The role of the axial anomaly in polarized deep inelastic scattering

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Based on Andrey Tarasov and Raju Venugopalan arXiv:2008.08104
Electron-Ion Collider (EIC)

- Polarized protons up to 275 GeV
- Nuclei up to Z/A*275 GeV
- Electrons up to 18 GeV
- \(\sqrt{s} = 20-140\) GeV
- Luminosity \(10^{34}\) cm\(^{-2}\)sec\(^{-1}\)
- First polarized electron - polarized proton collider
- First electron-nucleus collider

https://www.bnl.gov/eic/
Deep inelastic scattering

Virtuality: measure of resolution power

\[ Q^2 = -q^2 \]

\[ \Delta x \sim \frac{1}{Q} \]

Bjorken variable: measure of momentum fraction of a struck parton

\[ x_B = \frac{Q^2}{2P \cdot q} \]

https://www.bnl.gov/eic/
Finding 1: An EIC can uniquely address three profound questions about nucleons—neutrons and protons—and how they are assembled to form the nuclei of atoms:

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise?
- What are the emergent properties of dense systems of gluons?
The proton’s spin puzzle

\[ \frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta G + L_q + L_g \]

• DIS experiments showed that quarks carry only about 30% of the proton’s spin \( \Delta \Sigma = 0.25 \sim 0.3 \)

• Failure of the constituent quark model to explain spin of the proton - spin crisis
DIS structure functions

DIS cross-section:
\[ \sigma \sim L_{\mu\nu} \tilde{W}_{\mu\nu} \]

Hadron tensor:
\[ W^{\mu\nu}(q, P, S) = \tilde{W}_{\text{sym.}}^{\mu\nu}(q, P) + i\tilde{W}_{\text{asym.}}^{\mu\nu}(q, P, S) \]

Structure function:
\[
\tilde{W}_{\mu\nu}(q, P, S) = \frac{2M_N}{P \cdot q} \epsilon_{\mu\nu\alpha\beta} q^\alpha \left\{ S^\beta g_1(x_B, Q^2) + \left[ S^\beta - \frac{(S \cdot q) P^\beta}{P \cdot q} \right] g_2(x_B, Q^2) \right\}
\]
Structure function $g_1$:

$$g_1(x_B, Q^2) = \frac{1}{2} \sum_f e_f^2 \left( \Delta q_f(x_B, Q^2) + \Delta \bar{q}_f(x_B, Q^2) \right)$$

Polarized parton distribution function (pdf):

$$\Delta q_f(x_B, Q^2) = \frac{1}{4\pi} \int dy^- e^{-i y^- x_B} P^+ \langle P, S | \bar{\Psi}_f(0, y^-, 0_\perp) \gamma^+ \gamma_5 \Psi_f(0) | P, S \rangle$$

$$W^{\mu\nu}(q, P, S) = \frac{1}{2\pi} \int d^4 x \, e^{iqx} \langle P, S | j^\mu(x) j^\nu(0) | P, S \rangle$$
First moment of $g_1$

$$\int_0^1 dx_B \ g_1(x_B, Q^2) = \frac{1}{18} \left( 3F + D + 2 \Sigma(Q^2) \right)$$

Combination of F and D can be measured in $\beta$-decay and hyperon decay experiments.

In term of helicity PDFs iso-singlet quark helicity is

$$\Sigma(Q^2) = \sum_f \int_0^1 dx_B \ (\Delta q_f(x_B, Q^2) + \Delta \bar{q}_f(x_B, Q^2))$$

Quark spin contribution is defined by iso-singlet axial vector current

$$S^\mu \Sigma(Q^2) = \frac{1}{M_N} \sum_f \langle P, S | \bar{\Psi}_f \gamma^\mu \gamma_5 \Psi_f | P, S \rangle \equiv \frac{1}{M_N} \langle P, S | J_5^\mu (0) | P, S \rangle$$
Iso-singlet axial vector current

\[ S^{\mu} \Sigma(Q^2) = \frac{1}{M_N} \langle P, S | J_5^{\mu}(0) | P, S \rangle \]

The current is not conserved. The anomaly equation:

\[ \partial^{\mu} J_5^{\mu}(x) = \frac{n_f \alpha_s}{2\pi} \text{Tr} \left( F_{\mu\nu}(x) \tilde{F}^{\mu\nu}(x) \right) = 2 n_f \partial_{\mu} K^{\mu} \]

Chern-Simons current:

\[ K_{\mu} = \frac{\alpha_s}{4\pi} \epsilon_{\mu\nu\rho\sigma} \left[ A^\nu_a \left( \partial^{\rho} A^\sigma_a - \frac{1}{3} g f_{abc} A^\rho_b A^\sigma_c \right) \right] \]
Why is $\Sigma$ small?

