Thermal modification of open-charm mesons from an effective hadronic theory

Glòria Montaña

University of Barcelona
Institute of Cosmos Sciences

[GM, Angels Ramos, Laura Tolos, Juan Torres-Rincon, Phys.Lett.B 806 (2020)]

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OUTLINE

1. Motivation
2. Hadronic molecular model: Effective hadronic interaction & Unitarization
3. Results I: Dynamically generated states at $T = 0$
4. Finite temperature corrections
5. Results II: Thermal modifications
6. Euclidean correlators: comparison with lattice QCD
7. Results III: Open-charm Euclidean correlators
8. Conclusions and Outlook
Motivation
→ Heavy-ion-collision (HIC) programmes in on-going and upcoming experimental facilities (RHIC, LHC, FAIR) highly demand the theoretical study of **hadronic properties under extreme conditions of temperature and density**.
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Theoretical tools to study the matter at high temperatures:

- Perturbative theories
- Lattice QCD
- Non-perturbative effective hadronic theories
MOTIVATION (I)

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**Quarkonia suppression**

- Color screening
- Comover scattering
MOTIVATION (II)

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→ **Spectral functions** can be directly calculated using effective hadronic theories within a unitarized approach.
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Motivation

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Spectral functions can be directly calculated using effective hadronic theories within a unitarized approach.

We focus on finite-temperature mesonic matter to study the high temperature (< \( T_c \)) and low density region of the QCD phase diagram (matter generated in HICs in RHIC and LHC).

Ground-state heavy mesons (e.g. \( D, D_s, D^*, D_s^* \)) modify their properties in hot matter.
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Consequences in the behavior of excited mesonic states, such as the non-strange $D_0^*(2300)$ and $D_1^*(2430)$ and the strange $D_{s0}^*(2317)$ and $D_{s1}^*(2460)$, dynamically generated in a heavy-light molecular model at finite temperature.
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→ Spectral functions can be obtained at unphysical meson masses and used to calculate **Euclidean correlators** to compare with lattice QCD results.
Hadronic molecular model: Effective hadronic interaction & Unitarization
Although the basic constituents in QCD are quarks and gluons, the **conventional quark model** (Gell-Mann, 1964; Zweig, 1964) has been very successful in describing hadron structure.

**QUARK MODEL**

**Baryon**

$q, q, q$

$p, n, \Lambda, \Delta, \Xi_c, ...$

**Meson**

$q, \bar{q}$

$\pi, K, D, B, J/\psi, ...$
EXOTIC HADRONS

There are many (excited) hadrons that do not accommodate in the $qqq$ or $q\bar{q}$ picture. Other configurations allowed by QCD, e.g., $qqq\bar{q}$, $qqqq\bar{q}$, etc., are called exotic.

Baryonic systems
Pentaquarks $qqq\bar{q}$

Compact pentaquark

Meson-baryon molecule

$\Lambda(1405)$, $\Lambda_c(2595)$, $\Lambda_c(2625)$, $P_c(4380)$, $P_c(4450)$, ...
**Mesonic systems**

**Tetraquarks** $qar{q}qar{q}$

- **Compact tetraquark**
- **Meson-meson molecule**

- Hybrid
- Glueball

$D_{0}^{*}(2300)$, $D_{s}^{*}(2317)$, $X(3872)$, charged $Z$ states, ...
EXOTIC HADRONS

Mesonic systems

Tetraquarks $q\bar{q}q\bar{q}$

Compact tetraquark

Meson-meson molecule

$D_0^*(2300), D_s^*(2317), X(3872), \text{charged } Z \text{ states, ...}$

Hadronic molecules are deuteron-like quasi-bound states of two mesons (tetraquark), a meson and a baryon (pentaquark) or two baryons (hexaquark).

→ Dynamically generated via multiple scattering of their meson/baryon components.

→ Located near threshold $m_1 + m_2$

→ Studied using effective hadronic theories.

→ Mesons and baryons are the degrees of freedom.
EFFECTIVE HADRONIC THEORY

The most general effective Lagrangian, up to a given order, consistent with the symmetries of the underlying theory.
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Heavy-light meson-meson interaction $\rightarrow$ Symmetries of QCD:

- **Chiral symmetry** in the limit $m_u, m_d, m_s \rightarrow 0$
- **Heavy-quark symmetry** in the limit $m_c, m_b \rightarrow \infty$

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Interaction mediated by the exchange of mesons.

**Interaction of $D(*)$ and $D_s(*)$ with light mesons:**

$\rightarrow$ Lagrangian at NLO in the chiral expansion and LO in the heavy-quark expansion

$$\mathcal{L} = \mathcal{L}_{\text{LO}} + \mathcal{L}_{\text{NLO}}$$
INTERACTION LAGRANGIAN (LO)

Lagrangian at NLO in the chiral expansion and LO in the heavy-quark expansion

\[ \mathcal{L} = \mathcal{L}_{\text{LO}} + \mathcal{L}_{\text{NLO}} \]

\[ \mathcal{L}_{\text{LO}} = \langle \nabla^\mu D \nabla_\mu D^\dagger \rangle - m_D^2 \langle DD^\dagger \rangle - \langle \nabla^\mu D^{*\nu} \nabla_\mu D^{*\dagger}_\nu \rangle + m_D^2 \langle D^{*\nu} D^{*\dagger}_\nu \rangle 
+ ig \langle D^{*\mu} u_\mu D^\dagger - Du_\mu D^{*\dagger}_\mu \rangle + \frac{g}{2m_D} \langle D^{*\mu} u_\alpha \nabla_\beta D^{*\dagger}_\nu - \nabla_\beta D^{*\mu} u_\alpha D^{*\dagger}_\nu \rangle \epsilon^{\mu\nu\alpha\beta} \]

\[ \nabla_\mu D^{(*)} = \partial_\mu D^{(*)} - D^{(*)} \Gamma^\mu \quad \Gamma^\mu = \frac{1}{2} (u^\dagger \partial_\mu u + u \partial_\mu u^\dagger) \]

\[ u_\mu = i (u^\dagger \partial_\mu u - u \partial_\mu u^\dagger) \]

\[ u = \sqrt{U} = \exp \left( \frac{i \Phi}{\sqrt{2f_\pi}} \right) \]

