A lattice QCD calculation of the off-forward Compton amplitude and generalised parton distributions

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For the CSSM/QCDSF/UKQCD collaboration

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The $H$ Compton form factor.

GPD moments.
1 Background:
What are GPDs? Why are we interested in lattice calculations?

2 Outline of method:
Novel application of Feynman-Hellmann methods

3 Established results:
Presented in AHG et al., PRD 105, 2022

4 New results
Preliminary!
1 GPDs—what are they and why are they interesting?

2 The Feynman-Hellmann method

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5 Conclusions and Outlook
Outline of the problem

- Extensions of parton distribution functions (PDFs), related to elastic form factors
- Contain a staggering amount of physical information: a solution to proton spin puzzle, the spatial distributions of hadron constituents, and more

**However...**

- Difficult to measure experimentally
- and difficult to calculate on the lattice

**In this talk:**

- a new lattice method to calculate GPDs (Feynman-Hellmann)
- with strong parallels to experimental measurements
What are generalised parton distributions?

Light-cone matrix element:

\[
\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | \bar{\psi}_q(-\lambda n/2) \gamma_\mu \psi_q(\lambda n/2) | P \rangle \\
= H^q(x, \xi, t) \bar{u}(P') \gamma_\mu u(P) \\
+ E^q(x, \xi, t) \bar{u}(P') \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2M} u(P).
\]

- \( H^q \) and \( E^q \) are helicity-conserving and -flipping GPDs (analogous \( F_1 \) and \( F_2 \)).
- \( t = (P' - P)^2 \) momentum transfer.
- \( x, \xi \) are momentum fractions.
### Measurement of GPDs

**Deeply virtual Compton scattering:**
\[ e^- + p \rightarrow e^- + p + \gamma. \]

**Measure the off-forward Compton amplitude**

**Compton form factors at large \(-q^2\)**

\[
\text{CFF} = \int_{-1}^{1} dx \left( \frac{1}{x - \xi + i\varepsilon} \pm \frac{1}{x + \xi + i\varepsilon} \right) \text{GPD}
\]

**Difficulties:**
- deconvolution problem,
- spanning kinematics,
- lack of theoretical constraints.

**Lattice calculations:**
- provide theoretical constraints,
- access unphysical kinematics \((\xi = 0)\),
- exclude models,

**Our aim:** calculate this OFCA with lattice QCD for \(\xi = 0\). Previous calculations: focus on leading-order.
Lattice QCD

QCD path integral

\[ \langle O \rangle = \frac{1}{Z} \int \mathcal{D}A_\mu \mathcal{D}\bar{\psi} \mathcal{D}\psi O e^{iS_{\text{QCD}}}. \]

To evaluate this numerically:

1. discrete spacetime,
2. Wick rotation \( t \to -i\tau, \ e^{iS_{\text{QCD}}} \to e^{-S_{\text{QCD}}^E}, \)
3. generate gauge configurations according to \( e^{-S_{\text{QCD}}^E}. \)

Then, the path integral can be evaluated as a weighted sum over gauge configurations:

\[ \langle O \rangle \approx \frac{1}{N} \sum_{i=0}^{N} O_i. \]
Why can’t we calculate parton distributions directly?

Wick rotated separation:

\[ x^2 = (-i\tau)^2 - |\vec{x}|^2 = -\tau^2 - |\vec{x}|^2 < 0. \]

All spacelike.
But parton distributions are lightlike correlation functions:

\[ \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P^{(i)} | \bar{\psi}_{q}(-\lambda n/2) \psi_{q}(\lambda n/2) | P \rangle, \]

since \( n^2 = 0 \). ∴ can only calculate related quantities.
**Hunt for lattice parton distributions**

**A lot of related quantities:**
- Moments from 3-pt functions
- Quasi- and pseudo-distributions
- Lattice cross sections
- Heavy-quark OPE
- and others...

**GPD studies:**
- Many 3-pt calculations of leading $n = 1, 2, 3$ moments (insufficient for full reconstruction)
Why the Compton amplitude?