Write quark helicity in terms of the conserved current (add and subtract $K_\mu$):

$$S^\mu \Sigma(Q^2) = S^\mu \Sigma(Q^2) - \frac{2n_f}{M_N} \langle P, S| K^\mu | P, S \rangle + \frac{2n_f}{M_N} \langle P, S| K^\mu | P, S \rangle$$

Introduce a conserved current $\tilde{J}_5^\mu$:

$$S^\mu \Sigma(Q^2) = S^\mu \tilde{\Sigma}(Q^2) + \frac{2n_f}{M_N} \langle P, S| K^\mu | P, S \rangle$$
Why is $\Sigma$ small?

Write quark helicity in terms of the conserved current (add and subtract $K_\mu$):

$$S^\mu \Sigma(Q^2) = S^\mu \tilde{\Sigma}(Q^2) + \frac{2n_f}{M_N} \langle P, S| K^\mu | P, S \rangle$$

where

$$S^\mu \tilde{\Sigma}(Q^2) = \frac{1}{M_N} \langle P, S| \tilde{J}^5_\mu | P, S \rangle, \quad \tilde{J}^5_\mu = J^5_\mu - 2n_f K_\mu$$

Quark and gluon contribution:

$$\tilde{\Sigma}(Q^2) = \Sigma(Q^2) - \frac{n_f \alpha_S}{2\pi} \Delta G$$

If $\Delta G$ is large, this would provide a natural explanation of the spin crisis.
Gluon spin contribution in OPE

\[ \Delta G(x_B, Q^2) = \int_0^1 dx_B \Delta g(x_B, Q^2) \]

In the Operator Product Expansion (OPE) the gluon spin is defined by a non-local gauge-invariant operator:

\[ \Delta g(x_B, Q^2) = \frac{2i}{x_B} \int \frac{d\xi}{2\pi} e^{-i\xi x_B} \langle P, S| \text{Tr}_c n_\alpha F^{\alpha\mu}(\xi n)n^\beta \tilde{F}_{\beta\mu}(0)| P, S \rangle \]

- \( \Delta G \) cannot be expressed by a local operator
- It becomes local only in the light-cone gauge and coincides with \( K^+ \) (\( \Delta G \) is no \( K_\mu \))
Resolving the proton’s spin crisis

\[
\int_{x_{\text{min}}} d^2q g(x, Q^2) \]

\[
Q^2 = 10 \text{ GeV}^2
\]

\[
DSSV 2014
\]

\[
Q^2 = 10 \text{ GeV}^2
\]

\[
g_1(x, Q^2)
\]

\[
\text{DSSV 2014}
\]

\[
\text{EIC pseudo-data}
\]

D. De Florian, R. Sassot, M. Stratmann, W. Vogelsang, PRL 113 (2014)
Resolving the proton’s spin crisis

\[
\frac{d}{d \ln Q^2} \begin{pmatrix}
\Delta \Sigma \\
\Delta \Gamma
\end{pmatrix} = \begin{pmatrix}
\Delta P_{\Sigma \Sigma}(a_s) & 0 \\
-\frac{1}{2N_f} \Delta P_{\Sigma \Sigma}(a_s) & 0
\end{pmatrix} \begin{pmatrix}
\Delta \Sigma \\
\Delta \Gamma
\end{pmatrix}
\]

\[
\Delta \Gamma(Q^2) \equiv a_s(Q^2) \Delta G(Q^2)
\]

- The evolution is controlled by a single anomalous dimension. This is a consequence of the anomaly equation for the iso-singlet axial current
- This suggests that \( \Delta \Sigma \) only mixes with itself

