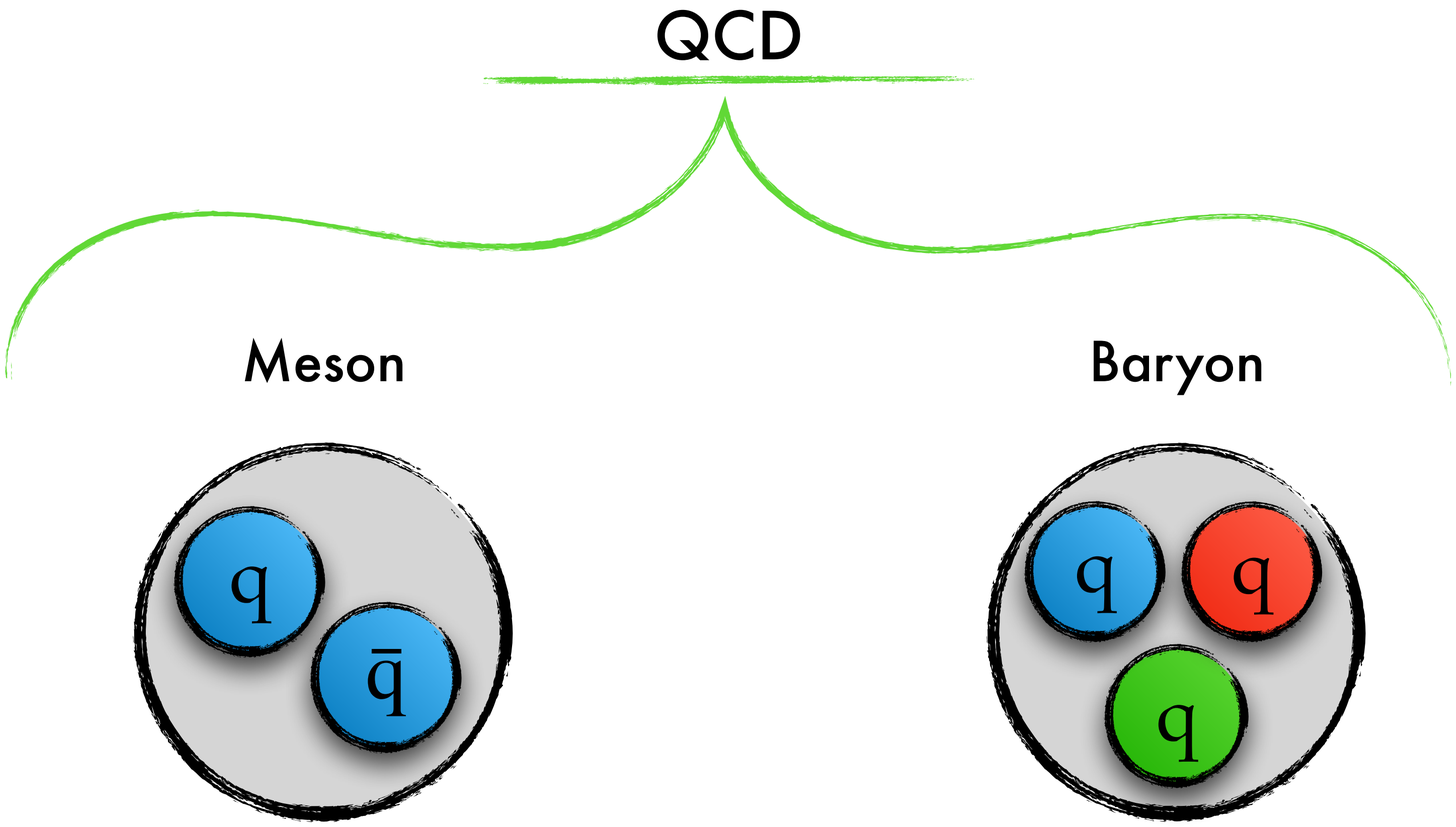
Constraining Hadronic Resonance Form Factors

Keegan Sherman[2], In collaboration with: Raúl Briceño[1,2], Andrew Jackura[1,2], and Felipe Ortega-Gama[1,3]
[1]Jefferson Lab, [2]Old Dominion University, [3]William & Mary

Hadron Structure

- Experiments show a rich spectrum of hadrons.

What is the structure of these states?



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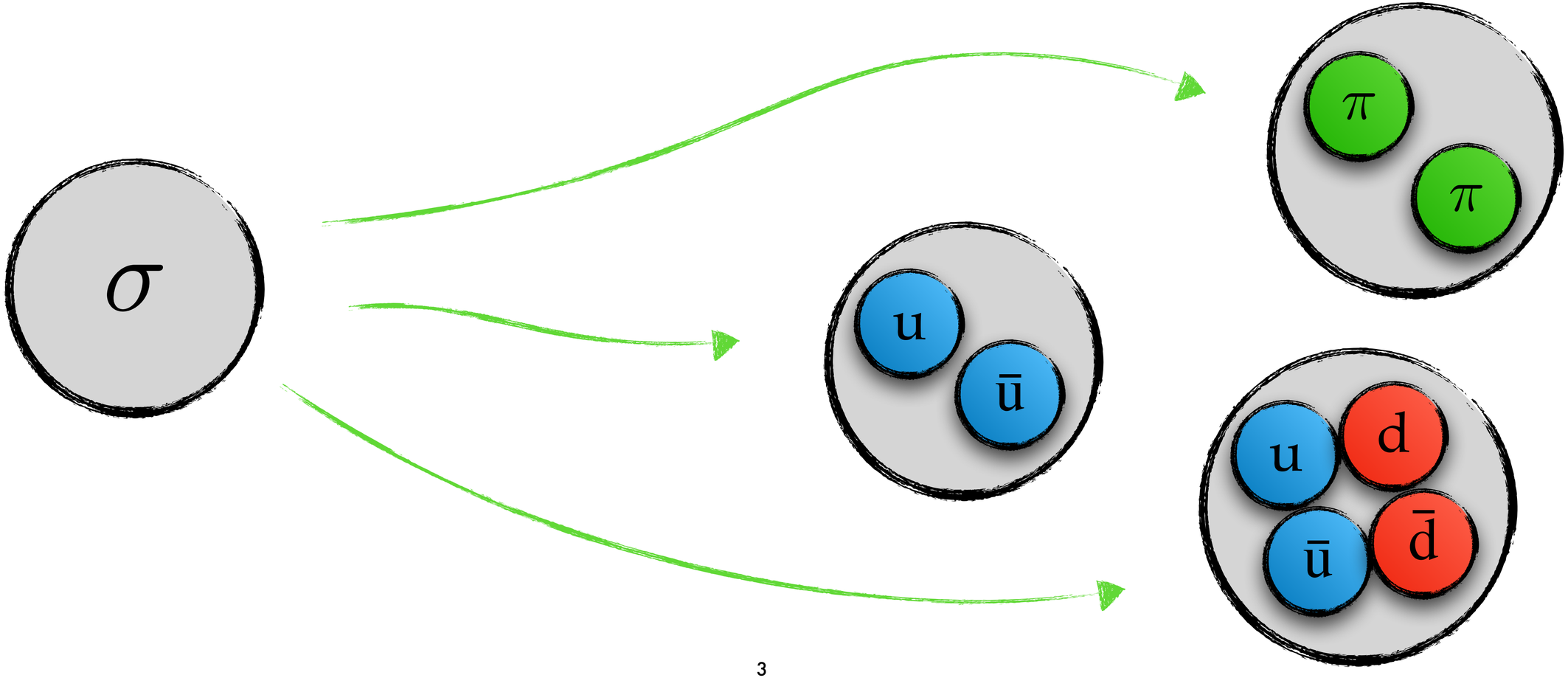
- QCD allows for more complex states.



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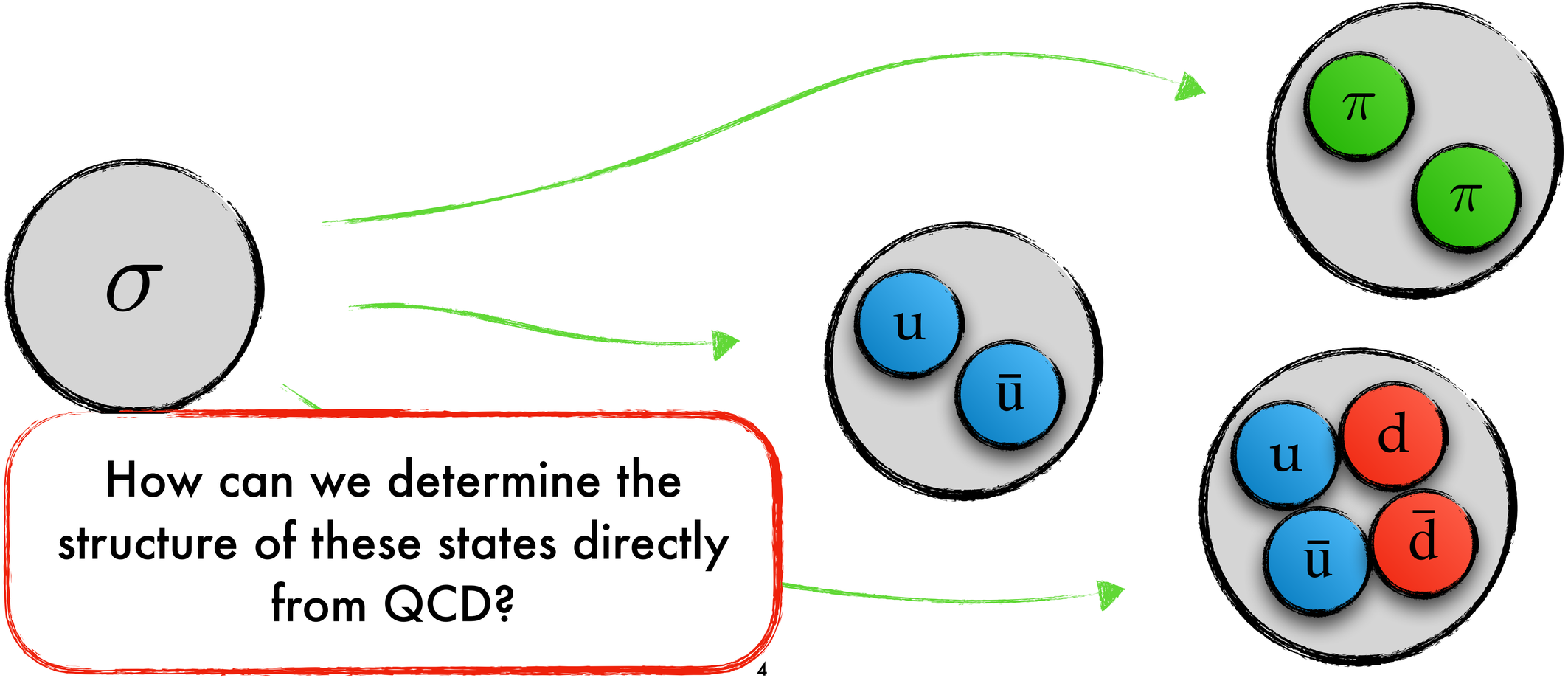

How can we determine the structure of these states directly from QCD?

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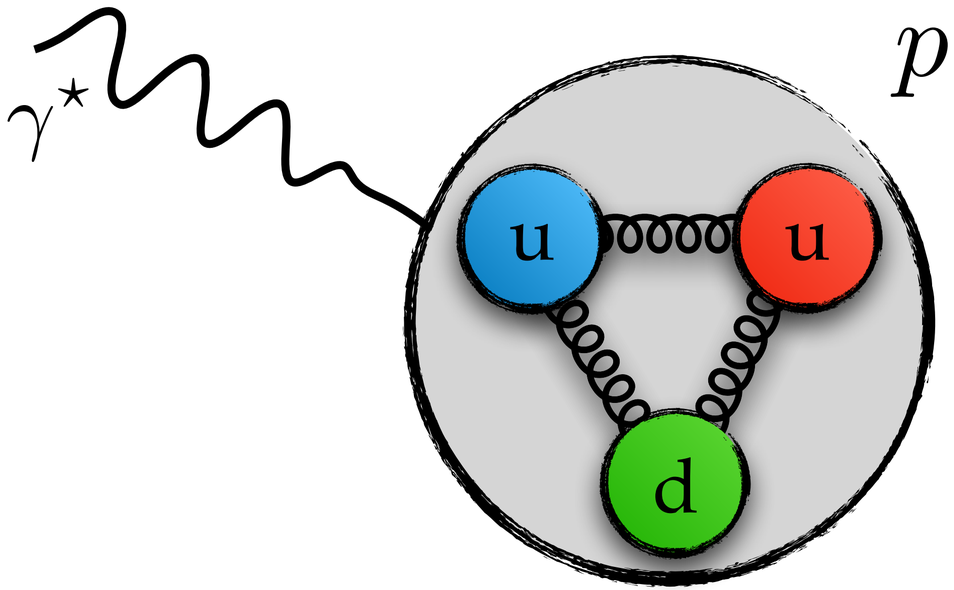

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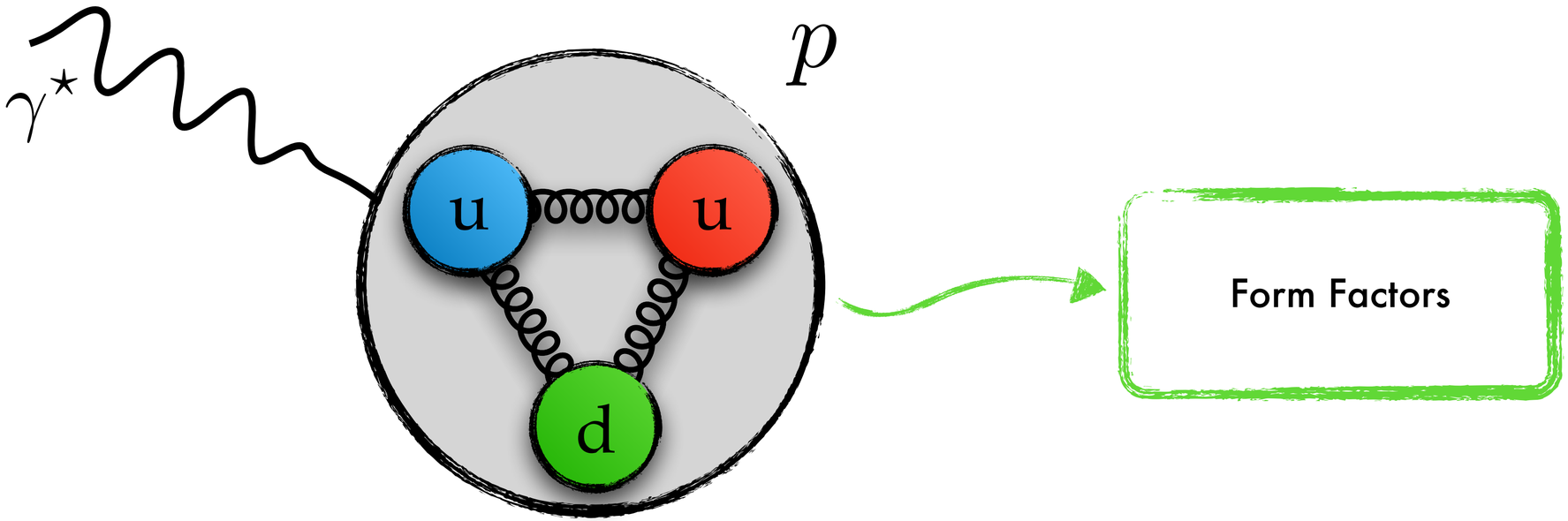
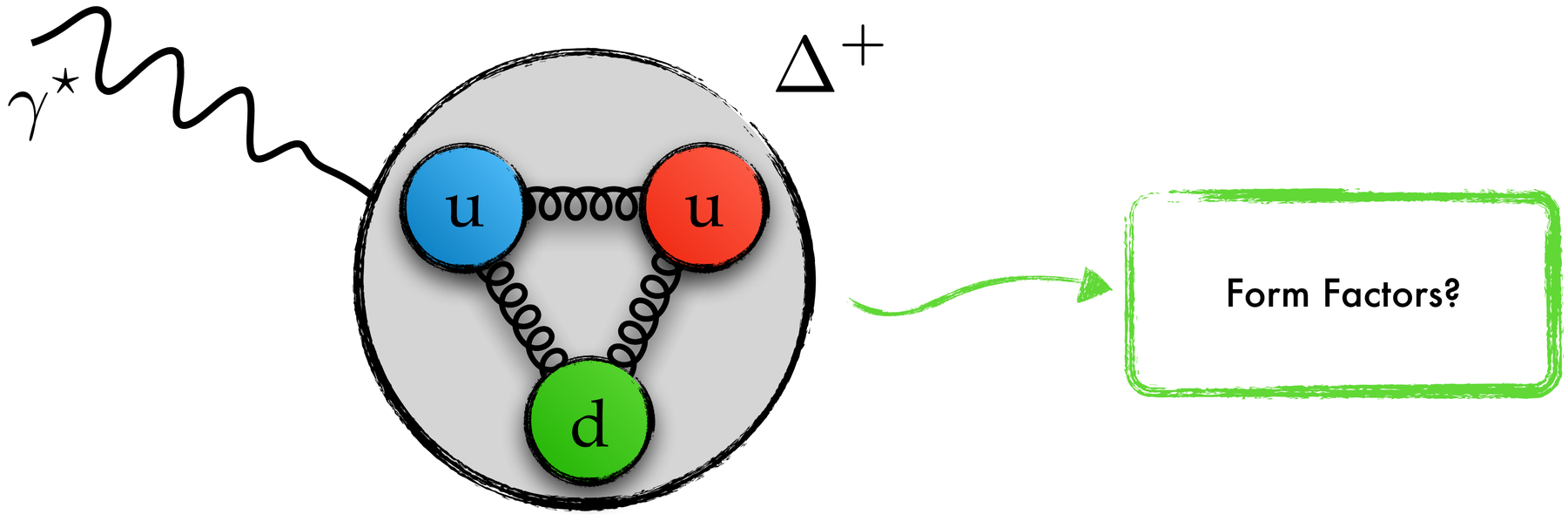
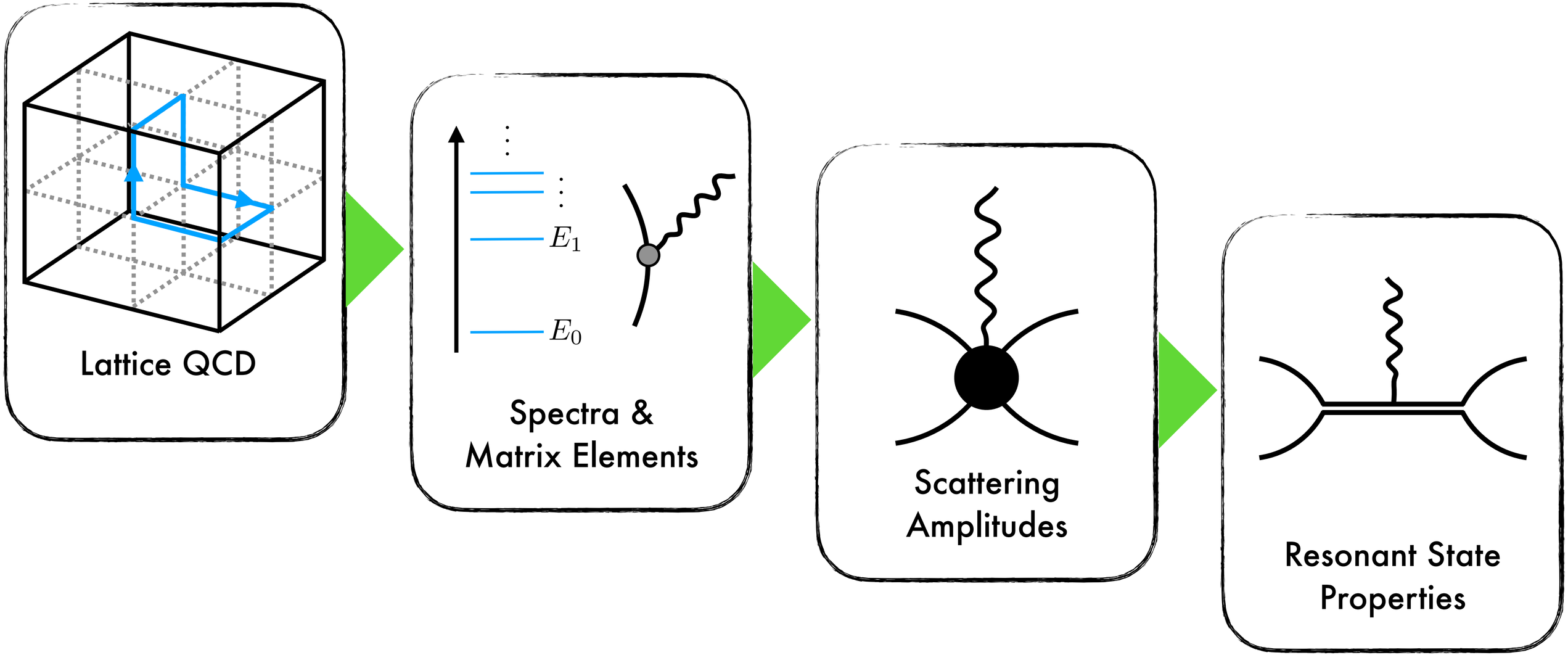


