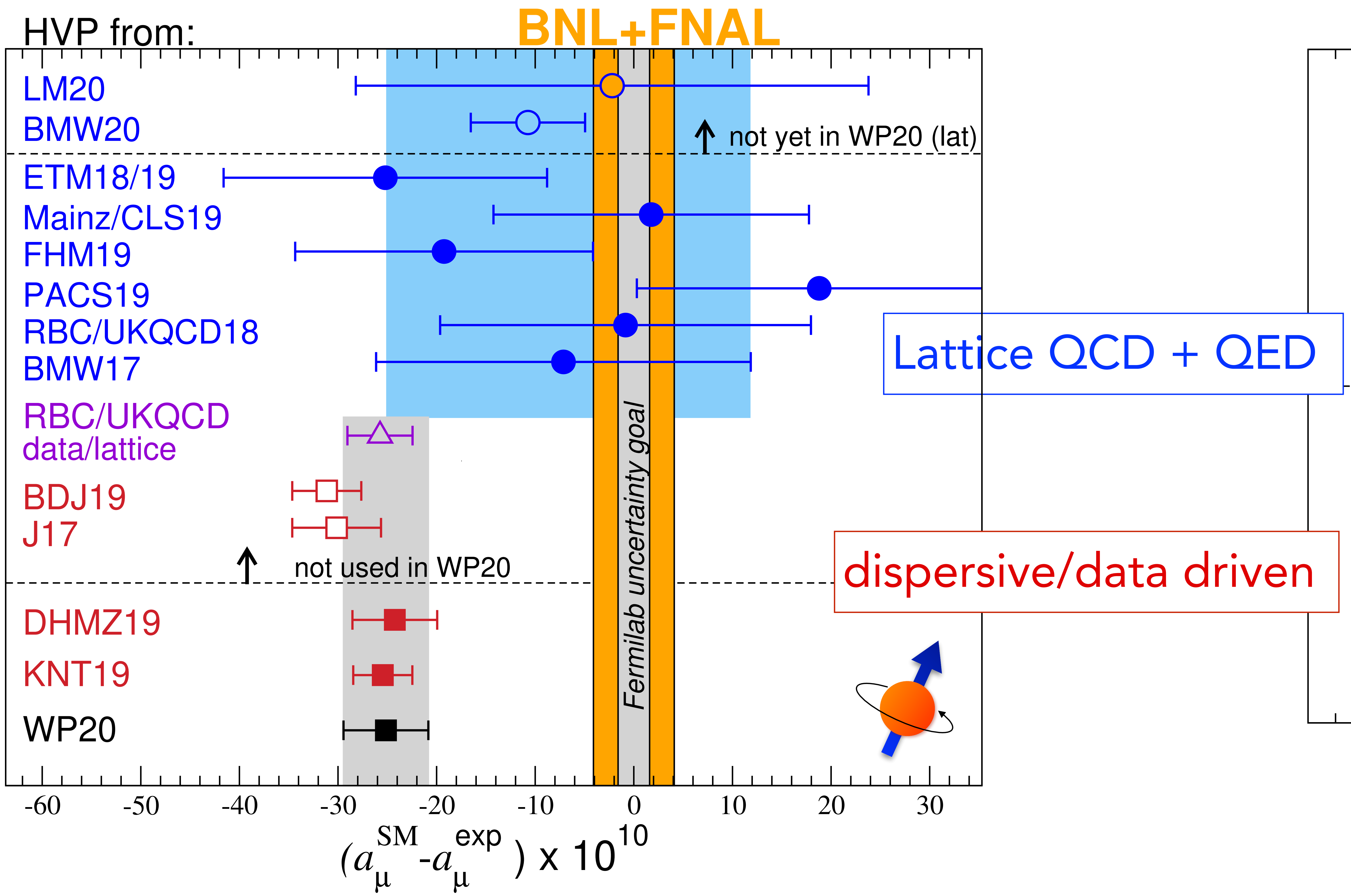
- ☆ ongoing activities: develop method average for Lattice HVP
- ☆ CMD-3 seminar (virtual): **27 March 2023** at 8:00am US CDT
- ☆ WP update with all available results ~ late 2023

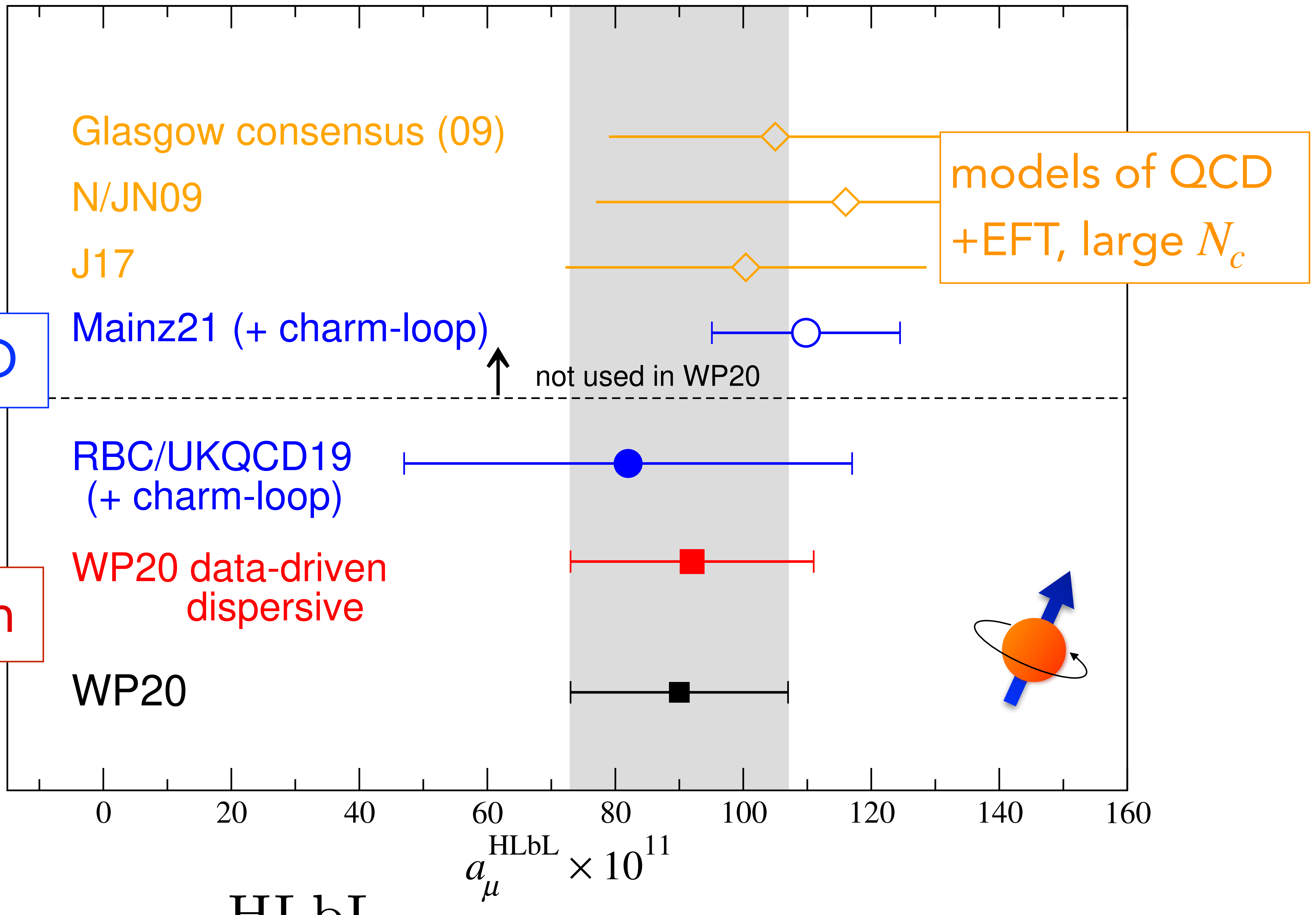
★ TI workshops:
Jun 2021 @ KEK (virtual)
Sep 2022 @ Higgscentre
Sep 2023 @ Bern
Summer 2024 @ KEK

Hadronic Corrections: Comparisons









$$a_\mu^{\rm HLbL}$$

$$a_\mu^{\rm SM} \quad a_\mu^{\rm HVP} + \left[a_\mu^{\rm QED} + a_\mu^{\rm Weak} + a_\mu^{\rm HLbL}\right]$$

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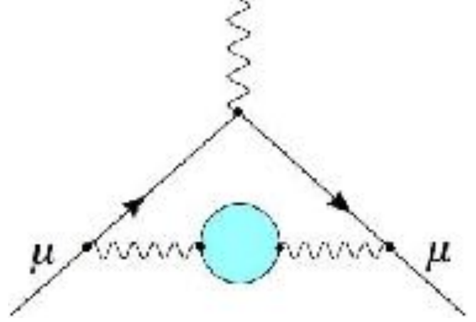
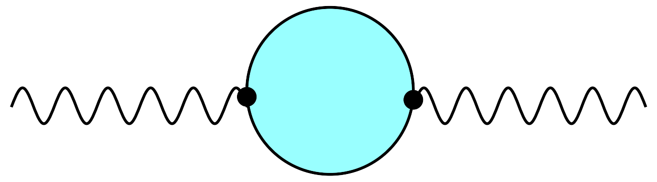


Hadronic vacuum polarization



$$\Pi_{\mu\nu} = \int d^4x e^{iqx} \langle j_\mu(x) j_\nu(0) \rangle = (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi(q^2) \qquad \hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$$

Leading order HVP correction:

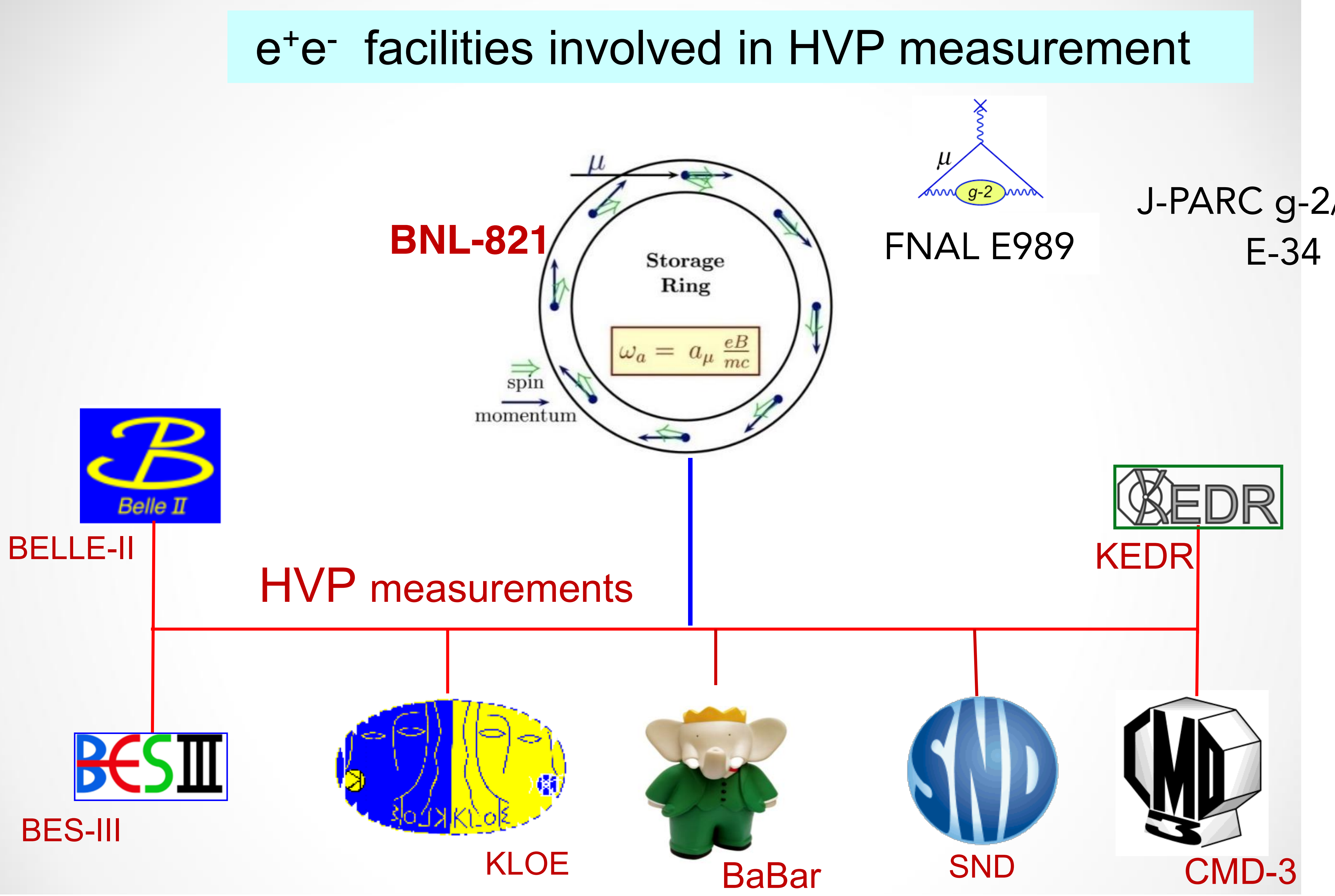
$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2)\, \hat{\Pi}(q^2)$$

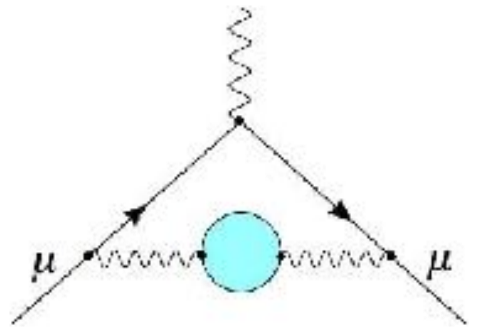
- Use optical theorem and dispersion relation to rewrite the integral in terms of the hadronic e^+e^- cross section:

$$a_\mu^{\mathrm{HVP,LO}} = \frac{m_\mu^2}{12\pi^3} \int ds \frac{\hat{K}(s)}{s} \sigma_{\exp}(s)$$

Dominant contributions from low energies;
$\pi^+\pi^-$ channel: 73% of total

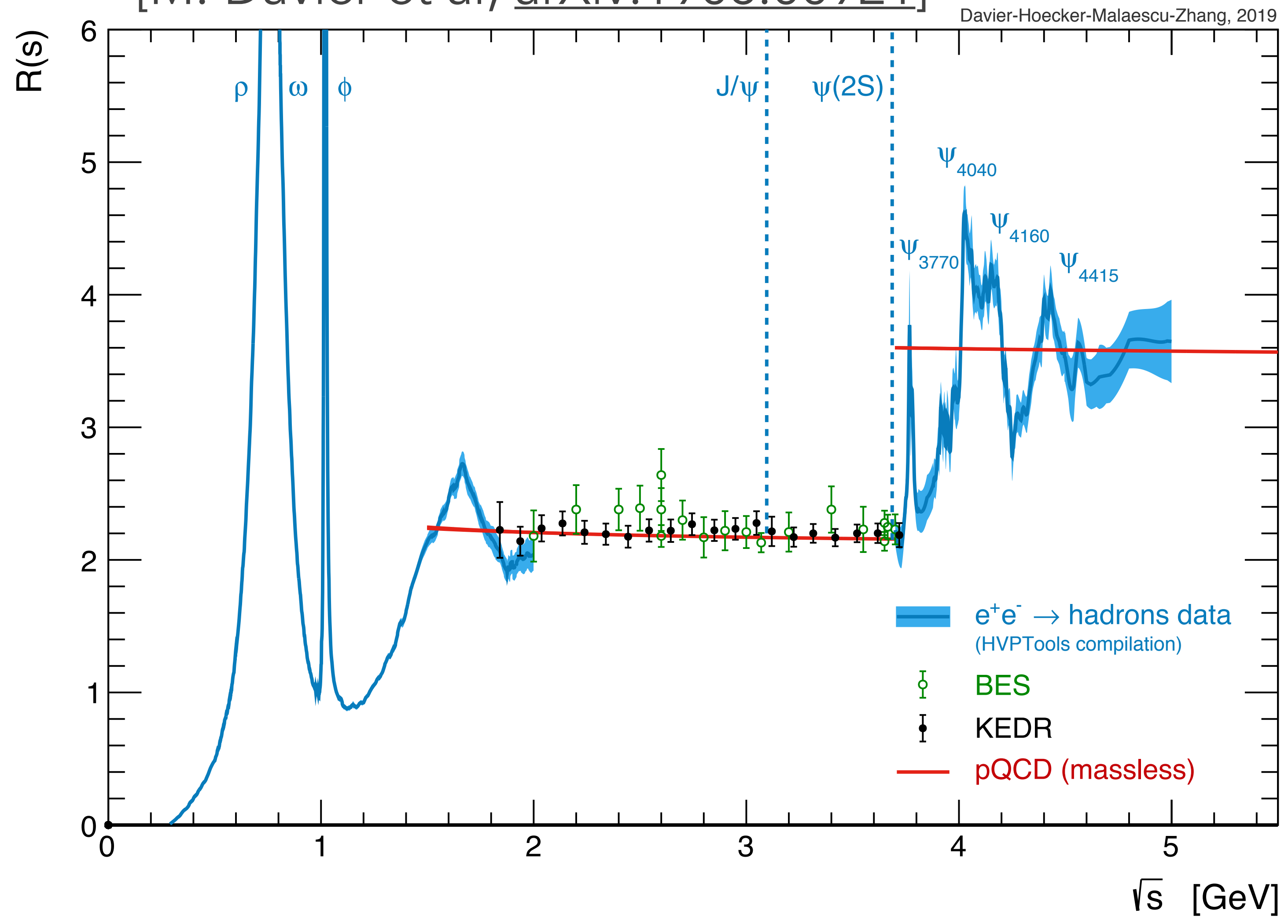
- Use **direct integration method**, summing up cross sections for all possible hadronic channels up to ~ 2 GeV

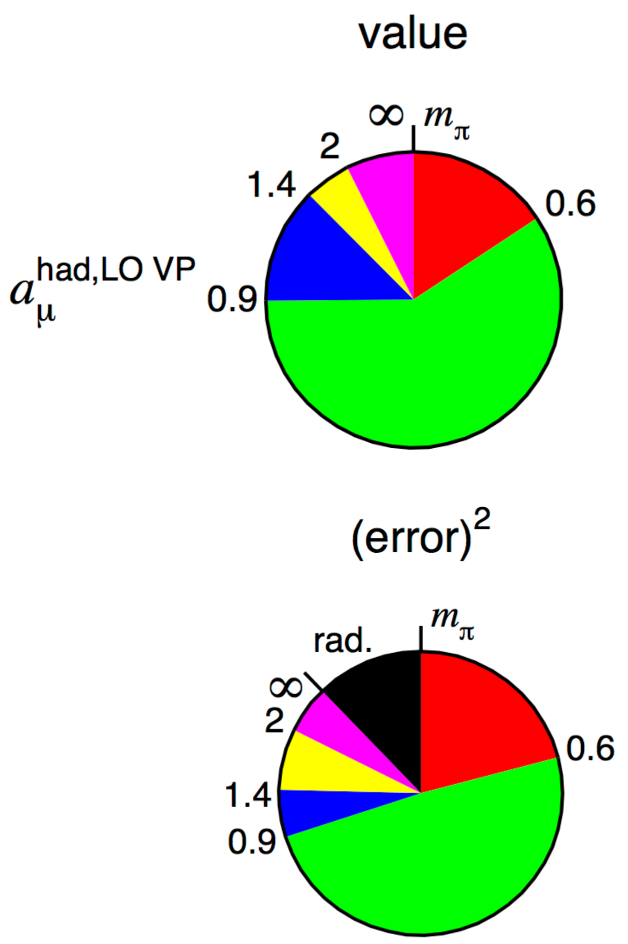




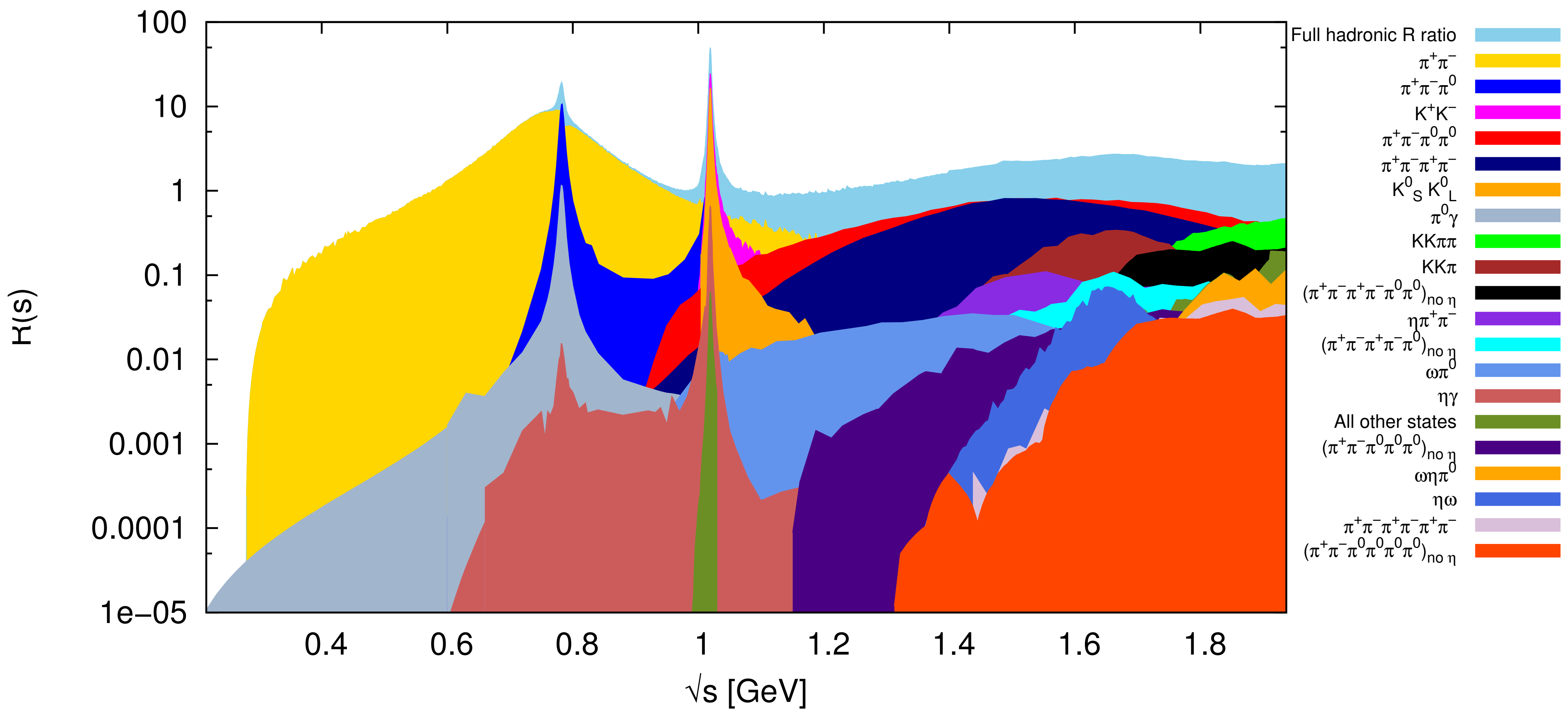
HVP: data-driven

[M. Davier et al, arXiv:1908.00921]





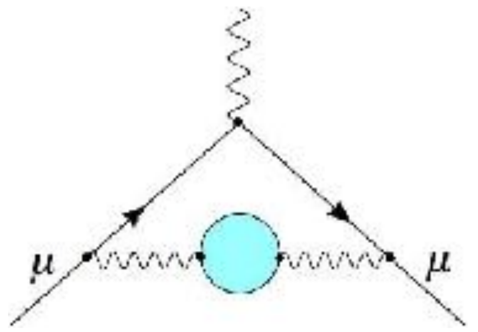
[A. Keshavarzi et al, arXiv:1802.02995]



