Quark orbital angular momentum in the proton from Lattice QCD

Michael Engelhardt, Jeremy Green, Nesreen Hasan, Taku Izubuchi, Christos Kallidonis, Stefan Krieg, Simonetta Liuti, Stefan Meinel, John Negele, Andrew Pochinsky, Abha Rajan, Marcel Rodekamp, Giorgio Silvi, Sergey Syritsyn

Acknowledgments:
Gauge ensembles provided by JLab/W&M Collaboration, and by RBC/UKQCD
Supported by U.S. DOE, Office of Science, Office of Nuclear Physics

Proton spin decompositions

$$\frac{1}{2} = \frac{1}{2}\sum_q \Delta q + \sum_q L_q + J_g \qquad \text{(Ji)}$$

$$\frac{1}{2} = \frac{1}{2}\sum_q \Delta q + \sum_q \mathcal{L}_q + \Delta g + \mathcal{L}_g \qquad \text{(Jaffe-Manohar)}$$

...and many more (in fact, we will see a continuous interpolation between the two ...)

There isn't one unique way of separating quark and gluon orbital angular momentum – the different decompositions have different, legitimate meanings.

Quark orbital angular momentum

Interpreting terms in the energy-momentum tensor:

$$L_q \quad \sim \quad -i\psi^\dagger(\vec{r}\times\vec{D})_z\psi$$

Can be obtained from $L_q = J_q - S_q$, where S_q and J_q can be related to GPDs (Ji sum rule) – this is the traditional method in Lattice QCD.

$$\mathcal{L}_q \quad \sim \quad -i\psi^\dagger(\vec{r}\times\vec{\partial})_z\psi \qquad \text{in light cone gauge}$$

Needs a different method - GTMDs.

Quark OAM and parton distributions

Ji sum rule:

$$L_3 = J_3 - S_3 = \frac{1}{2}\int dx\, x\,(H+E) - \frac{1}{2}\int dx\, \widetilde{H}$$

Twist-2 GTMD F_{14}:

$$L_3 = -\int dx \int d^2k_T\, \frac{k_T^2}{M^2}\, F_{14}$$

$\longrightarrow$ from Ji to Jaffe-Manohar OAM

Twist-3 GPD $\widetilde{E}_{2T}$:

$$L_3 = (L_3 + 2S_3) - 2S_3 = -\int dx\, x\widetilde{E}_{2T} - \int dx\, \widetilde{H}$$

Note: A version of this approach was first noted by M. Polyakov, denoting the relevant twist-3 GPD variously as G_2 or G_3. Relation to OAM derived using OPE. Here, will review alternative approach based on GTMDs.

Note: All distribution functions above taken in the forward limit!

Equation of motion relation

$$-x\widetilde{E}_{2T} - \widetilde{H} = -\int d^2k_T \, \frac{k_T^2}{M^2} \, F_{14} + \widetilde{M} \qquad \Longrightarrow \qquad L_3 = -\int dx \int d^2k_T \frac{k_T^2}{M^2} F_{14} = -\int dx \, x \, \widetilde{E}_{2T} - \int dx \, \widetilde{H}$$

(forward limit)

How? Consider correlator (straight gauge link $\mathcal{U}$)

$$W^{\Gamma}_{\Lambda'\Lambda} = \frac{1}{2} \int \frac{dz^- \, d^2z_T}{(2\pi)^3} e^{ixP^+z^- - ik_T \cdot z_T} \, \langle p', \Lambda' \mid \bar{\psi}(-z/2) \, \Gamma \, \mathcal{U} \, \psi(z/2) \mid p, \Lambda \rangle \big|_{z^+=0}$$

Use $\Gamma = i\sigma^{i+}\gamma^5$ and quark field equation of motion $\longrightarrow$ $(i\not{D} - m)\psi = 0$

to generate (note zero skewness throughout)

$$ik^+ \epsilon^{ij} W^{\gamma^j}_{\Lambda'\Lambda} + \frac{\Delta^i}{2} W^{\gamma^+\gamma^5}_{\Lambda'\Lambda} - i\epsilon^{ij} k^j W^{\gamma^+}_{\Lambda'\Lambda} + \mathcal{M}^i_{\Lambda'\Lambda} = 0$$

Form combination $\frac{\Delta^i}{\Delta_T^2} \left(W^{\Gamma}_{++} - W^{\Gamma}_{--} \right)$, insert parametrizations of $W^{\Gamma}_{\Lambda'\Lambda}$ in terms of GTMDs

Integrate over k_T, identify GPDs

Lorentz invariance relation

$$\frac{d}{dx}\int d^2k_T\,\frac{k_T^2}{M^2}\,F_{14} \;=\; \widetilde{E}_{2T} + H + E$$

How? Consider completely unintegrated correlator (straight gauge link $\mathcal{U}$)

$$\mathcal{W}^{\Gamma}_{\Lambda'\Lambda} = \frac{1}{2}\int\frac{d^4z}{(2\pi)^4}e^{ik\cdot z}\langle P+\Delta/2,\Lambda'|\bar{\psi}\left(-\frac{z}{2}\right)\Gamma\mathcal{U}\psi\left(\frac{z}{2}\right)|P-\Delta/2,\Lambda\rangle$$

It can be parametrized in terms of Lorentz invariant amplitudes $A(k^2, k\cdot P, k\cdot\Delta, \Delta^2, P\cdot\Delta)$.

Its k^- integral is the GTMD correlator, $\qquad W^{\Gamma}_{\Lambda'\Lambda} = \int dk^-\,\mathcal{W}^{\Gamma}_{\Lambda'\Lambda}$

There are more GTMDs than A-amplitudes $\longrightarrow$ relations between GTMDs induced.

Combine with EoM relation,

$$x\widetilde{E}_{2T} = -\,\widetilde{H} + \int d^2k_T\,\frac{k_T^2}{M^2}\,F_{14} - \widetilde{M} \qquad\Longrightarrow\qquad L_3 = -\int dx\int d^2k_T\frac{k_T^2}{M^2}F_{14} = \frac{1}{2}\int dx\,x\,(H+E) - \frac{1}{2}\int dx\,\widetilde{H}$$

(forward limit)

Quark Orbital Angular Momentum from Wigner Distribution

$$L_3^{\mathcal{U}} = \int dx \int d^2k_T \int d^2r_T\, (r_T \times k_T)_3\, \mathcal{W}^{\mathcal{U}}(x, k_T, r_T)$$

Wigner distribution

$$= -\int dx \int d^2k_T\, \frac{k_T^2}{M^2}\, F_{14}(x, k_T^2, k_T\cdot\Delta_T, \Delta_T^2)\Big|_{\Delta_T = 0}$$

Generalized transverse momentum-dependent parton distribution (GTMD)

$$= \epsilon_{ij}\frac{\partial}{\partial z_{T,i}}\frac{\partial}{\partial \Delta_{T,j}}\,\langle p', S \mid \overline{\psi}(-z/2)\gamma^+\mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\Big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}$$

P, S in 3-direction, $P \to \infty$
$p' = P + \Delta_T/2$, $p = P - \Delta_T/2$

Connection to GTMDs – A. Metz, M. Schlegel, C. Lorcé, B. Pasquini . . .

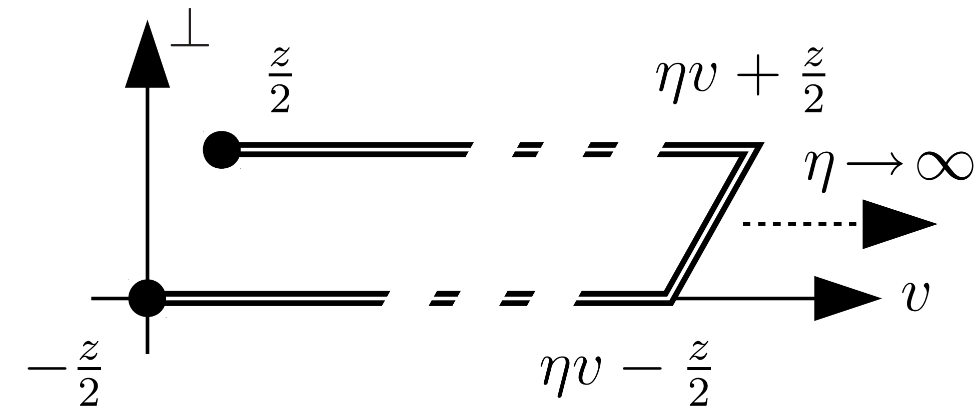
Direct evaluation of quark orbital angular momentum

$$\frac{L_3^{\mathcal{U}}}{n} = \frac{\epsilon_{ij}\frac{\partial}{\partial z_{T,i}}\frac{\partial}{\partial \Delta_{T,j}}\,\langle p', S \mid \overline{\psi}(-z/2)\gamma^{+}\mathcal{U}[-z/2,z/2]\psi(z/2) \mid p, S\rangle\big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}}{\langle p', S \mid \overline{\psi}(-z/2)\gamma^{+}\mathcal{U}[-z/2,z/2]\psi(z/2) \mid p, S\rangle\big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}}$$