[Kolomeitsev and Lutz (2004)]
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\[ + ig \langle D^\mu u^\mu D^\dagger \rangle - D^\mu u^\mu D^\dagger \rangle + \frac{g}{2m_D} \langle D^\mu u_\alpha \nabla_\beta D^\dagger \rangle - \nabla_\beta D^\mu u_\alpha D^\dagger \rangle \epsilon^{\mu\nu\alpha\beta} \]

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+ ig \langle D^*_{\mu} u_{\mu} D^\dagger \rangle - Du^\mu D^*_{\mu} \rangle 
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\[ D \text{ mesons:} \]
\[ D = (D^0 \quad D^+ \quad D^+_s) , \quad D^*_{\mu} = (D^{*0} \quad D^{*+} \quad D^{*+}_s)_{\mu} \]

\[ \mathbf{D}_i \quad \mathbf{D}_j \]

\[ \Phi = \begin{pmatrix} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta \\ \frac{1}{\sqrt{2}} \pi^- + \frac{1}{\sqrt{6}} \eta \\ \frac{1}{\sqrt{2}} \pi^+ + \frac{1}{\sqrt{6}} \eta \\ K^- \\ K^0 \\ \sqrt{2} \eta \end{pmatrix} \]

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INTERACTION LAGRANGIAN (NLO)

Lagrangian at NLO in the chiral expansion and LO in the heavy-quark expansion

\[ \mathcal{L} = \mathcal{L}_{\text{LO}} + \mathcal{L}_{\text{NLO}} \]

\[ \mathcal{L}_{\text{NLO}} = - h_0 \langle DD^\dagger \rangle \langle \chi_+ \rangle + h_1 \langle D\chi_+ D^\dagger \rangle + h_2 \langle DD^\dagger \rangle \langle u^\mu u_\mu \rangle 
+ h_3 \langle Du^\mu u_\mu D^\dagger \rangle + h_4 \langle \nabla_\mu D \nabla_\nu D^\dagger \rangle \langle u^\mu u_\nu \rangle + h_5 \langle \nabla_\mu D \{ u^\mu, u_\nu \} \nabla_\nu D^\dagger \rangle 
+ \tilde{h}_0 \langle D^{*\mu} D^{*\dagger}_\mu \rangle \langle \chi_+ \rangle - \tilde{h}_1 \langle D^{*\mu} \chi_+ D^{*\dagger}_\mu \rangle - \tilde{h}_2 \langle D^{*\mu} D^{*\dagger}_\mu \rangle \langle u^\nu u_\nu \rangle 
- \tilde{h}_3 \langle D^{*\mu} u^\nu u_\nu D^{*\dagger}_\mu \rangle - \tilde{h}_4 \langle \nabla_\mu D^{*\alpha} \nabla_\nu D^{\dagger*}_\alpha \rangle \langle u^\mu u_\nu \rangle - \tilde{h}_5 \langle \nabla_\mu D^{*\alpha} \{ u^\mu, u_\nu \} \nabla_\nu D^{\dagger*}_\alpha \rangle \]

\[ \chi_+ = u^\dagger \chi u^\dagger + u \chi u \quad \chi = \text{diag}(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2) \quad \text{LECs} : \ h_0, \ldots, 5, \ \tilde{h}_0, \ldots, 5 \]

[Liu, Orginos, Guo, Hanhart and Meißner (2013)]
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[Albaladejo, Fernandez-Soler, Guo and Nieves (2017)]
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SCATTERING AMPLITUDE

\[ \mathcal{L} = \mathcal{L}_{\text{LO}} + \mathcal{L}_{\text{NLO}} \]

Tree-level scattering amplitude of \( D^{(*)}, D_s^{(*)} \) mesons with \( \pi, K, \bar{K}, \eta \) mesons:

\[
V^{ij}(s, t, u) = \frac{1}{f_\pi^2} \left[ \frac{1}{4} C^{ij}_{LO} (s - u) - 4 C^{ij}_0 h_0 + 2 C^{ij}_1 h_1 \right. \\
- 2 C^{ij}_{24} \left( 2 h_2 (p_2 \cdot p_4) + h_4 ((p_1 \cdot p_2)(p_3 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3)) \right) \\
+ 2 C^{ij}_{35} \left( h_3 (p_2 \cdot p_4) + h_5 ((p_1 \cdot p_2)(p_3 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3)) \right) \]

\( D_i(p_1) \quad D_j(p_3) \)

\( \Phi_i(p_2) \quad \Phi_j(p_4) \)
Tree-level scattering amplitude of $D^{(*)}$, $D_s^{(*)}$ mesons with $\pi, K, \vec{K}, \eta$ mesons:

$$V^{ij}(s, t, u) = \frac{1}{f_\pi^2} \left[ \frac{1}{4} C_{LO}^{ij} (s - u) - 4 C_{0}^{ij} h_0 + 2 C_{1}^{ij} h_1 ight. $$

$$- \left. 2 C_{24}^{ij} \left( 2 h_2 (p_2 \cdot p_4) + h_4 \left( (p_1 \cdot p_2)(p_3 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3) \right) \right) \right]$$

$$+ \left. 2 C_{35}^{ij} \left( h_3 (p_2 \cdot p_4) + h_5 \left( (p_1 \cdot p_2)(p_3 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3) \right) \right) \right]$$

$C_{LO,0,1,24,35}^{ij}$: isospin coefficients for the transition $i \rightarrow j$
### SCATTERING AMPLITUDE

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\]

LECsfitted to LQCD data of scattering lengths

[Guo, Liu, Meißner, Oller Rusetsky (2019)]
UNITARIZATION

Bethe-Salpeter equation in coupled-channels:

\[ D_i D_j = D_i D_j + D_i D_k D_j + D_i D_k D_l D_j + \ldots \]

\[ T_{ij} = V_{ij} + V_{ik} G_k V_{kj} + V_{ik} G_k V_{kl} G_l V_{lj} + \ldots \]
UNITARIZATION