- total hadronic cross section σ_{had} from > 100 data sets in more than 35 channels summed up to ~ 2GeV
- $\sqrt{s} > 2$ GeV: inclusive data + pQCD + narrow resonances
- σ_{had} defined to include real & virtual photons
- direct integration method: no need to specify resonances (ρ,…)
- two independent compilations (DHMZ, KNT)

Tensions between BaBar and KLOE data sets:

- Cross checks using analyticity and unitarity relating pion form factor to $\pi\pi$ scattering
- Combinations of data sets affected by tensions between measurements and treatment of correlations:
  ➠ conservative merging procedure



HVP: data-driven

In 2020 WP:

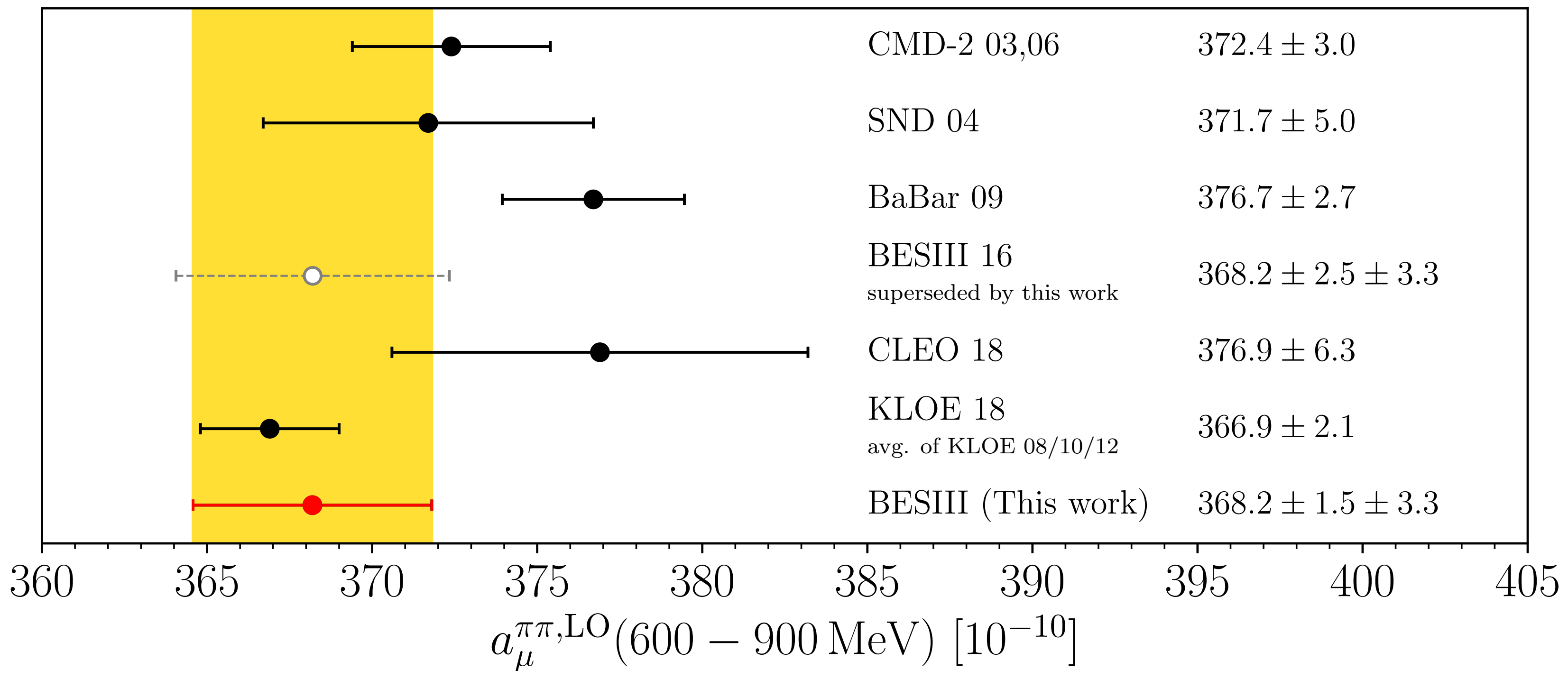
Conservative merging procedure to obtain a realistic assessment of the underlying uncertainties:

- account for tensions between data sets
- account for differences in methodologies for compilation of experimental inputs
- include correlations between systematic errors
- cross checks from unitarity & analyticity constraints
  [Colangelo et al, 2018; Anantharayan et al, 2018; Davier et al, 2019; Hoferichter et al, 2019]
- Full NLO radiative corrections [Campanario et al, 2019]

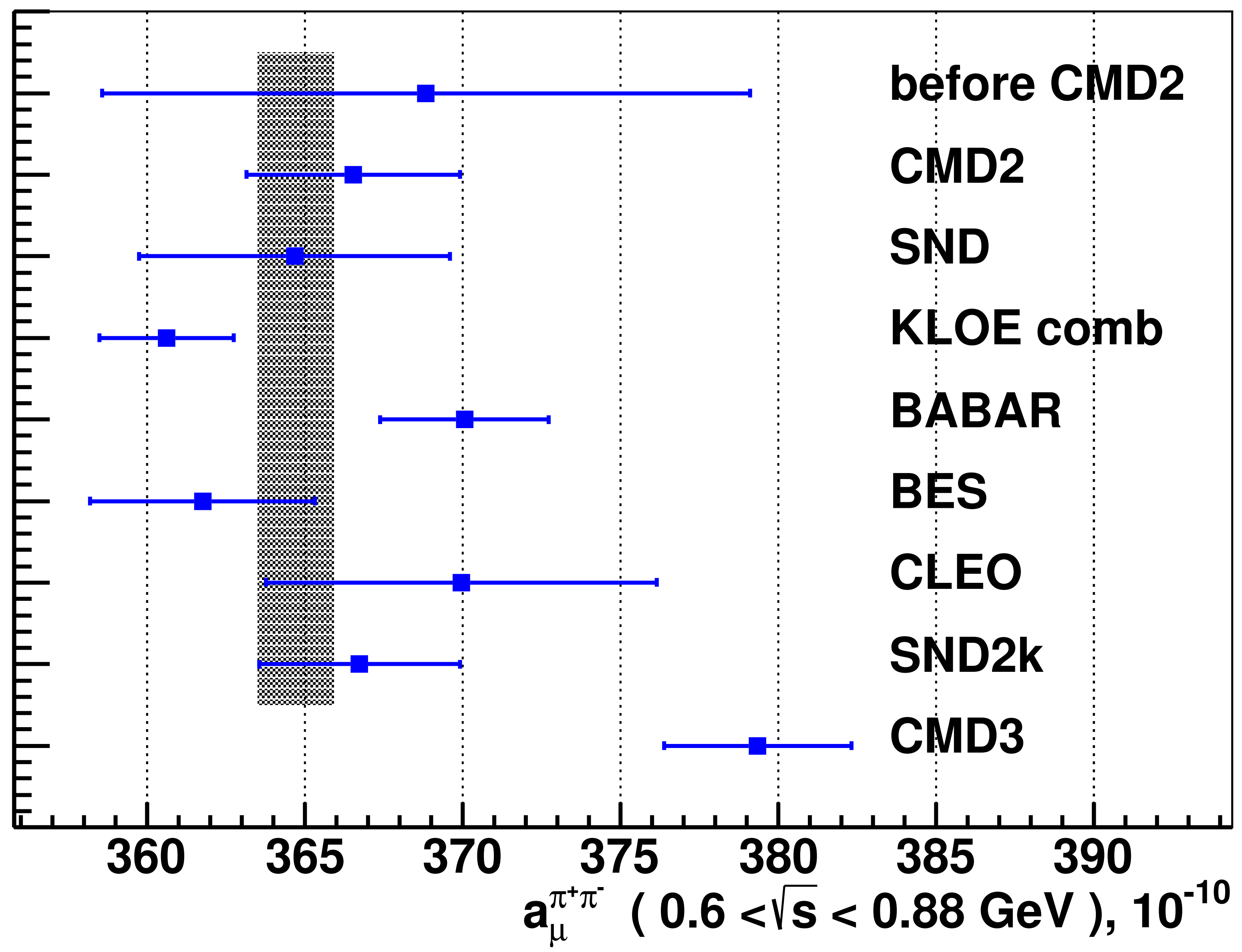
$$a_\mu^{\rm HVP,LO} = 693.1\,(2.8)_{\rm exp}\,(0.7)_{\rm DV+pQCD}\,(2.8)_{\rm BaBar-KLOE} \times 10^{-10}$$

$$= 693.1\,(4.0) \times 10^{-10}$$

[M. Ablikim et al (BES III), arXiv:2009.05011]

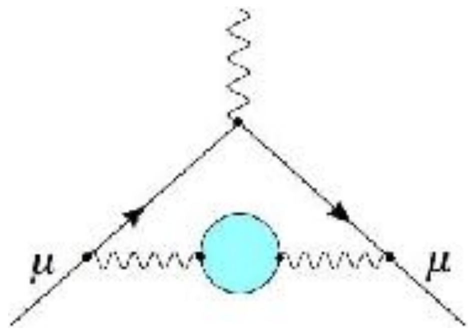


New: from CMD-3 [F. Ignatov et al, arXiv:2302.08834]



A new puzzle!

- discrepancies between experiments now $\gtrsim (3-5)\,\sigma$ need to be understood/resolved
- (virtual) scientific seminar + discussion panel on CMD-3 measurement
  March 27 (8:00 –11:00 am US CDT)
  Discussions are continuing!
- 6th Muon g-2 Theory Initiative workshop (4-8 Sep 2023, Bern)



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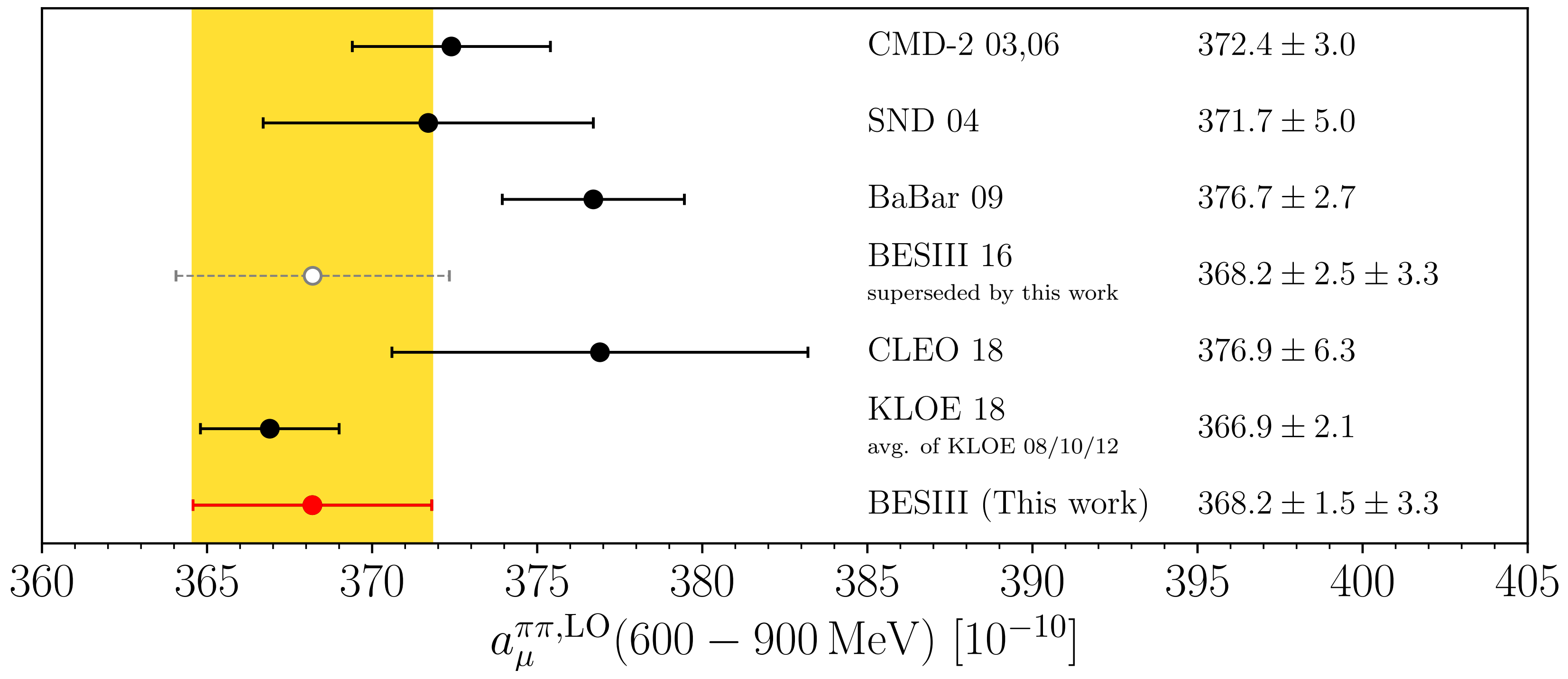
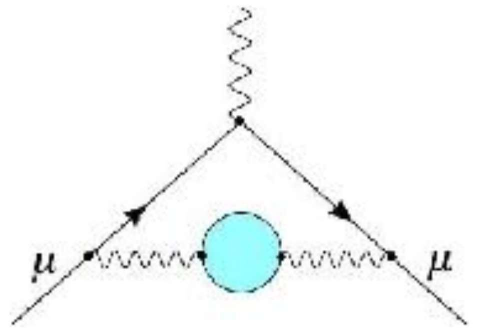


Ongoing work on experimental inputs:

- BaBar: new analysis of large data set in $\pi\pi$ channel, also $\pi\pi\pi$, other channels
- KLOE: new analysis of large data in $\pi\pi$ channel, other channels
- SND: new results for $\pi\pi$ channel, other channels in progress
- BESIII: new results in 2021 for $\pi\pi$ channel, continued analysis also for $\pi\pi\pi$, other channels
- Belle II: arXiv:2207.06307 (Snowmass WP)
  Better statistics than BaBar or KLOE; similar or better systematics for low-energy cross sections
- Most collaborations proceeding with blind analyses



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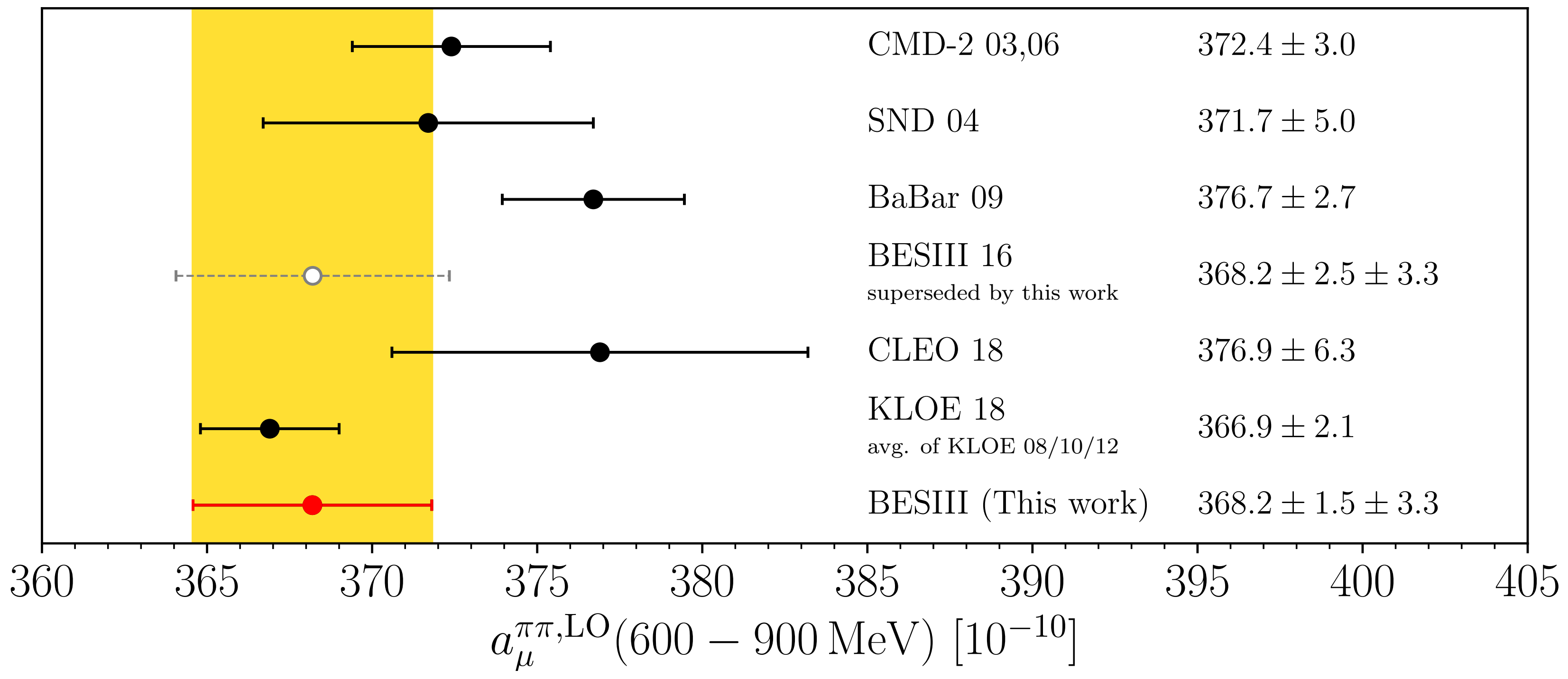
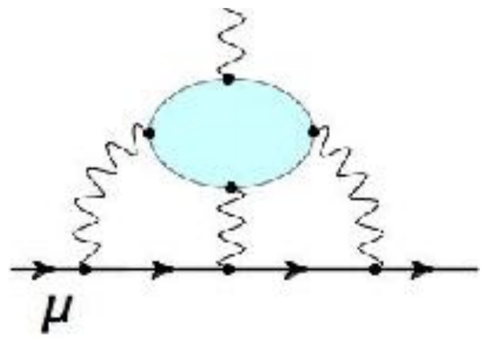


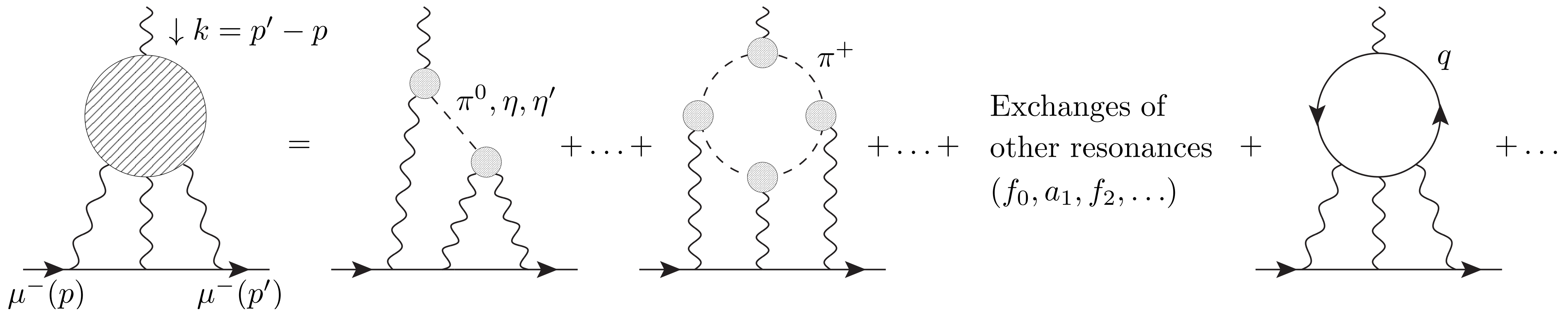
Ongoing work on theoretical aspects:

- Developing NNLO Monte Carlo generators (STRONG 2020 workshop https://agenda.infn.it/event/28089/) [➠ appendix]
- radiative corrections using FsQED (scalar QED + pion form factor)
- charge asymmetry (CMD-3 measurement) vs radiative corrections [Ignatov + Lee, arXiv:2204.12235]
- development of new dispersive treatment of radiative corrections in $\pi\pi$ channel [Colangelo at al, arXiv2207.03495]
- new focus on structure-dependent NLO effects: source of difference between ISR and direct scan measurements? [Strong 2020 workshop]
- including τ decay data: requires nonperturbative evaluation of IB correction [M. Bruno et al, arXiv:1811.00508]

If the differences between experiments are resolved:
data-driven evaluations of HVP with $\sim 0.3\,\%$ feasible by ~2025



Hadronic Light-by-light

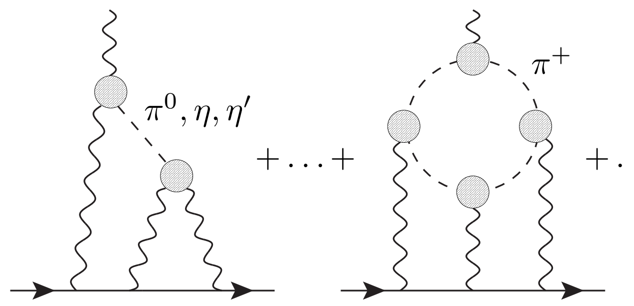


Dispersive approach:

[Colangelo at al, 2014; Pauk & Vanderhaegen 2014; …]

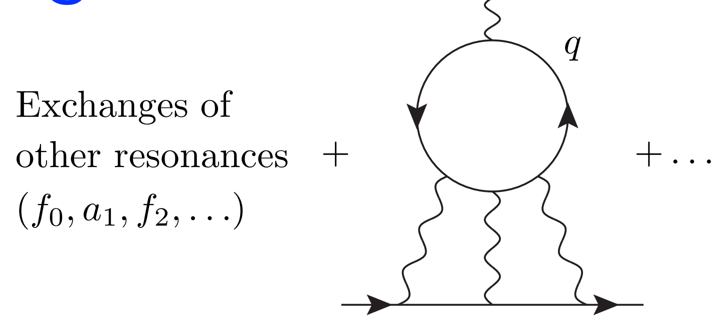
- ✦ model independent
- ✦ significantly more complicated than for HVP
- ✦ provides a framework for data-driven evaluations
- ✦ can also use lattice results as inputs

Dominant contributions ($\approx 75\,\%$ of total):



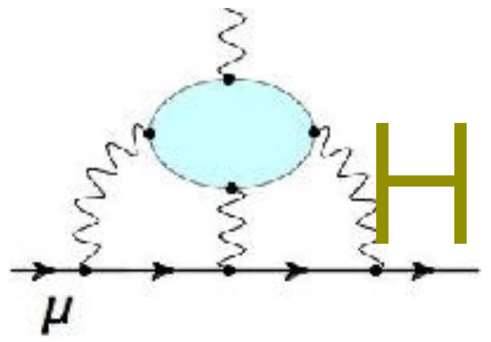
- ✦ Well quantified with $\approx 6\,\%$ uncertainty
- ✦ η, η' pole contributions: Canterbury approximants only
- ✦ Ongoing work: consolidation of η, η' pole contributions using disp. relations and LQCD

Subleading contributions ($\approx 25\,\%$ of total):



- ✦ Not yet well known
  - ➠ dominant contribution to total uncertainty
- ✦ Ongoing work:
  - Implementation of short-distance constraints (now at 2-loop)
  - DR implementation for axial vector contributions
  - new $q_4 = 0$ DR program for higher spin intermediate states [Luedtke @ Higgscentre workshop with Procura and Stoffer, in progress]
  - Mainz and BESIII ramping up $\gamma^{(*)}\gamma^*$ programs [A. Denig and C. Redmer @ Higgscentre workshop]

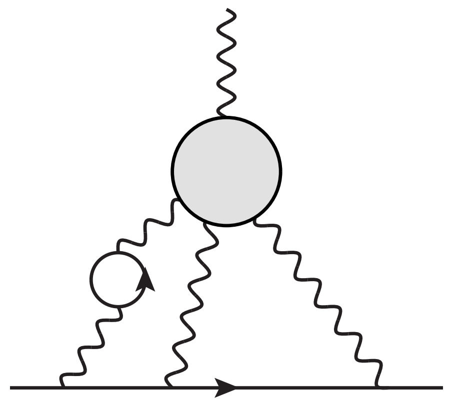
Dispersive, data-driven evaluation of HLbL with $\leq 10\,\%$ total uncertainty feasible by ~2025.



