



UNIVERSITAT DE
BARCELONA



ICCUB



EXCELENCIA
MARÍA
DE MAEZTU



Thermal modification of open-charm mesons from an effective hadronic theory

Glòria Montaña

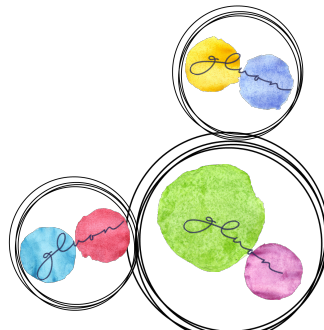
University of Barcelona
Institute of Cosmos Sciences

[GM, Angels Ramos, Laura Tolos, Juan Torres-Rincon, Phys.Lett.B 806 (2020)]

[GM, Angels Ramos, Laura Tolos, Juan Torres-Rincon, arXiv:2007.12601]

[GM, Olaf Kaczmarek, Laura Tolos, Angels Ramos, arXiv:2007.15690]

Theory Seminar - Jefferson Lab
September 14, 2020



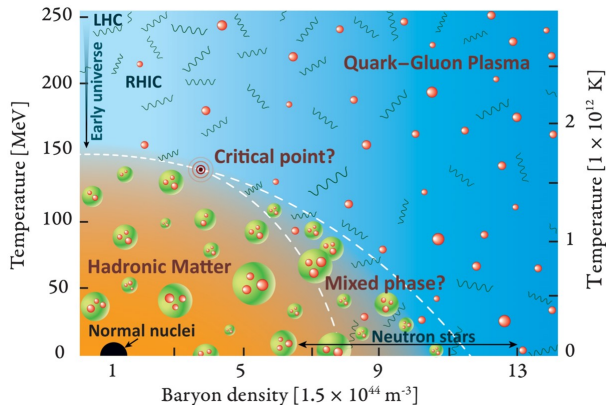
OUTLINE

1. Motivation
2. Hadronic molecular model:
Effective hadronic interaction & Unitarization
3. Results I:
Dynamically generated states at $T = 0$
4. Finite temperature corrections
5. Results II: Thermal modifications
6. Euclidean correlators: comparison with lattice QCD
7. Results III: Open-charm Euclidean correlators
8. Conclusions and Outlook

Motivation

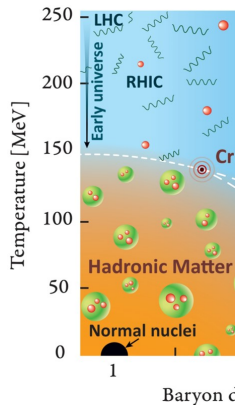
MOTIVATION (I)

- Heavy-ion-collision (HIC) programmes in on-going and upcoming experimental facilities (RHIC, LHC, FAIR) highly demand the theoretical study of **hadronic properties under extreme conditions of temperature and density**.



MOTIVATION (I)

→ Heavy-ion-collision (HIC) programmes in on-going and upcoming experimental facilities (RHIC, LHC, FAIR) highly demand the theoretical study of **hadronic properties under extreme conditions of temperature and density**.



Theoretical tools to study the matter at high temperatures:

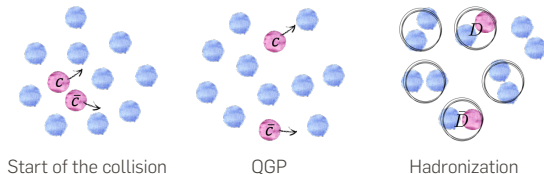
- Perturbative theories
- Lattice QCD
- Non-perturbative effective hadronic theories

MOTIVATION (I)

- Heavy-ion-collision (HIC) programmes in on-going and upcoming experimental facilities (RHIC, LHC, FAIR) highly demand the theoretical study of **hadronic properties under extreme conditions of temperature and density**.
- Due to the large mass and relaxation time of the c quark, **charmed mesons are a powerful probe of the QGP**.

MOTIVATION (I)

- Heavy-ion-collision (HIC) programmes in on-going and upcoming experimental facilities (RHIC, LHC, FAIR) highly demand the theoretical study of **hadronic properties under extreme conditions of temperature and density**.
- Due to the large mass and relaxation time of the c quark, **charmed mesons are a powerful probe of the QGP**.



Quarkonia suppression

- Color screening
- Comover scattering



Comover scattering

MOTIVATION (II)

- Properties of hadrons and their thermal modification are contained in their spectral functions.

MOTIVATION (II)

- Properties of hadrons and their thermal modification are contained in their spectral functions.
- **Spectral functions** can be directly calculated using effective hadronic theories within a unitarized approach.

MOTIVATION (II)

- Properties of hadrons and their thermal modification are contained in their spectral functions.
- **Spectral functions** can be directly calculated using effective hadronic theories within a unitarized approach.
- We focus on **finite-temperature mesonic matter** to study the high temperature ($< T_c$) and low density region of the QCD phase diagram (matter generated in HICs in RHIC and LHC).

MOTIVATION (II)

- Properties of hadrons and their thermal modification are contained in their spectral functions.
- **Spectral functions** can be directly calculated using effective hadronic theories within a unitarized approach.
- We focus on **finite-temperature mesonic matter** to study the high temperature ($< T_c$) and low density region of the QCD phase diagram (matter generated in HICs in RHIC and LHC).
- **Ground-state heavy mesons** (e.g. D , D_s , D^* , D_s^*) modify their properties in hot matter.

MOTIVATION (II)

- Properties of hadrons and their thermal modification are contained in their spectral functions.
- **Spectral functions** can be directly calculated using effective hadronic theories within a unitarized approach.
- We focus on **finite-temperature mesonic matter** to study the high temperature ($< T_c$) and low density region of the QCD phase diagram (matter generated in HICs in RHIC and LHC).
- **Ground-state heavy mesons** (e.g. D , D_s , D^* , D_s^*) modify their properties in hot matter.
- Consequences in the behavior of **excited mesonic states**, such as the non-strange $D_0^*(2300)$ and $D_1^*(2430)$ and the strange $D_{s0}^*(2317)$ and $D_{s1}^*(2460)$, dynamically generated in a heavy-light molecular model at finite temperature.

MOTIVATION (II)

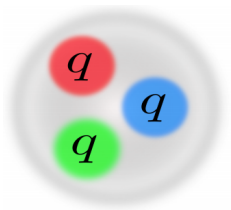
- Properties of hadrons and their thermal modification are contained in their spectral functions.
- **Spectral functions** can be directly calculated using effective hadronic theories within a unitarized approach.
- We focus on **finite-temperature mesonic matter** to study the high temperature ($< T_c$) and low density region of the QCD phase diagram (matter generated in HICs in RHIC and LHC).
- **Ground-state heavy mesons** (e.g. D , D_s , D^* , D_s^*) modify their properties in hot matter.
- Consequences in the behavior of **excited mesonic states**, such as the non-strange $D_0^*(2300)$ and $D_1^*(2430)$ and the strange $D_{s0}^*(2317)$ and $D_{s1}^*(2460)$, dynamically generated in a heavy-light molecular model at finite temperature.
- Spectral functions can be obtained at unphysical meson masses and used to calculate **Euclidean correlators** to compare with lattice QCD results.

Hadronic molecular model:
Effective hadronic interaction & Unitarization

QUARK MODEL

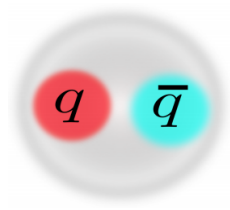
Although the basic constituents in QCD are quarks and gluons, the **conventional quark model** (Gell-Mann, 1964; Zweig, 1964) has been very successful in describing hadron structure.

Baryon



$p, n, \Lambda, \Delta, \Xi_c, \dots$

Meson



$\pi, K, D, B, J/\psi, \dots$

EXOTIC HADRONS

There are many (excited) hadrons that do not accommodate in the qqq or $q\bar{q}$ picture.

Other configurations allowed by QCD, e.g., $q\bar{q}q\bar{q}$, $qqqq\bar{q}$, etc., are called **exotic**.

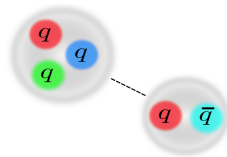
Baryonic systems

Pentaquarks $qqqq\bar{q}$

Compact pentaquark



Meson-baryon molecule



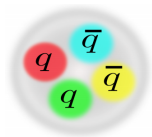
$\Lambda(1405)$, $\Lambda_c(2595)$, $\Lambda_c(2625)$, $P_c(4380)$, $P_c(4450)$, ...

EXOTIC HADRONS

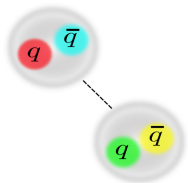
Mesonic systems

Tetraquarks $q\bar{q}q\bar{q}$

Compact
tetraquark



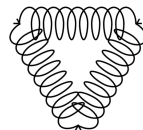
Meson-meson
molecule



Hybrid



Glueball

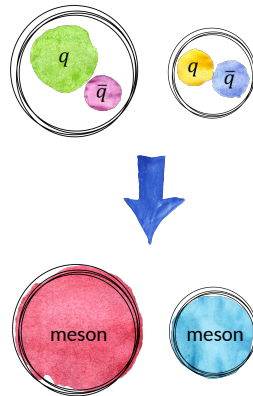


$D_0^*(2300)$, $D_s^*(2317)$, $X(3872)$, charged Z
states, ...

HADRONIC MOLECULES

Hadronic molecules are deuteron-like **quasi-bound states** of two mesons (tetraquark), a meson and a baryon (pentaquark) or two baryons (hexaquark).

- Dynamically generated via multiple scattering of their meson/baryon components.
- Located near threshold $m_1 + m_2$
- Studied using effective hadronic theories.
- Mesons and baryons are the degrees of freedom.



EFFECTIVE HADRONIC THEORY

The most general effective Lagrangian, up to a given order, **consistent with the symmetries** of the underlying theory.

EFFECTIVE HADRONIC THEORY

The most general effective Lagrangian, up to a given order, **consistent with the symmetries** of the underlying theory.

Heavy-light meson-meson interaction → Symmetries of QCD :

- **Chiral symmetry** in the limit $m_u, m_d, m_s \rightarrow 0$
- **Heavy-quark symmetry** in the limit $m_c, m_b \rightarrow \infty$

Mesons and baryons are the degrees of freedom.

Interaction mediated by the exchange of mesons.

EFFECTIVE HADRONIC THEORY

The most general effective Lagrangian, up to a given order, **consistent with the symmetries** of the underlying theory.

Heavy-light meson-meson interaction → Symmetries of QCD :

- **Chiral symmetry** in the limit $m_u, m_d, m_s \rightarrow 0$
- **Heavy-quark symmetry** in the limit $m_c, m_b \rightarrow \infty$

Mesons and baryons are the degrees of freedom.

Interaction mediated by the exchange of mesons.

Interaction of $D^{(*)}$ and $D_s^{(*)}$ with light mesons:

→ Lagrangian at NLO in the chiral expansion and LO in the heavy-quark expansion

$$\mathcal{L} = \mathcal{L}_{\text{LO}} + \mathcal{L}_{\text{NLO}}$$

INTERACTION LAGRANGIAN (LO)

Lagrangian at NLO in the chiral expansion and LO in the heavy-quark expansion

$$\mathcal{L} = \mathcal{L}_{\text{LO}} + \mathcal{L}_{\text{NLO}}$$

$$\begin{aligned} \mathcal{L}_{\text{LO}} = & \langle \nabla^\mu D \nabla_\mu D^\dagger \rangle - m_D^2 \langle D D^\dagger \rangle - \langle \nabla^\mu D^{*\nu} \nabla_\mu D_\nu^{*\dagger} \rangle + m_D^2 \langle D^{*\nu} D_\nu^{*\dagger} \rangle \\ & + i g \langle D^{*\mu} u_\mu D^\dagger - D u^\mu D_\mu^{*\dagger} \rangle + \frac{g}{2m_D} \langle D_\mu^* u_\alpha \nabla_\beta D_\nu^{*\dagger} - \nabla_\beta D_\mu^* u_\alpha D_\nu^{*\dagger} \rangle \epsilon^{\mu\nu\alpha\beta} \end{aligned}$$

$$\begin{aligned} \nabla_\mu D^{(*)} &= \partial_\mu D^{(*)} - D^{(*)} \Gamma_\mu & \Gamma_\mu &= \frac{1}{2} (u^\dagger \partial_\mu u + u \partial_\mu u^\dagger) \\ u_\mu &= i (u^\dagger \partial_\mu u - u \partial_\mu u^\dagger) & u &= \sqrt{U} = \exp \left(\frac{i \Phi}{\sqrt{2} f_\pi} \right) \end{aligned}$$

[Kolomeitsev and Lutz (2004)]

[Lutz and Soyeur (2008)]

[Guo, Hanhart and Meißner (2009)]

[Geng, Kaiser, Martin-Camalich and Weise (2010)]

INTERACTION LAGRANGIAN (LO)