Hadron Structure

- How can we determine the structure of these states directly from QCD?
  - Can probe stable hadrons
  - What about unstable hadrons?



Hadronic Properties from QCD



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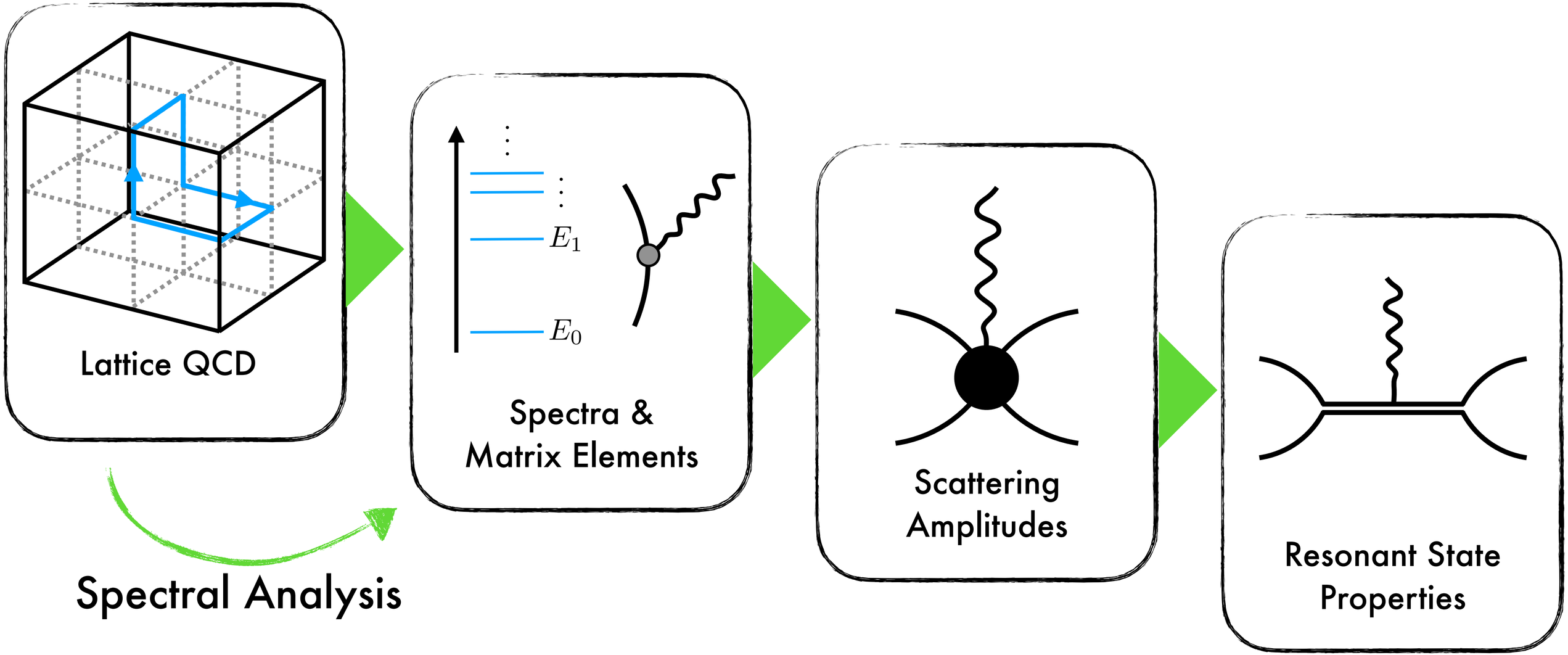

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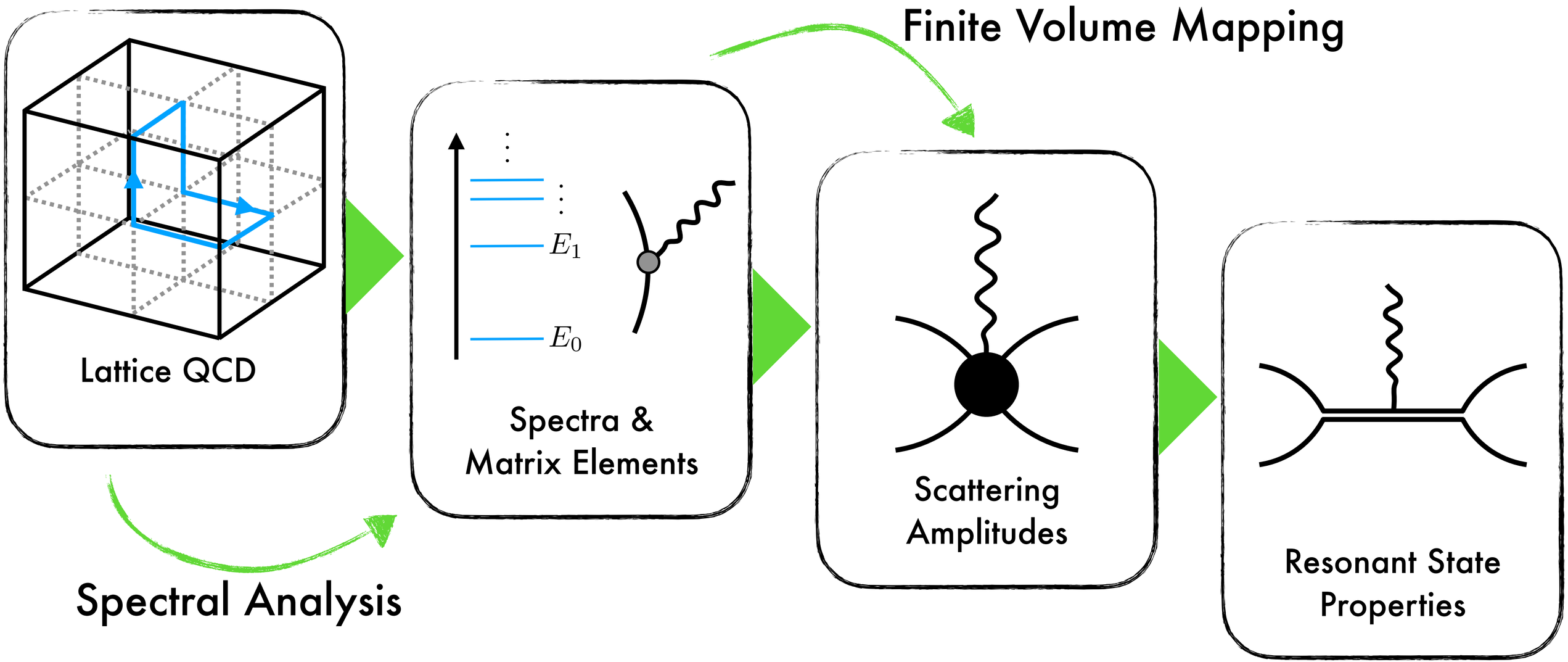
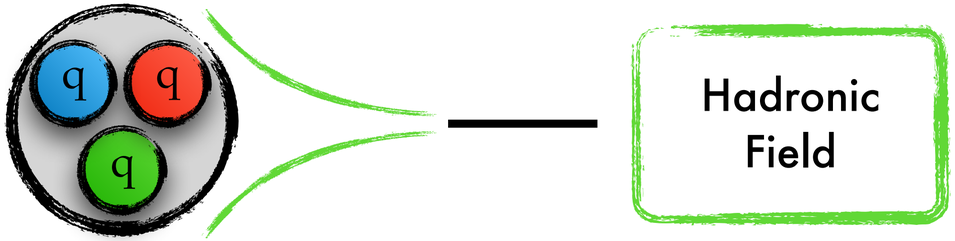


Finite Volume Mappings

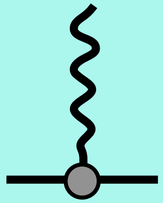


Mapping between finite volume and $2 + \mathcal{J} \to 2$ amplitude

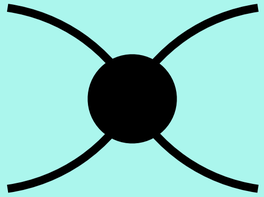
$$\langle m|\mathcal{J}|n\rangle = \frac{1}{L^3}\mathcal{W}_{L,\mathrm{df}}\sqrt{\mathcal{R}_{L,m}\mathcal{R}_{L,n}}$$

$$\mathcal{W}_{L,\mathrm{df}} = \mathcal{W}_{\mathrm{df}} + \mathcal{M}\cdot f \cdot G_L \cdot \mathcal{M}$$

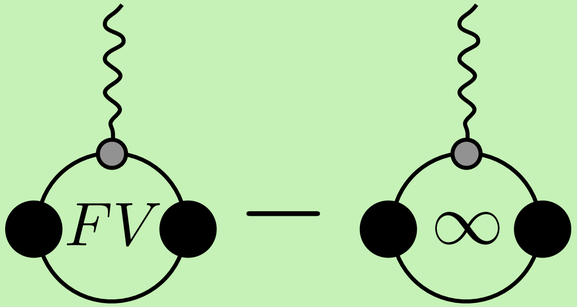
Single particle form factor $f =$



$2 \to 2$ Scattering Amplitude $\mathcal{M} =$

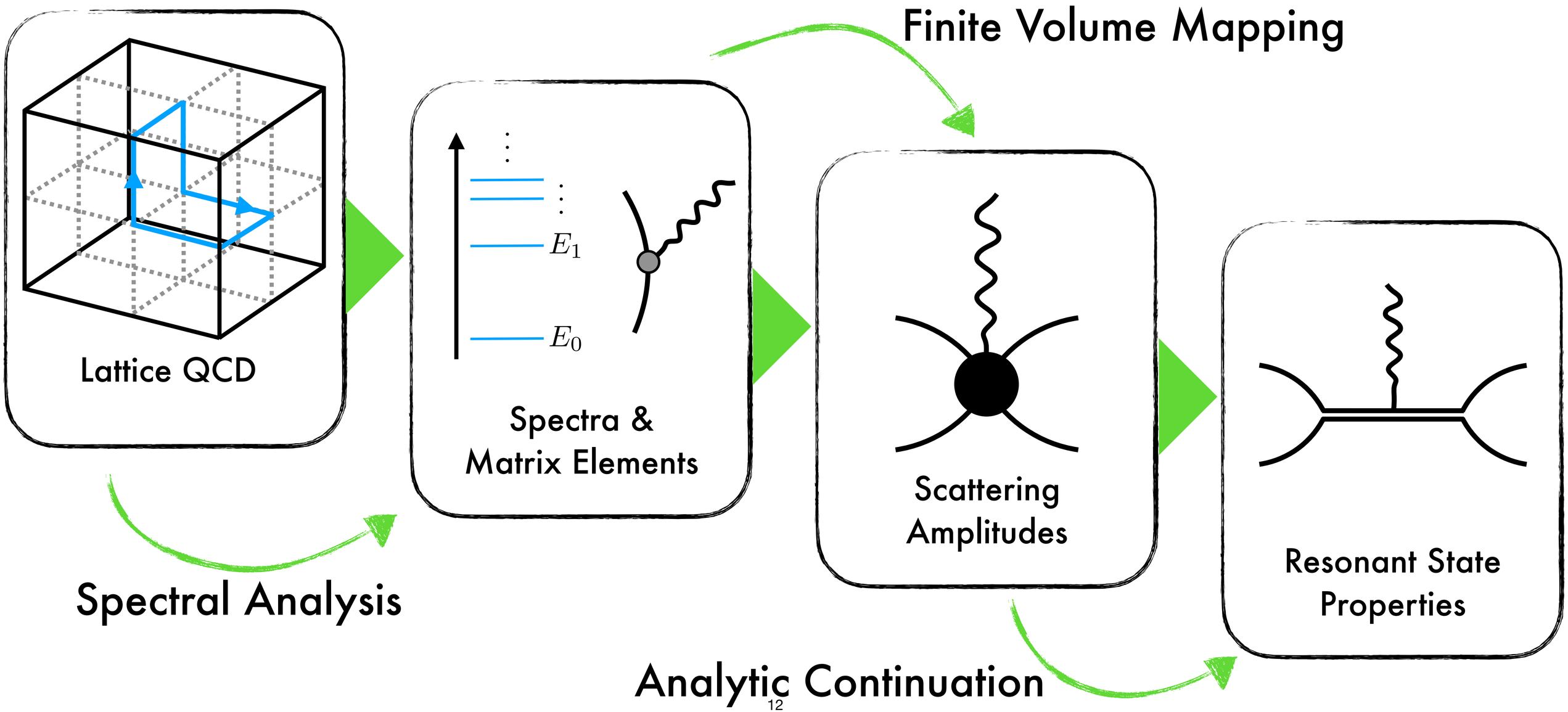


Finite volume geometric function $G_L =$

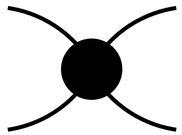


[Briceño, Hansen 2016]
[Baroni, Briceño, Hansen, Ortega-Gama 2019]

Hadronic Properties from QCD



Outline

- Two-body scattering
  - $i\mathcal{M} =$ 
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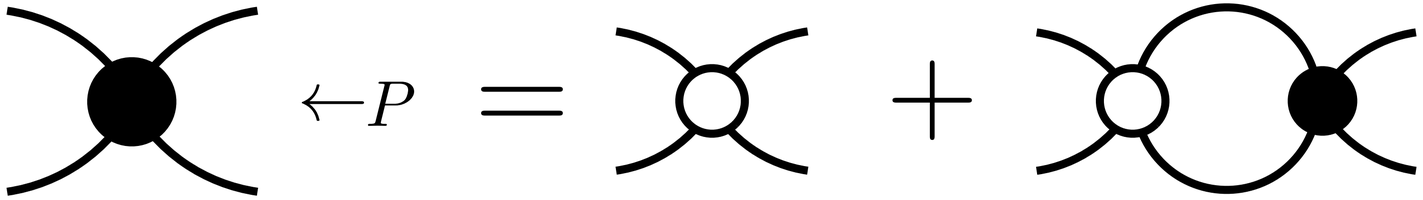
Two-Body Scattering

$i\mathcal{M} =$ 

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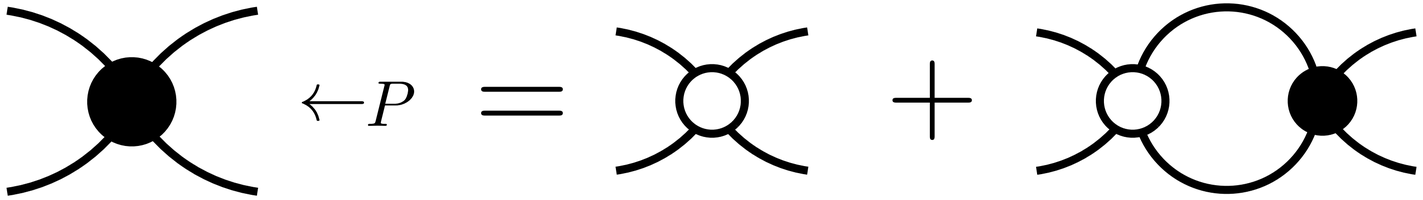
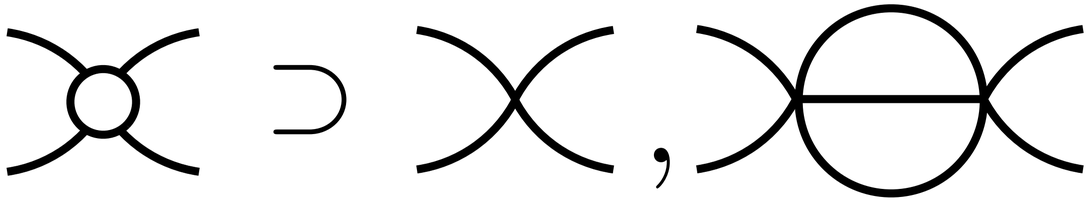
$i\mathcal{M} =$ 