- 3-pt moments and quasi leading-order
- But in DVCS expt., hard scale relatively small: $Q^2 < 10 \text{ GeV}^2$
- Unknown subtraction function, $S_1$

**From lattice OFCA, can we get:**
- $Q^2$ dependence [Latt. 2021 PoS 324]
- higher-order terms [Latt. 2019 PoS 278]
- subtraction function [Latt. 2021 PoS 028]

For the **forward** ($P = P'$) Compton amplitude, we have calculated these properties with Feynman-Hellmann.
Summary

• GPDs contain LOTS of physical information.
• They are hard to measure from experiment.
• They are hard to calculate on the Lattice.
• We want lattice calculation of Compton amplitude (more overlap with experiment) → GPDs
1. GPDs—what are they and why are they interesting?

2. The Feynman-Hellmann method

3. Established results
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4. New Results
   Preliminary!

5. Conclusions and Outlook
Why Feynman-Hellmann?

Lattice OFCA:

\[ T_{\mu\nu} = \sum_{z_\mu} e^{i(q+q') \cdot z} \langle P' | T \{ j_\mu(z) j_\nu(0) \} | P \rangle. \]

Requires 4-pt function:

\[ \langle \chi(z_4) j_\mu(z_3) j_\nu(z_2) \chi^\dagger(z_1) \rangle \]

- New inversion for each \((\tau, \tau')\)
- Large \(T\): \(\tau_{\text{sink}} \gg \tau', \tau_{\text{srce}} \ll \tau\)
- Expensive!

Feynman-Hellmann perturbed Dirac operator:

\[ iD - m \rightarrow iD - m - \lambda_1 \mathcal{J}(q_1) - \lambda_2 \mathcal{J}(q_2) \]

Expand around \(\lambda_1, \lambda_2 = 0\):

\[
\begin{align*}
\text{Graph} & = \sum_j \lambda_j \sum_{\tau_1} \mathcal{J}_j(\tau_1) + \sum_{j,k} \lambda_j \lambda_k \sum_{\tau_1 \geq \tau_2} \mathcal{J}_k(\tau_2) \mathcal{J}_j(\tau_1) + \mathcal{O}(\lambda^3)
\end{align*}
\]

Then \(\lambda_1 \lambda_2\) term will be OFCA.
In more detail

Calculate perturbed quark propagator:

\[ S_{(\lambda_1, \lambda_2)}(x_n - x_m) = \left[ M - \lambda_1 \cos(\bar{q}_1 \cdot \bar{x}) \gamma_3 - \lambda_2 \cos(\bar{q}_2 \cdot \bar{x}) \gamma_3 \right]^{-1}_{n,m}. \]

Two couplings, \( \lambda_1, \lambda_2 \); two momenta, \( \bar{q}_1 \) and \( \bar{q}_2 \); choose \( \gamma_3 \), which gives \( T_{33} \) component.

\[ S_{\bar{\lambda}} = \underbrace{S}_{\text{unperturbed}} + \sum_i \lambda_i S J_3(\bar{q}_i) S + \sum_{i,j} \lambda_i \lambda_j S J_3(\bar{q}_i) S J_3(\bar{q}_j) S + \mathcal{O}(\lambda^3) \]

Put into nucleon propagator: \( G_{d\bar{d}} \sim \langle S^u S^u S^d \rangle \), or \( G_{u\bar{u}} \sim \langle S^u S^u S^d \rangle \).

\[ G_{(\lambda,\lambda)} + G_{(-\lambda,-\lambda)} - G_{(-\lambda,\lambda)} - G_{(\lambda,-\lambda)} \approx \lambda^2 + \mathcal{O}(\lambda^4) \]

The \( (\lambda_1)^2 \) and \( (\lambda_2)^2 \) terms give forward Compton amplitudes. The \( \lambda_1 \lambda_2 \) term gives OFCA.
Off-forward Compton amplitude from Feynman-Hellmann

Ground state saturation

\[ |\vec{p}| = |\vec{p} + \vec{q}_1 - \vec{q}_2| \] (equal energy but momentum transfer)

**Feynman-Hellmann relation**

\[
R_\lambda \equiv \frac{G(\lambda,\lambda) + G(-\lambda,-\lambda) - G(-\lambda,\lambda) - G(\lambda,-\lambda)}{G_0} \simeq 2\lambda^2 \tau \frac{T_{33}}{E_N}.
\]