Lagrangian at NLO in the chiral expansion and LO in the heavy-quark expansion

$$\mathcal{L} = \mathcal{L}_{\text{LO}} + \mathcal{L}_{\text{NLO}}$$

$$\begin{aligned} \mathcal{L}_{\text{LO}} = & \langle \nabla^\mu D \nabla_\mu D^\dagger \rangle - m_D^2 \langle D D^\dagger \rangle - \langle \nabla^\mu D^{*\nu} \nabla_\mu D_\nu^{*\dagger} \rangle + m_D^2 \langle D^{*\nu} D_\nu^{*\dagger} \rangle \\ & + i g \langle D^{*\mu} u_\mu D^\dagger - D u^\mu D_\mu^{*\dagger} \rangle + \frac{g}{2m_D} \langle D_\mu^* u_\alpha \nabla_\beta D_\nu^{*\dagger} - \nabla_\beta D_\mu^* u_\alpha D_\nu^{*\dagger} \rangle \epsilon^{\mu\nu\alpha\beta} \end{aligned}$$

$$\begin{aligned} \nabla_\mu D^{(*)} &= \partial_\mu D^{(*)} - D^{(*)} \Gamma_\mu & \Gamma_\mu &= \frac{1}{2} (u^\dagger \partial_\mu u + u \partial_\mu u^\dagger) \\ u_\mu &= i (u^\dagger \partial_\mu u - u \partial_\mu u^\dagger) & u &= \sqrt{U} = \exp \left(\frac{i \Phi}{\sqrt{2} f_\pi} \right) \end{aligned}$$

[Kolomeitsev and Lutz (2004)]

[Lutz and Soyeur (2008)]

[Guo, Hanhart and Meißner (2009)]

[Geng, Kaiser, Martin-Camalich and Weise (2010)]

INTERACTION LAGRANGIAN (LO)

Lagrangian at NLO in the chiral expansion and LO in the heavy-quark expansion

$$\mathcal{L} = \mathcal{L}_{\text{LO}} + \mathcal{L}_{\text{NLO}}$$

$$\begin{aligned} \mathcal{L}_{\text{LO}} = & \langle \nabla^\mu D \nabla_\mu D^\dagger \rangle - m_D^2 \langle D D^\dagger \rangle - \langle \nabla^\mu D^{*\nu} \nabla_\mu D_\nu^{*\dagger} \rangle + m_D^2 \langle D^{*\nu} D_\nu^{*\dagger} \rangle \\ & + i g \langle D^{*\mu} u_\mu D^\dagger - D u^\mu D_\mu^{*\dagger} \rangle + \frac{g}{2m_D} \langle D_\mu^* u_\alpha \nabla_\beta D_\nu^{*\dagger} - \nabla_\beta D_\mu^* u_\alpha D_\nu^{*\dagger} \rangle \epsilon^{\mu\nu\alpha\beta} \end{aligned}$$

$$\begin{aligned} \nabla_\mu D^{(*)} &= \partial_\mu D^{(*)} - D^{(*)} \Gamma_\mu & \Gamma_\mu &= \frac{1}{2} (u^\dagger \partial_\mu u + u \partial_\mu u^\dagger) \\ u_\mu &= i (u^\dagger \partial_\mu u - u \partial_\mu u^\dagger) & u &= \sqrt{U} = \exp \left(\frac{i \Phi}{\sqrt{2} f_\pi} \right) \end{aligned}$$

[Kolomeitsev and Lutz (2004)]

[Lutz and Soyeur (2008)]

[Guo, Hanhart and Meißner (2009)]

[Geng, Kaiser, Martin-Camalich and Weise (2010)]

INTERACTION LAGRANGIAN (LO)

Lagrangian at NLO in the chiral expansion and LO in the heavy-quark expansion

$$\mathcal{L} = \mathcal{L}_{\text{LO}} + \mathcal{L}_{\text{NLO}}$$

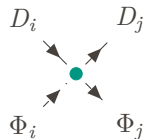
$$\begin{aligned} \mathcal{L}_{\text{LO}} = & \langle \nabla^\mu D \nabla_\mu D^\dagger \rangle - m_D^2 \langle D D^\dagger \rangle - \langle \nabla^\mu D^{*\nu} \nabla_\mu D_\nu^{*\dagger} \rangle + m_D^2 \langle D^{*\nu} D_\nu^{*\dagger} \rangle \\ & + i g \langle D^{*\mu} u_\mu D^\dagger - D u^\mu D_\mu^{*\dagger} \rangle + \frac{g}{2m_D} \langle D_\mu^* u_\alpha \nabla_\beta D_\nu^{*\dagger} - \nabla_\beta D_\mu^* u_\alpha D_\nu^{*\dagger} \rangle \epsilon^{\mu\nu\alpha\beta} \end{aligned}$$

D mesons:

$$D = (D^0 \quad D^+ \quad D_s^+), \quad D_\mu^* = (D^{*0} \quad D^{*+} \quad D_s^{*+})_\mu$$

Light mesons:

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}$$



INTERACTION LAGRANGIAN (NLO)

Lagrangian at NLO in the chiral expansion and LO in the heavy-quark expansion

$$\mathcal{L} = \mathcal{L}_{\text{LO}} + \mathcal{L}_{\text{NLO}}$$

$$\begin{aligned} \mathcal{L}_{\text{NLO}} = & - h_0 \langle DD^\dagger \rangle \langle \chi_+ \rangle + h_1 \langle D\chi_+ D^\dagger \rangle + h_2 \langle DD^\dagger \rangle \langle u^\mu u_\mu \rangle \\ & + h_3 \langle Du^\mu u_\mu D^\dagger \rangle + h_4 \langle \nabla_\mu D \nabla_\nu D^\dagger \rangle \langle u^\mu u^\nu \rangle + h_5 \langle \nabla_\mu D \{u^\mu, u^\nu\} \nabla_\nu D^\dagger \rangle \\ & + \tilde{h}_0 \langle D^{*\mu} D_\mu^{*\dagger} \rangle \langle \chi_+ \rangle - \tilde{h}_1 \langle D^{*\mu} \chi_+ D_\mu^{*\dagger} \rangle - \tilde{h}_2 \langle D^{*\mu} D_\mu^{*\dagger} \rangle \langle u^\nu u_\nu \rangle \\ & - \tilde{h}_3 \langle D^{*\mu} u^\nu u_\nu D_\mu^{*\dagger} \rangle - \tilde{h}_4 \langle \nabla_\mu D^{*\alpha} \nabla_\nu D_\alpha^{*\dagger} \rangle \langle u^\mu u^\nu \rangle - \tilde{h}_5 \langle \nabla_\mu D^{*\alpha} \{u^\mu, u^\nu\} \nabla_\nu D_\alpha^{*\dagger} \rangle \end{aligned}$$

$$\chi_+ = u^\dagger \chi u^\dagger + u \chi u \quad \chi = \text{diag}(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2) \quad \text{LECs : } h_{0,\dots,5}, \tilde{h}_{0,\dots,5}$$

[Liu, Orginos, Guo, Hanhart and Meißner (2013)]

[Tolos and Torres-Rincon (2013)]

[Albaladejo, Fernandez-Soler, Guo and Nieves (2017)]

[Guo, Liu, Meißner, Oller and Rusetsky (2019)]

INTERACTION LAGRANGIAN (NLO)

Lagrangian at NLO in the chiral expansion and LO in the heavy-quark expansion

$$\mathcal{L} = \mathcal{L}_{\text{LO}} + \mathcal{L}_{\text{NLO}}$$

$$\begin{aligned} \mathcal{L}_{\text{NLO}} = & -h_0 \langle DD^\dagger \rangle \langle \chi_+ \rangle + h_1 \langle D\chi_+ D^\dagger \rangle + h_2 \langle DD^\dagger \rangle \langle u^\mu u_\mu \rangle \\ & + h_3 \langle Du^\mu u_\mu D^\dagger \rangle + h_4 \langle \nabla_\mu D \nabla_\nu D^\dagger \rangle \langle u^\mu u^\nu \rangle + h_5 \langle \nabla_\mu D \{u^\mu, u^\nu\} \nabla_\nu D^\dagger \rangle \\ & + \tilde{h}_0 \langle D^{*\mu} D_\mu^{*\dagger} \rangle \langle \chi_+ \rangle - \tilde{h}_1 \langle D^{*\mu} \chi_+ D_\mu^{*\dagger} \rangle - \tilde{h}_2 \langle D^{*\mu} D_\mu^{*\dagger} \rangle \langle u^\nu u_\nu \rangle \\ & - \tilde{h}_3 \langle D^{*\mu} u^\nu u_\nu D_\mu^{*\dagger} \rangle - \tilde{h}_4 \langle \nabla_\mu D^{*\alpha} \nabla_\nu D_\alpha^{*\dagger} \rangle \langle u^\mu u^\nu \rangle - \tilde{h}_5 \langle \nabla_\mu D^{*\alpha} \{u^\mu, u^\nu\} \nabla_\nu D_\alpha^{*\dagger} \rangle \end{aligned}$$

$$\chi_+ = u^\dagger \chi u^\dagger + u \chi u \quad \chi = \text{diag}(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2) \quad \text{LECs : } h_{0,\dots,5}, \tilde{h}_{0,\dots,5}$$

[Liu, Orginos, Guo, Hanhart and Meißner (2013)]

[Tolos and Torres-Rincon (2013)]

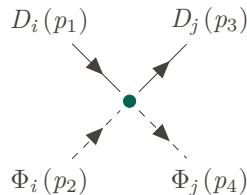
[Albaladejo, Fernandez-Soler, Guo and Nieves (2017)]

[Guo, Liu, Meißner, Oller and Rusetsky (2019)]

SCATTERING AMPLITUDE

$$\mathcal{L} = \mathcal{L}_{\text{LO}} + \mathcal{L}_{\text{NLO}}$$

Tree-level scattering amplitude of $D^{(*)}$, $D_s^{(*)}$ mesons with π , K , \bar{K} , η mesons:

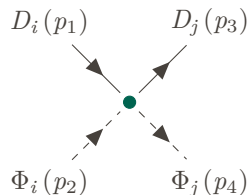


$$\begin{aligned} V^{ij}(s, t, u) = & \frac{1}{f_\pi^2} \left[\frac{1}{4} C_{\text{LO}}^{ij} (s - u) - 4 C_0^{ij} h_0 + 2 C_1^{ij} h_1 \right. \\ & - 2 C_{24}^{ij} \left(2 h_2 (p_2 \cdot p_4) + h_4 ((p_1 \cdot p_2)(p_3 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3)) \right) \\ & \left. + 2 C_{35}^{ij} \left(h_3 (p_2 \cdot p_4) + h_5 ((p_1 \cdot p_2)(p_3 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3)) \right) \right] \end{aligned}$$

SCATTERING AMPLITUDE

$$\mathcal{L} = \mathcal{L}_{\text{LO}} + \mathcal{L}_{\text{NLO}}$$

Tree-level scattering amplitude of $D^{(*)}$, $D_s^{(*)}$ mesons with π , K , \bar{K} , η mesons:



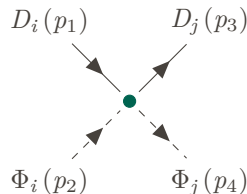
$$\begin{aligned} V^{ij}(s, t, u) = & \frac{1}{f_\pi^2} \left[\frac{1}{4} C_{\text{LO}}^{ij} (s - u) - 4 C_0^{ij} h_0 + 2 C_1^{ij} h_1 \right. \\ & - 2 C_{24}^{ij} \left(2 h_2 (p_2 \cdot p_4) + h_4 ((p_1 \cdot p_2)(p_3 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3)) \right) \\ & \left. + 2 C_{35}^{ij} \left(h_3 (p_2 \cdot p_4) + h_5 ((p_1 \cdot p_2)(p_3 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3)) \right) \right] \end{aligned}$$

$C_{\text{LO},0,1,24,35}^{ij}$: isospin coefficients for the transition $i \rightarrow j$

SCATTERING AMPLITUDE

$$\mathcal{L} = \mathcal{L}_{\text{LO}} + \mathcal{L}_{\text{NLO}}$$

Tree-level scattering amplitude of $D^{(*)}$, $D_s^{(*)}$ mesons with π , K , \bar{K} , η mesons:



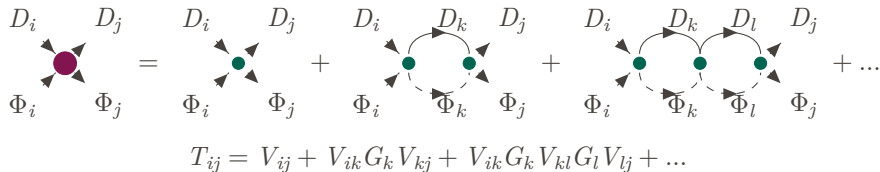
$$\begin{aligned} V^{ij}(s, t, u) = & \frac{1}{f_\pi^2} \left[\frac{1}{4} C_{\text{LO}}^{ij} (s - u) - 4 C_0^{ij} h_0 + 2 C_1^{ij} h_1 \right. \\ & - 2 C_{24}^{ij} \left(2 h_2 (p_2 \cdot p_4) + h_4 ((p_1 \cdot p_2)(p_3 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3)) \right) \\ & \left. + 2 C_{35}^{ij} \left(h_3 (p_2 \cdot p_4) + h_5 ((p_1 \cdot p_2)(p_3 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3)) \right) \right] \end{aligned}$$

LECs fitted to LQCD data of scattering lengths

[Guo, Liu, Meißner, Oller Rusetsky (2019)]

