Theoretical Simulation of the Virtual Meson Production in the Forward Direction

Chueng-Ryong Ji
North Carolina State University

May, 23, 2022
Better Work in Forward Direction

\[ t = \Delta^2 = -\frac{\xi^2 M^2 + \Delta_{\perp}^2}{1 - \xi}; \Delta^+ (\equiv \Delta^0 + \Delta^3) = \xi P^+; \Delta_{\perp}^2 > \Delta_{\perp \text{min}}^2 \neq 0 \]
Analysis of virtual meson production in a (1 + 1)-dimensional scalar field model

Yongwoo Choi,1,* Ho-Meoyng Choi,2,† Chueng-Ryong Ji,3,‡ and Yongseok Oh,1,4,§

1Department of Physics, Kyungpook National University, Daegu 41566, Korea
2Department of Physics Education, Teachers College, Kyungpook National University, Daegu 41566, Korea
3Department of Physics, North Carolina State University, Raleigh, North Carolina 27695-8202, USA
4Asia Pacific Center for Theoretical Physics, Pohang, Gyeongbuk 37673, Korea

(Received 10 December 2021; accepted 30 March 2022; published 17 May 2022)

Light-front dynamic analysis of the longitudinal charge density using the solvable scalar field model in (1 + 1) dimensions

Yongwoo Choi,1 Ho-Meoyng Choi,2,* Chueng-Ryong Ji,3,‡ and Yongseok Oh,1,4,§
Outline

• Dirac’s Proposition for Relativistic Dynamics
  Instant Form Dynamics (IFD) vs. Light-Front Dynamics (LFD)
• Link between IFD and LFD: IMF $\neq$ LFD
• Virtual Meson Production off a Scalar Target
• Benchmarking GPD Applicability in Forward Direction
• GPD Sum Rule and Valence/Nonvalence Decomposition
• Conclusion and Outlook
Dirac’s Proposition for Relativistic Dynamics

\[ p^0 = \sqrt{p^2 + m^2} \]
\[ p^- = p^0 - p^3 \]
\[ (p^1, p^2) \leftrightarrow p_\perp \]
\[ p^3 \leftrightarrow p^+ = p^0 + p^3 \]

Energy-Momentum Dispersion Relations

Except zero-modes

\[ k_1^+ = k_2^+ = 0 \]
Interpolation between IFD and LFD

\[ x^- = \frac{x^0 - x^3}{\sqrt{2}} \]
\[ x^+ = \frac{x^0 + x^3}{\sqrt{2}} \]
\[ x^\pm = \cos \delta x^0 + \sin \delta x^3 \]
\[ x^- = \sin \delta x^0 - \cos \delta x^3 \]

\[ (IFD) \quad 0 \leq \delta \leq \frac{\pi}{4} \quad (LFD) \]
\[ 1 \geq C \equiv \cos(2\delta) \geq 0 \]

K. Hornbostel, PRD45, 3781 (1992) – RQFT
C. Ji and S. Rey, PRD53, 5815 (1996) – Chiral Anomaly
C. Ji and A. Suzuki, PRD87, 065015 (2013) – Scattering Amps
B. Ma and C. Ji, PRD104, 036004 (2021) – QCD_{1+1}
\[ \delta = 0 \]
\[ p_0 = p^0 \]
\[ -p_3 = p^3 \]

0 \( \delta < \pi/4 \)
\[ p_+ = p^0 \cos \delta - p^3 \sin \delta \]
\[ p_- = p^0 \sin \delta + p^3 \cos \delta \]

\[ \delta = \pi/4 \]
\[ p_+ = p^- \]
\[ p_- = p^+ \]

\[ \frac{1}{2q^0} \left( \frac{1}{p_1^0 + p_2^0 - q^0} - \frac{1}{p_1^0 + p_2^0 + q^0} \right) \]

\[ \frac{1}{2\omega_q} \left( \frac{1}{p_1^0 + \frac{8q_-}{c} - \omega_q} - \frac{1}{p_1^0 + \frac{8q_- + \omega_q}{c}} \right) \]

\[ \omega_q = \sqrt{q^2 + C \left( \frac{q_1^2}{c^2} + m^2 \right)} \]