Role of the gauge link $\mathcal{U}$:

Y. Hatta, M. Burkardt:

- Straight $\mathcal{U}[-z/2, z/2] \longrightarrow$ Ji OAM
- Staple-shaped $\mathcal{U}[-z/2, z/2] \longrightarrow$ Jaffe-Manohar OAM
- Difference is torque accumulated due to final state interaction



Direct evaluation of quark orbital angular momentum

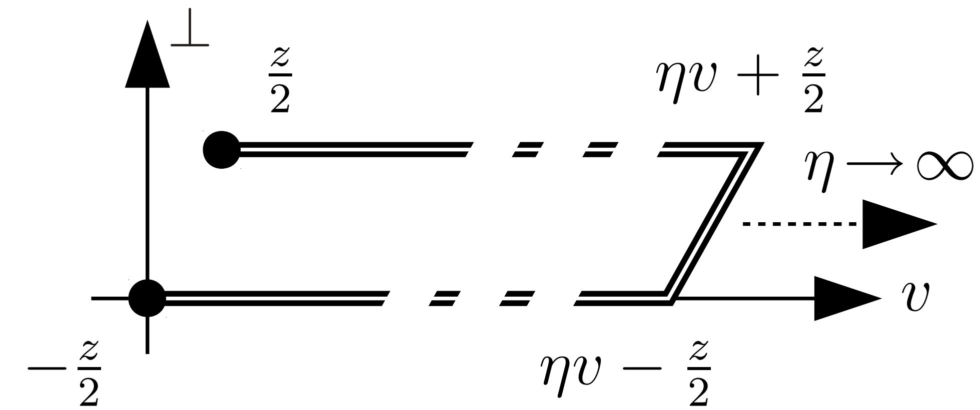
$$\frac{L_3^{\mathcal{U}}}{n} = \frac{\epsilon_{ij}\frac{\partial}{\partial z_{T,i}}\frac{\partial}{\partial \Delta_{T,j}} \langle p', S \mid \overline{\psi}(-z/2)\gamma^+ \mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle \big|_{z^+=z^-=0,\ \Delta_T=0,\ z_T\to 0}}{\langle p', S \mid \overline{\psi}(-z/2)\gamma^+ \mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle \big|_{z^+=z^-=0,\ \Delta_T=0,\ z_T\to 0}}$$

Role of the gauge link $\mathcal{U}$:

Direction of staple taken off light cone (rapidity divergences)

Characterized by Collins-Soper parameter $\hat{\zeta} \equiv \frac{P \cdot v}{|P||v|}$

Are interested in $\hat{\zeta} \longrightarrow \infty$; synonymous with $P \longrightarrow \infty$ in the frame of the lattice calculation $(v = e_3)$



Ensemble details

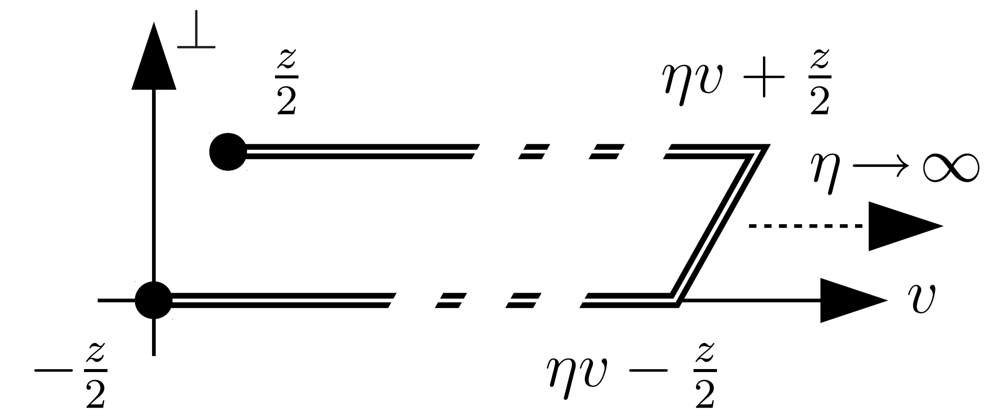
Clover ensemble provided by R. Edwards, B. Joó and K. Orginos (JLab / W&M Collaboration)

$L^3 \times T$	a(fm)	M_π (MeV)	M_N (GeV)	#conf.	#meas.
$32^3 \times 96$	0.11403(77)	317(2)(2)	1.077(8)	967	23208

Direct evaluation of quark orbital angular momentum

$$\frac{L_3^{\mathcal{U}}}{n} = \frac{\epsilon_{ij}\frac{\partial}{\partial z_{T,i}}\frac{\partial}{\partial \Delta_{T,j}}\langle p', S \mid \overline{\psi}(-z/2)\gamma^+ \mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}}{\langle p', S \mid \overline{\psi}(-z/2)\gamma^+ \mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}}$$

Parameters to consider: Δ, z, $\hat{\zeta}$, η



Direct evaluation of quark orbital angular momentum

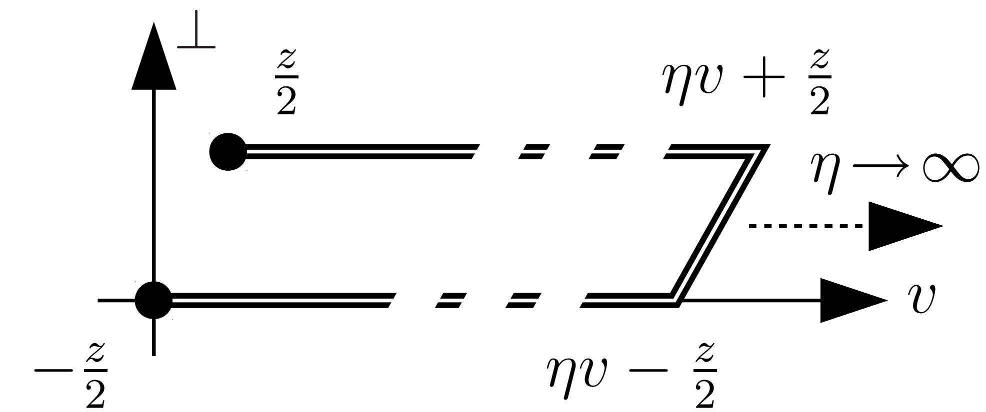
$$\frac{L_3^{\mathcal{U}}}{n} = \frac{\epsilon_{ij}\frac{\partial}{\partial z_{T,i}}\frac{\partial}{\partial \Delta_{T,j}} \langle p', S \mid \overline{\psi}(-z/2)\gamma^+ \mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\big|_{z^+=z^-=0,\ \Delta_T=0,\ z_T\to 0}}{\langle p', S \mid \overline{\psi}(-z/2)\gamma^+ \mathcal{U}[-z/2, z/2]\psi(z/2) \mid p, S\rangle\big|_{z^+=z^-=0,\ \Delta_T=0,\ z_T\to 0}}$$

- Derivative w.r.t. Δ_T taken using direct derivative method (Rome method)
- Derivative w.r.t. z_T taken as finite difference over lattice spacing a (cutoff on transverse momenta same as lattice cutoff)