UNITARIZATION

Bethe-Salpeter equation in coupled-channels:



$$T_{ij} = V_{ij} + V_{ik} G_k V_{kj} + V_{ik} G_k V_{kl} G_l V_{lj} + \dots$$

UNITARIZATION

Bethe-Salpeter equation in coupled-channels:

$$D_i \quad D_j \\ \Phi_i \quad \Phi_j = D_i \quad D_j \\ \Phi_i \quad \Phi_j + D_i \quad D_k \quad D_j \\ \Phi_i \quad \Phi_k \quad \Phi_j + D_i \quad D_k \quad D_l \quad D_j \\ \Phi_i \quad \Phi_k \quad \Phi_l \quad \Phi_j + \dots$$

$$T_{ij} = V_{ij} + V_{ik} G_k V_{kj} + V_{ik} G_k V_{kl} G_l V_{lj} + \dots$$

$$D_i \quad D_j \\ \Phi_i \quad \Phi_j = D_i \quad D_j \\ \Phi_i \quad \Phi_j + D_i \quad D_k \quad D_j \\ \Phi_i \quad \Phi_k \quad \Phi_j$$

$$T_{ij} = V_{ij} + V_{ik} G_k T_{kj} \quad \rightarrow \quad T = (1 - VG)^{-1} V \quad (\text{On-shell factorization})$$

LOOP FUNCTION

The two-meson propagator

$$G_k = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 - m_{D,k}^2 + i\varepsilon} \frac{1}{(P - q)^2 - m_{\Phi,k}^2 + i\varepsilon}$$

has to be regularized:

- Dimensional regularization: subtraction constants $a_k(\mu)$
- **Cutoff regularization** with $|\vec{q}| < \Lambda$

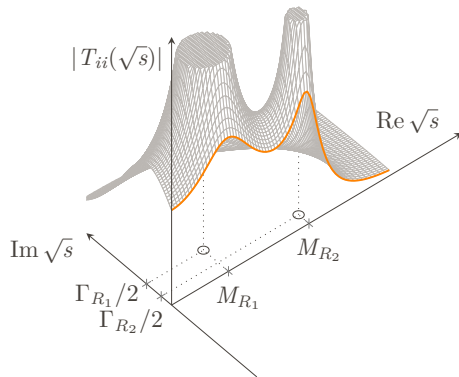
At threshold: $\sqrt{s_{\text{thr}}} = m_1 + m_2$

$$G_{\text{DR}}(\sqrt{s_{\text{thr}}}, a_l(\mu)) = G_{\Lambda}(\sqrt{s_{\text{thr}}}, \Lambda)$$

DYNAMICALLY GENERATED STATES

- Identification of **states** in the unitarized scattering amplitudes
- **Analytical continuation** and **poles** in the complex-energy plane
- Bound states, resonances and virtual states in different **Riemann sheets**

- Mass $M_R = \text{Re } \sqrt{s_R}$
- Half-width $\frac{\Gamma_R}{2} = \text{Im } \sqrt{s_R}$
- Coupling constants $|g_i|$
- Compositeness $X_i = \left| g_i^2 \frac{\partial G_i(z_p)}{\partial z} \right|$



Results I:
Dynamically generated states at $T = 0$

COUPLED CHANNELS

In isospin basis:

| (S, I) | Channels $J^P = 0^- \oplus 0^-$ | Threshold (MeV) |
|--------------------|------------------------------------|-----------------|
| $(-1, 0)$ | $D\bar{K}$ | 2364.88 |
| $(-1, 1)$ | $D\bar{K}$ | 2364.88 |
| $(0, \frac{1}{2})$ | $D\pi$ | 2005.28 |
| | $D\eta$ | 2415.10 |
| | $D_s\bar{K}$ | 2463.98 |
| $(0, \frac{3}{2})$ | $D\pi$ | 2005.28 |
| $(1, 0)$ | DK | 2364.88 |
| | $D_s\eta$ | 2516.20 |
| $(1, 1)$ | $D_s\pi$ | 2106.38 |
| | DK | 2364.88 |
| $(2, \frac{1}{2})$ | D_sK | 2463.98 |

| (S, I) | Channels $J^P = 1^- \oplus 0^-$ | Threshold (MeV) |
|--------------------|------------------------------------|-----------------|
| $(-1, 0)$ | $D^*\bar{K}$ | 2504.20 |
| $(-1, 1)$ | $D^*\bar{K}$ | 2504.20 |
| $(0, \frac{1}{2})$ | $D^*\pi$ | 2146.59 |
| | $D^*\eta$ | 2556.42 |
| | $D_s^*\bar{K}$ | 2607.84 |
| $(0, \frac{3}{2})$ | $D^*\pi$ | 2146.59 |
| $(1, 0)$ | D^*K | 2504.20 |
| | $D_s^*\eta$ | 2660.06 |
| $(1, 1)$ | $D_s^*\pi$ | 2250.24 |
| | D^*K | 2504.20 |
| $(2, \frac{1}{2})$ | D_s^*K | 2607.84 |

COUPLED CHANNELS

In isospin basis:

| (S, I) | Channels $J^P = 0^- \oplus 0^-$ | Threshold (MeV) |
|--------------------|------------------------------------|-----------------|
| $(-1, 0)$ | $D\bar{K}$ | 2364.88 |
| $(-1, 1)$ | $D\bar{K}$ | 2364.88 |
| $(0, \frac{1}{2})$ | $D\pi$ | 2005.28 |
| | $D\eta$ | 2415.10 |
| | $D_s\bar{K}$ | 2463.98 |
| $(0, \frac{3}{2})$ | $D\pi$ | 2005.28 |
| $(1, 0)$ | DK | 2364.88 |
| | $D_s\eta$ | 2516.20 |
| $(1, 1)$ | $D_s\pi$ | 2106.38 |
| | DK | 2364.88 |
| $(2, \frac{1}{2})$ | D_sK | 2463.98 |

| (S, I) | Channels $J^P = 1^- \oplus 0^-$ | Threshold (MeV) |
|--------------------|------------------------------------|-----------------|
| $(-1, 0)$ | $D^*\bar{K}$ | 2504.20 |
| $(-1, 1)$ | $D^*\bar{K}$ | 2504.20 |
| $(0, \frac{1}{2})$ | $D^*\pi$ | 2146.59 |
| | $D^*\eta$ | 2556.42 |
| | $D_s^*\bar{K}$ | 2607.84 |
| | $D^*\pi$ | 2146.59 |
| $(0, \frac{3}{2})$ | $D^*\pi$ | 2146.59 |
| $(1, 0)$ | D^*K | 2504.20 |
| | $D_s^*\eta$ | 2660.06 |
| | $D_s^*\pi$ | 2250.24 |
| $(1, 1)$ | D^*K | 2504.20 |
| $(2, \frac{1}{2})$ | D_s^*K | 2607.84 |

COUPLED CHANNELS

In isospin basis:

| (S, I) | Channels $J^P = 0^- \oplus 0^-$ | Threshold (MeV) |
|--------------------|------------------------------------|-----------------|
| $(-1, 0)$ | $D\bar{K}$ | 2364.88 |
| $(-1, 1)$ | $D\bar{K}$ | 2364.88 |
| $(0, \frac{1}{2})$ | $D\pi$ | 2005.28 |
| | $D\eta$ | 2415.10 |
| | $D_s\bar{K}$ | 2463.98 |
| $(0, \frac{3}{2})$ | $D\pi$ | 2005.28 |
| $(1, 0)$ | DK | 2364.88 |
| | $D_s\eta$ | 2516.20 |
| $(1, 1)$ | $D_s\pi$ | 2106.38 |
| | DK | 2364.88 |
| $(2, \frac{1}{2})$ | D_sK | 2463.98 |

| (S, I) | Channels $J^P = 1^- \oplus 0^-$ | Threshold (MeV) |
|--------------------|------------------------------------|-----------------|
| $(-1, 0)$ | $D^*\bar{K}$ | 2504.20 |
| $(-1, 1)$ | $D^*\bar{K}$ | 2504.20 |
| $(0, \frac{1}{2})$ | $D^*\pi$ | 2146.59 |
| | $D^*\eta$ | 2556.42 |
| | $D_s^*\bar{K}$ | 2607.84 |
| $(0, \frac{3}{2})$ | $D^*\pi$ | 2146.59 |
| $(1, 0)$ | D^*K | 2504.20 |
| | $D_s^*\eta$ | 2660.06 |
| $(1, 1)$ | $D_s^*\pi$ | 2250.24 |
| | D^*K | 2504.20 |
| $(2, \frac{1}{2})$ | D_s^*K | 2607.84 |

DYNAMICALLY GENERATED STATES

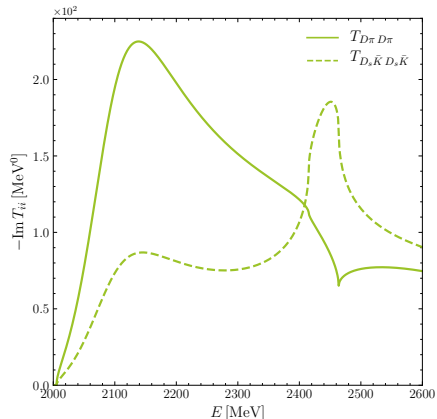
Scalars ($J^P = 0^+$): $D_0^*(2300)$ and $D_{s0}^*(2317)$

| (S, I) | RS | M_R (MeV) | $\Gamma_R/2$ (MeV) | $ g_i $ (GeV) | χ_i |
|--------------------|-------------|----------------|-----------------------|---------------------------|----------------------------|
| $(0, \frac{1}{2})$ | $(-, +, +)$ | 2081.9 | 86.0 | $ g_{D\pi} = 8.9$ | $\chi_{D\pi} = 0.40$ |
| | | | | $ g_{D\eta} = 0.4$ | $\chi_{D\eta} = 0.00$ |
| | $(-, -, +)$ | 2529.3 | 145.4 | $ g_{D_s\bar{K}} = 5.4$ | $\chi_{D_s\bar{K}} = 0.05$ |
| | | | | $ g_{D\pi} = 6.7$ | $\chi_{D\pi} = 0.10$ |
| $(1, 0)$ | $(+, +)$ | 2252.5 | 0.0 | $ g_{D\eta} = 9.9$ | $\chi_{D\eta} = 0.40$ |
| | | | | $ g_{D_s\bar{K}} = 19.4$ | $\chi_{D_s\bar{K}} = 1.63$ |
| | $(+, +)$ | 2252.5 | 0.0 | $ g_{DK} = 13.3$ | $\chi_{DK} = 0.66$ |
| | | | | $ g_{D_s\eta} = 9.2$ | $\chi_{D_s\eta} = 0.17$ |

Experimental values:

$$M_R = 2300 \pm 19 \text{ MeV}, \quad \Gamma_R = 274 \pm 40 \text{ MeV}$$

$$M_R = 2317.8 \pm 0.5 \text{ MeV}, \quad \Gamma_R < 3.8 \text{ MeV}$$



DYNAMICALLY GENERATED STATES

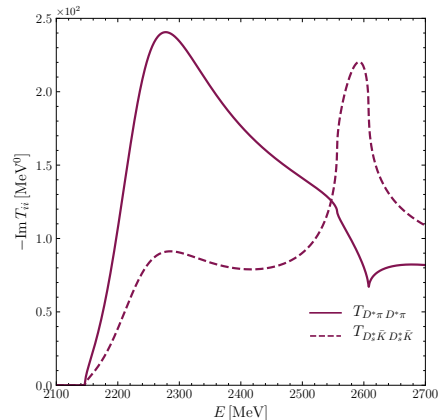
Axial vectors ($J^P = 1^+$): $D_1^*(2430)$ and $D_{s1}^*(2460)$

| (S, I) | RS | M_R (MeV) | $\Gamma_R/2$ (MeV) | $ g_i $ (GeV) | χ_i |
|--------------------|-------------|----------------|-----------------------|-----------------------------|------------------------------|
| $(0, \frac{1}{2})$ | $(-, +, +)$ | 2222.3 | 84.7 | $ g_{D^*\pi} = 9.5$ | $\chi_{D^*\pi} = 0.40$ |
| | | | | $ g_{D^*\eta} = 0.4$ | $\chi_{D^*\eta} = 0.00$ |
| | | | | $ g_{D_s^*\bar{K}} = 5.7$ | $\chi_{D_s^*\bar{K}} = 0.05$ |
| | $(-, -, +)$ | 2654.6 | 117.3 | $ g_{D^*\pi} = 6.5$ | $\chi_{D^*\pi} = 0.09$ |
| | | | | $ g_{D^*\eta} = 10.0$ | $\chi_{D^*\eta} = 0.40$ |
| | | | | $ g_{D_s^*\bar{K}} = 18.5$ | $\chi_{D_s^*\bar{K}} = 1.47$ |
| $(1, 0)$ | $(+, +)$ | 2393.3 | 0.0 | $ g_{D^*K} = 14.2$ | $\chi_{D^*K} = 0.68$ |
| | | | | $ g_{D_s^*\eta} = 9.7$ | $\chi_{D_s^*\eta} = 0.17$ |

Experimental values:

$$M_R = 2427 \pm 40 \text{ MeV}, \quad \Gamma_R = 384_{-110}^{+130} \text{ MeV}$$