$i\mathcal{K}_0 =$ 

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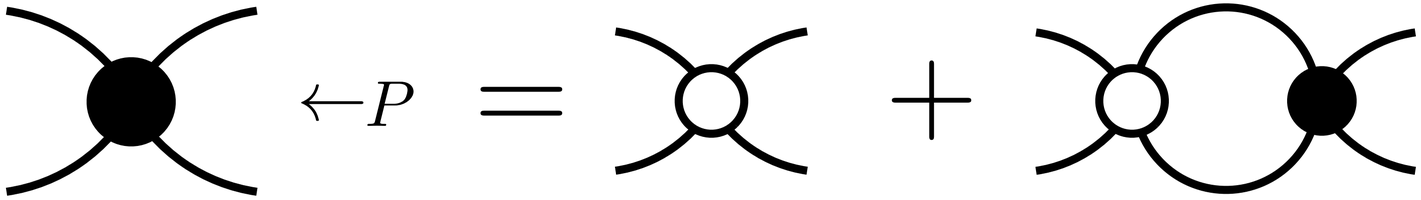
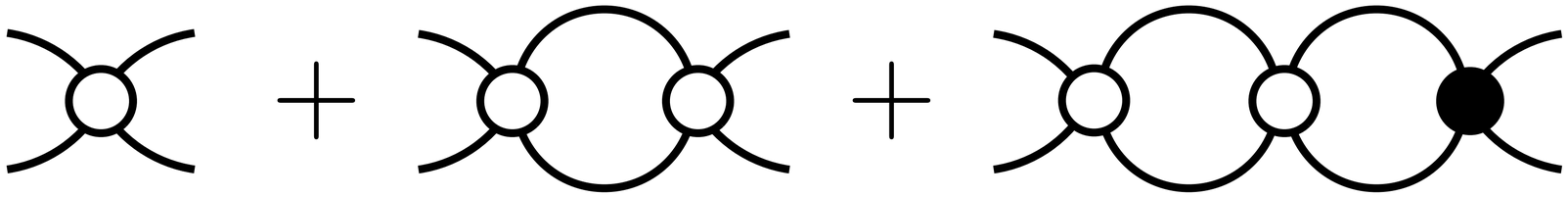
$i\mathcal{M} =$ 

$=$ 

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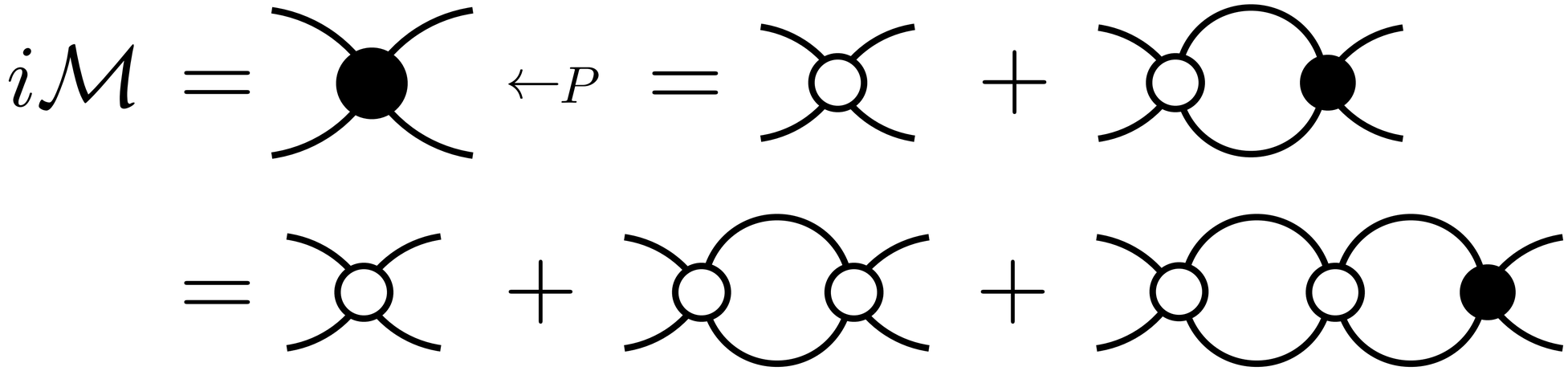
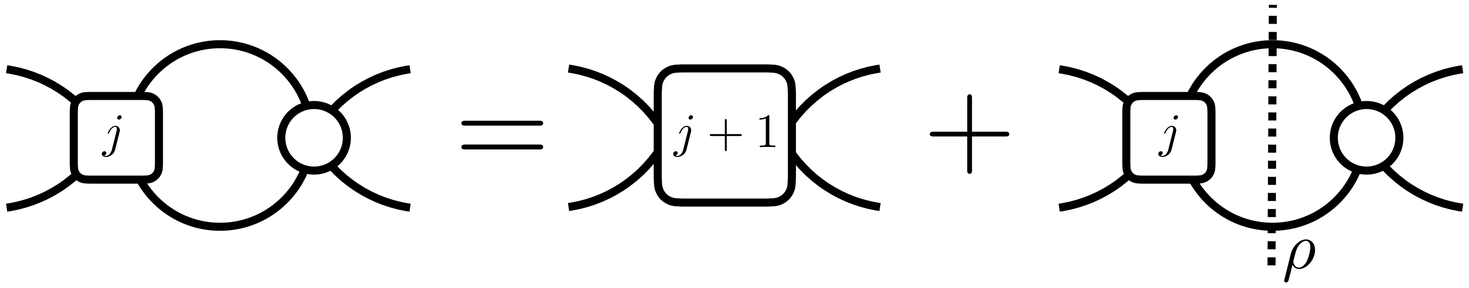


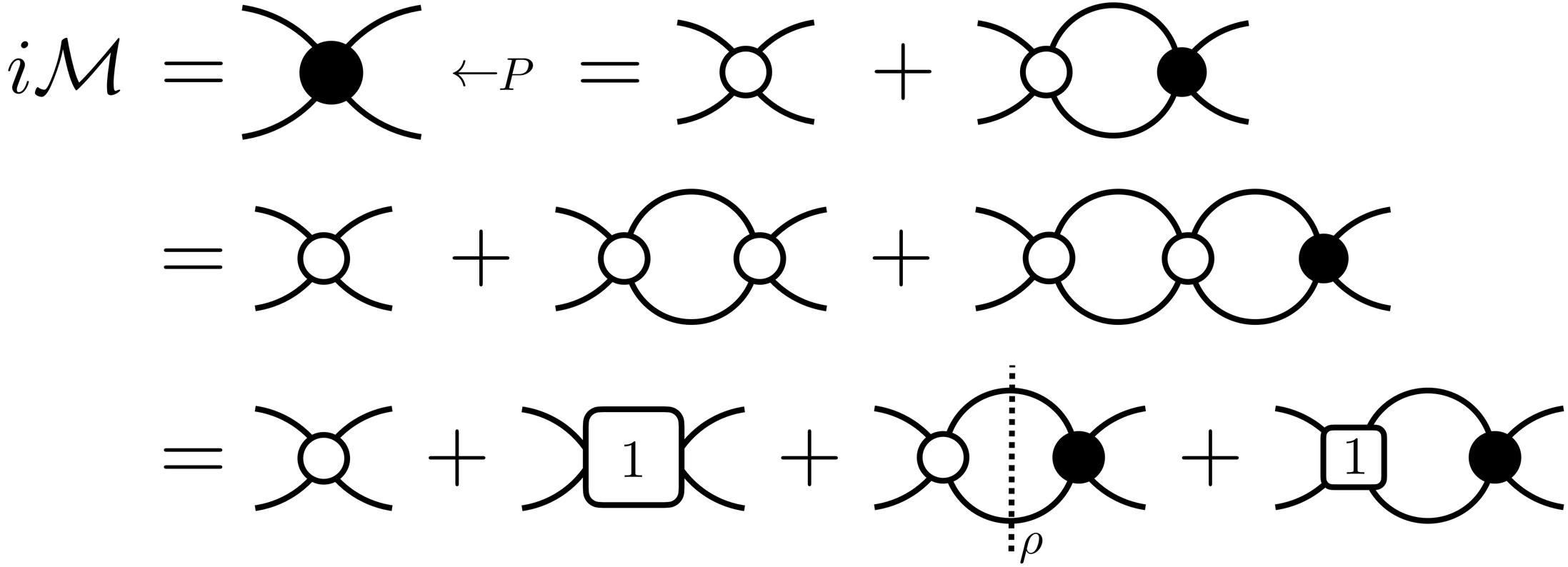
Iterated loop identity



$$\rho = \frac{\xi q^\star}{8\pi\sqrt{s}}$$

$$q^\star \sim \sqrt{s - s_{th}}$$

Two-Body Scattering



Two-Body Scattering

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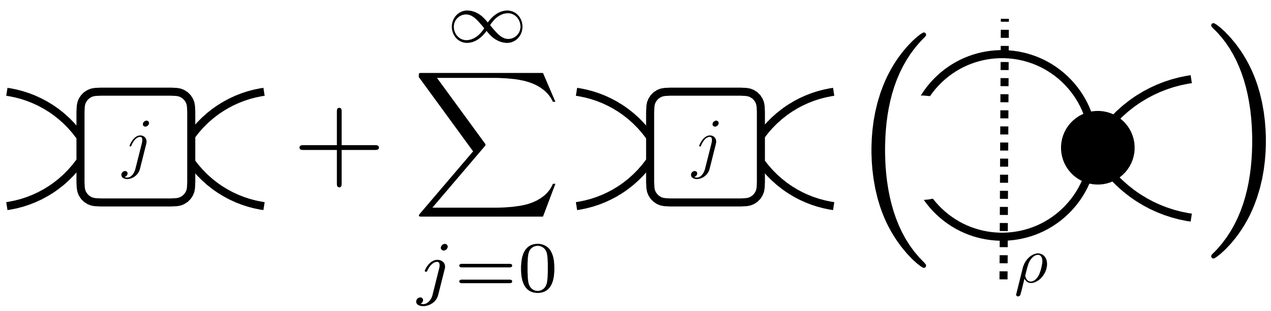
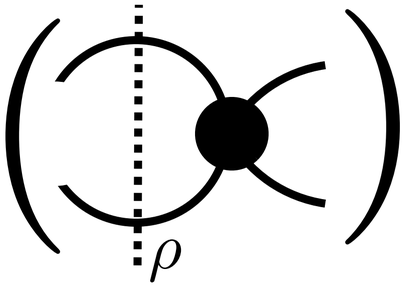


$$\equiv i\mathcal{K} + i\mathcal{K}\rho i\mathcal{M}$$

Two-Body Scattering

$$i\mathcal{M} = \sum_{j=0}^{\infty}$$

$$+ \sum_{j=0}^{\infty}$$



$$\equiv i\mathcal{K} + i\mathcal{K}\rho i\mathcal{M}$$

$$i\mathcal{M} = i\mathcal{K}\frac{1}{1 - i\rho\mathcal{K}} = \frac{i}{\mathcal{K}^{-1} - i\rho}$$

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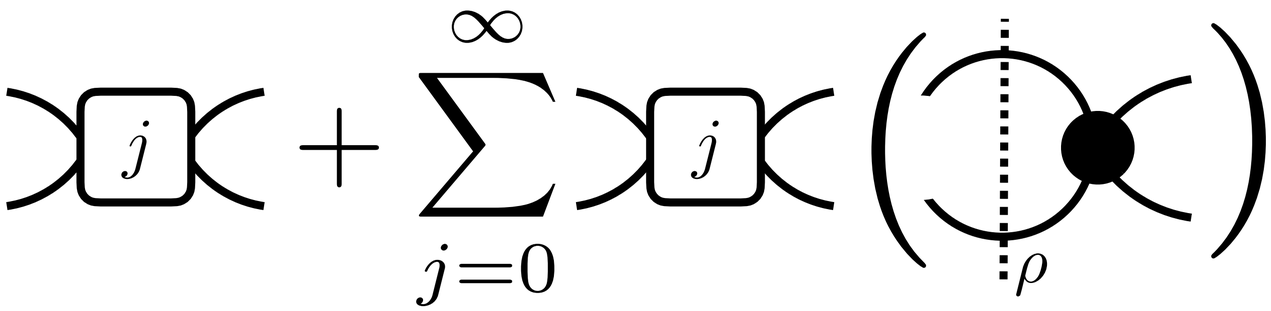

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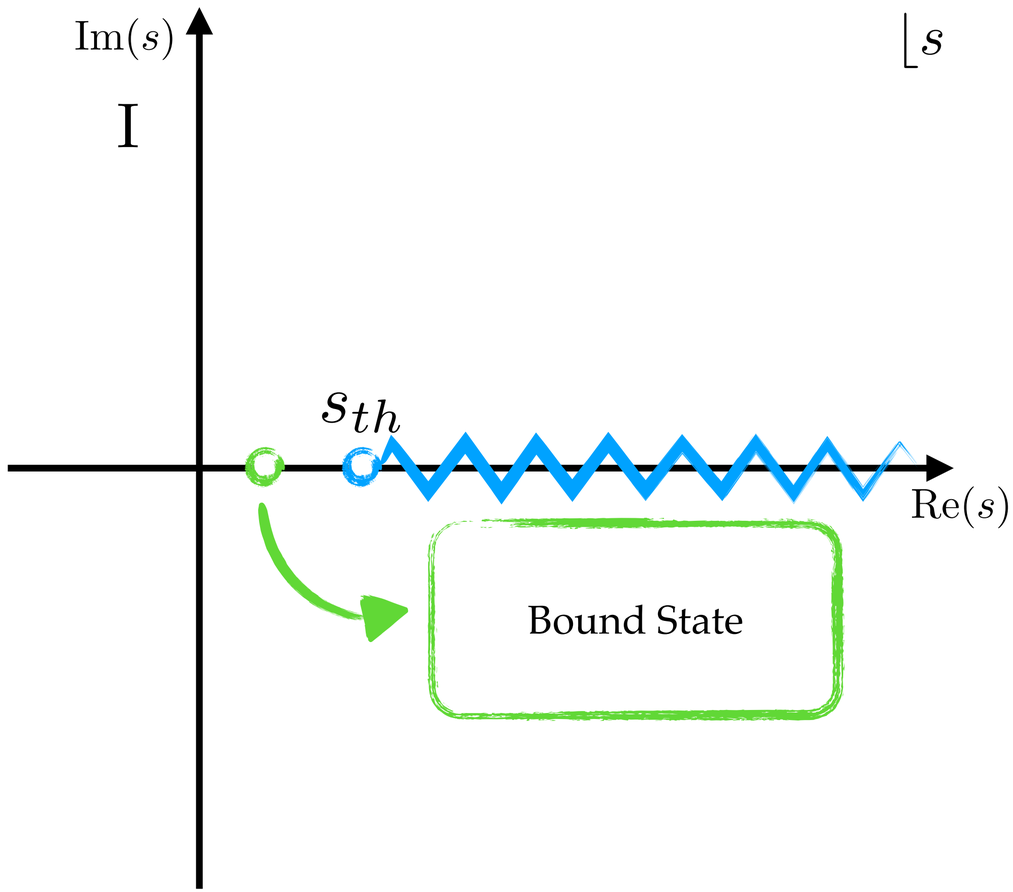
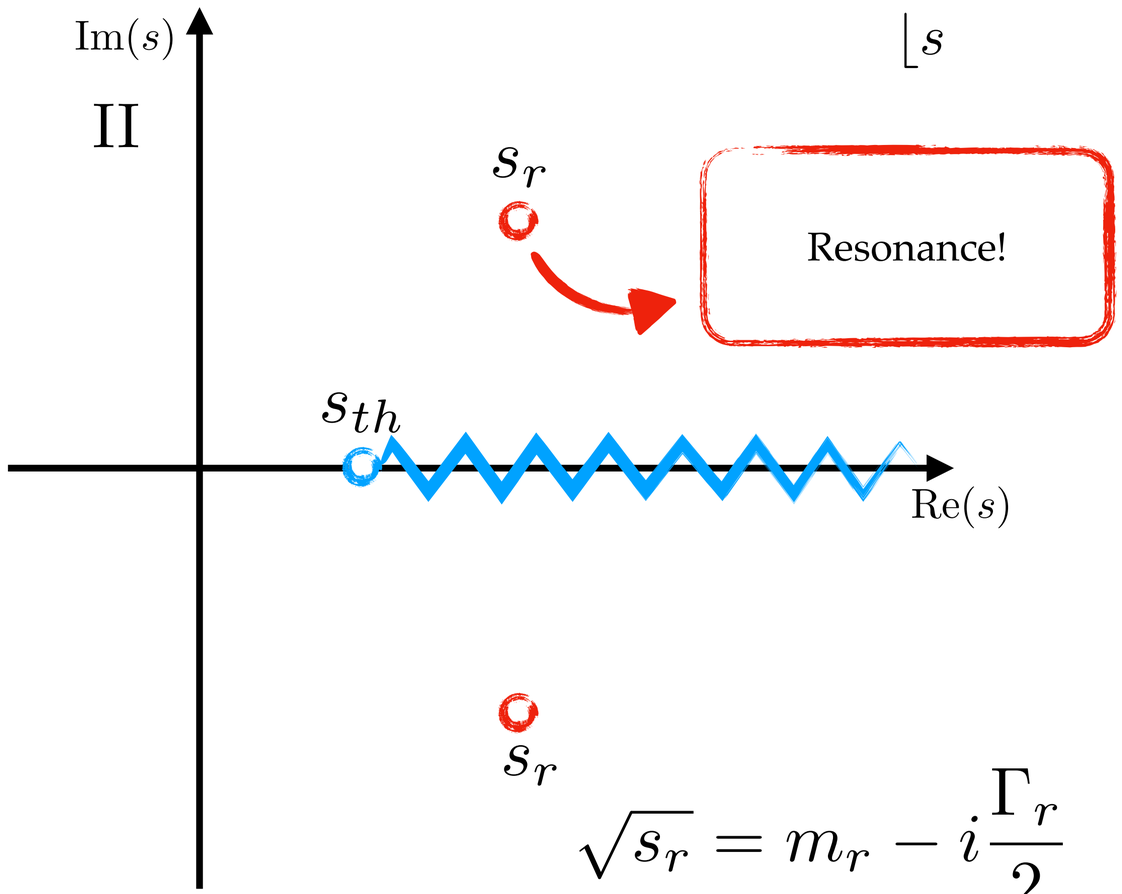
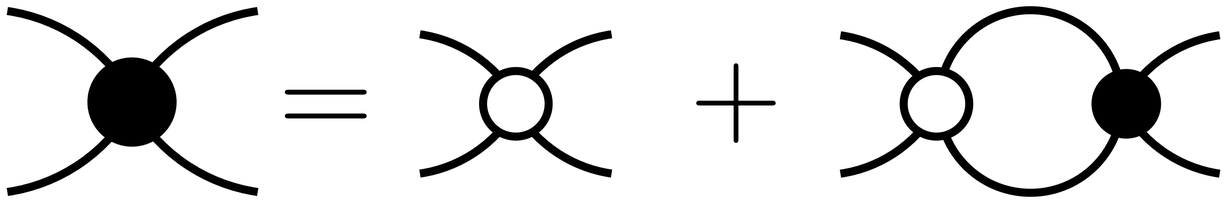
$$\mathrm{Im}(\mathcal{M}) \sim \sqrt{s - s_{th}}$$