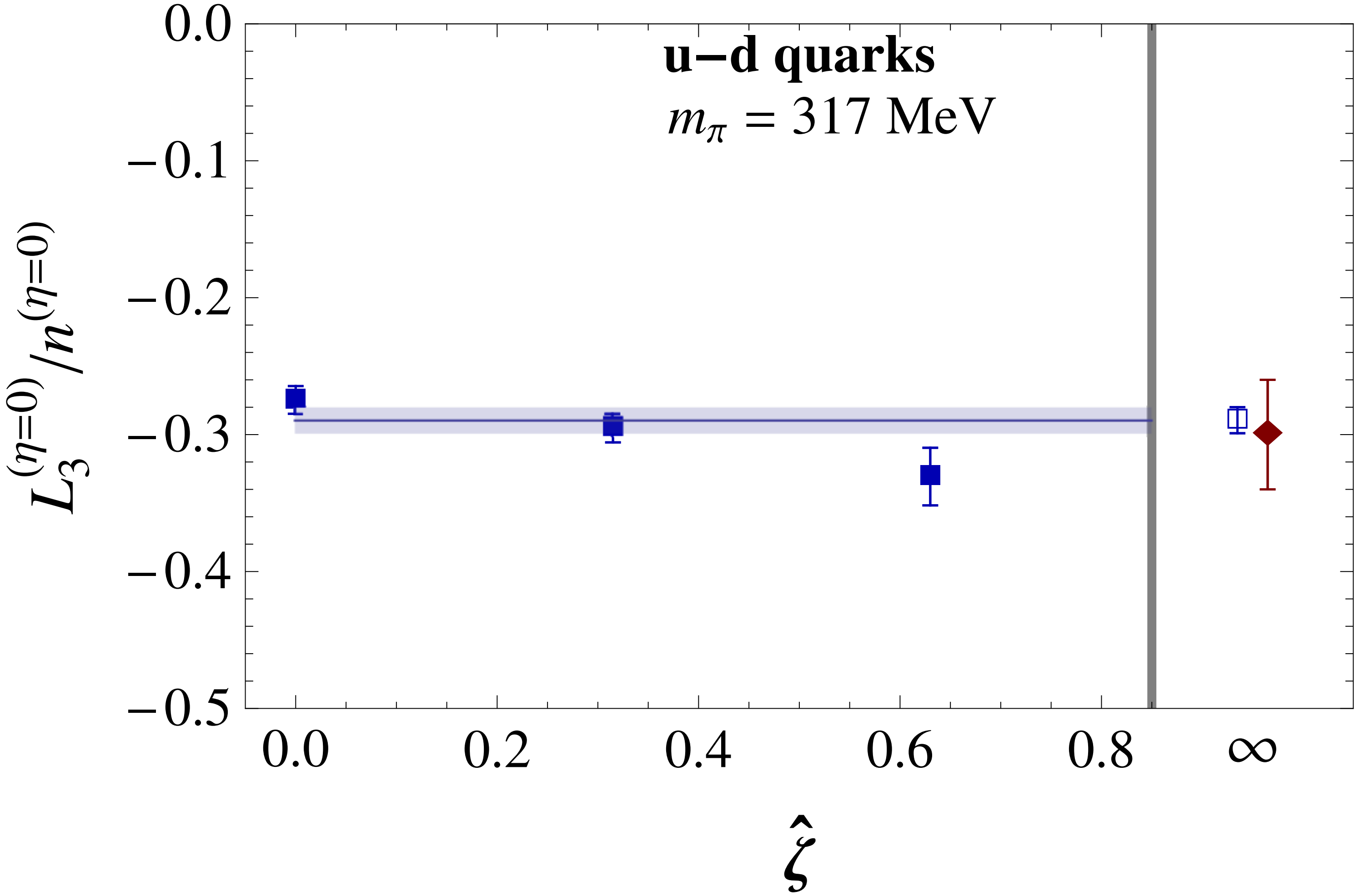
Direct evaluation of quark orbital angular momentum

$$\frac{L_3^{\mathcal{U}}}{n} = \frac{\epsilon_{ij} \frac{\partial}{\partial z_{T,i}} \frac{\partial}{\partial \Delta_{T,j}} \langle p', S \mid \overline{\psi}(-z/2)\gamma^+ \mathcal{U}[-z/2, z/2] \psi(z/2) \mid p, S\rangle \Big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}}{\langle p', S \mid \overline{\psi}(-z/2)\gamma^+ \mathcal{U}[-z/2, z/2] \psi(z/2) \mid p, S\rangle \Big|_{z^+=z^-=0\,,\ \Delta_T=0\,,\ z_T\to 0}}$$

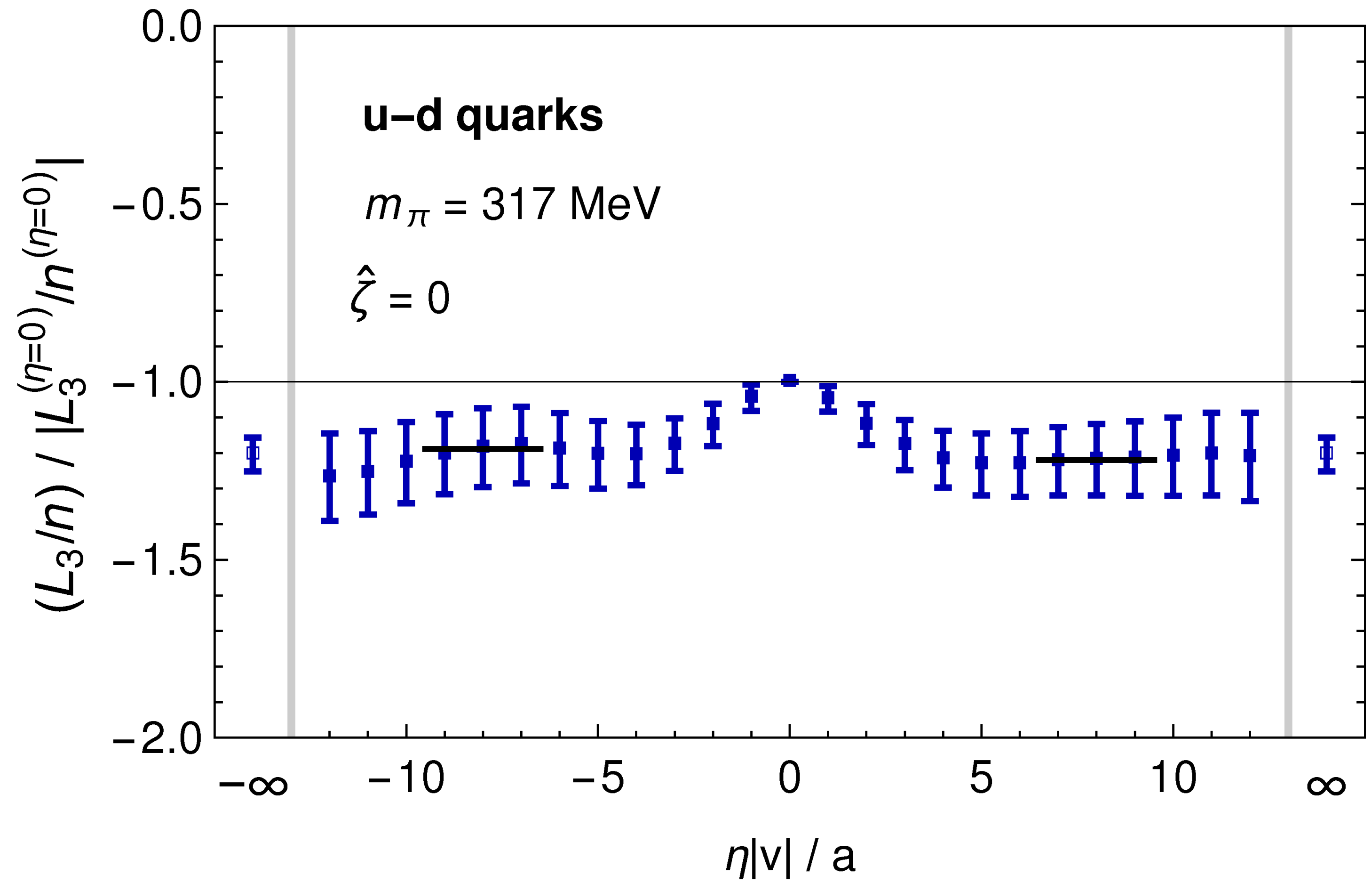
Remaining parameters to consider: $\hat{\zeta}$, η



