Jet angularities in dijet production in proton-proton and heavy-ion collisions at RHIC

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in collaboration with Yang-Ting Chien, Daniel Reichelt and Steffen Schumann

Based on arXiv:2404.04168
(submitted to JHEP)
QCD Lagrangian is very simple:

\[ \mathcal{L}_{QCD} = \sum_q \left( \bar{\psi}_q \gamma^\mu \left[ \delta_{ij} \partial_\mu + ig (G_\mu^a t_a)_{ij} \right] \psi_q - m_q \bar{\psi}_q \psi_q \right) - \frac{1}{4} G_{\mu\nu}^a G^{\mu\nu}_a \]

However, QCD is very complicated:

- Analytical calculations can be performed only in high-energy limit where coupling is small.
- Has a lot of open questions (e.g., the origin of confinement, CP-violation, collective behavior etc).
- Despite significant improvements in the accuracy of analytical calculations and lattice simulations, a lot still has to be done.
Jets and their connections to QCD

Jets are collimated clusters of particles which connect physics at the microscopic (much smaller than 1 fm) and the macroscopic scales (1 – 10 m).
Jet angularity is defined as
\[ \lambda_\alpha = \sum_{i \in \text{jet}} \frac{p_{t,i}}{p_{t,jet}} \left( \frac{\Delta R_{ij}}{R} \right)^\alpha, \quad \kappa = 1, \quad \alpha > 0 \]

- Sum runs over all particles inside the jet
- Jet radius \( R \)
- Rapidity-azimuth distance \( \Delta R_{i,jet} \)
- IRC (infrared and collinear) safe observable!

**LHA (Les Houches Angularity):** \( \alpha = 1/2 \)

**Jet Width:** \( \alpha = 1 \)

**Jet Thrust:** \( \alpha = 2 \)
Impact of Multiple Partonic Interactions

- Protons are composite objects so several (semi-)hard partonic interactions can occur per one pp collision!
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- MPI cause multiple uniform soft emissions which “contaminate” jet substructure
SoftDrop can be used to remove soft radiation from MPI:

Jet cone and jet clustering tree

SoftDrop removes soft radiation!
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1. Recluster jet into two subjets
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2. Check if one branch is much softer than the other one using the SoftDrop condition

\[
\frac{\min(p_{ti}, p_{tj})}{p_{ti} + p_{tj}} > z_{\text{cut}} \left( \frac{\Delta R_{ij}}{R} \right)^\beta
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3. If true, discard the softest branch and repeat; otherwise stop

Here $z_{cut}$ and $\beta$ control the intensity of grooming.
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Here \(z_{\text{cut}}\) and \(\beta\) control the intensity of grooming.
A “standard” 2-to-2 QCD process cannot be used for jet substructure calculations (no substructure!)

So one needs to take 2-to-2 process and add more emissions to it

Jet substructure can be studied already for the 2-to-3 processes

It is called fixed-order (FO) calculation.

However, higher order corrections e.g. 2-to-4, 2-to-5 etc. in general are difficult to calculate
Resummation: leading log (LL)

2-to-3 cross section gets a di-log enhancement:

\[ d\sigma \sim d(\log \theta^2) d(\log z) \]

Let's define a simple IRC safe jet substructure observable:

\[ \tau = z\theta^2 \]

In case of multiple emissions:

\[ \tau = \sum_{i=\text{gluon}} z_i \theta_i^2 \]
For more details see: 1709.06195

Multiple gluon emissions exponentiate:

\[ P_g(x < \tau) = \exp \left( -\frac{\alpha_s}{\pi} \frac{C_F}{2} \log^2 \tau \right) \]

Similar expression can be obtained for quark emissions:

\[ P_q(x < \tau) = \exp \left( -\frac{\alpha_s}{\pi} \frac{C_A}{2} \log^2 \tau \right) \]

Note that both expressions are finite if \( \tau \to 0 \) whereas FO result diverges!
In general:

\[ P_{q/g} = 1 + \alpha_s \left( c_{22} L^2 + c_{21} L + \ldots \right) + \alpha_s^2 \left( c_{24} L^4 + c_{23} L^3 + \ldots \right) + \ldots \]

Both LL and NLL resummation can be performed separately for quark and gluon production channels!