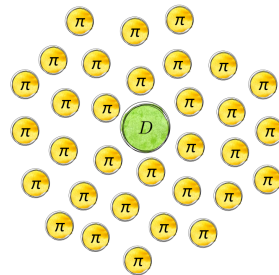
$$M_R = 2459.5 \pm 0.6 \text{ MeV}, \quad \Gamma_R < 3.5 \text{ MeV}$$



Finite temperature corrections

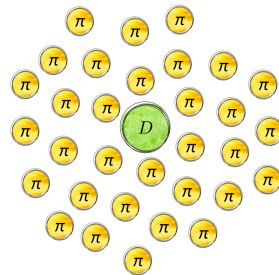
MESONIC BATH

- Mesonic matter at a temperature $0 < T < T_c$
- Vanishing baryon density
- Heavy mesons behave as Brownian particles
- Modification of the properties of the D mesons



MESONIC BATH

- Mesonic matter at a temperature $0 < T < T_c$
- Vanishing baryon density
- Heavy mesons behave as Brownian particles
- Modification of the properties of the D mesons



Both **production** and **absorption** processes of heavy-light pairs are possible

MODIFICATION OF HEAVY MESONS IN A HOT MEDIUM

Imaginary time formalism

- Sum over Matsubara frequencies

$$q^0 \rightarrow \omega_n = \frac{i}{\beta} 2\pi n, \quad \int \frac{d^4 q}{(2\pi^4)} \rightarrow \frac{i}{\beta} \sum_n \int \frac{d^3 q}{(2\pi)^3} \quad (\text{bosons})$$

MODIFICATION OF HEAVY MESONS IN A HOT MEDIUM

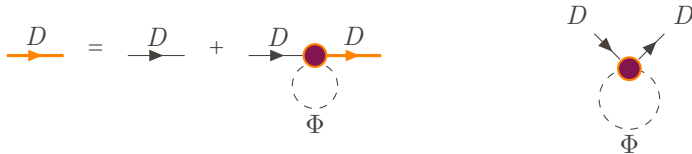
Imaginary time formalism

- Sum over Matsubara frequencies

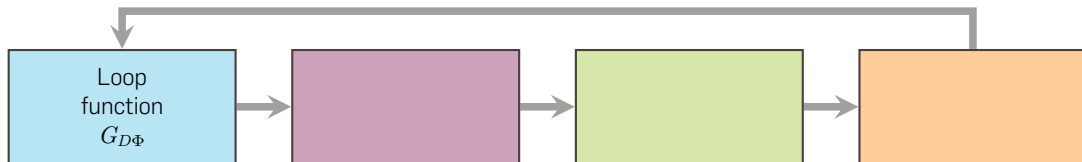
$$q^0 \rightarrow \omega_n = \frac{i}{\beta} 2\pi n, \quad \int \frac{d^4 q}{(2\pi^4)} \rightarrow \frac{i}{\beta} \sum_n \int \frac{d^3 q}{(2\pi)^3} \quad (\text{bosons})$$

Dressing the mesons in the loop function

- Self-energy corrections
- Pion mass slightly varies below $T_c \rightarrow$ only heavy meson is dressed

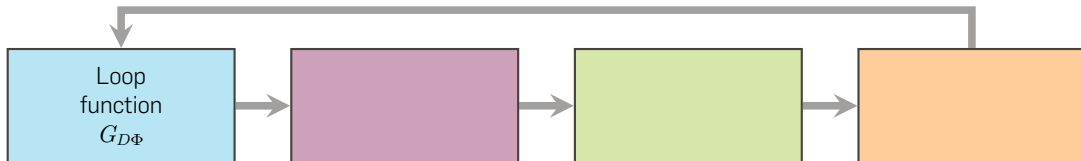


SELF-CONSISTENCY



$$G_{D\Phi}(E, \vec{p}; T) = \int \frac{d^3 q}{(2\pi)^3} \int d\omega \int d\omega' \frac{S_D(\omega, \vec{q}; T) S_\Phi(\omega', \vec{p} - \vec{q}; T)}{E - \omega - \omega' + i\varepsilon} [1 + f(\omega, T) + f(\omega', T)]$$

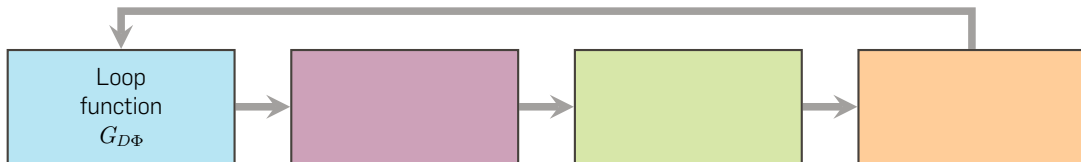
SELF-CONSISTENCY



$$G_{D\Phi}(E, \vec{p}; T) = \int \frac{d^3 q}{(2\pi)^3} \int d\omega \int d\omega' \frac{S_D(\omega, \vec{q}; T) S_{\Phi}(\omega', \vec{p} - \vec{q}; T)}{E - \omega - \omega' + i\varepsilon} [1 + f(\omega, T) + f(\omega', T)]$$

Spectral functions: $S_D, S_{\Phi} \rightarrow \frac{\omega_{\Phi}}{\omega'} \delta(\omega'^2 - \omega_{\Phi}^2), \quad \omega_{\Phi} = \sqrt{q_{\Phi}^2 + m_{\Phi}^2}$

SELF-CONSISTENCY

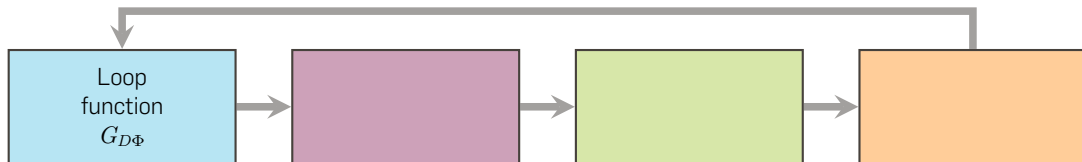


$$G_{D\Phi}(E, \vec{p}; T) = \int \frac{d^3 q}{(2\pi)^3} \int d\omega \int d\omega' \frac{S_D(\omega, \vec{q}; T) S_{\Phi}(\omega', \vec{p} - \vec{q}; T)}{E - \omega - \omega' + i\varepsilon} [1 + f(\omega, T) + f(\omega', T)]$$

Spectral functions: $S_D, S_{\Phi} \rightarrow \frac{\omega_{\Phi}}{\omega'} \delta(\omega'^2 - \omega_{\Phi}^2), \quad \omega_{\Phi} = \sqrt{q_{\Phi}^2 + m_{\Phi}^2}$

Bose distribution function at T: $f(\omega, T) = \frac{1}{e^{\omega/T} - 1}$

SELF-CONSISTENCY



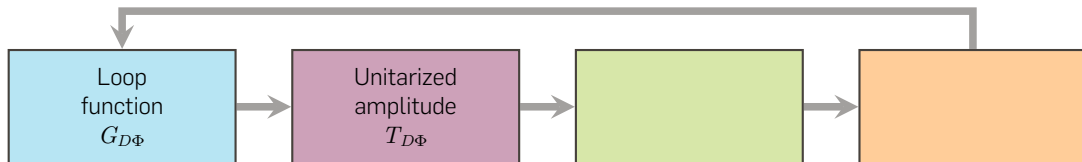
$$G_{D\Phi}(E, \vec{p}; T) = \int \frac{d^3 q}{(2\pi)^3} \int d\omega \int d\omega' \frac{S_D(\omega, \vec{q}; T) S_{\Phi}(\omega', \vec{p} - \vec{q}; T)}{E - \omega - \omega' + i\epsilon} [1 + f(\omega, T) + f(\omega', T)]$$

Spectral functions: $S_D, S_{\Phi} \rightarrow \frac{\omega_{\Phi}}{\omega'} \delta(\omega'^2 - \omega_{\Phi}^2), \quad \omega_{\Phi} = \sqrt{q_{\Phi}^2 + m_{\Phi}^2}$

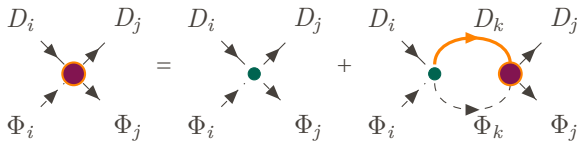
Bose distribution function at T: $f(\omega, T) = \frac{1}{e^{\omega/T} - 1}$

Regularized with a cutoff $|\vec{q}| < \Lambda$

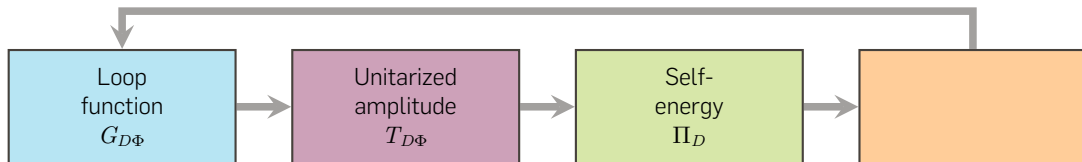
SELF-CONSISTENCY



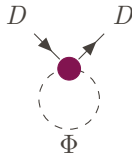
$$T_{ij} = V_{ij} + V_{ik} G_k T_{kj}$$



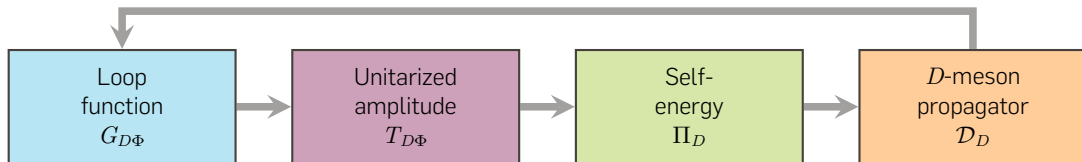
SELF-CONSISTENCY



$$\Pi_D(E, \vec{p}; T) = -\frac{1}{\pi} \int \frac{d^3 q}{(2\pi)^3} \int d\Omega \frac{E}{\omega_\Phi} \frac{f(\Omega, T) - f(\omega_\Phi, T)}{E^2 - (\omega_\Phi - \Omega)^2 + i\varepsilon} \text{Im } T_{D\Phi}(\Omega, \vec{p} + \vec{q}; T)$$



SELF-CONSISTENCY

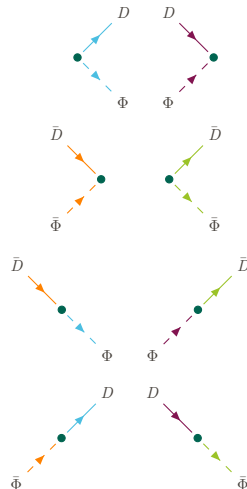


$$S_D(\omega, \vec{q}; T) = -\frac{1}{\pi} \text{Im} \left[\mathcal{D}_D(\omega, \vec{q}; T) \right] = -\frac{1}{\pi} \text{Im} \left(\frac{1}{\omega^2 - \vec{q}^2 - m_D^2 - \Pi_D(\omega, \vec{q}; T)} \right)$$

The diagram shows the Dyson equation for the D -meson propagator. On the left, an orange arrow labeled D represents the full propagator. This is equal to the sum of two terms on the right. The first term is a black arrow labeled D , representing the bare propagator. The second term is a black arrow labeled D entering a purple vertex, from which a dashed loop labeled Φ emerges, and then an orange arrow labeled D exits the vertex, representing the self-energy correction.