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D_i D_j = D_i D_j + D_i D_k D_j + D_i D_k D_l D_j + \ldots
\]

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T_{ij} = V_{ij} + V_{ik} G_k V_{kj} + V_{ik} G_k V_{kl} G_l V_{lj} + \ldots
\]

\[
T_{ij} = T_{ij} + V_{ik} G_k T_{kj} \rightarrow T = (1 - VG)^{-1} V \quad \text{(On-shell factorization)}
\]
The two-meson propagator

\[ G_k = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 - m_{D,k}^2 + i\varepsilon} \frac{1}{(P - q)^2 - m_{\Phi,k}^2 + i\varepsilon} \]

has to be regularized:

- Dimensional regularization: subtraction constants \( a_k(\mu) \)
- **Cutoff regularization** with \( |\vec{q}| < \Lambda \)

At threshold: \( \sqrt{s_{\text{thr}}} = m_1 + m_2 \)

\[ G_{\text{DR}}(\sqrt{s_{\text{thr}}}, a_l(\mu)) = G_{\Lambda}(\sqrt{s_{\text{thr}}}, \Lambda) \]
Dynamically generated states

→ Identification of states in the unitarized scattering amplitudes
→ Analytical continuation and poles in the complex-energy plane
→ Bound states, resonances and virtual states in different Riemann sheets

• Mass \( M_R = \text{Re} \sqrt{s_R} \)
• Half-width \( \frac{\Gamma_R}{2} = \text{Im} \sqrt{s_R} \)
• Coupling constants \( |g_i| \)
• Compositeness \( X_i = \left| g_i^2 \frac{\partial G_i(z_p)}{\partial z} \right| \)
Results I: Dynamically generated states at $T = 0$
In isospin basis:

<table>
<thead>
<tr>
<th>$(S, I)$</th>
<th>Channels</th>
<th>Threshold (MeV)</th>
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</tr>
</thead>
<tbody>
<tr>
<td>$(-1, 0)$</td>
<td>$D\bar{K}$</td>
<td>2364.88</td>
<td>$(-1, 0)$</td>
<td>$D^*\bar{K}$</td>
<td>2504.20</td>
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<tr>
<td>$0, \frac{1}{2}$</td>
<td>$D\pi$</td>
<td>2005.28</td>
<td>$(0, \frac{1}{2})$</td>
<td>$D^*\pi$</td>
<td>2146.59</td>
</tr>
<tr>
<td>$D\eta$</td>
<td>2415.10</td>
<td></td>
<td>$D^*\eta$</td>
<td>2556.42</td>
<td></td>
</tr>
<tr>
<td>$D_{s}\bar{K}$</td>
<td>2463.98</td>
<td></td>
<td>$D_{s}^*\bar{K}$</td>
<td>2607.84</td>
<td></td>
</tr>
<tr>
<td>$(0, \frac{3}{2})$</td>
<td>$D\pi$</td>
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<td></td>
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<td>$(1, 1)$</td>
<td>$D_{s}\pi$</td>
<td>2106.38</td>
<td>$(1, 1)$</td>
<td>$D_{s}^*\pi$</td>
<td>2250.24</td>
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<td>$DK$</td>
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### COUPLED CHANNELS

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<td>$J^P = 1^-$</td>
<td>$D_s\pi$</td>
<td>2106.38</td>
</tr>
<tr>
<td>$J^P = 1^-$</td>
<td>$D\bar{K}$</td>
<td>2364.88</td>
</tr>
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<td>$D_s\bar{K}$</td>
<td>2463.98</td>
</tr>
</tbody>
</table>

### Results II

<table>
<thead>
<tr>
<th>$(S, I)$</th>
<th>Channels</th>
<th>Threshold (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J^P = 0^-$</td>
<td>$D^*\bar{K}$</td>
<td>2504.20</td>
</tr>
<tr>
<td>$J^P = 0^-$</td>
<td>$D^*\bar{K}$</td>
<td>2504.20</td>
</tr>
<tr>
<td>$J^P = 0^-$</td>
<td>$D^*\pi$</td>
<td>2146.59</td>
</tr>
<tr>
<td>$J^P = 0^-$</td>
<td>$D^*\eta$</td>
<td>2556.42</td>
</tr>
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<td>$J^P = 0^-$</td>
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<td>2607.84</td>
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<td>2660.06</td>
</tr>
<tr>
<td>$J^P = 1^-$</td>
<td>$D_s^*\pi$</td>
<td>2250.24</td>
</tr>
<tr>
<td>$J^P = 1^-$</td>
<td>$D_s^*\bar{K}$</td>
<td>2607.84</td>
</tr>
</tbody>
</table>
### COUPLED CHANNELS

**In isospin basis:**

<table>
<thead>
<tr>
<th>((S, I))</th>
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<th>Threshold (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-1, 0))</td>
<td>(D\bar{K})</td>
<td>2364.88</td>
</tr>
<tr>
<td>((-1, 1))</td>
<td>(D\bar{K})</td>
<td>2364.88</td>
</tr>
<tr>
<td>((0, \frac{1}{2}))</td>
<td>(D\pi)</td>
<td>2005.28</td>
</tr>
<tr>
<td></td>
<td>(D\eta)</td>
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<tr>
<td></td>
<td>(D_s\bar{K})</td>
<td>2463.98</td>
</tr>
<tr>
<td>((0, \frac{3}{2}))</td>
<td>(D\pi)</td>
<td>2005.28</td>
</tr>
<tr>
<td>((1, 0))</td>
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</tr>
<tr>
<td></td>
<td>(D_s\eta)</td>
<td>2516.20</td>
</tr>
<tr>
<td>((1, 1))</td>
<td>(D_s\pi)</td>
<td>2106.38</td>
</tr>
<tr>
<td></td>
<td>(DK)</td>
<td>2364.88</td>
</tr>
<tr>
<td>((2, \frac{1}{2}))</td>
<td>(D_sK)</td>
<td>2463.98</td>
</tr>
</tbody>
</table>