Therefore, resummed expressions can be used to define “quark” and “gluon” jets!

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Resummation: next-to-leading log (NLL)

In general:

\[
P_{q/g} = 1 + \alpha_s \left( \frac{c_{22} L^2}{\alpha_s^2} + \frac{c_{21} L + \ldots}{c_{24} L^4} + \frac{c_{23} L^3 + \ldots}{\alpha_s^2 c_{24} L^4} \right) + \ldots
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Resummation and matching to fixed order (FO) results

\[ Q + \bar{Q} \rightarrow Q + \bar{Q} + g \]

\[ Q + \bar{Q} \rightarrow Q + \bar{Q} + \sum_{\text{soft, collinear}} g \]

FO calculations

Resummation
Resummation and matching to fixed order (FO) results

Matching to FO results:
- Excludes double counting between overlapping phase space regions.
- Provides finite results at small values of observable of interest.
- Matching “quark” and “gluon” jet contributions can be done separately which leads to NLL' accuracy level.

\[ \tau = \frac{z \theta^2}{\cal{C}} \]
CAESAR approach by Banfi, Salam and Zanderighi

CAESAR allows to automate resummation for each observable that can be parametrized as

$$\Sigma_{\text{res}}(v) = \sum_{\delta} \Sigma^\delta_{\text{res}}(v), \text{ with}$$

$$\Sigma^\delta_{\text{res}}(v) = \int d\mathcal{B}_\delta \frac{d\sigma^\delta}{d\mathcal{B}_\delta} \exp \left[ - \sum_{i \in \delta} R_i^{B^\delta}(L) \right] \mathcal{P}^{B^\delta}(L) \mathcal{S}^{B^\delta}(L) \mathcal{F}^{B^\delta}(L) \mathcal{H}^\delta(B_\delta),$$

- Born cross section $\frac{d\sigma^\delta}{d\mathcal{B}_\delta}$
- Soft function $\mathcal{S}$
- Ratio of PDFs $\mathcal{P}$
- Multiple emission function $\mathcal{F}$
- Collinear radiator $R_i$
- Kinematic cuts $\mathcal{H}$

CAESAR = Computer Automated Expert Semi-Analytical Resummer, see the original paper by A. Banfi, G. Salam and G. Zanderighi [0407286]
The resummation is performed at NLL accuracy level.

The NLL results are matched to NLO FO results leading to NLO+NLL' accuracy level.

The calculations are automated and are available as a CAESAR resummation plugin to SHERPA MC 2404.04168, 2112.09545, 2104.06920.
✓ Our results for jet angularities are at highest available accuracy NLO+NLL’ level

✓ Result are available for LHA, Jet Width and Jet Thrust (for ungroomed and groomed jets)

✓ The increase in accuracy of calculation reduces the size of uncertainty bands. Usually data has smaller errorbars (at least at the LHC) hence further improvement in accuracy of the calculation is desirable (though it is very challenging)
Parton-level (PL) vs. hadron-level (HL) distributions

- Perturbative calculations do not describe physics at low energy scales
- Two major NP-contributions are due to MPI and hadronization
- Unlike the LHC case, at RHIC the NP-effects dominate
Parton-level vs. hadron-level distributions

- Perturbative calculations do not describe physics at low energy scales
- Two major NP-contributions are due to MPI and hadronization
- Unlike the LHC case, at RHIC the NP-effects dominate
- Large NP-contributions are predominantly coming from fragmentation of jets made out of a single parton causing bin-bigration
MPI multiplicity at RHIC

![Graph showing MPI multiplicity distribution](image)

$p p \to jj$, $p_{T1} > 30 \text{ GeV}$, $p_{T2} > 20 \text{ GeV}$, $R_0 = 0.4$, $|y_{1,2}| < 0.7$