Altarelli and Lampe (1990)
Vogt, Moch, Rogal, Vermaseren (2008)
DeFlorian, Vogelsang (2019)
Alternative picture: Anomalous chiral Ward identities

\[ \Sigma(Q^2) = \frac{1}{3m_N} \Delta C_1^S(\alpha_S) \left( g_{QQ} \chi(0) + g_{\eta NN} \sqrt{\chi'(0)} \right) \]

Topological charge density:

\[ Q(x) = \frac{\alpha_S}{8\pi} Tr \left( F_{\mu\nu} \tilde{F}^{\mu\nu} \right) \]

Topological susceptibility:

\[ \chi_{YM}(p^2) = i \int dx \, e^{i p \cdot x} \langle 0 | T(Q(x)Q(0)) | 0 \rangle \]

In the chiral limit \( \chi(0) \to 0 \), the slope \( \chi'(0) \) is estimated to be small. Topological charge screening of spin.

Shore, Veneziano (1990)
Narison, Shore, Veneziano, hep-ph/9812333
G. M. Shore, hep-ph/0701171
Perturbative and non-perturbative interplay

The key role of the anomaly is seen from the structure of the triangle graph in the off-forward limit

$$\frac{1}{M_N} \langle P', S | J_5^\mu(0) | P, S \rangle = \Sigma(Q^2, t) S^\mu + h(Q^2, t) \cdot S \cdot l^\mu$$

In the forward limit only $\Sigma$ contribute, $h$ doesn’t have a pole

The triangle diagram has an infrared pole and $\kappa(t) \propto F \widetilde{F}$

$$\frac{1}{M_N} \langle P', S | J_5^\mu(0) | P, S \rangle = \frac{l \cdot S \cdot l^\mu}{l^2} \kappa(Q^2, t) + \left( S^\mu - \frac{l \cdot S \cdot l^\mu}{l^2} \right) \lambda(Q^2, t)$$

R. L. Jaffe, A. Manohar


R. L. Jaffe

A. Manohar

Pseudoscalar contribution

exact
OPE vs. exact off-forward calculation

Exact result of the off-forward calculation is divergent:

In the forward OPE calculation the result is finite:

\[
\lim_{m^2/p^2 \to 0} \Gamma_5^+ = \mp \frac{\alpha_s N_f p^+}{\pi}
\]

Perturbative and non-perturbative interplay

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Pseudoscalar contribution

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exact
Perturbative and non-perturbative interplay

The infrared pole of the triangle diagram must be cancelled by a pole in the non-perturbative contribution:

$$\Sigma(Q^2) = \frac{n_f \alpha_s}{2\pi M_N} \lim_{l_\mu \to 0} \langle P', S \mid \frac{1}{il \cdot s} \text{Tr} \left( F \tilde{F} \right) (0) \mid P, S \rangle$$

The result is manifestly gauge invariant!

The existence of the infrared pole of the triangle diagram has not been addressed in pQCD calculations. In the OPE in the forward limit the triangle diagram is finite, i.e. it doesn’t have the infrared pole and $F \tilde{F}$.
The triangle diagram

In the Feynman diagram approach the calculation of the triangle anomaly is ambiguous.

\[ \gamma_\mu \gamma_5 \]

\[ k + p \]

\[ k - q \]

\[ p^\alpha \]

\[ q^\beta \]

\[ \neq \]

\[ k \]

\[ k - p - q \]

\[ p^\alpha \]

\[ q^\beta \]

A brute force calculation yields a wrong result. The value of the triangle diagram depends on how the loop momentum is parametrized. The “correct” parametrization can be achieved by imposing the vector Ward identities.

Our approach is to use the worldline formalism which doesn’t have these ambiguities.

Linearly divergent integrals!
The worldline formalism

One-loop effective action:

\[ \Gamma_{QED}[A] = \ln \text{Det} [\slashed{\not{\partial}} + A] \]

Heat-kernel regularization:

\[ \Gamma_{QED}[A] = -\frac{1}{2} \text{Tr} \int_0^\infty \frac{dT}{T} \exp \left\{ -T \left[ (p_\mu + eA_\mu)^2 - \frac{e}{2} F_{\mu\nu} \sigma^{\mu\nu} \right] \right\} \]