**Channels \(J^P = 1^- \oplus 0^-\)**

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<td>2607.84</td>
</tr>
</tbody>
</table>
Dynamically generated states

Scalars ($J^P = 0^+$): $D_0^*(2300)$ and $D_{s0}^*(2317)$

| $(S, I)$ | RS   | $M_R$  | $\Gamma_R / 2$ | $|g_i|$ | $\chi_i$ |
|---------|------|--------|----------------|--------|----------|
| $(0, \frac{1}{2})$ | $(-, +, +)$ | 2081.9 | 86.0 | $|g_{D\pi}| = 8.9$ | $\chi_{D\pi} = 0.40$ |
|         |      |        |                |        |          | $|g_{D\eta}| = 0.4$ | $\chi_{D\eta} = 0.00$ |
|         |      |        |                |        |          | $|g_{DSK}| = 5.4$ | $\chi_{DSK} = 0.05$ |
|         | $(-, -, +)$ | 2529.3 | 145.4 | $|g_{D\pi}| = 6.7$ | $\chi_{D\pi} = 0.10$ |
|         |      |        |                |        |          | $|g_{D\eta}| = 9.9$ | $\chi_{D\eta} = 0.40$ |
|         |      |        |                |        |          | $|g_{DSK}| = 19.4$ | $\chi_{DSK} = 1.63$ |
| $(1, 0)$ | $(+, +)$ | 2252.5 | 0.0 | $|g_{DK}| = 13.3$ | $\chi_{DK} = 0.66$ |
|         |      |        |                |        |          | $|g_{DS\eta}| = 9.2$ | $\chi_{DS\eta} = 0.17$ |

Experimental values:

$M_R = 2300 \pm 19$ MeV, $\Gamma_R = 274 \pm 40$ MeV

$M_R = 2317.8 \pm 0.5$ MeV, $\Gamma_R < 3.8$ MeV
DYNAMICALLY GENERATED STATES

Axial vectors ($J^P = 1^+$): $D_1^*(2430)$ and $D_{s1}^*(2460)$

| $(S, I)$ | RS   | $M_R$  | $\Gamma_R/2$ | $|g_i|$  | $\chi_i$ |
|---------|------|--------|--------------|--------|---------|
| $(0, \frac{1}{2})$ | $(-, +, +)$ | 2222.3 | 84.7 | $|g_D^*\pi| = 9.5$ | $\chi_D^*\pi = 0.40$ |
|          | $(-, -, +)$ | 2654.6 | 117.3 | $|g_D^*\eta| = 0.4$ | $\chi_D^*\eta = 0.00$ |
| $(1, 0)$ | $(+, +)$ | 2393.3 | 0.0  | $|g_{D_s^*K}| = 5.7$ | $\chi_{D_s^*K} = 0.05$ |
|          |        |        |      | $|g_{D_s^*\eta}| = 6.5$ | $\chi_{D_s^*\eta} = 0.09$ |
|          |        |        |      | $|g_{D_s^*K}| = 10.0$ | $\chi_{D_s^*K} = 0.40$ |
|          |        |        |      | $|g_{D_s^*\eta}| = 18.5$ | $\chi_{D_s^*\eta} = 1.47$ |

Experimental values:

$M_R = 2427 \pm 40$ MeV, $\Gamma_R = 384_{-110}^{+130}$ MeV

$M_R = 2459.5 \pm 0.6$ MeV, $\Gamma_R < 3.5$ MeV
Finite temperature corrections
→ Mesonic matter at a temperature $0 < T < T_c$
→ Vanishing baryon density
→ Heavy mesons behave as Brownian particles
→ Modification of the properties of the $D$ mesons
MESONIC BATH

→ Mesonic matter at a temperature $0 < T < T_c$
→ Vanishing baryon density
→ Heavy mesons behave as Brownian particles
→ Modification of the properties of the $D$ mesons

Both production and absorption processes of heavy-light pairs are possible
MODIFICATION OF HEAVY MESONS IN A HOT MEDIUM

Imaginary time formalism

- Sum over Matsubara frequencies

\[ q^0 \rightarrow \omega_n = \frac{i}{\beta} \frac{2\pi n}{2\pi^4}, \quad \int \frac{d^4 q}{(2\pi^4)} \rightarrow \frac{i}{\beta} \sum_n \int \frac{d^3 q}{(2\pi)^3} \quad \text{(bosons)} \]
MODIFICATION OF HEAVY MESONS IN A HOT MEDIUM

**Imaginary time formalism**

- Sum over Matsubara frequencies

\[ q^0 \rightarrow \omega_n = \frac{i}{\beta} 2\pi n, \quad \int \frac{d^4 q}{(2\pi^4)} \rightarrow \frac{i}{\beta} \sum_n \int \frac{d^3 q}{(2\pi^3)} \quad (\text{bosons}) \]

**Dressing the mesons in the loop function**

- Self-energy corrections
- Pion mass slightly varies below \( T_c \) \( \rightarrow \) only heavy meson is dressed

\[ D \quad = \quad D + \quad D \quad \quad \Phi \quad \quad \Phi \]

\[ D \quad \quad \quad D \]
SELF-CONSISTENCY

\[ G_{D\Phi}(E, \vec{p}; T) = \int \frac{d^3 q}{(2\pi)^3} \int d\omega \int d\omega' \frac{S_D(\omega, \vec{q}; T)S_{\Phi}(\omega', \vec{p} - \vec{q}; T)}{E - \omega - \omega' + i\varepsilon} [1 + f(\omega, T) + f(\omega', T)] \]
SELF-CONSISTENCY

\[ G_{D\Phi}(E, \vec{p}; T) = \int \frac{d^3 q}{(2\pi)^3} \int d\omega \int d\omega' \frac{S_D(\omega, \vec{q}; T)S_\Phi(\omega', \vec{p} - \vec{q}; T)}{E - \omega - \omega' + i\varepsilon} \left[ 1 + f(\omega, T) + f(\omega', T) \right] \]