HLbL: dispersive

Comparison:

Contribution	PdRV(09) [471]	N/JN(09) [472, 573]	J(17) [27]	Our estimate
π^0, η, η'-poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K-loops/boxes	−19(19)	−19(13)	−20(5)	−16.4(2)
S-wave $\pi\pi$ rescattering	−7(7)	−7(2)	−5.98(1.20)	−8(1)
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	–	–	–	– 1(3)
tensors	–	–	1.1(1)	
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)
u, d, s-loops / short-distance	–	21(3)	20(4)	15(10)
c-loop	2.3	–	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)



NLO HLbL contribution:

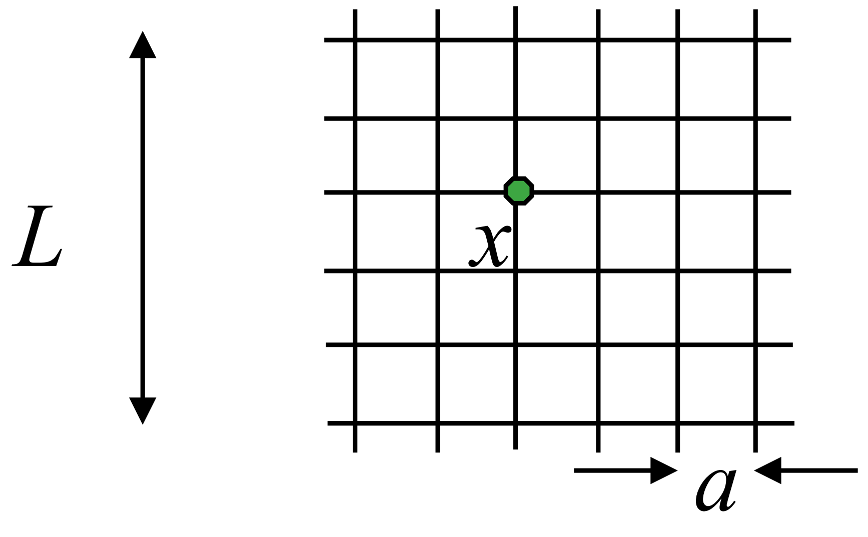
$$a_\mu^{\text{HLbL,NLO}} = 2\,(1) \times 10^{-11}$$

Outline

- Introduction
- Fermilab and JPARC muon g-2 experiments
- SM contribution to g-2
- Muon g-2 Theory Initiative
- dispersive/data driven methods
  - Hadronic Vacuum Polariztion (HVP)
  - Hadronic Light-by-Light (HLbL)
- lattice QCD
  - lattice HVP
  - lattice HLbL
- Connections
- Summary and Outlook

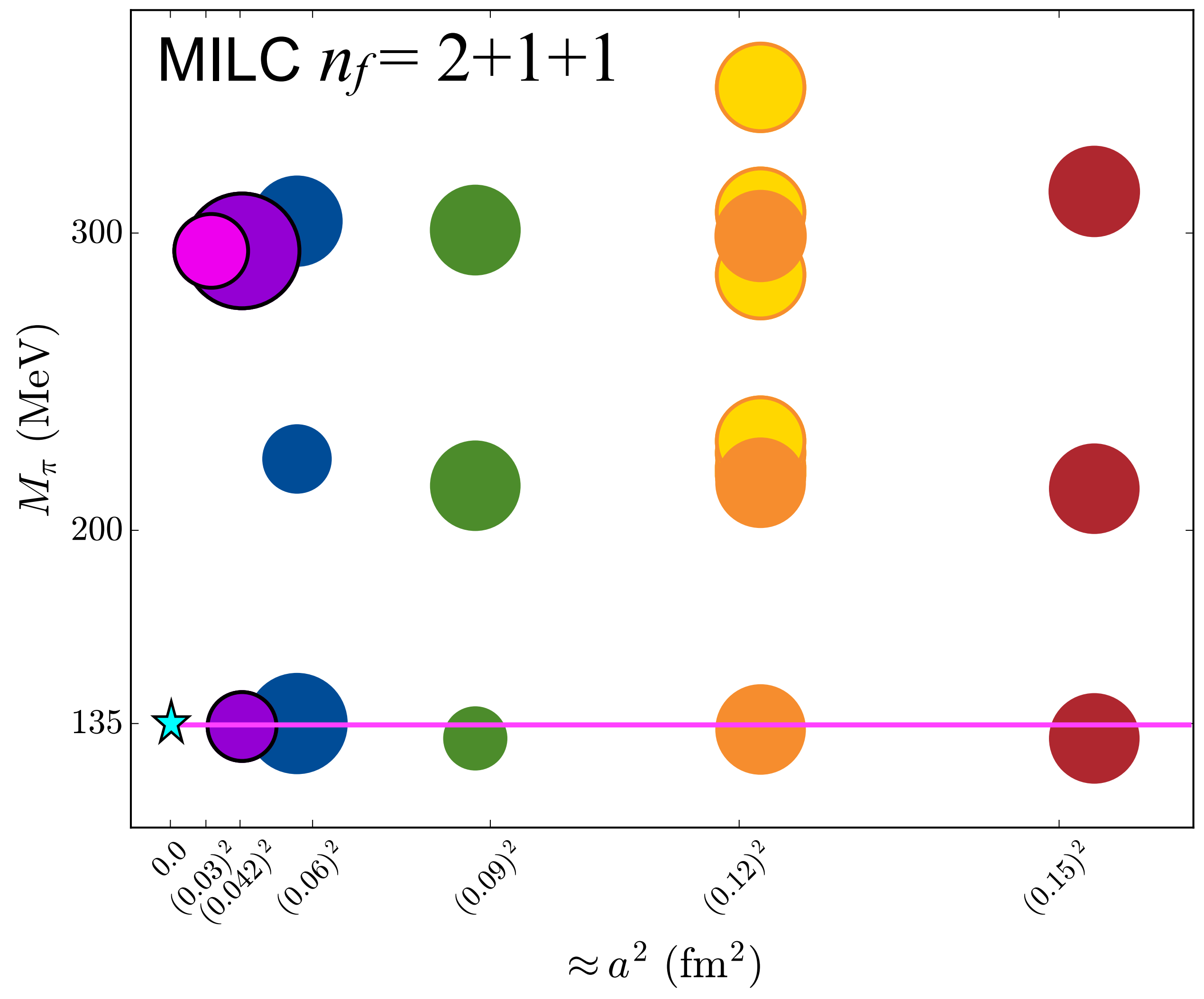
Lattice QCD Introduction

$$\mathcal{L}_{\rm QCD} = \sum_f \bar{\psi}_f (\not{D} + m_f)\psi_f + \frac{1}{4}{\rm tr} F_{\mu\nu}F^{\mu\nu}$$



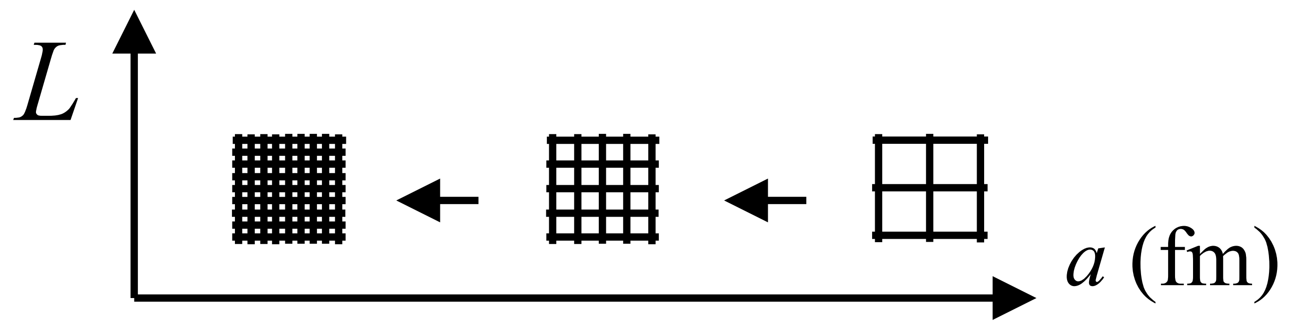
- discrete Euclidean space-time (spacing a)
  derivatives → difference operators, etc…
- finite spatial volume (L)
- finite time extent (T)

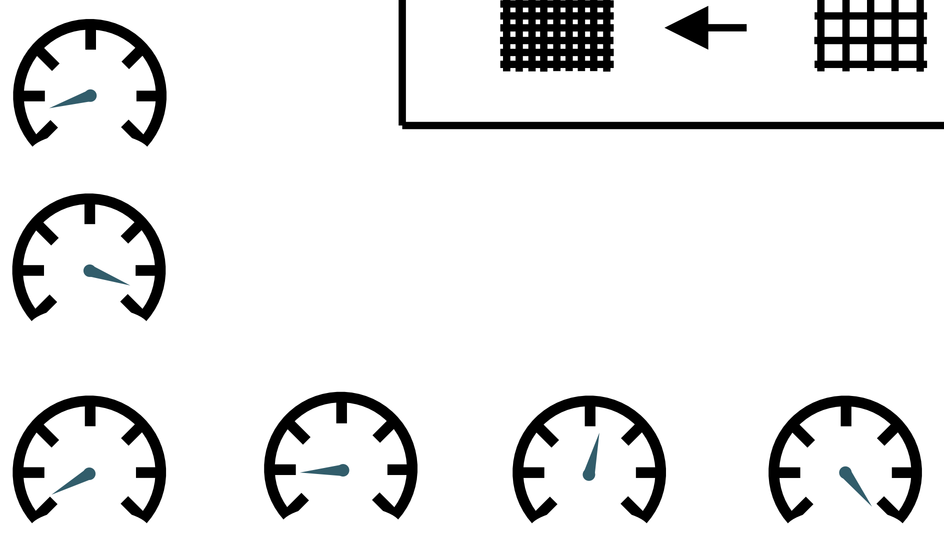
Integrals are evaluated numerically using monte carlo methods.



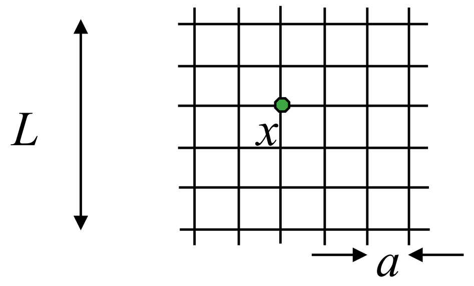
adjustable parameters

- lattice spacing: $a \to 0$
- finite volume, time: $L \to \infty,\ T > L$
- quark masses (m_f): $M_{H,{\rm lat}} = M_{H,{\rm exp}}$
  tune using hadron masses
  extrapolations/interpolations
  $m_f \to m_{f,{\rm phys}}$





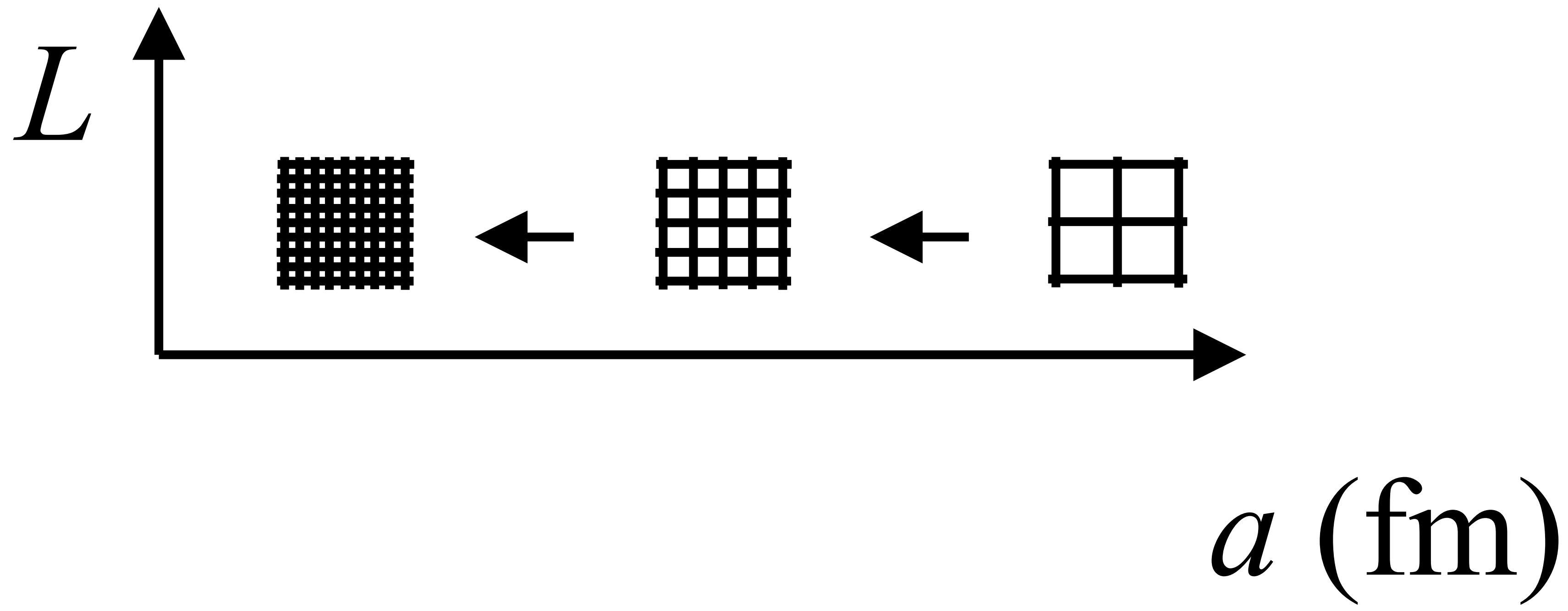
m_{ud} m_s m_c m_b

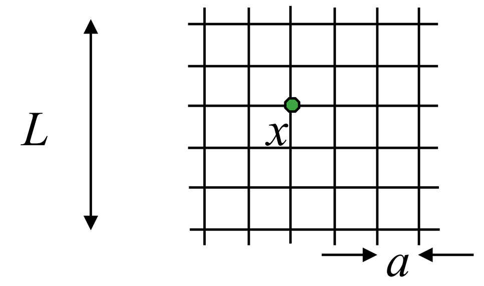


Lattice QCD Introduction

discretization effects — continuum extrapolation

- typical momentum scale of quarks gluons inside hadrons: $\sim\Lambda_{\mathrm{QCD}}$
- make a small to separate the scales: $\Lambda_{\mathrm{QCD}} \ll 1/a$
- Symanzik EFT: $\langle\mathcal{O}\rangle^{\mathrm{lat}} = \langle\mathcal{O}\rangle^{\mathrm{cont}} + O(a\Lambda)^n$, $n \geq 2$
  - provides functional form for extrapolation (depends on the details of the lattice action)
  - can be used to build improved lattice actions
  - can be used to anticipate the size of discretization effects





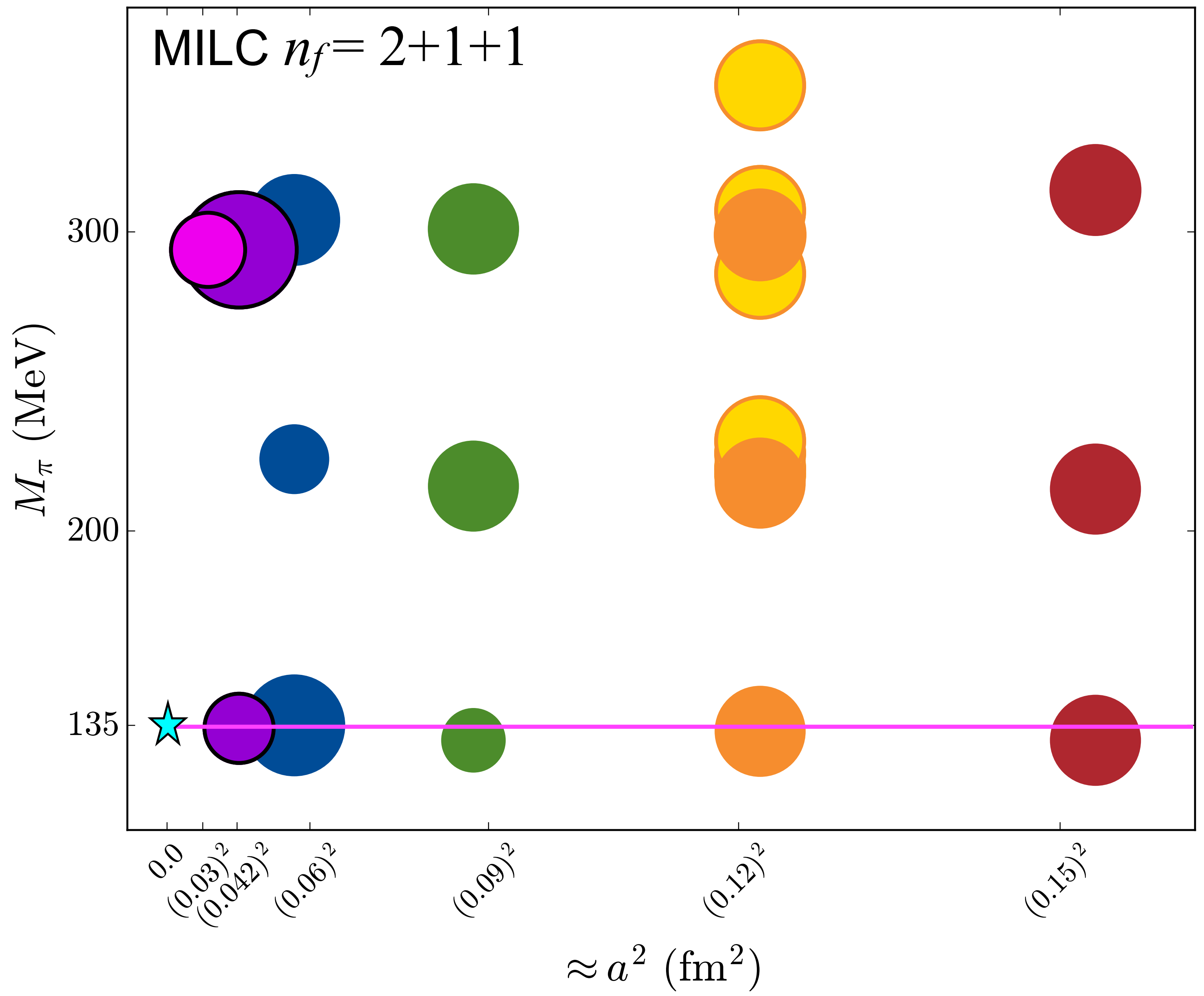
Lattice QCD Introduction

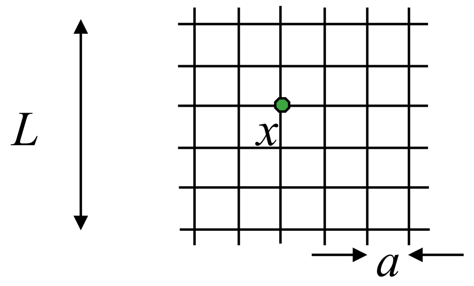
systematic error analysis

…of lattice spacing, chiral, heavy quark, and finite volume effects is based on Effective Field Theory (EFT) descriptions of QCD ➞ ab initio

- finite a: Symanzik EFT
- light quark masses: Chiral Perturbation Theory
- heavy quarks: HQET
- finite L: finite volume EFT

In practice:
stability and control over systematic errors depends on the lattice action(s) employed, underlying simulation parameters, available computational resources, analysis choices, …





Lattice QCD Introduction

The State of the Art

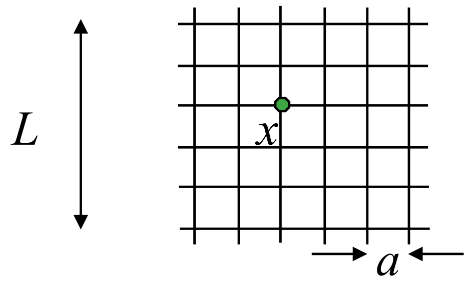
Lattice QCD calculations of simple quantities (with at most one stable meson in initial/final state) that **quantitatively account for all systematic effects** (discretization, finite volume, renormalization,…) in some cases with

- sub percent precision.
- total errors that are commensurate (or smaller) than corresponding experimental uncertainties.