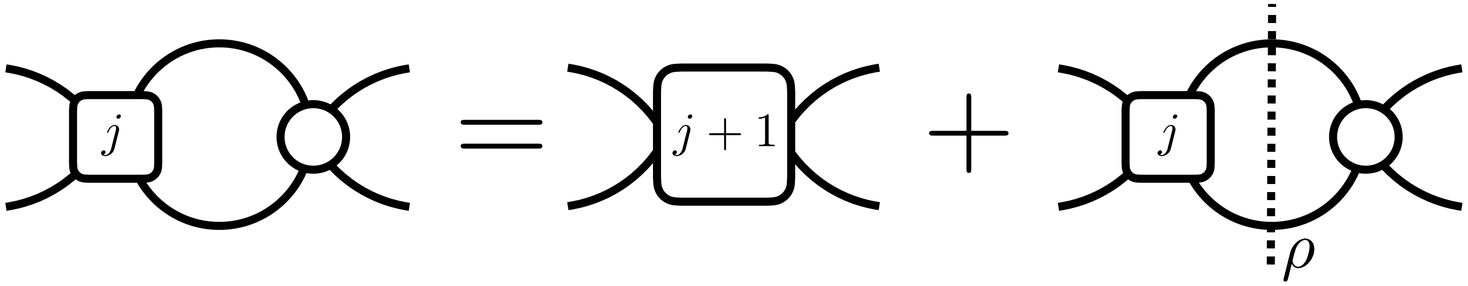




$$\sqrt{s_r} = m_r - i\frac{\Gamma_r}{2}$$

Two-Body Scattering - Take aways

$i\mathcal{M} =$ 



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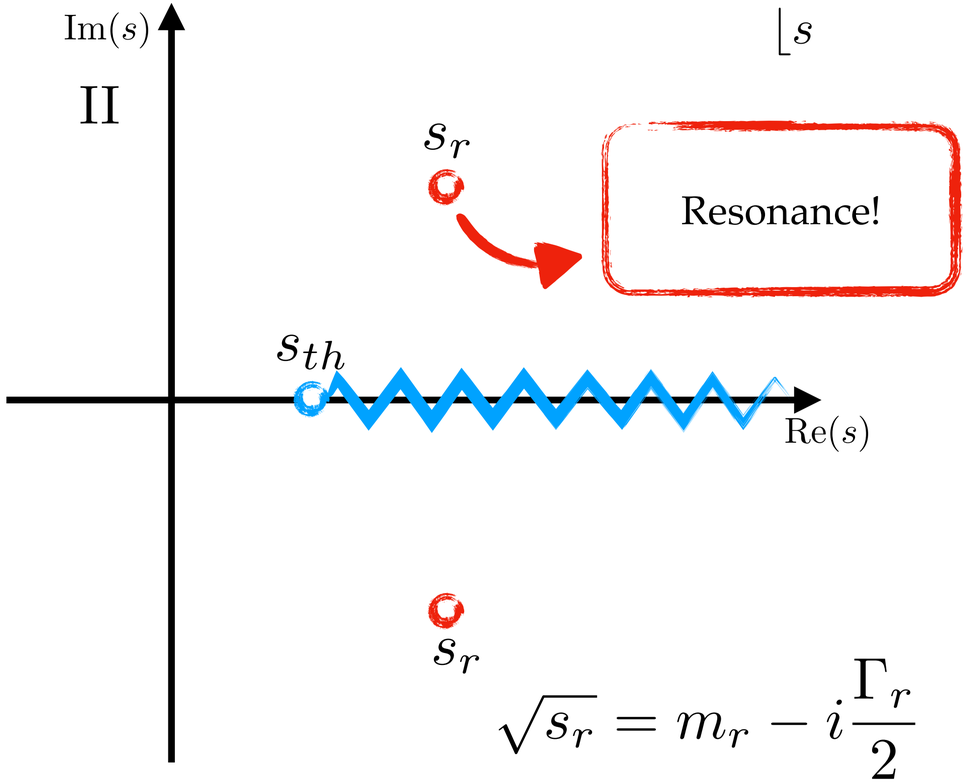

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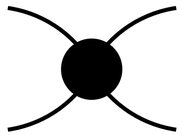
  - $i\mathcal{M} =$ 
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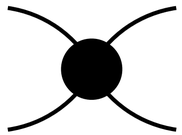
  - $i\mathcal{M} =$ 
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Breit-Wigner Model

- Model the K-Matrix with an s-wave Breit-Wigner.

$$\mathcal{M} = \frac{1}{\mathcal{K}^{-1} - i\rho}$$

Model Dependent

$$\mathcal{K} = \frac{1}{\rho}\frac{E^\star \Gamma(E^\star)}{m_0^2 - E^{\star 2}} \qquad \Gamma(E^\star) = \frac{g^2}{6\pi}\frac{m_0^2}{E^{\star 2}} q^\star$$

$\mathcal{M}$ Amplitude

$$m_1 = \pi$$
$$m_2 = n$$
$$m_0 = 1.23 \text{ GeV}$$

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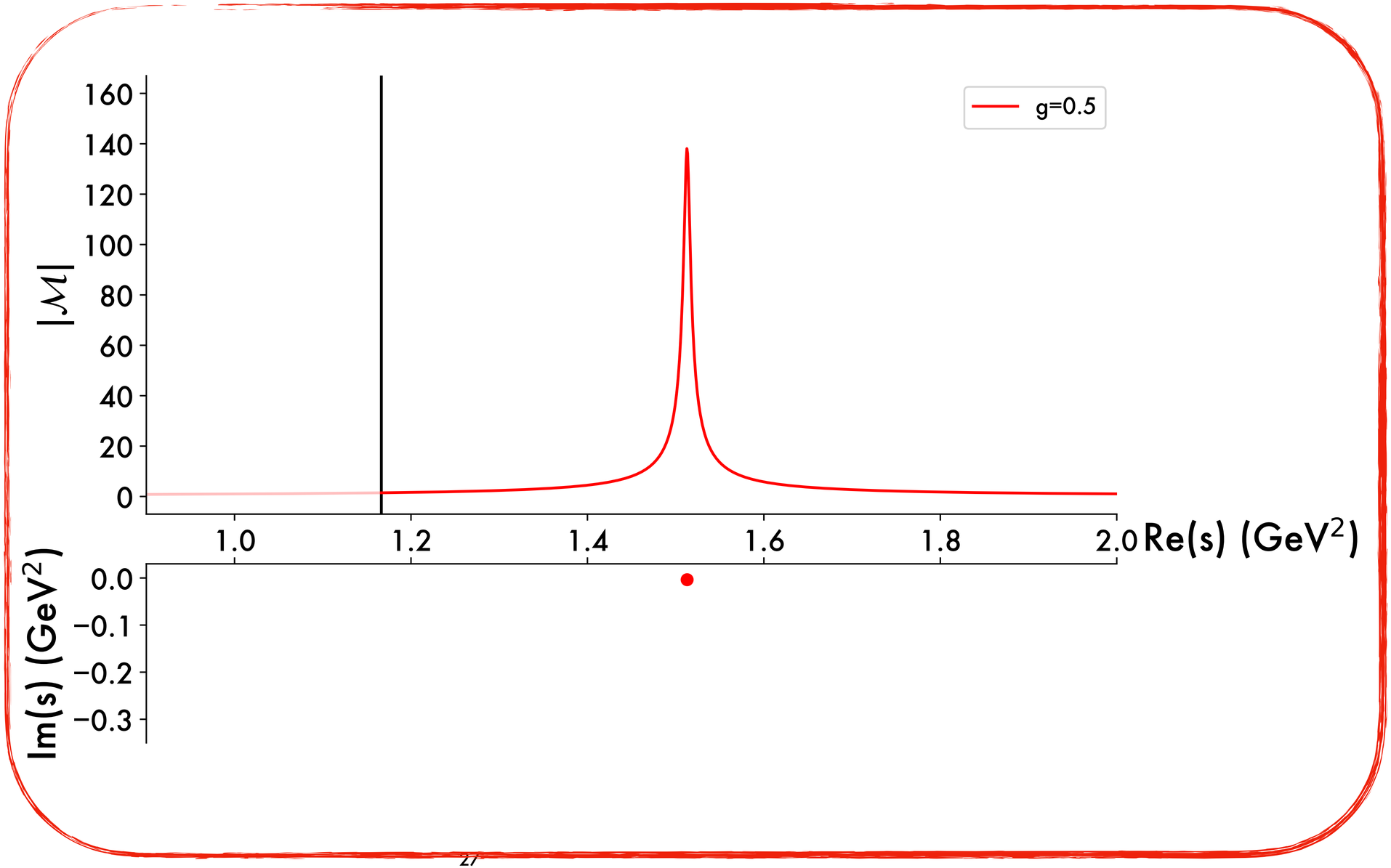

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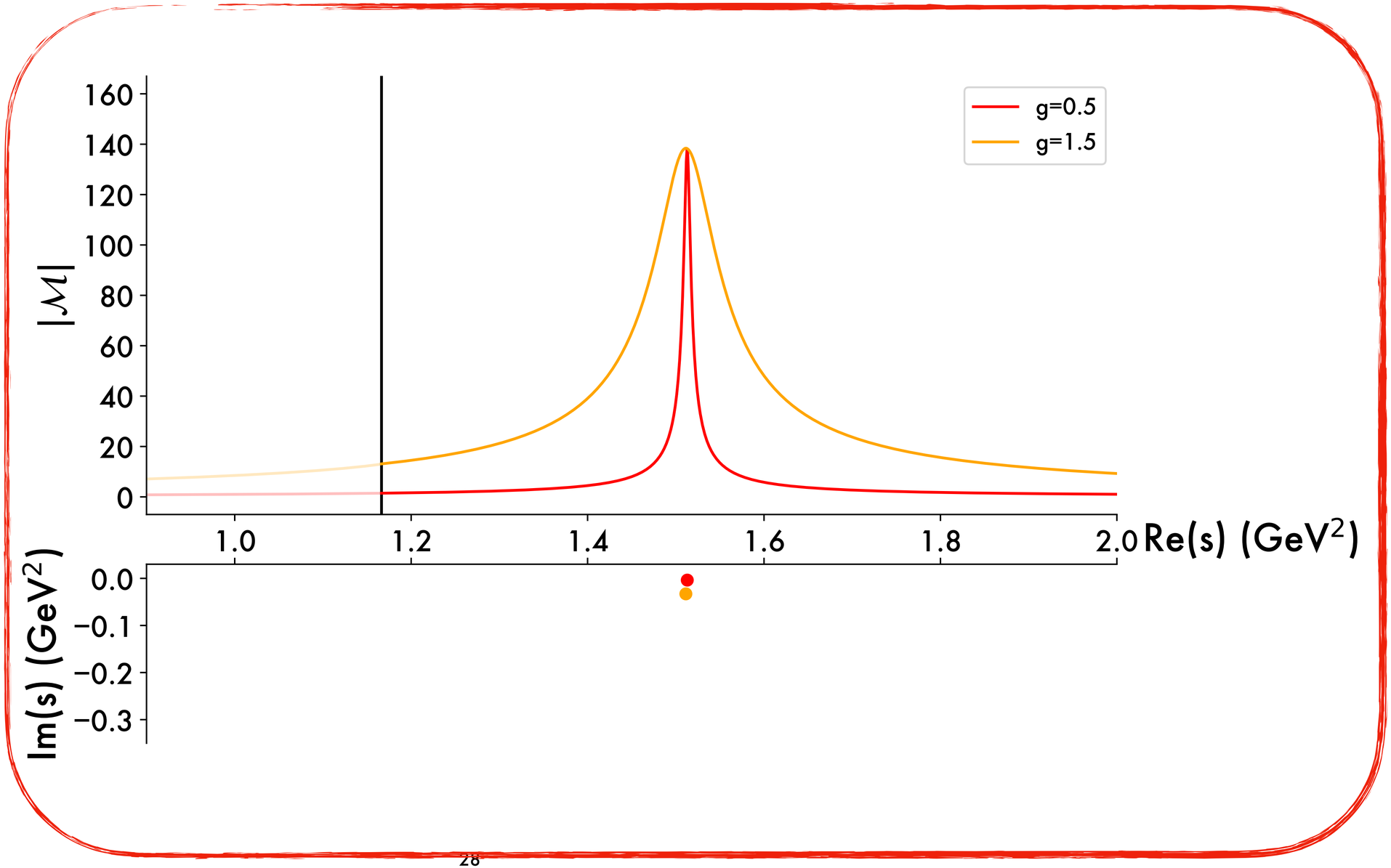

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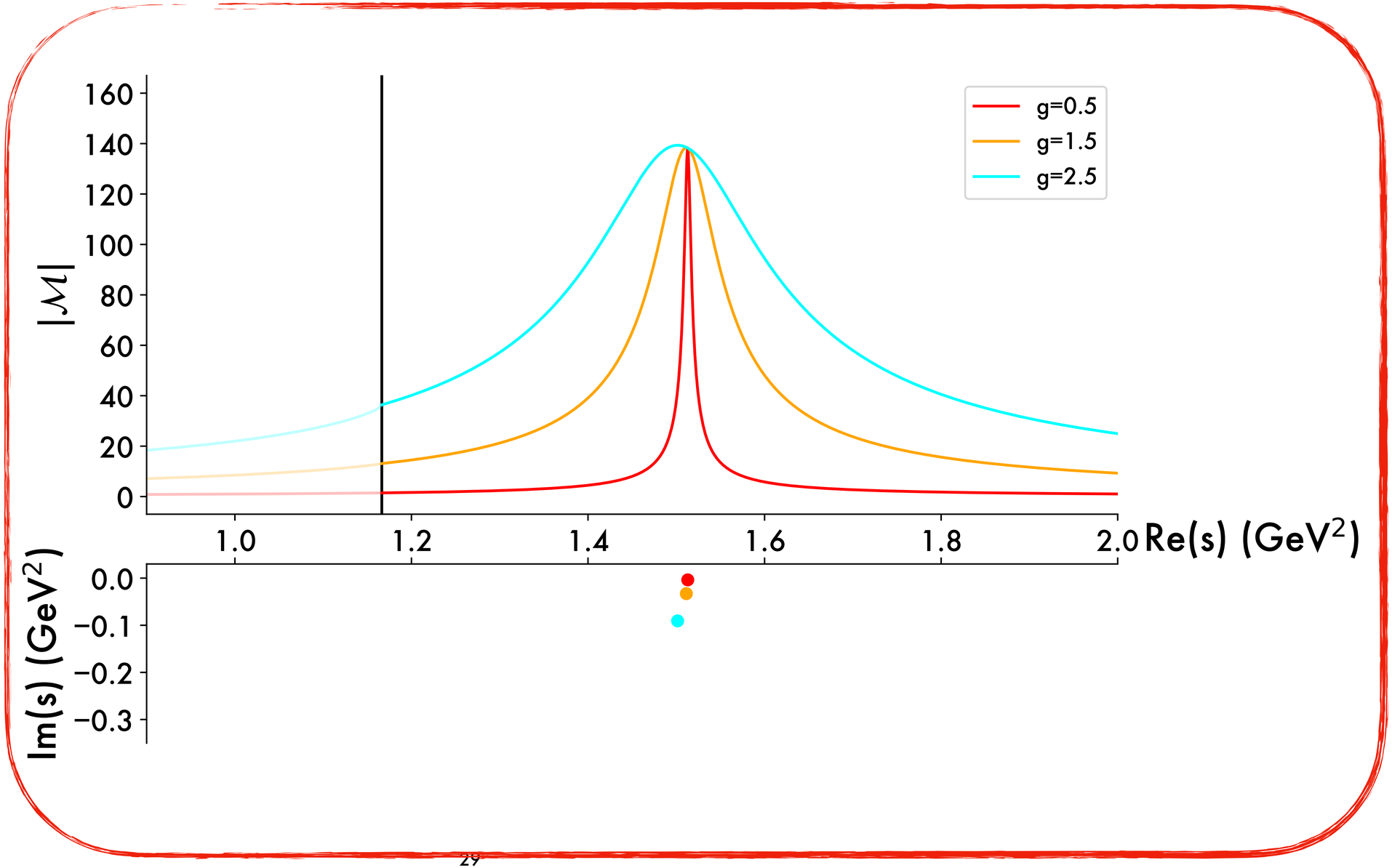

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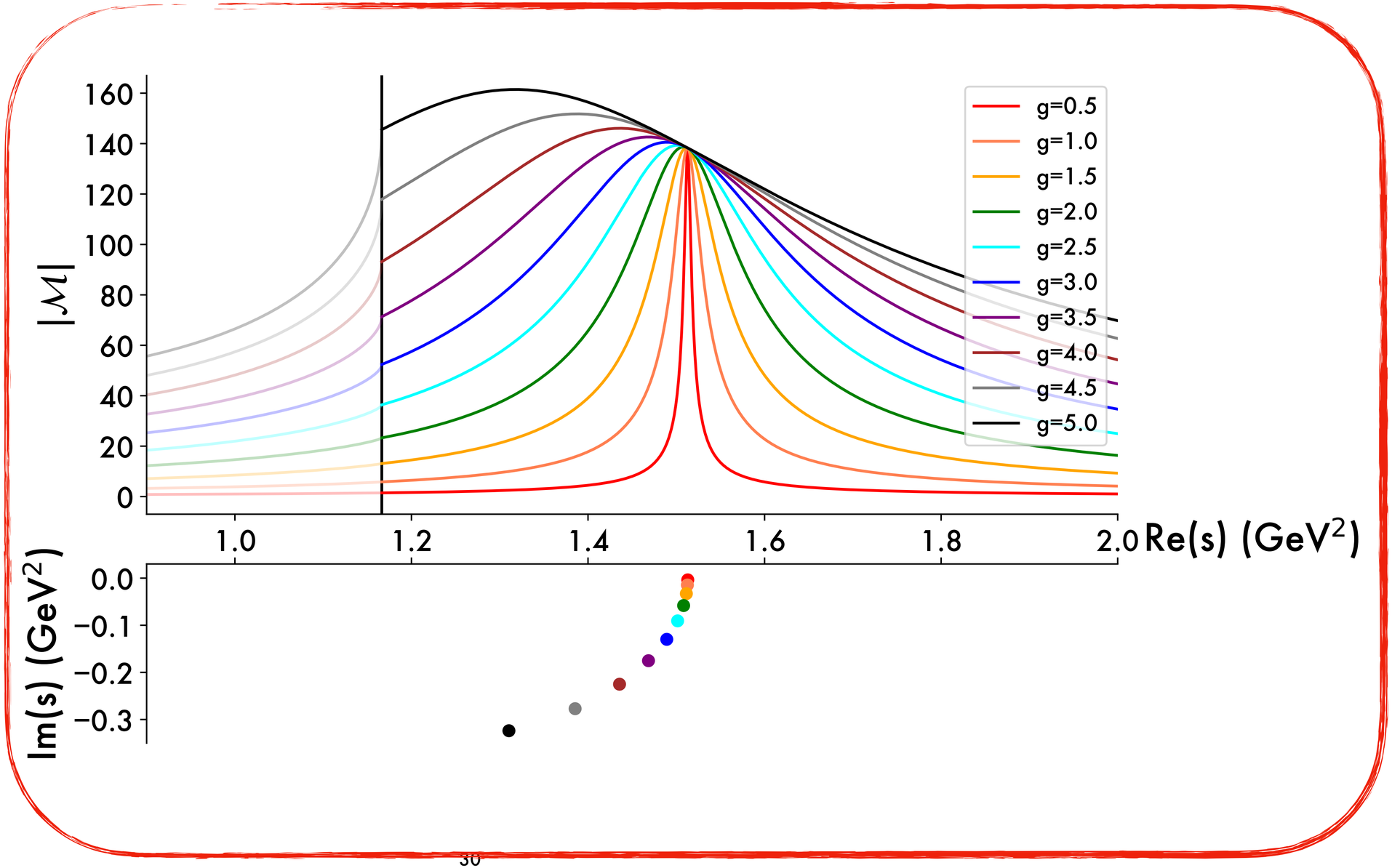