PHYSICAL INTERPRETATION OF THE THERMAL BATH

$$\begin{aligned}
 G_{D\Phi}(E, \vec{p}; T) \sim & \left\{ \frac{\text{bath} \rightarrow \text{bath} + D\Phi \quad \text{bath} + D\Phi \rightarrow \text{bath}}{[1 + f(\omega_D, T)][1 + f(\omega_\Phi, T)] - f(\omega_D, T)f(\omega_\Phi, T)} \right. \\
 & \left. \frac{E - \omega_D - \omega_\Phi + i\varepsilon}{+ \frac{\text{bath} + \bar{D}\bar{\Phi} \rightarrow \text{bath} \quad \text{bath} \rightarrow \text{bath} + \bar{D}\bar{\Phi}}{f(\omega_D, T)f(\omega_\Phi, T) - [1 + f(\omega_D, T)][1 + f(\omega_\Phi, T)]}} \right. \\
 & \left. + \frac{\text{bath} + \bar{D} \rightarrow \text{bath} + \Phi \quad \text{bath} + \Phi \rightarrow \text{bath} + \bar{D}}{f(\omega_D, T)[1 + f(\omega_\Phi, T)] - f(\omega_\Phi, T)[1 + f(\omega_D, T)]} \right. \\
 & \left. + \frac{\text{bath} + \bar{\Phi} \rightarrow \text{bath} + D \quad \text{bath} + D \rightarrow \text{bath} + \bar{\Phi}}{f(\omega_\Phi, T)[1 + f(\omega_D, T)] - f(\omega_D, T)[1 + f(\omega_\Phi, T)]} \right\} \\
 & \frac{E - \omega_D + \omega_\Phi + i\varepsilon}{}
 \end{aligned}$$



At zero temperature $f(\omega, T = 0) = 0$

PHYSICAL INTERPRETATION OF THE THERMAL BATH

$$\begin{aligned}
 G_{D\Phi}(E, \vec{p}; T) \sim & \left\{ \frac{[1 + f(\omega_D, T)][1 + f(\omega_\Phi, T)] - f(\omega_D, T)f(\omega_\Phi, T)}{E - \omega_D - \omega_\Phi + i\varepsilon} \right. \\
 & + \frac{f(\omega_D, T)f(\omega_\Phi, T) - [1 + f(\omega_D, T)][1 + f(\omega_\Phi, T)]}{E + \omega_D + \omega_\Phi + i\varepsilon} \\
 & + \frac{f(\omega_D, T)[1 + f(\omega_\Phi, T)] - f(\omega_\Phi, T)[1 + f(\omega_D, T)]}{E + \omega_D - \omega_\Phi + i\varepsilon} \\
 & \left. + \frac{f(\omega_\Phi, T)[1 + f(\omega_D, T)] - f(\omega_D, T)[1 + f(\omega_\Phi, T)]}{E - \omega_D + \omega_\Phi + i\varepsilon} \right\}
 \end{aligned}$$

First branch cut
($T = 0$ unitary cut):

$$E \geq (m_D + m_\Phi)$$

Additional branch cut
(Landau cut):

$$E \leq (m_D - m_\Phi)$$

At zero temperature $f(\omega, T = 0) = 0$

Results II: Thermal modifications

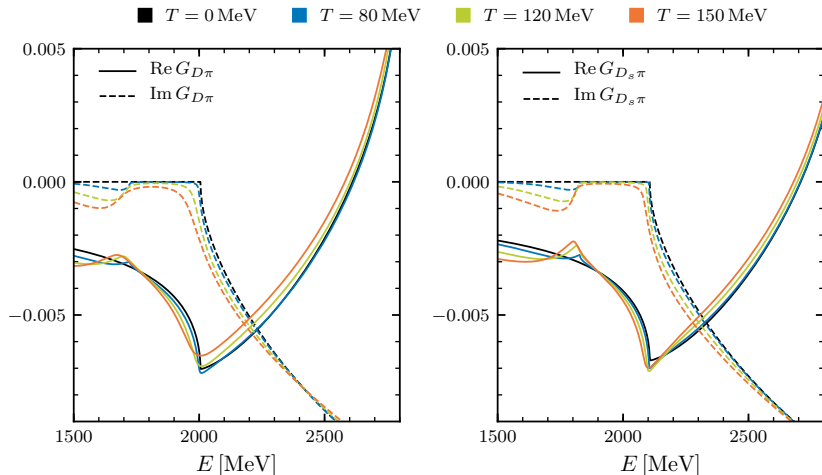
LOOP FUNCTIONS

Pionic bath

D and D_s with
light mesons

Unitary cut:
 $E \geq (m_D + m_\pi)$

Landau cut:
 $E \leq (m_D - m_\pi)$



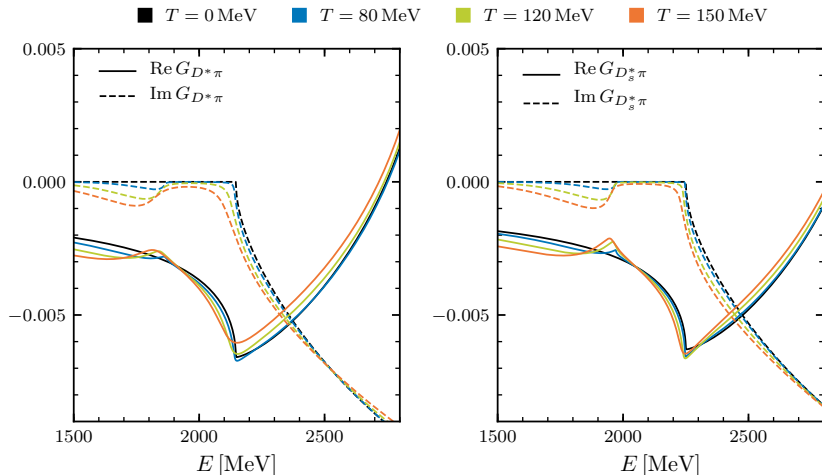
LOOP FUNCTIONS

Pionic bath

D^* and D_s^* with
light mesons

Unitary cut:
 $E \geq (m_{D^*} + m_\pi)$

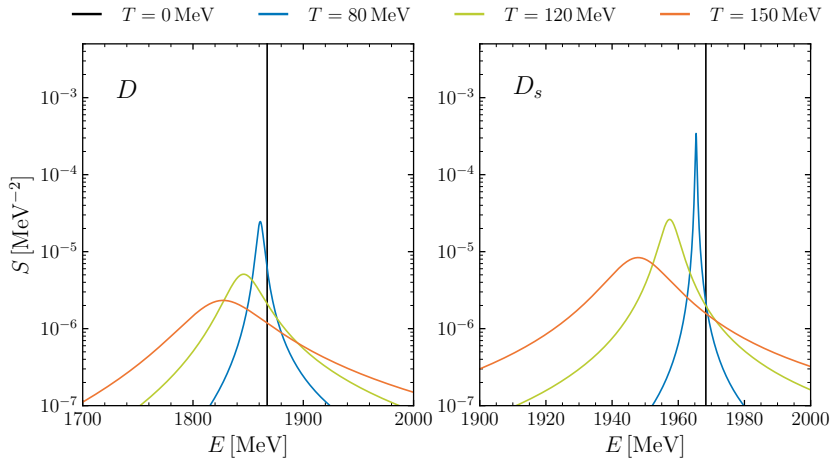
Landau cut:
 $E \leq (m_{D^*} - m_\pi)$



SPECTRAL FUNCTIONS

Open-charm
**pseudoscalar
mesons**
in a pionic bath

$$S_D(\omega, \vec{q}; T) = -\frac{1}{\pi} \text{Im } \mathcal{D}_D(\omega, \vec{q}; T)$$

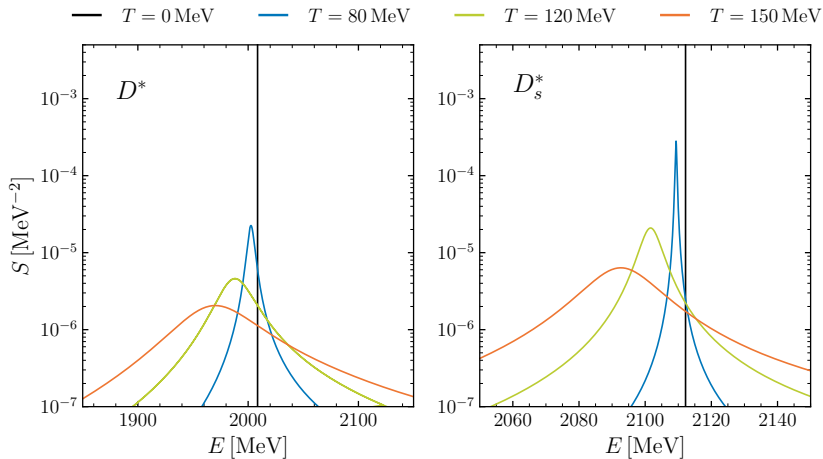


Mass shift and width acquisition of the D and D_s mesons in a thermal bath

SPECTRAL FUNCTIONS

Open-charm
vector mesons
in a pionic bath

$$S_D(\omega, \vec{q}; T) = -\frac{1}{\pi} \text{Im} \mathcal{D}_D(\omega, \vec{q}; T)$$



Mass shift and width acquisition of the D^* and D_s^* mesons in a thermal bath

DYNAMICALLY GENERATED STATES

Scalars ($J^P = 0^+$):

T-matrix in sector $(C, S, I) = (1, 0, 1/2)$

→ Two-pole structure of the $D_0^*(2300)$

Experimental values:

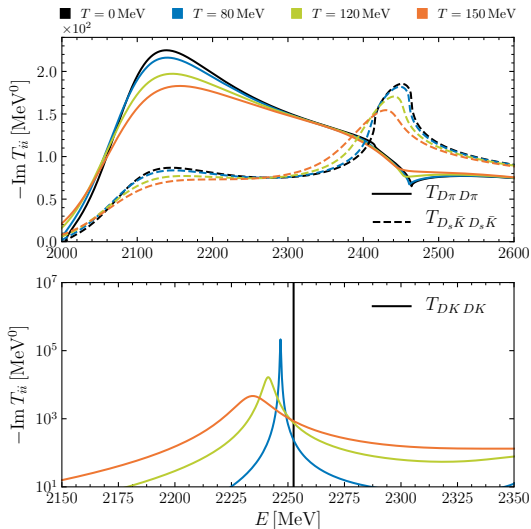
$$M_R = 2300 \pm 19 \text{ MeV}, \quad \Gamma_R = 274 \pm 40 \text{ MeV}$$

T-matrix in sector $(C, S, I) = (1, 1, 0)$

→ $D_{s0}^*(2317)$

Experimental values:

$$M_R = 2317.8 \pm 0.5 \text{ MeV}, \quad \Gamma_R < 3.8 \text{ MeV}$$



DYNAMICALLY GENERATED STATES

Axial vectors ($J^P = 1^+$):

T-matrix in sector $(C, S, I) = (1, 0, 1/2)$

→ Two-pole structure of the $D_1^*(2430)$

Experimental values:

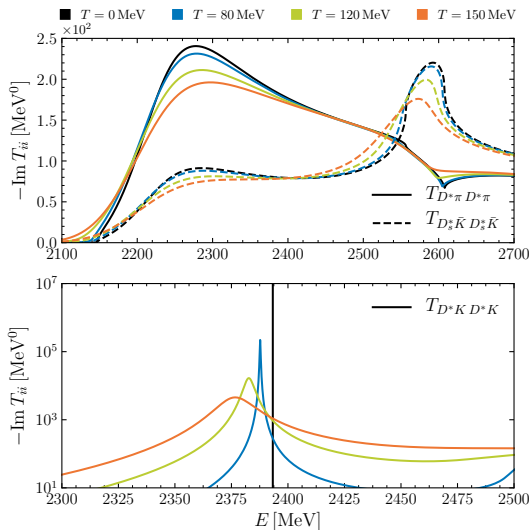
$$M_R = 2427 \pm 40 \text{ MeV}, \quad \Gamma_R = 384_{-110}^{+130} \text{ MeV}$$

T-matrix in sector $(C, S, I) = (1, 1, 0)$

→ $D_{s1}^*(2460)$

Experimental values:

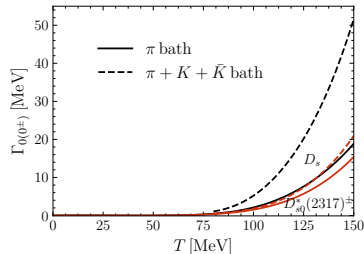
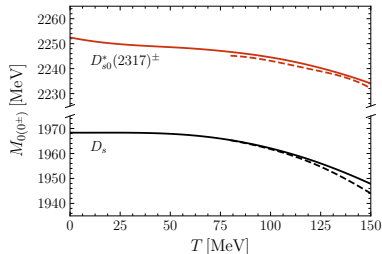
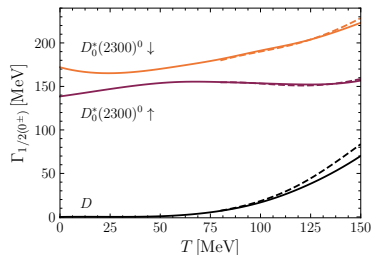
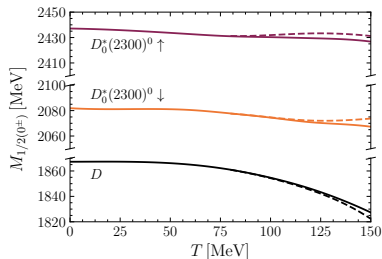
$$M_R = 2459.5 \pm 0.6 \text{ MeV}, \quad \Gamma_R < 3.5 \text{ MeV}$$



CHIRAL PARTNERS

Evolution of masses and widths of the open-charm mesons in a pionic (or $\pi + K + \bar{K}$) bath

$$I(J^P) = \frac{1}{2}(0^\pm), 0(0^\pm)$$

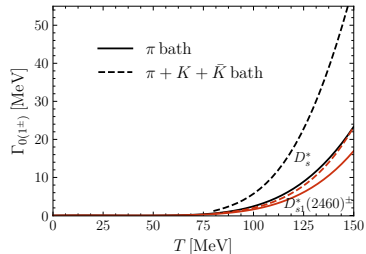
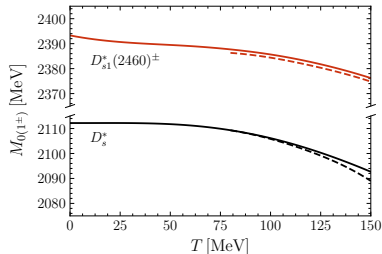
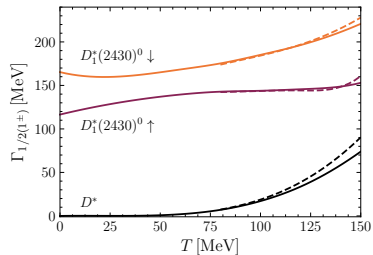
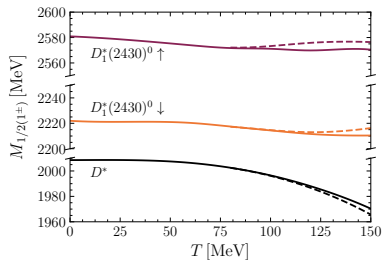