\[ C = \cos 2\delta \]
\[ S = \sin 2\delta \]

\[ \frac{8q_- + \omega_q}{C} \rightarrow \frac{2}{C} \rightarrow \frac{2q_-}{2q_-} + O(C) \]
\[ \rightarrow \infty \text{ as } C \rightarrow 0 \]
Infinite Momentum Frame (IMF) Approach

\[
\frac{1}{E_1 + E_2 - Eq}
\]

\[
1 - \frac{1}{Eq + E_3 + E_4}
= - \frac{1}{Eq + E_1 + E_2}
\rightarrow 0
\]

S. Weinberg, PR158, 1638 (1967)
“Dynamics at Infinite Momentum”

Note that this is still in the instant form (IFD).
\[ \Sigma(a) + \Sigma(b) = 1/(s-m^2) \]; \( s = 2 \text{ GeV}^2 \), \( m = 1 \text{ GeV} \)

J-shape peak & valley:

\[ P_z = -\sqrt{\frac{s(1-C)}{2C}} \]; \( C = \cos(2\delta) \)

As \( C \to 0 \), \( P^+ = P^0 + P_z \to 0 \) leads to LF Zero-modes.
Virtual Meson Production off a Scalar Target

Salient Features

• No interference with the Bethe-Heitler process
• Consistency between our benchmark BSA prediction for $0^{-+}$ meson production off the scalar target with the data of the exclusive coherent electroproduction of the $\pi^0$ off $^4$He measured at JLab Hall B
• General formulation of hadronic amplitudes in Meson Production off the Scalar Target ($0^{++}$ vs. $0^{-+}$)
• Comparison/Contrast with the leading twist GPD formulation.
Beam-Spin Asymmetry of Exclusive Coherent

Electroproduction of the $\pi^0$ Off $^4$He

Frank Thanh Cao, Ph.D.

University of Connecticut, 2019

To understand the partonic structure of nucleons in nuclei, extracting the beam spin asymmetry (BSA) from exclusive processes is an important measurement to get at the so-called Generalized Parton Distributions (GPDs) that describe the partons behavior inside the nucleon. In particular, BSA in Deeply Virtual Meson Production (DVMP) can offer valuable constraints on the transverse GPDs which are not accessible through Deeply Virtual Compton Scattering (DVCS).

......

This benchmark measurement is in agreement with symmetry arguments presented in a recent theoretical formulation [2] that offers a framework complementary to that of the GPDs and gives confidence in the assumptions made for future studies of exclusive nuclear processes.
\[ A_{LU} (\phi) = A_{LU}^{90^\circ} \sin \phi \]

\[ A_{LU}^{90^\circ} = -1.08 \pm 3.22 \text{ (stat.)} \pm 2.83 \text{ (sys.)} \% \]

4-26-2019

Beam-Spin Asymmetry of Exclusive Coherent Electroproduction of the \( \pi^0 \) Off 4He

Frank Thanh Cao

University of Connecticut - Storrs, frankcao@gmail.com
\[ \langle |M|^2 \rangle = \left( \frac{e^2}{q^2} \right)^2 \mathcal{L}^{\mu\nu} \mathcal{H}_{\mu\nu} \]

\[ \mathcal{L}^{\mu\nu} = q^2 \left[ g^{\mu\nu} + \frac{2}{q^2} (k^{\mu} k^{\nu} + k'^{\mu} k'^{\nu}) \right] + 2i\hbar \epsilon^{\mu\nu\alpha\beta} k_{\alpha} k'_{\beta} \]

\[ \mathcal{H}_{\mu\nu} = J^\dagger_{\mu} J_{\nu} \]

\[ \mathcal{H}_{\mu\nu} \neq \mathcal{H}_{\nu\mu} \]
\[ J_{PS}^{\mu} = F_{PS} \epsilon^{\mu \nu \alpha \beta} q_{\nu} \bar{P}_{\alpha} \Delta_{\beta} \]

\[ \mathcal{H}_{\mu \nu} = J_{\mu}^\dagger J_{\nu} \]