Progress due to a virtuous cycle of theoretical developments, improved algorithms/methods and increases in computational resources (``Moore's law")

Scope of LQCD calculations is increasing due to continual development of new methods:

- nucleon matrix elements
- nonleptonic kaon decays ($K \to \pi\pi$, ϵ',…)
- resonances, scattering ($\pi\pi \to \rho$,…)
- long-distance effects (ΔM_K, …)
- QED corrections
- radiative decay rates
- structure: PDFs, GPDs, TMDs, …
- inclusive decay rates ($B \to X_c \ell \nu$,…)
- …



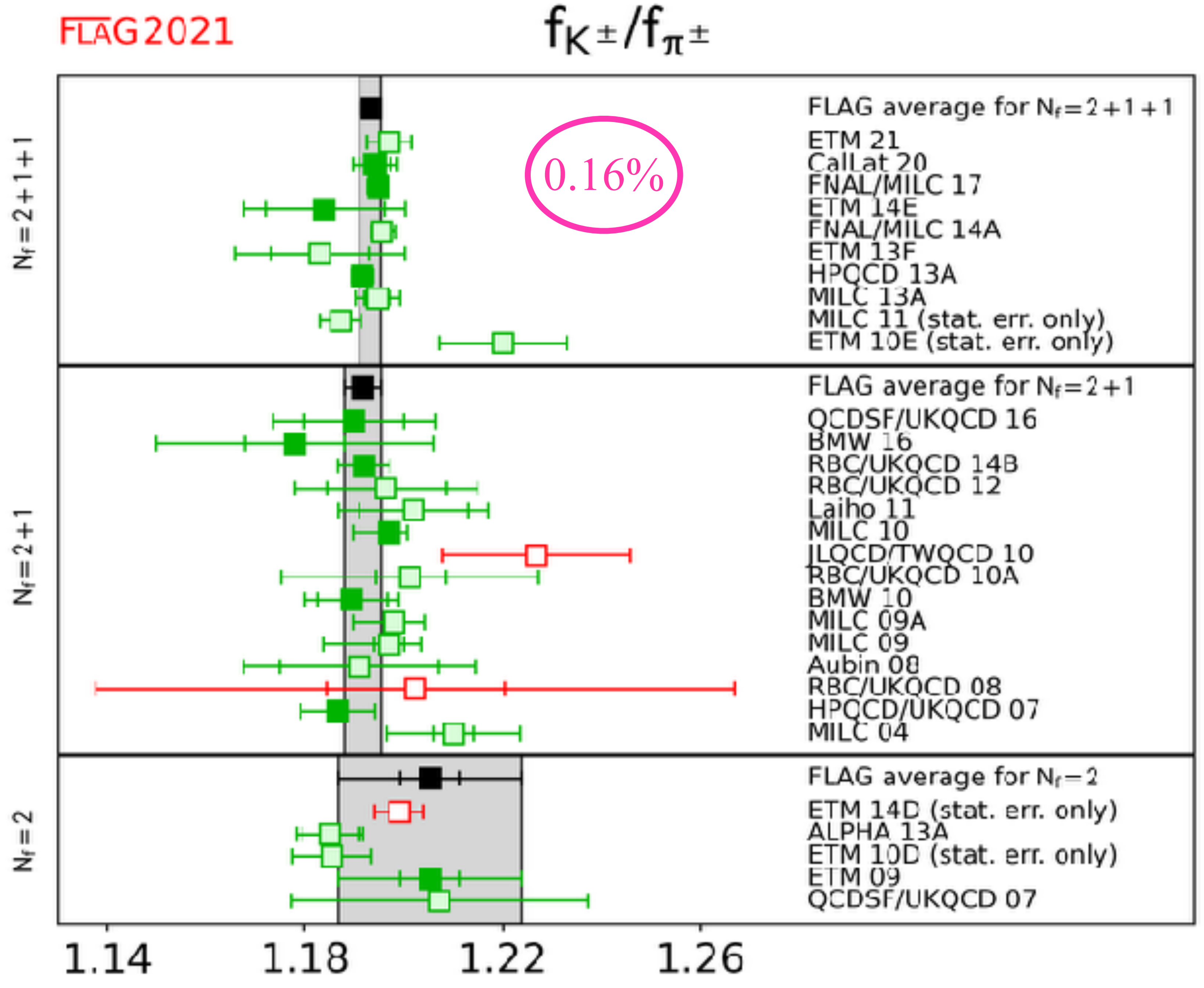
Lattice QCD Introduction

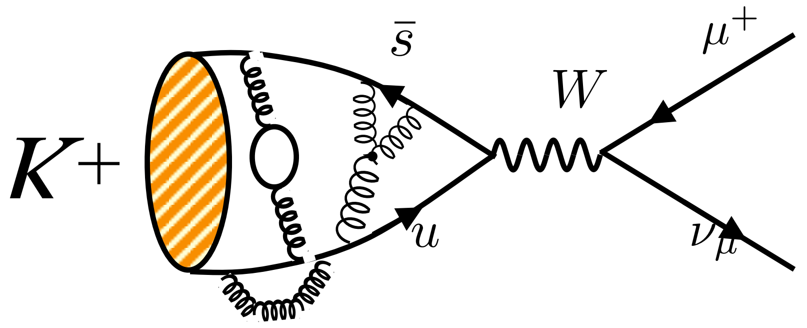
https://www.usqcd.org/documents/13flavor.pdf and [J. Butler et al, arXiv:1311.1076]

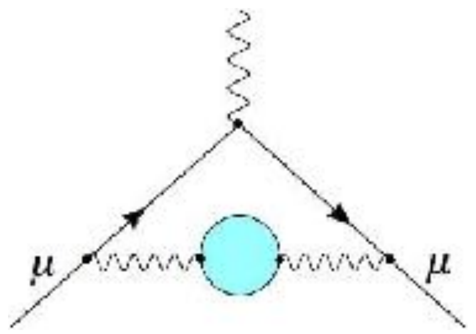
Quantity	CKM element	2013 expt. error	2007 forecast lattice error	2013 lattice error	forecast lattice error	2021 FLAG Average
f_K/f_π	$\|V_{us}\|$	0.2%	0.5%	0.4%	0.15%	0.18 %
$f_+^{K\pi}(0)$	$\|V_{us}\|$	0.2%	–	0.4%	0.2%	0.18 %
f_D	$\|V_{cd}\|$	4.3%	5%	2%	$< 1\%$	0.3 %
f_{D_s}	$\|V_{cs}\|$	2.1%	5%	2%	$< 1\%$	0.2 %
$D \to \pi\ell\nu$	$\|V_{cd}\|$	2.6%	–	4.4%	2%	0.7 %
$D \to K\ell\nu$	$\|V_{cs}\|$	1.1%	–	2.5%	1%	0.6 %
$B \to D^*\ell\nu$	$\|V_{cb}\|$	1.3%	–	1.8%	$< 1\%$	~1.5 % [fro
$B \to \pi\ell\nu$	$\|V_{ub}\|$	4.1%	–	8.7%	2%	~3 %
f_B	$\|V_{ub}\|$	9%	–	2.5%	$< 1\%$	0.7 % (0.6
ξ	$\|V_{ts}/V_{td}\|$	0.4%	2–4%	4%	$< 1\%$	1.3 %
Δm_s	$\|V_{ts}V_{tb}\|^2$	0.24%	7–12%	11%	5%	4.5 %
B_K	$\mathrm{Im}(V_{td}^2)$	0.5%	3.5–6%	1.3%	$< 1\%$	1.3 %

[from 2212.12648]

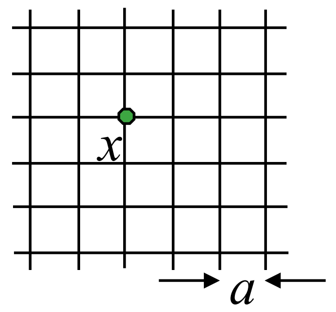
QED threshold:
QED corrections important/ dominant source of theory error in SM predictions



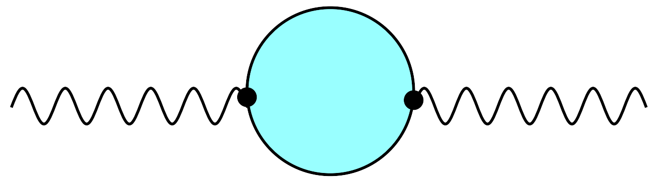




Lattice HVP: Introduction



[B. Lautrup, A. Peterman, E. de Rafael, Phys. Rep 1972; E. de Rafael, Phys. Let. B 1994; T. Blum, PRL 2002]



$\hat{\Pi}(q^2)$

Leading order HVP correction:

$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2)\, \hat{\Pi}(q^2)$$

- Calculate $a_\mu^{\mathrm{HVP,LO}}$ in Lattice QCD

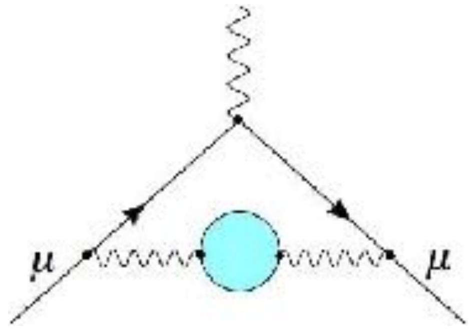
Compute correlation function: $C(t) = \frac{1}{3} \sum_{i,x} \langle j_i(x,t) j_i(0,0) \rangle$

and $\hat{\Pi}(Q^2) = 4\pi^2 \int_0^\infty dt\, C(t) \left[t^2 - \frac{4}{Q^2} \sin^2\left(\frac{Qt}{2}\right)\right]$

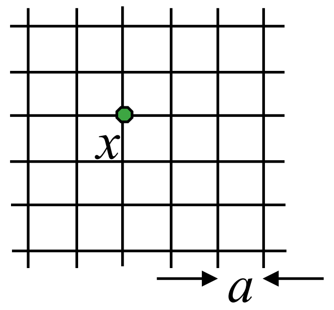
[D. Bernecker and H. Meyer, arXiv:1107.4388, EPJA 2011]

Obtain $a_\mu^{\mathrm{HVP,LO}}$ from an integral over Euclidean time:

$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{w}(t)\, C(t)$$

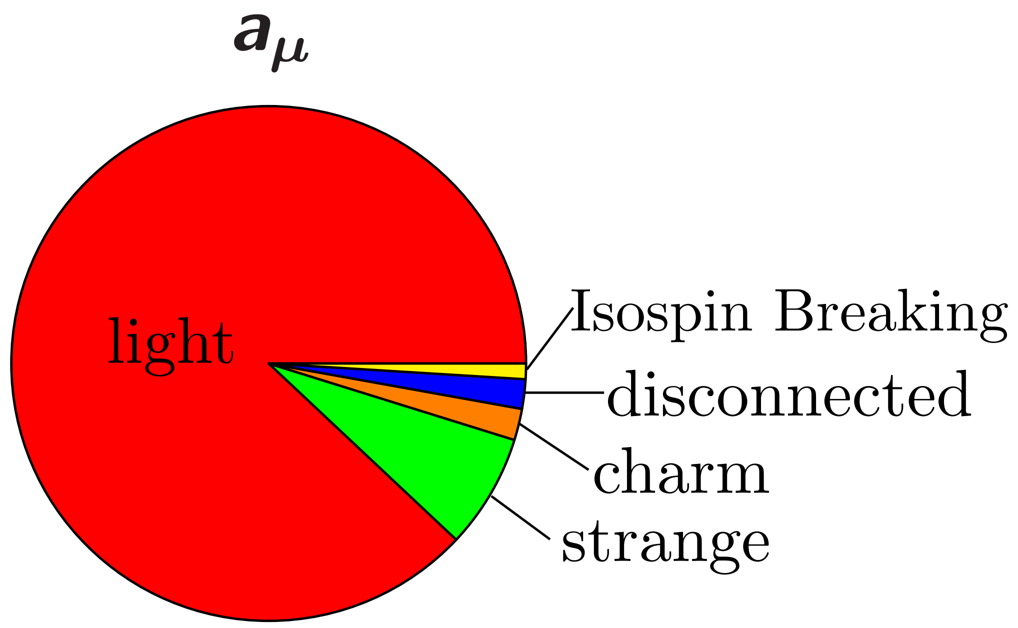


Lattice HVP: Introduction



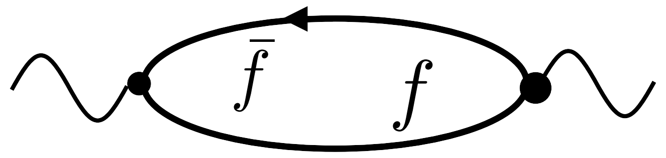
Calculate $a_\mu^{\rm HVP}$ in Lattice QCD:

$$a_\mu^{\rm HVP,LO} = \sum_f a_{\mu,f}^{\rm HVP,LO} + a_{\mu,\rm disc}^{\rm HVP,LO}$$

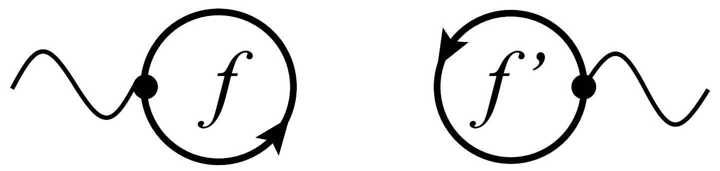


- Separate into connected for each quark flavor + disconnected contributions (gluon and sea-quark background not shown in diagrams)

  Note: almost always $m_u = m_d$

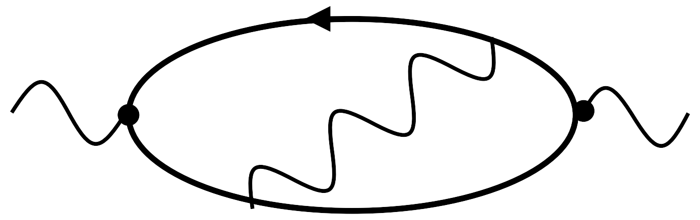
$f = ud, s, c, b$ $\quad \sum_f$ 

- light-quark connected contribution: $a_\mu^{\rm HVP,LO}(ud)$ ~90% of total
- s,c,b-quark contributions $a_\mu^{\rm HVP,LO}(s,c,b)$ ~8%, 2%, 0.05% of total

$+$ 

- disconnected contribution: $a_{\mu,\rm disc}^{\rm HVP,LO}$ ~2% of total

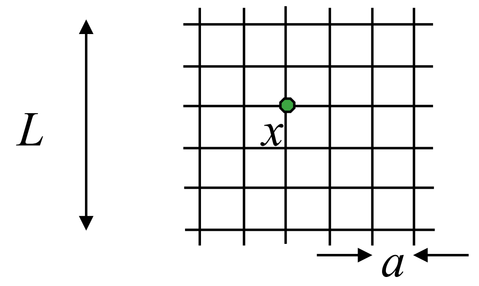
- need to add QED and strong isospin breaking ($\sim m_u - m_d$) corrections:

$+$  $+ \ldots$

- Isospinbreaking (QED + $m_u \neq m_d$) corrections: $\delta a_\mu^{\rm HVP,LO}$ ~1% of total

$$a_\mu^{\rm HVP,LO} = a_\mu^{\rm HVP,LO}(ud) + a_\mu^{\rm HVP,LO}(s) + a_\mu^{\rm HVP,LO}(c) + a_{\mu,\rm disc}^{\rm HVP,LO} + \delta a_\mu^{\rm HVP,LO}$$

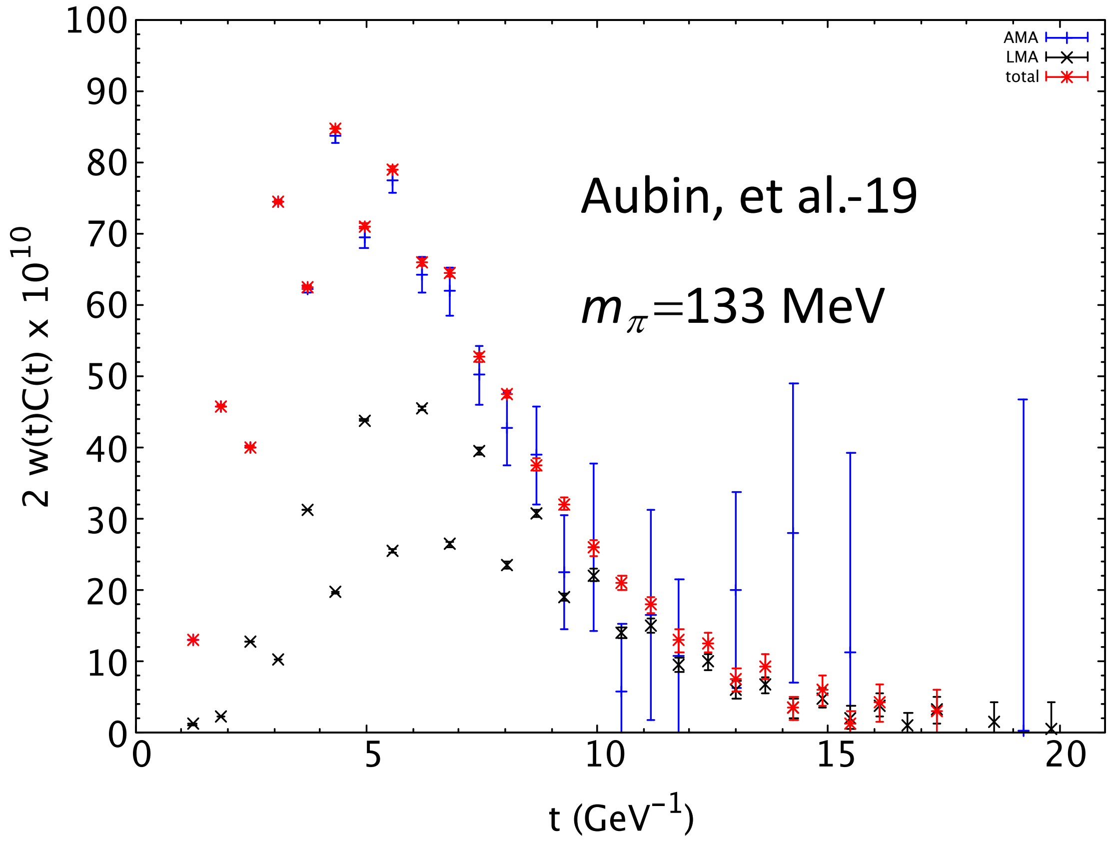
Long-distance tail



$$G(t) = \frac{1}{3}\sum_{i,x}\langle j_i(x,t)\, j_i(0,0)\rangle$$

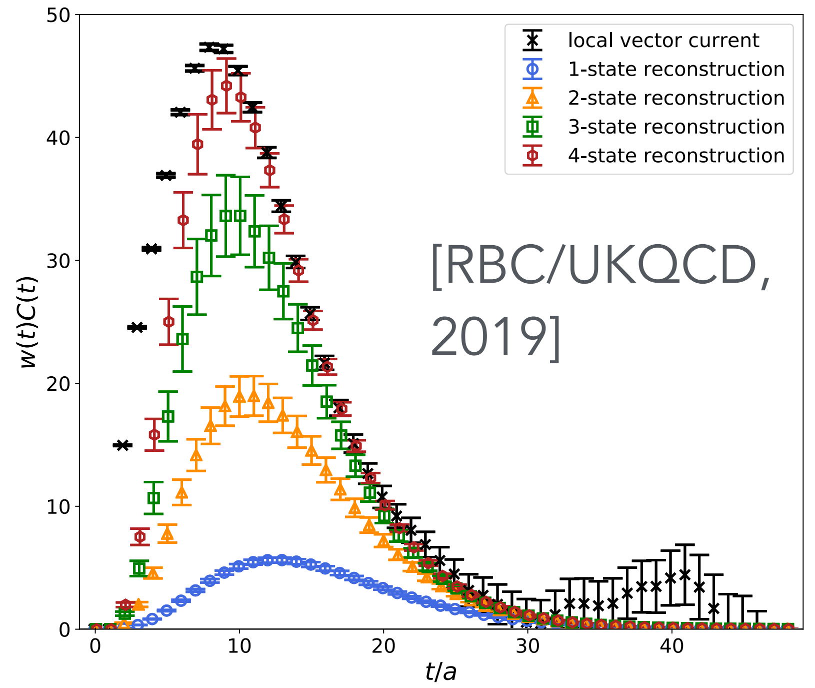
- Use noise reduction methods (AMA, LMA,…):

  Aubin et al, RBC/UKQCD, BMWc, Mainz, …



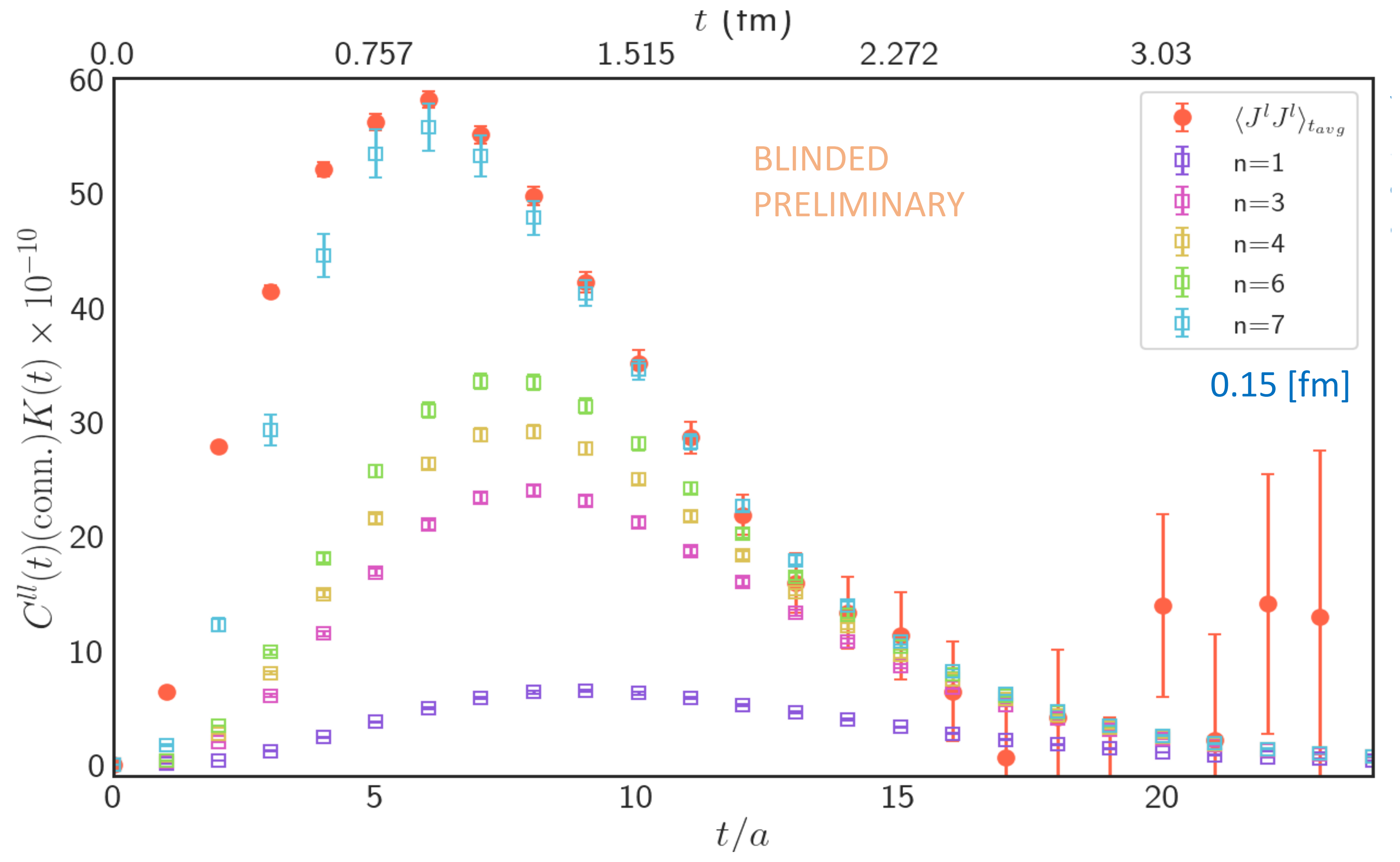
- Spectral reconstruction (RBC/UKQCD, Mainz):
  - ✦ obtain low-lying finite-volume spectrum (E_n, A_n) in dedicated study using additional operators that couple to two-pion states
  - ✦ use to reconstruct $G(t > t_c)$
  - ✦ can be used to improve bounding method:

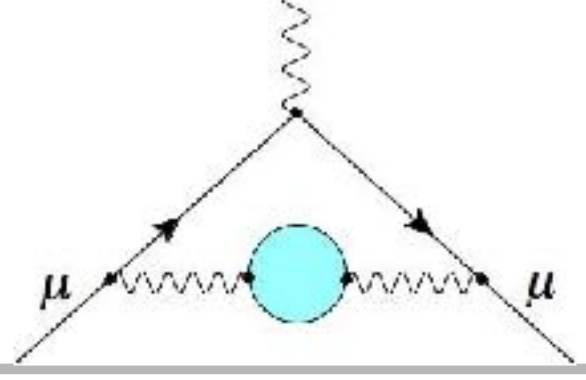
$$G(t) \to G(t) - \sum_{n=0}^{N} A_n^2\, e^{-E_n t}$$