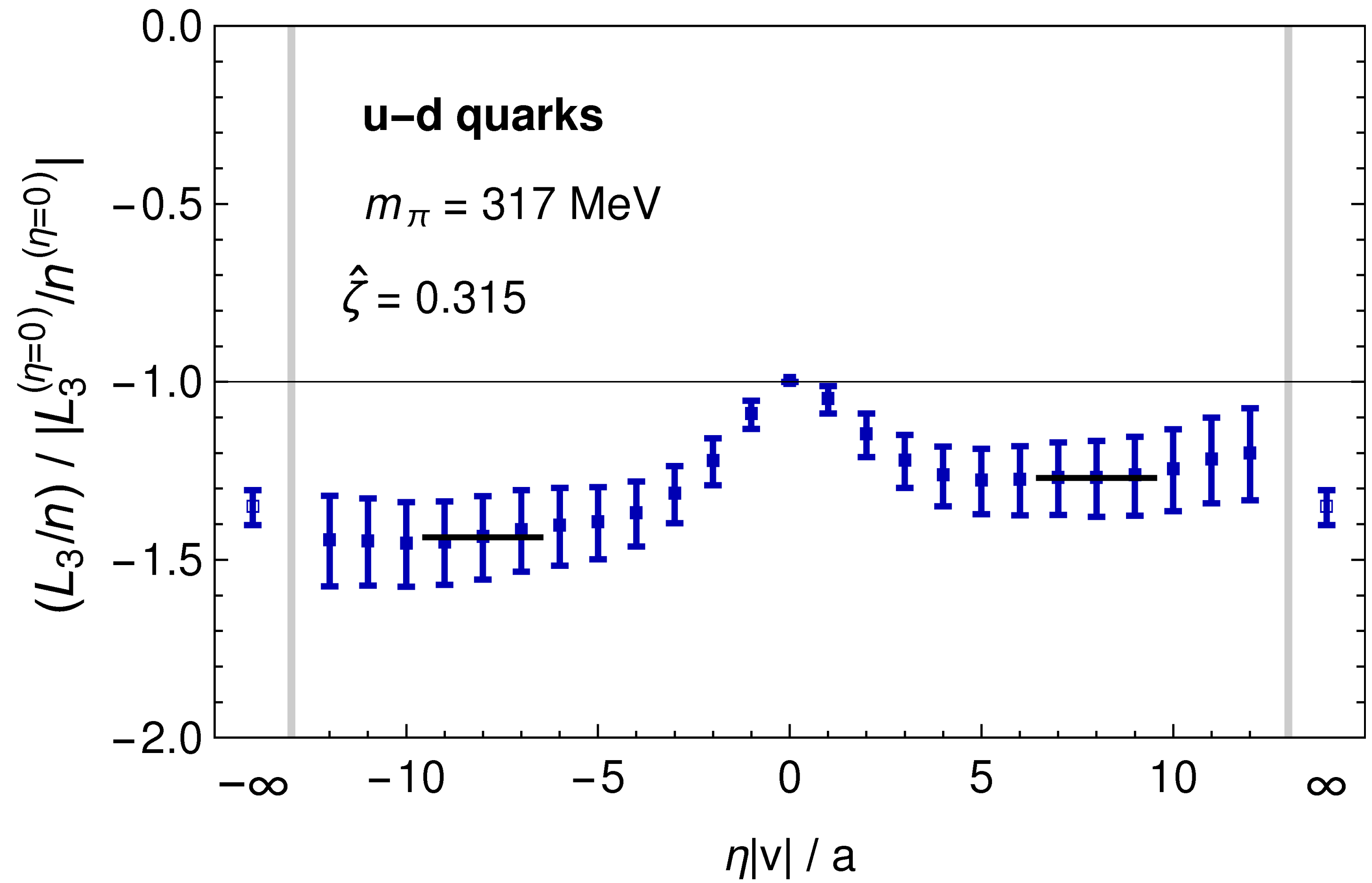
Ji quark orbital angular momentum: $\eta = 0$



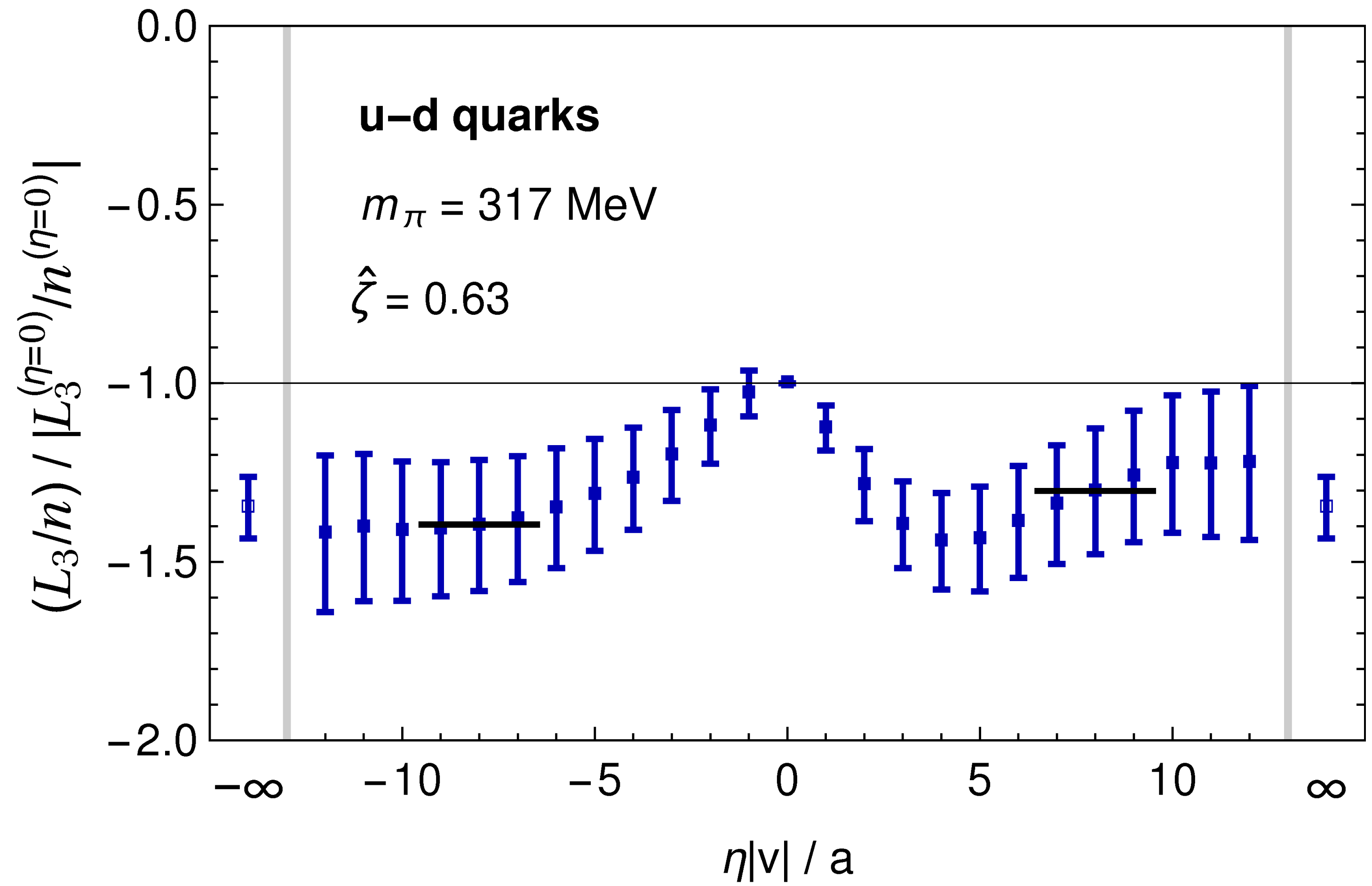
From Ji to Jaffe-Manohar quark orbital angular momentum