[GM, A. Ramos, L. Tolos, J. Torres-Rincon, arXiv:2007.12601]

CHIRAL PARTNERS

Evolution of masses and widths of the open-charm mesons in a pionic (or $\pi + K + \bar{K}$) bath

$$I(J^P) = \frac{1}{2}(1^\pm), 0(1^\pm)$$



[GM, A. Ramos, L. Tolos, J. Torres-Rincon, arXiv:2007.12601]

Euclidean correlators: comparison with lattice QCD

FROM SPECTRAL FUNCTIONS TO EUCLIDEAN CORRELATORS

Spectral function → **Euclidean correlator**

$$G_E(\tau, \vec{p}; T) = \int_0^\infty d\omega \, K(\tau, \omega; T) \, \rho(\omega, \vec{p}; T)$$

FROM SPECTRAL FUNCTIONS TO EUCLIDEAN CORRELATORS

Spectral function → **Euclidean correlator**

$$G_E(\tau, \vec{p}; T) = \int_0^\infty d\omega \, K(\tau, \omega; T) \, \rho(\omega, \vec{p}; T)$$

Spectral function $\rho(\omega, \vec{p}; T)$

FROM SPECTRAL FUNCTIONS TO EUCLIDEAN CORRELATORS

Spectral function → **Euclidean correlator**

$$G_E(\tau, \vec{p}; T) = \int_0^\infty d\omega K(\tau, \omega; T) \rho(\omega, \vec{p}; T)$$

Spectral function $\rho(\omega, \vec{p}; T)$

$$K(\tau, \omega; T) = \frac{\cosh[\omega(\tau - \frac{1}{2T})]}{\sinh(\frac{\omega}{2T})}$$

FROM SPECTRAL FUNCTIONS TO EUCLIDEAN CORRELATORS

Spectral function → Euclidean correlator

$$G_E(\tau, \vec{p}; T) = \int_0^\infty d\omega K(\tau, \omega; T) \rho(\omega, \vec{p}; T)$$

Spectral function $\rho(\omega, \vec{p}; T)$

$$K(\tau, \omega; T) = \frac{\cosh[\omega(\tau - \frac{1}{2T})]}{\sinh(\frac{\omega}{2T})}$$

Euclidean correlator → Spectral function (ill-posed)

- Bayesian methods (e.g. MEM)
- Fitting Ansätze

FROM SPECTRAL FUNCTIONS TO EUCLIDEAN CORRELATORS

Spectral function → Euclidean correlator

$$G_E(\tau, \vec{p}; T) = \int_0^\infty d\omega K(\tau, \omega; T) \rho(\omega, \vec{p}; T)$$

Spectral function $\rho(\omega, \vec{p}; T)$

$$K(\tau, \omega; T) = \frac{\cosh[\omega(\tau - \frac{1}{2T})]}{\sinh(\frac{\omega}{2T})}$$

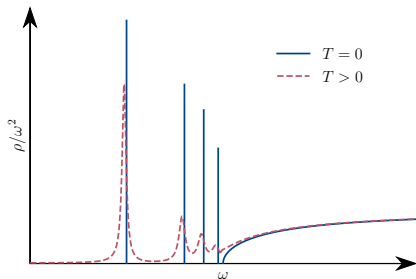
Euclidean correlator → Spectral function (ill-posed)

- Bayesian methods (e.g. MEM)
- Fitting Ansätze

Reconstructed
correlator

$$G_E^r(\tau; T, T_r) = \int_0^\infty d\omega K(\tau, \omega; T) \rho(\omega; T_r) \rightarrow \frac{G_E(\tau; T)}{G_E^r(\tau; T, T_r)}$$

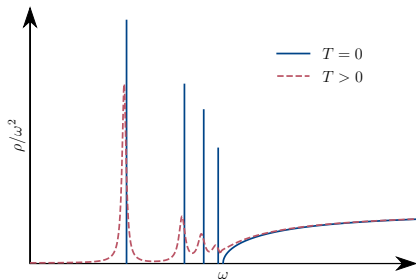
EUCLIDEAN CORRELATORS WITH EFT



$$S_D(\omega, \vec{q}; T) = -\frac{1}{\pi} \text{Im} \mathcal{D}_D(\omega, \vec{q}; T)$$
$$= -\frac{1}{\pi} \text{Im} \left(\frac{1}{\omega^2 - \vec{q}^2 - M_D^2 - \Pi_D(\omega, \vec{q}; T)} \right)$$

at unphysical meson masses (used in the lattice)

EUCLIDEAN CORRELATORS WITH EFT

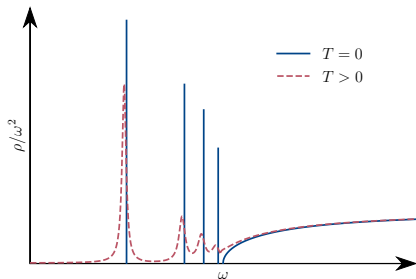


$$S_D(\omega, \vec{q}; T) = -\frac{1}{\pi} \text{Im} \mathcal{D}_D(\omega, \vec{q}; T)$$
$$= -\frac{1}{\pi} \text{Im} \left(\frac{1}{\omega^2 - \vec{q}^2 - M_D^2 - \Pi_D(\omega, \vec{q}; T)} \right)$$

at unphysical meson masses (used in the lattice)

Ground-state: $\rho_{\text{gs}}(\omega; T) = M_D^4 S_D(\omega; T)$

EUCLIDEAN CORRELATORS WITH EFT



$$S_D(\omega, \vec{q}; T) = -\frac{1}{\pi} \text{Im } \mathcal{D}_D(\omega, \vec{q}; T)$$

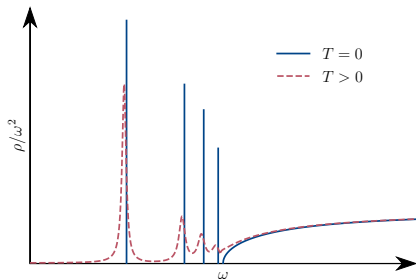
$$= -\frac{1}{\pi} \text{Im} \left(\frac{1}{\omega^2 - \vec{q}^2 - M_D^2 - \Pi_D(\omega, \vec{q}; T)} \right)$$

at unphysical meson masses (used in the lattice)

Ground-state: $\rho_{\text{gs}}(\omega; T) = M_D^4 S_D(\omega; T)$

Full: $\rho(\omega; T) = \rho_{\text{gs}}(\omega; T) + a \rho_{\text{cont}}(\omega; T)$

EUCLIDEAN CORRELATORS WITH EFT



$$S_D(\omega, \vec{q}; T) = -\frac{1}{\pi} \text{Im } \mathcal{D}_D(\omega, \vec{q}; T)$$

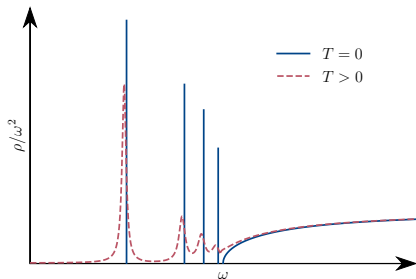
$$= -\frac{1}{\pi} \text{Im} \left(\frac{1}{\omega^2 - \vec{q}^2 - M_D^2 - \Pi_D(\omega, \vec{q}; T)} \right)$$

at unphysical meson masses (used in the lattice)

Ground-state: $\rho_{\text{gs}}(\omega; T) = M_D^4 S_D(\omega; T)$

Full: $\rho(\omega; T) = \rho_{\text{gs}}(\omega; T) + a \rho_{\text{cont}}(\omega; T)$

EUCLIDEAN CORRELATORS WITH EFT



$$S_D(\omega, \vec{q}; T) = -\frac{1}{\pi} \text{Im} \mathcal{D}_D(\omega, \vec{q}; T)$$

$$= -\frac{1}{\pi} \text{Im} \left(\frac{1}{\omega^2 - \vec{q}^2 - M_D^2 - \Pi_D(\omega, \vec{q}; T)} \right)$$

at unphysical meson masses (used in the lattice)

Ground-state: $\rho_{\text{gs}}(\omega; T) = M_D^4 S_D(\omega; T)$

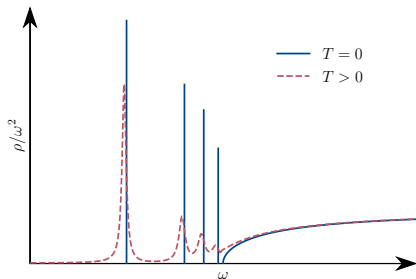
Full: $\rho(\omega; T) = \rho_{\text{gs}}(\omega; T) + a \rho_{\text{cont}}(\omega; T)$

$$\rho_{\text{cont}}(\omega; T) = \frac{3}{32\pi} \sqrt{\left(\frac{m_1^2 - m_2^2}{\omega^2} + 1\right)^2 - \frac{4m_2^2}{\omega^2} \omega^2}$$

$$\times 2 \left(1 - \frac{(m_1 - m_2)^2}{\omega^2}\right)$$

$$\times [n(-\omega_0, T) - n(\omega - \omega_0, T)] \theta(\omega - (m_1 + m_2))$$

EUCLIDEAN CORRELATORS WITH EFT



$$m_1 = m_c = 1.5 \text{ GeV}$$

$$m_2 = m_l = 0, m_2 = m_s = 100 \text{ MeV}$$

$$S_D(\omega, \vec{q}; T) = -\frac{1}{\pi} \text{Im} \mathcal{D}_D(\omega, \vec{q}; T)$$

$$= -\frac{1}{\pi} \text{Im} \left(\frac{1}{\omega^2 - \vec{q}^2 - M_D^2 - \Pi_D(\omega, \vec{q}; T)} \right)$$

at unphysical meson masses (used in the lattice)

$$\text{Ground-state: } \rho_{\text{gs}}(\omega; T) = M_D^4 S_D(\omega; T)$$

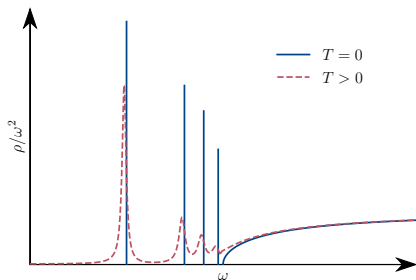
$$\text{Full: } \rho(\omega; T) = \rho_{\text{gs}}(\omega; T) + a \rho_{\text{cont}}(\omega; T)$$

$$\rho_{\text{cont}}(\omega; T) = \frac{3}{32\pi} \sqrt{\left(\frac{m_1^2 - m_2^2}{\omega^2} + 1\right)^2 - \frac{4m_2^2}{\omega^2} \omega^2}$$

$$\times 2 \left(1 - \frac{(m_1 - m_2)^2}{\omega^2}\right)$$

$$\times [n(-\omega_0, T) - n(\omega - \omega_0, T)] \theta(\omega - (m_1 + m_2))$$

EUCLIDEAN CORRELATORS WITH EFT



a : weight factor

$$S_D(\omega, \vec{q}; T) = -\frac{1}{\pi} \text{Im} \mathcal{D}_D(\omega, \vec{q}; T)$$

$$= -\frac{1}{\pi} \text{Im} \left(\frac{1}{\omega^2 - \vec{q}^2 - M_D^2 - \Pi_D(\omega, \vec{q}; T)} \right)$$

at unphysical meson masses (used in the lattice)

Ground-state: $\rho_{\text{gs}}(\omega; T) = M_D^4 S_D(\omega; T)$

Full: $\rho(\omega; T) = \rho_{\text{gs}}(\omega; T) + a \rho_{\text{cont}}(\omega; T)$

$$\rho_{\text{cont}}(\omega; T) = \frac{3}{32\pi} \sqrt{\left(\frac{m_1^2 - m_2^2}{\omega^2} + 1\right)^2 - \frac{4m_2^2}{\omega^2} \omega^2}$$

$$\times 2 \left(1 - \frac{(m_1 - m_2)^2}{\omega^2}\right)$$

$$\times [n(-\omega_0, T) - n(\omega - \omega_0, T)] \theta(\omega - (m_1 + m_2))$$

Results III: Open-charm Euclidean correlators

EFT SPECTRAL FUNCTIONS AT UNPHYSICAL MASSES

$$m_\pi = 384 \text{ MeV}$$

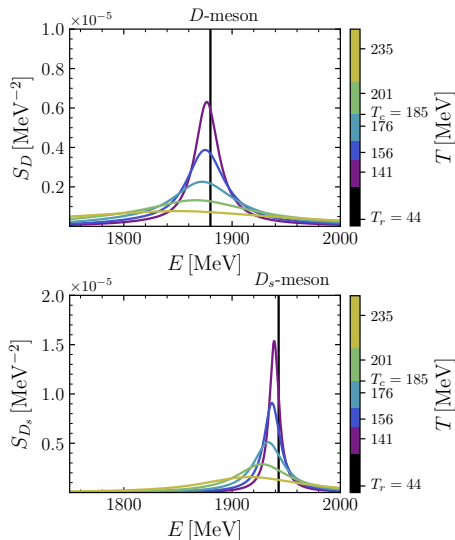
$$m_K = 546 \text{ MeV}$$

$$m_\eta = 589 \text{ MeV}$$

$$m_D = 1880 \text{ MeV}$$

$$m_{D_s} = 1943 \text{ MeV}$$

[Kelly, Rothkopf, Skullerud
(2018)]