Spectral functions: \( S_D, S_\Phi \rightarrow \frac{\omega_\Phi}{\omega} \delta(\omega'^2 - \omega_\Phi^2), \quad \omega_\Phi = \sqrt{q_\Phi^2 + m_\Phi^2} \)
\[ G_{D\Phi}(E, \vec{p}; T) = \int \frac{d^3 q}{(2\pi)^3} \int d\omega \int d\omega' \frac{S_D(\omega, \vec{q}; T) S_\Phi(\omega', \vec{p} - \vec{q}; T)}{E - \omega - \omega' + i\varepsilon} \left[ 1 + f(\omega, T) + f(\omega', T) \right] \]

Spectral functions: \[ S_D, \quad S_\Phi \rightarrow \frac{\omega_\Phi}{\omega} \delta(\omega' \omega_\Phi^2 - \omega_\Phi^2), \quad \omega_\Phi = \sqrt{q_\Phi^2 + m_\Phi^2} \]

Bose distribution function at T: \[ f(\omega, T) = \frac{1}{e^{\omega/T} - 1} \]
SELF-CONSISTENCY

\[ G_{D\Phi}(E, \vec{p}; T) = \int \frac{d^3 q}{(2\pi)^3} \int d\omega \int d\omega' \frac{S_D(\omega, \vec{q}; T) S_{\Phi}(\omega', \vec{p} - \vec{q}; T)}{E - \omega - \omega' + i\varepsilon} \left[ 1 + f(\omega, T) + f(\omega', T) \right] \]

Spectral functions: \( S_D, \quad S_{\Phi} \rightarrow \frac{\omega_{\Phi}^2}{\omega_{\Phi}} \delta(\omega r^2 - \omega_{\Phi}^2), \quad \omega_{\Phi} = \sqrt{q_{\Phi}^2 + m_{\Phi}^2} \)

Bose distribution function at T: \( f(\omega, T) = \frac{1}{e^{\omega/T} - 1} \)

Regularized with a cutoff \(|\vec{q}| < \Lambda\)
SELF-CONSISTENCY

\[ T_{ij} = V_{ij} + V_{ik} G_k T_{kj} \]
\[ \Pi_D(E, \vec{p}; T) = -\frac{1}{\pi} \int \frac{d^3 q}{(2\pi)^3} \int d\Omega \frac{E}{\omega_\Phi} \frac{f(\Omega, T) - f(\omega_\Phi, T)}{E^2 - (\omega_\Phi - \Omega)^2 + i\epsilon} \text{Im} T_{D\Phi}(\Omega, \vec{p} + \vec{q}; T) \]
SELF-CONSISTENCY

\[ S_D(\omega, \vec{q}; T) = -\frac{1}{\pi} \text{Im} \left( \mathcal{D}_D(\omega, \vec{q}; T) \right) = -\frac{1}{\pi} \text{Im} \left( \frac{1}{\omega^2 - \vec{q}^2 - m_D^2 - \Pi_D(\omega, \vec{q}; T)} \right) \]

\[ D = D + D \]

Loop function \( G_{D\Phi} \) → Unitarized amplitude \( T_{D\Phi} \) → Self-energy \( \Pi_D \) → \( D \)-meson propagator \( \mathcal{D}_D \)
PHYSICAL INTERPRETATION OF THE THERMAL BATH

\[ G_{D\Phi}(E, \vec{p}; T) \sim \left\{ \begin{array}{ll}
\text{bath} \to \text{bath} + D\Phi & \text{bath} + D\Phi \to \text{bath} \\
[1 + f(\omega_D, T)][1 + f(\omega_\Phi, T)] - f(\omega_D, T)f(\omega_\Phi, T) & \end{array} \right. \\
E - \omega_D - \omega_\Phi + i\varepsilon
\]

\[ \begin{array}{ll}
\text{bath} + \overline{D\Phi} \to \text{bath} & \text{bath} \to \text{bath} + \overline{D\Phi} \\
[1 + f(\omega_D, T)][1 + f(\omega_\Phi, T)] - f(\omega_D, T)f(\omega_\Phi, T) & \\
E + \omega_D + \omega_\Phi + i\varepsilon
\end{array} \]

At zero temperature \( f(\omega, T = 0) = 0 \)
PHYSICAL INTERPRETATION OF THE THERMAL BATH

\[ G_{D\Phi}(E, \vec{p}; T) \sim \left\{ \begin{array}{l}
\frac{[1 + f(\omega_D, T)][1 + f(\omega, T)] - f(\omega_D, T)f(\omega, T)}{E - \omega_D - \omega + i\varepsilon} \\
+ \frac{f(\omega_D, T)f(\omega, T) - [1 + f(\omega_D, T)][1 + f(\omega, T)]}{E + \omega_D + \omega + i\varepsilon} \\
+ \frac{f(\omega, T) [1 + f(\omega, T)] - f(\omega, T) [1 + f(\omega_D, T)]}{E + \omega_D - \omega + i\varepsilon} \\
+ \frac{f(\omega, T) [1 + f(\omega, T)] - f(\omega_D, T) [1 + f(\omega, T)]}{E - \omega_D + \omega + i\varepsilon}
\end{array} \right. \]

First branch cut
\((T = 0 \text{ unitary cut)}:\)
\[ E \geq (m_D + m_\Phi) \]

Additional branch cut
\((\text{Landau cut)}:\)
\[ E \leq (m_D - m_\Phi) \]

At zero temperature \(f(\omega, T = 0) = 0\)
Results II: Thermal modifications
LOOP FUNCTIONS

Pionic bath

\( D \) and \( D_s \) with light mesons

Unitary cut:
\[ E \geq (m_D + m_\Phi) \]

Landau cut:
\[ E \leq (m_D - m_\Phi) \]
Pionic bath

$D^*$ and $D_s^*$ with light mesons

Unitary cut:
$E \geq (m_{D^*} + m_\Phi)$

Landau cut:
$E \leq (m_{D^*} - m_\Phi)$
Open-charm \textbf{pseudoscalar mesons} in a pionic bath

\[ S_D(\omega, \vec{q}; T) = -\frac{1}{\pi} \text{Im} \mathcal{D}_D(\omega, \vec{q}; T) \]