Shaun Lahert (Fermilab-HPQCD-MILC) @ Lattice 2021

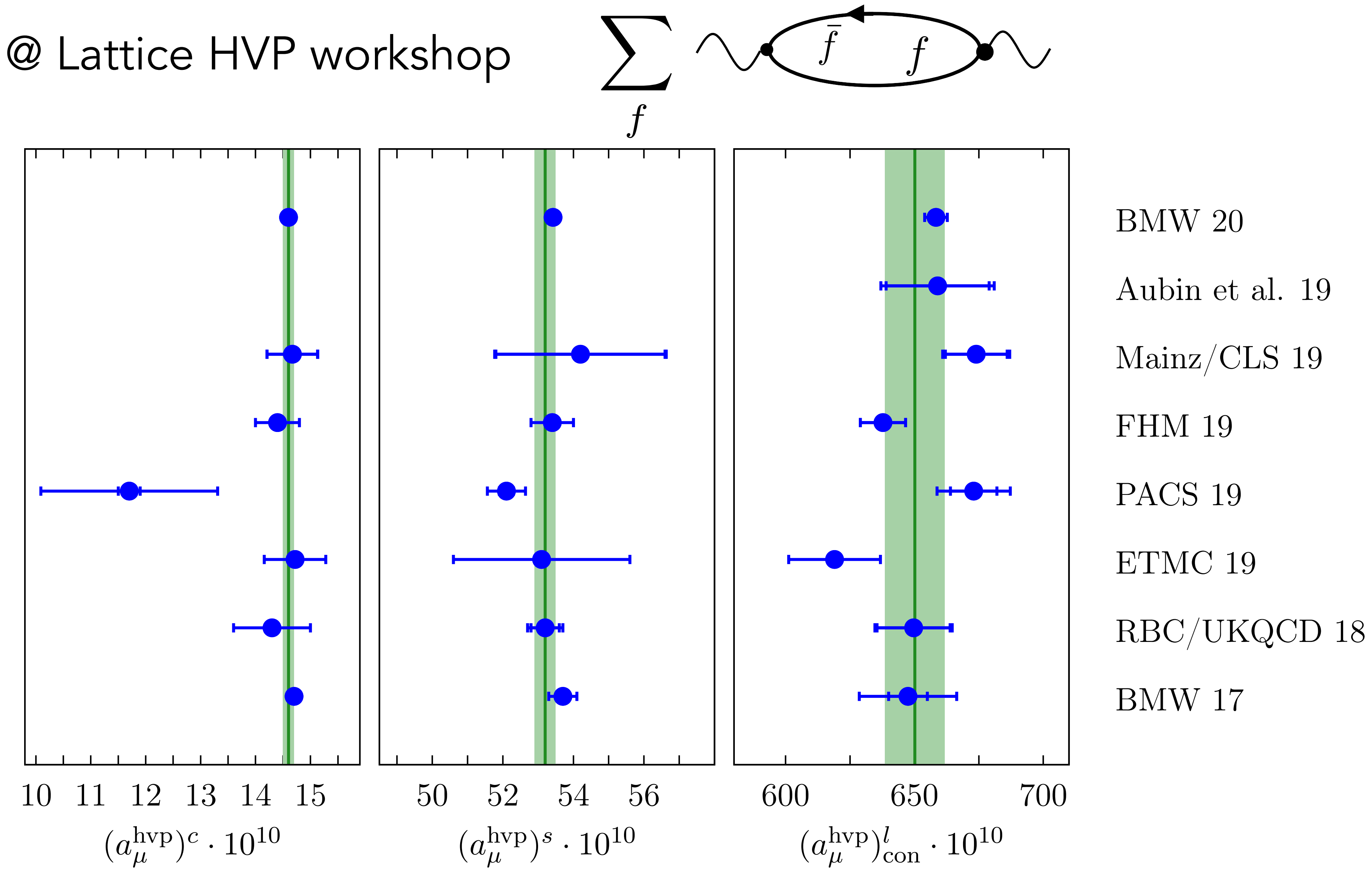
- First calculation with staggered multi-pion operators





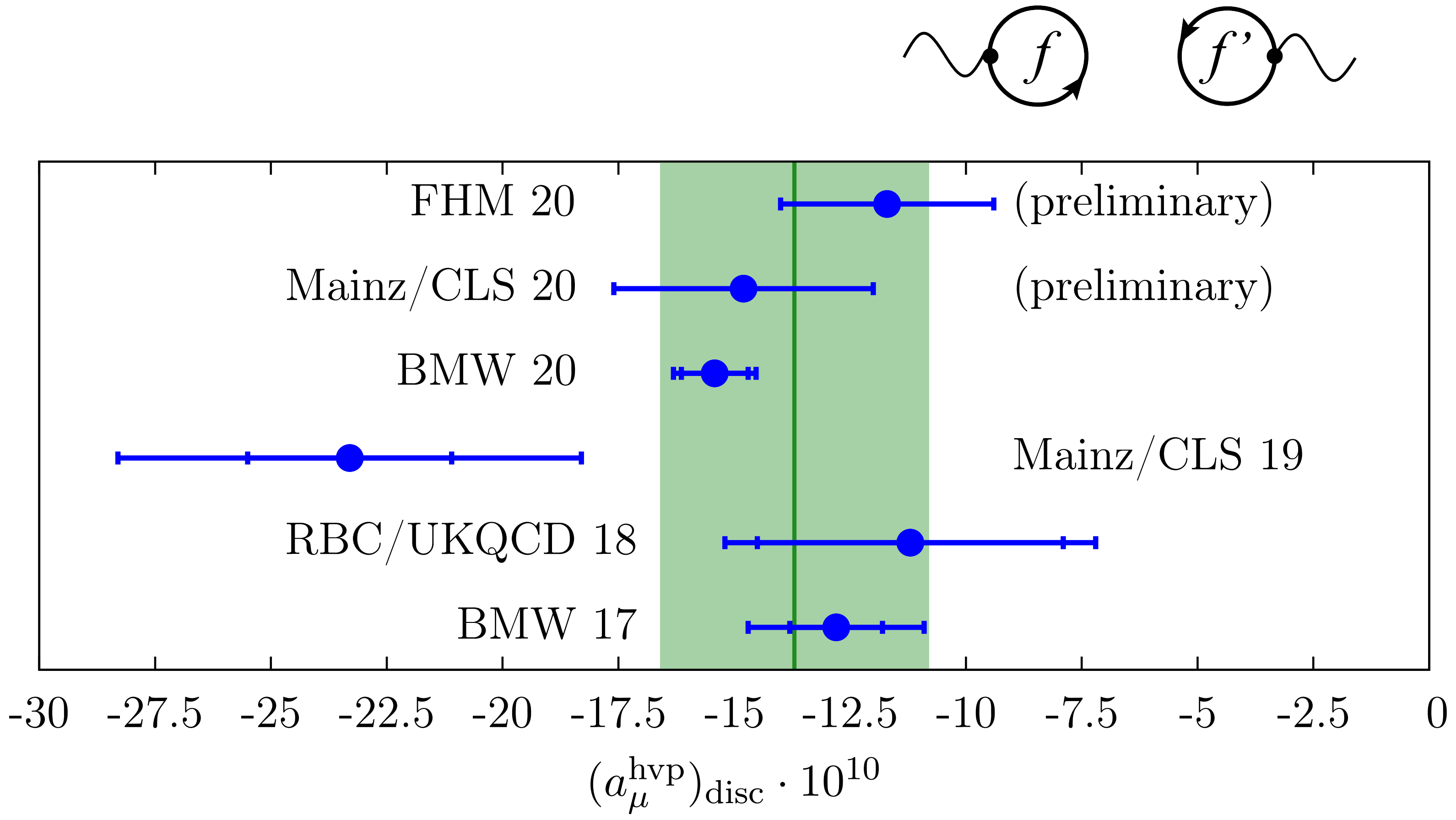
Lattice HVP: subleading corrections

H. Wittig @ Lattice HVP workshop



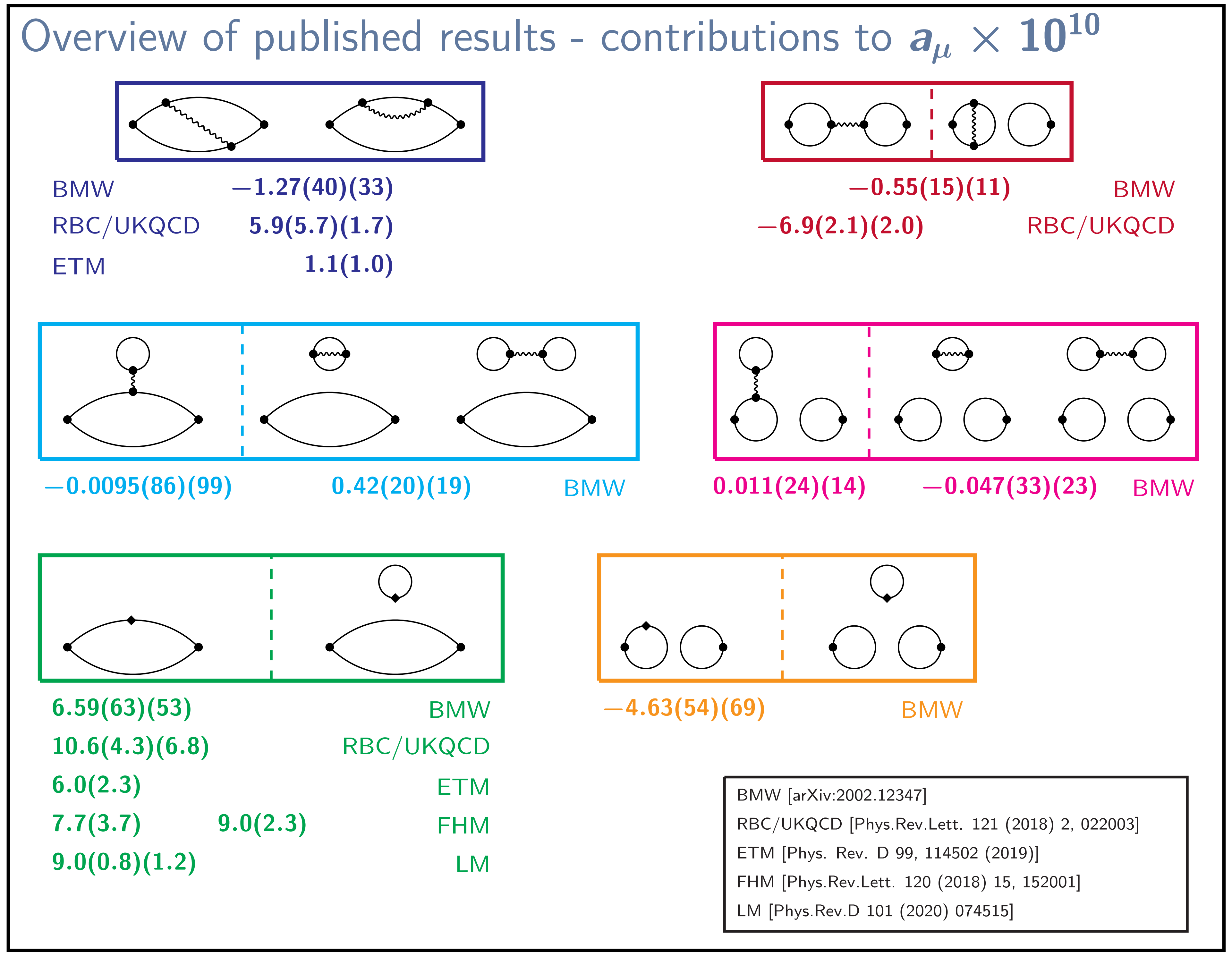
- Charm, strange contributions already well determined.
- Mild tensions for light contribution

Ongoing efforts by FNAL-HPQCD-MILC RBC/UKQCD, Mainz

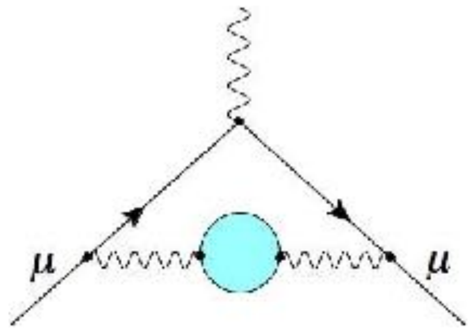


Consistent results with increasing precision

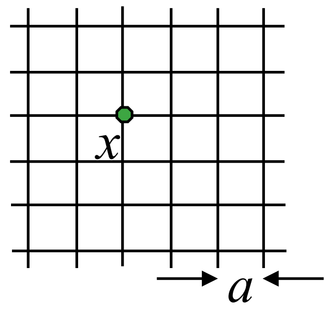
V. Gülpers @ Lattice HVP workshop



- Some tensions between lattice results for individual contributions.
- Large cancellations between individual contributions: $\delta a_\mu^{\rm IB} \lesssim 1\,\%$

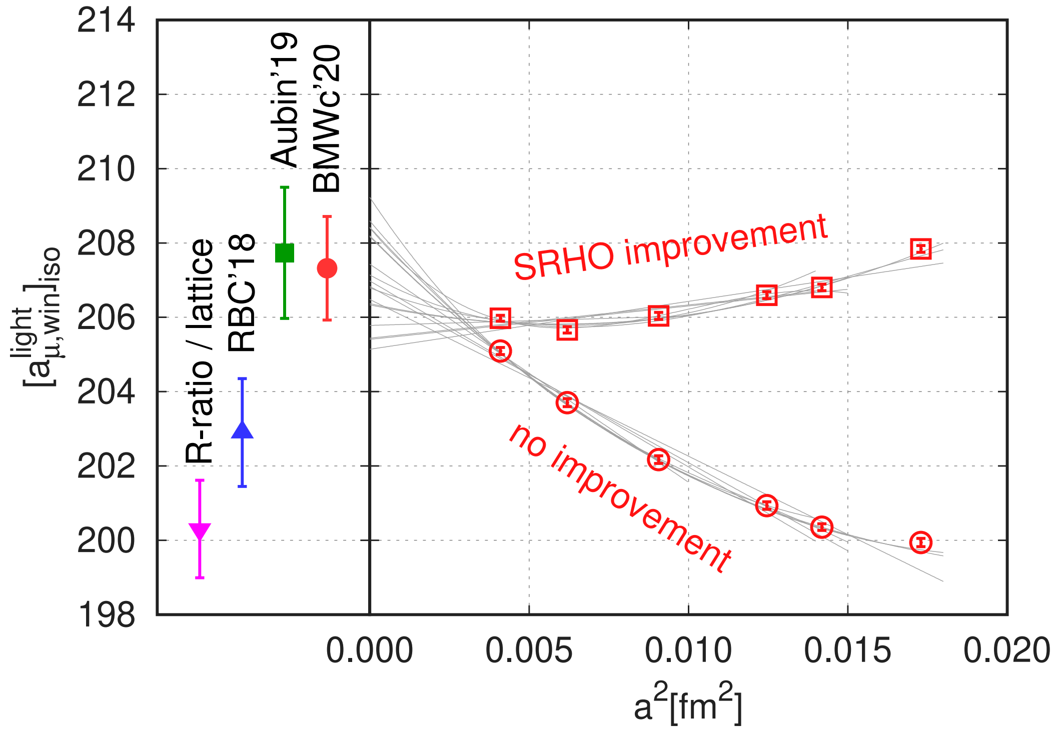


HVP: lattice



In 2020 WP:

- Lattice HVP average at $2.6\,\%$ total uncertainty:
  $a_\mu^{\rm HVP,LO} = 711.6\,(18.4) \times 10^{10}$
- BMW 20 (published in 2021)
  first LQCD calculation with sub-percent $(0.8\,\%)$ error
  in tension with data-driven HVP (2.1σ)
- Further tensions for intermediate window



-3.7σ tension with data-driven evaluation

-2.2σ tension with RBC/UKQCD18

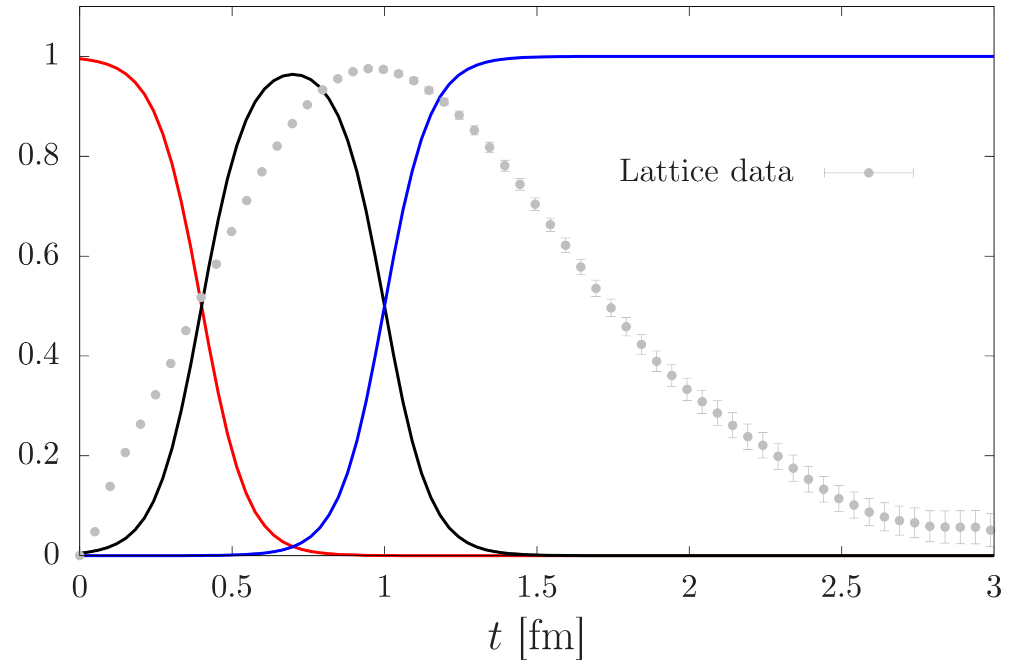
$$a_\mu^{\rm HVP,LO} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{w}(t)\, C(t)$$

- Use windows in Euclidean time to consider the different time regions separately. [T. Blum et al, arXiv:1801.07224, 2018 PRL]

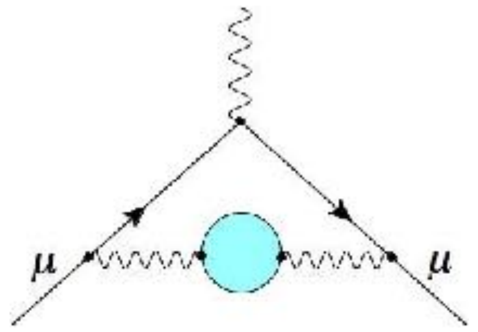
Short Distance (SD)	$t: 0 \to t_0$
Intermediate (W)	$t: t_0 \to t_1$
Long Distance (LD)	$t: t_1 \to \infty$

$t_0 = 0.4\,\text{fm}, t_1 = 1.0\,\text{fm}$

H. Wittig @ Lattice 2021



- disentangle systematics/statistics from long distance/FV and discretization effects
- intermediate window: easy to compute in lattice QCD & using disperse approach:
- Internal cross check: compute each window separately (in continuum, infinite volume limits,…) and combine: $a_\mu = a_\mu^{\rm SD} + a_\mu^{\rm W} + a_\mu^{\rm LD}$



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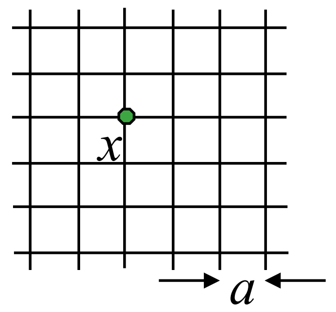

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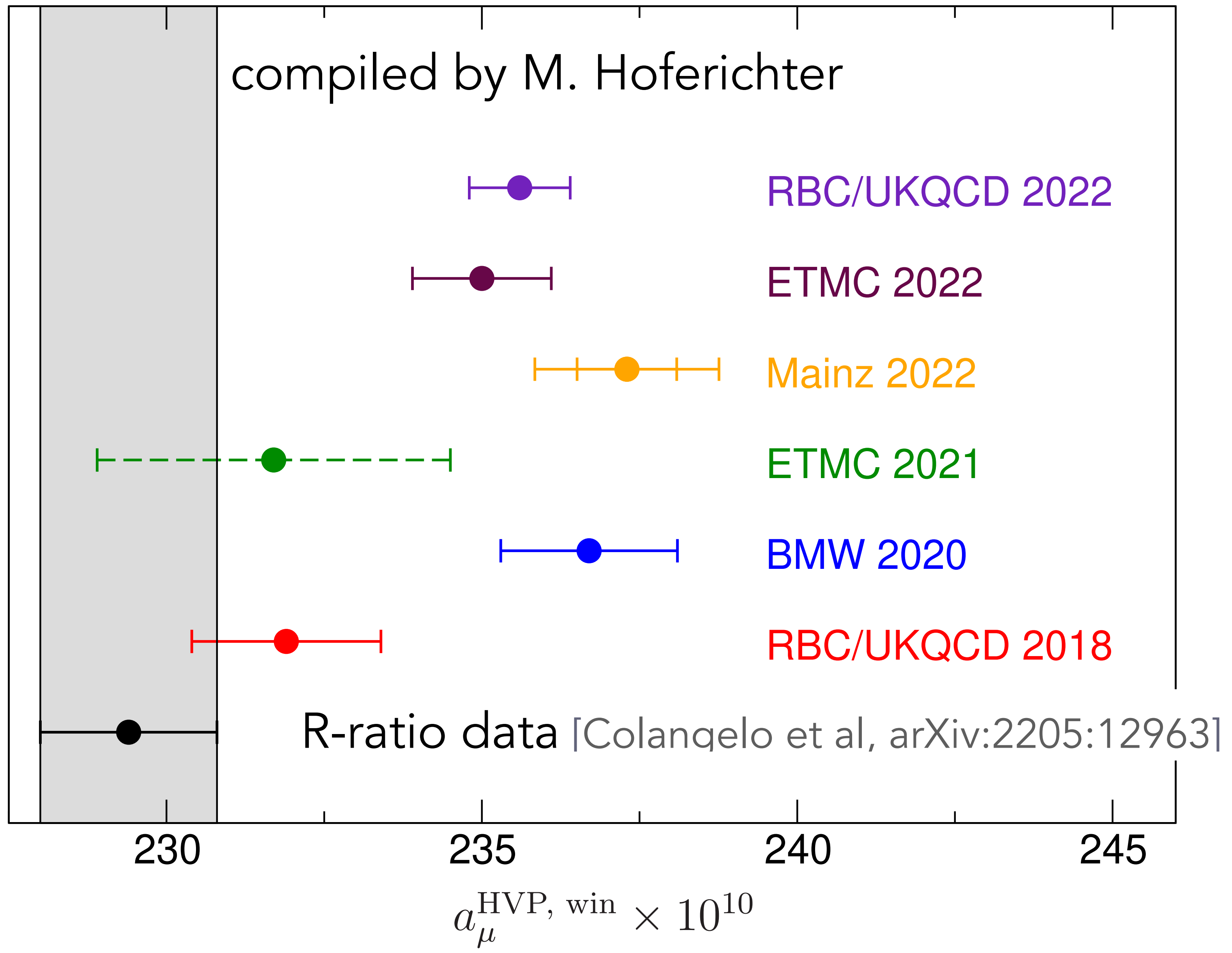
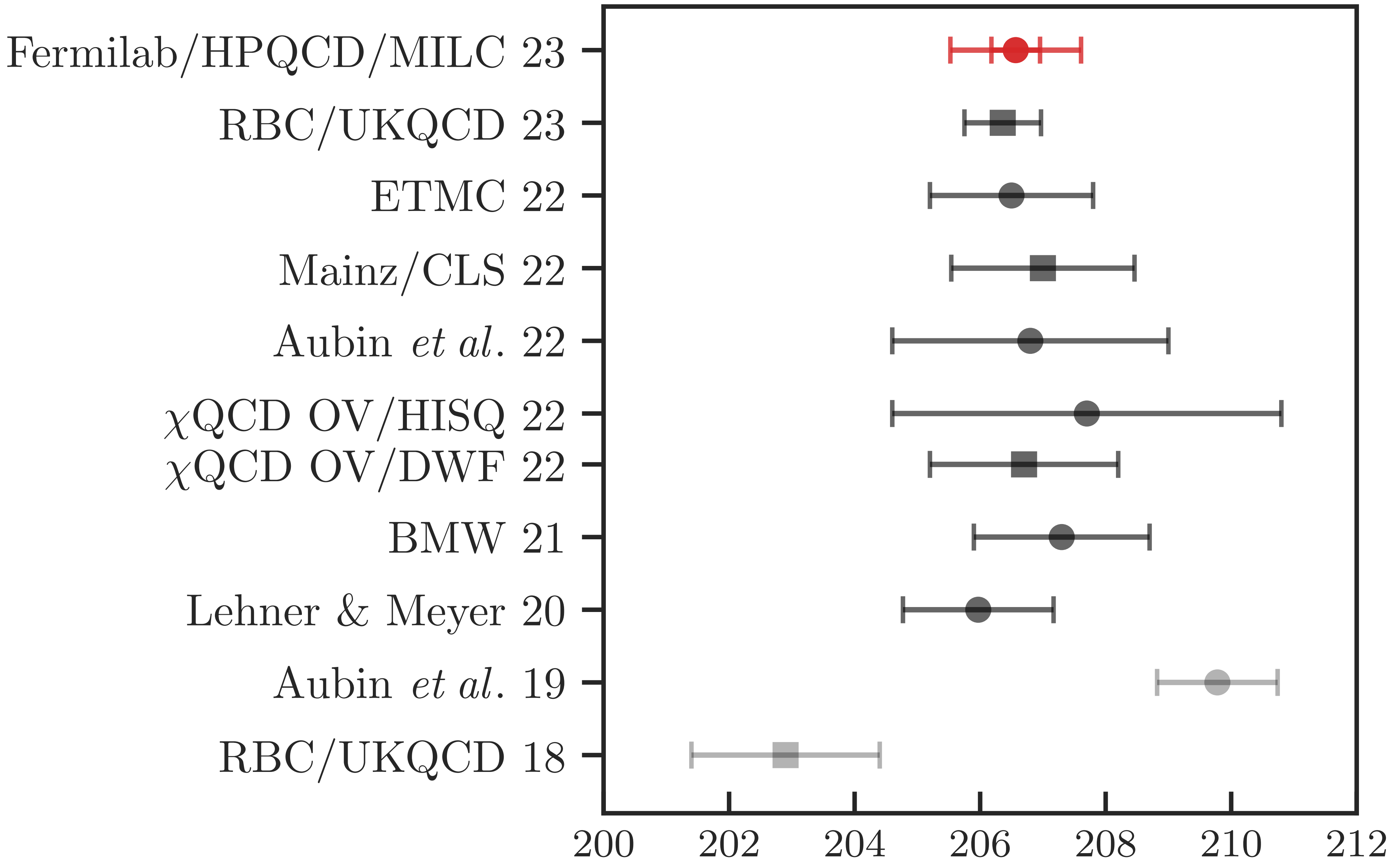
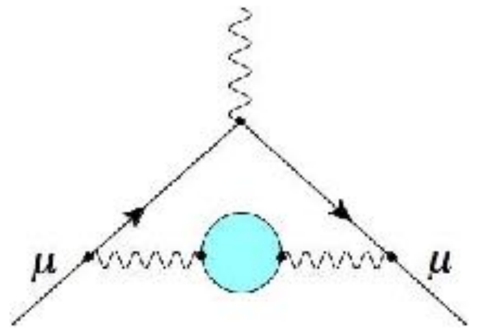