- \( T_{33} \) is OFCA for \( \mu = \nu = 3 \).
- Linear in \( \tau \), Euclidean time; quadratic in \( \lambda \).
Fits and signal quality

\[ R_\lambda(\tau) \simeq 2\lambda^2 \tau \frac{T_{33}}{E_N}. \]

Extract \( T_{33} \) for a given sink momentum, \( \mathbf{p} \). Now, what do we do with it?
Forward Compton amplitude from Feynman-Hellmann

**Forward Compton amplitude:**

\[
T_{\mu\nu}(p, q) = \left(-\eta_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1(\omega, Q^2)
+ \left(p_\mu - \frac{p \cdot q}{q^2} q_\mu\right) \left(p_\nu - \frac{p \cdot q}{q^2} q_\nu\right) \frac{F_2(\omega, Q^2)}{p \cdot q},
\]

- Previously calculated with Feynman-Hellmann [CSSM/QCDSF PRL 118 (2017), PRD 102 (2020)]
- Same calculation but with one \(\lambda\) and \(\vec{q}\).
- Extract \(T_{33}(\vec{p}, \vec{q})\), forward Compton amplitude.

Spin-independent:

\[
F_1(\omega, Q^2) = F_1(0, Q^2) = 2\omega^2 \int_0^1 dx \frac{2xF_1(x, Q^2)}{1 - x^2\omega^2 - i\epsilon},
\]
\[
F_2(\omega, Q^2) = 4\omega \int_0^1 dx \frac{F_2(x, Q^2)}{1 - x^2\omega^2 - i\epsilon}
\]

- From FH, \(F_{1,2}\) and subtraction function \(S_1(Q^2) = F_1(0, Q^2)\)
- Inverse Bjorken variable, \(\omega = 2\vec{p} \cdot \vec{q}/q^2\)
- DIS structure functions \(F_{1,2}\) become PDFs at \(Q^2 \rightarrow \infty\).
Parton distribution moments

On lattice, Euclidean CA. To relate to Minkowski,

\[ E_X(\vec{p} \pm \vec{q}) > E_N(\vec{p}), \quad \Rightarrow \quad \omega = \left| \frac{2\vec{p} \cdot \vec{q}}{q^2} \right| < 1 \]

For \(|\omega| < 1\),

\[ \overline{F}_1(\omega, Q^2) = 2\omega^2 \int_0^1 dx \frac{2xF_1(x, Q^2)}{1 - x^2\omega^2 - i\epsilon}, \]

\[ \overline{F}_1(\omega, Q^2) = 4 \sum_{n=2,4,6} \omega^n M_n(Q^2) \]

\[ M_n(Q^2) \xrightarrow{Q^2 \to \infty} \int_0^1 dxx^{n-1} q(x). \]

Euclidean Compton amplitude \(\to\) moments of physical Compton amplitude.
Perturbative expansion of OFCA

Off-forward is very similar, but more complicated...\[
T_{\mu\nu} = \frac{1}{2P \cdot \bar{q}} \left[ - \left( h \cdot \bar{q} H_1 + e \cdot \bar{q} \xi_1 \right) g_{\mu\nu} + \frac{1}{P \cdot \bar{q}} \left( h \cdot \bar{q} H_2 + e \cdot \bar{q} \xi_2 \right) \bar{P}_\mu \bar{P}_\nu + H_3 h_{\{\mu} \bar{P}_{\nu\}} \right] + \ldots
\]

OFCA has 18 Compton form factors, but we want to relate these to GPDs. Lots of perturbative expansions of OFCA. A classic from X. Ji, PRL 78 (1997):

\[
T^{\mu\nu}(P, q; P', q') = -\frac{1}{2} \left( n^\mu \tilde{n}^\nu + n^\nu \tilde{n}^\mu - g^{\mu\nu} \right) \int_{-1}^{1} dx \left( \frac{1}{x - \xi + i\epsilon} + \frac{1}{x + \xi + i\epsilon} \right) \times \left[ H(x, \xi, t) \bar{u}(P') \tilde{n} u(P) + E(x, \xi, t) \bar{u}(P') i\sigma^{\alpha\beta} \tilde{n}_\alpha \Delta_\beta \frac{2M}{2M} u(P) \right].
\]