Mass shift and width acquisition of the \textbf{D} and \textbf{D}_s mesons in a thermal bath
SPECTRAL FUNCTIONS

Open-charm vector mesons in a pionic bath

\[ S_D(\omega, \vec{q}; T) = -\frac{1}{\pi} \text{Im} \mathcal{D}_D(\omega, \vec{q}; T) \]

Mass shift and width acquisition of the \( D^* \) and \( D_s^* \) mesons in a thermal bath
Scalars ($J^P = 0^+$):

T-matrix in sector $(C, S, I) = (1, 0, 1/2)$
→ Two-pole structure of the $D_0^*(2300)$

Experimental values:
$M_R = 2300 \pm 19$ MeV, $\Gamma_R = 274 \pm 40$ MeV

T-matrix in sector $(C, S, I) = (1, 1, 0)$
→ $D_{s0}^*(2317)$

Experimental values:
$M_R = 2317.8 \pm 0.5$ MeV, $\Gamma_R < 3.8$ MeV
Axial vectors \((J^P = 1^+)\):

\[ T\text{-matrix in sector } (C, S, I) = (1, 0, 1/2) \]

\[ \rightarrow \text{ Two-pole structure of the } D_1^*(2430) \]

Experimental values:
\[ M_R = 2427 \pm 40 \text{ MeV}, \quad \Gamma_R = 384^{+130}_{-110} \text{ MeV} \]

\[ T\text{-matrix in sector } (C, S, I) = (1, 1, 0) \]

\[ \rightarrow D_{s1}^*(2460) \]

Experimental values:
\[ M_R = 2459.5 \pm 0.6 \text{ MeV}, \quad \Gamma_R < 3.5 \text{ MeV} \]
CHIRAL PARTNERS

Evolution of masses and widths of the open-charm mesons in a pionic
(or $\pi + K + \bar{K}$) bath

$I(J^P) = \frac{1}{2}(0^\pm), 0(0^\pm)$

Evolution of masses and widths of the open-charm mesons in a pionic (or $\pi + K + \bar{K}$) bath

$$I(J^P) = \frac{1}{2}(1^{\pm}), 0(1^{\pm})$$

Euclidean correlators: comparison with lattice QCD
FROM SPECTRAL FUNCTIONS TO EUCLIDEAN CORRELATORS

Spectral function $\rightarrow$ Euclidean correlator

$$G_E(\tau, \vec{p}; T) = \int_0^\infty d\omega \ K(\tau, \omega; T) \ \rho(\omega, \vec{p}; T)$$
FROM SPECTRAL FUNCTIONS TO EUCLIDEAN CORRELATORS

Spectral function $\rightarrow$ Euclidean correlator

$$G_E(\tau, \vec{p}; T) = \int_0^\infty d\omega \ K(\tau, \omega; T) \ \rho(\omega, \vec{p}; T)$$

Spectral function $\rho(\omega, \vec{p}; T)$
FROM SPECTRAL FUNCTIONS TO EUCLIDEAN CORRELATORS

Spectral function $\rightarrow$ Euclidean correlator

$$G_E(\tau, \vec{p}; T) = \int_0^\infty d\omega K(\tau, \omega; T) \rho(\omega, \vec{p}; T)$$

Spectral function $\rho(\omega, \vec{p}; T)$

$$K(\tau, \omega; T) = \frac{\cosh[\omega(\tau - \frac{1}{2T})]}{\sinh(\frac{\omega}{2T})}$$
**Spectral function** \( \rightarrow \) **Euclidean correlator**

\[
G_E(\tau, \vec{p} \mid T) = \int_0^\infty d\omega \ K(\tau, \omega \mid T) \ \rho(\omega, \vec{p} \mid T)
\]

Spectral function \( \rho(\omega, \vec{p} \mid T) \)

\[
K(\tau, \omega \mid T) = \frac{\cosh[\omega(\tau - \frac{1}{2T})]}{\sinh(\frac{\omega}{2T})}
\]

**Euclidean correlator** \( \rightarrow \) **Spectral function** (ill-posed)

- Bayesian methods (e.g. MEM)
- Fitting Ansätze
FROM SPECTRAL FUNCTIONS TO EUCLIDEAN CORRELATORS

**Spectral function** $\rightarrow$ **Euclidean correlator**

\[ G_E(\tau, \vec{p}; T) = \int_0^\infty d\omega \, K(\tau, \omega; T) \, \rho(\omega, \vec{p}; T) \]

Spectral function $\rho(\omega, \vec{p}; T)$

\[ K(\tau, \omega; T) = \frac{\cosh[\omega(\tau - \frac{1}{2T})]}{\sinh(\frac{\omega}{2T})} \]

**Euclidean correlator** $\rightarrow$ **Spectral function** (ill-posed)

- Bayesian methods (e.g. MEM)
- Fitting Ansätze

Reconstructed correlator

\[ G^r_E(\tau; T, T_r) = \int_0^\infty d\omega K(\tau, \omega; T)\rho(\omega; T_r) \rightarrow \frac{G_E(\tau; T)}{G^r_E(\tau; T, T_r)} \]
EUCLIDEAN CORRELATORS WITH EFT

\[ S_D(\omega, \bar{q}; T) = -\frac{1}{\pi} \text{Im} D_D(\omega, \bar{q}; T) \]
\[ = -\frac{1}{\pi} \text{Im} \left( \frac{1}{\omega^2 - \bar{q}^2 - M_D^2 - \Pi_D(\omega, \bar{q}; T)} \right) \]

at unphysical meson masses (used in the lattice)
EUCLIDEAN CORRELATORS WITH EFT

\[ S_D(\omega, \bar{q}; T) = \frac{-1}{\pi} \text{Im} \, D_D(\omega, \bar{q}; T) \]
\[ = \frac{-1}{\pi} \text{Im} \left( \frac{1}{\omega^2 - \bar{q}^2 - M_D^2 - \Pi_D(\omega, \bar{q}; T)} \right) \]

at unphysical meson masses (used in the lattice)