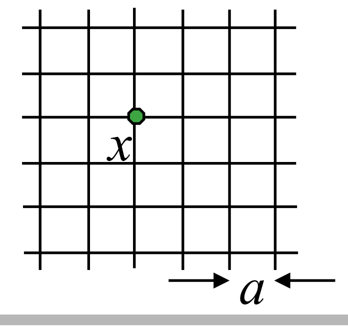
- new results in 2022 for intermediate window, a_μ^W from six different lattice groups.
- Most recently announced unblinded results by RBC/UKQCD and Fermilab/HPQCD/MILC
- lattice-only comparison of light-quark connected contribution to intermediate window:



- Fermilab/HPQCD/MILC: arXiv:2301.08274
- RBC/UKQCD 23: arXiv:2301.08696
- ETM: arXiv:2206.15084
- Mainz: arXiv:2206.06582
- Aubin et al: arXiv:2204.12256
- χQCD: arXiv:2204.01280
- BMW 21: arXiv:2002.12347
- L&M: arXiv:2003.04177
- Aubin et al: arXiv:2204.12256
- RBC/UKQCD 18: arXiv:1801.07224



HVP: lattice



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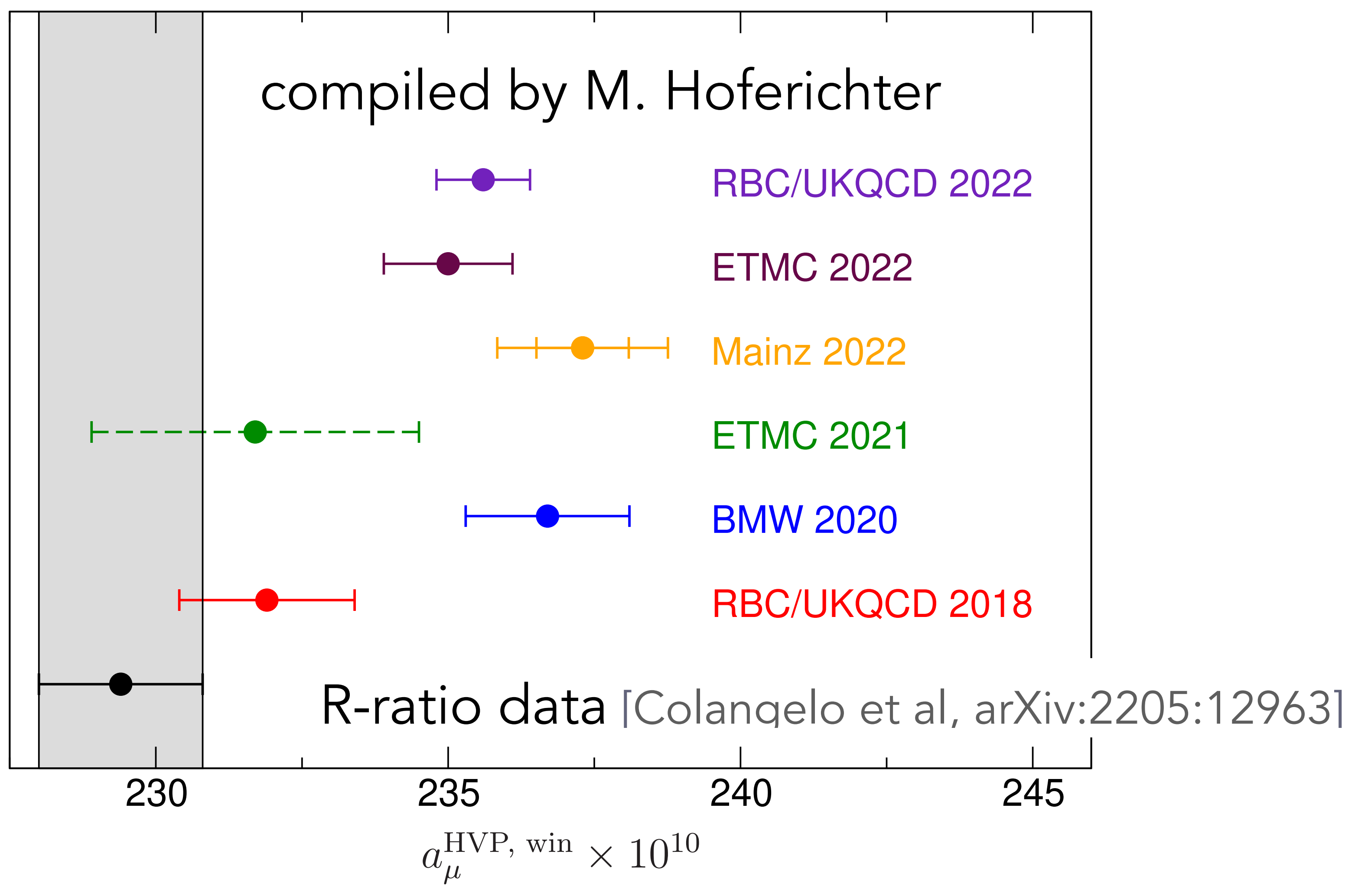


Note: int window ~ 1/3 of $a_\mu^{\text{HVP,LO}}$

Still need independent LQCD calculations of all windows and sub-leading contributions.

Ongoing work:

Evaluations of short-distance windows [ETMC, RBC/UKQCD]

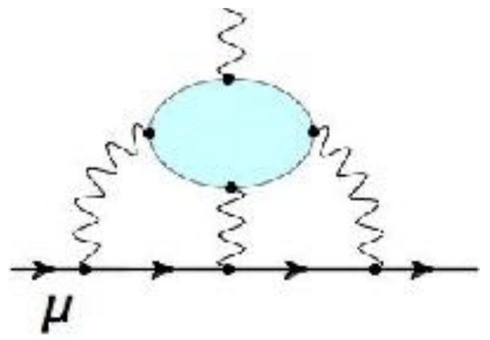
Proposals for computing more windows:

- Use linear combinations of finer windows to locate the tension (if it persists) in $\sqrt{s}$ [Colangelo et al, arXiv:12963]
- Use larger windows, excluding the long-distance region $t \gtrsim 2\,\text{fm}$ to maximize the significance of any tension [Davies at at, arXiv:2207.04765]

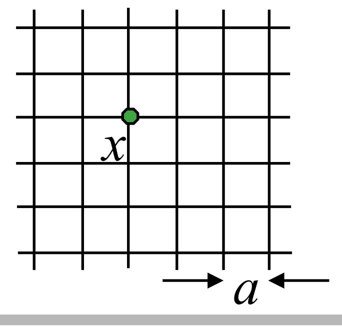
For total HVP:

- independent lattice results at sub-percent precision: coming soon!
- Including $\pi\pi$ states for refined long-distance computation (Mainz, RBC/UKQCD, FNAL/MILC)
- include smaller lattice spacings to test continuum extrapolations (needs adequate computational resources)

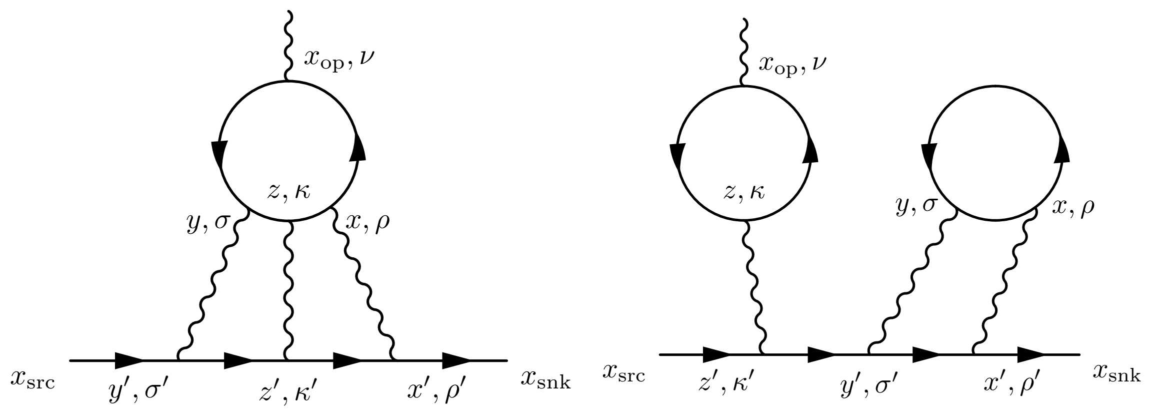
If results are consistent, Lattice HVP (average) with $\sim 0.5\,\%$ errors feasible by 2025

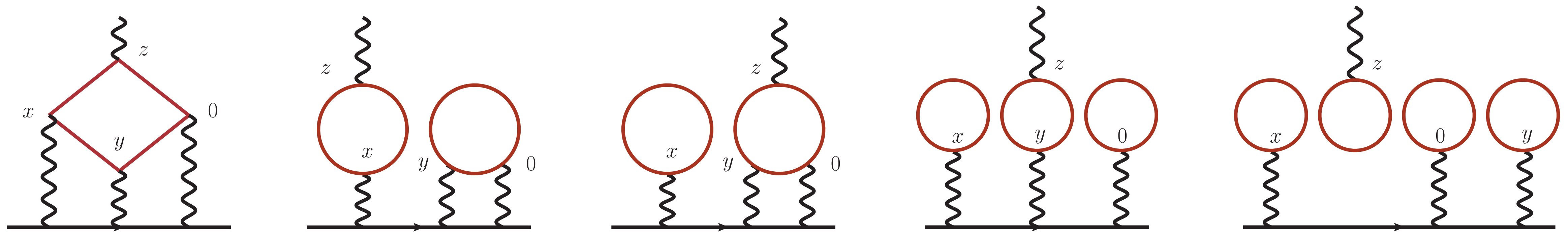


Hadronic Light-by-light



Lattice QCD+QED: Two independent and complete direct calculations of a_μ^{HLbL}





- RBC/UKQCD
  [T. Blum et al, arXiv:1610.04603, 2016 PRL; arXiv:1911.08123, 2020 PRL]
- QCD + $\mathrm{QED_L}$ (finite volume)

  DWF ensembles at/near phys mass,
  $a \approx 0.08 - 0.2\,\mathrm{fm}$, $L \sim 4.5 - 9.3\,\mathrm{fm}$

- Mainz group
  [E. Chao et al, arXiv:2104.02632]
- QCD + QED (infinite volume & continuum)

  CLS (2+1 Wilson-clover) ensembles
  $m_\pi \sim 200 - 430\,\mathrm{MeV}$, $a \approx 0.05 - 0.1\,\mathrm{fm}$, $m_\pi L > 4$

- Cross checks between RBC/UKQCD & Mainz approaches in White Paper at unphysical pion mass
- Both groups are continuing to improve their calculations, adding more statistics, lattice spacings, physical mass ensemble (Mainz)
- update from RBC/UKQCD [T. Blum @ Higgscentre workshop] preliminary results from QCD + QED (inf.)

Lattice HLbL results with $10\,\%$ total uncertainty feasible by ~2025

Connections

$$\sigma(e^+e^- \to \text{hadrons}) \quad \Leftrightarrow \quad a_\mu^{\text{HVP}} \quad \Leftrightarrow \quad \Delta\alpha_{\text{had}}(M_Z^2)$$

Martin Hoferichter @ Lattice HVP workshop

- $\Delta\alpha_{\text{had}}(M_Z^2)$ also depends on the hadronic vacuum polarization function, and can be written as an integral over $\sigma(e^+e^- \to \text{hadrons})$, but weighted towards higher energies.
- a shift in a_μ^{HVP} also changes $\Delta\alpha_{\text{had}}(M_Z^2)$: ➠ EW fits [Passera, et al, 2008, Crivellin et al 2020, Keshavarsi et al 2020, Malaescu & Scott 2020]
  If the shift in a_μ^{HVP} is in the low-energy region ($\lesssim 1\,\text{GeV}$), the impact on $\Delta\alpha_{\text{had}}(M_Z^2)$ and EW fits is small.
- A shift in a_μ^{HVP} from low ($\lesssim 2\,\text{GeV}$) energies
  ➠ $\sigma(e^+e^- \to \pi\pi)$
  must satisfy unitarity & analyticity constraints ➠ $F_\pi^V(s)$
  can be tested with lattice calculations
  [Colangelo, Hoferichter, Stoffer, arXiv:2010.07943]

Hadronic running of α and global EW fit

	e^+e^- KNT, DHMZ	EW fit HEPFit	EW fit GFitter	guess based on BMWc
$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) \times 10^4$	276.1(1.1)	270.2(3.0)	271.6(3.9)	277.8(1.3)
difference to e^+e^-		-1.8σ	-1.1σ	$+1.0\sigma$

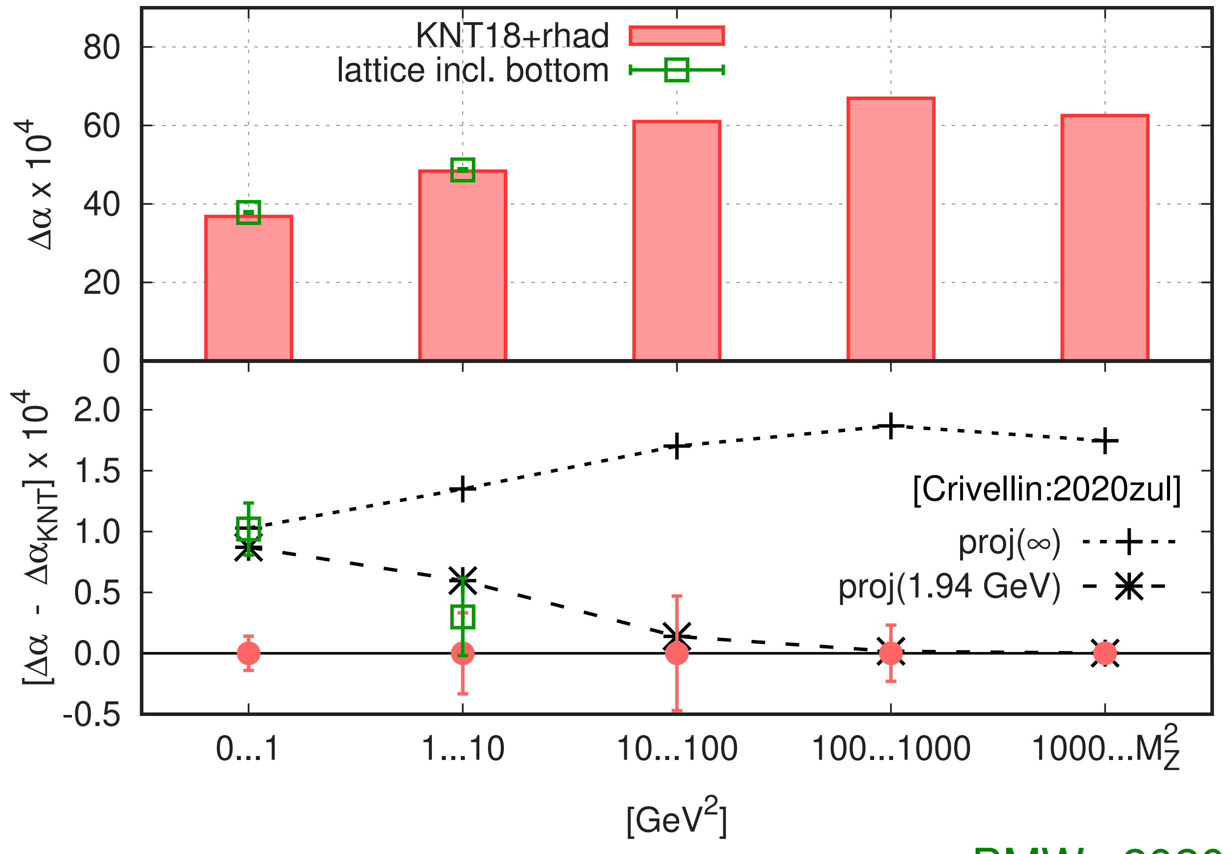
- **Time-like formulation**:

$$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) = \frac{\alpha M_Z^2}{3\pi} P \int_{s_{\text{thr}}}^{\infty} ds \frac{R_{\text{had}}(s)}{s(M_Z^2 - s)}$$

- **Space-like formulation**:

$$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) = \frac{\alpha}{\pi}\hat{\Pi}(-M_Z^2) + \frac{\alpha}{\pi}\left(\hat{\Pi}(M_Z^2) - \hat{\Pi}(-M_Z^2)\right)$$

- Global EW fit
  - Difference between HEPFit and GFitter implementation mainly treatment of M_W
  - Pull goes into **opposite direction**



BMWc 2020

More in talks by M. Passera, B. Malaescu (phenomenology) and K. Miura, T. San José (lattice)

Connections

$$\sigma(e^+e^- \to \text{hadrons}) \quad \Leftrightarrow \quad a_\mu^{\text{HVP}} \quad \Leftrightarrow \quad \Delta\alpha_{\text{had}}(M_Z^2)$$

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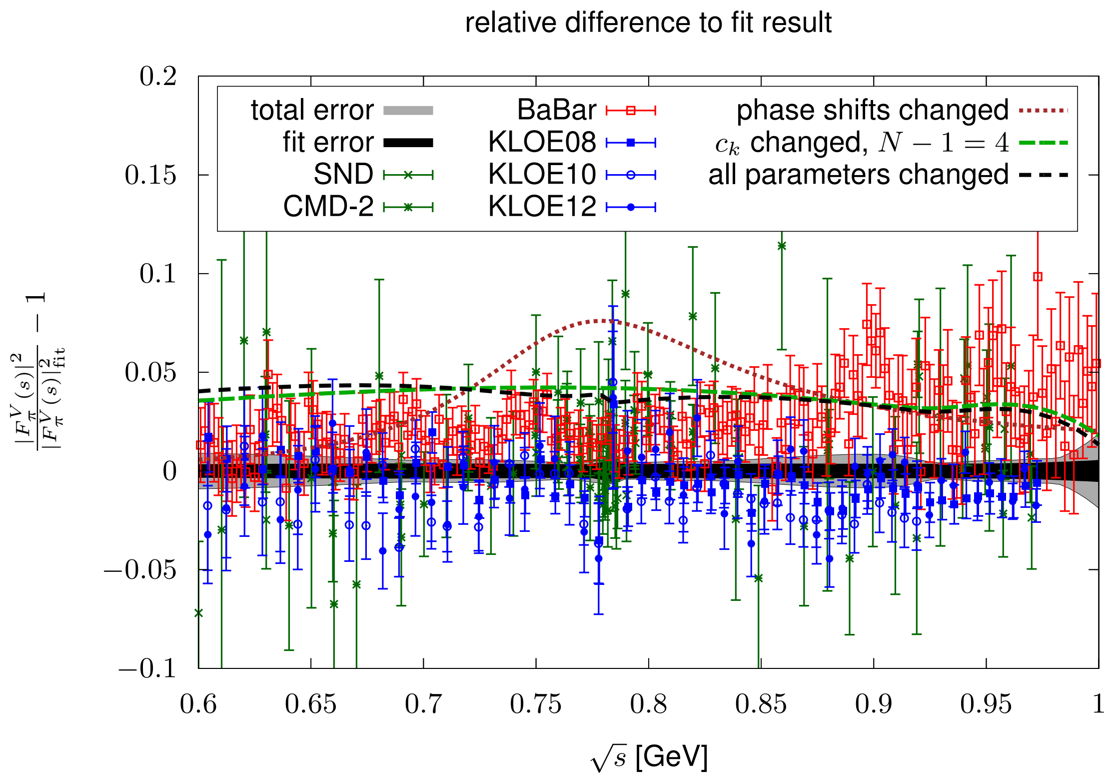
Peter Stoffer @ Lattice HVP workshop

Constraints on the two-pion contribution to HVP — arXiv:2010.07943 [hep-ph]

Modifying $a_\mu^{\pi\pi}|_{\leq 1\,\text{GeV}}$

- "low-energy" scenario: local changes in cross section of $\sim 8\%$ **around ρ**
- "high-energy" scenario: impact on **pion charge radius** and space-like VFF $\Rightarrow$ chance for **independent lattice-QCD checks**
- requires **factor ~ 3 improvement** over χQCD result:
  $\langle r_\pi^2 \rangle = 0.433(9)(13)\,\text{fm}^2$
  $\to$ arXiv:2006.05431 [hep-ph]



Summary

- consistent results from independent, precise LQCD calculations for light-quark connected contribution to **intermediate window** $a_\mu^{\rm W}$ (~ 1/3 of $a_\mu^{\rm HVP,LO}$) ➠ $3-4\,\sigma$ tension with data-driven results?
- still need independent LQCD results for long-distance contribution, total HVP: coming soon
  ➠ develop method average for lattice HVP results, **assess tensions (if any)** with data-driven average
- Programs and plans in place to improve by 2025:
  - data-driven HVP: **if differences are resolved/understood,** $\sim 0.3\,\%$
    new measurements from BaBar, KLOE, Belle II, …. will shed light on current discrepancies
    (blind analyses are paramount!)
  - lattice HVP: **if no tensions** between independent lattice results, $\sim 0.5\,\%$
  - dispersive HLbL and lattice HLbL: no puzzles, steady progress, $\sim 10\,\%$
- **IF tensions/differences between data-driven HVP and lattice HVP are resolved,** SM prediction will likely match precision goal of the Fermilab experiment.
- IF NOT, will need detailed comparisons, explore connections between HVP, $\sigma(e^+e^-)$, $\Delta\alpha$, global EW fits.