EFT SPECTRAL FUNCTIONS AT UNPHYSICAL MASSES

$$m_\pi = 384 \text{ MeV}$$

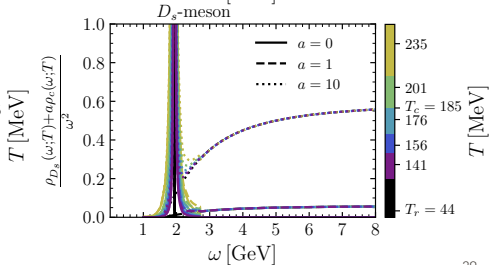
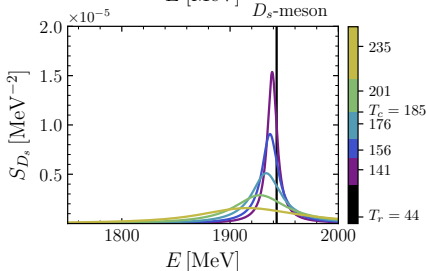
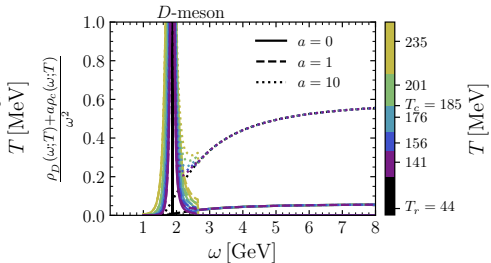
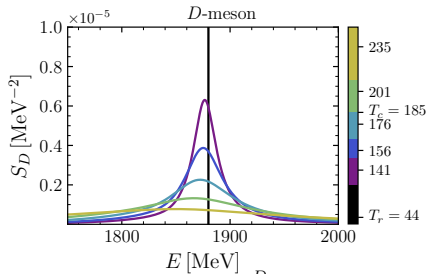
$$m_K = 546 \text{ MeV}$$

$$m_\eta = 589 \text{ MeV}$$

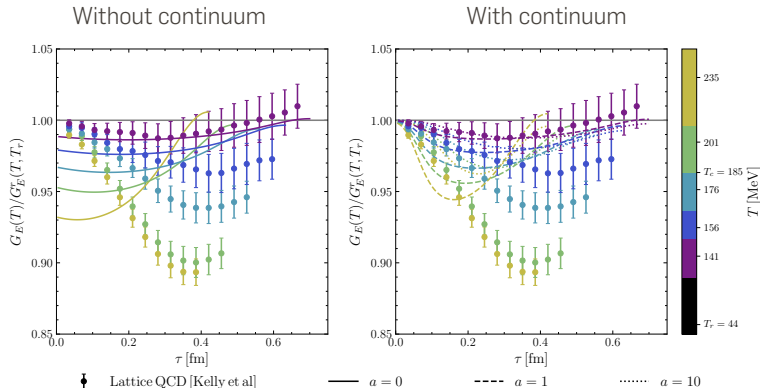
$$m_D = 1880 \text{ MeV}$$

$$m_{D_s} = 1943 \text{ MeV}$$

[Kelly, Rothkopf, Skullerud
(2018)]



COMPARISON WITH LQCD

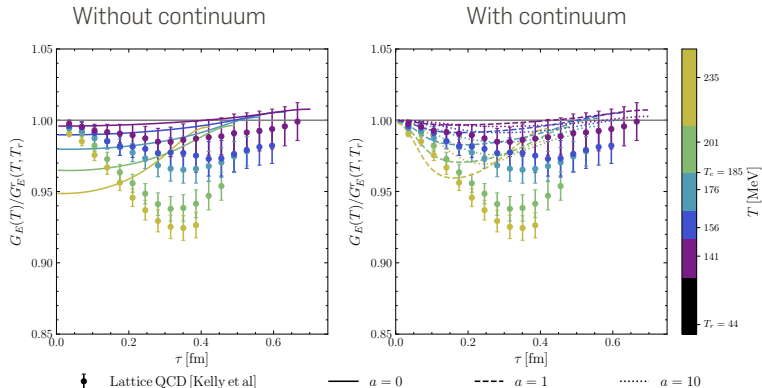
D meson

- Inclusion of the continuum improves the matching at small τ
- Very good agreement at the lowest temperature. At larger temperatures: excited states?
- Close and above T_c the EFT breaks down.

[GM, O. Kaczmarek, L. Tolos, A. Ramos, arXiv:2007.15690]

COMPARISON WITH LQCD

D_s meson



- Inclusion of the continuum improves the matching at small τ
- Very good agreement at the lowest temperature. At larger temperatures: excited states?
- Close and above T_c the EFT breaks down.

[GM, O. Kaczmarek, L. Tolos, A. Ramos, arXiv:2007.15690]

Conclusions and Outlook

CONCLUSIONS AND OUTLOOK

- We have introduced **finite-temperature corrections** to the description of the interaction of open charm mesons with light mesons in a self-consistent manner.

CONCLUSIONS AND OUTLOOK

- We have introduced **finite-temperature corrections** to the description of the interaction of open charm mesons with light mesons in a self-consistent manner.
- We have obtained **spectral functions** at various temperatures below T_c .

CONCLUSIONS AND OUTLOOK

- We have introduced **finite-temperature corrections** to the description of the interaction of open charm mesons with light mesons in a self-consistent manner.
- We have obtained **spectral functions** at various temperatures below T_c .
- The **mass** of the charmed $D^{(*)}$ - and $D_s^{(*)}$ -mesons **decreases** with temperature (~ 40 MeV for the $D^{(*)}$ and ~ 20 MeV for the $D_s^{(*)}$ at $T = 150$ MeV) while they **acquire a substantial width** (~ 70 MeV for the $D^{(*)}$ and ~ 20 MeV for the $D_s^{(*)}$ at $T = 150$ MeV).

CONCLUSIONS AND OUTLOOK

- We have introduced **finite-temperature corrections** to the description of the interaction of open charm mesons with light mesons in a self-consistent manner.
- We have obtained **spectral functions** at various temperatures below T_c .
- The **mass** of the charmed $D^{(*)}$ - and $D_s^{(*)}$ -mesons **decreases** with temperature (~ 40 MeV for the $D^{(*)}$ and ~ 20 MeV for the $D_s^{(*)}$ at $T = 150$ MeV) while they **acquire a substantial width** (~ 70 MeV for the $D^{(*)}$ and ~ 20 MeV for the $D_s^{(*)}$ at $T = 150$ MeV).
- The **dynamically generated resonances** shift their mass and get wider as temperature increases. Still far from chiral degeneracy.

CONCLUSIONS AND OUTLOOK

- We have introduced **finite-temperature corrections** to the description of the interaction of open charm mesons with light mesons in a self-consistent manner.
- We have obtained **spectral functions** at various temperatures below T_c .
- The **mass** of the charmed $D^{(*)}$ - and $D_s^{(*)}$ -mesons **decreases** with temperature (~ 40 MeV for the $D^{(*)}$ and ~ 20 MeV for the $D_s^{(*)}$ at $T = 150$ MeV) while they **acquire a substantial width** (~ 70 MeV for the $D^{(*)}$ and ~ 20 MeV for the $D_s^{(*)}$ at $T = 150$ MeV).
- The **dynamically generated resonances** shift their mass and get wider as temperature increases. Still far from chiral degeneracy.
- The effect of the addition of **kaons in the bath** is mild.

CONCLUSIONS AND OUTLOOK

- We have introduced **finite-temperature corrections** to the description of the interaction of open charm mesons with light mesons in a self-consistent manner.
- We have obtained **spectral functions** at various temperatures below T_c .
- The **mass** of the charmed $D^{(*)}$ - and $D_s^{(*)}$ -mesons **decreases** with temperature (~ 40 MeV for the $D^{(*)}$ and ~ 20 MeV for the $D_s^{(*)}$ at $T = 150$ MeV) while they **acquire a substantial width** (~ 70 MeV for the $D^{(*)}$ and ~ 20 MeV for the $D_s^{(*)}$ at $T = 150$ MeV).
- The **dynamically generated resonances** shift their mass and get wider as temperature increases. Still far from chiral degeneracy.
- The effect of the addition of **kaons in the bath** is mild.
- We have calculated **Euclidean correlators** from spectral functions at the unphysical masses used in the lattice. **Well below T_c there is a good agreement with LQCD results**, whereas close or above T_c the discrepancy indicates the missing contribution of higher-excited states.

CONCLUSIONS AND OUTLOOK

In the near future we aim to:

→ Include thermal **modification of light mesons**

CONCLUSIONS AND OUTLOOK

In the near future we aim to:

- Include thermal **modification of light mesons**
- Extend our calculations to the **bottom** sector

CONCLUSIONS AND OUTLOOK

In the near future we aim to:

- Include thermal **modification of light mesons**
- Extend our calculations to the **bottom** sector
- Explore the **hidden heavy-flavor** sector

CONCLUSIONS AND OUTLOOK

In the near future we aim to:

- Include thermal **modification of light mesons**
- Extend our calculations to the **bottom** sector
- Explore the **hidden heavy-flavor** sector
- Study **transport properties** of heavy mesons at finite temperature

CONCLUSIONS AND OUTLOOK

In the near future we aim to:

- Include thermal **modification of light mesons**
- Extend our calculations to the **bottom** sector
- Explore the **hidden heavy-flavor** sector
- Study **transport properties** of heavy mesons at finite temperature
- Further test our results against **lattice QCD** calculations and make predictions at the physical point

Thank you!

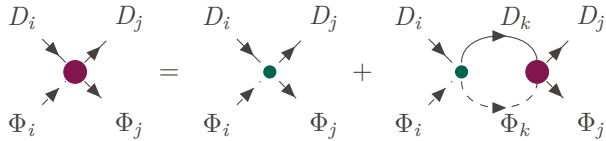
Gràcies!

Backup slides

UNITARIZED T -MATRIX AT $T \neq 0$

$$T_{ij} = V_{ij} + V_{ik} G_k T_{kj}$$

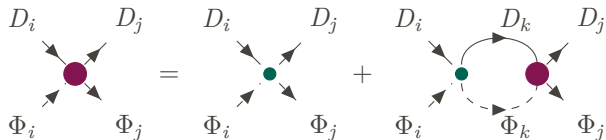
- Iteration 1: Undressed mesons



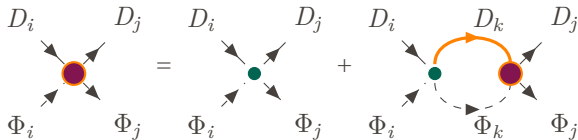
UNITARIZED T -MATRIX AT $T \neq 0$

$$T_{ij} = V_{ij} + V_{ik} G_k T_{kj}$$

- Iteration 1: Undressed mesons



- Iteration > 1 : Dressed heavy meson



LOOP FUNCTION AT $T > 0$

$$G_{D\Phi}(E, \vec{p}; T) = \int \frac{d^3 q}{(2\pi)^3} \int d\omega \int d\omega' \frac{S_D(\omega, \vec{q}; T) S_\Phi(\omega', \vec{p} - \vec{q}; T)}{E - \omega - \omega' + i\varepsilon} \cdot [1 + f(\omega, T) + f(\omega', T)]$$

LOOP FUNCTION AT $T > 0$

$$G_{D\Phi}(E, \vec{p}; T) = \int \frac{d^3 q}{(2\pi)^3} \int d\omega \int d\omega' \frac{S_D(\omega, \vec{q}; T) S_\Phi(\omega', \vec{p} - \vec{q}; T)}{E - \omega - \omega' + i\varepsilon} \cdot [1 + f(\omega, T) + f(\omega', T)]$$

Spectral functions:

$$S_D, \quad S_\Phi \rightarrow \frac{\omega_\Phi}{\omega'} \delta(\omega'^2 - \omega_\Phi^2), \quad \omega_\Phi = \sqrt{q_\Phi^2 + m_\Phi^2}$$

LOOP FUNCTION AT $T > 0$

$$G_{D\Phi}(E, \vec{p}; T) = \int \frac{d^3 q}{(2\pi)^3} \int d\omega \int d\omega' \frac{S_D(\omega, \vec{q}; T) S_\Phi(\omega', \vec{p} - \vec{q}; T)}{E - \omega - \omega' + i\varepsilon} \cdot [1 + f(\omega, T) + f(\omega', T)]$$

Spectral functions:

$$S_D, \quad S_\Phi \rightarrow \frac{\omega_\Phi}{\omega'} \delta(\omega'^2 - \omega_\Phi^2), \quad \omega_\Phi = \sqrt{q_\Phi^2 + m_\Phi^2}$$

Bose distribution function at T:

$$f(\omega, T) = \frac{1}{e^{\omega/T} - 1}$$

LOOP FUNCTION AT $T > 0$

$$G_{D\Phi}(E, \vec{p}; T) = \int \frac{d^3 q}{(2\pi)^3} \int d\omega \int d\omega' \frac{S_D(\omega, \vec{q}; T) S_\Phi(\omega', \vec{p} - \vec{q}; T)}{E - \omega - \omega' + i\varepsilon} \cdot [1 + f(\omega, T) + f(\omega', T)]$$