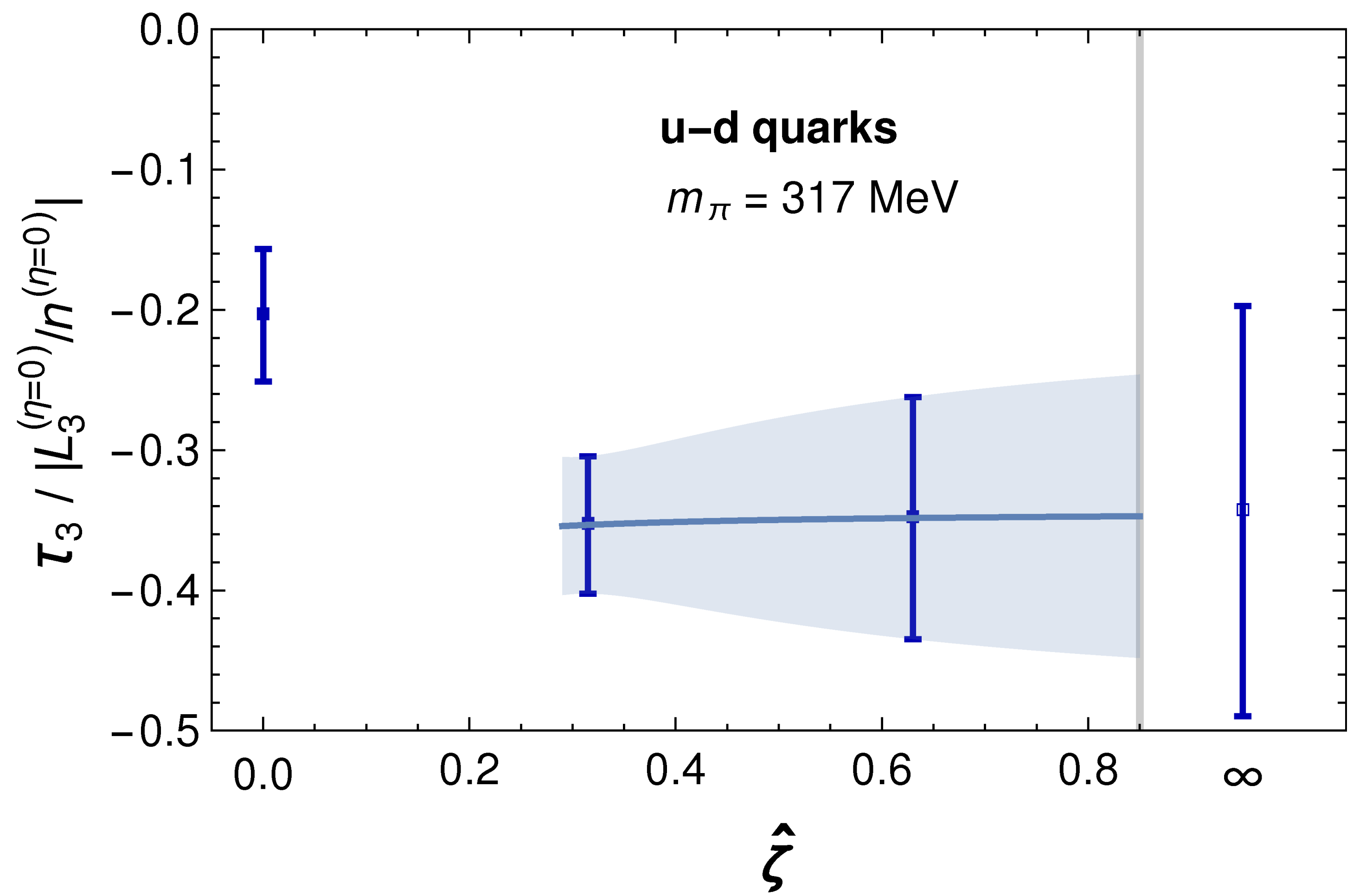
From Ji to Jaffe-Manohar quark orbital angular momentum



From Ji to Jaffe-Manohar quark orbital angular momentum



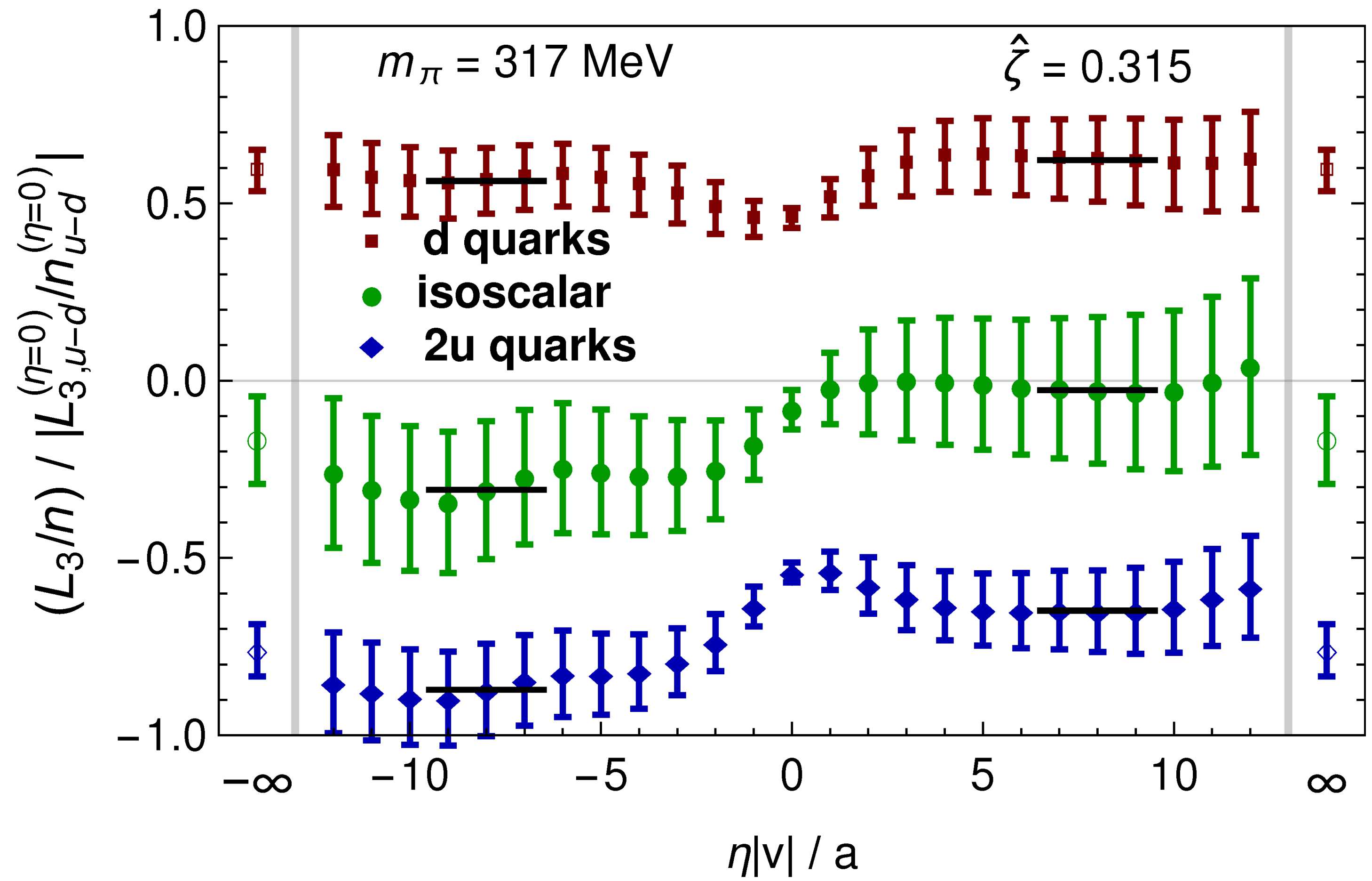
Final state interaction torque – extrapolation in $\hat{\zeta}$



$$\tau_3 = (L_3^{(\eta=\infty)}/n^{(\eta=\infty)}) - (L_3^{(\eta=0)}/n^{(\eta=0)})$$

Integrated torque accumulated by struck quark leaving proton

Flavor separation – from Ji to Jaffe-Manohar quark orbital angular momentum



Extraction of $\tilde{E}_{2T}$ moment from QCD matrix element

$$L_3 + 2S_3 = -\int dx\, x \widetilde{E}_{2T} = 2\int dx\, x \int d^2k_T \left(\frac{k_T \cdot \Delta_T}{\Delta_T^2} F_{27} + F_{28} \right) = -iP^+ \int dx\, x \int d^2k_T \epsilon_{ij} \frac{\Delta^i}{\Delta_T^2} \left(W_{++}^{\gamma^j} - W_{--}^{\gamma^j} \right)$$

(all in forward limit – note $W_{\Lambda'\Lambda}^{\gamma^i}$ is parametrized in terms of 8 twist-3 GTMDs $F_{21}, \ldots, F_{28}$).

$$L_3 + 2S_3 = \epsilon_{ij}\, \frac{1}{2}\, \frac{\partial}{\partial(z \cdot P)}\, \frac{\partial}{\partial \Delta^i} \langle P + \Delta_T/2, +|\, \bar{\psi}(-z/2)\gamma^j\, \mathcal{U}\, \psi(z/2)\, |P - \Delta_T/2, +\rangle|_{z^+=z^-=0, \Delta_T=0, z_T \to 0}$$

