Impact of finite volume and external magnetic field on the properties of quark matter

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In this presentation, we are going to discuss about,

- **Introduction**
  - The Big-Bang
  - QCD phase diagram
  - Theoretical Approaches

- **Polyakov chiral quark mean field model**
  - Lagrangian density
  - Thermodynamic potential
  - Polyakov loop

- **What do we need to study and why?**
  - Finite volume effects
  - External magnetic field

- **Results**

- **Conclusion**
The Big-Bang
Theoretical Approaches

- **Lattice QCD simulations**: It is a non-perturbative application of field theory based on the Feynman path integral technique.

- **MIT bag model**: Hadrons are considered to be composed of weakly interacting quarks confined within a finite region referred to as the "bag."

- **Nambu Jona Lasinio (NJL) and Polyakov NJL (PNJL) model**: focusing on the interaction between quarks and anti-quarks through a four-point interaction term in the Lagrangian density.

- **Chiral perturbation theory**: Includes the expansion in power of momentum and quark masses.

AND Many More.......
Chiral SU(3) Quark Mean Field Model

Lagrangian density

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{q0} + \mathcal{L}_{qm} + \mathcal{L}_{\Sigma\Sigma} + \mathcal{L}_{VV} + \mathcal{L}_{SB} + \mathcal{L}_{\Delta m} + \mathcal{L}_h. \]  

- \( \mathcal{L}_{q0} \) is the free part of massless quarks.
- \( \mathcal{L}_{qm} \) quark meson interaction term.
- \( \mathcal{L}_{\Sigma\Sigma} \) scalar meson self-interaction term (\( \sigma, \zeta, \chi \) and \( \delta \) fields).
- \( \mathcal{L}_{VV} \) vector meson self-interaction term (\( \omega, \rho \) and \( \phi \) fields).
- \( \mathcal{L}_{SB}, \mathcal{L}_{\Delta m} \) and \( \mathcal{L}_h \) are explicit symmetry breaking terms.
Thermodynamical potential density

\[
\Omega = \sum_{i=u,d,s} \frac{-2k_B T \gamma_i}{(2\pi)^3} \int_0^\infty d^3 k \left[ \ln(1 + e^{-(E_i^*(k) - \nu_i)/k_B T}) \right. \\
\left. + \ln(1 + e^{-(E_i^*(k) + \nu_i)/k_B T}) \right] - \mathcal{L}_M,
\]  

(2)

- \( \mathcal{L}_M = \mathcal{L}_{\Sigma\Sigma} + \mathcal{L}_{VV} + \mathcal{L}_{SB} \).
- \( E_i^*(k) = \sqrt{m_i^* + k^2} \) is the effective single particle energy of quarks.
- \( m_i^* = -g_i^\sigma \sigma - g_i^\zeta \zeta - g_i^\delta \delta + m_{i0} \) is effective constituent quark mass.
- \( \nu_i^* = \mu_i - g_i^\omega \omega - g_i^\phi \phi - g_i^\rho \rho \) is effective chemical potential.
- \( \gamma_i \) is spin degeneracy factor for quarks (\( \gamma_i = 3 \)) and electrons (\( \gamma_i = 1 \)).
Polyakov Chiral SU(3) quark mean field model

Polyakov loop

\[ \Phi(\tilde{x}) = \frac{\text{Tr}_c L}{N_C}, \quad (3) \]

and its conjugate

\[ \bar{\Phi}(\tilde{x}) = \frac{\text{Tr}_c L^\dagger}{N_C}. \quad (4) \]

Total lagrangian density

\[ \mathcal{L}_{\text{PCQMF}} = \mathcal{L}_{\text{eff}} - \]
Polyakov Chiral SU(3) quark mean field model

Polyakov loop

$$\Phi(\tilde{x}) = (\text{Tr}_c L)/N_C,$$  \hspace{1cm} (3)

and its conjugate

$$\bar{\Phi}(\tilde{x}) = (\text{Tr}_c L^\dagger)/N_C.$$ \hspace{1cm} (4)

Total lagrangian density

$$\mathcal{L}_{PCQMF} = \mathcal{L}_{\text{eff}} - U(\Phi(\tilde{x}), \bar{\Phi}(\tilde{x}), T),$$ \hspace{1cm} (5)