One can introduce bosonic and fermionic coherent states and construct a functional integral representation:

\[ \int d^4x \left| x \right\rangle \left\langle x \right| = \int d^4p \left| p \right\rangle \left\langle p \right| = 1; \quad \left\langle x | p \right\rangle = e^{ipx} \]

\[ i \int |\xi\rangle \langle \xi| d^2\xi = -i \int |\bar{\xi}\rangle \langle \bar{\xi}| d^2\bar{\xi} = 1; \quad \langle \xi | \bar{\xi} \rangle = e^{\sum_{i=1}^2 \xi_i \bar{\xi}_i} ; \quad \langle \bar{\xi} | \xi \rangle = e^{\sum_{i=1}^2 \bar{\xi}_i \xi_i} \]

Grassmann numbers

Bern, Kosower, NPB 379 (1992) 145
Strassler, NPB 385 (1992) 145
D’Hoker, Gagne (1996)
Berezin, Marinov (1976)
Barducci (1977)
The triangle anomaly in the worldline formalism

\[ \Gamma[A, A_5] = -\frac{1}{2} \text{Tr}_c \int_0^\infty \frac{dT}{T} \int Dx \int_{AP} D\psi \]

\[ \times \exp \left\{ - \int_0^T d\tau \left( \frac{1}{4} \dot{x}^2 + \frac{1}{2} \psi_\mu \dot{\psi}^\mu + ig \dot{x}^\mu A_\mu - ig \dot{\psi}^\mu \psi_\nu F_{\mu\nu} - 2i\psi_5 \dot{\psi}^\mu \psi_\mu \psi_\nu A_5^\nu + i\psi_5 \partial_\mu A_5^\mu + (D - 2) A_5^2 \right) \right\} \]

The triangle anomaly:

\[ \langle P', S \mid J_5^\kappa \mid P, S \rangle = \int d^4y \frac{\partial}{\partial A_{5\kappa}(y)} \Gamma[A, A_5] \bigg|_{A_5=0} e^{ily} \equiv \Gamma_5^\kappa[l] \]

Functional integrals over trajectories of a point-like particle

Free “propagation”

Vector coupling

Axial coupling

The triangle anomaly in the worldline formalism

\[ \Gamma_5^{\kappa\alpha\beta}[l, k_2, k_4] = -\frac{ig^2}{2} \int_0^\infty \frac{dT}{T} \int Dx \int_{AP} D\psi \int_0^T d\tau_l \psi_5 \left( il^\kappa + 2\psi_4^\kappa \dot{x}_l \cdot \psi_l \right) e^{ilx_l} \]

\[ \times \int_0^T d\tau_2 \int_0^T d\tau_4 \left( \dot{x}_2^\alpha + 2i\psi_2^\alpha \psi_2^\lambda k_2^\lambda \right) e^{ik_2x_2} \left( \dot{x}_4^\beta + 2i\psi_4^\beta \psi_4^\eta k_4^\eta \right) e^{ik_4x_4} \exp \left\{ -\int_0^T d\tau \left( \frac{1}{4} \dot{x}^2 + \frac{1}{2} \psi_\mu \dot{\psi}^\mu \right) \right\} \]

- Calculation of the functional integrals is straightforward and doesn’t contain any ambiguity
- \( \psi_5 \) changes anti-periodic boundary condition of the Grassmann functional integral to periodic

Axial current

Vector currents

Tarasov, Venugopalan (2020)
The triangle anomaly in the worldline formalism

\[ \Gamma_5^\kappa[l] = \frac{1}{4\pi^2} \int \frac{d^4k_2}{(2\pi)^4} \int \frac{d^4k_4}{(2\pi)^4} \text{Tr}_c F_{\alpha\beta}(k_2) \tilde{F}^{\alpha\beta}(k_4) (2\pi)^4 \delta^4(l + k_2 + k_4) \]

We reproduce famous infrared pole of the anomaly

\[ S^\mu \int_0^1 dx_B \left. g_1(x_B, Q^2) \right|_{Q^2 \to \infty} \]

\[ = \sum_f e_f^2 \frac{\alpha_s}{2i\pi M_N} \lim_{l_\mu \to 0} \frac{l^\mu}{l^2} \langle P', S | \text{Tr}_c F_{\alpha\beta}(0) \tilde{F}^{\alpha\beta}(0) | P, S \rangle \]

The infrared pole must be cancelled by a pseudoscalar contribution
The anomaly contribution in $g_1$