Ground-state: \[ \rho_{gs}(\omega; T) = M_D^4 S_D(\omega; T) \]
EUCLIDEAN CORRELATORS WITH EFT

\[ S_D(\omega, \bar{q}; T) = -\frac{1}{\pi} \text{Im} D_D(\omega, \bar{q}; T) \]
\[ = -\frac{1}{\pi} \text{Im} \left( \frac{1}{\omega^2 - \bar{q}^2 - M_D^2 - \Pi_D(\omega, \bar{q}; T)} \right) \]

at unphysical meson masses (used in the lattice)

Ground-state: \( \rho_{gs}(\omega; T) = M_D^4 S_D(\omega; T) \)

Full: \( \rho(\omega; T) = \rho_{gs}(\omega; T) + a \rho_{\text{cont}}(\omega; T) \)
EUCLIDEAN CORRELATORS WITH EFT

\[
S_D(\omega, \bar{q}; T) = -\frac{1}{\pi} \text{Im} D_D(\omega, \bar{q}; T) = -\frac{1}{\pi} \text{Im} \left( \frac{1}{\omega^2 - \bar{q}^2 - M_D^2 - \Pi_D(\omega, \bar{q}; T)} \right)
\]

at unphysical meson masses (used in the lattice)

Ground-state: \( \rho_{gs}(\omega; T) = M_D^4 S_D(\omega; T) \)

Full: \( \rho(\omega; T) = \rho_{gs}(\omega; T) + a \rho_{cont}(\omega; T) \)
EUCLIDEAN CORRELATORS WITH EFT

\[ S_D(\omega, \vec{q}; T) = -\frac{1}{\pi} \text{Im} \mathcal{D}_D(\omega, \vec{q}; T) \]

\[ = -\frac{1}{\pi} \text{Im} \left( \frac{1}{\omega^2 - \vec{q}^2 - M_D^2 - \Pi_D(\omega, \vec{q}; T)} \right) \]

at unphysical meson masses (used in the lattice)

Ground-state: \( \rho_{gs}(\omega; T) = M_D^4 S_D(\omega; T) \)

Full: \( \rho(\omega; T) = \rho_{gs}(\omega; T) + a \rho_{cont}(\omega; T) \)

\[ \rho_{cont}(\omega; T) = \frac{3}{32\pi} \sqrt{\left( \frac{m_1^2 - m_2^2}{\omega^2} + 1 \right)^2 - 4m_2^2} \frac{\omega^2}{\omega^2} \]

\[ \times 2 \left( 1 - \frac{(m_1 - m_2)^2}{\omega^2} \right) \]

\[ \times [n(-\omega_0, T) - n(\omega - \omega_0, T)] \theta(\omega - (m_1 + m_2)) \]
EUCLIDEAN CORRELATORS WITH EFT

\[ S_D(\omega, \vec{q}; T) = -\frac{1}{\pi} \text{Im} D_D(\omega, \vec{q}; T) \]
\[ = -\frac{1}{\pi} \text{Im} \left( \frac{1}{\omega^2 - \vec{q}^2 - M_D^2 - \Pi_D(\omega, \vec{q}; T)} \right) \]

at unphysical meson masses (used in the lattice)

Ground-state: \( \rho_{gs}(\omega; T) = M_D^4 S_D(\omega; T) \)

Full: \( \rho(\omega; T) = \rho_{gs}(\omega; T) + a \rho_{\text{cont}}(\omega; T) \)

\( \rho_{\text{cont}}(\omega; T) = \frac{3}{32\pi} \sqrt{\left( \frac{m_1^2 - m_2^2}{\omega^2} + 1 \right)^2 - \frac{4m_2^2}{\omega^2}} \omega^2 \)
\[ \times 2 \left( 1 - \frac{(m_1 - m_2)^2}{\omega^2} \right) \]
\[ \times [n(-\omega_0, T) - n(\omega - \omega_0, T)] \theta(\omega - (m_1 + m_2)) \]

- \( m_1 = m_c = 1.5 \text{ GeV} \)
- \( m_2 = m_l = 0, m_2 = m_s = 100 \text{ MeV} \)
EUCLIDEAN CORRELATORS WITH EFT

\[ S_D(\omega, \vec{q}; T) = -\frac{1}{\pi} \text{Im} \ D_D(\omega, \vec{q}; T) \]

\[ = -\frac{1}{\pi} \text{Im} \left( \frac{1}{\omega^2 - \vec{q}^2 - M_D^2 - \Pi_D(\omega, \vec{q}; T)} \right) \]

at unphysical meson masses (used in the lattice)

Ground-state: \( \rho_{gs}(\omega; T) = M_D^4 S_D(\omega; T) \)

Full: \( \rho(\omega; T) = \rho_{gs}(\omega; T) + a \ \rho_{\text{cont}}(\omega; T) \)

\[ \rho_{\text{cont}}(\omega; T) = \frac{3}{32\pi} \sqrt{\left( \frac{m_1^2 - m_2^2}{\omega^2} + 1 \right)^2 - \frac{4m_2^2}{\omega^2}} \omega^2 \]

\[ \times 2 \left( 1 - \frac{(m_1 - m_2)^2}{\omega^2} \right) \]

\[ \times [n(-\omega_0, T) - n(\omega - \omega_0, T)] \theta(\omega - (m_1 + m_2)) \]
Results III: Open-charm Euclidean correlators
EFT SPECTRAL FUNCTIONS AT UNPHYSICAL MASSES

\[ m_\pi = 384 \text{ MeV} \]
\[ m_K = 546 \text{ MeV} \]
\[ m_\eta = 589 \text{ MeV} \]
\[ m_D = 1880 \text{ MeV} \]
\[ m_{D_s} = 1943 \text{ MeV} \]

[Kelly, Rothkopf, Skullerud (2018)]
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\( D \) meson

- Inclusion of the continuum improves the matching at small \( \tau \).
- Very good agreement at the lowest temperature. At larger temperatures: excited states?
- Close and above \( T_c \) the EFT breaks down.