Modified thermodynamical potential density

$$\Omega_{PCQMF} = -2k_B T \sum_{u,d,s} \int_0^\infty \frac{d^3 k}{(2\pi)^3} \left[ \ln(1 + e^{-3(E^*_i(k)-\nu_i)/k_B T}} \right] + U(\Phi, \bar{\Phi}, T),$$ \hspace{1cm} (6)
here, $\mathcal{U}(\Phi(\tilde{x}), \bar{\Phi}(\tilde{x}), T)$ is temperature dependent Polyakov loop effective potential,

$$
\frac{\mathcal{U}(\Phi, \bar{\Phi}, T)}{T^4} = -\frac{a(T)}{2} \Phi \bar{\Phi} + b(T) \ln \left[ 1 - 6 \Phi \bar{\Phi} + 4(\Phi^3 + \bar{\Phi}^3) - 3(\Phi \bar{\Phi})^2 \right],
$$

(7)

with $T$-dependent parameters:

$$
a(T) = a_0 + a_1 \left( \frac{T_0}{T} \right) + a_2 \left( \frac{T_0}{T} \right)^2, \quad b(T) = b_3 \left( \frac{T_0}{T} \right)^3.
$$

(8)

<table>
<thead>
<tr>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$b_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.51</td>
<td>-2.47</td>
<td>15.2</td>
<td>-1.75</td>
</tr>
</tbody>
</table>

Table: Parameters in Polyakov effective potential
Coupled equations of motion:

\[
\frac{\partial \Omega}{\partial \sigma} = k_0 \chi^2 \sigma - 4k_1 \left( \sigma^2 + \zeta^2 + \delta^2 \right) \sigma - 2k_2 \left( \sigma^3 + 3\sigma \delta^2 \right) - 2k_3 \chi \sigma \zeta \\
- \frac{d}{3} \chi^4 \left( \frac{2\sigma}{\sigma^2 - \delta^2} \right) + \left( \frac{\chi}{\chi_0} \right)^2 m_\pi^2 f_\pi - \left( \frac{\chi}{\chi_0} \right)^2 m_\omega \omega^2 \frac{\partial m_\omega}{\partial \sigma} \\
- \left( \frac{\chi}{\chi_0} \right)^2 m_\rho \rho^2 \frac{\partial m_\rho}{\partial \sigma} - \sum g^i_\sigma \rho^s_i = 0, \quad (9)
\]

\[
\frac{\partial \Omega}{\partial \zeta} = k_0 \chi^2 \zeta - 4k_1 \left( \sigma^2 + \zeta^2 + \delta^2 \right) \zeta - 4k_2 \zeta^3 - k_3 \chi \left( \sigma^2 - \delta^2 \right) - \frac{d}{3} \chi^4 \\
+ \left( \frac{\chi}{\chi_0} \right)^2 \left[ \sqrt{2} m_K^2 f_K - \frac{1}{\sqrt{2} m_\pi^2 f_\pi} \right] - \left( \frac{\chi}{\chi_0} \right)^2 m_\phi \phi^2 \frac{\partial m_\phi}{\partial \zeta} - \sum g^i_\zeta \rho^s_i = 0, \quad (10)
\]

\[
\frac{\partial \Omega}{\partial \delta} = k_0 \chi^2 \delta - 4k_1 \left( \sigma^2 + \zeta^2 + \delta^2 \right) \delta - 2k_2 \left( \delta^3 + 3\sigma^2 \delta \right) + 2k_3 \chi \delta \zeta \\
+ \frac{2}{3} d \chi^4 \left( \frac{\delta}{\sigma^2 - \delta^2} \right) - \sum g^i_\delta \rho^s_i = 0, \quad (11)
\]
Coupled equations of motion:

\[
\frac{\partial \Omega}{\partial \chi} = k_0 \chi (\sigma^2 + \zeta^2 + \delta^2) - k_3 (\sigma^2 - \delta^2) \zeta + \chi^3 \left[ 1 + \ln \left( \frac{\chi^4}{\chi_0^4} \right) \right]
\]

\[
+ (4k_4 - d)\chi^3 - \frac{4}{3} d\chi^3 \ln \left( \left( \frac{\sigma^2 - \delta^2}{\sigma_0^2 \zeta_0} \right) \left( \frac{\chi}{\chi_0} \right) \right)
\]