OPE expansion of the product of two electromagnetic currents (hadronic tensor):

\[ i \int d^4x \ e^{i q x} T(J_\mu(x) J_\nu(0)) \approx [\text{symmetric terms under } \mu \leftrightarrow \nu] \]

\[-i \epsilon_{\mu \nu \lambda \sigma} q^\lambda \sum_n \frac{1}{2} [1 - (-)^n] \left( \frac{2}{-q^2} \right)^n q_{\mu_1} \ldots q_{\mu_{n-1}} \sum_i E_{1,i}^n(-q^2, g) \]

\[ \times R_{1,i}^{\sigma \mu_1 \ldots \mu_{n-1}} - i(\epsilon_{\mu \rho \lambda \sigma} q^\rho - \epsilon_{\nu \rho \lambda \sigma} q_\nu q^\rho - q^2 \epsilon_{\mu \nu \lambda \sigma}) \]

\[ \times \sum_n \frac{1}{2} [1 - (-)^n] \frac{n-1}{n} \left( \frac{2}{-q^2} \right)^n q_{\mu_1} \ldots q_{\mu_{n-2}} \sum_i E_{2,i}^n(-q^2, g) R_{2,i}^{\lambda \sigma \mu_1 \ldots \mu_{n-2}} \]

In the Bjorken limit when $Q^2 \to \infty$ the leading contribution corresponds to $n = 1$ and is defined by the axial current

\[ R_{1,\psi}^n = (i)^{n-1} \overline{\psi} \gamma_5 \gamma^\sigma D^{\mu_1} \ldots D^{\mu_{n-1}} \psi \]

Kodaira (1980)
The box diagram in the worldline approach

\[ \Gamma_{A}^{\mu \nu}[k_1, k_3] = \int \frac{d^4k_2}{(2\pi)^4} \int \frac{d^4k_4}{(2\pi)^4} \Gamma_{A}^{\mu \nu \alpha \beta}[k_1, k_3, k_2, k_4] \text{Tr}_c(\tilde{A}_{\alpha}(k_2)\tilde{A}_{\beta}(k_4)) \]

Hadronic tensor  
Box diagram

Worldline representation of the box diagram:

\[ \Gamma_{A}^{\mu \nu \alpha \beta}[k_1, k_3, k_2, k_4] \equiv -\frac{g^2 e^2 e_f^2}{2} \int_0^\infty \frac{dT}{T} \int \mathcal{D}x \int \mathcal{D}\psi \exp \left\{ -\int_0^T d\tau \left( \frac{1}{4} \dot{x}^2 + \frac{1}{2} \psi \cdot \dot{\psi} \right) \right\} \]

\[ \times \left[ V_1^\mu(k_1)V_3^\nu(k_3)V_2^\alpha(k_2)V_4^\beta(k_4) - (\mu \leftrightarrow \nu) \right] \]

where the vector current

\[ V_i^\mu(k_i) \equiv \int_0^T d\tau_i (\dot{x}_i^\mu + 2i\psi_i^\mu k_j \cdot \psi_j) e^{ik_i \cdot x_i} \]
The box diagram in the Bjorken limit

In the Bjorken limit $Q^2 \to \infty$ and $x_B$ is fixed

\[
\Gamma_A^{\mu\nu\alpha\beta}[k_1, k_3, k_2, k_4] \bigg|_{Q^2 \to \infty} = -2 \frac{g^2 e^2 e_f^2}{\pi^2} \epsilon^{\mu\nu\eta\kappa} (k_{1\eta} - k_{3\eta})(2\pi)^4 \delta^{(4)} \left( \sum_{i=1}^{4} k_i \right) \\
\times \prod_{k=2}^{4} \int_0^1 d\beta_k \delta(1 - \sum_{k=2,3,4} \beta_k) \frac{1}{(k_1 + k_2)^2 \beta_3 + k_1^2 \beta_4 + k_3^2 \beta_2} \frac{(k_2^\kappa + k_4^\kappa) \epsilon^{\alpha\beta\sigma\lambda} k_{2\sigma} k_{4\lambda}}{(k_2 + k_4)^2}
\]

Kinematic factor, dependence on $x_B$ and $Q^2$
The structure function $g_1$ in the Bjorken limit:

$$S^\mu g_1(x_B, Q^2) \bigg|_{Q^2 \to \infty} = \sum_f e_f^2 \frac{\alpha_s}{i\pi M_N} \int_{x_B}^1 \frac{dx}{x} \left(1 - \frac{x_B}{x}\right) \int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_\mu \to 0} \frac{l^\mu}{l^2} \langle P', S|\text{Tr}_c F_{\alpha\beta}(\xi n) \tilde{F}^{\alpha\beta}(0)|P, S\rangle + O\left(\frac{\Lambda^2_{QCD}}{Q^2}\right)$$

The structure function $g_1$ is dominated by the triangle anomaly, hence $g_1$ is topological.