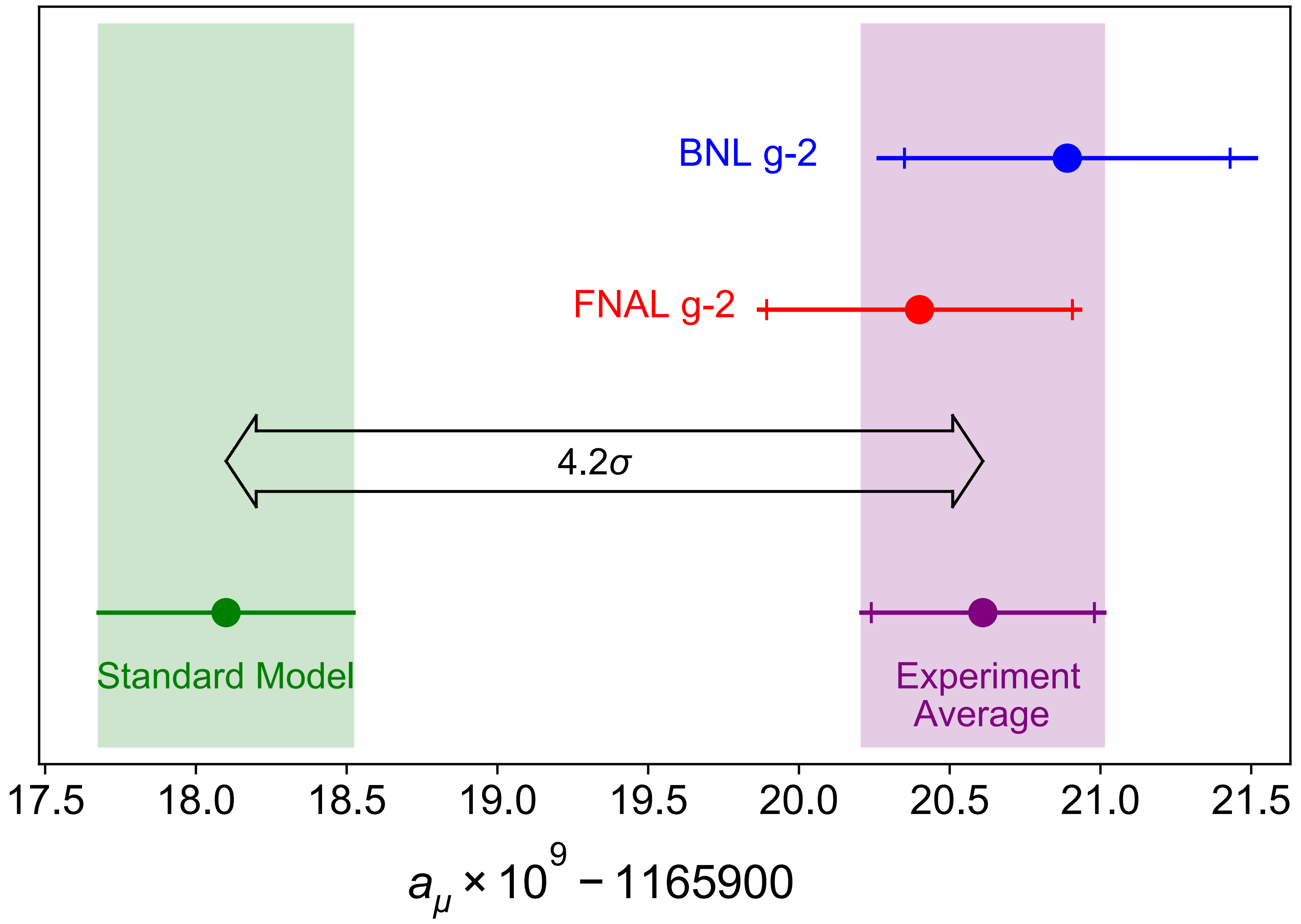
➠ **continued coordination by Theory Initiative: workshops, WPs, …**

Beyond the SM possibilities

a_μ is loop-induced, conserves CP & flavor, flips chirality.

The difference between Exp-SM is large:

$$\Delta a_\mu = 251(59) \times 10^{-11} \; > \; a_\mu(\mathrm{EW})$$



Generically expect:

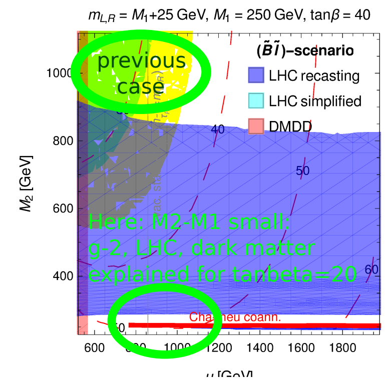
$$a_\mu^{\mathrm{NP}} \sim a_\mu^{\mathrm{EW}} \times \frac{M_W^2}{\Lambda^2} \times \text{couplings}$$

Can be accommodated by many BSM theories (800+ papers)

D. Stöckinger @ g-2 Days (http://pheno.csic.es/g-2Days21/)

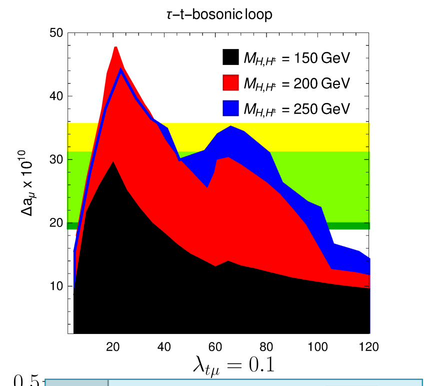
SUSY: MSSM, MRSSM

- MSugra. . . many other generic scenarios
- Bino-dark matter+some coannihil.+mass splittings
- Wino-LSP+specific mass patterns



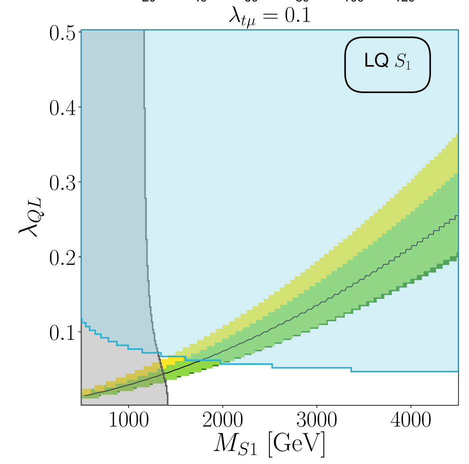
Two-Higgs doublet model

- Type I, II, Y, Type X(lepton-specific), flavour-aligned



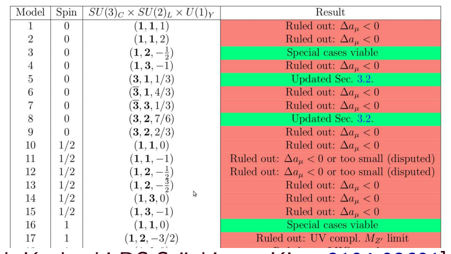
Lepto-quarks, vector-like leptons

- scenarios with muon-specific couplings to μ_L and μ_R



Simple models (one or two new fields)

- Mostly excluded
- light N.P. (ALPs, Dark Photon, Light $L_\mu - L_\tau$)



[Athron,Balazs,Jacob,Kotlarski,DS,Stöckinger-Kim, 2104.03691]

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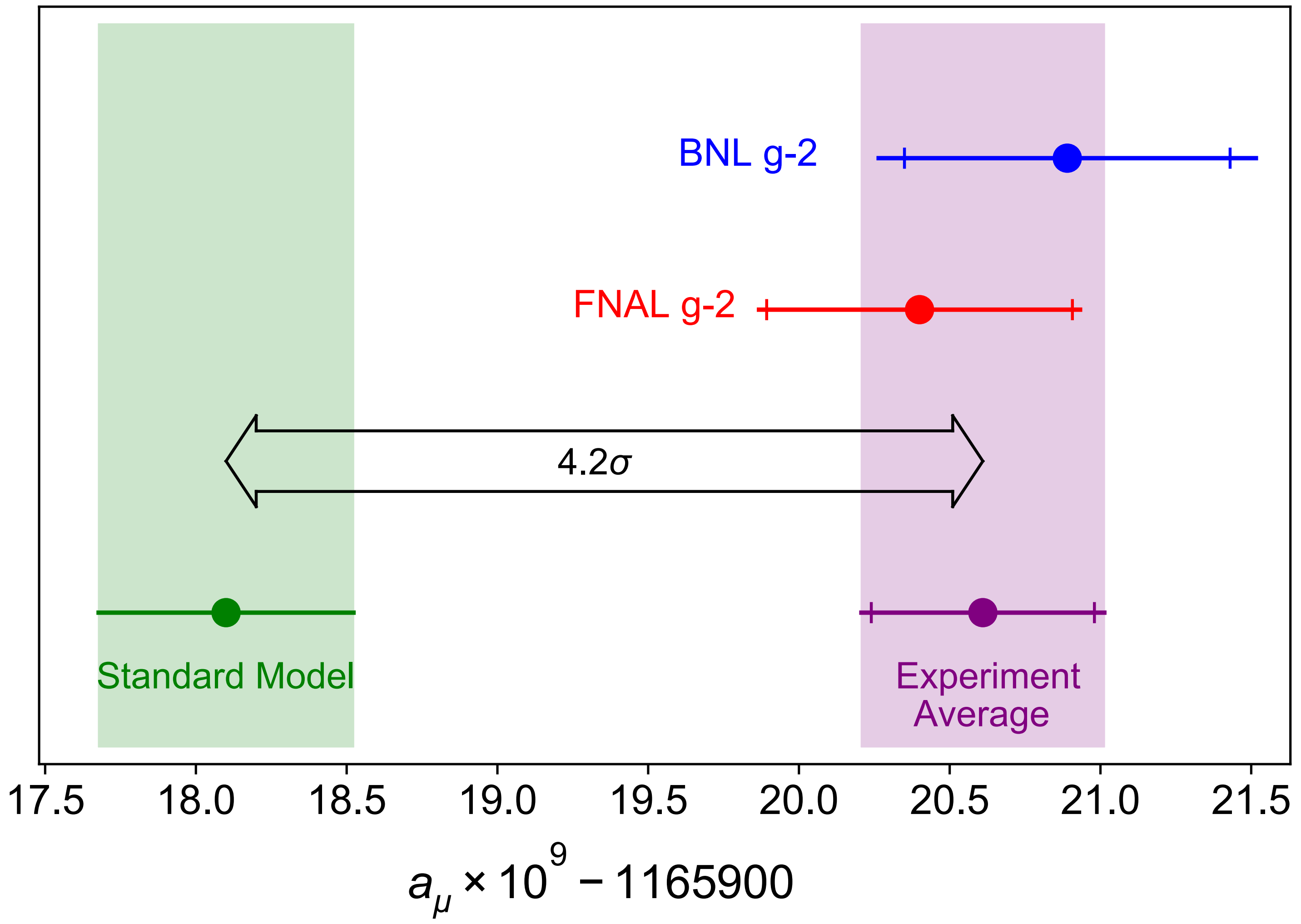

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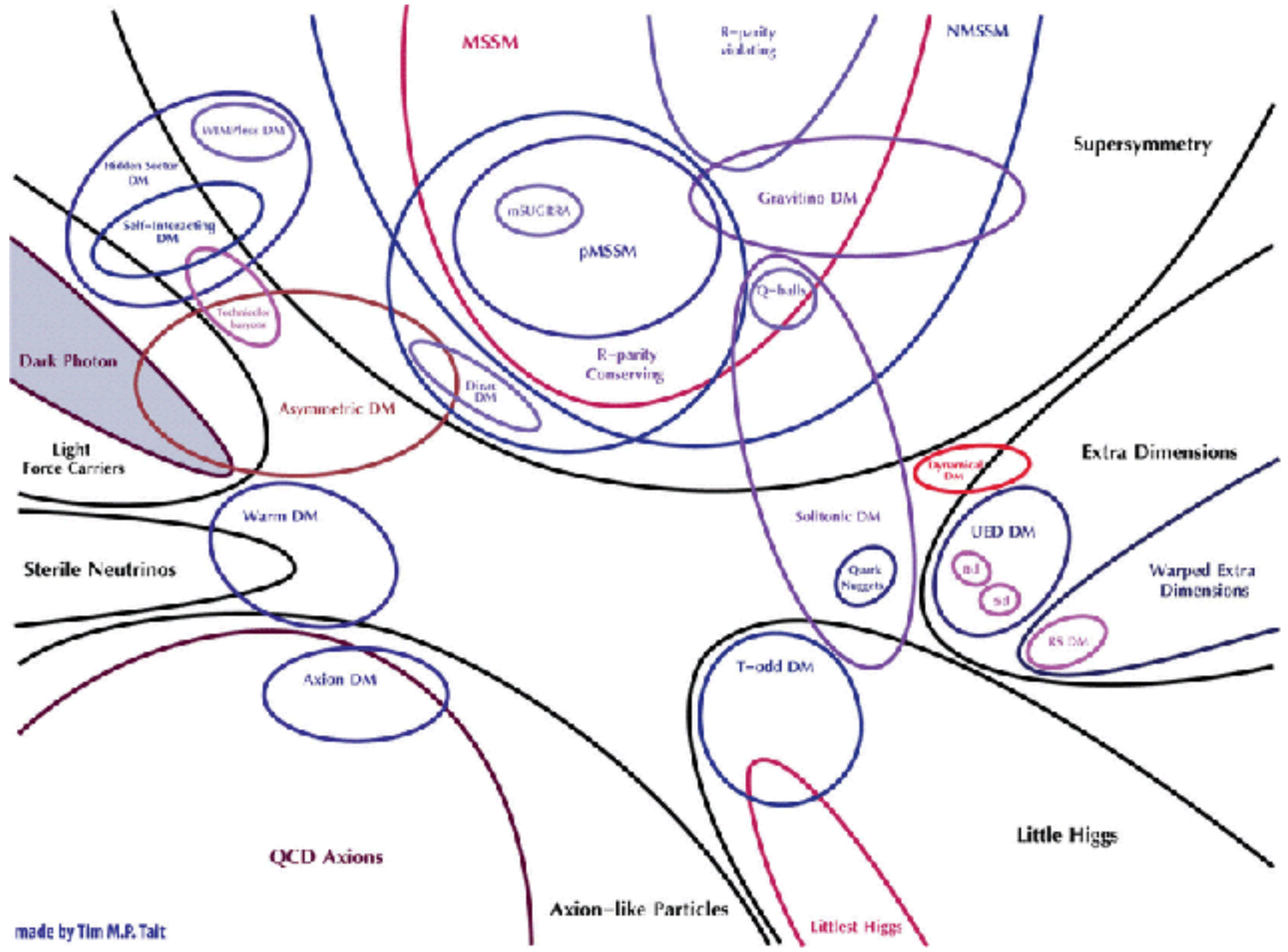
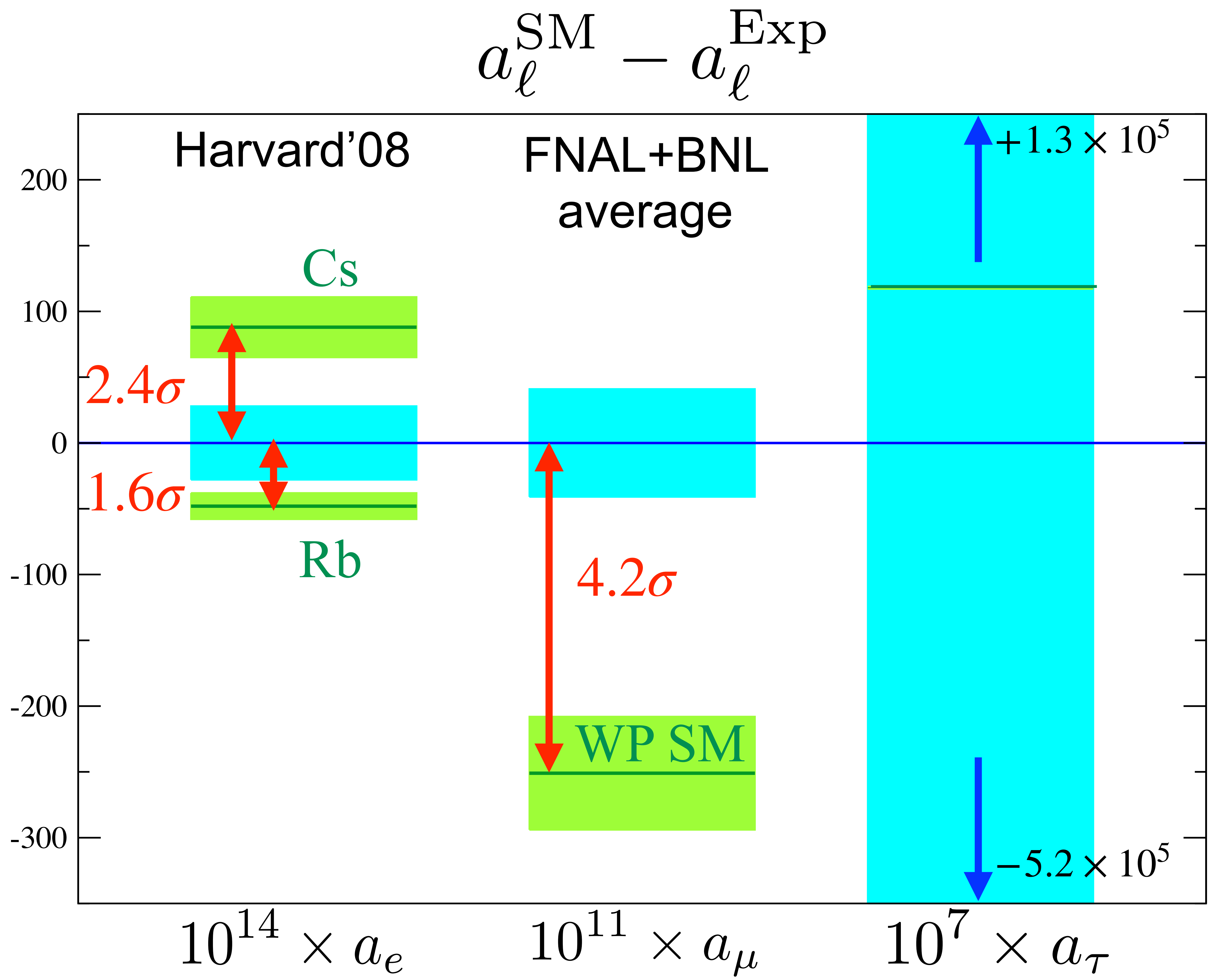


- Can new physics hide in the low-energy $\sigma(e^+e^- \to \pi\pi)$ cross section?
  ➠ No [Luzio, et al, arXiv:2112.08312]
- New boson at ~ 1GeV decays into $\mu^+\mu^-, e^+e^-$, affects $\sigma(e^+e^- \to \pi\pi)$ indirectly [L. Darmé et al, arXiv:2112.09139]
- Neutral, long-lived hadrons, heretofore undetected? [Farrar, arXiv:2206.13460]
- Z' at < 1 GeV, coupling to 1st gen matter particles [Coyle, Wagner, arXiv:2305.02354]

Outlook

- Experimental program beyond 2025:
  - J-PARC: Muon g-2/EDM
  - CERN: MUonE
  - Fermilab: future muon campus experiments?
  - Belle II, BESIII, Novosibirsk,…
  - Chiral Belle (?)
- Data-driven/dispersive program beyond 2025:
  - development of NNLO MC generators
  - for HLbL, improved experimental/lattice inputs together with further development of dispersive approach
- MUonE will provide a space-like determination of HVP
- Lattice QCD beyond 2025:
  - access to future computational resources (coming Exascale) will enable improvements of all errors (statistical and systematic)
  - concurrent development of better methods and algorithms (gauge-field sampling, noise reduction) will accelerate progress
  - **beyond g-2: a rich program relevant for all areas of HEP**

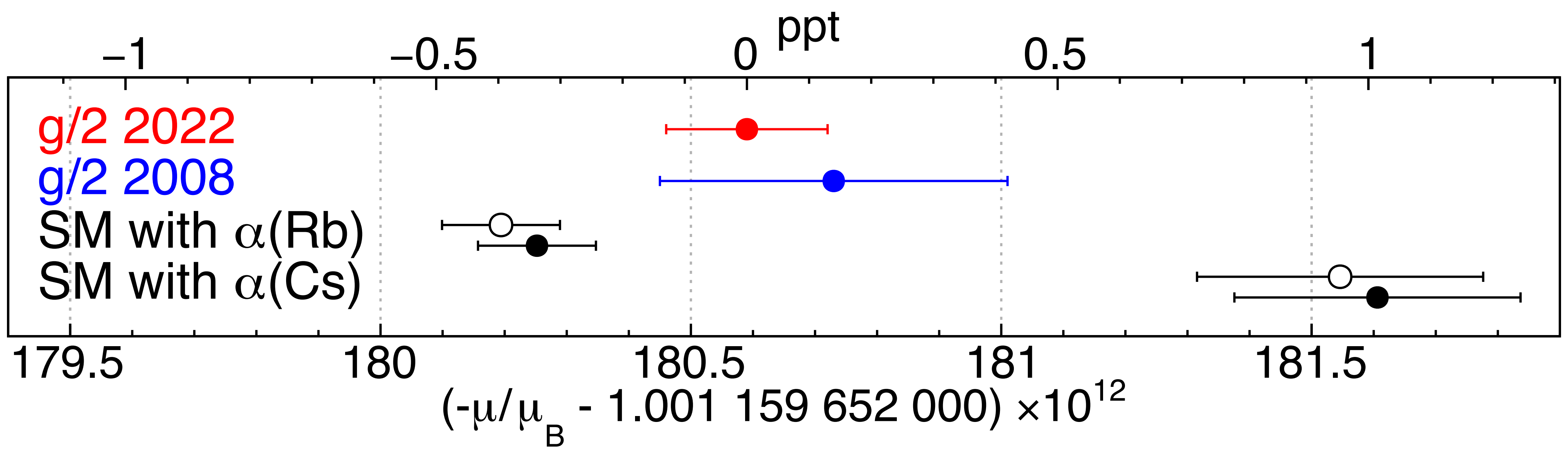
Lepton moments summary



Sensitivity to heavy new physics: $a_\ell^{NP} \sim \frac{m_\ell^2}{\Lambda^2}$

$(m_\mu/m_e)^2 \sim 4 \times 10^4$

[X. Fan et al (Gabrielse group), arXiv:2209.13084]



Cs: α from Berkeley group [Parker et al, Science 360, 6385 (2018)]

Rb: α from Paris group [Morel et al, Nature 588, 61–65(2020)]

Prospects for tau moment measurement:

Chiral Belle arXiv:2205.12847

- use polarized e^- beam
- with $40ab^{-1}$ measurement of a_τ at 10^{-5} feasible
- with more statistics measurement at 10^{-6} possible

Outlook

$m_{ud},\, m_u/m_d,\, m_s$ ⇐ $a_\mu^{\text{HVP LO}}$ a_μ^{HLbL} d_n MEs for light nuclei

$m_c,\, m_b$ $\langle \bar{B}_q^0 | \mathcal{O}_i^{\Delta B=2} | B_q^0 \rangle$ PDFs, GPDs, TMDs,.. $\langle NN | O_i | NN \rangle$

$\alpha_{\overline{MS}}(m_z)$ $\langle \bar{D}^0 | \mathcal{O}_i^{\Delta C=2} | D^0 \rangle$ $\Lambda_b \to (p, \Lambda_c, \Lambda)\, \ell\nu$ $D_s \to \ell\nu\gamma$ $B \to \pi\pi\, \ell\nu\ \ldots$

$\hat{B}_K, \ldots$ nucleon form factors, .. $K^+ \to \ell^+\nu\,(\gamma)$ $B \to X_c \ell \nu$, other inclusive decay rates, …

$f_{B_{(s)}}\ , \ldots$ $g_A,\, g_T,\, g_S$ $\pi\pi \to \rho$ $\pi \to \ell^+\ell^-,\ K \to \gamma\gamma$

$f^{K\to\pi}(q^2)$ $f_{+,0,T}^{B\to D}(q^2), \ldots$ $\langle \pi\pi_{(I=0)} | \mathcal{H}^{\Delta S=1} | K^0 \rangle$ $K^+ \to \pi^+\ell^+\ell^-$ …

$f_{K^\pm}$ $f_{+,0,T}^{B\to\pi}(q^2), \ldots$ $\langle \pi\pi_{(I=2)} | \mathcal{H}^{\Delta S=1} | K^0 \rangle$ $\Delta M_K,\, \epsilon_K$ $K^+ \to \pi^+\nu\bar{\nu}$ Complexity

✓ LQCD flagship results

↑ First complete LQCD results, large(ish) errors

↑ First results, physical params, incomplete systematics

↑ new methods, pilot projects, unphysical kinematics

↑ new ideas, first studies

Thank you!