- But \( n^\mu \) and \( \tilde{n}^\mu \) are lightlike vectors.
- Almost all published expansions of OFCA use light-cone kinematics (especially for nucleon).
- But it can’t be compared to a Euclidean lattice calculation.
GPD moments from the Compton amplitude

\[ T_{\mu\nu} = \frac{1}{2\bar{P} \cdot \bar{q}} \left[ -\left( h \cdot \bar{q} \mathcal{H}_1 + e \cdot \bar{q} \mathcal{E}_1 \right) g_{\mu\nu} + \frac{1}{\bar{P} \cdot \bar{q}} \left( h \cdot \bar{q} \mathcal{H}_2 + e \cdot \bar{q} \mathcal{E}_2 \right) \bar{P}_\mu \bar{P}_\nu + \mathcal{H}_3 h_{\{\mu} \bar{P}_{\nu\}} \right] + \ldots \]

My master’s:

- Matching non-perturbative tensor decomposition (e.g. Tarrach) to leading-order GPD moments
- Even though we’re interested in non-perturbative structure, want high-energy limit \((\bar{Q}^2 \to \infty)\) to guide us.

\[ \mathcal{H}_1(\bar{\omega}, t) - S_1 = \int_{-1}^{1} dx \frac{2x}{(x - 1/\bar{\omega})^2 + i\varepsilon} H(x, t) \quad \mathcal{E}_1(\bar{\omega}, t) + S_1 = \int_{-1}^{1} dx \frac{2x}{(x - 1/\bar{\omega})^2 + i\varepsilon} E(x, t) \]

\[ = 2 \sum_{n=2,4,6} \bar{\omega}^n A_{n,0}(t), \quad |\bar{\omega}| < 1 \]

\[ = 2 \sum_{n=2,4,6} \bar{\omega}^n B_{n,0}(t), \quad |\bar{\omega}| < 1 \]

To fit GPD moments

Feynman-Hellmann

\[ \sum_{\text{spins}} \Gamma u' T^{33} \tilde{u} \propto \sum_{n=2,4,6} \bar{\omega}^n \left[ N_A^{A} A_{n,0}(t) + N_B^{B} B_{n,0}(t) \right] + \mathcal{O}\left( \frac{1}{Q^2} \right). \]
Summary

- Feynman-Hellmann efficient method to calculate four-point functions in lattice QCD.
- Can be extended to off-forward kinematics.
- We can extract the off-forward Compton amplitude, and relate it to GPDs.
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2 The Feynman-Hellmann method

3 Established results
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4 New Results
   Preliminary!

5 Conclusions and Outlook
Calculation Details

<table>
<thead>
<tr>
<th>$N_f$</th>
<th>$\kappa_u$</th>
<th>$\kappa_d$</th>
<th>$L^3 \times T$</th>
<th>$a$</th>
<th>$M_{\pi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 + 1</td>
<td>0.1209</td>
<td>32$^3 \times 64$</td>
<td>0.07</td>
<td>470</td>
<td></td>
</tr>
</tbody>
</table>

- SU(3) flavour symmetric point—$u, d$ and $s$ quarks have same mass.
- Heavy pion mass: $\sim 470$ MeV, compared to the physical point $\sim 140$ MeV ($\pi^+$).

**Feynman-Hellmann details:**

- To isolate $\lambda_1 \lambda_2$, we calculate perturbed propagators with:
  $$(\lambda, \lambda), \quad (-\lambda, -\lambda), \quad (\lambda, -\lambda), \quad (-\lambda, \lambda).$$
- Two magnitudes: $\lambda = 0.0125, 0.025$.
- Each set of perturbed propagators, insert two momenta: $\vec{q}_1$ and $\vec{q}_2$.
- These momenta define the kinematics accessible for the calculation.
Kinematics

Our momentum scalars are:

- \( \bar{\omega} = \frac{4\vec{p} \cdot (\vec{q}_1 + \vec{q}_2)}{(\vec{q}_1 + \vec{q}_2)^2}, \quad \xi \propto \vec{q}_1^2 - \vec{q}_2^2. \)

- We always choose \( |\vec{q}_1| = |\vec{q}_2|, \) so \( \xi = 0. \)

- Momentum transfers:

\[
t = -(\vec{q}_1 - \vec{q}_2)^2, \quad \bar{Q}^2 = \frac{1}{4} (\vec{q}_1 + \vec{q}_2)^2.
\]

\( t \) determines how off-forward, while \( \bar{Q}^2 \to \infty \) isolates leading-order GPDs.