\[
\frac{2\chi^2}{m^2_{\pi} \pi} \left[ m^2_{\pi} f_{\pi} \sigma + \left( \sqrt{2} m^2_K f_K - \frac{1}{\sqrt{2}} m^2_{\pi} f_{\pi} \right) \zeta \right] - \frac{\chi^2}{\chi_0^2} (m_{\omega}^2 \omega^2 + m_{\rho}^2 \rho^2) = 0, \quad (12)
\]

\[
\frac{\partial \Omega}{\partial \omega} = \frac{\chi^2}{\chi_0^2} m_{\omega}^2 \omega + 4g_{4\omega}^3 + 12g_{4\omega}^2 \rho^2 - \sum_{i=u,d} g_{\omega}^i \rho_{i}^\nu = 0, \quad (13)
\]

\[
\frac{\partial \Omega}{\partial \rho} = \frac{\chi^2}{\chi_0^2 m_{\rho}^2 \rho + 4g_{4\rho}^3 + 12g_{4\omega}^2 \rho^2} - \sum_{i=u,d} g_{\rho}^i \rho_{i}^\nu = 0, \quad (14)
\]

\[
\frac{\partial \Omega}{\partial \phi} = \frac{\chi^2}{\chi_0^2} m_{\phi}^2 \phi + 8g_{4\phi}^3 - \sum_{i=s} g_{\phi}^i \phi_{i}^\nu = 0. \quad (15)
\]
\[
\frac{\partial \Omega}{\partial \Phi} = \left[ \frac{-a(T)\Phi}{2} - \frac{6b(T)(\Phi - 2\Phi^2 + \Phi^2\bar{\Phi})}{1 - 6\Phi\bar{\Phi} + 4(\Phi^3 + \Phi^3) - 3(\Phi\bar{\Phi})^2} \right] T^4 - \sum_{i=u,d,s} \frac{2k_B T N_C}{(2\pi)^3}
\]

\[
\int_0^\infty d^3k \left[ \frac{e^{-2(E_i^*-\nu_i)/k_B T}}{(1 + e^{-3(E_i^*-\nu_i)/k_B T} + 3\Phi e^{-(E_i^*-\nu_i)/k_B T} + 3\bar{\Phi} e^{-2(E_i^*-\nu_i)/k_B T})} \right]
\]

\[
= 0, \quad (16)
\]

\[
\frac{\partial \Omega}{\partial \bar{\Phi}} = \left[ \frac{-a(T)\bar{\Phi}}{2} - \frac{6b(T)(\Phi - 2\bar{\Phi}^2 + \bar{\Phi}^2\Phi)}{1 - 6\Phi\bar{\Phi} + 4(\Phi^3 + \Phi^3) - 3(\Phi\bar{\Phi})^2} \right] T^4 - \sum_{i=u,d,s} \frac{2k_B T N_C}{(2\pi)^3}
\]

\[
\int_0^\infty d^3k \left[ \frac{e^{-(E_i^*+\nu_i)/k_B T}}{1 + e^{-3(E_i^*+\nu_i)/k_B T} + 3\Phi e^{-(E_i^*+\nu_i)/k_B T} + 3\bar{\Phi} e^{-2(E_i^*+\nu_i)/k_B T}} \right]
\]

\[
= 0, \quad (17)
\]
In the presence of the finite magnetic field, the total effective energy of the quarks is modified as

$$E_i^* = \sqrt{p_z^2 + m_i^*^2 + |q_i| (2n + 1 - \Upsilon)} B,$$

and

$$p = \sqrt{p_z^2 + |q_i| (2n + 1 - \Upsilon)} B.$$  

Thus, the total thermodynamical potential is altered, and the term giving the contribution of quarks and antiquarks interaction is written as

$$\Omega_{q\bar{q}} = - \sum_{i=u,d,s} |q_i| BT \frac{2\pi}{\alpha_k} \int_{-\infty}^{\infty} dp_z \left( \ln g_i^+ + \ln g_i^- \right).$$
Finite volume effects