Centre for Theoretical Higgs Physics

UKQCD

KEK

Fermilab

Institute for Nuclear Theory

Illinois

The University of Edinburgh

NVIDIA

STORJ

NEC

Atos

U.S. Department of Energy, Office of Science

University of Washington

RIKEN BNL Research Center

epcc

URA Universities Research Association

STRONG 2020

Fermi Research Alliance LLC

HIM Helmholtz Helmholtz-Institut Mainz

SFB 1044 The Low-Energy Frontier of the Standard Model

PRISMA

IP3

UCONN Department of Physics

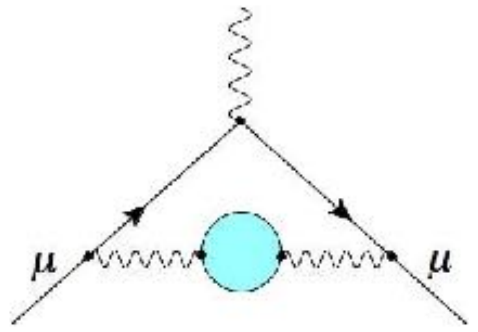
JGU Johannes Gutenberg Universität Mainz

Appendix

Updated WP Summary Table

Contribution	Value $\times 10^{11}$	References
Experimental average (E989+E821)	116592061(41)	Phys.Rev.Lett. 124, 141801
HVP LO (e^+e^-)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	−98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	12.4(1)	Ref. [8]
HVP LO (lattice, $udsc$)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	2(1)	Ref. [31]
HLbL (lattice, uds)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	90(17)	Refs. [18–30, 32]
QED	116 584 718.931(104)	Refs. [33, 34]
Electroweak	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	92(18)	Refs. [18–32]
Total SM Value	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	251(59)	

website: https://muon-gm2-theory.illinois.edu



HVP: data-driven

Conservative merging procedure to obtain a realistic assessment of the underlying uncertainties:

[B. Malaescu @ INT g-2 workshop]

Detailed comparisons by-channel and energy range between direct integration results:

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	−0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	−0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	−0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
$K_S K_L$	12.82(0.06)(0.18)(0.15)	13.04(19)	−0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	−0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
[1.8, 3.7] GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	−1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	−0.08
[3.7, ∞) GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	$694.0(1.0)(3.5)(1.6)(0.1)_\psi(0.7)_{\text{DV+QCD}}$	692.8(2.4)	1.2

Energy range	ACD18	CHS18	DHMZ19	DHMZ19′	KNT19
≤ 0.6 GeV		110.1(9)	110.4(4)(5)	110.3(4)	108.7(9)
≤ 0.7 GeV		214.8(1.7)	214.7(0.8)(1.1)	214.8(8)	213.1(1.2)
≤ 0.8 GeV		413.2(2.3)	414.4(1.5)(2.3)	414.2(1.5)	412.0(1.7)
≤ 0.9 GeV		479.8(2.6)	481.9(1.8)(2.9)	481.4(1.8)	478.5(1.8)
≤ 1.0 GeV		495.0(2.6)	497.4(1.8)(3.1)	496.8(1.9)	493.8(1.9)
[0.6, 0.7] GeV		104.7(7)	104.2(5)(5)	104.5(5)	104.4(5)
[0.7, 0.8] GeV		198.3(9)	199.8(0.9)(1.2)	199.3(9)	198.9(7)
[0.8, 0.9] GeV		66.6(4)	67.5(4)(6)	67.2(4)	66.6(3)
[0.9, 1.0] GeV		15.3(1)	15.5(1)(2)	15.5(1)	15.3(1)
≤ 0.63 GeV	132.9(8)	132.8(1.1)	132.9(5)(6)	132.9(5)	131.2(1.0)
[0.6, 0.9] GeV		369.6(1.7)	371.5(1.5)(2.3)	371.0(1.6)	369.8(1.3)
$[\sqrt{0.1}, \sqrt{0.95}]$ GeV		490.7(2.6)	493.1(1.8)(3.1)	492.5(1.9)	489.5(1.9)

Include constraints using unitarity & analyticity for $\pi\pi$ and $\pi\pi\pi$ channels

[CHS 2018, Colangelo et al, arXiv:1810.00007; HHKS19, Hoferichter et al, arXiv:1907.01556]

$$\Rightarrow a_\mu^{\text{HVP,LO}} = 693.1\,(2.8)_{\text{exp}}\,(2.8)_{\text{sys}}\,(0.7)_{\text{DV+pQCD}} \times 10^{-10} = 693.1\,(4.0) \times 10^{-10}$$

Efforts on Radiative Corrections for low energy $e^+e^- \to$ hadrons

Workstop+Conference in Zurich 5-9 June 2023 (LOC: A. Signer, G. Stagnitto, Y. Ulrich)

Three-day in-person (Workstop) +
a three half day hybrid conference

5 Working Groups:

- WP1: Leptonic processes at NNLO
- WP2: Form factor contributions at N3LO
- WP3: Processes with hadrons
- WP4: Parton showers
- WP5: Experimental input

Radiative corrections and Monte Carlo tools for low-energy hadronic cross sections in e^+e^- collisions

5–9 Jun 2023
University of Zurich
Europe/Zurich timezone

Enter your search term

- Overview
- Timetable
- Contribution List
- Code of Conduct

Contact

yannick.ulrich@durham....

In this workstop, we will discuss radiative corrections and Monte Carlo tools for low-energy hadronic cross sections in e^+e^- collisions. This is to be seen as part of the Strong 2020 effort. We will cover

- leptonic processes at NNLO and beyond
- processes with hadrons
- parton shower
- experimental input

Each area will be given at least half a day, starting with an open 1h seminar followed by a lengthy discussion.

Just like previous workstops, we try to gather a small number of theorists who actively work on this topic to make very concrete progress. It should not just be about giving talks, but to actually learn from each other and put together the jigsaw pieces.

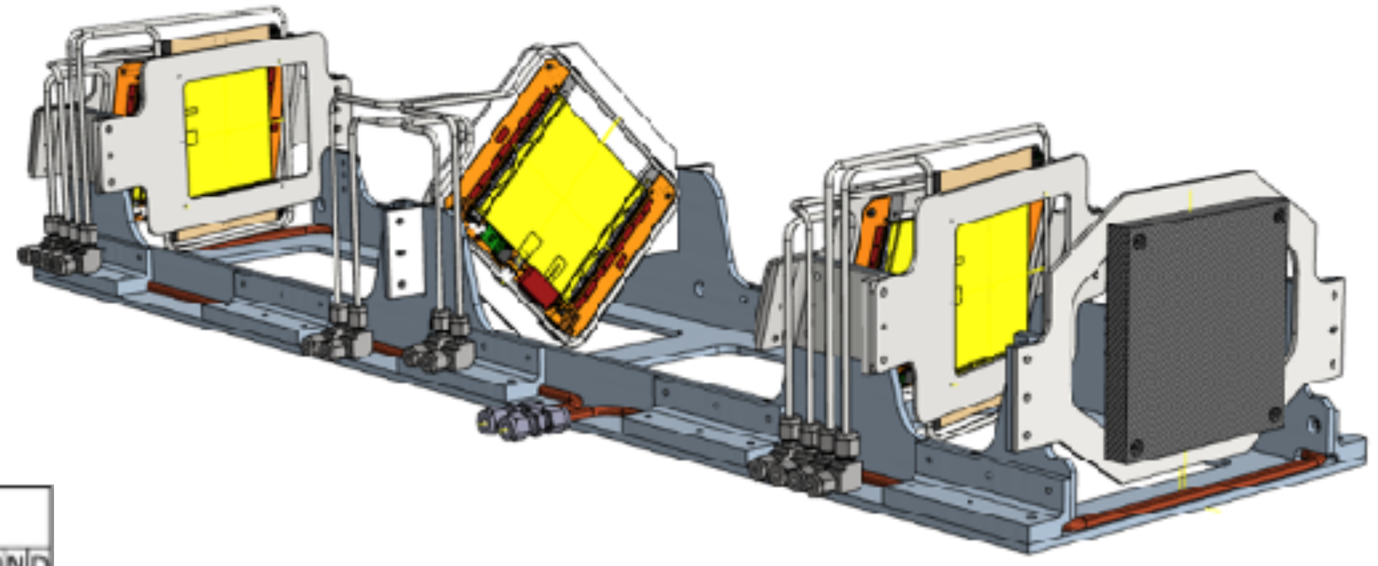
Additionally to the workstop that is only by-invite only, there is a broader conference directly following the workstop.

Final goal: full NNLO MC. Aim to write a report by Autumn 2023

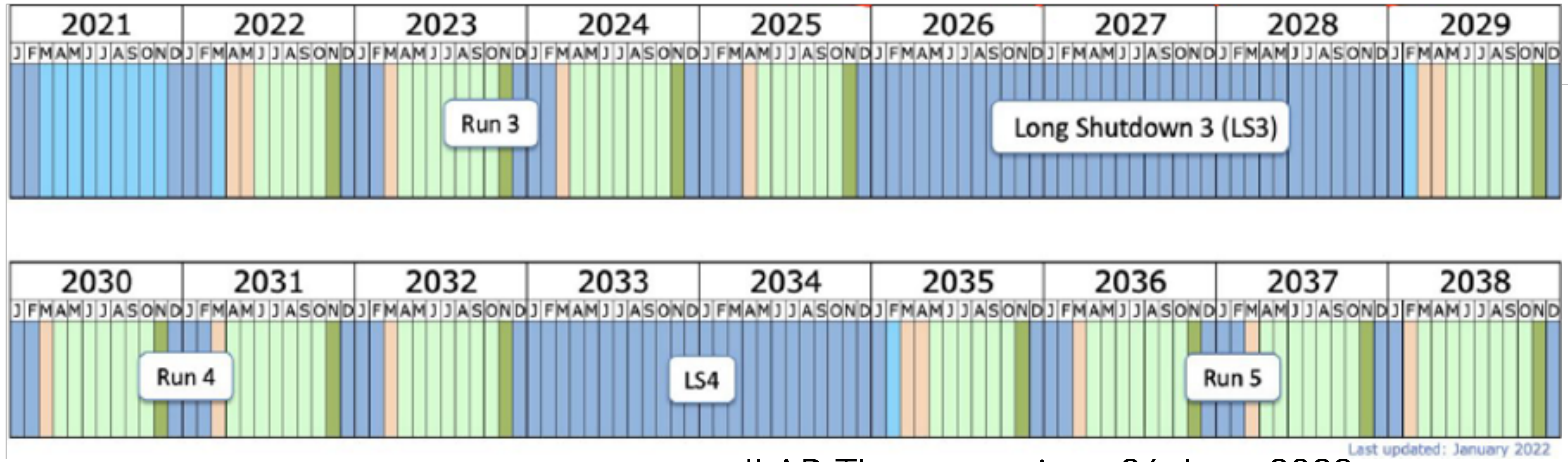
Alternative measurement of HVP for a_μ^{HVP} : MUonE at CERN

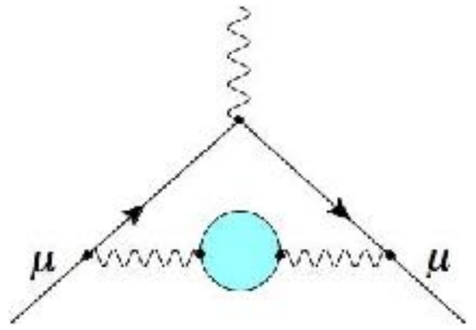
- Space-like determination of a_μ^{HVP} at <0.5% through the scattering of 160 GeV muons on electron target
- Much progress in the last years, inc. detector optimization and development of $\mu - e$ (N)NLO MC
- Staged approach towards the full experiment: one station (2022), two stations (2023); possible 10 stations before LS3 (2026) (2% accuracy)
- Technical proposal towards full experiment in preparation
- Growing interest from both experimental and theory community

-*C. M. Carloni Calame et al PLB 746 (2015) 325*
-*G. Abbiendi et al Eur.Phys.J.C* 77 (2017) 3, 139
-LoI https://cds.cern.ch/record/2677471/files/SPSC-I-252.pdf



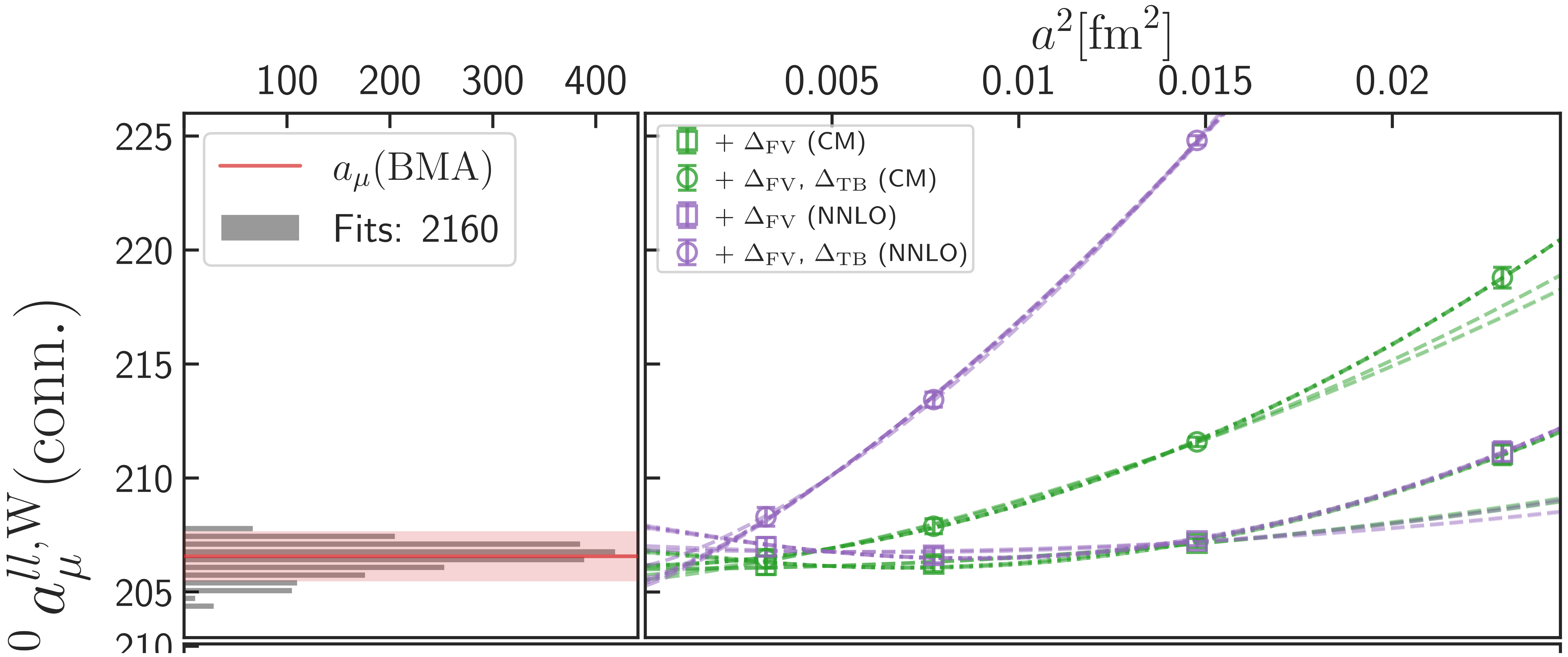
LHC schedule

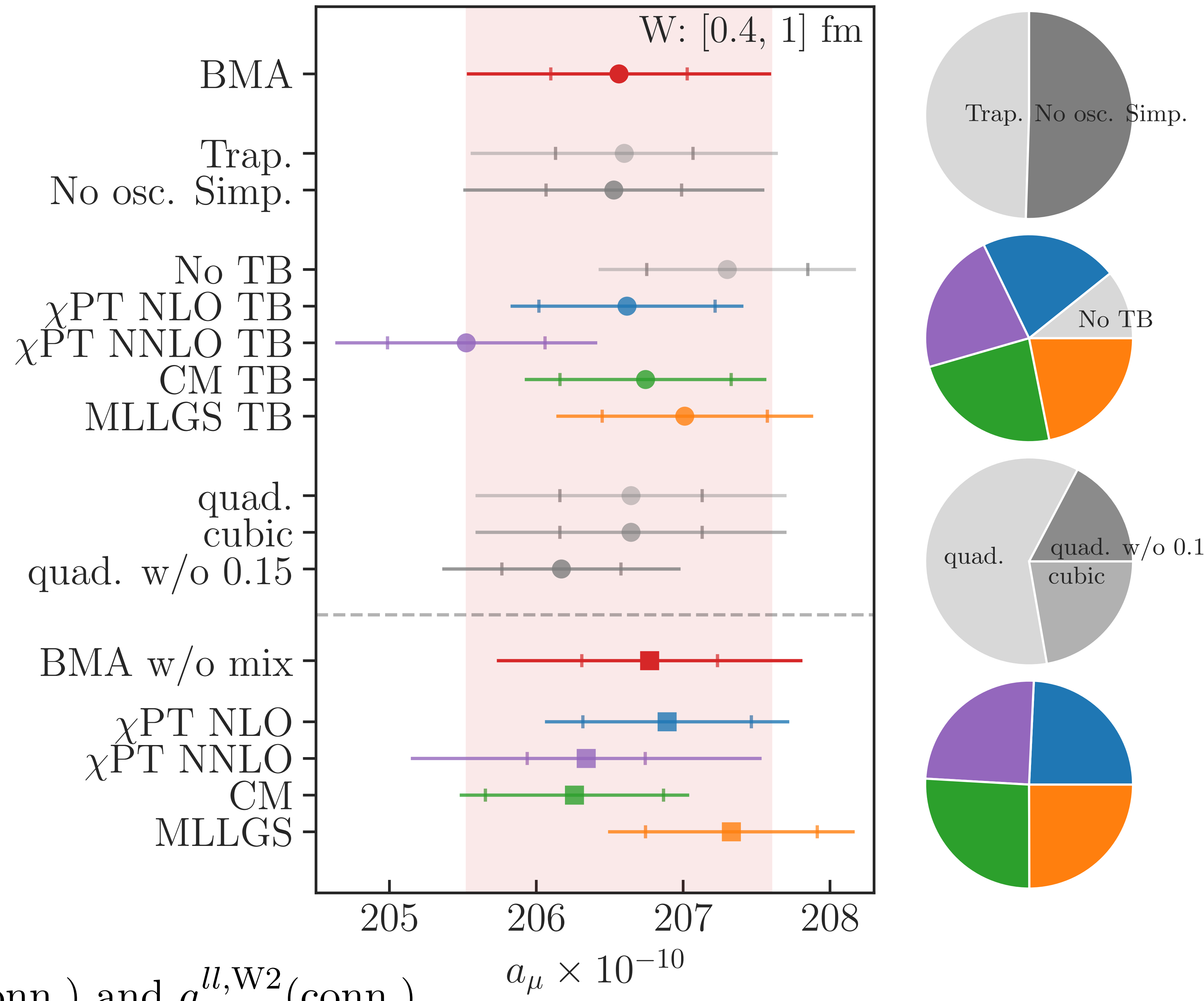




Intermediate window (ud)

Fermilab/HPQCD/MILC: arXiv:2301.08274, 2023 PRD





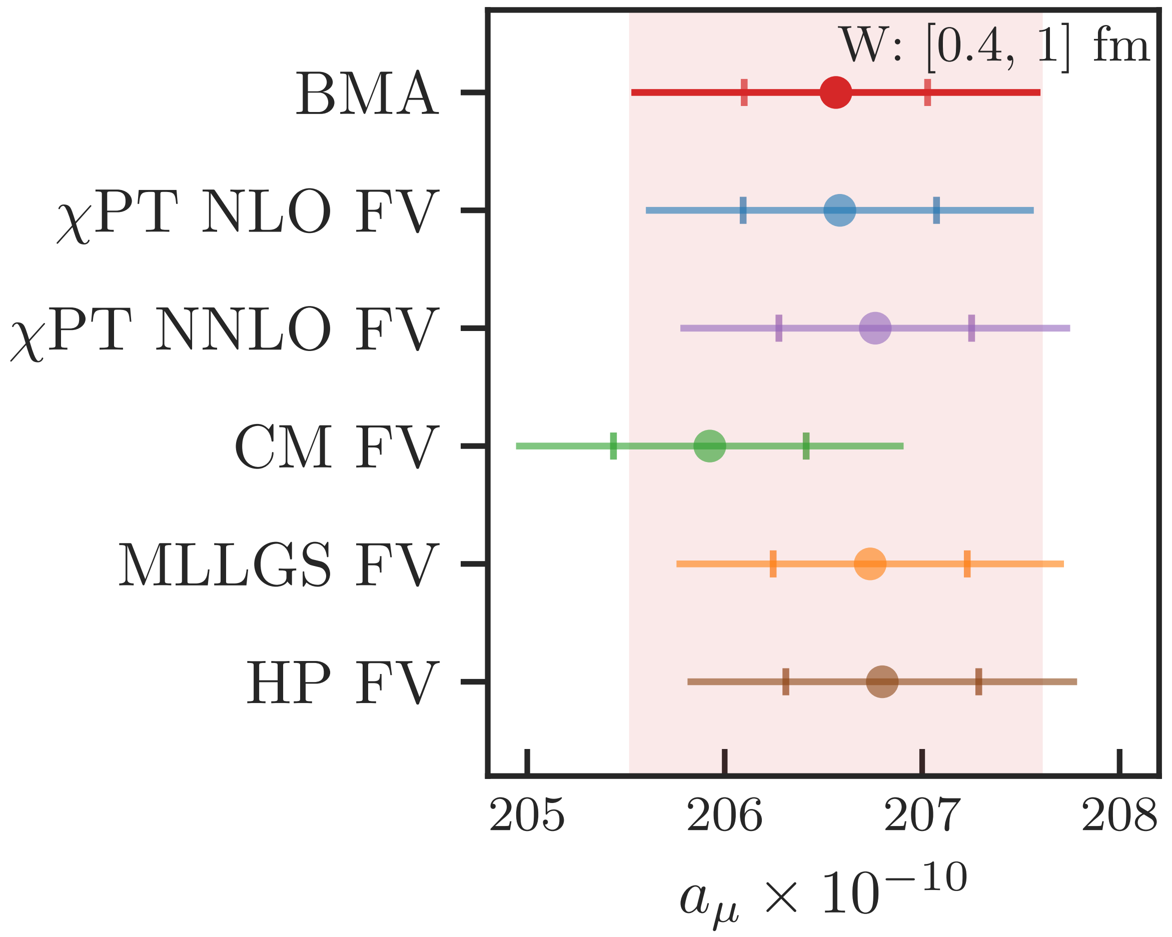


TABLE IV. Approximate error budgets for $a_\mu^{ll,\mathrm{W}}$(conn.) and $a_\mu^{ll,\mathrm{W2}}$(conn.).

Source	$\delta a_\mu^{ll,\mathrm{W}}$(conn.) (%)	$\delta a_\mu^{ll,\mathrm{W2}}$(conn.) (%)
Monte Carlo statistics	0.19	2.44
Continuum extrapolation ($a \to 0$, Δ_{TB})	0.34	1.05
Finite-volume correction (Δ_{FV})	0.16	0.23
Pion-mass adjustment (Δ_{M_π})	0.06	0.96
Scale setting (w_0 (fm), w_0/a)	0.21	1.28
Current renormalization (Z_V)	0.17	0.16
Total	0.50%	3.18%