Note: we need to keep source/sink momenta equal magnitude: \( |\vec{p}| = |\vec{p} + \vec{q}_2 - \vec{q}_1| \)

Right: \( \bar{\omega} = 0.8, \bar{Q}^2 = 25, t = -4 \) (lattice units).
Data Sets

<table>
<thead>
<tr>
<th>Set</th>
<th>$q_1$, $q_2$</th>
<th>$t$ [GeV$^2$]</th>
<th>$Q^2$ [GeV$^2$]</th>
<th>$N_{\text{meas}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>$(1, 5, 1), (-1, 5, 1)$</td>
<td>$-1.10$</td>
<td>$7.13$</td>
<td>$996$</td>
</tr>
<tr>
<td>#2</td>
<td>$(4, 2, 2), (2, 4, 2)$</td>
<td>$-2.20$</td>
<td>$6.03$</td>
<td>$996$</td>
</tr>
</tbody>
</table>

Note that the two data sets have different $\bar{Q}^2$.

- Can compare this the forward Compton amplitude, from K. U. Can et al PRD 102 (2020).
- In those results, $Q^2 = 7.13$ GeV$^2$, $t = 0$. 
Applying Feynman-Hellmann

By varying the sink momentum, $\vec{p}$, we vary scaling variable:

$$\tilde{\omega} = \frac{4\vec{p} \cdot (\vec{q}_1 + \vec{q}_2)}{(\vec{q}_1 + \vec{q}_2)^2}$$
Compton Amplitude

- Red curve is forward Compton amplitude: $t = 0$, $Q^2 = 7.13$ GeV$^2$ (PRD 102, 2020)
- up quarks, with unpolarised spin-parity projector: $T_\uparrow + T_\downarrow$
\[
\overline{T}^{\text{unpol}}(\bar{\omega}, t) = 2 \sum_{n=2,4,6}^{\infty} \bar{\omega}^n [A_{n,0}^u(t) + \frac{t}{2m(E + m)} B_{n,0}^u(t)]
\]
- Dominated by $\mathcal{H}$ CFF, equivalently $A_{n,0}$ moments
- Decrease with $-t$. 
Fitting Moments

Fit our data to

\[ f_{N_{\text{max}}} (\bar{\omega}) = M_2 \bar{\omega}^2 + M_4 \bar{\omega}^4 + \ldots + M_{2N_{\text{max}}} \bar{\omega}^{2N_{\text{max}}} \]

To prevent over-fitting, use MCMC with monotonic decreasing priors (not rigorous):

\[ M_2 \geq M_4 \geq \ldots \geq M_{2N_{\text{max}}} - 2 \geq M_{2N_{\text{max}}} \geq 0. \]

Extract the linear combination of moments

\[ M_n(t) \approx A_{n,0}(t) + \frac{t}{8m^2} B_{n,0}(t). \]

- Fit \( N_{\text{max}} = 3; \) limited by number of \( \bar{\omega} \) points
- \( n = 4 \) moments never calculated before!
- \( n = 2 \) consistent with 3-pt moments at similar pion mass
Summary

- Preliminary calculation was successful
- Extracted linear combination of GPD moments ($A_{n,0}$ and $B_{n,0}$)

However,

- We would like $A$ and $B$ moments (equivalently $\mathcal{H}$ and $\mathcal{E}$ CFFs) separately
- More $\bar{\omega}$ values
- Better motivated fitting priors (not monotonic)
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New data

<table>
<thead>
<tr>
<th>Set</th>
<th>$t$ [GeV$^2$]</th>
<th>$Q^2$ [GeV$^2$]</th>
<th>$N_{\text{meas}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
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<td>10000</td>
</tr>
<tr>
<td>#2</td>
<td>$-0.29$</td>
<td>4.79</td>
<td>1000</td>
</tr>
<tr>
<td>#3</td>
<td>$-0.57$</td>
<td>4.86</td>
<td>1000</td>
</tr>
<tr>
<td></td>
<td>$-1.14$</td>
<td>4.86</td>
<td>1000</td>
</tr>
</tbody>
</table>

For $L^3 \times T = 48^3 \times 96$, $m_\pi = 410$ MeV.