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Conclusions and Outlook
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→ The effect of the addition of **kaons in the bath** is mild.

→ We have calculated **Euclidean correlators** from spectral functions at the unphysical masses used in the lattice. **Well below Tc there is a good agreement with LQCD results**, whereas close or above $T_c$ the discrepancy indicates the missing contribution of higher-excited states.
In the near future we aim to:

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→ Further test our results against *lattice QCD* calculations and make predictions at the physical point
Thank you!
Gràcies!
Backup slides
UNITARIZED $T$-MATRIX AT $T \neq 0$

$$T_{ij} = V_{ij} + V_{ik} G_k T_{kj}$$

- Iteration 1: Undressed mesons

$$D_i D_j = D_i D_j + D_i D_k D_k D_j$$
UNITARIZED $T$-MATRIX AT $T \neq 0$

$$T_{ij} = V_{ij} + V_{ik}G_kT_{kj}$$

- Iteration 1: Undressed mesons

- Iteration $> 1$: Dressed heavy meson
LOOP FUNCTION AT $T > 0$

$$G_{D\Phi}(E, \vec{p}; T) = \int \frac{d^3 q}{(2\pi)^3} \int d\omega \int d\omega' \frac{S_D(\omega, \vec{q}; T)S_\Phi(\omega', \vec{p} - \vec{q}; T)}{E - \omega - \omega' + i\varepsilon} \cdot [1 + f(\omega, T) + f(\omega', T)]$$
LOOP FUNCTION AT $T > 0$

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Spectral functions:

$$S_D, \quad S_\Phi \rightarrow \frac{\omega_\Phi}{\omega'} \delta(\omega'^2 - \omega^2_\Phi), \quad \omega_\Phi = \sqrt{q^2_\Phi + m^2_\Phi}$$
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Bose distribution function at $T$:

\[
f(\omega, T) = \frac{1}{e^{\omega/T} - 1}
\]
LOOP FUNCTION AT $T > 0$

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Regularized with a cutoff $|\vec{q}| < \Lambda$
SELF-ENERGY AT $T > 0$

$$\Pi_D(E, \vec{p}; T) = -\frac{1}{\pi} \int \frac{d^3 q}{(2\pi)^3} \int d\Omega \frac{E}{\omega_\Phi} \frac{f(\Omega, T) - f(\omega_\Phi, T)}{E^2 - (\omega_\Phi - \Omega)^2 + i\varepsilon} \text{Im} T_D(\Omega, \vec{p} + \vec{q}; T)$$
SPECTRAL FUNCTION

\[ S_D(\omega, \vec{q}; T) = -\frac{1}{\pi} \text{Im} \ D_D(\omega, \vec{q}; T) = -\frac{1}{\pi} \text{Im} \left( \frac{1}{\omega^2 - \vec{q}^2 - m_D^2 - \Pi_D(\omega, \vec{q}; T)} \right) \]

Iteration 1

\[ D = D + D \]

Iteration > 1

\[ D = D + D \]
SELF-CONSISTENCY

Loop function

\[ G_{D\Phi} \]
SELF-CONSISTENCY

Loop function $G_{D\Phi}$ → Unitarized amplitude $T_{D\Phi}$
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Loop function $G_{D\Phi}$ → Unitarized amplitude $T_{D\Phi}$ → Self-energy $\Pi_D$ → $D$-meson propagator $\mathcal{D}_D$

$$D = D + \Phi$$
Pionic bath

$D$ and $D_s$ with light mesons

First opening:
$E \geq (m_D - 2m_\pi)$

Second opening:
$E \leq (m_D + 2m_\pi)$
Pionic bath

$D^*$ and $D_s^*$ with light mesons

First opening:
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Second opening:
$E \leq (m_D + 2m_\pi)$
**LATTICE EUCLIDEAN CORRELATORS**

**Temperature of the system on a lattice** of size $N^3_\sigma \times N_\tau \rightarrow$ related to the temporal extent:

$$T = \frac{1}{aN_\tau}, \quad a : \text{Lattice spacing}$$

Euclidean correlators of some operators $\hat{O}$:

$$\langle \hat{O}_1(\tau, \vec{x})\hat{O}_2(0, \vec{0}) \rangle = \frac{1}{Z} \int \mathcal{D}[U]\mathcal{D}[\bar{\psi}, \psi]\hat{O}_2[U, \bar{\psi}, \psi]\hat{O}_1[U, \bar{\psi}, \psi]e^{-S_F[U, \bar{\psi}, \psi] - S_G[U]}$$
**LATTICE EUCLIDEAN CORRELATORS**

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$\mathcal{Z}$ : partition function
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$S_F[U, \bar{\psi}, \psi]$ and $S_G[U]$ : fermion and gluon parts of the discretized QCD action
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**Meson Euclidean temporal correlator:**

$$\langle J(\tau, \vec{x}) J(0, \vec{0}) \rangle = -\frac{1}{Z} \int D[U] e^{-S_G[U]} \det[M] \text{Tr} [ \Gamma_H \ M^{-1}(0, \vec{0}; \tau, \vec{x}) \ \Gamma_H \ M^{-1}(\tau, \vec{x}; 0, \vec{0}) ],$$
**LATTICE EUCLIDEAN CORRELATORS**

**Temperature of the system on a lattice** of size $N^3 \times N_T \rightarrow$ related to the temporal extent:

$$T = \frac{1}{aN_T}, \quad a : \text{Lattice spacing}$$

Euclidean correlators of some operators $\hat{O}$:

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$\Gamma_H = 1, \gamma_5, \gamma_\mu, \gamma_5 \gamma_\mu : \text{for scalar, pseudoscalar, vector and axial vector channels}$
LATTICE EUCLIDEAN CORRELATORS

**Temperature of the system on a lattice** of size $N_\sigma^3 \times N_\tau \rightarrow$ related to the temporal extent:

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$M^{-1}$ : fermion propagator