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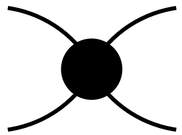
  - $i\mathcal{M} =$ 
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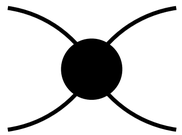
  - $i\mathcal{M} =$ 
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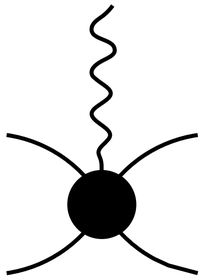
  - $i\mathcal{W} =$ 
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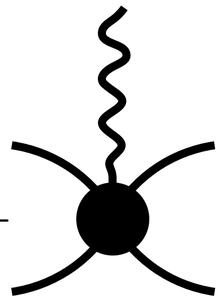
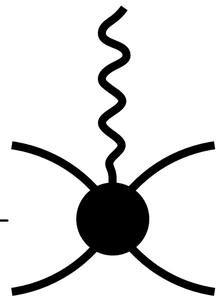
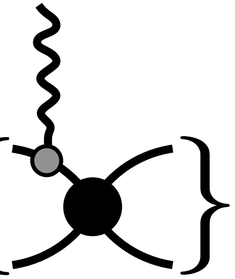
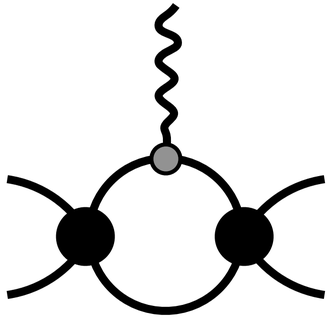
$2 + \mathcal{J} \to 2$ **Amplitude**

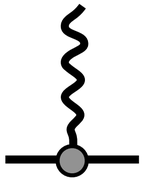
$i\mathcal{W} =$ $P_f \leftarrow$  $\leftarrow P_i$ $= i\mathcal{W}_{1B} + i\mathcal{W}_{\cancel{1B}}$

[Briceño, Hansen, 2015]

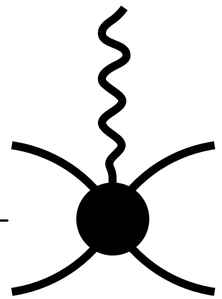
$2 + \mathcal{J} \to 2$ **Amplitude**

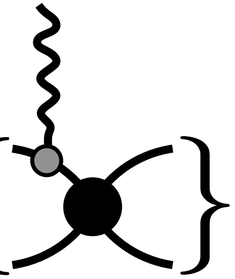
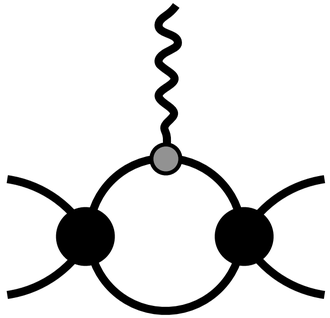
$i\mathcal{W} = P_f \leftarrow$  $\leftarrow P_i =$

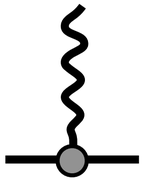
$=\sum\{$  $\}+$ 

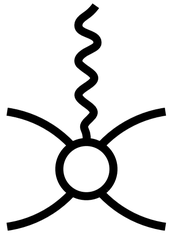
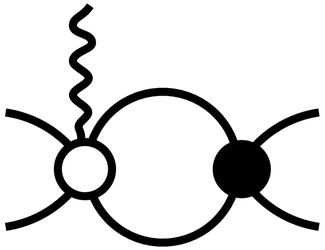
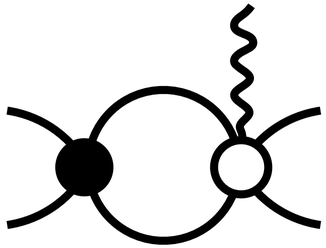
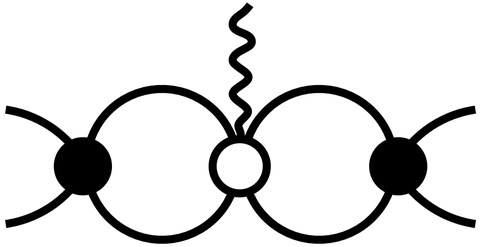
$if =$ 

$2 + \mathcal{J} \to 2$ **Amplitude**

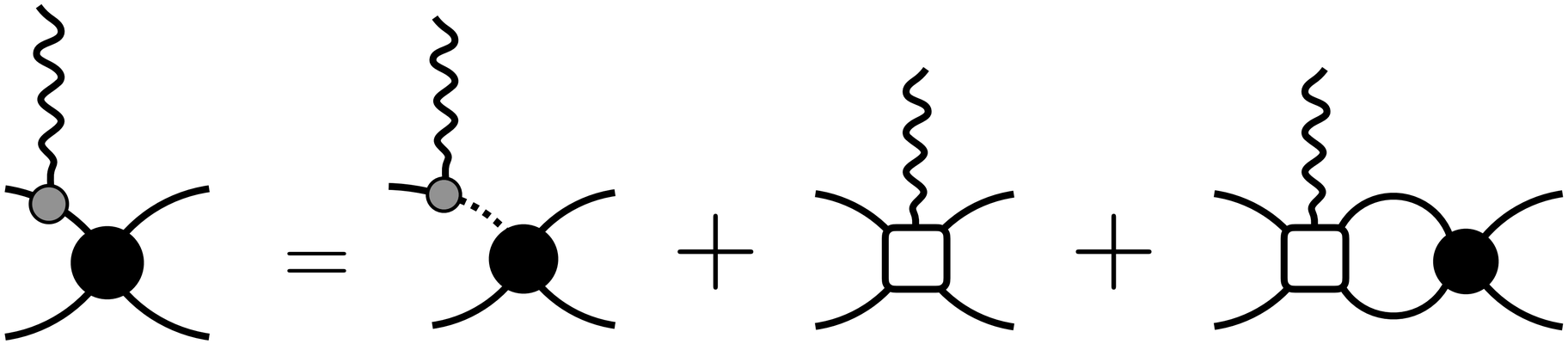
$i\mathcal{W} = P_f \leftarrow$  $\leftarrow P_i =$

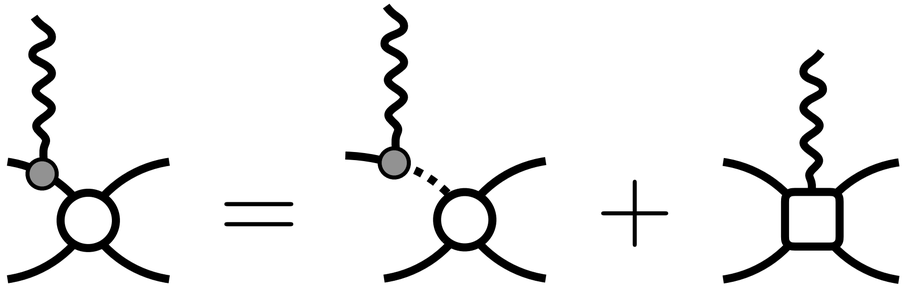
$=\sum\{$  $\}+$ 

$if =$ 

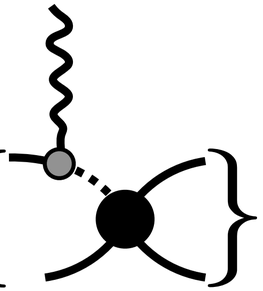
$+$  $+$  $+$  $+$ 

$2 + \mathcal{J} \to 2$ **Amplitude**



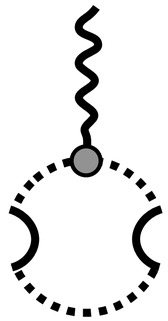


$2 + \mathcal{J} \to 2$ **Amplitude**

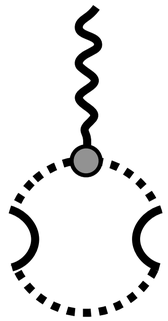
$i\mathcal{W} = \sum \{$  $\} + i\mathcal{W}_{\rm df}$

$$i\mathcal{W}_{\rm df} = \mathcal{M}\left[i\mathcal{A} + if\mathcal{G}\right]\mathcal{M}$$

Triangle Diagram - Scalar Current

$$if\mathcal{G} =$$  $$= if\int \frac{d^4k}{(2\pi)^4}\frac{i}{k^2-m_2^2+i\epsilon}\frac{1}{(P_f-k)^2-m_1^2+i\epsilon}\frac{1}{(P_i-k)^2-m_1^2+i\epsilon}$$

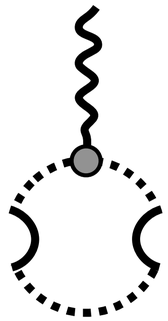
Triangle Diagram - Scalar Current

$ifG =$  $= if \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m_2^2 + i\epsilon} \frac{1}{(P_f - k)^2 - m_1^2 + i\epsilon} \frac{1}{(P_i - k)^2 - m_1^2 + i\epsilon}$

$$\sim if \left[\log\left(\frac{1 + z_f^\star + i\epsilon}{1 - (z_f^\star + i\epsilon)} \right) + \log\left(\frac{1 + z_i^\star + i\epsilon}{1 - (z_i^\star + i\epsilon)} \right) \right]$$

Triangle Diagram - Scalar Current

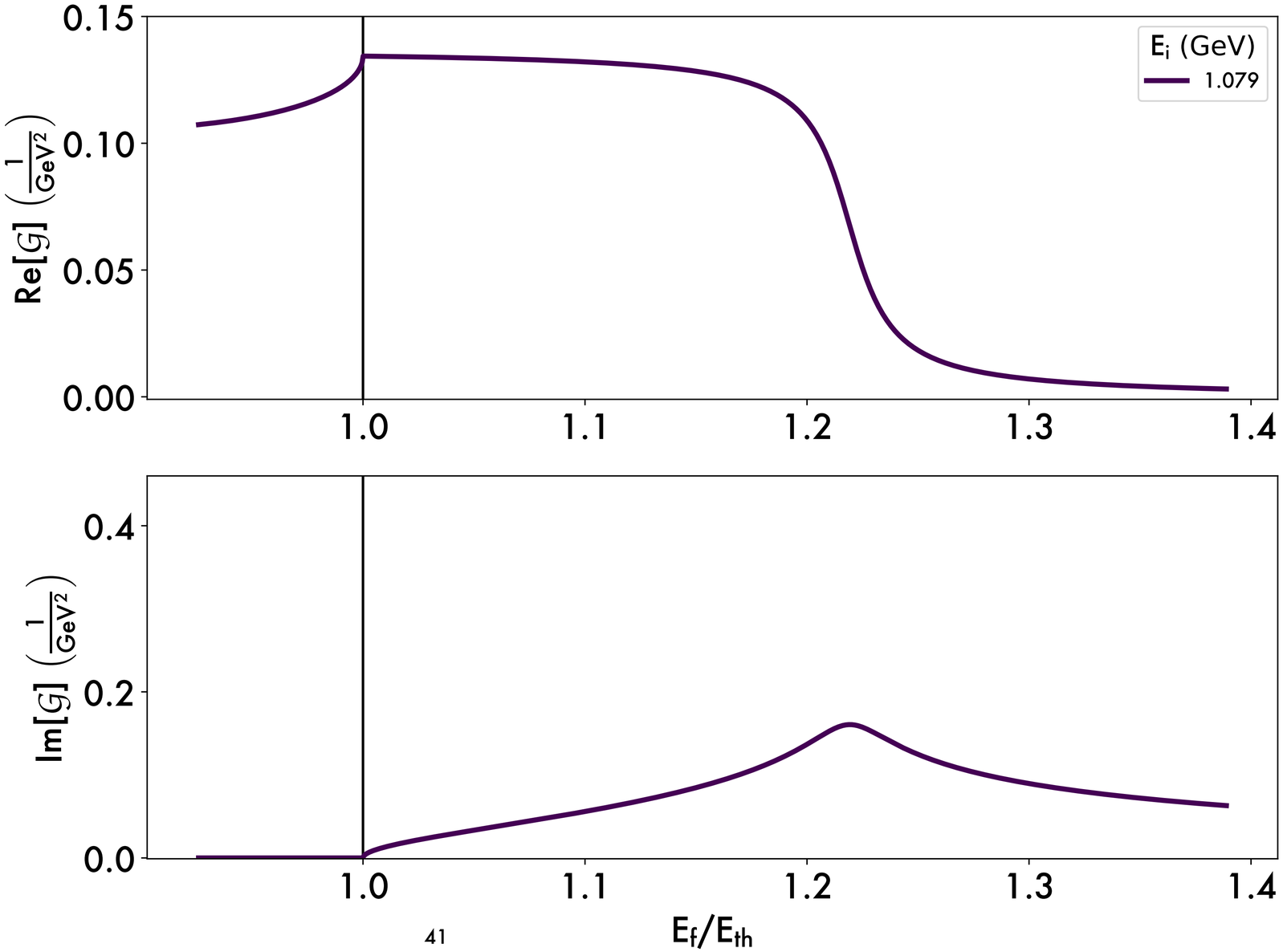
$$i f \mathcal{G} =$$



$$m_1 = \pi$$
$$m_2 = n$$

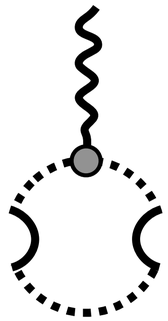
$|\mathbf{P}_i| = 0$

$|\mathbf{P}_f| = 2\pi/15$



Triangle Diagram - Scalar Current

$$i f \mathcal{G} =$$



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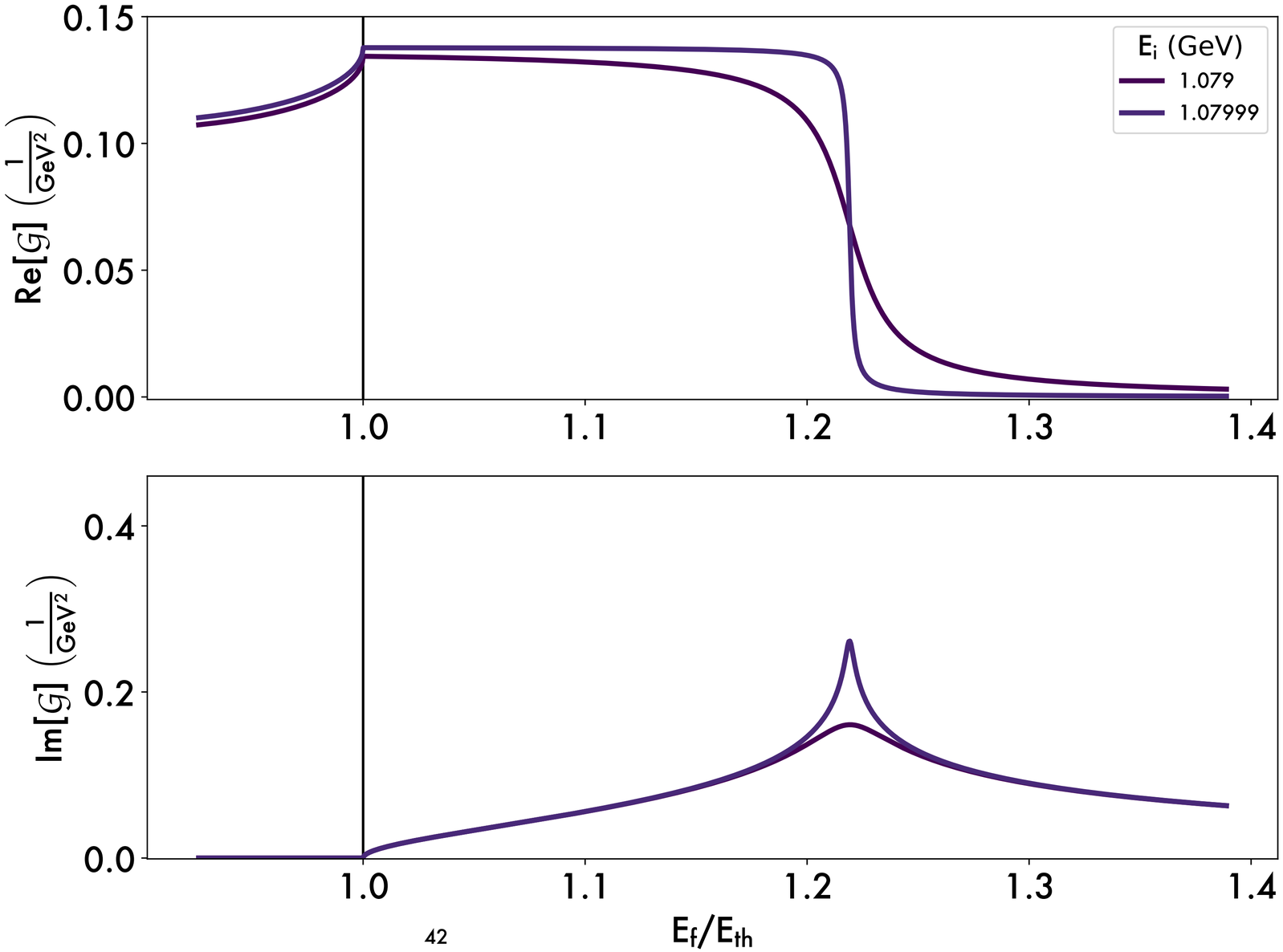
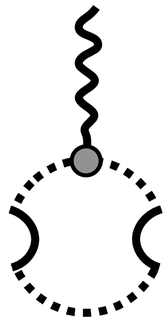