### Table I. PYTHIA 8 settings and tuning parameters.

<table>
<thead>
<tr>
<th>Setting</th>
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<tr>
<td><strong>PDF.pSet</strong></td>
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Detroit PYTHIA tune *(2110.09447)*
MPI multiplicity at RHIC

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*Detroit tune parameter choice essential means suppression of MPI production at RHIC!*
Impact of MPI on jet substructure

Different MPI tunes lead to somewhat different shapes of jet angularities
Incorporation of non-perturbative corrections

- Hadron level to parton level ratio:
  one simulates the same observable at parton and hadron levels then
  multiplies resummed predictions by the ratio
  \[
  \lambda_{\alpha}^{HL} = \lambda_{\alpha}^{PL} \times \left( \frac{\lambda_{\alpha}^{HL,MC}}{\lambda_{\alpha}^{PL,MC}} \right)
  \]

- Shape functions, as in the work of Korchemsky, Sterman arXiv:hep-ph/9902341
  where the hadron level result is given by a convolution
  \[
  \frac{d\sigma}{d\lambda_{\alpha}^{HL}} = \int d\epsilon \ f(\epsilon) \int d\lambda_{\alpha}^{PL} \frac{d\sigma}{d\lambda_{\alpha}^{PL}} \delta \left( \lambda_{\alpha}^{HL} - \lambda_{\alpha}^{PL} - C_{\alpha, z_{cut}}^{\beta} \epsilon_{\alpha}^{\beta} \right)
  \]

- Parton-to-hadron transfer matrices, see arXiv:2112.09545
  \[
  \sigma^{HL}(\vec{v}_h) = T(\vec{v}_h|\vec{v}_p) \times \sigma^{PL}(\vec{v}_p)
  \]
The transfer matrices can be extracted from MC simulations.
Parton-to-hadron transition via transfer matrices (TM)

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- One needs to "put event generation on pause" when parton shower reach non-perturbative scale and calculate $\lambda^{PL}_{\alpha}$. 
Parton-to-hadron transition via transfer matrices (TM)

- The transfer matrices can be extracted from MC simulations.
- One needs to “put event generation on pause” when parton shower reach non-perturbative scale and calculate $\lambda_{\alpha}^{PL}$.
- After that one “resume” event generation and calculate $\lambda_{\alpha}^{HL}$.
- The correlations between $\lambda_{\alpha}^{PL}$ and $\lambda_{\alpha}^{HL}$ are used to build TMs.
Transfer matrices (TM)

- Transfer matrices were obtained with SHERPA MC
- The information on correlation between partons and hadrons in each event is embedded
- The clearly visible off-diagonal structures indicate strong bin-migration caused by non-perturbative effects
- Unlike the approach of the shape functions the TM are not bounded to any particular functional form
The approach based upon HL/PL ratio neglects bin migration. Therefore, does not provide correct shift of the distribution.

\[ \lambda_{\alpha}^{HL} = \lambda_{\alpha}^{PL} \times \left( \frac{\lambda_{\alpha}^{HL,MC}}{\lambda_{\alpha}^{PL,MC}} \right) \]

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Main predictions: NLO+NLL’+NP results

- We corrected our NLO+NLL’ results for non-perturbative effects using TM approach.
- Our approach is more accurate than standard MC@NLO SHERPA simulations (SHERPA parton shower is at LL accuracy).
- Our uncertainty estimate is more accurate and includes variation of larger number of parameters.
- Our results are automated and available as a resummation plugin to SHERPA.
Impact of Quark-Gluon Plasma (QGP)

- We expect a new state of matter (called QGP) to be born in AA and pA collisions.
- Particles produced via hard QCD interaction and parton shower can interact with the QGP scattering centers.
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- QGP may have a significant impact on jet substructure generally known as jet quenching.
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- We expect a new state of matter (called QGP) to be born in AA and pA collisions.
- QGP may have a significant impact on jet substructure generally known as jet quenching.
- Particles produced via hard QCD interaction and parton shower can interact with the QGP scattering centers.
- Thermalization of QGP creates a huge soft background.
There are different MC models of jet-medium interactions, e.g. HIJING, JEWEL, PQM, HYBRID, JETSCAPE, Q-Pythia, LBT…

We used two light-weighted Pythia6 based MC models: Q-Pythia and JEWEL.