Spectral functions:

$$S_D, \quad S_\Phi \rightarrow \frac{\omega_\Phi}{\omega'} \delta(\omega'^2 - \omega_\Phi^2), \quad \omega_\Phi = \sqrt{q_\Phi^2 + m_\Phi^2}$$

Bose distribution function at T:

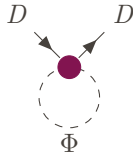
$$f(\omega, T) = \frac{1}{e^{\omega/T} - 1}$$

Regularized with a cutoff $|\vec{q}| < \Lambda$

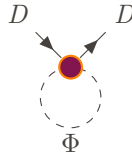
SELF-ENERGY AT $T > 0$

$$\Pi_D(E, \vec{p}; T) = -\frac{1}{\pi} \int \frac{d^3 q}{(2\pi)^3} \int d\Omega \frac{E}{\omega_\Phi} \frac{f(\Omega, T) - f(\omega_\Phi, T)}{E^2 - (\omega_\Phi - \Omega)^2 + i\varepsilon} \text{Im } T_{D\Phi}(\Omega, \vec{p} + \vec{q}; T)$$

Iteration 1



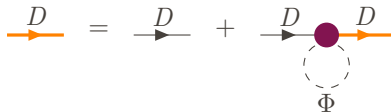
Iteration > 1



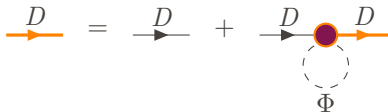
SPECTRAL FUNCTION

$$S_D(\omega, \vec{q}; T) = -\frac{1}{\pi} \text{Im} \mathcal{D}_D(\omega, \vec{q}; T) = -\frac{1}{\pi} \text{Im} \left(\frac{1}{\omega^2 - \vec{q}^2 - m_D^2 - \Pi_D(\omega, \vec{q}; T)} \right)$$

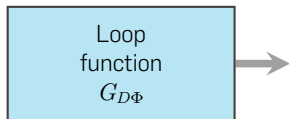
Iteration 1



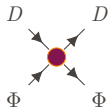
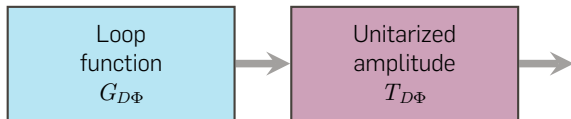
Iteration > 1



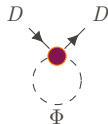
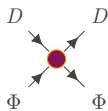
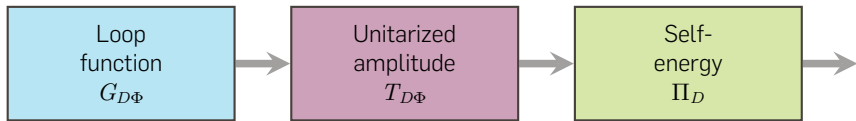
SELF-CONSISTENCY



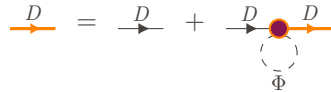
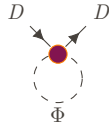
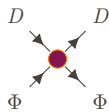
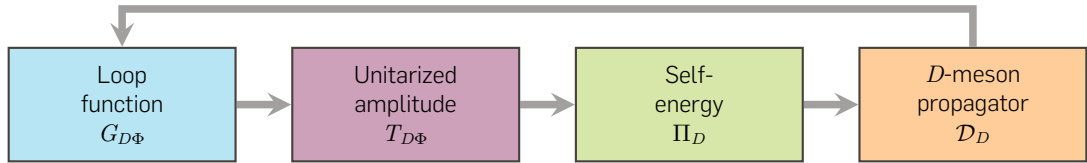
SELF-CONSISTENCY



SELF-CONSISTENCY



SELF-CONSISTENCY



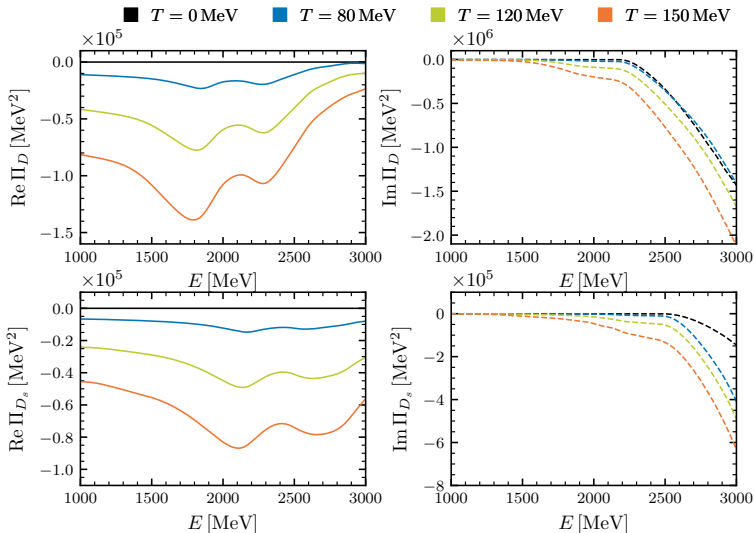
SELF-ENERGY

Pionic bath

D and D_s with
light mesons

First opening:
 $E \geq (m_D - 2m_\pi)$

Second opening:
 $E \leq (m_D + 2m_\pi)$



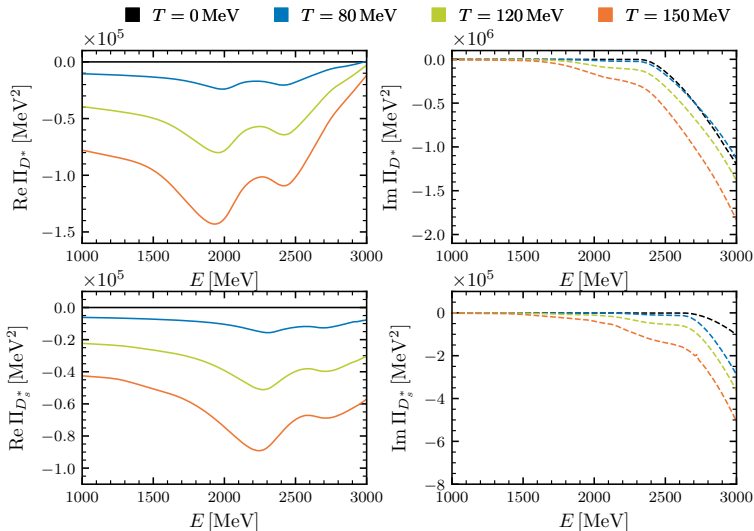
SELF-ENERGY

Pionic bath

D^* and D_s^* with
light mesons

First opening:
 $E \geq (m_D - 2m_\pi)$

Second opening:
 $E \leq (m_D + 2m_\pi)$



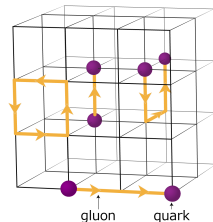
LATTICE EUCLIDEAN CORRELATORS

Temperature of the system on a lattice of size $N_\sigma^3 \times N_\tau \rightarrow$ related to the temporal extent:

$$T = \frac{1}{aN_\tau}, \quad a : \text{Lattice spacing}$$

Euclidean correlators of some operators \hat{O} :

$$\langle \hat{O}_1(\tau, \vec{x}) \hat{O}_2(0, \vec{0}) \rangle = \frac{1}{Z} \int \mathcal{D}[U] \mathcal{D}[\bar{\psi}, \psi] \hat{O}_2[U, \bar{\psi}, \psi] \hat{O}_1[U, \bar{\psi}, \psi] e^{-S_F[U, \bar{\psi}, \psi] - S_G[U]}$$



LATTICE EUCLIDEAN CORRELATORS

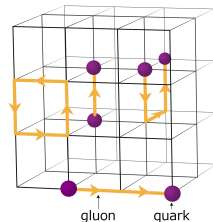
Temperature of the system on a lattice of size $N_\sigma^3 \times N_\tau \rightarrow$ related to the temporal extent:

$$T = \frac{1}{aN_\tau}, \quad a : \text{Lattice spacing}$$

Euclidean correlators of some operators \hat{O} :

$$\langle \hat{O}_1(\tau, \vec{x}) \hat{O}_2(0, \vec{0}) \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[U] \mathcal{D}[\bar{\psi}, \psi] \hat{O}_2[U, \bar{\psi}, \psi] \hat{O}_1[U, \bar{\psi}, \psi] e^{-S_F[U, \bar{\psi}, \psi] - S_G[U]}$$

\mathcal{Z} : partition function



LATTICE EUCLIDEAN CORRELATORS

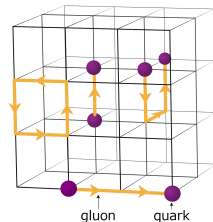
Temperature of the system on a lattice of size $N_\sigma^3 \times N_\tau \rightarrow$ related to the temporal extent:

$$T = \frac{1}{aN_\tau}, \quad a : \text{Lattice spacing}$$

Euclidean correlators of some operators \hat{O} :

$$\langle \hat{O}_1(\tau, \vec{x}) \hat{O}_2(0, \vec{0}) \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[U] \mathcal{D}[\bar{\psi}, \psi] \hat{O}_2[U, \bar{\psi}, \psi] \hat{O}_1[U, \bar{\psi}, \psi] e^{-S_F[U, \bar{\psi}, \psi] - S_G[U]}$$

$S_F[U, \bar{\psi}, \psi]$ and $S_G[U]$: fermion and gluon
 parts of the discretized QCD action



LATTICE EUCLIDEAN CORRELATORS

Temperature of the system on a lattice of size $N_\sigma^3 \times N_\tau \rightarrow$ related to the temporal extent:

$$T = \frac{1}{aN_\tau}, \quad a : \text{Lattice spacing}$$

Euclidean correlators of some operators \hat{O} :

$$\langle \hat{O}_1(\tau, \vec{x}) \hat{O}_2(0, \vec{0}) \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[U] \mathcal{D}[\bar{\psi}, \psi] \hat{O}_2[U, \bar{\psi}, \psi] \hat{O}_1[U, \bar{\psi}, \psi] e^{-S_F[U, \bar{\psi}, \psi] - S_G[U]}$$

Meson Euclidean temporal correlator:

$$\langle J(\tau, \vec{x}) J(0, \vec{0}) \rangle = -\frac{1}{\mathcal{Z}} \int \mathcal{D}[U] e^{-S_G[U]} \det[M] \text{Tr} [\Gamma_H M^{-1}(0, \vec{0}; \tau, \vec{x}) \Gamma_H M^{-1}(\tau, \vec{x}; 0, \vec{0})],$$

LATTICE EUCLIDEAN CORRELATORS

Temperature of the system on a lattice of size $N_\sigma^3 \times N_\tau \rightarrow$ related to the temporal extent:

$$T = \frac{1}{aN_\tau}, \quad a : \text{Lattice spacing}$$

Euclidean correlators of some operators \hat{O} :

$$\langle \hat{O}_1(\tau, \vec{x}) \hat{O}_2(0, \vec{0}) \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[U] \mathcal{D}[\bar{\psi}, \psi] \hat{O}_2[U, \bar{\psi}, \psi] \hat{O}_1[U, \bar{\psi}, \psi] e^{-S_F[U, \bar{\psi}, \psi] - S_G[U]}$$

Meson Euclidean temporal correlator:

$$\langle J(\tau, \vec{x}) J(0, \vec{0}) \rangle = -\frac{1}{\mathcal{Z}} \int \mathcal{D}[U] e^{-S_G[U]} \det[M] \text{Tr} [\Gamma_H M^{-1}(0, \vec{0}; \tau, \vec{x}) \Gamma_H M^{-1}(\tau, \vec{x}; 0, \vec{0})],$$

$\Gamma_H = 1, \gamma_5, \gamma_\mu, \gamma_5 \gamma_\mu$: for scalar, pseudoscalar, vector and axial vector channels

LATTICE EUCLIDEAN CORRELATORS

Temperature of the system on a lattice of size $N_\sigma^3 \times N_\tau \rightarrow$ related to the temporal extent:

$$T = \frac{1}{aN_\tau}, \quad a : \text{Lattice spacing}$$

Euclidean correlators of some operators \hat{O} :

$$\langle \hat{O}_1(\tau, \vec{x}) \hat{O}_2(0, \vec{0}) \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[U] \mathcal{D}[\bar{\psi}, \psi] \hat{O}_2[U, \bar{\psi}, \psi] \hat{O}_1[U, \bar{\psi}, \psi] e^{-S_F[U, \bar{\psi}, \psi] - S_G[U]}$$

Meson Euclidean temporal correlator:

$$\langle J(\tau, \vec{x}) J(0, \vec{0}) \rangle = -\frac{1}{\mathcal{Z}} \int \mathcal{D}[U] e^{-S_G[U]} \det[M] \text{Tr} [\Gamma_H M^{-1}(0, \vec{0}; \tau, \vec{x}) \Gamma_H M^{-1}(\tau, \vec{x}; 0, \vec{0})],$$

M^{-1} : fermion propagator