Triangle Diagram - Scalar Current

$$i f \mathcal{G} =$$



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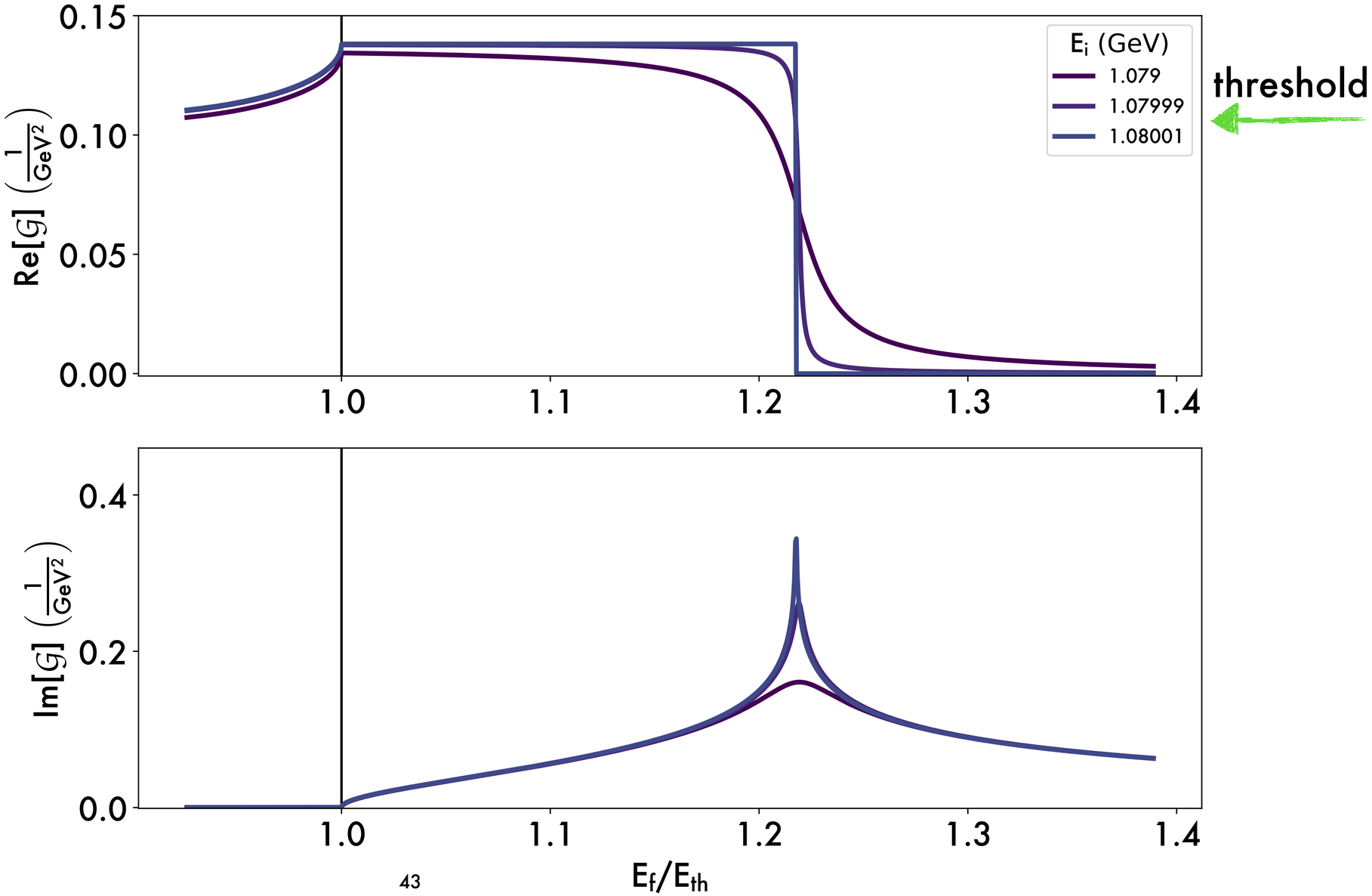
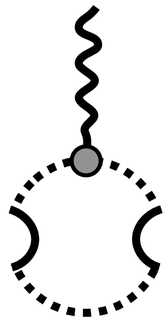


Triangle Diagram - Scalar Current

$$i f \mathcal{G} =$$



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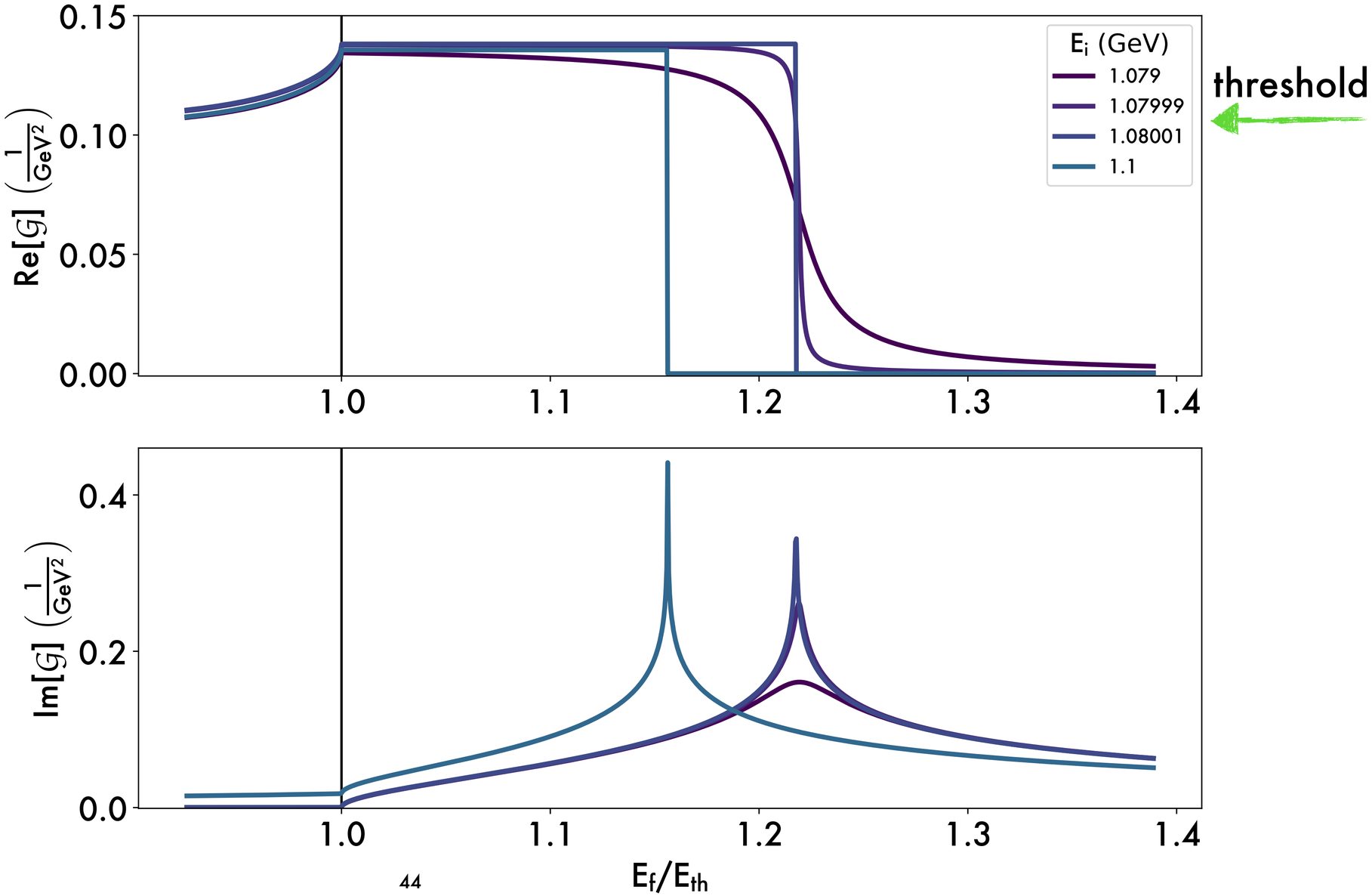
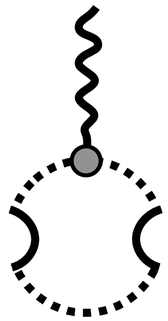


Triangle Diagram - Scalar Current

$$i f \mathcal{G} =$$



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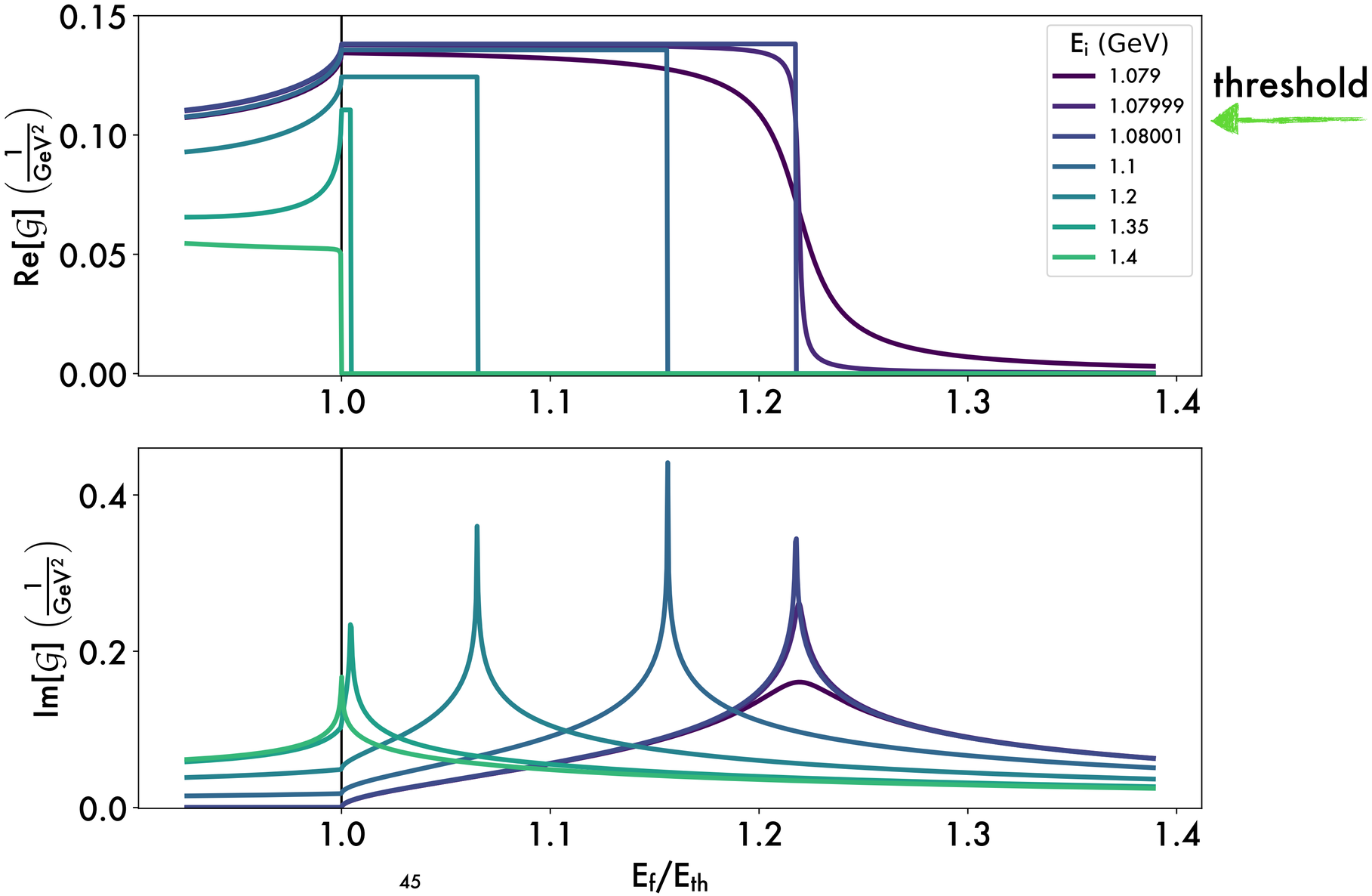
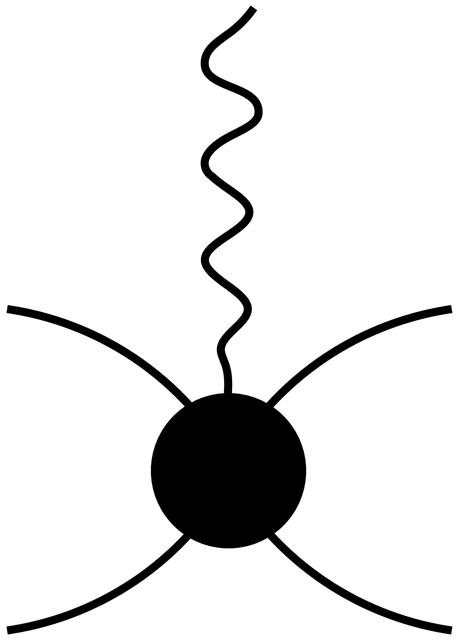
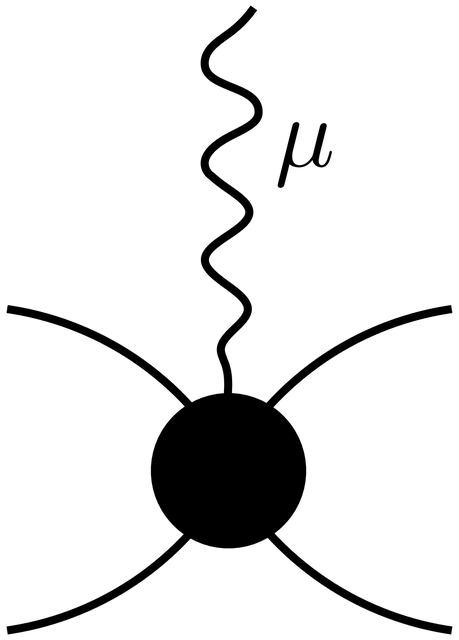


$2 + \mathcal{J} \to 2$ **Amplitude**



$$i\mathcal{W}_{\mathrm{df}} = \mathcal{M}\left[i\mathcal{A} + if\mathcal{G}\right]\mathcal{M}$$

$2 + \mathcal{J} \to 2$ **Amplitude**

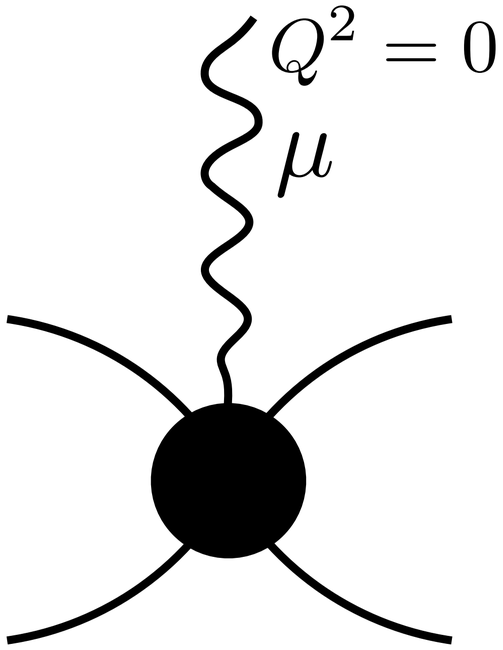


$$i\mathcal{W}^{\mu}_{\mathrm{df}} = \mathcal{M}\left[i\mathcal{A}^{\mu} + if\mathcal{G}^{\mu}\right]\mathcal{M}$$

Current Conservation

[Briceño, Hansen, Jackura 2019]

$2 + \mathcal{J} \to 2$ **Amplitude**



$$i\mathcal{W}^{\mu}_{\mathrm{df}} = \mathcal{M}\left[i\mathcal{A}^{\mu} + if\mathcal{G}^{\mu}\right]\mathcal{M}$$

Current Conservation

Ward-Takahashi Identity

$$\mathcal{W}^{\mu}_{\mathrm{df}} = 2P^{\mu}Q_0\frac{\partial}{\partial s}\mathcal{M}(s)$$

[Briceño, Hansen, Jackura 2019]

$\mathcal{A}$ Plots - Vector Current

$$m_1 = \pi$$
$$m_2 = n$$
$$m_0 = 1.23 \text{ GeV}$$

Ward-Takahashi Identity

$$\mathcal{W}^{\mu}_{\text{df}} = 2P^{\mu}Q_0 \frac{\partial}{\partial s}\mathcal{M}(s)$$