Q-Pythia: is using modified Altarelli-Parisi splitting functions (BDMPS-Z formalism)

JEWEL: 2-to-2 rescatterings between parton shower partons and QGP scattering centers is included in parton shower evolution.
Impact of medium effects

Q-Pythia is, essentially, Pythia-6 with modified parton shower

Parton shower is unitary and hence conserves energy
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- Therefore, there is no energy exchange between “QGP medium” and parton shower which has drastic consequences
Impact of medium effects

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- Parton shower is unitary and hence conserves energy
- As a consequence there is no energy exchange between “QGP medium” and parton shower which has drastic consequences
- JEWEL, in turn, “injects” QGP particles into parton shower evolution which leads to energy exchange between QGP and parton shower
Solid understanding of the vacuum case is required
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Solid understanding of the vacuum case is required.

Neither Q-Pythia pp nor JEWEL pp predictions do not agree with our NLO+NLL’+NP results!
We obtained NLO+NLL’ accuracy level results for 3 different types of jet angularities: Les-Houches Angularity (LHA), Jet Width and Jet Thrust for both groomed and ungroomed jets. These results are automated (as a SHERPA MC plugin) and are available on request.

We found that MPI at RHIC are strongly suppressed and the non-perturbative contribution to the jet substructure is given mostly by hadronization (fragmentation) effects.

The hadronization corrections were incorporated via transfer matrices extracted from SHERPA MC.

At RHIC hadronization causes strong bin-migration (significantly off-diagonal transfer matrices).

Jet substructure can be used to test jet quenching models however a solid understanding of the vacuum case is required.

The obtained AA results suggest necessity to improve the Q-Pythia model.
THANK YOU FOR LISTENING!
Soft and collinear limits

If emitted gluon is soft:
\[ d\sigma \approx C_F \frac{\alpha_S}{\pi} \frac{(2 \sin \theta/2)^2 d(2 \sin \theta/2)^2}{\left[(2 \sin \theta/2)^2 + m_Q^2/E_Q^2\right]^2} \frac{dz}{z} \]

If emitted gluon is soft and collinear:
\[ d\sigma \approx C_F \frac{\alpha_S}{\pi} \frac{\theta^2 d\theta^2}{\left[\theta^2 + m_Q^2/E_Q^2\right]^2} \frac{dz}{z} \]

If emitting quark is massless:
\[ d\sigma \approx C_F \frac{\alpha_S}{\pi} \frac{d\theta^2}{\theta^2} \frac{dz}{z} \quad d\sigma \sim d(\log \theta^2) d(\log z) \]
Lund Plane Projection and dead cone effect

\[ d\sigma \approx C_F \frac{\alpha_S}{\pi} \frac{(2 \sin \theta/2)^2 d(2 \sin \theta/2)^2}{[ (2 \sin \theta/2)^2 + \theta_D^2]^2} \frac{dz}{z} \approx C_F \frac{\alpha_S}{\pi} \frac{\theta^2 d\theta^2}{[\theta^2 + \theta_D^2]^2} \frac{dz}{z} \]

\[ \theta_D = \lim_{E_0 \to 0} \left( \frac{2m_Q}{\sqrt{s}} \right) = m_Q/E_Q \]

\[ d\sigma \approx d(\log \theta^2) d(\log z) \]

\[ d\sigma \approx \left( \frac{\theta}{\theta_D} \right)^2 d \left( \frac{\theta}{\theta_D} \right)^2 d(\log z) \]

- Dead cone effect is a general property of gauge theories (e.g. also exist in QED)
- The QCD predictions were made a long time ago Dokshitzer et all 91, Ellis et al 96
- But the first direct observation came only recently 2106.05713 (ALICE)
- Jet substructure observables (in particular Lund Plane projection) are sensitive to the dead cone effect!
Bin migration caused by NP-contributions (LHC)

Credits: G. Soyer and S. Schumann