\[ = |F_{PS}|^2 \epsilon_{\mu \alpha \beta \gamma} \epsilon_{\nu \alpha' \beta' \gamma'} q^\alpha \bar{P}^\beta \Delta \gamma q^{\alpha'} \bar{P}^{\beta'} \Delta \gamma' \]

\[ = \mathcal{H}_{\nu \mu} \]

\[ \epsilon^{\mu \nu \alpha \beta} k_{\alpha} k'_{\beta} \mathcal{H}_{\mu \nu} = 0 \]

\[ \frac{d\sigma_{PS}^{h=+1} - d\sigma_{PS}^{h=-1}}{d\sigma_{PS}^{h=+1} + d\sigma_{PS}^{h=-1}} = 0 \]
Pseudoscalar($0^{-+}$) Meson vs. Scalar($0^{++}$) Meson

\[ \varepsilon^{\mu\nu\alpha\beta} \quad \text{vs.} \quad d^{\mu\nu\alpha\beta} = g^{\mu\nu} g^{\alpha\beta} - g^{\mu\alpha} g^{\nu\beta} \]

C.Ji & B.Bakker, PoS QCDEV2017,038(2017); B.Bakker & C.Ji, Few Body Syst. 58,no.1,8(2017)

\[ F_{PS}(Q^2, t, x) \quad J^\mu_S = (S_q q_{\alpha} + S_{\bar{P}} \bar{P}_{\alpha}) d^{\mu\nu\alpha\beta} q_{\beta} \Delta_{\nu} \]

\[ J^\mu_{PS} = F_{PS} \varepsilon^{\mu\nu\alpha\beta} q_{\nu} \bar{P}_{\alpha} \Delta_{\beta} \]

\[ F_1 = S_q - S_{\bar{P}} \]
\[ F_2 = S_{\bar{P}} \]

\[ F_1(Q^2, t, x) \quad F_2(Q^2, t, x) \]

\[ J^\mu_S = F_1(q^2 \Delta^\mu - q^\mu q \cdot \Delta) + F_2[(\bar{P} \cdot q + q^2)\Delta^\mu - (\bar{P}^\mu + q^\mu)q \cdot \Delta] \]

\[ q \quad ; \quad \bar{P} = P + P' \quad ; \quad \Delta = P - P' = q' - q \]
Scalar Field Model Simulation of VMP in Forward Direction

\[ \gamma^* + \text{He} \rightarrow f_0(980) + \text{He} \]

\[ \mathcal{M}_{\text{tot}}^{\mu(1+1)} = [(\Delta \cdot q)q^\mu - q^2\Delta^\mu]F \]

Two more amplitudes for the charged target, but not for the neutral target
<table>
<thead>
<tr>
<th></th>
<th>$0 &lt; k^+ &lt; -q^+$</th>
<th>$-q^+ &lt; k^+ &lt; \Delta^+$</th>
<th>$\Delta^+ &lt; k^+ &lt; p^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>S</strong></td>
<td><img src="image1" alt="Diagram" /></td>
<td><img src="image2" alt="Diagram" /></td>
<td><img src="image3" alt="Diagram" /></td>
</tr>
<tr>
<td>0 &lt; k^+ &lt; q^+</td>
<td><img src="image4" alt="Diagram" /></td>
<td><img src="image5" alt="Diagram" /></td>
<td><img src="image6" alt="Diagram" /></td>
</tr>
<tr>
<td>U</td>
<td><img src="image7" alt="Diagram" /></td>
<td><img src="image8" alt="Diagram" /></td>
<td><img src="image9" alt="Diagram" /></td>
</tr>
<tr>
<td>0 &lt; k^+ &lt; q^+</td>
<td><img src="image10" alt="Diagram" /></td>
<td><img src="image11" alt="Diagram" /></td>
<td><img src="image12" alt="Diagram" /></td>
</tr>
<tr>
<td>C</td>
<td><img src="image13" alt="Diagram" /></td>
<td><img src="image14" alt="Diagram" /></td>
<td><img src="image15" alt="Diagram" /></td>
</tr>
</tbody>
</table>
Compton Form Factor