Renormalize using number of valence quarks n

$$2\, P^j\, n = \langle P, +|\, \bar{\psi}(-z/2)\gamma^j\, \mathcal{U}\, \psi(z/2)\, |P, +\rangle\Big|_{z^+=z^-=0, z_T \to 0}$$

i.e., ultimately determine ratio $(L_3 + 2S_3)/n$

Setting up a lattice calculation

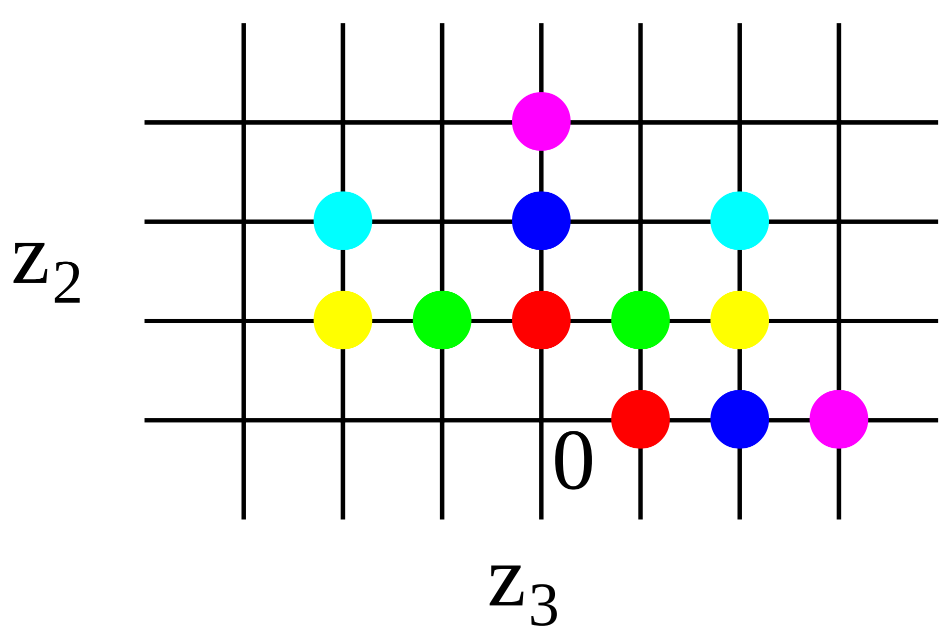
Lattice frame is boosted with respect to phenomenology frame; use invariants $z \cdot P$, z^2

Original frame: $z^+ = 0, \quad z \cdot P = z^- P^+, \quad z^2 = -z_T^2$

Lattice frame: $z_0 = 0, \quad z \cdot P = -z_3 P_3, \quad z^2 = -z_3^2 - z_T^2$

Use Rome direct derivative method to perform $\partial / \partial \Delta^i$

Perform $\partial / \partial z_3$ via finite difference at constant z^2



Setting up a lattice calculation

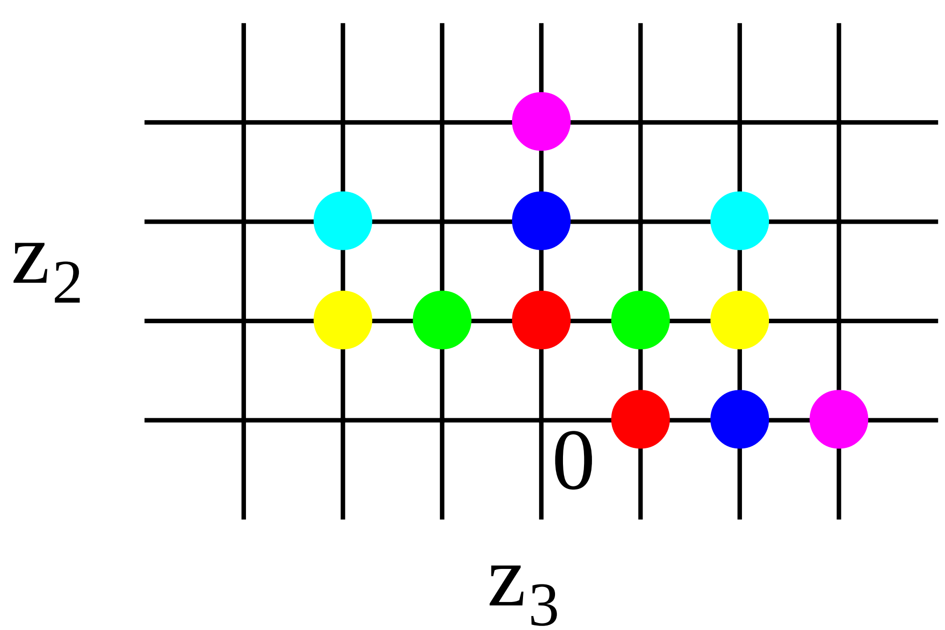
Lattice frame is boosted with respect to phenomenology frame; use invariants $z \cdot P$, z^2

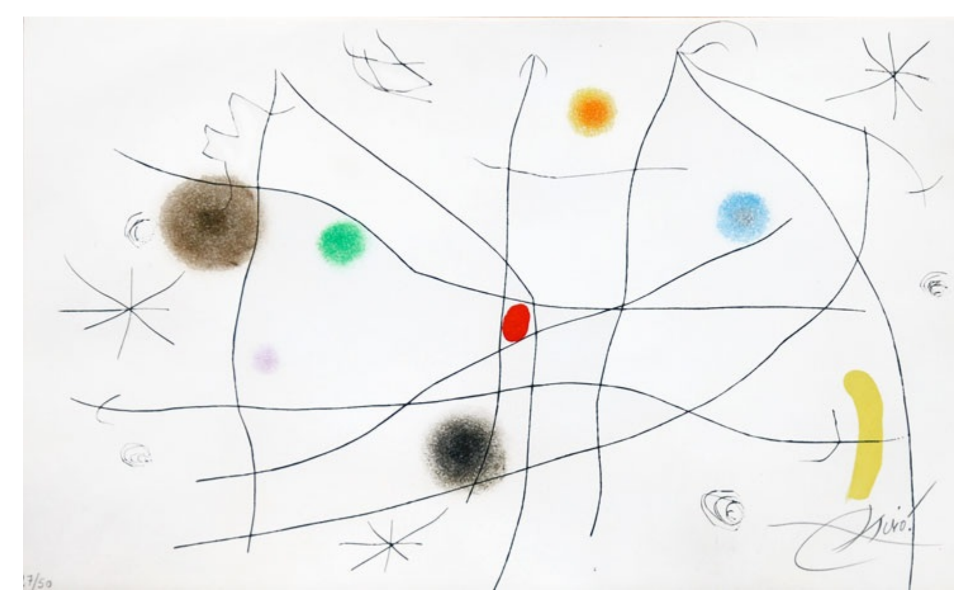
Original frame: $z^+ = 0, \quad z \cdot P = z^- P^+, \quad z^2 = -z_T^2$