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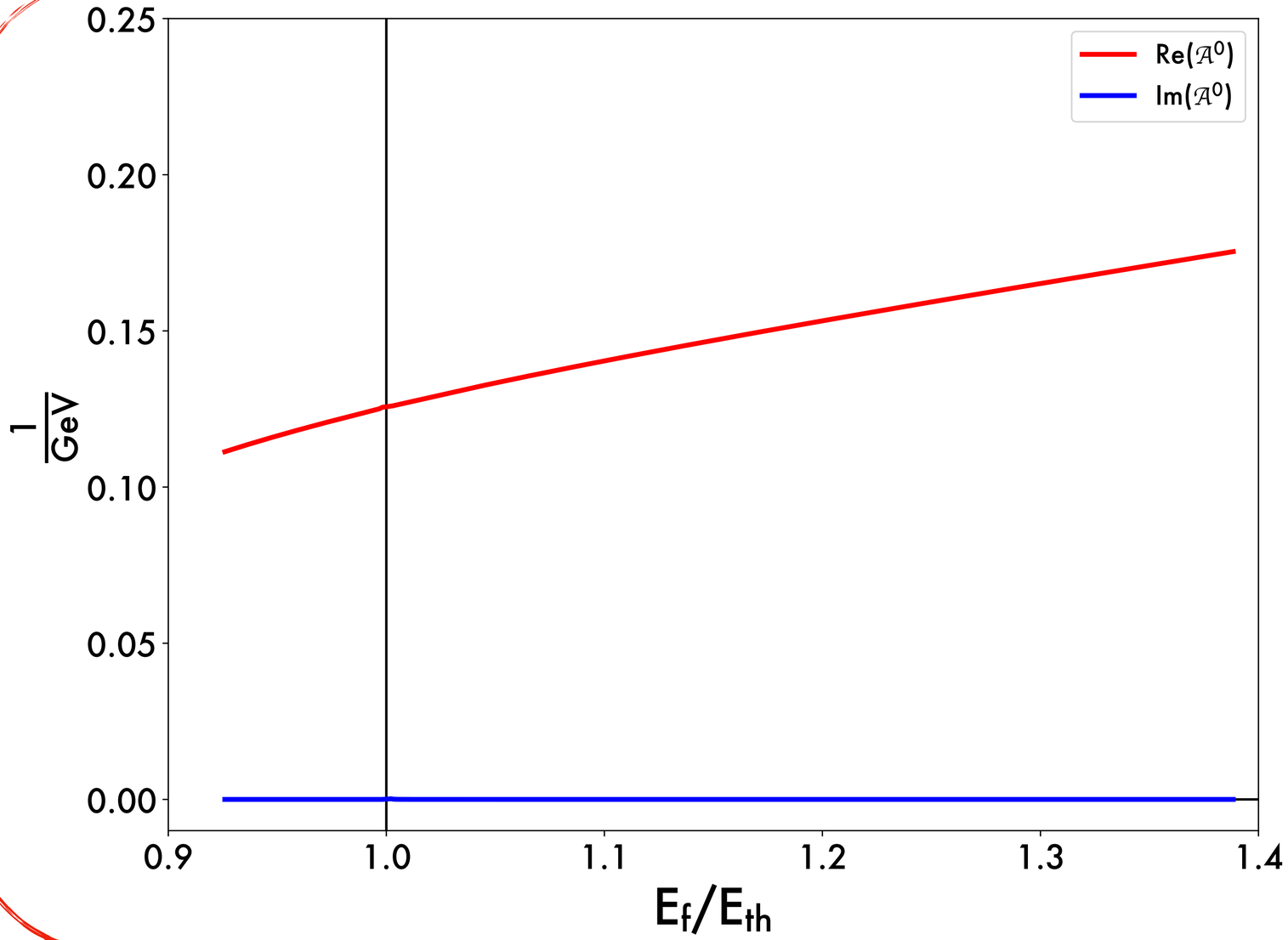
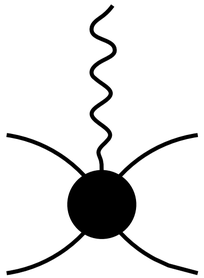


Outline

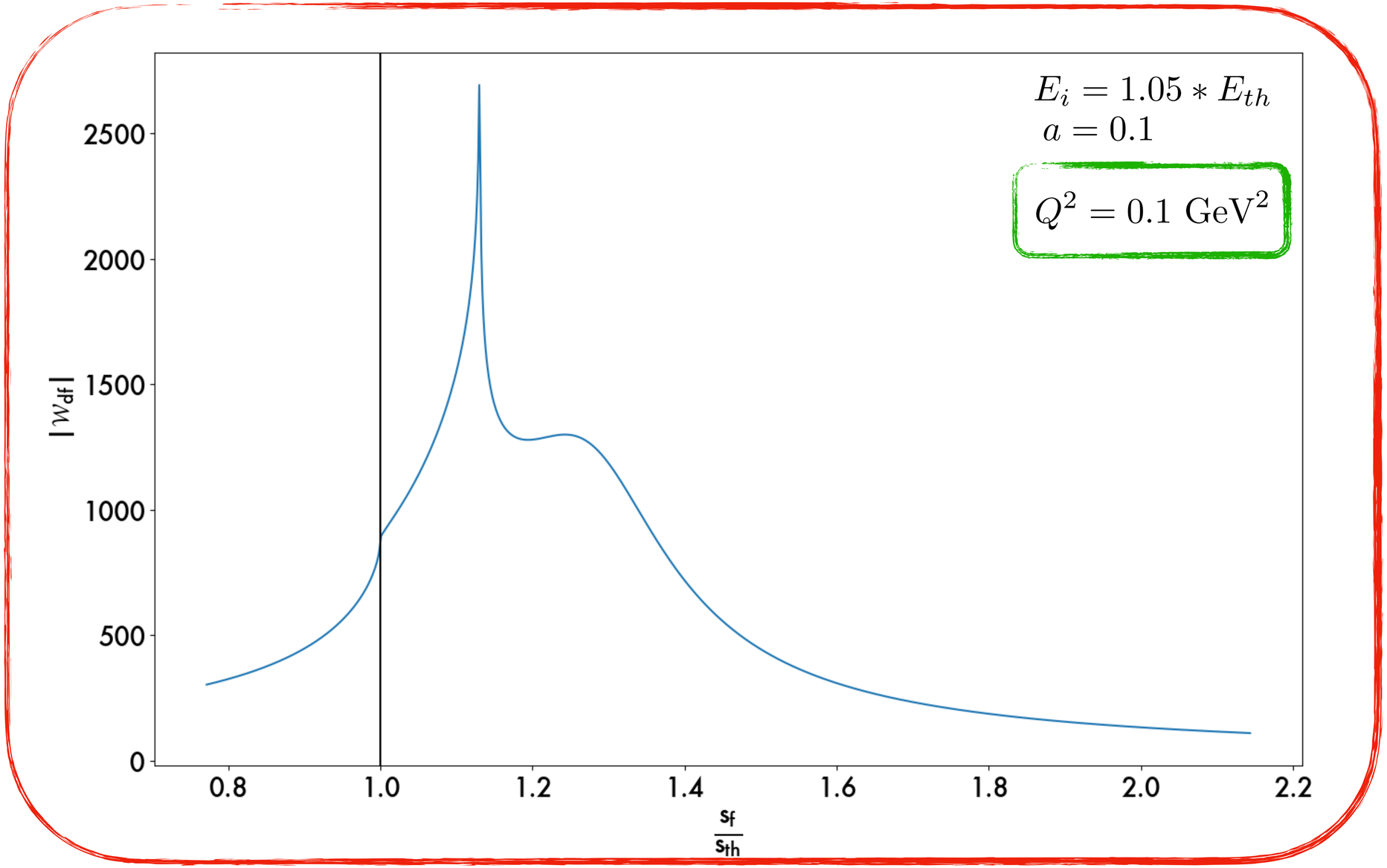
- Two-body scattering
- Two-body scattering with one external current
  - $i\mathcal{W} =$ 
    - Toy model plots
- Resonance form factors

$\mathcal{W}_{\mathrm{df}}$ Amplitude - Scalar Current

$$m_1 = \pi$$
$$m_2 = n$$
$$m_0 = 1.23 \text{ GeV}$$

$$f(Q^2) = \frac{m_0^2}{m_0^2 + Q^2}$$

$$\mathcal{A}(Q^2) = a f(Q^2)$$



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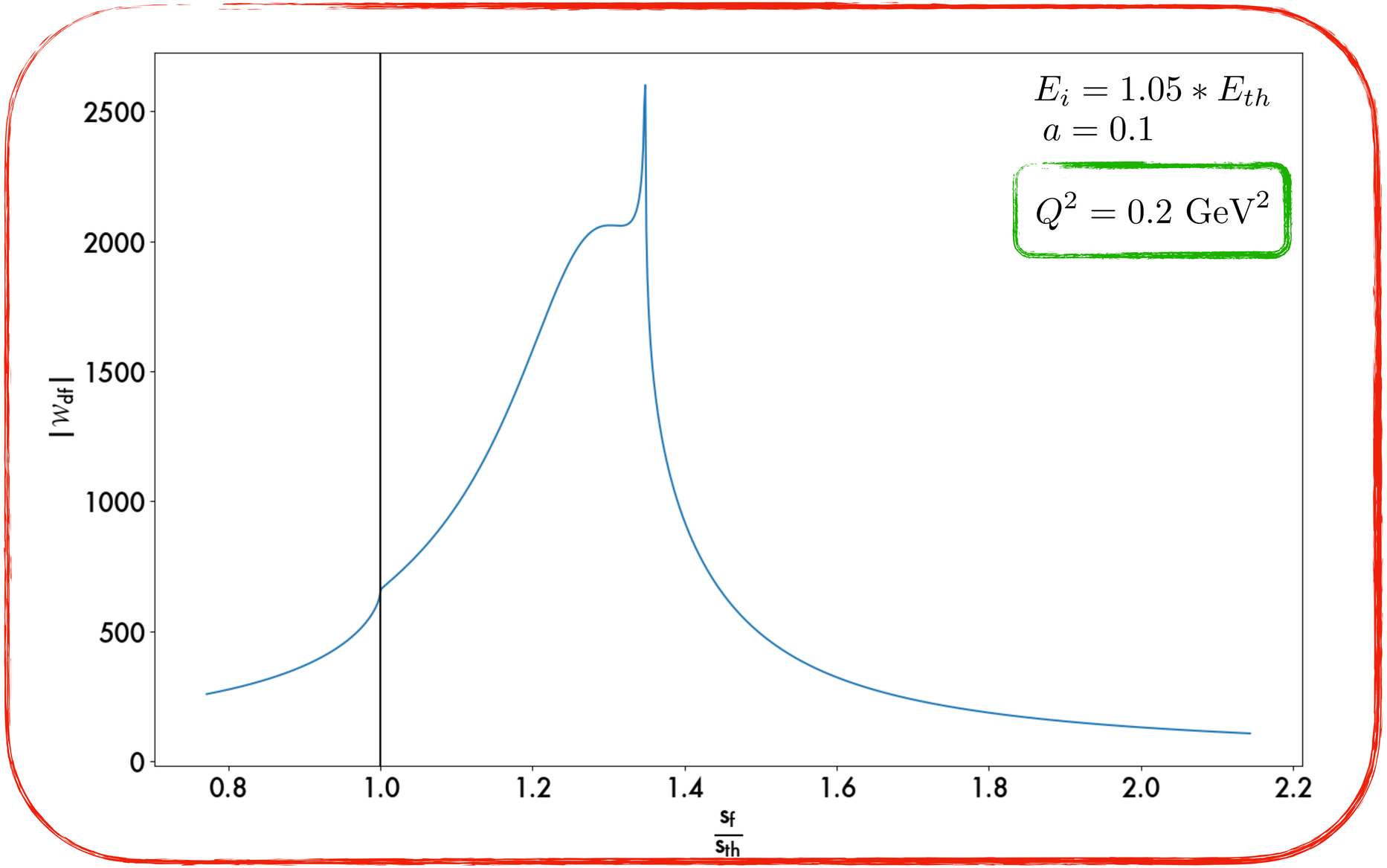

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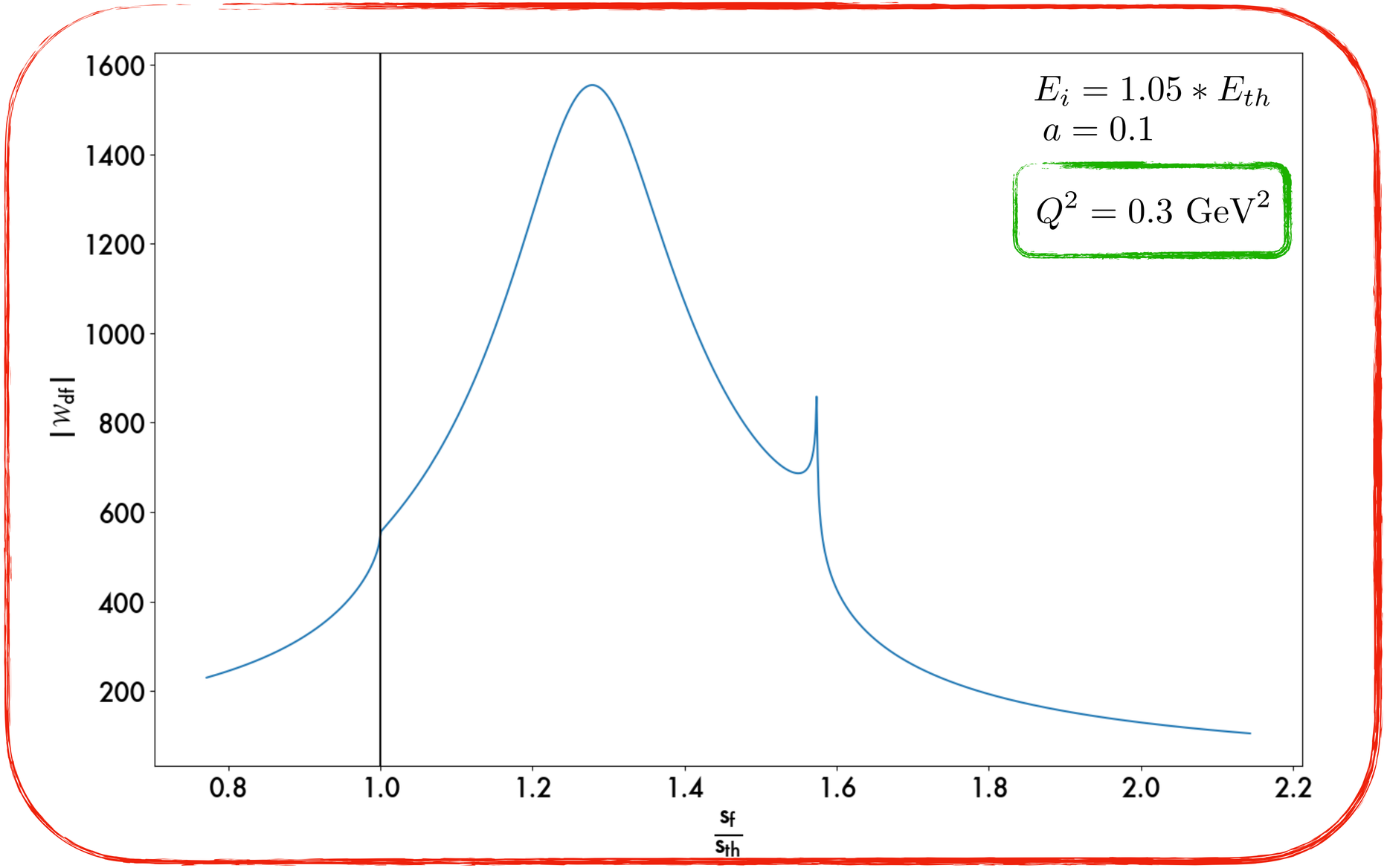

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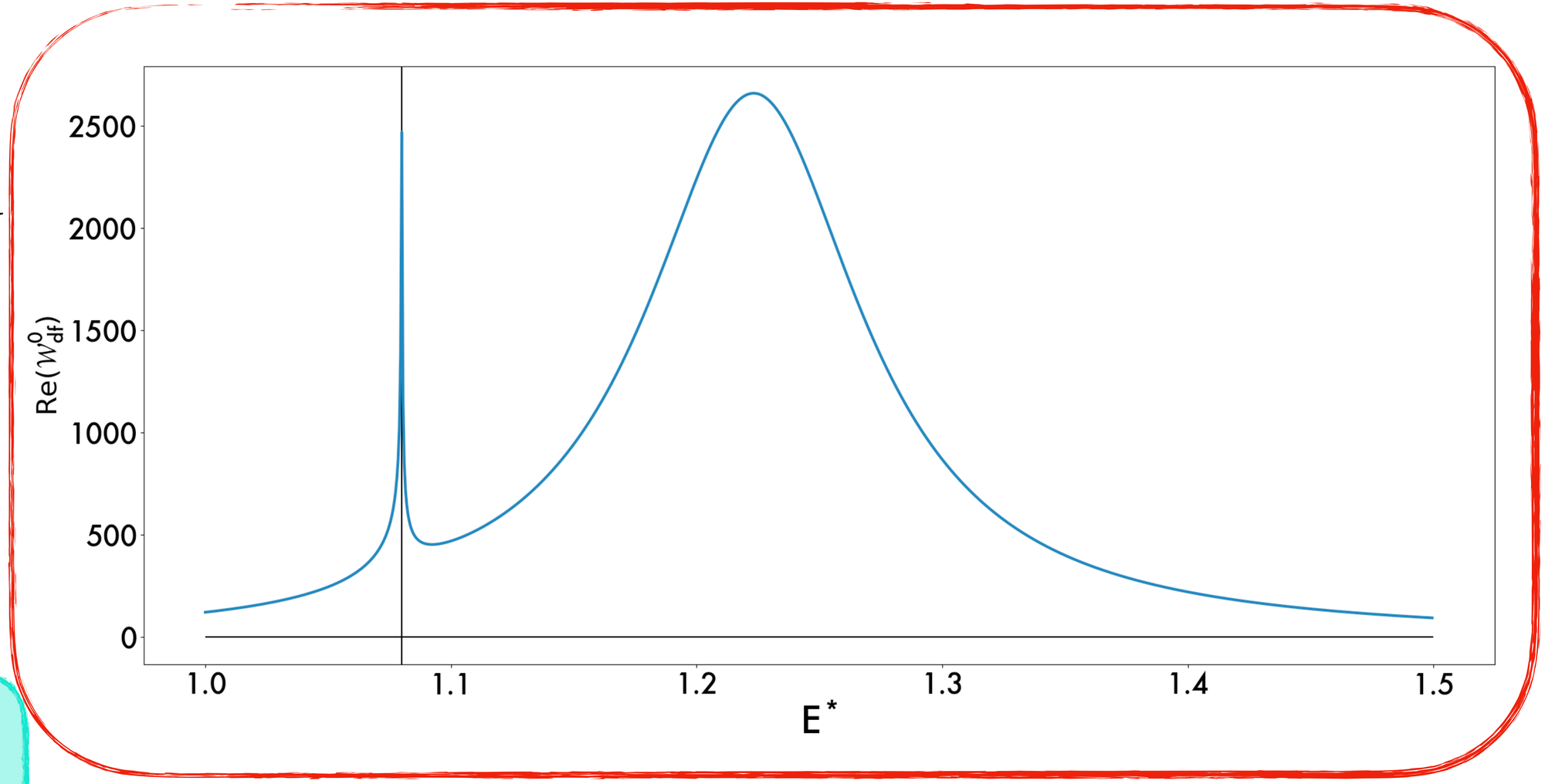
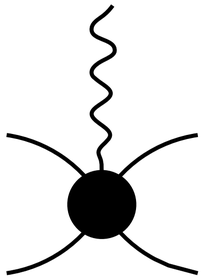


Ward-Takahashi Identity

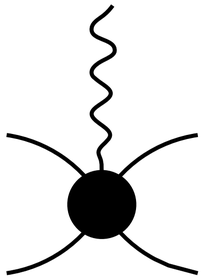
$$\mathcal{W}^{\mu}_{\rm df} = 2P^{\mu}Q_0\frac{\partial}{\partial s}\mathcal{M}(s)$$