First moment of $g_1$:

$$S^\mu \int_0^1 dx_B g_1(x_B, Q^2) \bigg|_{Q^2 \to \infty} = \sum_f e_f^2 \frac{\alpha_s}{2i\pi M_N} \lim_{l_\mu \to 0} \frac{l^\mu}{l^2} \langle P', S|\text{Tr}_c F_{\alpha\beta}(0) \tilde{F}^{\alpha\beta}(0)|P, S\rangle$$

matches calculation of the triangle diagram.

The pole must be removed by contribution of the pseudo scalar sector.
The structure function $g_1$ in the Regge limit

In the Regge limit $x_B \to 0$ and $Q^2$ is fixed

$$S^\mu g_1(x_B, Q^2) \bigg|_{Q^2 \to \infty} = \sum_f e_f^2 \frac{\alpha_s}{\pi M_N} \int_{x_B}^1 \frac{dx}{x} \int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_\mu \to 0} \frac{l^\mu}{l^2} \langle P', S| \text{Tr}_c F_{\alpha\beta}(\xi n) \tilde{F}^{\alpha\beta}(0)| P, S \rangle$$

In Regge asymptotics: $x > x_B$ and $x_B \to 0$

The result for $g_1(x_B, Q^2)$ is formally identical in Bjorken and Regge limits

$g_1(x_B, Q^2)$ is topological in both asymptotic limits of QCD

$$u_2 - u_4 \sim \frac{Q_s^2}{2xP \cdot q} \sim x_B$$

Shock-wave approximation
Pseudoscalar contribution

To take into account contribution of the pseudoscalar sector we use the most general form of the imaginary part of the effective action

\[ W_\mathcal{S}[\Phi, \Pi, A] = \frac{1}{8} \int_{-1}^{1} d\alpha \int_0^\infty dT \mathcal{N} \int Dx \, D\psi \, \text{tr}_c \, \mathcal{J}(0) \, \mathcal{P} \, e^{-\int_0^T d\tau \mathcal{L}_\alpha} \]

where the worldline Lagrangian

\[ \mathcal{L}_\alpha = \frac{\dot{x}^2}{2\mathcal{E}} + \frac{1}{2} \psi_A \dot{\psi}_A - i \dot{x} \cdot A + \frac{i}{2} \mathcal{E} \psi_\mu F_{\mu\nu} \psi_\nu + \frac{1}{2} \mathcal{E} \alpha^2 \Phi^2 + \frac{1}{2} \mathcal{E} \Pi^2 
+ i \alpha \mathcal{E} \psi_\mu \psi_6 D_\mu \Phi + i \mathcal{E} \psi_\mu \psi_5 D_\mu \Pi + \alpha \mathcal{E} \psi_5 \psi_6 \left[ \Pi, \Phi \right] + \text{axial vector coupling} \]

D'Hoker, Gagne, hep-th/9508131

We want to perform a perturbative analysis to better understand how the anomalous Ward identities are satisfied.
Summary

- We revisit the role of the chiral “triangle” anomaly in deeply inelastic scattering (DIS) of electrons off polarized protons employing a powerful worldline formalism.

- We demonstrate how the triangle anomaly appears at high energies in the DIS box diagram.

- We show that the anomaly appears in both the Bjorken limit of large $Q^2$ and in the Regge limit of small $x_B$.

- We find that the infrared pole in the anomaly arises in both limits.

- The leading contribution to $g_1$, in both asymptotics, is given by the expectation value of the topological charge density (it’s relation to $\Delta G$ is unclear), which is a generalization of a result previously argued by Jaffe and Manohar for the first moment of $g_1$.

- The cancellation of this pole involves a subtle interplay of perturbative and nonperturbative physics that is deeply related to the $U_A(1)$ problem in QCD.