\[ \mathcal{M}_{\text{tot}}^{(1+1)} = \left[ (\Delta \cdot q) q^\mu - q^2 \Delta^\mu \right] \mathcal{F} \]

\[ \mathcal{F}_{c}^{\text{VMP}}(Q^2, t) = \frac{\mathcal{M}_{\text{charged}}^\mu}{(\Delta \cdot q) q^\mu - q^2 \Delta^\mu} \]

\[ x_{Bj} = \frac{2t}{t(1 + \mu_s + \tau) - \sqrt{t(t - 4M_T^2)[(1 + \mu_s + \tau)^2 - 4\tau]} \left( t + \sqrt{t^2 - 4tM_T^2} \right)} \]

\[ \zeta = \frac{1}{2M_T^2} \left( t + \sqrt{t^2 - 4tM_T^2} \right) \]

\[ \mu_s = M_3^2 / Q^2 \quad \text{and} \quad \tau = -t / Q^2 \]
DVMP Reduction to GPD in S-channel with $+$ Current

\[ \mathcal{M}_s^\mu \sim \int \frac{d^2k}{(2\pi)^2} \frac{1}{k^2 - m^2} \frac{2k^\mu + q^\mu}{(k+q)^2 - m^2} \frac{1}{(k-\Delta)^2 - m^2} \frac{1}{(k-p)^2 - M^2} \]

For a plus current of a virtual photon,

\[ \frac{2k^+ + q^+}{(k^+ + q^+)(k^- - k^-)} \frac{1}{k^2 - m^2} \frac{1}{(k-\Delta)^2 - m^2} \frac{1}{(k-p)^2 - M^2} \]

where \( k^- = -q^- + \frac{m_{Q_1}}{k^+ + q^+} - i \frac{\epsilon}{k^+ + q^+} \)

For large \( Q^2 \):

\[ \frac{1}{x - \zeta} \frac{\zeta'}{Q^2} \frac{1}{k^2 - m^2} \frac{2k^+ - \Delta^+}{p^+} \frac{1}{(k-\Delta)^2 - m^2} \frac{1}{(k-p)^2 - M^2} \]
DVMP Reduction to GPD in U-channel with + Current

\[ M_u^\mu \sim \begin{pmatrix} \frac{d^2k}{(2\pi)^2} \frac{1}{k^2 - m^2} \frac{2k^\mu - \Delta^\mu - q'^\mu}{(k - q')^2 - m^2} \end{pmatrix} \begin{pmatrix} 1 \\ (k - \Delta)^2 - m^2 \\ (k - p)^2 - M^2 \end{pmatrix} \]

For a plus current of a virtual photon,

\[ \frac{2k^+ - \Delta^+ - q'^+}{(k^+ - q'^+)(k^- - k^-)} \frac{1}{k^2 - m^2} \frac{1}{(k - \Delta)^2 - m^2} \frac{1}{(k - p)^2 - M^2} \]

where \( k^- = q^- + \frac{m_{Q^2}}{k^+ - q'^+} - \frac{\epsilon}{k^+ - q'^+} \)

For large \( Q^2 \):

\[ \frac{1}{x} \frac{\zeta'}{Q^2} \frac{2k^+ - \Delta^+}{p^+} \frac{1}{(k - \Delta)^2 - m^2} \frac{1}{(k - p)^2 - M^2} \]
\begin{align*}
\mathcal{M}_{\text{Leading}}^+ &= \frac{1}{4\pi} \frac{\zeta'}{Q^2} \left( \frac{1}{x - \zeta} - \frac{1}{x} \right) \frac{1}{k^2 - m^2} \frac{2k^+ - \Delta^+}{p^+} \frac{1}{(k - \Delta)^2 - m^2} \frac{1}{(k - p)^2 - M^2} \\
A^+ &= (\Delta \cdot q)^+ q^+ - q^2 \Delta^+ = \frac{1}{2} Q^2 \zeta p^+ \left[ 1 + \frac{t}{Q^2} + \cdots \right] \\
\frac{\mathcal{M}_{\text{Leading}}^+}{A_{\text{Leading}}^+} &= \frac{1}{2\pi} \frac{1}{Q^4} \left( \frac{1}{x - \zeta} - \frac{1}{x} \right) \frac{1}{k^2 - m^2} \frac{2k^+ - \Delta^+}{p^+} \frac{1}{(k - \Delta)^2 - m^2} \frac{1}{(k - p)^2 - M^2}
\end{align*}