[Briceño, Hansen, Jackura 2019]

Outline

- Two-body scattering
- Two-body scattering with one external current
  - $i\mathcal{W} =$ 
    - Toy model plots
- Resonance form factors

Outline

- Two-body scattering
- Two-body scattering with one external current
  - $i\mathcal{W} =$ 
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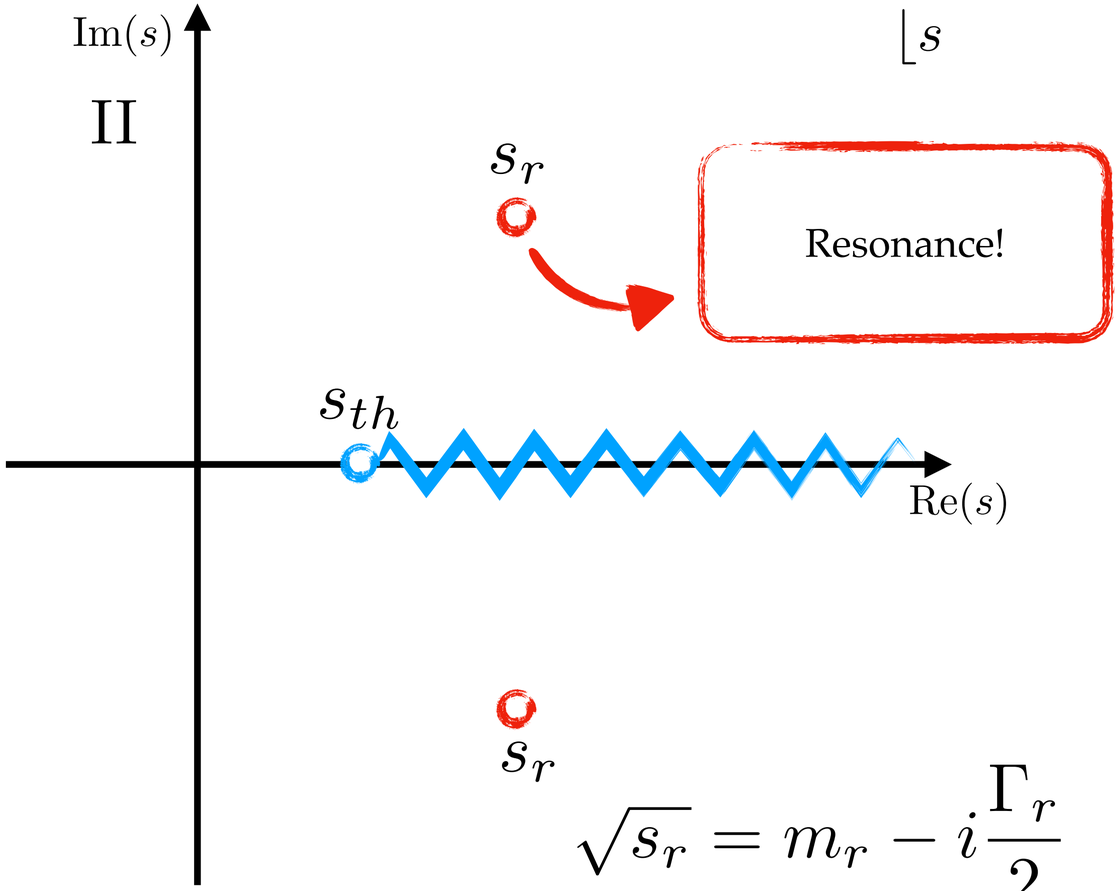
Outline

- ★ Two-body scattering
- ★ Two-body scattering with one external current
- ○ Resonance form factors

Resonance Form Factors

$$\lim_{s \to s_r} \mathcal{M}^{\mathrm{II}}(s) = -\frac{c^2}{s - s_r}$$

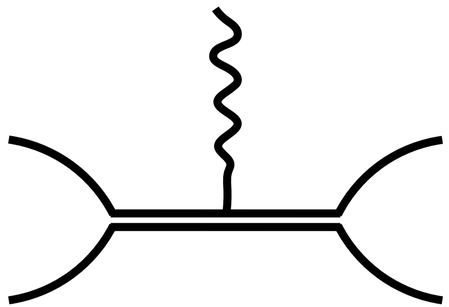


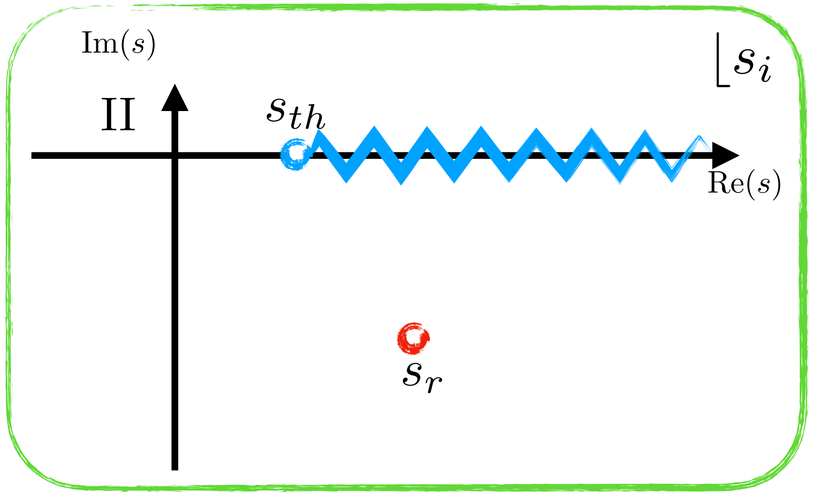
$$\sqrt{s_r} = m_r - i\frac{\Gamma_r}{2}$$

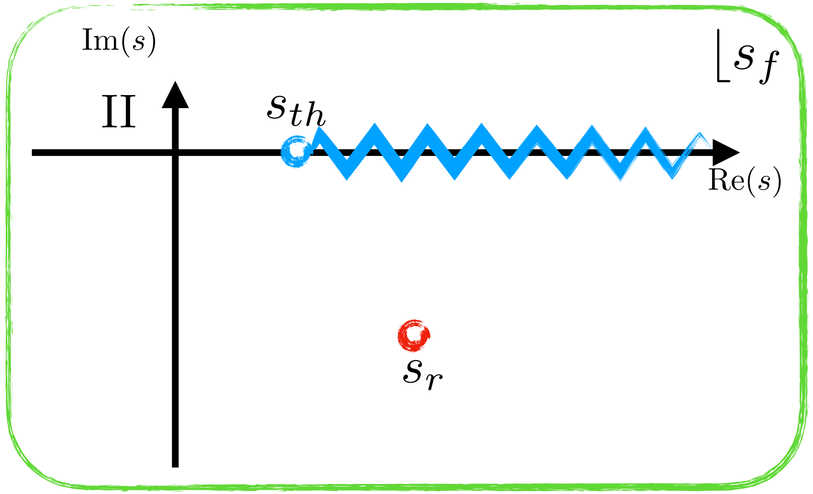
Resonance Form Factors

$$i\mathcal{W}_{\rm df} = \mathcal{M}\left[i\mathcal{A} + if\mathcal{G}\right]\mathcal{M}$$

$$\lim_{s_i, s_f \to s_r} \mathcal{W}^{\rm II}_{\rm df}(s) = \frac{-c}{s_i - s_r} f_r(Q^2) \frac{-c}{s_f - s_r}$$





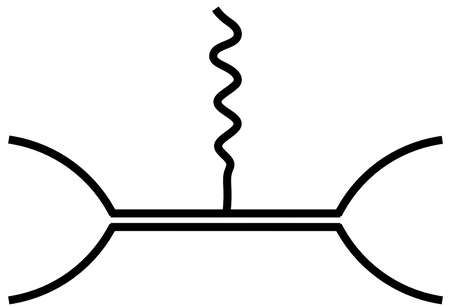


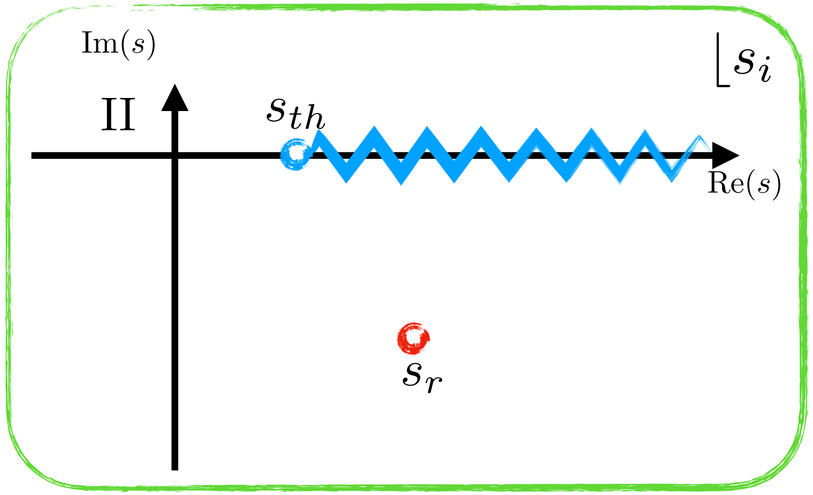
Resonance Form Factors

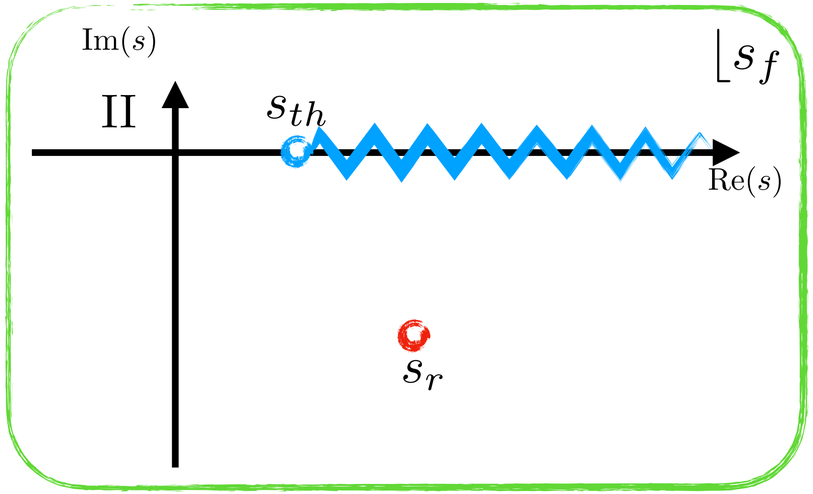
$$i\mathcal{W}_{\rm df} = \mathcal{M}\left[i\mathcal{A} + if\mathcal{G}\right]\mathcal{M}$$

$$\lim_{s_i, s_f \to s_r} \mathcal{W}^{\rm II}_{\rm df}(s) = \frac{-c}{s_i - s_r} f_r(Q^2) \frac{-c}{s_f - s_r}$$

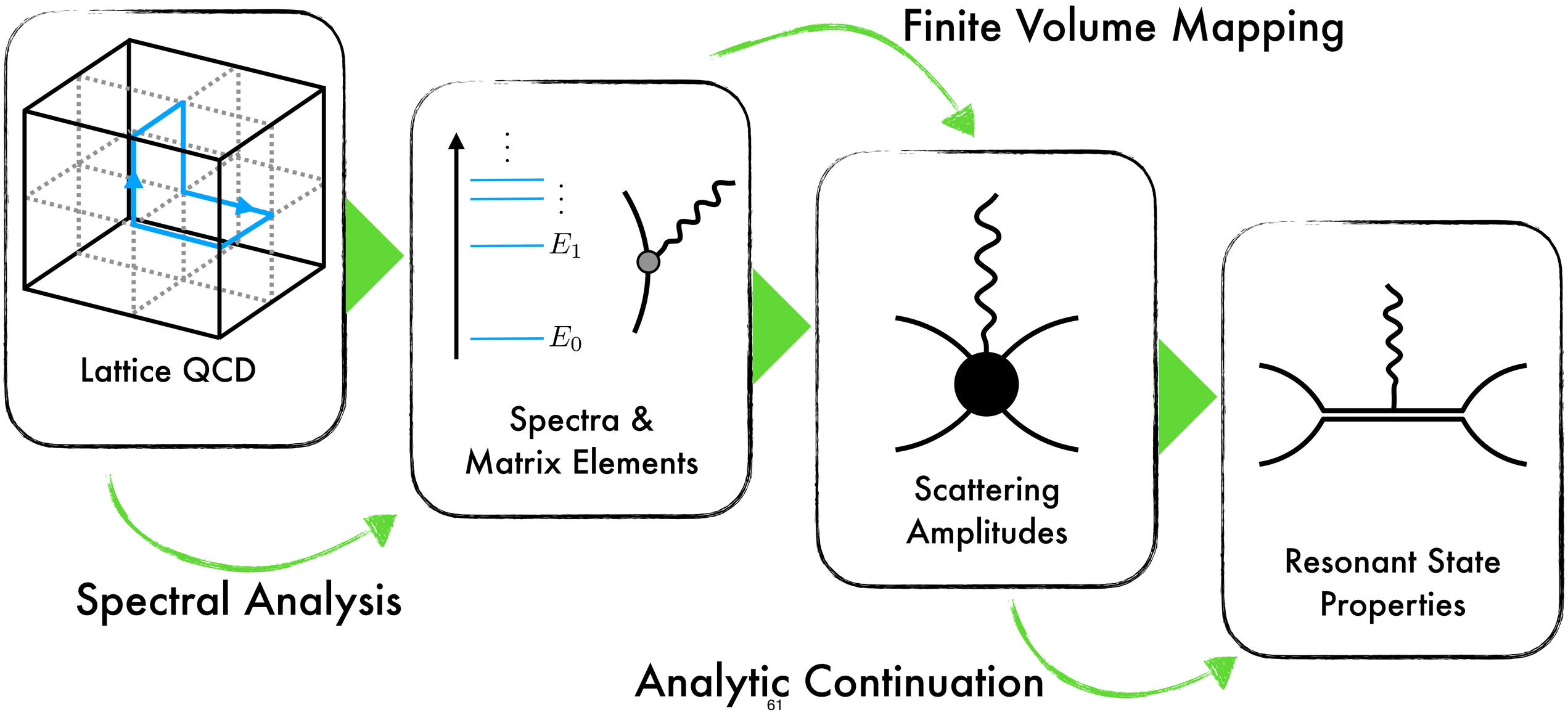
$$\implies f_r(Q^2) = c^2[\mathcal{A}(Q^2) + f(Q^2)\mathcal{G}^{\rm II}(Q^2)]$$







Hadronic Properties from QCD

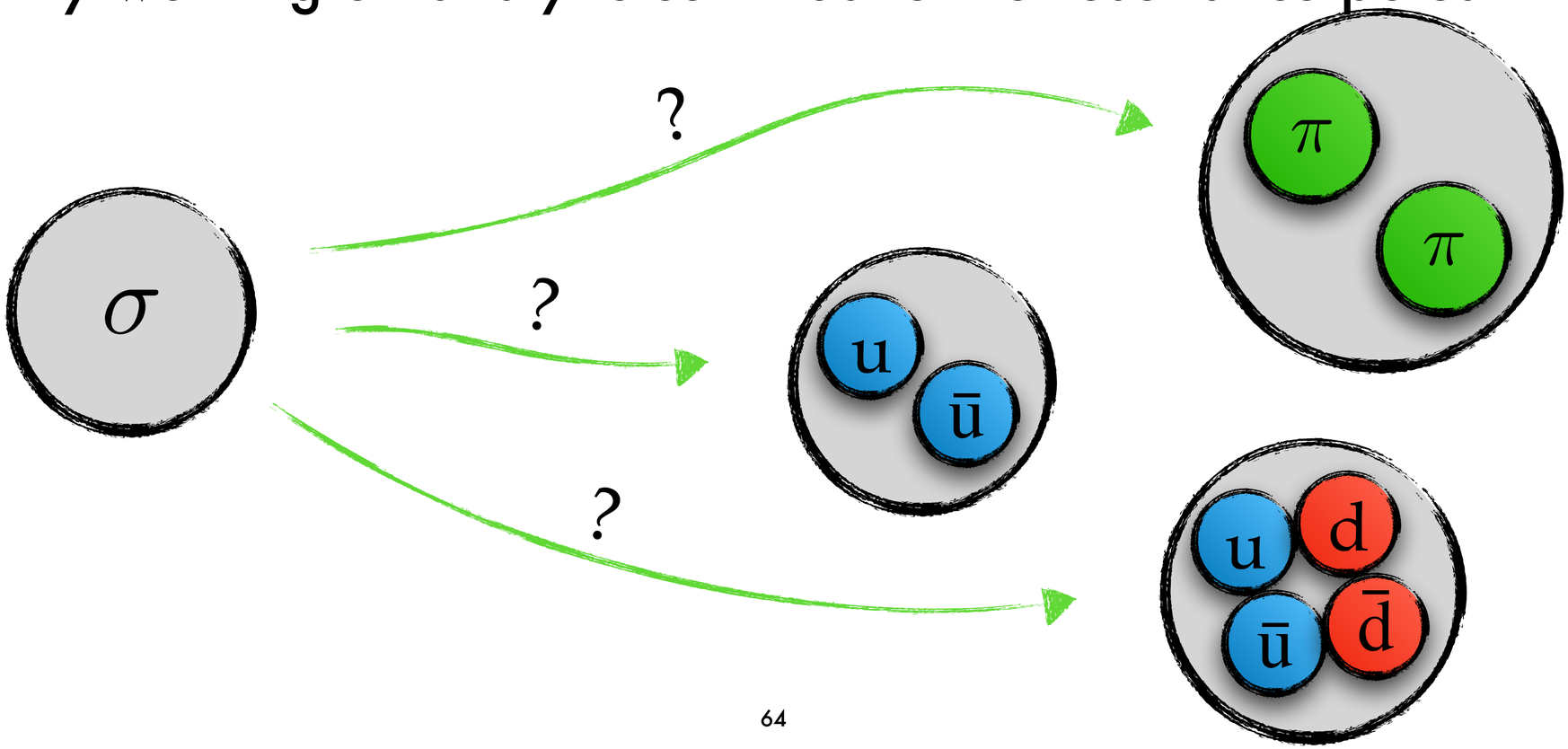


Outline

- ☆ Two-body scattering review
- ☆ Two-body scattering with one external current
- ○ Resonance form factors

Conclusion and Outlook

- On-shell representation of $i\mathcal{W}$
  - Full understanding of analytic structure
  - Currently working on analytic continuation to resonance poles

Conclusion and Outlook

- On-shell representation of $i\mathcal{W}$
  - Full understanding of analytic structure
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