Lattice frame: $z_0 = 0, \quad z \cdot P = -z_3 P_3, \quad z^2 = -z_3^2 - z_T^2$

Use Rome direct derivative method to perform $\partial/\partial\Delta^i$

Perform $\partial/\partial z_3$ via finite difference at constant z^2





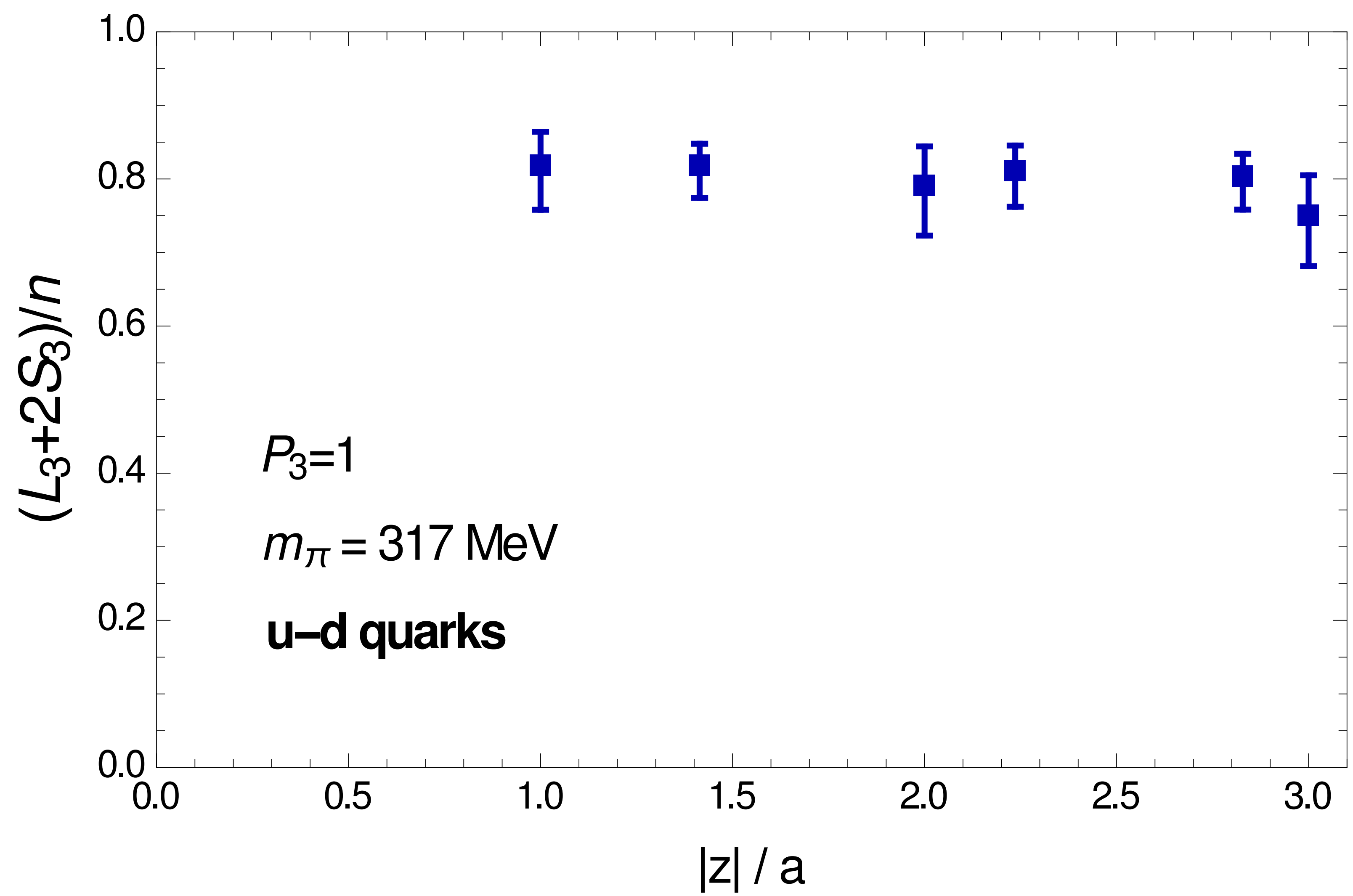
Joan Miró, print, 1974,
www.masterworksfineart.com

Ensemble: same as before

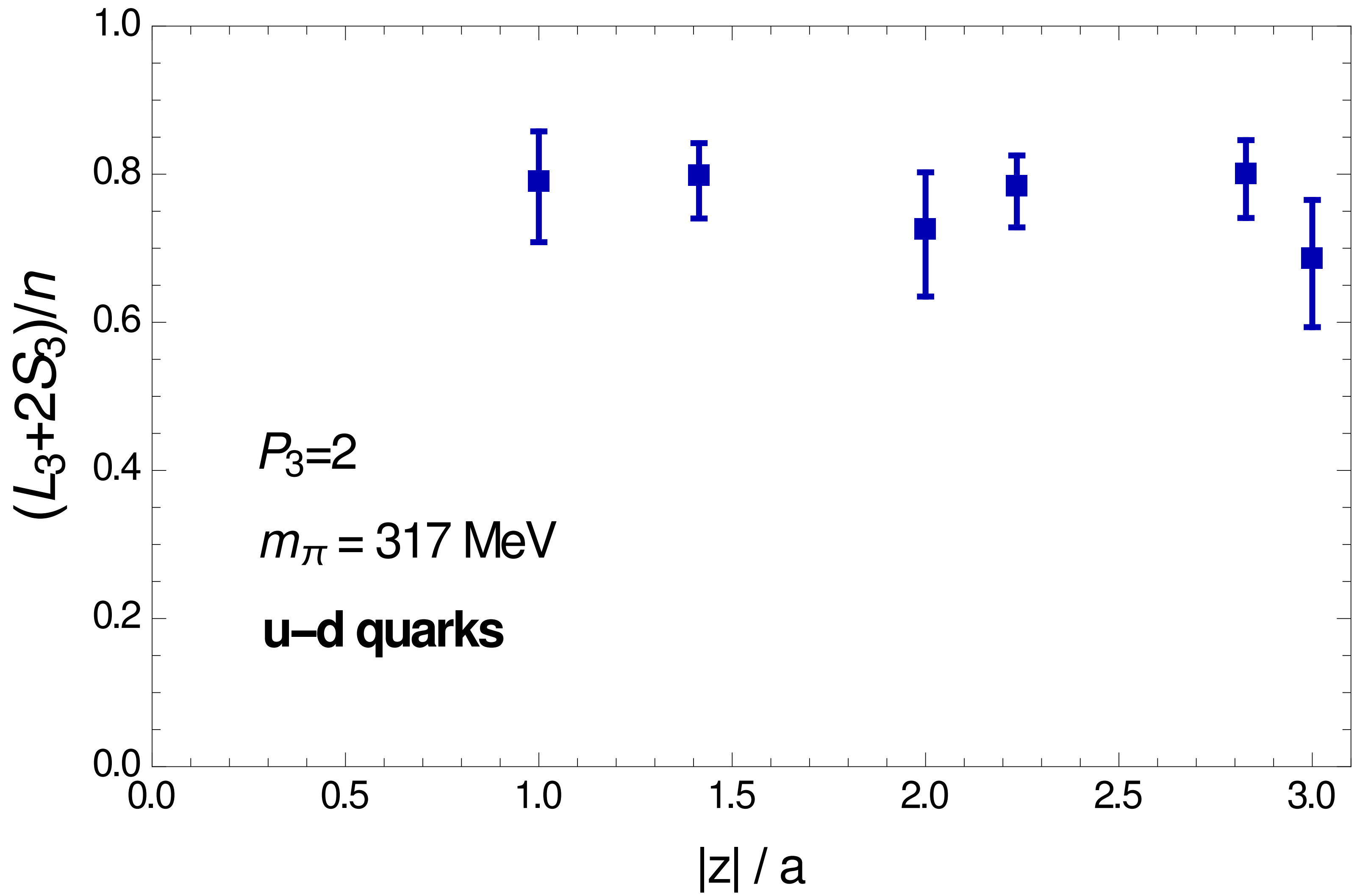
Clover fermion ensemble provided by R. Edwards, B. Joó and K. Orginos (JLab / W&M Collaboration)

$L^3 \times T$	a(fm)	M_π (MeV)	M_N (GeV)	#conf.	#meas.
$32^3 \times 96$	0.11403(77)	317(2)(2)	1.077(8)	967	23224