DVMP Reduction to GPD with - Current works as well.

\begin{align*}
\mathcal{M}_{\text{Leading}}^- &= \frac{1}{2\pi} \frac{1}{Q^4} \left( \frac{1}{x - \zeta} - \frac{1}{x} \right) (2x - \zeta) \frac{1}{k^2 - m^2} \frac{1}{(k - \Delta)^2 - m^2} \frac{1}{(k - p)^2 - M^2}
\end{align*}
\[ M^\mu = A^\mu F \]

\[ \mathcal{M}^\mu_{\text{Leading}} \sim \frac{q^\mu - 2q'^\mu}{q^2} \left( \frac{1}{x - \zeta} - \frac{1}{x} \right) \frac{1}{k^2 - m^2} \frac{1}{(k - \Delta)^2 - m^2} \frac{1}{(k - p)^2 - M^2} \]

\[ (\Delta \cdot q) \ q^\mu - q^2 \ \Delta^\mu \rightarrow \frac{Q^2}{2} (2 \ q'^\mu - q^\mu) \]

Gauge Invariance works asymptotically:

\[ q' \cdot q \rightarrow q^2 / 2 \]
\[ \left( \Delta \cdot q \right) q^\mu - q^2 \Delta^\mu \]

\[ \frac{Q^2}{2} \left( 2 \ q'^\mu - q^\mu \right) \]
\[ M_{s+u}^{\pm DVMP} = \frac{e Q_1 \zeta}{4\pi Q^2} \int_0^1 dx \left( \frac{1}{x - \zeta} - \frac{1}{x} \right) H(\zeta, x, t) \]
$t = t_{\text{th}} \simeq -0.7593 \text{ GeV}^2$

$t = -2 \text{ GeV}^2$

$-t/Q^2 \lesssim 0.1$
\[ F_M(t) = \int_0^1 \frac{dx}{1 - \zeta/2} H(\zeta, x, t). \]

\[ H(\zeta, x, t) = \begin{cases} 
H_{\text{ERBL}}(\zeta, x, t), & \text{for } 0 \leq x \leq \zeta, \\
H_{\text{DGLAP}}(\zeta, x, t), & \text{for } \zeta \leq x \leq 1 
\end{cases} \]

\[ J_S^\mu(0) = (p + p')^\mu F_M^S(q^2) \]
Decomposition of the Form Factor

\[ \sum_{i=V, NV} J_i^\mu(t) = (P + P')^\mu \sum_{i=V, NV} F_i(t) = (2P^\mu - \Delta^\mu) \sum_{i=V, NV} F_i(t) \]

\[
J_V^\mu(t) = \int_{\Delta^+}^{P^+} dk^+ \int dk^- \frac{2k^\mu - \Delta^\mu}{D_V} \\
J_{NV}^\mu(t) = \int_0^{\Delta^+} dk^+ \int dk^- \frac{2k^\mu - \Delta^\mu}{D_{NV}}
\]

\[
\frac{2k^\mu - \Delta^\mu}{2P^\mu - \Delta^\mu} = \frac{2x - \xi}{2 - \xi} \quad \text{only if} \quad \Delta^\mu = \xi P^\mu \quad \text{or} \quad q^\mu = q'^\mu + \xi P^\mu
\]

Note here that \((q - q')^2 = \Delta^2 = t = \xi^2 M^2 > 0\) while \(t < 0\) in DVMP.
Conclusion and Outlook

- Unless small $|t|/Q^2$, “Cat’s ears” contribution should not be neglected.
- Sum rule correspondence between DGLAP/ERBL GPDs and Valence/Nonvalence contributions to the form factor works only for a certain current component.
- Form factor decomposition depends on the current component although the form factor itself is independent of the choice of the current component. *(Democracy in current components)*
- 3+1 D extension with BSA investigation is underway.
- Application to the energy-momentum tensor decomposition appears feasible.