Benefits of new calculation:
- Many more $\bar{\omega}$ values (larger lattice)
- More $-t$ values and smaller
- Can isolate $\mathcal{H}_1$ and $\mathcal{E}_1$ and therefore $A$ and $B$ moments

New kinematics:
- Choose momentum transfer $\vec{q}_1 - \vec{q}_2$ in same direction as EM current
- Allows use to access a linear combination of $\mathcal{H}_1$ and $\mathcal{E}_1$ (only 2 CFFs)
- Use two different spin-parity projectors— isolate each CFF.
Separating $\mathcal{H}$ and $\mathcal{E}$

New kinematics:

$$\text{tr}\left\{ \Gamma \bar{u}' T_{33} u \right\} = N_f^h \mathcal{H}_1 + N_f^e \mathcal{E}_1$$

$\Gamma$ is spin-parity projector.

Then, similar to elastic FFs,

$$\left( \begin{array}{c} \Re T_{33}^{\text{unpol}} \\ \Im T_{33}^{\text{pol}} \end{array} \right) = \left( \begin{array}{cc} N_h^{\text{unpol}} & N_e^{\text{unpol}} \\ N_h^{\text{pol}} & N_e^{\text{pol}} \end{array} \right) \left( \begin{array}{c} \mathcal{H}_1 \\ \mathcal{E}_1 \end{array} \right)$$

Linear combination of $\mathcal{H}$ and $\mathcal{E}$ not as orthogonal as we’d like, but still workable

Isovector results for $t = -0.57 \text{ GeV}^2$. 

Alec Hannaford Gunn

GPDs from Feynman-Hellmann

CSSM/QCDSF/UKQCD collaboration

33 / 37
Model-independent GPD positivity constraints [Pobylitsa, PRD 65, 2002]:

\[ |A_{n,0}(t)| \leq a_n, \quad |B_{n,0}(t)| \leq \frac{2m}{\sqrt{-t}} a_n \]

- Note: different \( m_\pi \) and \( \bar{Q}^2 \approx 5 \text{ GeV}^2 \)
- Different systematics between all three
- But still consistent—less so with quasi at larger \(-t\)
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Conclusion:

- New method to calculate OFCA $\rightarrow$ GPDs
- Allows strong parallels with experiment: scaling, subtraction function, higher-twist
- Calculation of leading moments of $\mathcal{H}_1$ and $\mathcal{E}_1$

Outlook:

- Beyond leading moments: GPD model fits, inversion methods
- Off-forward analogue of $\mathcal{F}_2$: test off-forward Callan-Gross—test higher-order
- More $\bar{Q}^2$ values: $2 - 10 \text{ GeV}^2$
- More $t$ values (drop equal energy condition, extend FH)
Thank you for listening—questions?
$\mathcal{E}_1$ moments

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure.png}
\caption{Comparison of $B_{u-d}^2(t)$ with different fits and models.}
\end{figure}

PRELIMINARY

$\vec{Q}^2 \approx 5 \text{ GeV}^2, 48^3 \times 96$

Lin 21 (quasi fit)

ETMC 20 2f

ETMC 20 2+1+1f
Markov chain Monte Carlo fits

Compare fits:
- $J$ is number of moments.
- Uniform priors are $[0, 100]$.
- For $32^3 \times 64$ dataset—few $\bar{w}$ values.
- Yet to repeat with new data set or new priors.
OPE predicts $S_1(Q^2) \rightarrow 0$ with $Q^2 \rightarrow \infty$. 
Subtraction Term

Subtraction function from DVCS; input for calculation of proton pressure distribution.

Varies with discretisation: see forthcoming proceedings.