- For this, a lower limit on momentum can be defined, i.e., \( k_{min} = \frac{\pi}{R} = \lambda \) for zero and finite magnetic field.
- The implication of finite size discretizes the four-momentum component, \((p_0, \vec{p})\) applying, \(p_0 \rightarrow \frac{2\pi}{\beta}(n + \frac{1}{2})\) and \(\vec{p} \rightarrow \frac{2\pi}{R}(n + c)\), here, \(n = 0, \pm 1, \pm 2\ldots\) and \(T = \frac{1}{\beta}\). Here, \(c = 0\) and \(1/2\) corresponds to periodic and anti-periodic boundary conditions, respectively.
- By employing the technique of analytic continuation, the discrete sum of energies can be written as a continuous integral over momentum states.
Results

(a) \( \mu_n = 0 \text{ MeV} \)

(b) \( \mu_n = 200 \text{ MeV} \)

(c) \( \sigma \) (MeV)

(d) \( \sigma \) (MeV)

(e) \( \delta \) (MeV)

(f) \( \delta \) (MeV)

(g) \( N \) (MeV)

(h) \( N \) (MeV)

\( R = \infty \quad R = 4 \text{ fm} \quad R = 5 \text{ fm} \quad R = 2 \text{ fm} \)
Results
Results

(a) $eB = 0 \text{ GeV}^2$, $R = \infty$

(b) $eB = 0 \text{ GeV}^2$, $R = \infty$

(c) $eB = 0 \text{ GeV}^2$, $R = 2 \text{ fm}$

(d) $eB = 0 \text{ GeV}^2$, $R = 2 \text{ fm}$

(e) $eB = 0.4 \text{ GeV}^2$, $R = \infty$

(f) $eB = 0.4 \text{ GeV}^2$, $R = \infty$

Legend: $\mu_B = 0 \text{ MeV}$, $\mu_B = 400 \text{ MeV}$, $\mu_B = 200 \text{ MeV}$, $\mu_B = 600 \text{ MeV}$
Results

(a) and (b) show the dependence of $T$ on $T$ and $\mu_q$ for different values of $eB$. The graphs illustrate the crossover and deconfinement phases for various magnetic fields and quark transition points.

- **Crossover**
  - $eB = 0$ GeV$^2$
  - $eB = 0.1$ GeV$^2$
  - $eB = 0.2$ GeV$^2$
  - $eB = 0.4$ GeV$^2$

- **Deconfinement**
  - $eB = 0$ GeV$^2$
  - $eB = 0.1$ GeV$^2$
  - $eB = 0.2$ GeV$^2$
  - $eB = 0.4$ GeV$^2$

- **CEP**
  - 1st order

- **Quark Transition**
  - $eB = 0$
  - $eB = 0.4$ GeV$^2$

- **Other Parameters**
  - $R = \infty$
  - $R = 5$ fm
  - $R = 4$ fm
  - $R = 2$ fm

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Results

(a) $eB = 0$

(b) $R = \infty$

(c) $eB = 0$ GeV

(d) $eB = 0.1$ GeV

(e) $eB = 0.2$ GeV

(f) $eB = 0.4$ GeV

- $\chi_4^B$
- $\chi_4^B$
- $\chi_4^B$
- $\chi_4^B$

- lattice data for $(N = 8)$
- lattice data for $(N = 12)$

- $R = \infty$
- $R = 5$ fm
- $R = 4$ fm
- $R = 2$ fm

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Results

(a) $eB = 0$
- $R = \infty$
- $R = 5 \text{ fm}$
- $R = 4 \text{ fm}$
- $R = 2 \text{ fm}$

(b) $R = \infty$
- $eB = 0 \text{ GeV}^2$
- $eB = 0.1 \text{ GeV}^2$
- $eB = 0.2 \text{ GeV}^2$
- $eB = 0.4 \text{ GeV}^2$

(c) $\chi_2^S$
- $T (\text{MeV})$

(d) $\chi_4^S$
- $T (\text{MeV})$

- lattice data for $(N_r = 8)$
- * lattice data for $(N_r = 12)$
Summary

- We have analyzed the impact of finite volume and external magnetic field on the thermodynamic properties using the Polyakov loop extended chiral SU(3) quark mean field model in the asymmetric quark matter.

- The results obtained have been compared with the lattice QCD simulations data for zero value of the magnetic field and infinite system size.

- In future work, we will calculate the fluctuations of conserved charges for varying magnetic field values and system volume for the non-zero value of chemical potential.

- The functional renormalization approach can be employed to study thermodynamic variables of quark matter beyond mean-field.