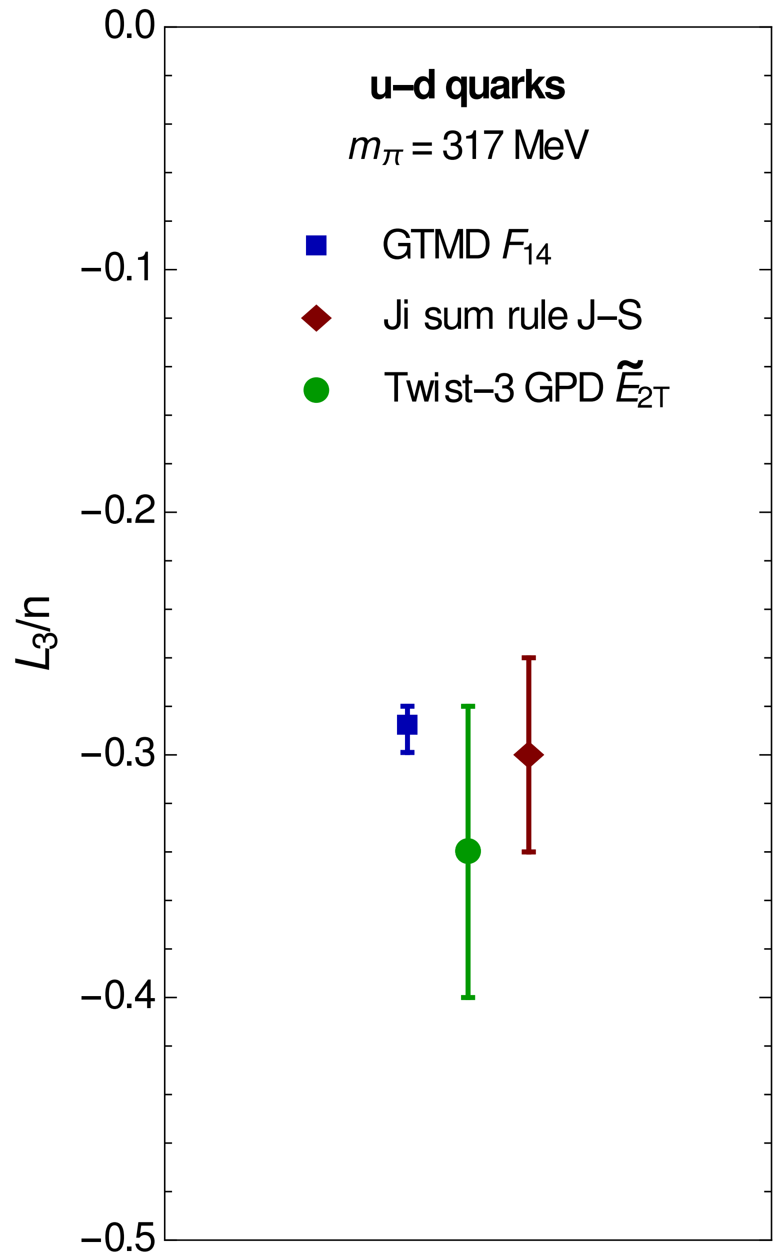
Dependence on z^2



Dependence on z^2



Result for L_3 and comparison to other approaches



- Combine with $2S_3$ previously obtained on same ensemble, same parameters (arXiv:1703.06703): $2S_3 = 1.18(2)$ (extrapolate $\widetilde{E}_{2T}$ to $P = 0$ to match).
- Different systematics within the various approaches appear to remain smaller than statistical uncertainty.
- Larger uncertainty of twist-3 GPD approach: Difference of large numbers, incomplete cancellation of fluctuations due to use of rotational symmetry in normalization.
- It is feasible to extract OAM from twist-3 GPDs, but higher effort compared to other approaches for comparable accuracy.

Direct evaluation of quark spin-orbit correlations

$$2L_3 S_3 \;=\; \int dx \int d^2 k_T \int d^2 r_T \, (r_T \times k_T)_3 \, \Sigma \;\, \mathcal{W}^{\mathcal{U}}_{\Sigma}(x, k_T, r_T) \qquad \text{Wigner distribution}$$

$$\frac{2L_3 S_3}{n} = \frac{\epsilon_{ij} \frac{\partial}{\partial z_{T,i}} \frac{\partial}{\partial \Delta_{T,j}} \, \langle P + \Delta_T/2 \mid \overline{\psi}(-z/2) \gamma^+ \gamma^5 \, \mathcal{U}[-z/2, z/2] \psi(z/2) \mid P - \Delta_T/2 \, \rangle \big|_{z^+ = z^- = 0\,,\; \Delta_T = 0\,,\; z_T \to 0}}{\langle P + \Delta_T/2 \mid \overline{\psi}(-z/2) \gamma^+ \mathcal{U}[-z/2, z/2] \psi(z/2) \mid P - \Delta_T/2 \, \rangle \big|_{z^+ = z^- = 0\,,\; \Delta_T = 0\,,\; z_T \to 0}}$$

P in 3-direction, $P \to \infty$

Renormalization: Form ratio with number of valence quarks n – note: are using Domain Wall Fermions!

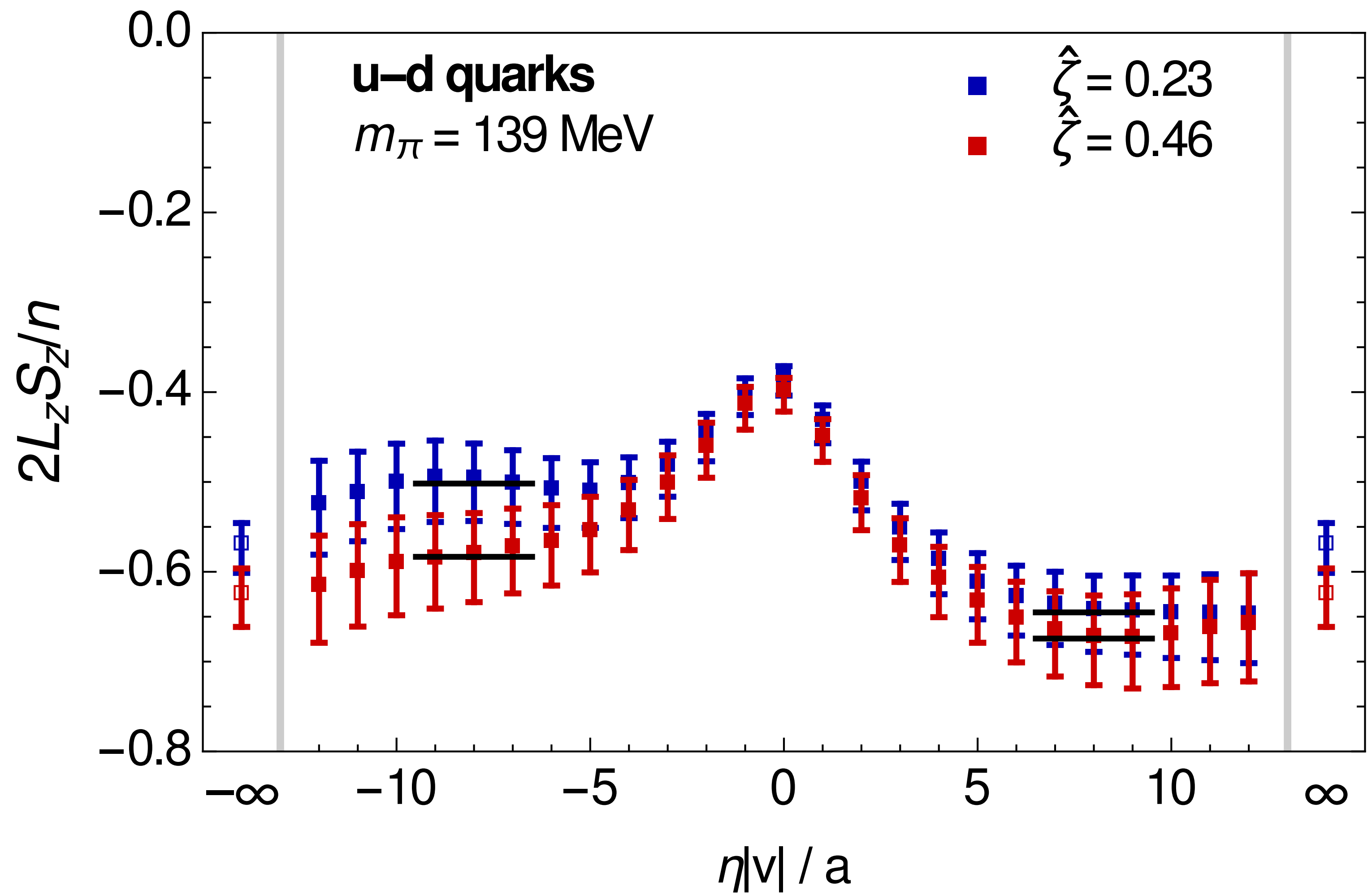
Connection to GTMDs: $$2L_3 S_3 = \int dx \int d^2 k_T \, \frac{k_T^2}{M^2} \, G_{11} \Big|_{\Delta_T = 0}$$

Ensemble details

DWF ensemble provided by the RBC/UKQCD collaboration

$L^3 \times T$	a(fm)	M_π (MeV)	#conf.	#meas.
$48^3 \times 96$	0.114	139	130	(33280 sloppy + 520 exact) AMA

Result



For comparison, in a polarized proton (from C. Alexandrou et al., PRD 101 (2020) 094513; 2003.08486):

$\langle L^u \rangle = -0.22(3)$, $\langle 2S^u \rangle = 0.86(2) \Rightarrow \langle L^u \rangle \langle 2S^u \rangle = -0.2$

$\langle L^d \rangle = 0.26(2)$, $\langle 2S^d \rangle = -0.42(2) \Rightarrow \langle L^d \rangle \langle 2S^d \rangle = -0.1$

$\Rightarrow \langle L^u \rangle \langle 2S^u \rangle - \langle L^d \rangle \langle 2S^d \rangle = -0.1$

... and this just in:

Lorentz invariance and equation of motion relations:

$$2L_3S_3 = \int dx \int d^2k_T \, \frac{k_T^2}{M^2} \, G_{11} \;=\; \frac{1}{2}\int dx \, x \, \widetilde{H} - \frac{1}{2}\int dx \, H + \frac{m}{2M}\int dx \, (2\widetilde{H}_T + E_T)$$

(in the forward limit). This is essentially the axial version of Ji's sum rule.

M. Rodekamp, at the physical point: $\int dx \, x \, \widetilde{H} \;=\; 0.21(2) \qquad \Longrightarrow \qquad 2L_3S_3 = -0.395(10)$

Sum rule satisfied.

Questions?